

Adaptive Projected Matrix Factorization method for data clustering

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ABSTRACT

Data clustering aims to group the data samples into clusters, and has attracted many researchers in a variety of multidisciplinary fields, such as machine learning and data mining. In order to capture the geometry structure, many methods perform clustering according to a predefined affinity graph. So the clustering performance is largely determined by the graph quality. Unfortunately, the graph quality cannot be guaranteed in various real-world applications. In this paper, an Adaptive Projected Matrix Factorization (APMF) method is proposed for data clustering. Our contributions are threefold: (1) instead of keeping the graph fixed, graph learning is taken as a part of the clustering procedure; (2) the clustering is performed in the projected subspace, so the noise in the input data space is alleviated; (3) an efficient and effective algorithm is developed to solve the proposed problem, and its convergence is proved. Extend experiments on nine real-world benchmarks validate the effectiveness of the proposed method, and verify its superiority against the state-of-the-art competitors.

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1. Introduction

Clustering is a fundamentally important task in the field of machine learning, and involves various practical applications, such as image classification [1], saliency detection [2] and signal processing [3]. Given the desired cluster number, the objective of clustering is to partition the data into different clusters, such that the samples within the same cluster have high similarity. In the past decades, a great number of techniques have proposed on clustering, such as *k*-means clustering [4], hierarchical clustering [5], multiview clustering [6,7], spectral clustering [8], non-negative matrix factorization [9], support vector clustering [10] and maximum margin clustering [11].

One of the most popular clustering method is non-negative matrix factorization (NMF). NMF approximates the data matrix with the product of two non-negative matrices, which can be considered to be the clustering centroid and indicator matrix respectively. And the clustering result is indicated by the index of the maximal element within each indicator vector. Since the non-negative constraints allow only additive operation (no subtractive and combina-

tion), the factorization leads to a part-based representation, which implies the perception mechanism of human brain [12]. So NMF is suitable to learn the parts of objects, and has shown good performance in image annotation [13], face recognition [14], crowd analysis [15–17] and document clustering [18].

Though NMF achieved prominent performance, it suffers from some problems. The main drawback of NMF is that it neglects the local correlation of data points, so it fails to perceive the data manifold. To tackle this problem, some graph-based NMF methods [12,14,19–23] have been proposed. By assuming that the points with close distances should be grouped into the same cluster, these approaches first construct a weighted affinity graph based on the data points' distances, and then perform matrix factorization according to the spectrum of the graph. So their performance highly depends on the quality of the affinity graph. However, in practical applications, the graph quality cannot be guaranteed because: (1) the points' distances in the input data space may be influenced by the noise; (2) the construction approach (0–1 weighting, Gaussian weighting) may be inappropriate. So the clustering performance tends to be adversely affected by the graph quality, and it's necessary to optimize the affinity graph in the clustering procedure.

Another major limitation of the existing NMF-based methods is that the factorization is accomplished in the input data space. According to Li et al. [24], data with high dimensionality is always lying within a low dimensional manifold. And in the desired low

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dimensional subspace, the noise within the original data is alleviated and more discriminative features are preserved. However, most existing methods process the input data directly. So they cannot capture the intrinsic data geometry accurately, and are sensitive to the data noise. In recent years, the success of subspace clustering methods [25–29] has proved that the exploration of subspace improves the robustness. However, most matrix factorization methods neglect this aspect.

To mitigate the above issues, we propose an Adaptive Projected Matrix Factorization (APMF) method in this paper, which inherits the merits of both spectral clustering and NMF. The data points are projected into a low dimensional subspace, so the noise in the input space is alleviated and more valuable features are captured. And the data graph is updated during the clustering procedure, so the graph quality is improved. In addition, the proposed method learns the indicator matrix directly without post-processing, so it achieves stable performance.

