Demographic Research Methods and the PyCCM library: Lecture Three

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Leverhulme Centre for Demographic Science

A lecture delivered at the Banco de la Republica, October 2025

Key Texts

The **key texts** of this course (perhaps in order of relevance?) are:

1. Preston, S.H, P. Heuveline and M. Guillot (2001) ‘Demography: Measuring and Modeling Population Processes’, Blackwell Publishers. **Chapter 5 is particularly relevant today!**
2. Wachter, K.W., (2014), ‘Essential Demographic Methods’, Harvard University Press.
3. Livi Bacci, M., (2012), ‘A Concise History of World Population’, Wiley-Blackwell.
4. Keyfitz, N. and Caswell, H., (2005), ‘Applied Mathematical Demography’, Springer.

Reading

There are four especially useful papers to read:

1. Bongaarts, J., & Feeney, G. (1998), 'On the Quantum and Tempo of Fertility', *Population and Development Review*, 271-291.
2. Bhrolchain, M. N. (1992), 'Period Paramount? A critique of the cohort approach to fertility', *Population and Development Review*, 599-629.
3. Mason, K. O. (1997), 'Explaining Fertility Transitions', *Demography*, 34(4), 443-454.
4. Morgan, S. P., & Taylor, M. G. (2006), 'Low Fertility at the Turn of the Twenty-first Century', *Annual Review of Sociology*, 32, 375-399.

Today...

What are we going to learn about today?

- A gentle introduction to fertility (including some ‘macro-level’ trends).
- Period and cohort measures of fertility (and debates around their use).
- Parity / birth order in fertility (but not the *effect* of order).
- Stability, Stationarity and more on (fertility) Transition.
- A *little* bit about (the fertility of) sociogenomics.

Importantly, we’ll learn about lots of **metrics** and **measurements**.

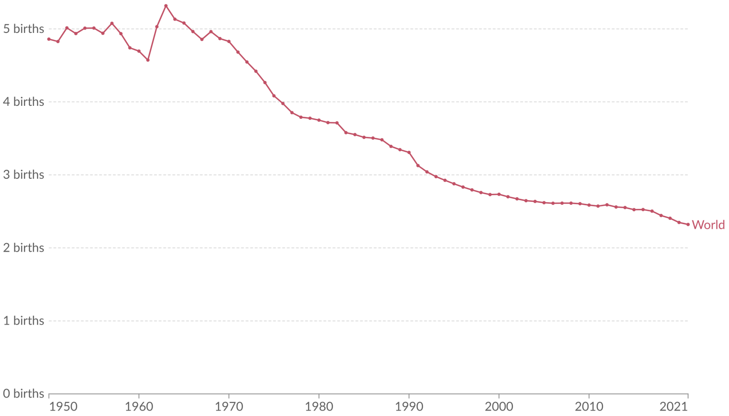
- Remember from earlier in the course: dēmos, graphō! Especially true here!

Introducing Fertility

- Populations change due to death, **births**, and migration.
  - We covered deaths last week, and we won't cover migration.
- We measured mortality using the (single-decrement) lifetable:
  - Death is a **non-repeatable** event.
  - Births are **repeatable** events.
- Specific measures have been devised considering this fact.
- Like mortality, we can measure both cohort and period fertility.
  - However, fertility shows stronger patterns of convergence than mortality.

Fertility rate: children per woman

The fertility rate[1], expressed as the number of children per woman, is based on age-specific fertility rates in one particular year.



Data source: United Nations, World Population Prospects (2022)

1. Fertility rate: The total fertility rate is a period metric. It summarizes fertility rates across all age groups in one particular year. For a given year, the total fertility rate represents the average number of children that would be born to a hypothetical woman if she (1) lived to the end of her childbearing years, and (2) experienced the same age-specific fertility rates throughout her whole reproductive life as the age-specific fertility rates seen in that particular year. It is different from the actual average number of children that women have. The fertility rate should not be confused with biological fertility, which is about the ability of a person to conceive. Read more: Fertility rate

Fertility rate: children per woman, 2021

Our World in Data

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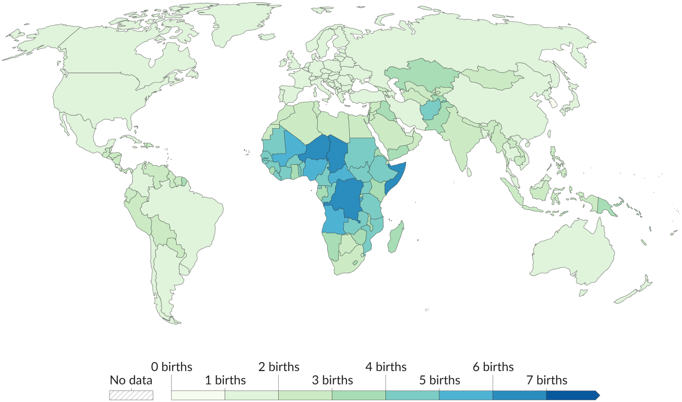

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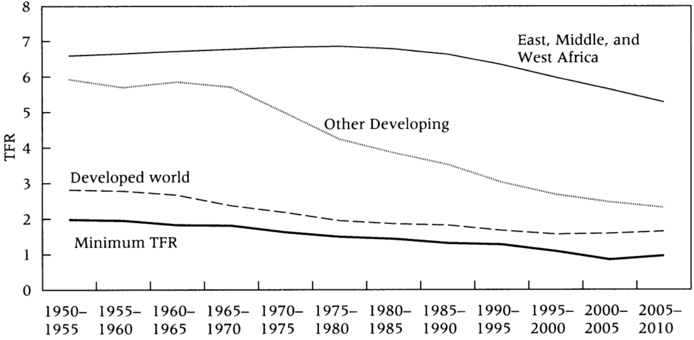
FIGURE 3 Total fertility, world regions and minimum national value, 1950–55 to 2005–10



SOURCE: United Nations (2009).

