

# Homework 3

a)  $F(x, y, z) = \sum (0, 1, 6, 7)$

$x$	$y$	$z$	0	1	1	1	0
0	0	0	1	1	0	0	
1	0	0	1	1	1	1	

$$F = x' \cdot y' + xy$$

b)  $F(x, y, z) = \sum (1, 4, 5, 6, 7)$

$x$	$y$	$z$	0	0	0	1	1	1	0
0	0	0	1	1	0	1	1	0	1
1	0	0	1	1	1	1	0	1	1

$$F = y'z + x$$

c)  $F(x, y, z) = xyz + x'y'z + xy'z' + xy'z'$

$x$	$y$	$z$	0	0	0	1	1	1	0
0	0	0	1	1	0	1	1	0	1
1	0	0	1	1	1	1	0	1	1

$$F = x'y'z + x'y'z' + xy$$

$$F = \sum (1, 4, 7, 6)$$

d)  $F(A, B, C) = ABC' + B'C + A'$

$A$	$B$	$C$	0	0	0	1	1	1	0
0	0	0	1	1	0	1	1	0	1
1	0	0	1	1	1	1	0	1	1

Simplify the following Boolean functions using 4 variables maps.

a)  $F(A, B, C, D) = \sum(0, 1, 2, 5, 7, 8, 9, 10, 13, 14)$

		CD		AB					
		00	01	11	10	00	01	11	10
AB		00	1	1	0	1	1	1	0
01		01	1	1	1	1	04	15	14
11		11	1	1	1	1	12	13	05
10		10	1	1	1	1	18	19	011

$\rightarrow$  ~~11~~ 11 11 11

		CD		AB					
		00	01	11	10	00	01	11	10
AB		00	1	1	0	2	1	1	0
01		01	04	15	14	6	04	15	14
11		11	12	13	05	14	12	13	14
10		10	18	19	011	10	18	19	10

$$F = C'D + DA' + B'C'D' + CD'A$$

b)  $F(A, B, C, D) = \sum(1, 3, 4, 5, 10, 12, 13)$

		CD		AB					
		00	01	11	10	00	01	11	10
AB		00	00	11	13	02	1	1	2
01		01	14	15	12	06	6	6	6
11		11	08	09	011	10	18	19	18
10		10	12	11	15	14	12	13	14

~~11~~ 11 11 11

		CD		AB					
		00	01	11	10	00	01	11	10
AB		00	1	1	0	2	1	1	0
01		01	6	15	7	6	6	6	6
11		11	18	19	15	16	18	19	16
10		10	8	9	11	10	10	11	10

$F = C'B + A'B'D + ABCD'$

c)  $F(a, b, c, d) = \sum m(1, 3, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31)$

		CD		AB					
		00	01	11	10	00	01	11	10
AB	00	00	01	12	11	12	1	1	2
01	14	15	14	15	16	15	6	6	6
11	92	013	115	114	115	114	18	19	18
10	03	09	110	111	110	111	8	9	10

$$F = C + A'B$$

d)  $F(w, x, y, z) = \sum m(0, 0, 1, 1, 1, 0, 0, 0, 0, 1)$

		YZ		WX			
		00	01	11	10	00	01
WX	00	1	0	1	1	1	0
01	1	4	05	1	6	0	1
11	6	12	0	13	15	0	14
10	1	8	9	1	0	11	1

$$F = X'Z' + YX'$$

3. Simplify the following Boolean function  $F$ , together with the ~~don't care~~ terms.

from d and then express the corresponding simplified function in sum of minterms.

$$a) F(x, y, z) = \sum (1, 4, 6)$$

$$\delta(x, y, z) = \sum (0, 2, 7)$$

	$xz$	$yz$	$xy$	$x$
$y$	00	01	11	10
$x$	0	1	1	0
0	1	X	X	X
1	X	1	X	1

$$F = z' + x'y' + \cancel{x}y$$

In order to do  $\Sigma$  of minterms we also have to count don't care minterms that get into simplification.