The main contributions of this paper are summarized as follows:

- (1) The graph learning is jointly incorporated in the NMF framework. By optimizing the data graph adaptively during the matrix factorization procedure, the geometric structure is explored and the graph quality is improved.
- (2) The data is projected to a low dimensional subspace, which preserves more discriminative features and contains less noise. Both the graph learning and clustering are accomplished in the projected subspace, so the data noise in the input space is avoided.
- (3) The proposed problem is solved with an efficient alternative algorithm, and no post-processing is involved. Experimental results demonstrate that the proposed method is effective and outperforms the representative competitors.

The rest of this paper is organized as follows. Section 2 reviews the related works on non-negative matrix factorization. Section 3 proposes the Adaptive Projective Matrix Factorization method, and introduces an efficient optimization algorithm. Experimental results are presented and theoretically discussed in Section 4. Finally Section 5 summarizes the conclusion.

2. Related work

In this section, we briefly review the original non-negative matrix factorization (NMF) and its graph-based variants, and discuss their advantages and limitations.

2.1. Non-negative matrix factorization

Given a data matrix $X = [x_1, x_2, \dots, x_n] \in \mathbb{R}^{d \times n}$ (d is the dimensionality and n is the number of samples), NMF aims to approximate X with two non-negative matrices :

$$X \approx FG^T, \quad (1)$$

where $F \in \mathbb{R}^{d \times k}$ and $G \in \mathbb{R}^{n \times k}$ are the clustering centroid and cluster indicator respectively, and k is the desired cluster number. So the objective function of NMF is defined as a closed form problem:

$$\min_{F \geq 0, G \geq 0} \|X - FG^T\|_F^2, \quad (2)$$

where $\|\cdot\|_F$ is the Frobenius norm. Benefited from the non-negative constraints, NMF obtains a part-based representation. This property provides a physiological and psychological interpretation for non-negative data [19], such as face images and documents. However, from Eq. (2) we can see that NMF just focus on the global data geometry, and ignore the local aspect. So some graph-based variants are proposed.

2.2. Graph-based variants of NMF

The investigation of data relationship is essential to handle the data with complicated structures. To this end, Cai et al. [19] proposed to impose the graph regularization term on NMF by solving the following problem:

$$\min_{F \geq 0, G \geq 0} \|X - FG^T\|_F^2 + \lambda \text{tr}(G^T L G), \quad (3)$$

where $\text{tr}()$ is the trace operator, $L = D - \frac{S^T + S}{2}$ is the Laplacian matrix, S is the affinity graph, and D is a diagonal matrix with $D_{ii} = \sum_j \frac{S_{ij} + S_{ji}}{2}$. The second term enforces that the samples with small distance should have similar indicator vectors. Thus, the local relationship between the points is exploited such that the clustering performance can be improved.

The residue error of each data point is squared in the objective function Eq. (3), so problem Eq. (3) is sensitive to outliers. By introducing the $\ell_{2,1}$ norm, Huang et al. [20] proposed a more robust version:

$$\min_{G \geq 0, G^T G = I} \|X - FG^T\|_{2,1} + \lambda \text{tr}(G^T L G), \quad (4)$$

where I is the identity matrix. The $\ell_{2,1}$ norm makes the objective function robust to outliers, and the constraint $G^T G = I$ ensures the optimal solution to be unique. However, the cluster result may be inaccurate if the affinity graph is constructed with low quality.

Instead of defining a simple graph, Zeng et al. [30] utilized the hyper graph L_{hyper} , where each edge connects more than two data points. And the objective function is denoted as

$$\min_{F \geq 0, G \geq 0} \|X - FG^T\|_F^2 + \lambda \text{tr}(G^T L_{\text{hyper}} G), \quad (5)$$

There are also some other methods [12,14,21–23] that introduced manifold regularization terms. However, the predefined graph may be with low quality, and affect the clustering performance adversely. Recently, Zhang et al. [31] proposed to update the graph in the clustering phase, and validated the rationality of performing graph learning.