Source: Wilson, Chris. 2011. Understanding global demographic convergence since 1950. *Population and Development Review*, 37, 375-388

Definitions

- Fertility, reproduction and fecundity are distinct in demography.
  - **Fertility:** specifically relates to *live* births.
  - **Reproduction**: intergenerational replacement (births/deaths).
  - **Fecundity**: The biological component of fertility.
- Age, marital status, parity are important dimensions of fertility analysis for demographers.
- The measurements we're about to talk about differentially take this into account.

Proximal Determinants

- Fertility is shaped by social and cultural forces.
  - These impact on biological processes/pathways to childbirth.
- These biosocial pathways or ‘channels’ are called ‘proximate determinants’ (Bongaarts and Potter, 1983):
  - Relative contraceptive use.
  - Access to and utilisation of abortion.
  - Postpartum infecundity (abstinance, breast feeding).
  - Marriage/partnership status.

The Female Perspective

- Fertility is a two-sex problem. But when demographers measure fertility:
  - It is most commonly related to women.
  - It most commonly refers only to live born infants.
- Some reasons for this include:
  - 'Mater semper certa est' (who knows their Latin?)
  - One pregnancy at a time.
  - Easier to delimit the set of ages in which births are given
  - Mothers often provide better data on their children than fathers.
- Male fertility is becoming more widely studied, especially in the the context of changing family norms.
  - This is something we comment on in a recent review in Population Studies.

Data Sources

- There are three major types of data source:
    1. **Vital registrations**: enables direct methods linking registered births.
    2. **Surveys**: birth or maternity histories (e.g. Demographic and Health Surveys).
    3. **Censuses**: lifetime fertility (children ever born) and recent fertility questions.
- No data are perfect! Bias and quality differ with each source.
    - Important to consider which are different with each source.
    - Lets see an example on the next slide (birth registrations).

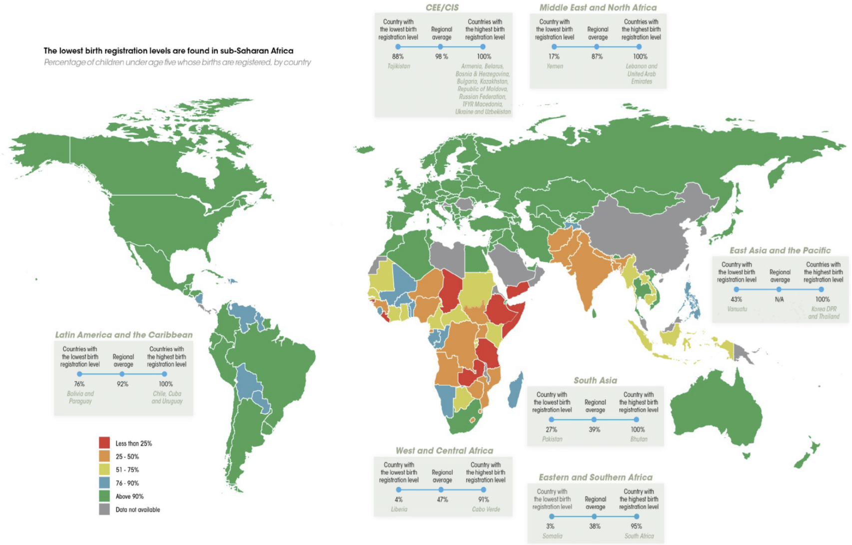


Figure: Source: 'Unicef – Every Child's Birth Right: Inequities and trends in birth registration'.

Period Fertility Measures

- We can define the **Crude Birth Rate (CBR)** as:

$$\text{CBR}[0,\ \text{T}] = \frac{\text{Number of births in } [0,\ \text{T}]}{\text{PY lived in } [0,\ \text{T}]} \tag{1}$$

- For those here last week; where did we see something similar before?
- Like last week, we're talking about an 'event' being converted into a 'rate'.
- The denominator should look especially familiar!
- Although in theory open to age-structural distortion, the CBR is usually a reasonably good overall indicator of fertility.
  - It is much less distorted than the crude death rate.

Period Fertility Measures (Cont.)

- To link the event of interest (birth) more clearly to the population-at-risk of experiencing the event, we use the **General Fertility Rate (GFR)**:

$$\mathrm{GFR}[0,\ \mathrm{T}] = \frac{\text{Number of births in } [0,\ \mathrm{T}]}{\text{PY lived in } [0,\ \mathrm{T}] \text{ by women aged } [15,\ 50)} \tag{2}$$

- Clearly, CBR and GFR are linked: $CBR[0,T] = GFR[0,T] \times_{35} C^F_{15}[0,T]$.

