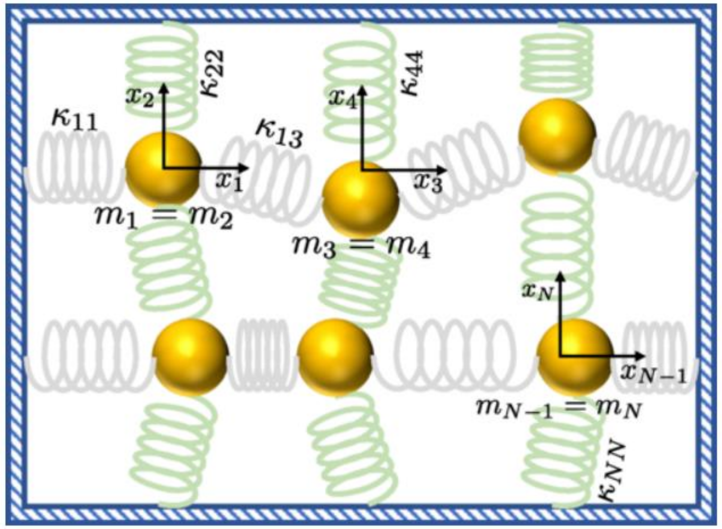


Network of classical coupled oscillators (Baboush et al, 2023, https://doi.org/10.1103/PhysRevX.13.041041 , [1])

The Abstract Oscillators Team:

Kerem Yurtseven
Cristina Radian
Viraj Daniel DSouza

Project Choice:

3. Development of Novel Quantum Algorithms --> *Womanium + Classiq*

Quantum Algorithm Choice:

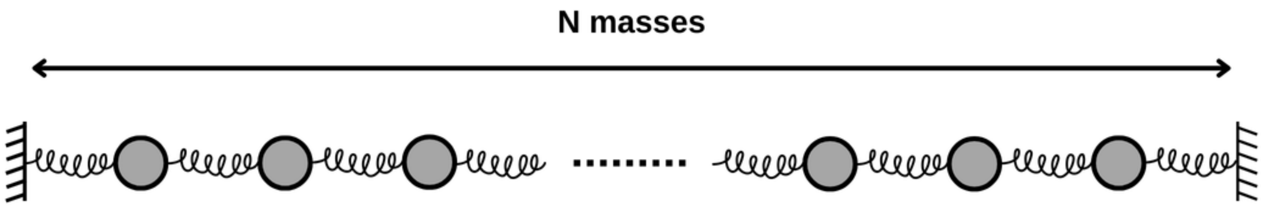
b) Exponential quantum speedup in simulating coupled classical oscillators.

Content:

1. Problem Statement
2. Solution Description
3. Success
4. Future scope
5. Bibliography and Acknowledgements

Problem Statement

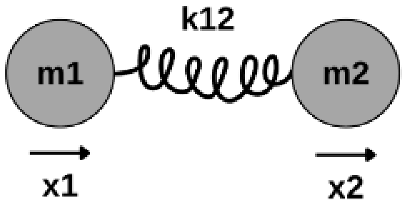
Building on the theoretical framework from the 2023 study by Babbush et al., our project seeks to translate the exponential quantum speedup in simulating coupled classical oscillators into a practical implementation using Classiq, compatible with current quantum hardware and software.



Project Importance: Achieving this practical quantum advantage would revolutionize fields like material science, climate modeling, and medical technology by enabling unprecedented accuracy and efficiency in complex simulations, driving innovation across multiple disciplines.