The shortcoming shared by all the above methods is that the clustering is performed in the input data space, where the valuable features may be corrupted by noise. So both the graph construction and matrix factorization tend to be affected. Li et al. [24] proved that the influence of data noise can be reduced in the subspace. So it is necessary to combine the subspace learning strategy into matrix factorization methods.

3. Methodology

In this section, we introduce the proposed Adaptive Projected Matrix Factorization (APMF), and design an efficient optimization algorithm. To capture the local data structure while improving the robustness, the proposed method jointly incorporates graph learning and subspace clustering into the matrix factorization framework.

3.1. Adaptive Projected Matrix Factorization method

In this part, the Adaptive Projected Matrix Factorization (APMF) method is presented. To find the optimal graph for clustering, we propose to update the graph when performing matrix factorization. So we have the following objective function:

$$\begin{aligned} \min_{F, G, S} & \|X - FG^T\|_{2,1} + \lambda \text{tr}(G^T L G) \\ & + \alpha \sum_{i,j} S_{ij}^2 \|(x_i - x_j)\|_2^2, \\ \text{s.t.} & \sum_j S_{ij} = 1, S_{ij} \geq 0, G^T G = I, G \geq 0, \end{aligned} \quad (6)$$

where α is a parameter. The third term enforces that the S_{ij} should be large if x_i and x_j are close to each other. So the affinity graph S can be updated according to the points' distances, and the local relationship within the data manifold can be captured. However, the clustering is still accomplished in the input data space. According to Li et al. [24], more discriminative features can be captured if the points are transformed into the desired subspace. So we project the data into subspace with a transformation matrix $W \in \mathbb{R}^{d \times m}$ ($m \ll d$). And the objective function is rewritten as

$$\begin{aligned} \min_{F, G, S, W} & \|W^T X - FG^T\|_{2,1} + \lambda \text{tr}(G^T LG) \\ & + \alpha \sum_{i,j} S_{ij}^2 \|W^T (x_i - x_j)\|_2^2, \\ \text{s.t.} & \sum_j S_{ij} = 1, S_{ij} \geq 0, G^T G = I, G \geq 0, W^T W = I, \end{aligned} \quad (7)$$

where the orthogonal constraint on W guarantees the uniqueness of W . In the optimization, we propose an alternative method to update each variable. Thus, both the matrix factorization and the graph learning are achieved in the learned subspace, and the merit of subspace clustering is inherited. Finally, we can obtain the cluster indicator G in the desired subspace, and the noise within the input space is avoided.

3.2. Optimization algorithm

Problem Eq. (7) is not convex, so we propose to solve it with the Augmented Lagrangian Multiplier (ALM) [32]. Because both the $\ell_{2,1}$ norm and Laplacian regularization depend on G , we introduce two auxiliary variables $E = W^T X - FG^T$ and $Z = G$. So problem Eq. (7) is rewritten as

$$\begin{aligned} \min_{E, Z, F, G, S, W, \Lambda_1, \Lambda_2, \mu} & \|E\|_{2,1} + \lambda \text{tr}(Z^T LG) \\ & + \alpha \sum_{i,j} S_{ij}^2 \|W^T (x_i - x_j)\|_2^2 \\ \text{s.t.} & E = W^T X - FG^T, Z = G, G^T G = I, \\ & \sum_j S_{ij} = 1, S_{ij} \geq 0, W^T W = I, \end{aligned} \quad (8)$$

which is equivalent to the following ALM problem

$$\begin{aligned} \min_{E, Z, F, G, S, W, \Lambda_1, \Lambda_2, \mu} & \|E\|_{2,1} + \lambda \text{tr}(Z^T LG) \\ & + \alpha \sum_{i,j} S_{ij}^2 \|W^T (x_i - x_j)\|_2^2 \\ & + \frac{\mu}{2} \|E - W^T X + FG^T + \frac{1}{\mu} \Lambda_1\|_F^2 \\ & + \frac{\mu}{2} \|Z - G + \frac{1}{\mu} \Lambda_2\|_F^2, \\ \text{s.t.} & Z \geq 0, G^T G = I, \sum_j S_{ij} = 1, S_{ij} \geq 0, W^T W = I, \end{aligned} \quad (9)$$