- **Age-specific fertility rates (ASFR)** relate period births to mothers of a certain age interval to the PY lived by such mothers:

$${}_{\mathrm{n}}\mathrm{F}_{\mathrm{x}}[0,\mathrm{T}] = \frac{\text{Births in } [0,\ \mathrm{T}] \text{ to women aged x to x+n}}{\text{PY lived in } [0,\ \mathrm{T}] \text{ by women aged x to x+n}} \tag{3}$$

Period Fertility Measures (Cont.)

- The most widely used measure is the **Total Fertility Rate (TFR)**.
  - It the sum of the age-specific rates over the range of childbearing ages.
    - This is otherwise known as the 'reproductive span'.
  - It is the average number of children a woman would bear if she experienced – at each age – the particular set of ASFRs.
  - This is under the condition that she survives throughout reproductive ages (where α is the minimum and β the is maximum age at childbearing).

$$\mathrm{TFR[0,T]} = \mathrm{n} \times \sum_{\mathrm{x}=\alpha}^{\beta-\mathrm{n}} {}_{\mathrm{n}}\mathrm{F}_{\mathrm{x}}[0,\mathrm{T}] \tag{4}$$

- It's an age-standardised measure that assumes all ages have same weight.
  - Note: the n is there to denote age-interval length.
- When computed as a period, it is in reference to a synthetic cohort.

Period Fertility Measures (Cont.)

- TFR gives an indication of the **quantum** of fertility (i.e # children).
- Demographers are also interested in the **tempo** (timing) of fertility.
- For this, the **Mean Age at Childbearing (MAC)** is commonly reported.

$$\mathrm{MAC}[0,\mathrm{T}] = \frac{\sum_{\mathrm{x}=\alpha}^{\beta-\mathrm{n}} (\mathrm{x} + \frac{\mathrm{n}}{2}) \times {}_{\mathrm{n}}\mathrm{F}_{\mathrm{x}}[0,\mathrm{T}]}{\sum_{\mathrm{x}=\alpha}^{\beta-\mathrm{n}} {}_{\mathrm{n}}\mathrm{F}_{\mathrm{x}}[0,\mathrm{T}]} \tag{5}$$

- As with last week / with mortality analysis, we see how much simpler life becomes with one year intervals!

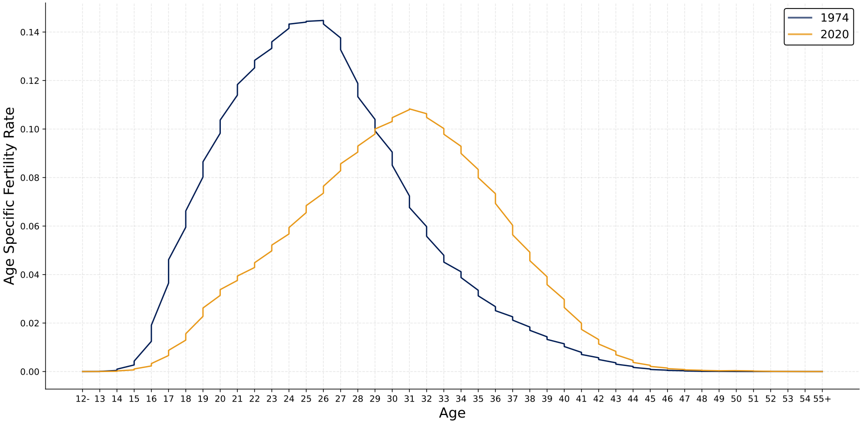
Period Fertility Measures (Cont.)

Class quiz: What's the difference between:

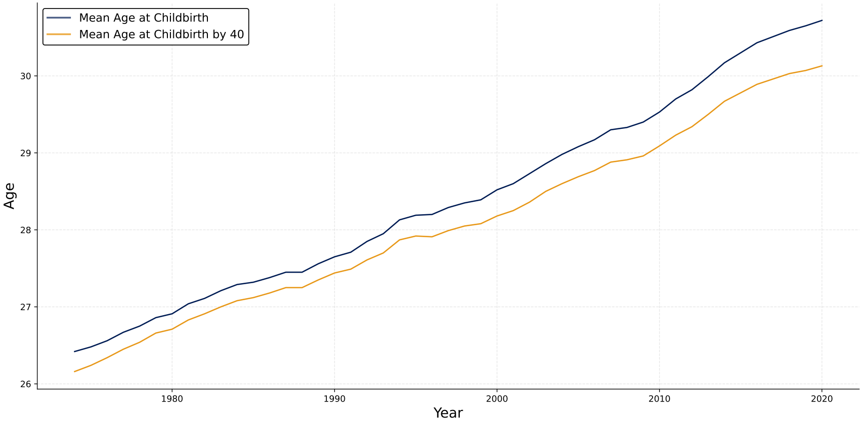
- The CBR?
- The GFR?
- The ASFR?
- The TFR?
- The MAC?

Is one of these more or less different than the others?

An Example of ASFRs (UK, 1974 and 2020)



An Example of MAC and MAC40 (UK)



Period Fertility Measures (Cont.)

In the absence of vital registration of birth events and exposures; two ways.