$$F = \sum (0, 1, 2, 4, 6)$$

$$b) F(A, B, C, D) = \sum (1, 5, 6, 7, 13)$$

$$\delta(A, B, C, D) = \sum (8, 9)$$

	$CD$	00	01	11	10
$AB$	00	0	1	3	2
0	00	X	1	5	7
1	01	1	X	15	14
10	10	X	9	10	12

$$A'B + C'DA' + C'DB$$

$$F = \sum (1, 4, 5, 7, 6, 13)$$

4. Simplify the following Boolean functions in product of sum.

$$a) F(A, B, C, D) = A'B'C'D + ABC + A'B'C'D' + A'B'C'D$$

According to DeMorgan theorem

$(a \cdot b)' = a' + b'$  to take complement in order to get sum of products

$$(A+B) \cdot (C+C') \cdot (A'+B'+C') \cdot (A+B+C'+D) \cdot (A+B+C'+D')$$

$$b) F(A, B, C, D) = \prod (2, 3, 4, 9, 11)$$

		CD		AB			
		00	01	11	10	01	02
		00	01	11	10	04	02
		11	12	13	15	14	
		10	8	09	11	10	

$$(A'B'C'D') \cdot (A'B'C + D) + \\ (A'B'C'D') \cdot (A'B'C + D') + \\ (A'B'C + D)$$

## 5. NAND/NOR Implementation

a) Simplify the following function and implement it with two-level NAND gate circuit.

$$F(A, B, C, D) = A'B'C'D' + C'D + AC'D = \\ = D(A'B'C' + C + AC') = D(A'B'C' + C + A)$$

\* Need to use K-Map not expression simplification

		CD		AB			
		00	01	11	10	01	02
		00	01	11	10	04	02
		11	11	11	11	11	11
		10	11	11	11	11	11

$$F = CD + B'D + AD$$

$$= D(A'B'C' + C + AC') = \\ = D(C'(A'B' + A) + C)$$

$b \cdot c + a$

Actually we can do both ways by way of K-Map and boolean expression simplification

