Drift Paper - Checking Assumptions - Sec 7

Craig L. Champlin, Using odometry drift to match ILI joint boundaries for run comparisons, International Journal of Pressure Vessels and Piping, Volume 213, 2025, 105351, ISSN 0308-0161, https://doi.org/10.1016/j.ijpvp.2024.105351. (https://www.sciencedirect.com/science/article/pii/S0308016124002291)

This section of the paper verifies the key assumptions from the derivation of Section 5

Assumptions:

1. Joint boundary spacing is regulaar
2. The drift is smooth (not in this file)
3. The drift is first order stationary with a trend

A note on stationarity

If a series is trend stationary...

1. the mean is a trend
2. variance is constant
3. there is no autocorrelation
4. the sequence contains information (is not random)

Resources:

- https://towardsdatascience.com/detecting-stationarity-in-time-series-data-d29e0a21e638
- https://arch.readthedocs.io/en/latest/

Setup and Import Packages

In [1]:

```
%pylab inline

import warnings
warnings.simplefilter("ignore")

import matplotlib.pyplot as plt
plt.rc("savefig", dpi=90)
plt.rc("font", family="sans-serif")
plt.rc("font", size=14)

import seaborn as sns
sns.set_style("darkgrid")

import numpy as np
import pandas as pd
```

```
Populating the interactive namespace from numpy and matplotlib
```

Import Data

In [2]:

```
# Import data

data = ('./Drift Paper Section 6.3 and 7 Data.csv')

colNames = ['odoA','event','drift','diff1']

eventTypes = pd.CategoricalDtype(categories=['agm',
                                            'bend',
                                            'bweld',
                                            'casing',
                                            'weld'], ordered=False)

dataTypes = {'odoA':np.float_,
             'event':eventTypes,
             'drift':np.float_,
             'diff1':np.float_}

df = pd.read_csv(data, skiprows=1, header=0, names=colNames)

df.name = 'Synthetic Data'
df.astype(dataTypes)
```

Out[2]:

	odoA	event	drift	diff1
0	279.53	weld	-61.97	0.08
1	304.78	casing	-61.14	0.83
2	324.62	bweld	-61.91	-0.77
3	331.07	weld	-61.93	-0.02
4	373.51	weld	-61.99	-0.06
...	...	...	...	...
194	8060.69	weld	-45.80	0.11
195	8073.38	casing	-45.80	0.01
196	8105.86	bweld	-45.57	0.23
197	8141.18	agm	-46.12	-0.54
198	8142.21	casing	-45.46	0.66

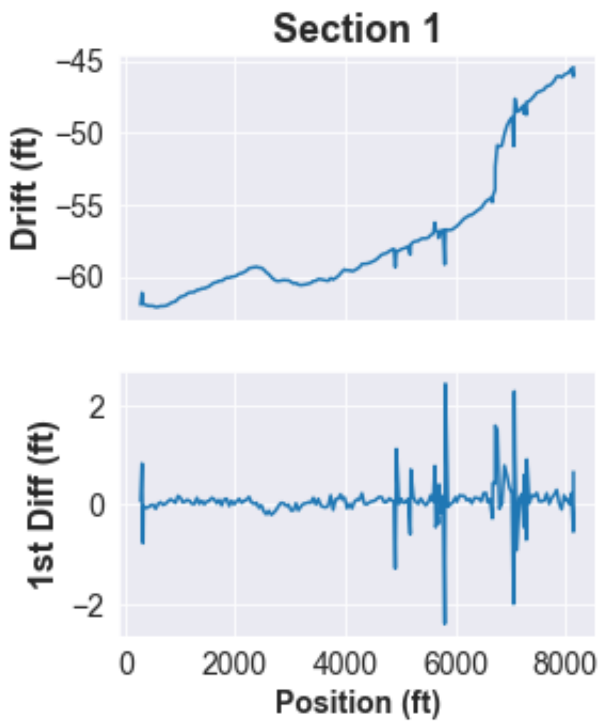
199 rows × 4 columns

Confirmation plot

In [3]:

```
plt.rc("figure", figsize=(4, 5))
fig, axs = plt.subplots(2,1, sharex=True, sharey=False)

axs[0].plot(df.odoA, df.drift)
axs[1].plot(df.odoA, df.diff1)

axs[0].set_title('Section 1', fontdict={'fontsize':18, 'fontweight':'bold'})
axs[1].set_xlabel("Position (ft)", fontdict={'fontsize':14, 'fontweight':'bol
axs[0].set_ylabel('Drift (ft)', fontdict={'fontsize':16, 'fontweight':'bold'}
axs[1].set_ylabel('1st Diff (ft)', fontdict={'fontsize':16, 'fontweight':'bol
```