Toy Problem

Configuration



- N = 2
- m1 = m2 = 1
- K12 = 1

Methodology

Pre-Processing

Pauli Decomposition

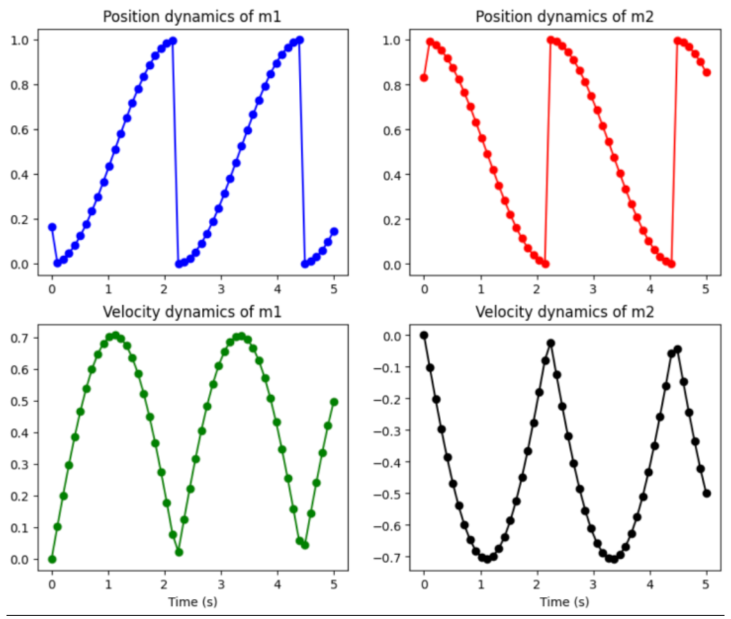
Hamiltonian Evolution

Post-Processing

- Exponentiation
- Suzuki Trotter
- Qubitization
- QSVT

Qubitization and QSVT: Block Encoding

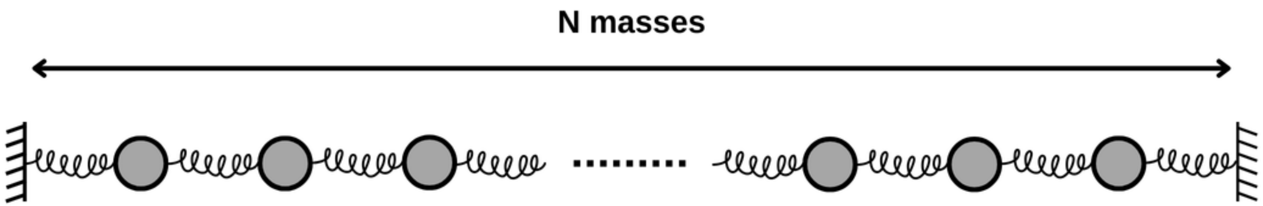
Results



Error at t = 0.5	Velocities	Positions
Mass 1	%0.01	%1.78
Mass 2	%4.21	%0.15

!PHASE ERROR!