By distributive Law:  $a+bc = (a+b) \cdot (a+c)$

Applying this to our expression

$$D((C'+C)(A'B' + A + C))$$

$$D = (C + A'B' + A) =$$

~~$\overline{ab} + C$~~  ~~181~~

$$= D(C + \underbrace{(A' + A)}_1 \cdot \underbrace{(B' + A)}_1) =$$

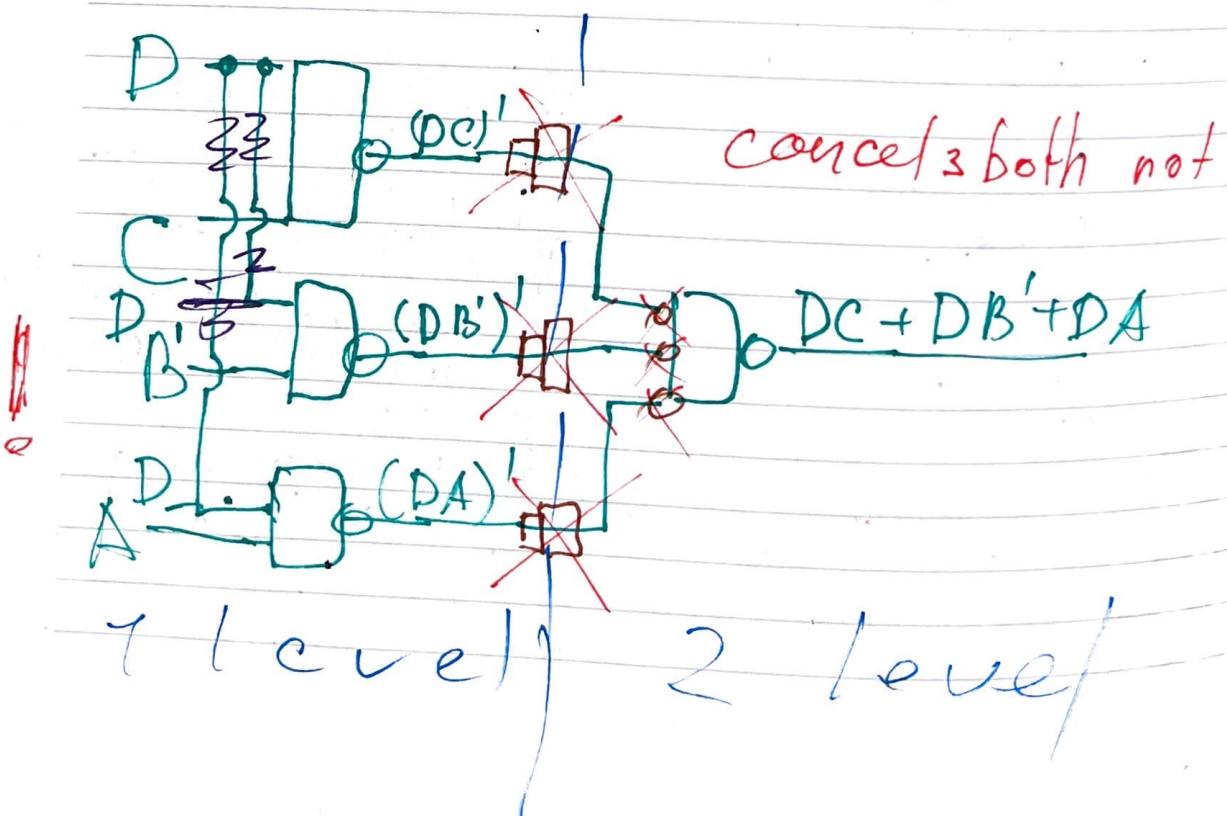
$$= D(C + B' + A) =$$

$$= DC + DB' + DA$$

Rules for obtaining 2-level NAND

Circuits for the simplified Boolean expression

1. Simplify Boolean Expression and express
2. Each product term of the function that has 2 more literals are connected to NAND gate each literal as input. This is first level
3. If the product term constitutes of a single literal, draw NAND gate for each single literal
4. Outputs from the first level NAND gates are passed to a single NAND gate in the second



b) Simplify the following functions and implement it with two-level NOR

$$F(w, x, y, z) = \sum(0, 3, 12, 15)$$

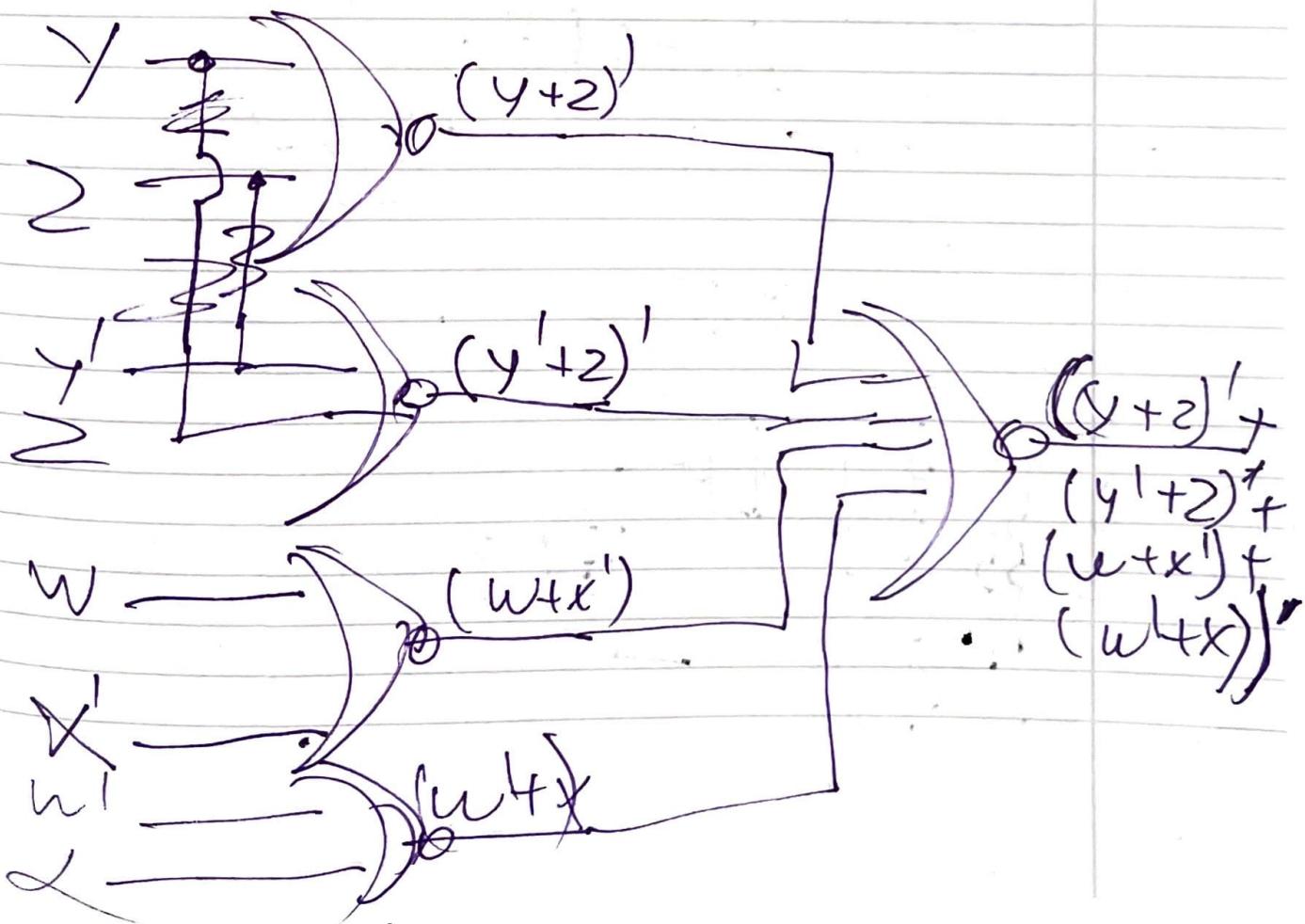
	00	01	11	10
00	1	0	1	0
01	0	0	0	0
11	1	1	1	0
10	0	0	0	0