Out[3]: Text(0, 0.5, '1st Diff (ft)')



Joint Length Histograms

In [4]:

```
# get a set of joint lengths by subtracting adjacent wheel counts

dfLen=df[['odoA']].copy()
dfLen['wc_shift']=df[['odoA']].copy().shift(periods=1, fill_value=324.62)
dfLen['Joint_Length']=dfLen['odoA']-dfLen['wc_shift']
display(dfLen)
```

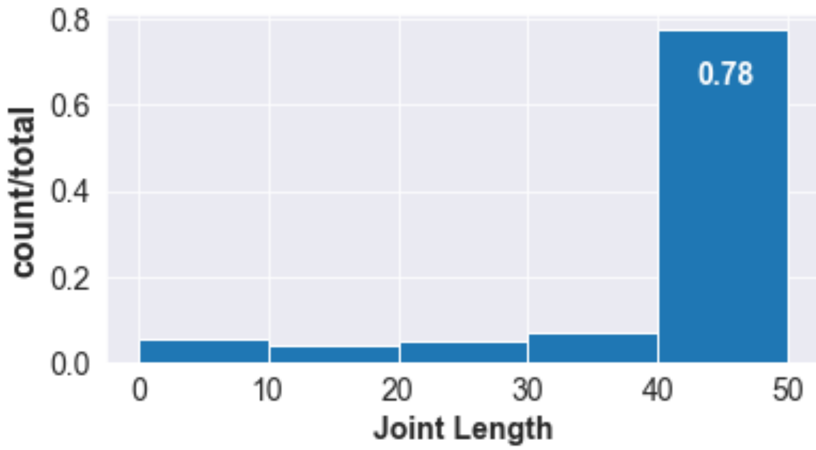
	odoA	wc_shift	Joint_Length
0	279.53	324.62	-45.09
1	304.78	279.53	25.25
2	324.62	304.78	19.84
3	331.07	324.62	6.45
4	373.51	331.07	42.44
...	...	...	...
194	8060.69	8018.78	41.91
195	8073.38	8060.69	12.69
196	8105.86	8073.38	32.48
197	8141.18	8105.86	35.32
198	8142.21	8141.18	1.03

199 rows × 3 columns

In [5]:

```
import statistics

numbins = 5
edge = 0.04

plt.rc("figure", figsize=(6, 3))
fig, axs = plt.subplots(1,1)

# axs.set_title('Joint Lengths', fontdict={'fontsize':18, 'fontweight':'bold'
axs.set_xlabel("Joint Length", fontdict={'fontsize':14, 'fontweight':'bold'})
axs.set_ylabel("count/total", fontdict={'fontsize':16, 'fontweight':'bold'})

axs.hist(dfLen['Joint_Length'],
                     bins=numbins,
                     range=(0,50),
                     weights=np.ones_like(dfLen['Joint_Length']) / len(dfLen[
                     density=False)

sz=0.78
axs.text(43, 0.65, '{:.2f}'.format(sz), fontdict={'color':'w', 'fontweight':'
```

Out[5]: Text(43, 0.65, '0.78')



Population Statistics

Mean and variance tell us a lot about the stationarity. Need a zero mean and constant variance. Also, no trend.

This is often done by splitting the sample into chunks and comparing them.