1. Direct: estimate using full birth histories from surveys (e.g DHS).
2. Indirect: draw on Censuses in the absence of vital registration/birth histories.
    - 'Own-children method': children present with mothers using household structure information.

Tempo

- Period measures of fertility suffer from **tempo distortions**.
- Tempo distortions arise when the MAC is changing.
- This distorts comparisons of the period TFR across years/populations.
- We therefore need to standardise for age patterns of childbearing.
  - If MAC is decreasing: period measures exaggerate fertility levels.
  - If MAC is increasing: period measures depress fertility levels.
- A Bongaarts-Feeney tempo adjusted TFR for birth order i is:

$$\mathrm{TFR_i'} = \mathrm{TFR_i}/(1 - \mathrm{r_i}) \tag{6}$$

r_i is the absolute change in tempo for birth order i (mean age, ith birth).

- If r_i >0 – fertility postponement – the $\mathrm{TFR_i'}$ corrects TFR by increasing it.
- If r_i <0 – fertility anticipation – the $\mathrm{TFR_i'}$ corrects TFR by decreasing it.

FIGURE 3 Observed and adjusted total fertility rate for the United States, 1950–90

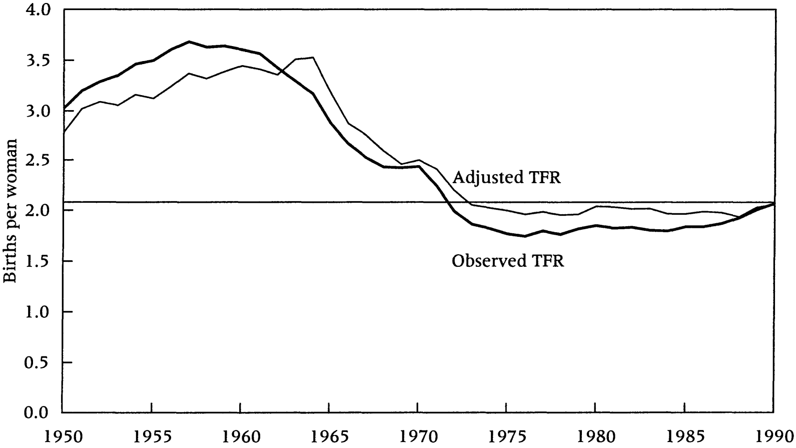


Figure 3 of Bongaarts and Feeney (1998): An excellent and highly recommended paper!

Parity and Birth Order

- Fertility is a cumulative process over the life course.
- First births precede second births which proceed third births, etc.
- The concept of **parity** – the number of live births a woman has had – can help us track this process.
- Parity-specific analyses provide important insights, such as:
    - Deviations from **natural fertility**: Is parity-specific control or fertility limitation based on parity widely practiced in the population?
    - In low fertility contexts: is the proportion of childlessness increasing? How are the numbers of people having at least one child decreasing?

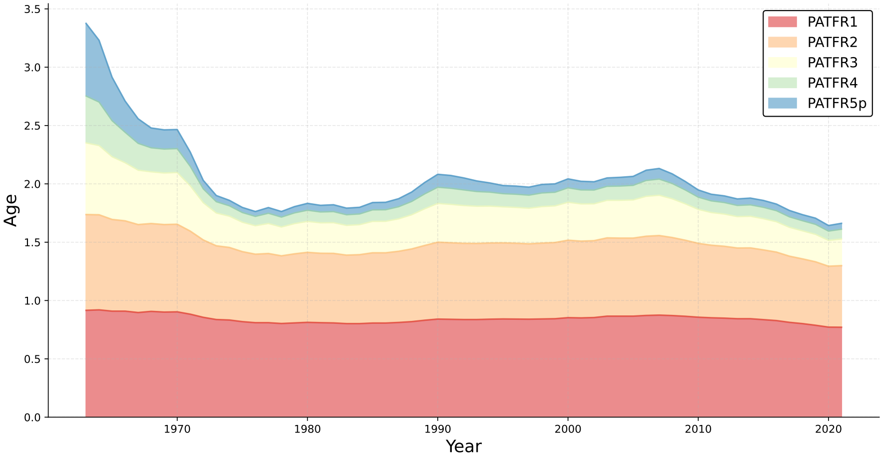
Order Specific Rates

- If we know the parity of births in vital registration data, we can estimate order-specific rates:

$$ {}_{\mathrm{n}}\mathrm{F_x}^{\mathrm{i}}[0,\mathrm{T}] = \frac{{}_{\mathrm{n}}\mathrm{B_x}^{\mathrm{i}}}{{}_{\mathrm{n}}\mathrm{W_x}} \tag{7} $$

$$ \mathrm{TFR} = \mathrm{n} \times \sum_{\mathrm{x}=\alpha}^{\beta-\mathrm{n}} {}_{\mathrm{n}}\mathrm{F_x}^{\mathrm{i}}[0,\mathrm{T}] \tag{8} $$

- We can similarly decompose the MAC by orders.



United States, Parity- and age-adjusted total fertility rate over time.

