CHAPTER EIGHT

Tracking GPS Signals

8.1 INTRODUCTION

One might think that the basic method of tracking a signal is to build a narrow-band filter around an input signal and follow it. In other words, while the frequency of the input signal varies over time, the center frequency of the filter must follow the signal. In the actual tracking process, the center frequency of the narrow-band filter is fixed, but a locally generated signal follows the frequency of the input signal. The phases of the input and locally generated signals are compared through a phase comparator. The output from the phase comparator passes through a narrow-band filter. Since the tracking circuit has a very narrow bandwidth, the sensitivity is relatively high in comparison with the acquisition method.

When there are phase shifts in the carrier due to the C/A code, as in a GPS signal, the code must be stripped off first as discussed in Section 7.5. The tracking process will follow the signal and obtain the information of the navigation data. If a GPS receiver is stationary, the expected frequency change due to satellite movement is very slow as discussed in Chapter 3. Under this condition, the frequency change of the locally generated signal is also slow; therefore, the update rate of the tracking loop can be low. In order to strip off the C/A code another loop is needed. Thus, to track a GPS signal two tracking loops are required. One loop is used to track the carrier frequency and is referred to as the carrier loop. The other one is used to track the C/A code and is referred to as the code loop.

In this chapter the basic loop concept will be discussed first. Two tracking methods will be discussed. The first one is the conventional tracking loop. The

Fundamentals of Global Positioning System Receivers: A Software Approach, Second Edition,
by James Bao-Yen Tsui
ISBN 0-471-70647-7

only unique point of this method is that the tracking loop will be presented in digital form and the tracking will be accomplished in software. The second method is referred to as the block adjustment of synchronizing signal (BASS) method. The BASS method is also implemented in software and the performance might be slightly sensitive to noise. The details of the two methods will be presented.

8.2 BASIC PHASE-LOCKED LOOPS[1-4]

In this section the basic concept of the phase-locked loop will be described, which includes the transfer function, the error transfer function, the noise bandwidth, and two types of input signals.

The main purpose of a phase-locked loop is to adjust the frequency of a local oscillator to match the frequency of an input signal, which is sometimes referred to as the reference signal. A basic phase-locked loop is shown in Figure 8.1.

Figure 8.1a shows the time domain configuration and Figure 8.1b shows the s-domain configuration, which is obtained from the Laplace transform. The input signal is $\theta_i(t)$ and the output from the voltage-controlled oscillator (VCO) is $\theta_f(t)$. The phase comparator $\sum$ measures the phase difference of these two signals. The amplifier k_0 represents the gain of the phase comparator and the low-pass filter limits the noise in the loop. The input voltage V_o to the VCO controls its output frequency, which can be expressed as

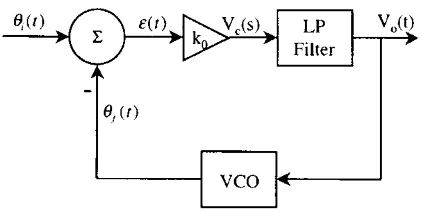
$$\omega_2(t) = \omega_0 + k_1 u(t) V_o \tag{8.1}$$

where ω_0 is the center angular frequency of the VCO, k_1 is the gain of the VCO, and $u(t)$ is a unit step function, which is defined as

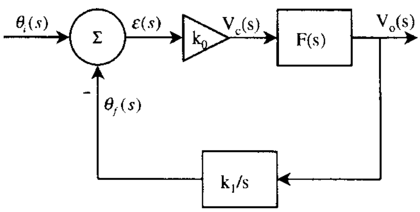
$$u(t) = \begin{cases} 0 & \text{for} \quad t < 0 \\ 1 & \text{for} \quad t > 0 \end{cases} \tag{8.2}$$

The phase angle of the VCO can be obtained by integrating Equation (8.1) as

$$\int_0^t \omega_2(t)dt = \omega_0 t + \theta_f(t) = \omega_0 t + \int_0^t k_1 u(t) V_o dt$$



(a) Time domain.



(b) S domain.