In [6]:

```
segs = [0, 0.25, 0.5, 0.75, 1]

print('Dataset : ', df.name, '\t Curve : ', 'diff1')

for i in range(1,len(segs)):

    x = df.loc[round(len(df.odoA)*segs[i-1]):round(len(df.odoA)*segs[i]-1),'o
    chunk = df.loc[round(len(df.odoA)*segs[i-1]):round(len(df.odoA)*segs[i]-1

    slope, intercept = np.polyfit(x,chunk,1)
    m = chunk.mean()
    s = chunk.std()

    print(i, ': mean = ', round(m,3), '\t std = ', round(s,1), '\t slope = ',
```

```
Dataset :  Synthetic Data         Curve :  diff1
1 : mean =  0.054        std =  0.2      slope =  0.0    intercept =  0.0
2 : mean =  0.011        std =  0.1      slope =  0.0    intercept =  -0.3
3 : mean =  0.068        std =  0.6      slope =  0.0    intercept =  -0.1
4 : mean =  0.198        std =  0.6      slope =  -0.0   intercept =  1.3
```

Autocorrelation

Documentation :
[https://www.statsmodels.org/stable/generated/statsmodels.tsa.stattools.acf.html#statsmodels.t

Parameters:

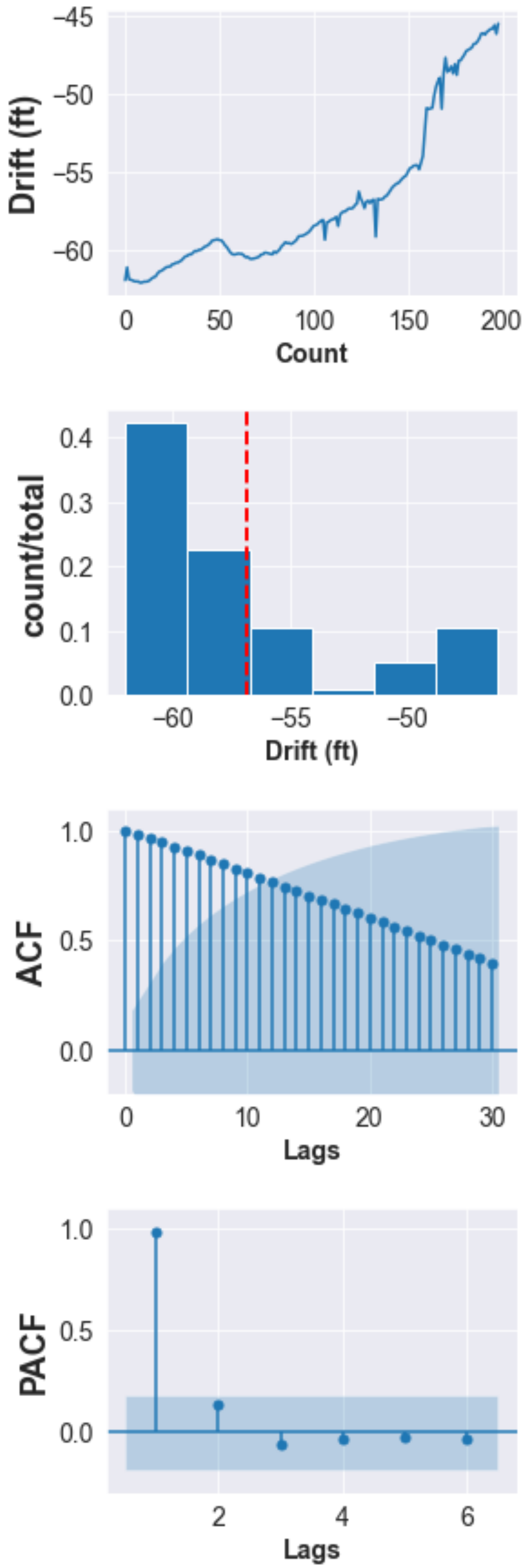
```
* x : the 1D time series data (no gaps)
* nlags : returned value includes lag 0, returns n+1 lags
* alpha : (scalar) returns confidence intervals, alpha = 0.05 =>
95% conf int
```