Parity Progression Ratios

- Parity-specfic analyses is carried out with Parity Progression Ratios (PPRs).
  - These were introduced by Henry (1953).
- PPRs capture the proportion of women progressing from one parity to next.
- Computed using census or survey data, most naturally applied to cohorts.

$$\text{PPR(i, i+1)} = \frac{\text{Number of women at parity i+1 or more}}{\text{Number of women at parity i or more}} = \frac{P_{i+1}}{P_i} \tag{9}$$

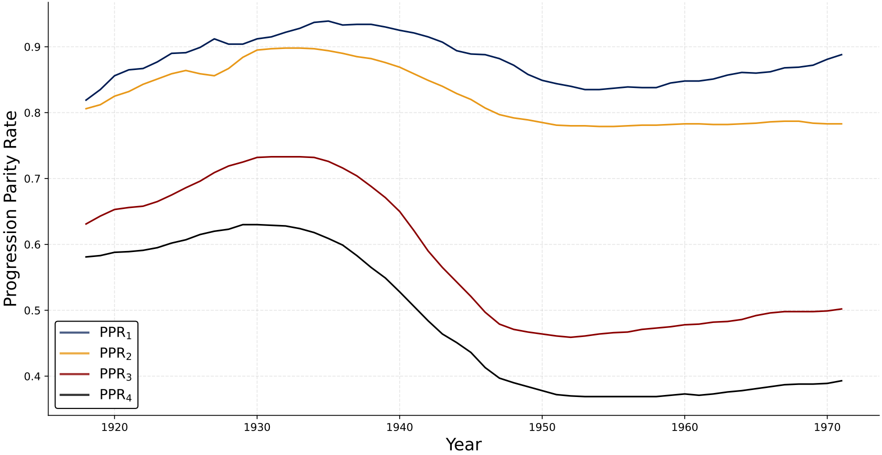
- We can also obtain the *cohort* TFR through the PPR:

$$\text{PPR(0, i)} = \frac{P_i}{P_0} \tag{10}$$

$$\text{TFR} = \sum_{i=1}^{I} \text{PPR (0, i)} \tag{11}$$

- This is usually calculated for cohorts who have completed child-bearing.

An Example of PPRs (USA, 1918-1971)



Parity-specific fertility control

- Increasingly concave PPRs indicate adoption of parity-specific control, i.e. increased stopping at fixed parities.
- Signals adoption of fertility control and deviation from 'natural fertility'.
  - The term 'natural fertility' was defined by French demographer Louis Henry.
  - Refers to the fertility level of a population in which no deliberate birth control practices are employed to limit or space births.
- Critique: fertility control can exist in many ways, i.e. proximate/social determinants, even without conscious family planning.
- This has typically led to:
  - Increased birth intervals (spacing).
  - Norms around marriage and sex.

Cohort Fertility

- In absence of mortality/migration, cohort TFR equals average total births to mothers in a given cohort.
- Cohort fertility describes the behaviour of a real cohort.
- Tempo distortion is not present in cohort fertility.
- However, cohort fertility is not necessarily as useful as period fertility.
  - Similar to mortality, cohorts are less relevant for monitoring current trends.
- Central conceptual debate in fertility analysis: period or cohort?

Reproduction and Intergenerational Renewal

- The ratio between female births/total births is usually taken around 0.490.
- This is seldom expressed as 104 male births per 100 female.
  - This is denoted as f_{ab} in Wachter.
- The **Gross Reproduction Rate (GRR)**: the size of the cohort of daughters relative to the cohort of mothers.
- Therefore, $\mathrm{GRR} = 0.49 \times \mathrm{TFR}$.
- When the SRB is distorted, the GRR provides insights.

Reproduction and Intergenerational Renewal (Cont.)

- The GRR can also be seen as the sum of age-specific fertility rates considering only daughters.

$$\mathrm{GRR}[0,\mathrm{T}] = \mathrm{n} \times \sum_{\mathrm{x}=\alpha}^{\beta-\mathrm{n}} {}_{\mathrm{n}}\mathrm{F}_{\mathrm{x}}[0,\mathrm{T}] \times \mathrm{f}_{\mathrm{ab}} \tag{12}$$

- Mortality might significantly affect reproduction and to what extent reproduction influences intergenerational replacement.

- To account for mortality, we introduce the **Net Reproduction Rate (NRR)**.

$$\mathrm{NRR}[0,\mathrm{T}] = \mathrm{n} \times \sum_{\mathrm{x}=\alpha}^{\beta-\mathrm{n}} {}_{\mathrm{n}}\mathrm{F}_{\mathrm{x}}[0,\mathrm{T}] \times \mathrm{f}_{\mathrm{ab}} \times \frac{{}_{\mathrm{n}}\mathrm{L}_{\mathrm{x}}^{\mathrm{f}}}{\mathrm{n} \times \mathrm{l}_0} \tag{13}$$

Reproduction and Intergenerational Renewal (Cont.)

- The NRR can be interpreted as the average number of daughters female members of a cohort would bear during their reproductive life, were they subject to the observed age-specific fertility and mortality.
  - If NRR > 1 the population grows under current mortality and fertility.
  - If NRR $= 1$ the population is constant under current mortality and fertility.
  - If NRR < 1 the population decreases under current mortality and fertility.
- Note: if GRR < 1, also NRR < 1 by definition.
- If mortality is high: GRR exceeds NRR.
- If mortality is high: replacement fertility is often much greater than 1 to be able to reproduce the next generation.