where $\Lambda_1 \in \mathbb{R}^{m \times n}$ and $\Lambda_2 \in \mathbb{R}^{k \times d}$ are ALM multipliers, and μ is a scalar parameter. Since problem Eq. (9) involves several variables to be optimized, we propose to solve it with an alternative algorithm. Firstly, F and G are initialized with the method in [20], and W and S are initialized with the strategy in [33]. Then we update one variable while keeping the others fixed.

Update E: to update E , we fix all the other variables, and then problem Eq. (9) becomes

$$\min_E \|E\|_{2,1} + \frac{\mu}{2} \|E - M\|_F^2, \quad (10)$$

where $M = W^T X - FG^T - \frac{1}{\mu} \Lambda_1$. As proved in [34], the optimal solution of problem Eq. (10) is

$$e_i = \begin{cases} \left(1 - \frac{1}{\mu \|m_i\|_2}\right) m_i, & \text{if } \|m_i\|_2 \geq \frac{1}{\mu}, \\ 0, & \text{else} \end{cases} \quad (11)$$

where e_i and m_i are the i -th column of E and M , respectively.

Update Z: when solving Z , problem Eq. (9) is reduced to

$$\min_{Z \geq 0} \lambda \text{tr}(Z^T LG) + \frac{\mu}{2} \|Z - G + \frac{1}{\mu} \Lambda_2\|_F^2. \quad (12)$$

Spanning the Frobenius norm and removing the irrelevant terms, we have

$$\min_{Z \geq 0} \|Z - P\|_F^2, \quad (13)$$

where $P = G - \frac{1}{\mu} \Lambda_2 - \frac{\lambda}{\mu} LG$. It is obvious that the optimal Z can be updated as

$$Z_{ij} = \max(P_{ij}, 0). \quad (14)$$

Update F: optimizing F yields the following problem

$$\min_F \|F - HG\|_F^2, \quad (15)$$

where $H = W^T X - E - \frac{1}{\mu} \Lambda_1$. Since there is no constraint on F , we can compute the optimal solution as $F = HG$.

Update G: for updating G , problem Eq. (9) is transformed to the following form:

$$\begin{aligned} \min_{G^T G = I} & \lambda \text{tr}(Z^T LG) + \frac{\mu}{2} \|E - W^T X + FG^T + \frac{1}{\mu} \Lambda_1\|_F^2 \\ & + \frac{\mu}{2} \|Z - G + \frac{1}{\mu} \Lambda_2\|_F^2, \end{aligned} \quad (16)$$

which can be further simplified to

$$\min_{G^T G = I} \|G - K\|_F^2, \quad (17)$$

where $K = (W^T X - E - \frac{1}{\mu} \Lambda_1)^T F + Z - \frac{\lambda}{\mu} LZ + \frac{1}{\mu} \Lambda_2$. According to [20], the optimal solution of the above problem is

$$G = UV^T, \quad (18)$$

where $U \in \mathbb{R}^{k \times d}$ and $V \in \mathbb{R}^{d \times d}$ are the left and right singular vectors of the compact SVD decomposition of G .

Update W: when updating W , we have

$$\min_{W^T W = I} \frac{\mu}{2} \|W^T X - Q\|_F^2 + \alpha \sum_{i,j} S_{ij}^2 \|W^T (x_i - x_j)\|_2^2, \quad (19)$$

where $Q = FG^T + E + \frac{1}{\mu} \Lambda_1$. Denoting a scatter matrix $\tilde{S} \in \mathbb{R}^{d \times d}$ as $\tilde{S} = \sum_{i,j} S_{ij}^2 (x_i - x_j)(x_i - x_j)^T$, then problem Eq. (20) becomes

$$\min_{W^T W = I} \text{tr}(W^T \tilde{S} W) - \frac{\mu}{\alpha} \text{tr}(W^T X Q), \quad (20)$$

which can be solved with the Generalized Power Iteration method [35].