In [8]:

```python
import statistics
from statsmodels.graphics.tsaplots import plot_acf
from statsmodels.graphics.tsaplots import plot_pacf
import matplotlib.gridspec as gridspec

col = 'drift'

size = [(6, 12), (4, 12)]
plt.rc("figure", figsize=size[1])

mu = df[[col]].mean()[0]
sigma = df[[col]].std()[0]

fig, axs = plt.subplots(4,1,
                        sharex=False, sharey=False,
                        layout='constrained',
                        gridspec_kw={'wspace': 0.1, 'hspace': 0.11})

i=0
# axs[0].set_title('Values')
axs[i].set_ylabel("Drift (ft)", fontdict={'fontsize':18, 'fontweight':'bold'}
axs[i].set_xlabel("Count", fontdict={'fontsize':14, 'fontweight':'bold'})
axs[i].plot(df[[col]])

i=1
axs[i].set_ylabel("count/total", fontdict={'fontsize':18, 'fontweight':'bold'
axs[i].set_xlabel("Drift (ft)", fontdict={'fontsize':14, 'fontweight':'bold'}
numbins = 6
edge = 0.04

axs[i].hist(df[[col]],
             bins=numbins,
             range=(df[[col]].quantile(edge)[0],df[[col]].quantile(1-edge)[0]
             weights=np.ones_like(df[[col]]) / len(df[[col]]),
             density=False)

axs[i].axvline(mu, color='r', linestyle='dashed', linewidth=2)
min_ylim, max_ylim = axs[i].get_ylim()


i=2
axs[i].set_ylabel("ACF", fontdict={'fontsize':18, 'fontweight':'bold'})
axs[i].set_xlabel("Lags", fontdict={'fontsize':14, 'fontweight':'bold'})
plot_acf(df[[col]], lags=30, alpha=0.01, ax=axs[i], zero=True, title=None)
axs[i].set_ylim([-0.2, 1.1])
# axs[i].set_xlim([-3, 40])

i=3
axs[i].set_ylabel("PACF", fontdict={'fontsize':18, 'fontweight':'bold'})
axs[i].set_xlabel("Lags", fontdict={'fontsize':14, 'fontweight':'bold'})
plot_pacf(df[[col]], lags=6, alpha=0.01, ax=axs[i], zero=False, title=None)
axs[i].set_ylim([-0.3,1.1])
```