$$\begin{aligned} F &= w'x'y'z' + w'x'yz + \\ &+ wx'y'z' + wx'yz = \\ &= w'x'(y'z' + yz) + wx' \\ &\quad (y'z' + yz) = (w'x' + \\ &\quad wx)(y'z' + yz) \end{aligned}$$

? why? have to be expressed in POS (XOR).

$$F' = y'z + yz' + w'x + ux'$$

$$\begin{aligned} (F')' &= (y'z + yz' + w'x + ux')' = \\ &= (y+z) \cdot (y'+z) \cdot (w+x') \cdot (w'+x) \end{aligned}$$



## 6. Multilevel NAND / NOR Implementation

a) Draw the multiple level circuit for the following expression:

$$W(X+Y+Z) + XYZ$$

1) Simplify expression in a form of

$$WX + WY + WZ + XYZ$$

~~NAND~~

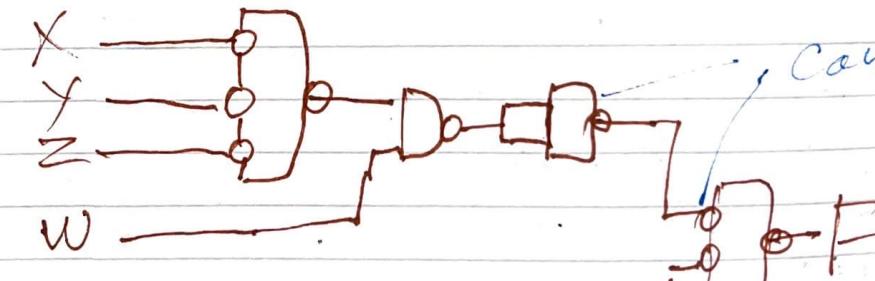
Not Gate by NAND



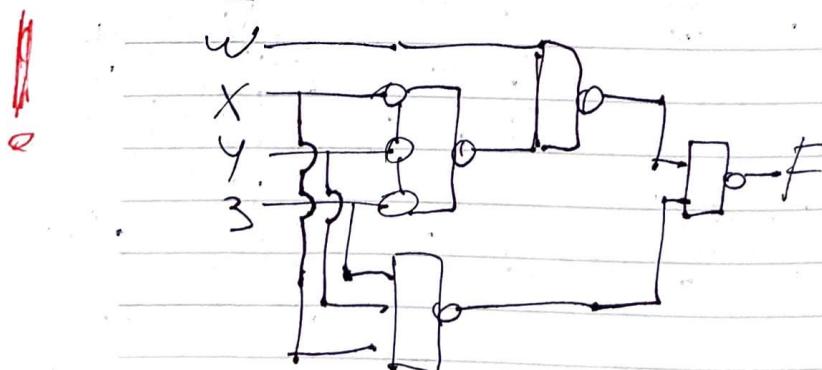
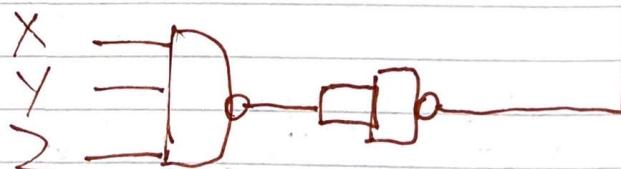
OR Gate by NAND