Update S: in order to optimize S , we transform the objective into the following problem:

$$\min_{S_{ij=1, S_{ij} \geq 0}} \frac{\lambda}{\alpha} \sum_{i,j} S_{ij} \|z_i - g_j\|_2^2 + \sum_{i,j} S_{ij}^2 \|W^T (x_i - x_j)\|_2^2, \quad (21)$$

where z_i is the i th column of Z , and g_j is the j th column of G . Problem Eq. (23) is independent for different i , so we optimize the following problem separately for each i :

$$\min_{S_{ij=1, S_{ij} \geq 0}} \frac{\lambda}{\alpha} \sum_j S_{ij} \|z_i - g_j\|_2^2 + \sum_j S_{ij}^2 \|W^T (x_i - x_j)\|_2^2. \quad (22)$$

Table 1
Description of datasets.

Datasets	Samples	Dimensionality	Classes
BA	1400	320	36
Jaffe	213	676	10
Yale	165	256	15
Dermatology	366	34	6
Ecoli	336	7	8
Ionosphere	351	34	2
Movement	360	90	15
Semeion	1593	256	10

Denoting a diagonal matrix $V \in \mathbb{R}^{n \times n}$ with $V_{jj} = \|W^T(x_i - x_j)\|_2^2$ and defining a column vector $u_i \in \mathbb{R}^{n \times 1}$ with its j th element as $\frac{\lambda}{2\alpha} \|z_i - g_j\|_2^2$, the above problem can be rewritten as

$$\min_{s_i^T \mathbf{1} = 1, s_i \geq 0} \frac{1}{2} s_i^T V s_i + s_i^T u_i, \quad (23)$$

where $s_i \in \mathbb{R}^{n \times 1}$ is a column vector with its j th element as S_{ij} , and $\mathbf{1} \in \mathbb{R}^{n \times 1}$ is a column vector with all its elements as 1. Problem Eq. (23) can be readily solved with an efficient method [36].

Update Λ_1 , Λ_2 and μ : according to [32], the ALM parameters Λ_1 , Λ_2 and μ are updated as

$$\begin{aligned} \Lambda_1 &= \Lambda_1 + \mu(W^T X - F G^T - E), \\ \Lambda_2 &= \Lambda_2 + \mu(Z - G), \\ \mu &= \rho \mu, \end{aligned} \quad (24)$$

where the parameter ρ is set as 1.5 in the experiments.

The details of the optimization algorithm are exhibited in Algorithm 1.

Algorithm 1 Optimization algorithm.

Input: Data matrix X , cluster number c , projection dimension m , parameters λ , α .

1: Initialize data graph S , matrices F and G , and transformation matrix W .

2: **repeat**

3: Update E with Eq. (11).

4: Update Z with Eq. (14)

5: Update F by solving problem (15).

6: Update G according to Eq. (18).

7: Update W by solving problem (20).

8: Update S by solving problem (23).

9: Update Λ_1 , Λ_2 and μ according to Eq. (24).

10: **until** Converge

Output: The learned graph S .

4. Experiments

In this section, experiments are conducted on real-world datasets to demonstrate the effectiveness of the proposed APMF.

The clustering accuracy (ACC), normalized mutual information (NMI) [20] and Purity [17] are taken as evaluation measurements.

Datasets: Eight real world benchmark datasets are employed, including one handwritten digit dataset, i.e., Binary Alphabet (BA) [37], two face image dataset, i.e., Jaffe [38] and Yale [39], and five datasets from UCI Machine Learning Repository [40], i.e., Dermatology, Ecoli, Ionosphere, Movement and Semeion. The details of the datasets are exhibited in Table 1.