Out[8]:

```
(-0.3, 1.1)
```



Drift (ft)
Count
count/total
Drift (ft)
ACF
Lags
PACF
Lags

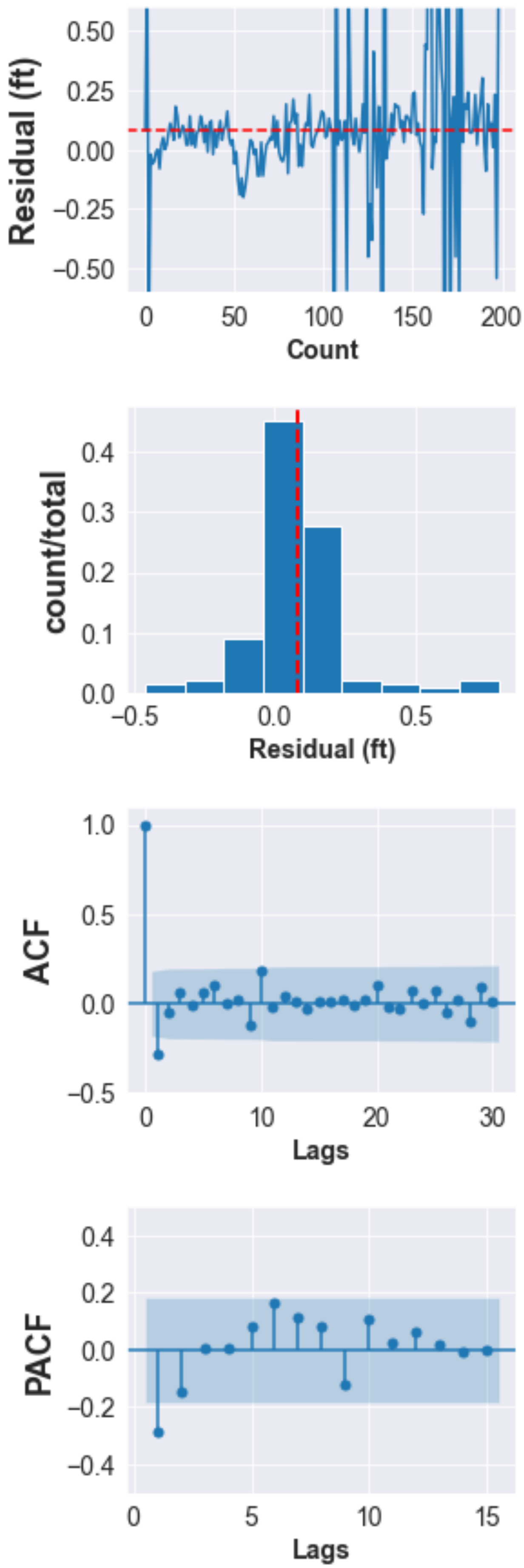
Partial Autocorrelation

In [10]:

```
col = 'diff1'

size = [(6, 12), (4, 12)]
plt.rc("figure", figsize=size[1])

# plt.rcParams["figure.constrained_layout.hspace"]

mu = df[[col]].mean()[0]
sigma = df[[col]].std()[0]

fig, axs = plt.subplots(4,1,
                        sharex=False, sharey=False,
                        layout='constrained',
                        gridspec_kw={'wspace': 0.1, 'hspace': 0.11})

i=0
# axs[0].set_title('Values')
axs[i].set_ylabel("Residual (ft)", fontdict={'fontsize':18, 'fontweight':'bol
axs[i].set_xlabel("Count", fontdict={'fontsize':14, 'fontweight':'bold'})
axs[i].plot(df[[col]])
axs[i].set_ylim([-0.6,0.6])

axs[i].axhline(mu, color='r', linestyle='dashed', linewidth=1.5)
# axs[i].text(0.2, mu + mu*0.5 - 0.27, r'$\mu: {:.2f}$'.format(mu))


i=1
# axs[1].set_title('Distribution')
axs[i].set_ylabel("count/total", fontdict={'fontsize':18, 'fontweight':'bold'
axs[i].set_xlabel("Residual (ft)", fontdict={'fontsize':14, 'fontweight':'bol
numbins = 9
edge = 0.04

axs[i].hist(df[[col]],
            bins=numbins,
            range=(df[[col]].quantile(edge)[0],df[[col]].quantile(1-edge)[0]
            weights=np.ones_like(df[[col]]) / len(df[[col]]),
            density=False)

axs[i].axvline(mu, color='r', linestyle='dashed', linewidth=2)
min_ylim, max_ylim = axs[i].get_ylim()
# axs[i].text(mu - mu*3.5, max_ylim*0.8, r'$\mu={:.2f}$'.format(mu))


i=2
axs[i].set_ylabel("ACF", fontdict={'fontsize':18, 'fontweight':'bold'})
axs[i].set_xlabel("Lags", fontdict={'fontsize':14, 'fontweight':'bold'})
plot_acf(df[[col]], lags=30, alpha=0.01, ax=axs[i], zero=True, title=None)
axs[i].set_ylim([-0.5, 1.1])
# axs[i].set_xlim([-3, 40])

i=3
axs[i].set_ylabel("PACF", fontdict={'fontsize':18, 'fontweight':'bold'})
```

```python
    axs[i].set_xlabel("Lags", fontdict={'fontsize':14, 'fontweight':'bold'})
    plot_pacf(df[[col]], lags=15, alpha=0.01, ax=axs[i], zero=False, title=None)
    axs[i].set_ylim([-0.5,0.5])
```

Out[10]: (-0.5, 0.5)



Residual (ft)
0.50
0.25
0.00
−0.25
−0.50
0
50
100
150
200
Count
count/total
0.4
0.3
0.2
0.1
0.0
−0.5
0.0
0.5
Residual (ft)
ACF
1.0
0.5
0.0
−0.5
0
10
20
30
Lags
PACF
0.4
0.2
0.0
−0.2
−0.4
0
5
10
15
Lags

Augmented DF w/ ARCH Toolbox

References:

- https://bashtage.github.io/arch/
- https://bashtage.github.io/arch/unitroot/generated/arch.unitroot.ADF.html#arch.unitroot.ADF

Hypotheses:

- H_0: The series is not stationary, it is a random walk
- H_a: The series is stationary, with no offset and n trend

One-sided test. Reject H_0 if $z \le z_{\alpha}$

Select Parameters:

- x: 1D data series
- trend : {"n", "c", "ct", "ctt"}, optional
  - n - no trend components
  - c - include a constanr (default)
  - ct - constant and linear time trend
  - ctt - constant and quadradic time trend

In [11]:

```
from arch.unitroot import ADF
col = 'diff1'

adf = ADF(df[[col]], lags = None, trend='c')
print(adf.summary().as_text())
```

```
   Augmented Dickey-Fuller Results
=====================================
Test Statistic                -12.966
P-value                         0.000
Lags                                1
-------------------------------------

Trend: Constant
Critical Values: -3.46 (1%), -2.88 (5%), -2.57 (10%)
Null Hypothesis: The process contains a unit root.
Alternative Hypothesis: The process is weakly stationary.
```

KPSS Test

The Kwiatkowski, Phillips, Schmidt and Shin (KPSS) stationarity test.

Reference:
https://bashtage.github.io/arch/unitroot/generated/arch.unitroot.KPSS.html#arch.unitroot.KPSS

Like ADF, the KPSS test is also a unit root test. This one is based around a linear regression, thus the test assumes the series is stationary. The null hypothesis is reversed from ADF. Null is that the sequence is stationary. The alternate is that it is stationary.

Hypotheses:

- H_0 : The series is stationary
- H_a : The series is non-stationary

One-sided test. Reject H_0 if $z \ge z_{\alpha}$

Parameters:

- x : 1D data series
- trend : {"c", "ct"}, optional
  - c - include a constant
  - ct - include a constant and a linear trend

In [14]:

```
# ALL events, full dataset

from arch.unitroot import KPSS
col = 'diff1'

display(df[[col]])

kpss = KPSS(df[[col]], lags = None, trend='c')
print(kpss.summary().as_text())
```

	diff1
0	0.08
1	0.83
2	-0.77
3	-0.02
4	-0.06
...	...
194	0.11
195	0.01
196	0.23
197	-0.54
198	0.66

199 rows × 1 columns

```
    KPSS Stationarity Test Results
=====================================
Test Statistic                  0.663
P-value                         0.016
Lags                                6
-------------------------------------

Trend: Constant
Critical Values: 0.74 (1%), 0.46 (5%), 0.35 (10%)
Null Hypothesis: The process is weakly stationary.
Alternative Hypothesis: The process contains a unit root.
```

In [53]:

```
# weld events only, full dataset

from arch.unitroot import KPSS
col = 'diff1'

display(df[[col]].loc[df['event'].isin(['weld'])])

kpss = KPSS(df[[col]].loc[df['event'].isin(['weld'])], lags = None, trend='c'
print(kpss.summary().as_text())
```

	diff1
0	0.08
3	-0.02
4	-0.06
5	-0.05
6	-0.01
...	...
190	0.03
191	-0.09
192	0.19
193	0.08
194	0.11

171 rows × 1 columns

```
    KPSS Stationarity Test Results
=====================================
Test Statistic                  0.907
P-value                         0.004
Lags                                5
-------------------------------------

Trend: Constant
Critical Values: 0.74 (1%), 0.46 (5%), 0.35 (10%)
Null Hypothesis: The process is weakly stationary.
Alternative Hypothesis: The process contains a unit root.
```

Zivot-Andrews Test

Allows one structural break in the series.

Reference:

https://bashtage.github.io/arch/unitroot/generated/arch.unitroot.ZivotAndrews.html#arch.unitroc

Hypotheses

- H_0: The series is not stationary, single structural break
- H_a: The series is stationary, with no offset and n trend

One-sided test. Reject H_0 if $z \le z_{\alpha}$

In [15]:

```python
from arch.unitroot import ZivotAndrews

col = 'diff1'

za = ZivotAndrews(df[[col]], max_lags = None, trend='c')
print(za.summary().as_text())
```

```
        Zivot-Andrews Results
=====================================
Test Statistic                -14.146
P-value                         0.000
Lags                                1
-------------------------------------

Trend: Constant
Critical Values: -5.28 (1%), -4.81 (5%), -4.57 (10%)
Null Hypothesis: The process contains a unit root with a single structural bre
ak.
Alternative Hypothesis: The process is trend and break stationary.
```