Stable Age Structures

- If a population's ASFR and ASMR remain constant – and the population is closed to migration – we obtain a **stable** population.
- In a stable population, the age structure is determined by its lifetable and long-term growth rate.
- The *proportions* in each age group in a stable population do not change.
  - However, the population size in those groups may change.
- **Stationary** populations are a special case of stable populations with no growth. In stationarity, population size is also constant (remember radix of the life table in which we assume a *constant* stream of births).

Stability

- Deviations in actual populations can be explained by particular events of migration and structural historic changes in rates.
- Stable populations help us see the long-run implications of today's rates for future age structures, or do retrospective analysis.
    - We'll talk a lot more about this next week!
- So far in this course, we've been in a world with unchanging vital rates.
- **Thought Exercise**: Imagine a country with a high TFR, which experiences overnight an immediate drop in fertility.
    - What is the effect on population?
- We would still see population growth due to the young age structure.
- This inherent potential for growth attributable to age structures is called **population momentum**.

"The phenomenon occurs because a history of high fertility has resulted in a high proportion of women in the reproductive ages, and these ensure high crude birth rates long after the age-specific rates have dropped"

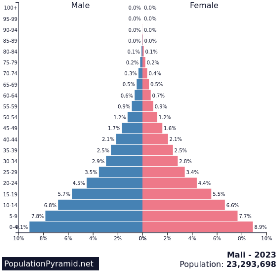
Keyfitz, N. (1971): 'On the Momentum of Population Growth'

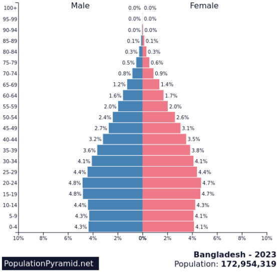
- Large proportions of young people grow even if each couple has only enough children to replace themselves.
- When transitioning, #young entering reproductive age still large.
- As these young people have children, overall number of births remains high.
- Momentum: the difference of present/future size of *stationary* population.
- This is why lowest-low fertility populations continue to experience growth.

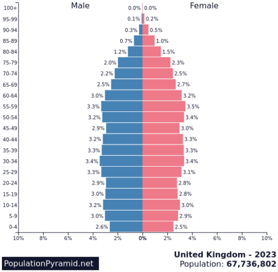
Fertility Transitions

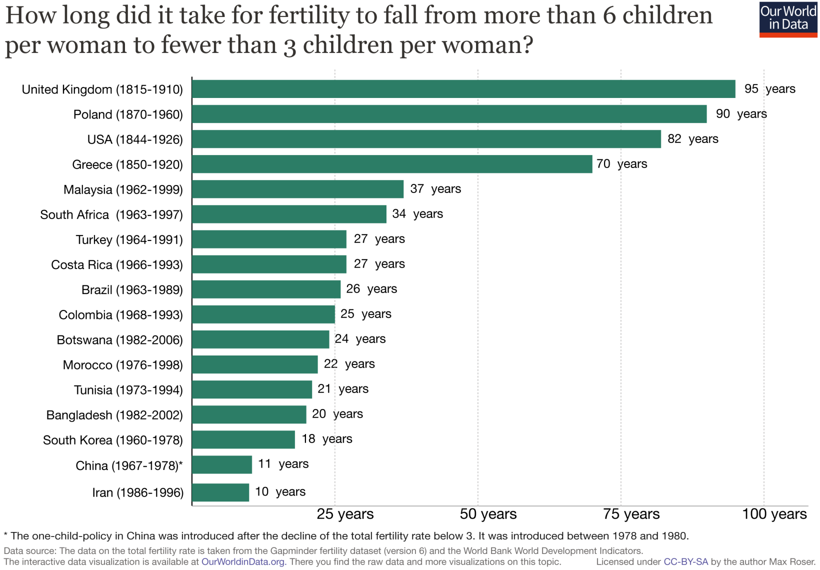
- In Africa, Asia, Latin America and the Caribbean, a decline of childbearing of women at older ages has led to $\downarrow$MAC.
- In Europe/Northern America, postponement of childbearing led to $\uparrow$MAC.
- **Transition** refers to long-term decline in the #children from $\geq$4 to ≤ 2.
- This marks a break from the past; from a pre-transition to transitional.
- But how is a transitional world different from a pre-transitional one?
  - Enter population pyramids, for the first time!

Transitions and Population Age Structures









- Earlier transitions took longer than more recent transitions.

Profiles of Fertility Transitions

- In terms of which countries transitioned when:
  - **Forerunners:** Europe and North America.
  - **Followers:** India, Sri Lanka, Korea, Taiwan, Chile, Maritius.
  - **Trailers:** parts of Africa, Latin America, Central and Eastern Asia.
  - **Latecomers:** Parts of Africa.
- Pre-decline levels of fertility generally lower for earlier declines.
- Gap between mortality and fertility decline is:
  - **Short** for forerunners (5-10 years)
  - **Medium** duration years for followers and trailers (30 years)
  - **Longest** for latecomers (45 years)
- Majority of world's population is has near-/below-replacement fertility, where fertility indicators show examples of global convergence.

Applied Fertility Analysis in Practice!