FIGURE 8.1 A basic phase-locked loop.

where

$$\theta_f(t) = \int_0^t k_1 u(t) V_o dt \tag{8.3}$$

The Laplace transform of $\theta_f(t)$ is

$$\theta_f(s) = V_o(s)\frac{k_1}{s} \tag{8.4}$$

From Figure 8.1b the following equations can be written.

$$V_c(s) = k_0\epsilon(s) = k_0[\theta_i(s) - \theta_f(s)] \tag{8.5}$$

$$V_o(s) = V_c(s)F(s) \tag{8.6}$$

$$\theta_f(s) = V_o(s)\frac{k_1}{s} \tag{8.7}$$

From these three equations one can obtain

$$\epsilon(s) = \theta_i(s) - \theta_f(s) = \frac{V_c(s)}{k_0} = \frac{V_o(s)}{k_0 F(s)} = \frac{s\theta_f(s)}{k_0 k_1 F(s)} \quad \text{or}$$

$$\theta_i(s) = \theta_f(s)\left(1 + \frac{s}{k_0 k_1 F(s)}\right) \tag{8.8}$$

where $\epsilon(s)$ is the error function. The transfer function $H(s)$ of the loop is defined as

$$H(s) \equiv \frac{\theta_f(s)}{\theta_i(s)} = \frac{k_0 k_1 F(s)}{s + k_0 k_1 F(s)} \tag{8.9}$$

The error transfer function is defined as

$$H_e(s) = \frac{\epsilon(s)}{\theta_i(s)} = \frac{\theta_i(s) - \theta_f(s)}{\theta_i(s)} = 1 - H(s) = \frac{s}{s + k_0 k_1 F(s)} \tag{8.10}$$

The equivalent noise bandwidth is defined as[1]

$$B_n = \int_0^\infty |H(j\omega)|^2 df \tag{8.11}$$

where ω is the angular frequency and it relates to the frequency f by $\omega = 2\pi f$.

In order to study the properties of the phase-locked loops, two types of input signals are usually studied. The first type is a unit step function as

$$\theta_i(t) = u(t) \quad \text{or} \quad \theta_i(s) = \frac{1}{s} \tag{8.12}$$

The second type is a frequency-modulated signal

$$\theta_i(t) = \Delta\omega t \quad \text{or} \quad \theta_i(s) = \frac{\Delta\omega}{s^2} \tag{8.13}$$

These two types of signals will be discussed in the next two sections.

8.3 FIRST-ORDER PHASE-LOCKED LOOP[1–4]

In this section, the first-order phase-locked loop will be discussed. A first-order phase-locked loop implies the denominator of the transfer function $H(s)$ is a first-order function of s. The order of the phase-locked loop depends on the order of the filter in the loop. For this kind of phase-locked loop, the filter function is

$$F(s) = 1 \tag{8.14}$$

This is the simplest phase-locked loop. For a unit step function input, the corresponding transfer function from Equation (8.9) becomes

$$H(s) = \frac{k_0 k_1}{s + k_0 k_1} \tag{8.15}$$

The denominator of $H(s)$ is a first order of s.

The noise bandwidth can be found as

$$\begin{aligned} B_n &= \int_0^\infty \frac{(k_0 k_1)^2 df}{\omega^2 + (k_0 k_1)^2} = \frac{(k_0 k_1)^2}{2\pi} \int_0^\infty \frac{d\omega}{\omega^2 + (k_0 k_1)^2} \\ &= \frac{(k_0 k_1)^2}{2\pi k_0 k_1} \tan^{-1}\left(\frac{\omega}{k_0 k_1}\right)_0^\infty = \frac{k_0 k_1}{4} \end{aligned} \tag{8.16}$$

With the input signal $\theta_i(s) = 1/s$, the error function can be found from Equation (8.10) as

$$\epsilon(s) = \theta_i(s) H_e(s) = \frac{1}{s + k_0 k_1} \tag{8.17}$$

The steady-state error can be found from the final value theorem of the Laplace transform, which can be stated as

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) \tag{8.18}$$

Using this relation, the final value of $\epsilon(t)$ can be found as

$$\lim_{t\to\infty} \epsilon(t) = \lim_{s\to 0} s\epsilon(s) = \lim_{s\to 0} \frac{s}{s + k_0 k_1} = 0 \tag{8.19}$$

With the input signal $\theta_i(s) = \Delta\omega/s^2$, the error function is

$$\epsilon(s) = \theta_i(s)H_e(s) = \frac{\Delta\omega}{s}\frac{1}{s + k_0k_1} \tag{8.20}$$

The steady-state error is

$$\lim_{t\to\infty} \epsilon(t) = \lim_{s\to 0} s\epsilon(s) = \lim_{s\to 0} \frac{\Delta\omega}{s + k_0k_1} = \frac{\Delta\omega}{k_0k_1} \tag{8.21}$$

This steady-state phase error is not equal to zero. A large value of k_0k_1 can make the error small. However, from Equation (8.15) the 3 dB bandwidth occurs at $s = k_0k_1$. Thus, a small final value of $\epsilon(t)$ also means large bandwidth, which contains more noise.

