Developing and validating the BIKER algorithm has provided a unique lense through which we can discuss riverine gas exchange theory. In that context, we next explore some considerations for future work on $k\_{600}$ upscaling in SWOT-observable rivers.

####4.2.1 Gas exchange under uniform and non-uniform flow conditions

Section 2.2 highlights that most upscaling studies to date have assumed uniform flow conditions (i.e. $S\_h \neq S\_0$) in order to train upscaling models using readily available slope data. However, the first-principles model previously used by @ulsethDistinctAirWater2019a and @raymondScalingGasTransfer2012a to define \*eD\* [@tsivoglouTracerMeasurementReaeration1976] does not make this simplifying assumption. Therefore, it is an open research question whether parameterizing $k\_{600}$ upscaling models via $S\_h$ can account for some of the unexplained residual variation in current upscaling models [@hallGasExchangeStreams2020]. Conveniently, SWOT will explicitly measure $S\_h$ at unprecendeted spatial and temporal resolutions and will be spatially joined to hydrography that provides $S\_0$. Thus, future researchers can use BIKER and SWOT in conjunction to directly answer this question.

####4.2.2 Bed roughness and gas exchange in SWOT rivers

Channel bed roughness affects riverine gas exchange mostly in high energy streams where slope is sufficiently steep for bubble-induced gas exchange to occur [@hallGasExchangeStreams2020]. @ulsethDistinctAirWater2019a showed bed roughness loosely scales with $k\_{600}$ in steep Alpine streams. However, they coarsely estimated bed roughness from arial imagery and to date most similar work has focused on labratory exercises [e.g. @chansonStreamReaerationNonuniform; @moogStreamReaerationNonuniform]. We argue that bed roughness is not controlling gas exchange in SWOT-observable rivers because they are so large and therefore flat. Instead, gas exchange in SWOT-observable rivers is presumably dominated by water-column, rather than bed friction, turbulence. We show in Figure S2 with the @ulsethDistinctAirWater2019a data that the 'effective bed roughness height' scales with $k\_{600}$ only in extremely steep streams (see Text S2 for the calculation of this bed roughness term). This relationship fundamentally breaks down in less steep rivers, and such steep slopes are functionally impossible in SWOT-observable rivers that are over 50m wide. This promising initial result indicates bed roughness controls some aspects of gas exchange and should be explicitly explored in future work, but is less relevant for BIKER's application to SWOT data. This is particularly important because small, steep rivers dominate global river networks and their GHG evasion [@horgbyUnexpectedLargeEvasion2019] due to the fractal nature of river systems [@tarbotonFractalNatureRiver1988].

####4.2.3 Wind-driven gas exchange in SWOT rivers

In wide rivers like those that SWOT will observe, wind begins to exert a non-trivial influence on gas exchange. It is well established that in lakes and the ocean, wind controls near-surface turbulence and thus gas exchange [@beaulieuControlsGasTransfer2012a; @readLakesizeDependencyWind2012]. Authors have argued that large rivers are a hybrid of the hydraulics-driven turbulence in small rivers and the wind-driven turbulence in lakes [@beaulieuControlsGasTransfer2012a]. As SWOT will measure only rivers wider than 50m, it follows that wind is likely exerting some influence on gas exchange in SWOT-observable reaches. Here, we opted to ignore wind effects in our upscaling model and in BIKER to favor global scalability and implementability for two reasons: 1) current upscaling efforts do not account for wind in their estimation of $k\_{600}$ either and 2) it is infeasible to parameterize every set of SWOT measurements with local wind data. Further, relying on an in situ understanding of wind defeats the purpose of BIKER for ungauged settings. Future work should explore the feasibility of assigning a Bayesian prior on wind speed for BIKER.

###3.1 Choosing a predictive model for \*k\*

To predict \*k\* from just the SWOT observables, a physical model for \*k\* is first selected. As mentioned in section 1, there are dozens of predictive models for \*k\* that have been devolped since the 1950s [@hallGasExchangeStreams2020]. Recently, many of these models were re-explored in rivers and streams by @wangPhysicallyBasedScaling2021, who significantly expanded the existing training datasets of field-measured $k\_{o\_2}$ by using the \*streamMetabolizer\* model to calibrate a form of equation 2 to high-fidelity in situ DO datasets [@applingOvercomingEquifinalityLeveraging2018] at 35 rivers across the United States. They showed that two $k\_{o\_2}$ models deduced from classic, process-based theories for mass transport yield approximately identical parameters when fit to ethier field measurements or this new dataset of simulated $k\_{o\_2}$. They also showed through cross-validation that these models are more robust to overfitting on specific sets of data than many of the other models tested. While these two models still exhibit large errors, they provide the best fit of those tested and potentially suggest that there are uniform scaling realtionships between certain hydraulic properties and $k\_{o\_2}$. @wangPhysicallyBasedScaling2021's better fit, and more parsimonious, of the two models is reprinted as equation 3 (with that paper's reported coefficients of determination $r^2$). $\alpha$ is the fitted parameter in the linear regression. Equation 3-2nd line was fit to 588 field measurements of $k\_{o\_2}$ while equation 3-third line was fit to 3,919 simulated $k\_{o\_2}$ values at 35 rivers.

$$\mathbf{(3)} \begin{eqnarray} k\_{o\_2} &=& \alpha U\_\star \\ &=& 47.67U\_\star (r^2:0.53, measured) \\ &=& 49.71U\_\star (r^2: 0.76, simulated) \end{eqnarray}$$

\*\*I'm struggling with this paragraph, and also don't know how to write it diplomatically\*\*

We implement this model within BIKER (using 48 as a reasonable value for $\alpha$ given both forms of equation 3) for a few reasons. First, @wangPhysicallyBasedScaling2021 suggest that an $\alpha$ of approximately 48 is uniform across rivers and streams, regardless of the training data used. This is encouraging for use within BIKER as BIKER is specifically designed to be as river-agnostic as possible and so we sought as generalized an equation as possible. Second, it yields a simple linear relationship across all rivers, regardless of their size or steepeness. This is not true of other \*k\* models, for example those based on \*eD\* (section 1). The relationship between \*eD\* and \*k\* is not necessairly linear, with reasonable performance from both linear and power-law models and generally poorer predictive performance for small \*k\* values [@raymondScalingGasTransfer2012a]. Further, recent work has shown that \*k\* does not scale with \*eD\* via the same statistical parameters across all river geomorphologies [@ulsethDistinctAirWater2019a]. We further explored the relationship presented by [@ulsethDistinctAirWater2019a] in the SWOT context, finding that the $k \sim eD$ relationship fundamentally breaks down in rivers with low \*eD\*, which corresponds to nearly all rivers that SWOT will observe (Figure S2). Therefore, an \*eD\* based predictive model is less useful in SWOT rivers. Third, we sought to explicitly predict $k\_{o\_2}$, as its use extends beyond constraining gas fluxes from rivers and into parameterizing river metabolism models (equation 2). We stress that most other predictive models for \*k\* could be implemented within BIKER. However, our goal of using BIKER on any river that is observable by SWOT necessitates that we pick an equation that is applicable across as many rivers as possible. Future work should look at using a river-specific \*k\* equation to pseudo-calibrate BIKER to a specific river, as well as the influence of wind-induced gas exchange (see section 5.2).