AND Gate by NAND

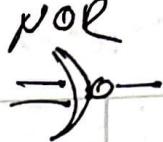


Cancelling

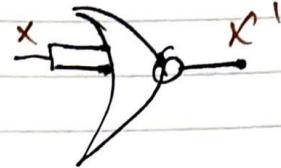


b). Draw multiple level NOR circuit

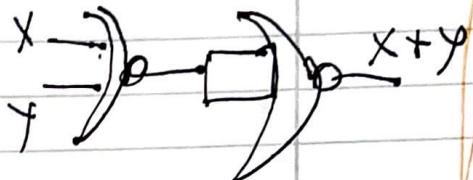
$$CD(B+C)A + (\neg BC' + \neg DE')$$



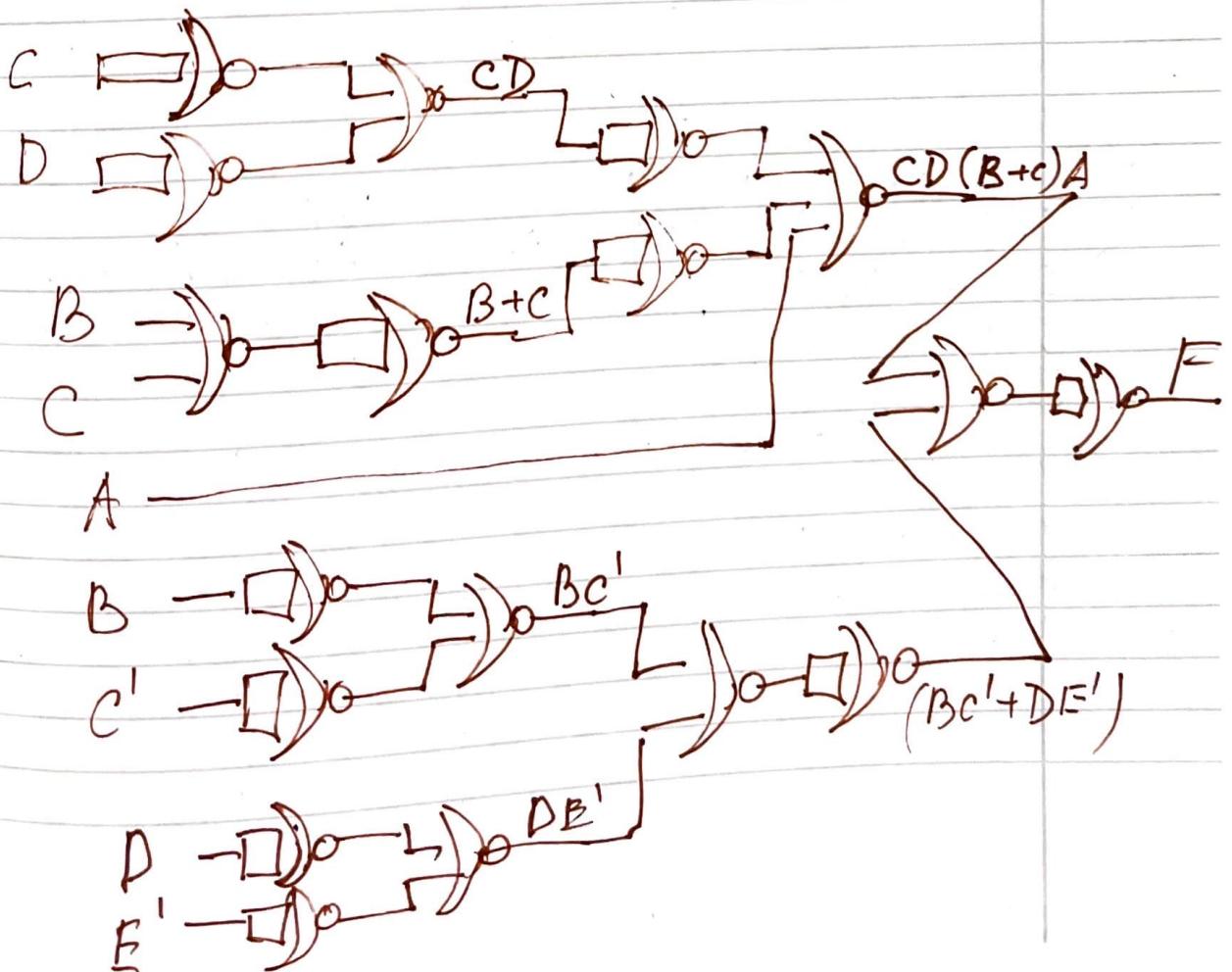
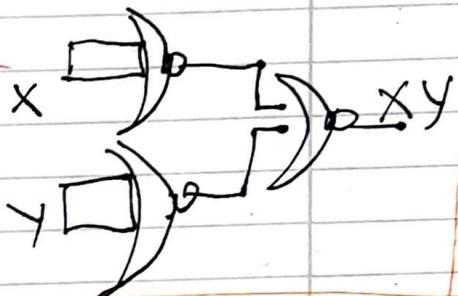
Not Gate by NOR