8.4 SECOND-ORDER PHASE-LOCKED LOOP(1–4)

A second-order phase-locked loop means the denominator of the transfer function $H(s)$ is a second-order function of s. One of the filters to make such a second-order phase-locked loop is

$$F(s) = \frac{s\tau_2 + 1}{s\tau_1} \tag{8.22}$$

Substituting this relation into Equation (8.9), the transfer function becomes

$$H(s) = \frac{\dfrac{k_0k_1\tau_2 s}{\tau_1} + \dfrac{k_0k_1}{\tau_1}}{s^2 + \dfrac{k_0k_1\tau_2 s}{\tau_1} + \dfrac{k_0k_1}{\tau_1}} \equiv \frac{2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{8.23}$$

where ω_n is the natural frequency, which can be expressed as

$$\omega_n = \sqrt{\frac{k_0k_1}{\tau_1}} \tag{8.24}$$

and ζ is the damping factor, which can be shown as

$$2\zeta\omega_n = \frac{k_0k_1\tau_2}{\tau_1} \quad \text{or} \quad \zeta = \frac{\omega_n\tau_2}{2} \tag{8.25}$$

The denominator of $H(s)$ is a second order of s.

The noise bandwidth can be found as[1]

$$B_n = \int_0^\infty |H(\omega)|^2 df = \frac{\omega_n}{2\pi}\int_0^\infty \frac{1+\left(2\zeta\dfrac{\omega}{\omega_n}\right)^2}{\left[1-\left(\dfrac{\omega}{\omega_n}\right)^2\right]^2+\left(2\zeta\dfrac{\omega}{\omega_n}\right)^2}d\omega$$

$$= \frac{\omega_n}{2\pi}\int_0^\infty \frac{1+4\zeta^2\left(\dfrac{\omega}{\omega_n}\right)^2}{\left(\dfrac{\omega}{\omega}\right)^4+2(2\zeta^2-1)\left(\dfrac{\omega}{\omega_n}\right)^2+1}d\omega = \frac{\omega_n}{2}\left(\zeta+\frac{1}{4\zeta}\right) \quad (8.26)$$

This integration can be found in the appendix at the end of this chapter.

The error transfer function can be obtained from Equation (8.10) as

$$H_e(s) = 1 - H(s) = \frac{s^2}{s^2+2\zeta\omega_n s+\omega_n^2} \quad (8.27)$$

When the input is $\theta_i(s) = 1/s$, the error function is

$$\epsilon(s) = \frac{s}{s^2+2\zeta\omega_n s+\omega_n^2} \quad (8.28)$$

The steady-state error is

$$\lim_{t\to\infty}\epsilon(t) = \lim_{s\to 0} s\epsilon(s) = 0 \quad (8.29)$$

When the input is $\theta_i(s) = 1/s^2$, the error function is

$$\epsilon(s) = \frac{1}{s^2+2\zeta\omega_n s+\omega_n^2} \quad (8.30)$$

The steady-state error is

$$\lim_{t\to\infty}\epsilon(t) = \lim_{s\to 0} s\epsilon(s) = 0 \quad (8.31)$$

In contrast to the first-order loop, the steady-state error is zero for the frequency-modulated signal. This means the second-order loop tracks a frequency-modulated signal and returns the phase comparator characteristic to the null point. The conventional phase-locked loop in a GPS receiver is usually a second-order one.

8.5 TRANSFORM FROM CONTINUOUS TO DISCRETE SYSTEMS[5,6]

In the previous sections, the discussion is based on continuous systems. In order to build a phase-locked loop in software for digitized data, the continuous system must be changed into a discrete system. This discussion is based on reference 5. The transfer from the continuous s-domain into the discrete z-domain is through bilinear transform as

$$s = \frac{2}{t_s}\frac{1 - z^{-1}}{1 + z^{-1}} \tag{8.32}$$

where t_s is the sampling interval. Substituting this relation in Equation (8.22) the filter is transformed to

$$F(z) = C_1 + \frac{C_2}{1 - z^{-1}} = \frac{(C_1 + C_2) - C_1 z^{-1}}{1 - z^{-1}} \tag{8.33}$$

where

$$C_1 = \frac{2\tau_2 - t_s}{2\tau_1}$$
$$C_2 = \frac{t_s}{\tau_1} \tag{8.34}$$

This filter is shown in Figure 8.2.

