Questions

1-

a) Give a recursive algorithm to solve the following recursive function (hint: use Fibonacci as a reference

$f(0) = 1;$
$f(1) = 2;$

$f(n) = 3\,f(n-1) + 4\,f(n-2);\ n > 1$

```
Int F(int n){
 if(n==0 || n==1) return n
    else return (3*F(n-1)+4*F(n-2)
}
```

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b) Solve $f(n)$ as a function of $n$ (using the method we used in class for Homogenous Equations). Do not solve for the constants.

$F(0) = 1; \quad F(1) = 2;$

$① \ f(n) - 3f(n-1) - 4f(n-2) = 0$

$② \ f(n+1) - 3f(n) - 4f(n-1) = 0$

$n = n+1$

$②-① \ f(n+1) - 4f(n) - f(n-1) + 4f(n-2) = 0$ homogeneous

$x^{n+1} - 4x^{n} - x^{n-1} + 4x^{n-2} = 0$

$x^{n-2}(x^{3} - 4x^{2} - x + 4) = 0$

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X

2- **Prove that the following statements are true (*T*) or false (*F*). (let $\log n = \log_2 n$). You must define first what you are trying to prove using the limit definition.**

a) $n^3/\log n \in O(n^k)$, for any integer constant $2 \le k \le 4$.

$g(n) \in O(F(n))$ $\lim_{n\to\infty} \frac{g(n)}{F(n)} = \text{const}, 0$ True

$\lim_{n\to\infty} \frac{\frac{n^3}{\log n}}{n^k}$

False for $k=2$ -2

$k=2$ $\lim_{n\to\infty} \frac{n^3}{n^2 \log n} = \frac{n}{\log n}$ - const True

$k=3$ $\lim \frac{n^3}{n^3 \log n} = 0$ - True

$k=4$ $\lim \frac{n^3}{n^4 \log n} = \frac{1}{n \log n} = 0$ True

b) $n + n \log n^k \in \Theta(n \log n)$, for any positive integer constant $k$.

$g(n) \in \Theta(F(n))$ $\lim_{n\to\infty} \frac{g(n)}{F(n)} = \text{const}$

$\lim_{n\to\infty} \frac{n + n\log n^k}{n \log n} = \lim_{n\to\infty} \frac{n}{n \log n} + \lim_{n\to\infty} \frac{kn \log n}{n \log n} = k$ True

$\to 0$

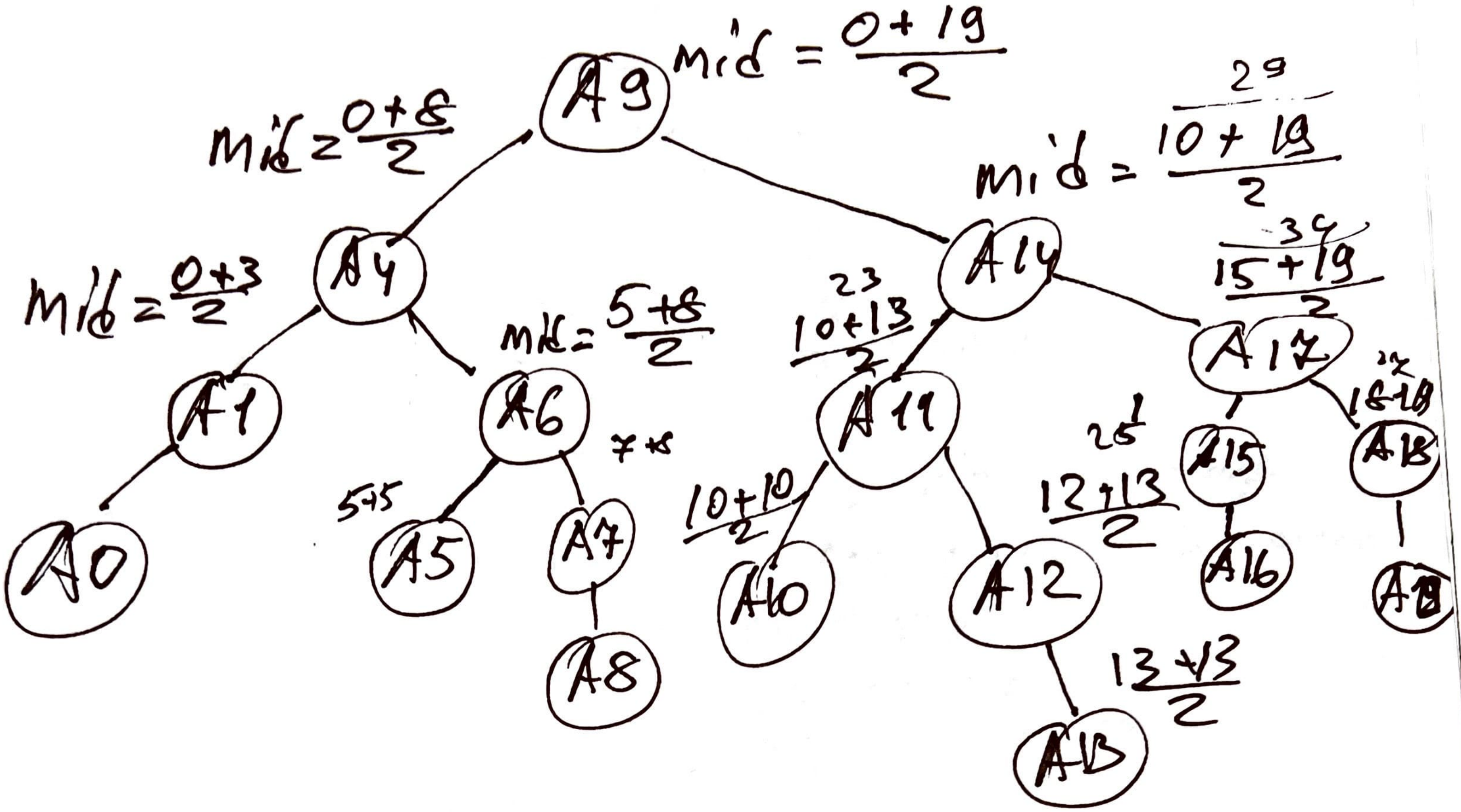
c) $n \log n^2 \in O(n^k)$, for any integer constant $k > 2$.