OR Gate by NOR



AND Gate by NOR



4. Derive circuits for 3 bit parity generator and four-bit parity using odd parity bit.

X	Y	Z	P
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

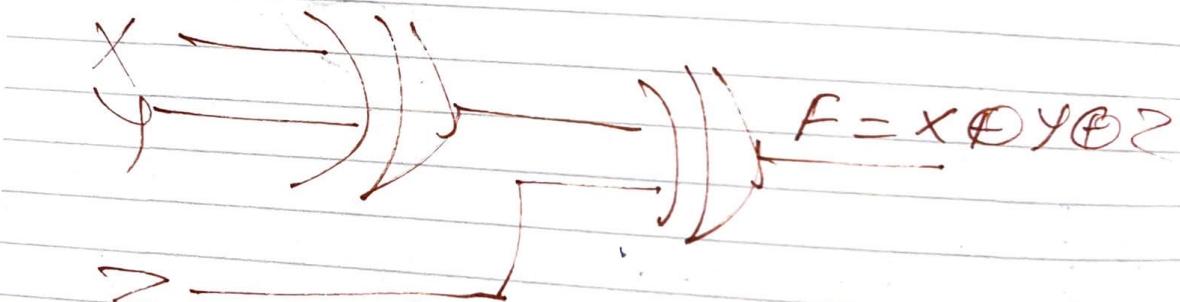
$$P = \sum m(0, 3, 5, 6)$$

$$= X'Y'Z' + X'YZ + XY'Z + XYZ$$

$$\times \begin{array}{|c|c|} \hline 0 & 0 \\ \hline 0 & 1 \\ \hline 1 & 0 \\ \hline 1 & 1 \\ \hline \end{array}$$

if  $Z_1Z_2Z_3Z_4$  is  $X$

$$X \oplus Y \oplus Z$$



X	Y	Z	P	CW
0	0	0	1	1
0	0	1	0	0
0	1	0	0	0
0	1	1	1	0
1	0	0	0	0
1	0	1	1	0
1	1	0	0	1
1	1	1	0	1

if 0 then is error  
1 then no error

$a$	$b$	$c$	$d$	$P \oplus c$	if $c = 0$ then error $\neq 0$ then no error
0	0	0	0	1 <del>0</del>	
0	0	0	1	0	$(a+b+c+d)'$
0	0	1	0	1	$(0+0)' = 1$
0	0	1	1	1	$(0+1)' = 0$
0	1	0	0	0	
0	1	0	1	1	
0	1	1	0	1	
0	1	1	1	0	
1	0	0	0	0	
1	0	0	1	1	
1	0	1	0	1	
1	0	1	1	0	
1	1	0	0	1	
1	1	0	1	0	
1	1	1	0	0	
1	1	1	1	1	