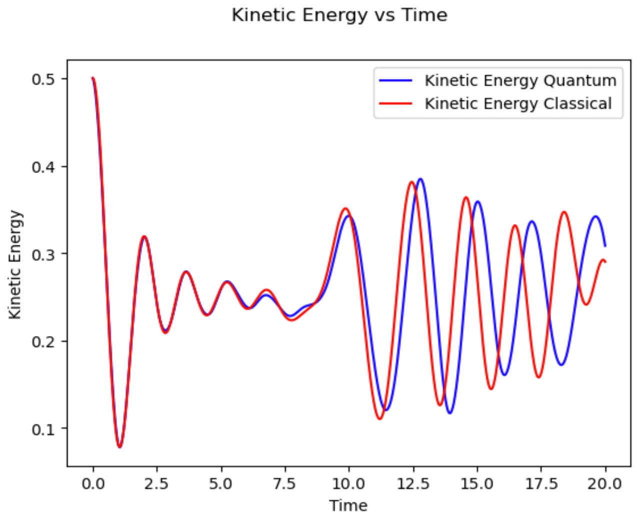
Kinetic Energy Estimation

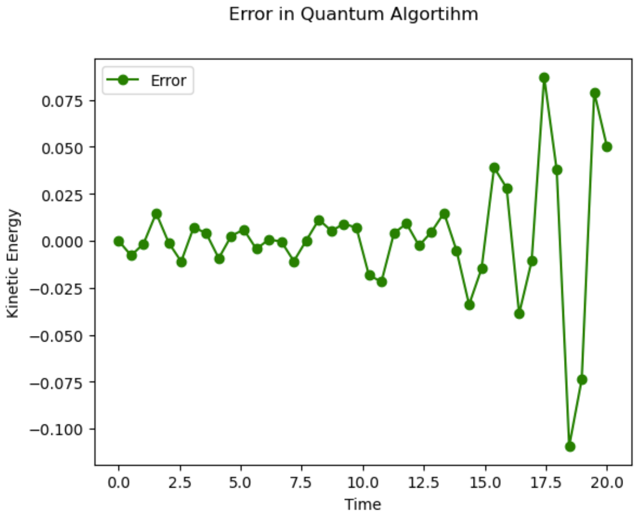


Real Example Problem 2 can be simulated with N = 8 masses to prove the final equation