$g(n) \in O(F(n))$ $\lim_{n\to\infty} \frac{g(n)}{F(n)} = \text{const}, 0$

$\lim_{n\to\infty} \frac{n \log n^2}{n^k} = \lim_{n\to\infty} \frac{2n \log n}{n^k}$ True

$k=3$ $\lim_{n\to\infty} \frac{2n \log n}{n^k} = \frac{2n\log n}{n^3} = \frac{2\log n}{n^2} \to 0$

3- Write the binary tree representation for the Binary Search for 20 elements and give the worst-case scenario.




$W(n) = \log_2 20 \simeq 5$

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4- Sort the list A[ ]={ 20, 13,4, 34, 5, 15, 90, 100, 75, 102, 112, 1}

a) Insertion Sort (count the number of comparisons not the swaps).

$-\infty$ 20 1 compariso

$-\infty$ 13 20 2 comparison

$-\infty$ 4 13 20 3 comp

$-\infty$ 4 13 20 34 1 comp

$-\infty$ 4 5 13 20 34 4 comp

$-\infty$ 4 5 13 15 20 34 3 comp

$-\infty$ 4 5 13 15 20 34 90 1 comp

$-\infty$ 4 5 13 15 20 34 90 100 1 comp

$-\infty$ 4 5 13 15 20 34 75 90 100 3 comp

$-\infty$ 4 5 13 15 20 34 75 90 100 102 1 comp

$-\infty$ 4 5 13 15 20 34 75 90 100 102 112 1 com

$-\infty$ 1 4 5 13 15 20 34 75 90 100 102 112 12

Total = 1 + 2 + 3 + 1 + 4 + 3 + 1 + 1 + 3 + 1 + 1 + 12 = 33 comparisons

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5- The worst-case of the algorithm Merge sort is defined by the recursive equation

$$W(1) = 1$$
$$W(n) = 2\,W(n/2) + n$$

Under the assumption that $n = 2^k$ (n is a power of 2), solve for W(n) as a function of n.

$w(n) - 2w(n/2) = n$ $\quad n = 2^k$

$w(2^k) - 2w(2^{k-1}) = 2^k$

(1) $w(k) - 2w(k-1) = 2^k$

(2) $w(k+1) - \underline{2w(k)} = 2^{k+1}$

$k \leftarrow k+1$

(3) $2w(k) - 4w(k-1) = 2^{k+1}$

(1) · 2

$-4w(k)$

(2) + (3) $\quad w(k+1) + 4w(k-1) = 0$

$x^{k+1} - 4x^k + 4x^{k-1} = 0$

$x^{k-1}(x^2 - 4x + 4) = 0$

2 2

$(x-2)(x-2)$ $(x+2)(x+2)$ $\quad x = -2 \quad x = 2$

$W(k) = C_1 \cdot 2^k + C_2 - 2^k$

$-4$