Competitors: In the experiments, the proposed APMF is compared with six state-of-the-arts, including k -means, Ratio Cut (RCut) [41] Normalized Cut (NCut) [42], NMF [9], Graph-regularized NMF (GNMF) [19] and Robust Manifold NMF (RM-NMF) [20]. For k -means, RCut, NCut and RMNMF, the self-tune Gaussian method [43] is used to construct the graph. For GNMF, the 0–1 weighting graph is used since it's more suitable [19]. Because k -means, RCut and NCut are sensitive to initialization, we perform them for 200 repetitions and report the average performance. In order to evaluate whether the data projection is effective, we further denote the method with objective Eq. (6) as Adaptive Matrix Factorization (AMF), and compare it with the proposed APMF. For a fair comparison, all the competitors utilize their optimal parameters. And for our APMF, the parameters λ and α are all set as 0.01, and the projection dimensionality m is determined empirically according to the data dimensionality d ($m \ll d$).

Performance: Tables 2–4 exhibits the clustering results of different methods. It can be seen that APMF has the highest ACC, NMI and Purity on all datasets, which implies the best performance. In particular, APMF outperforms the competitors a lot on Jaffe and Ionosphere. k -means and NMF fails in most cases, since they can not perceive the local data structure. RCut, NCut, GNMF and RM-NMF show better performance because they exploit the manifold structure. However, they are limited to learn the optimal affinity graph for clustering, and easily affected by the noise in the input data space. Moreover, k -means, RCut and NCut are sensitive to the initialization. AMF shows good performance on some datasets because it learns the data graph adaptively. But it fails to find the low dimensional subspace, so it's easily affected by data noise. The comparison between AMF and APMF demonstrates that the learning of subspace does improve the clustering performance. The proposed APMF takes graph learning as a part of the clustering stage, and learns the local data structure in the desired subspace. So it achieves the best results.

In addition, we evaluate whether the projected subspace preserves discriminant features. We randomly select five samples from the Yale dataset. With the learned optimal transformation matrix W , the larger value of $\|W_i\|_2$ indicates that the i th feature is more valuable [33]. So we select the most important {32, 64, 96, ..., 256} features by sorting $\|W_i\|_2$, and show the selected features in Fig. 1. Each pixel is a feature, the selected ones maintain their original pixel value, and the unselected ones are shown in white. From left to right, the feature number is {32, 64, 96, ..., 256}, respectively. And we can see in Fig. 1 that the transformation matrix W captures the most discriminative features, such as eyes, mouth and nose. And the skin pixels, which can be considered as the back-

Table 2
ACC of different methods on eight real-world datasets. The best results are in bold face.

Datasets	k -means	RCut	NCut	NMF	GNMF	RMNMF	AMF	APMF
BA	0.41	0.15	0.42	0.17	0.20	0.40	0.44	0.46
Jaffe	0.75	0.77	0.80	0.67	0.47	0.80	0.77	0.92
Yale	0.43	0.24	0.51	0.35	0.32	0.38	0.48	0.57
Dermatology	0.71	0.65	0.92	0.73	0.61	0.72	0.95	0.96
Ecoli	0.64	0.63	0.52	0.35	0.44	0.70	0.67	0.76
Ionosphere	0.70	0.52	0.69	0.63	0.64	0.72	0.71	0.83
Movement	0.44	0.40	0.46	0.37	0.26	0.39	0.49	0.49
Semeion	0.57	0.25	0.52	0.37	0.36	0.44	0.67	0.68