$$\left| \hat{k}_V(t) - \frac{K_V(t)}{E} \right| \leq \epsilon$$

Params: m = 1, k = 1, Suzuki Trotter



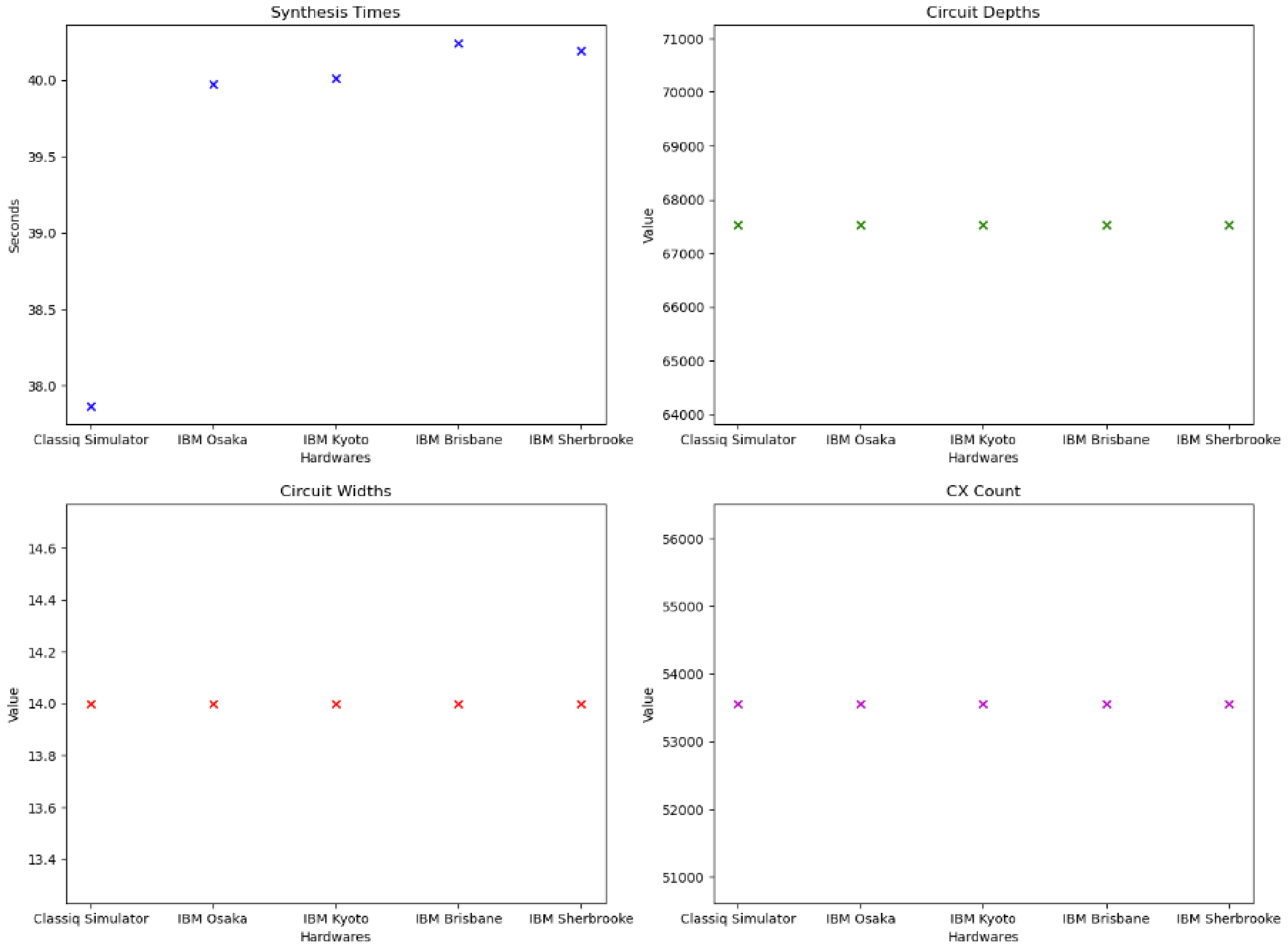


Resource Estimation

Real Quantum Hardwares are critical to understand the importance and reality of quantum algorithms

Compared the following simulator and IBM Quantum Hardware for Kinetic Energy Estimation

- Classiq Simulator
- IBM Osaka
- IBM Kyoto
- IBM Sherbrooke
- IBM Brisbane



Resource Estimation with Suzuki Trotter

Optimization

All of the hardwares give same circuit parameters expect synthesis time. So, choose IBM Osaka with lowest time and try to optimize it caring the maximum qubit of hardware

IBM Osaka	No Optimization	Depth Optimization	Width Optimization
Synthesis Time	39.97 s	66.69 s	67.00 s
Depth	67537	67537	67537
Width	14	14	14
CX Gate Number	53561	53561	53561