- Two large GWAS of fertility by our team (HRB1 and HRB2):

'Genome-wide analysis identifies 12 loci influencing human reproductive behavior'

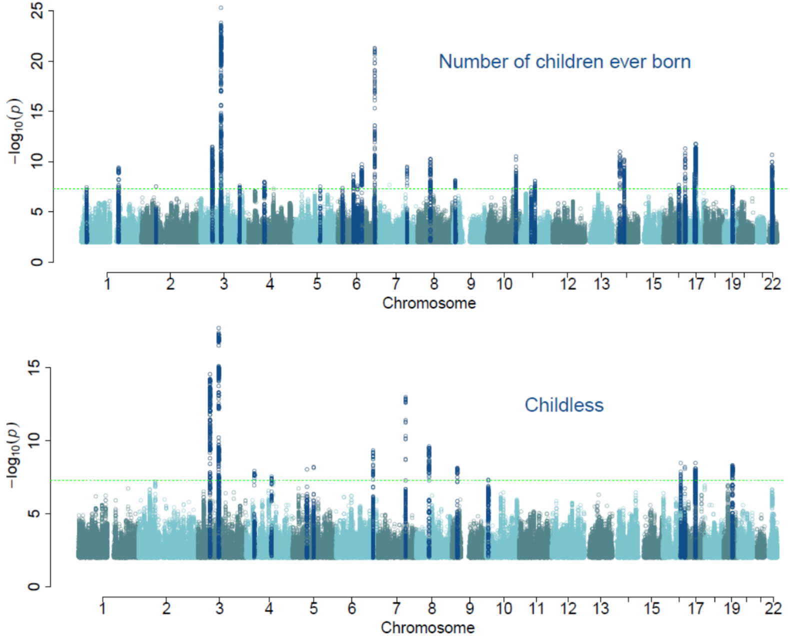
Barban et al. (2016, Nature Genomics)

and

'Genome-wide analysis identifies genetic effects on reproductive success and ongoing natural selection at the FADS locus'

Mathieson et al. (2023, Nature Human Behaviour)

- HRB1 – for NEB and AFB – finds 12 SNPs in 343,072 individuals.
- HRB2 – for NEB/Childnessness – finds 43 SNPs in 785,604. individuals.


Number of children ever born
$-\log_{10}(p)$
0
5
10
15
20
25
1
2
3
4
5
6
7
8
9
10
12
13
15
17
19
22
Chromosome
Childless
$-\log_{10}(p)$
0
5
10
15
1
2
3
4
5
6
7
8
9
10
11
12
13
15
17
19
22
Chromosome

Second Demographic Transition (SDT)

- Post–transition regime emphasising autonomy and individual choice.
- Value shifts: secularisation, self-realisation, gender equality.
- Later marriage and childbearing become widespread norms.
- Growth of cohabitation, nonmarital births, and union dissolution.
- Rising childlessness and diversification of family forms.

SDT and Fertility Patterns

- **Tempo:** postponement raises mean age at childbearing $\Rightarrow$ period TFR falls.

$$\mathrm{TFR}' = \frac{\mathrm{TFR}}{1-r}, \qquad r = \Delta\mathrm{MAC}$$

- **Quantum:** incomplete recuperation at older ages lowers completed fertility.
- **Heterogeneity:** larger educational and regional differences emerge.
- **Parity control:** stopping at low orders increases childlessness and one-child families.

$$\mathrm{CFR} = \sum_{i \geq 1} \mathrm{PPR}(0, i)$$

How do we account for Fertility in PyCCM?

Fertility Smoothing: Overview

- Smooths the **Total Fertility Rate (TFR)** $T(t)$ over time.
- Moves from a baseline level T_0 toward a target level $T^{\star}$.
- Adjustment occurs gradually over C projection years.
- Uses the last observed **Age-Specific Fertility Rates (ASFRs)** ${}_nF_x(t_0)$.
- Defines weights $w_x = {}_nF_x(t_0)/T_0$; these sum to 1 across ages.
- Future fertility at age x: ${}_nF_x(t) = w_x T(t)$ for all $t > t_0$.

Targets and Horizons

- Each unit d has a target TFR $T_d^\star$ (from `target_tfrs.csv`).
- If missing, a global default $T_{\mathrm{default}}^\star$ is applied.
- National target $T_{\mathsf{nat}}^\star =$ exposure-weighted mean of units.
- Exposure $E_d(t) =$ female person–years aged 15–49 in unit d.
- Weights $\omega_d(t) = E_d(t)/\sum_j E_j(t)$ reflect population size.
- Unit convergence horizon C_d defines years to reach $T_d^\star$;
  if absent, default horizon C is used.

Smoothing Schedules

- $S(t) =$ fraction of the fertility gap closed by year t.
- TFR evolves as $T(t) = (1 - S(t))T_0 + S(t)T^\star$.
- **Exponential schedule:**
  $S(t) = 1 - e^{-\kappa t}$, where $\kappa = -\ln(1 - s_{\text{target}})/C$.
- **Logistic schedule:**
  $S(t) = \{1 + \exp[-\sigma(t/C - m)]\}^{-1}$, midpoint m, slope σ.
- Typically $S(C) \approx 0.99$, i.e. 99% of the gap closed by year C.