Table 3

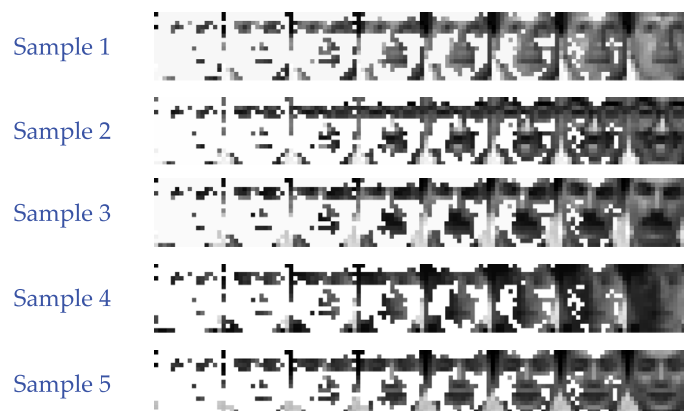
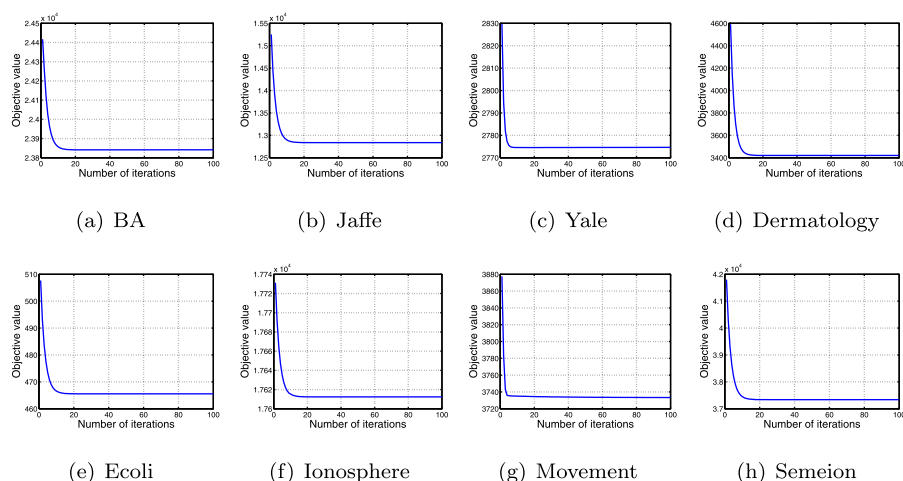
NMI of different methods on eight real-world datasets. The best results are in bold face.

Datasets	<i>k</i> -means	RCut	NCut	NM	GNMF	RMNMF	AMF	APMF
BA	0.57	0.24	0.57	0.29	0.35	0.52	0.58	0.59
Jaffe	0.82	0.81	0.85	0.69	0.52	0.77	0.78	0.90
Yale	0.52	0.30	0.56	0.42	0.38	0.47	0.55	0.61
Dermatology	0.81	0.68	0.88	0.64	0.50	0.85	0.90	0.91
Ecoli	0.58	0.56	0.48	0.21	0.57	0.51	0.58	0.62
Ionosphere	0.12	0.12	0.20	0.15	0.16	0.13	0.23	0.29
Movement	0.56	0.51	0.60	0.40	0.27	0.49	0.59	0.62
Semeion	0.52	0.22	0.48	0.32	0.34	0.34	0.58	0.59

Table 4

Purity of different methods on eight real-world datasets. The best results are in bold face.

Datasets	<i>k</i> -means	RCut	NCut	NMF	GNMF	RMNMF	AMF	APMF
BA	0.44	0.18	0.45	0.17	0.37	0.46	0.46	0.48
Jaffe	0.77	0.80	0.84	0.69	0.48	0.80	0.78	0.92
Yale	0.45	0.27	0.52	0.35	0.28	0.39	0.50	0.58
Dermatology	0.8	0.72	0.93	0.75	0.61	0.73	0.94	0.96
Ecoli	0.81	0.84	0.79	0.51	0.54	0.80	0.83	0.86
Ionosphere	0.70	0.64	0.72	0.64	0.64	0.72	0.71	0.83
Movement	0.47	0.43	0.49	0.38	0.26	0.42	0.50	0.50
Semeion	0.60	0.31	0.56	0.39	0.40	0.46	0.67	0.69

**Fig. 1.** Features captured by the transformation matrix W . From to right, the selected feature number is {32, 64, 96, 128, 160, 192, 224, 256}. Most discriminative features, such as eyes, nose and mouth, are preserved.**Fig. 2.** Convergence curves on different datasets.