The VCO in the phase-locked loop is replaced by a direct digital frequency synthesizer and its transfer function $N(z)$ can be used to replace the result in Equation (8.7) with

$$N(z) = \frac{\theta_f(z)}{V_o(z)} \equiv \frac{k_1 z^{-1}}{1 - z^{-1}} \tag{8.35}$$

Using the same approach as Equation (8.8), the transfer function $H(z)$ can be written as

$$H(z) = \frac{\theta_f(z)}{\theta_i(z)} = \frac{k_0 F(z) N(z)}{1 + k_0 F(z) N(z)} \tag{8.36}$$

Substituting the results of Equations (8.33) and (8.35) into the above equation, the result is

$$H(z) = \frac{k_0 k_1 (C_1 + C_2) z^{-1} - k_0 k_1 C_1 z^{-2}}{1 + [k_0 k_1 (C_1 + C_2) - 2] z^{-1} + (1 - k_0 k_1 C_1) z^{-2}} \tag{8.37}$$

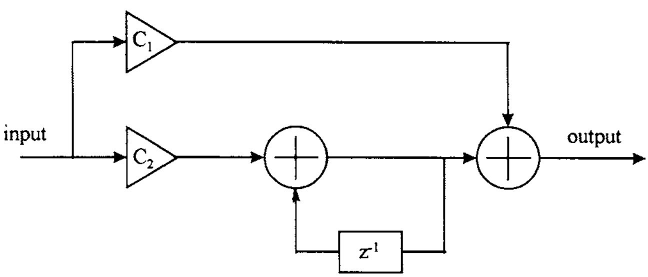


FIGURE 8.2 Loop filter.

By applying bilinear transform in Equation (8.32) to Equation (8.23), the result can be written as,

$$H(z) = \frac{[4\zeta\omega_n + (\omega_n t_s)^2] + 2(\omega_n t_s)^2 z^{-1} + [(\omega_n t_s)^2 - r\zeta\omega_n t_s]z^{-2}}{[4 + 4\zeta\omega_n t_s + (\omega_n t_s)^2] + [2(\omega_n t_s)^2 - 8]z^{-1} + [4 - 4\zeta\omega_n t_s + (\omega_n t_s)^2]z^{-2}} \tag{8.38}$$

By equating the denominator polynomials in the above two equations, C_1 and C_2 can be found as

$$C_1 = \frac{1}{k_0 k_1} \frac{8\zeta\omega_n t_s}{4 + 4\zeta\omega_n t_s + (\omega_n t_s)^2}$$
$$C_2 = \frac{1}{k_0 k_1} \frac{4(\omega_n t_s)^2}{4 + 4\zeta\omega_n t_s + (\omega_n t_s)^2} \tag{8.39}$$

The applications of these equations will be discussed in the next two sections.

In reference 6 a third-order phase-locked loop is also implemented. The filter is implemented in digital format and the result can be used for phase-locked loop designs, but it is not included in this book.

8.6 CARRIER AND CODE TRACKING[4]

Before discussing the usage of the above equations, let us concentrate on the tracking of GPS signals. The input to a conventional phase-locked loop is usually a continuous wave (cw) or frequency-modulated signal and the frequency of the VCO is controlled to follow the frequency of the input signal. In a GPS receiver the input is the GPS signal and a phase-locked loop must follow (or track) this signal. However, the GPS signal is a bi-phase coded signal. The carrier and code frequencies change due to the Doppler effect, which is caused by the motion of the GPS satellite as well as from the motion of the GPS receiver as discussed in Chapter 3. In order to track the GPS signal, the C/A code information must be removed. As a result, it requires two phase-locked loops to track a GPS signal. One loop is to track the C/A code and the other one is to track the carrier frequency. These two loops must be coupled together as shown in Figure 8.3.

In Figure 8.3, the C/A code loop generates three outputs: an early code, a late code, and a prompt code. The prompt code is applied to the digitized input signal and strips the C/A code from the input signal. Stripping the C/A code means to multiply the C/A code to the input signal with the proper phase as shown in Figure 7.1. The output will be a cw signal with phase transition caused only by the navigation data. This signal is applied to the input of the carrier loop. The output from the carrier loop is a cw with the carrier frequency of the input signal. This signal is used to strip the carrier from the digitized input signal, which means using this signal to multiply the input signal. The output is a signal with only a C/A code and no carrier frequency, which is applied to the input of the code loop.