ASFR Reconstruction and Checks

- Reconstructed fertility by age: ${}_nF_x(t) = w_x T(t)$.
- ${}_nF_x(t) =$ births to women aged x–$x{+}n$ divided by exposure.
- Invariant check: $\sum_x n_{xn} F_x(t) = T(t)$ (exact by construction).
- Validation tolerance: error $< 10^{-6}$ allowed in code.
- Negative births forced to zero; exposures $< \varepsilon$ ignored.
- Weights renormalised so $\sum_x n_x w_x = 1$ after cleaning.

Interpretation and Limits

- Age pattern (w_x) fixed; only total level $T(t)$ changes.
- Unit variation arises from $(T_d^\star, C_d)$ differences.
- National path changes with exposure-weighted aggregation.
- No age-shift: timing (tempo) effects not modelled here.
- Baseline year t_0 determines the enduring fertility shape.
- Smoothing thus controls level convergence, not age pattern evolution.

Family Dynamics and Fertility

- Union states $s \in \{\mathrm{U}, \mathrm{C}, \mathrm{M}\}$ shape exposure to childbearing.
- Exposure shares $p_x^{(s)}$ and state-specific fertility ${}_nF_x{}^{(s)}$:

$$
{}_nF_x = \sum_s p_x^{(s)}\, {}_nF_x{}^{(s)}
$$

- Longer singlehood reduces exposure at prime reproductive ages.
- Union instability lengthens birth intervals and lowers progression ratios.
- Supportive policies (childcare, leave, flexibility) can aid recuperation.

Conclusion (Fertility)

- We've learned a lot so far!!
  - Note the myriad ways that Fertility can be mentioned.
  - As with mortality, important differences between period/cohort!
  - See the suggested reading, too.
- We *haven't* broken births into marrital status/contraception type/abortion: see Preston et al. for this, i.e. the TMFR.
- We haven't talked about the more sociological/family effects of birth order.

Migration: The Third Component of a CCM

- Alongside births and deaths, migration drives population change.
- Alters size, age structure, and regional distribution.
- Central to both subnational and national projections.
- Captured explicitly in the population balance:

$$K_{t+n} = K_t + B - D + I - E.$$

Flows, Stocks, and Measurement

- **Flows:** movements during $[t, t+n)$.
- **Stocks:** migrants resident at time t.
- Internal $(d_1 \leftrightarrow d_2)$ and international flows.
- Net migration: $M_{x,t} = I_{x,t} - E_{x,t}$.
- Measured with varying duration and coverage.

Migration Rates

- Crude in–migration rate:

$$\mathrm{CIR} = \frac{I_{[0,T]}}{PY_{[0,T]}}.$$

- Crude out–migration rate:

$$\mathrm{CER} = \frac{E_{[0,T]}}{PY_{[0,T]}}.$$

- Net migration rate:

$$\mathrm{NMR} = \frac{I - E}{PY}.$$

- Age–specific rates remove age structure bias.

Age Patterns of Migration

- Strong hump at young adult ages.
- Peaks at education and job entry.
- Family formation and retirement produce secondary peaks.
- PyCCM holds these shapes when scaling migration flows.

PyCCM Inputs: Migration Data

- Inputs: population, immigration (`flujo_inmigracion`), emigration (`flujo_emigracion`).
- Keys: year, age group, sex, value, department.
- Ages harmonised (0–4,...,80+) and sex recoded.
- Values coerced to numeric and pivoted to wide form.
- Missing flows filled with zero; national totals aggregated.

PyCCM Computation: Net Flows and Rates

- Net migration by age and sex: $M_{x,t} = I_{x,t} - E_{x,t}$.
- Population exposure $P_{x,t}$ from census or projections.
- Net migration rate:
$$
{}_{\mathrm{n}}\mathrm{M}_{\mathrm{x}} = \frac{M_{x,t}}{P_{x,t}}.
$$
- Rates and counts stored by department, age, sex, year.
- Used in forward projection step for each unit.

Integrating Migration into Projection

- Base population $K_{x,t}$ adjusted with net migration.
- Continuous approximation:

$$K_{x,t+1} = K_{x,t} + B_{x,t} - D_{x,t} + M_{x,t}.$$

- PyCCM uses a **half–interval exposure method**.
- Adds half of migration before, half after deaths/births.
- Prevents over– or under–counting during the year.

Half–Year Convention and Projection Total

- Mid–year correction improves demographic balance.
- Exposure updated as:

$$P^*_{x,t} = P_{x,t} + \tfrac{1}{2} M_{x,t}.$$

- End–of–year population:

$$K_{x,t+1} = K_{x,t} + B_{x,t} - D_{x,t} + \tfrac{1}{2} M_{x,t}.$$

- Later add remaining $\frac{1}{2} M_{x,t}$ to close interval.
- Ensures total growth $=$ natural change $+$ full migration.
- Method aligns with fertility and mortality exposures.

Conclusion

Today we've learned about Fertility and Mortality!

We've also learnt about how they feed into the PyCCM library.

Next, we have a computer lab. We're going to learn about:

- How to calculate ASFRs.
- How to calculate TFRs from ASFRs.
- How to smooth TFRs towards a target.
- How to then decompose those future TFRs back into ASFRs.
- How to incorporate (half) of the migration data into the exposures.