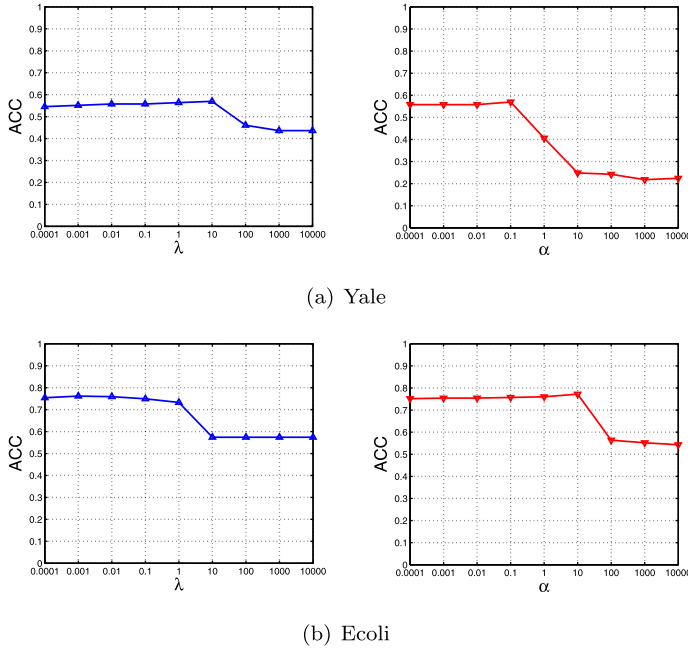


Fig. 3. The influence of λ and α on (a) Yale and (b) Ecoli datasets.

ground noise, are dropped in most cases. So the data projection operation helps to capture the valuable features while reducing the influence of noise.

Convergence: We further discuss the convergence behavior of APMF. During the optimization of each parameter, a global or local optimal solution is obtained. So the ALM objective value in problem Eq. (9) decreases monotonously in each iteration, and finally converges to a local optima. In addition, as μ grows exponentially with the number of iterations, the third and fourth terms in the ALM objective function will be close to 0. So problem Eq. (9) converges to the original objective function in problem Eq. (7) asymptotically. Therefore, the convergence of problem Eq. (7) is guaranteed. Fig. 2 visualizes the convergence curves of the original objective value on different datasets. As shown in the figure, the optimization algorithm converges within ten iterations, which verifies the effectiveness of the optimization.

Parameter sensitivity: There are two regularization parameters, λ and α , in the proposed method. λ controls the weight of the manifold learning term, and α balances the graph learning term. We investigate their impacts on the final clustering performance. Fig. 3 shows the ACC curves of APMF with varying λ and α respectively. It can be seen that the performance of APMF is very stable when λ and α are within the range of $\{0.0001, 0.001, \dots, 0.1\}$. When the values of λ and α are larger than 1, the approximate error in problem Eq. (7) becomes large, so the performance decreases. Thus, we set λ and α as 0.01 throughout the experiments.

5. Conclusions

In this paper, a new clustering method called Adaptive Projected Matrix Factorization (APMF) has been put forward. The proposed method jointly combines the graph learning method into the NMF framework, so it is able to learn the optimal affinity graph for clustering. In addition, the proposed method takes advantage of subspace clustering by projecting the input data into the desired subspace, so the noise in the input data space is alleviated and more discriminative features are captured. Moreover, an efficient algorithm is designed to optimize the proposed problem with

proved convergence. Extensive experiments on several datasets validates the superior performance of APMF.

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