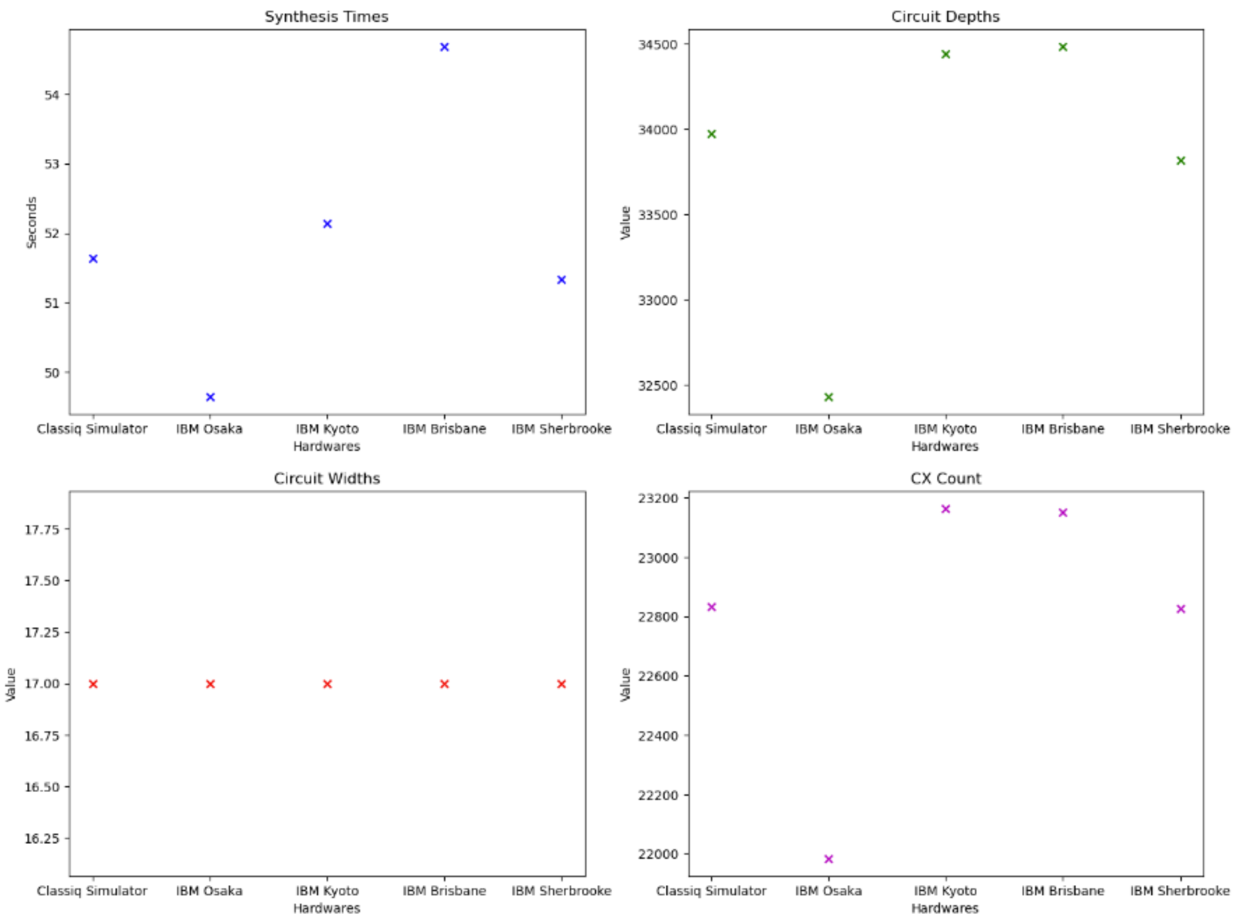
NO OPTIMIZATION
CHANGE EVOLUTION METHOD

Qubitization

Imply Qubitization to use in Kinetic Energy Estimation, yet set N = 2 due to very long synthesis and execution times

Compared the following simulator and IBM Quantum Hardware for Kinetic Energy Estimation

- Classiq Simulator
- IBM Osaka
- IBM Kyoto
- IBM Sherbrooke
- IBM Brisbane



Resource Estimation with Qubitization

Optimization with Qubitization

Hardwares differ a lot in depth and CX gate number and IBM Osaka is the most appropriate amongst them. So, choose IBM Osaka with lowest time and try to optimize it caring the maximum qubit (127) of hardware

IBM Osaka	No Optimization	Depth Optimization	Width Optimization	Depth Opt. 127 Max
Synthesis Time	49.65 s	189.37 s	818.17 s	2350.92 s
Depth	32342	19767	56581	19744
Width	17	1273	15	127
CX Gate Number	21984	14836	39098	14816

SUCCESSFUL OPTIMIZATION

Achievements

- Toy examples are successfully simulated and compared with classical results
- Multiple evolution methods are implemented with considerable accuracy
- A real problem from the paper is illustrated and theoretical assertions are supported with simulations
- Resource estimation and hardware comparison are made for the real problem
- Most appropriate hardware is chosen and tried to get optimized with different evolution methods
- An optimal transpiled circuit is created for the implementation of the algorithm to real quantum hardwares

Future Scope and Issues

- Post-processing the global phase is a problem which results in estimation only up to a global phase
- In case of singular B matrices, it is not certain to find the position of the masses at every time
- For higher number of masses, both classical and quantum algorithms can have problems
- Optimizations can be further improved for real quantum hardware
- Real quantum hardware simulations with higher number of masses can be conducted
- Post-processing for the results of real quantum hardware executions can be implemented
- The algorithm can be generalized for the use of different subjects such as electric circuit simulations, molecular simulations, climate modelling, material science etc.

Bibliography and Acknowledgements

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This project uses Classiq Github by Classiq.