

1 Problem 1

1.1 (i)

If we let $SSE = \frac{1}{N} \sum_{t=1}^N (r^t - (w_1 x^t + w_0))^2$, then we want to minimize the SSE by setting the derivative with respect to w_0 and w_1 to zero:

$$\frac{\partial SSE}{\partial w_0} = \frac{1}{N} \sum_{t=1}^N 2(r^t - w_1 x^t - w_0)(-1) = 0 \quad (1)$$

We can get rid of the $\frac{2}{N}$ by multiplying each side by $\frac{N}{2}$ and carry the summations through to arrive at:

$$\sum_{t=1}^N w_0 + \sum_{t=1}^N w_1 x^t = \sum_{t=1}^N r^t \quad (2)$$

$$Nw_0 + w_1 \sum_{t=1}^N x^t = \sum_{t=1}^N r^t \quad (3)$$

and similarly for w_1 :

$$\frac{\partial SSE}{\partial w_1} = \frac{1}{N} \sum_{t=1}^N 2(r^t - w_1 x^t - w_0)(-x^t) = 0 \quad (4)$$

we can get rid of $\frac{2}{N}$ by multiplying each side by $\frac{N}{2}$ and carry the summations through to arrive at:

$$w_0 \sum_{t=1}^N x^t + w_1 \sum_{t=1}^N (x^t)^2 = \sum_{t=1}^N x^t r^t \quad (5)$$

Now divide both sides of (3) by N:

$$w_0 + w_1 \bar{x}^t = \bar{r}^t \quad (6)$$

We'll use (3) and (5) to solve a system of equations. Multiply (3) by $\sum_{t=1}^N x^t$ and multiply (5) by N

$$Nw_0 \sum_{t=1}^N x^t + w_1 \left(\sum_{t=1}^N x^t \right)^2 = \sum_{t=1}^N x^t \sum_{t=1}^N r^t \quad (7)$$

$$Nw_0 \sum_{t=1}^N x^t + Nw_1 \sum_{t=1}^N (x^t)^2 = N \sum_{t=1}^N x^t r^t \quad (8)$$

subtract (7) and (8)

$$Nw_1 \sum_{t=1}^N (x^t)^2 - w_1 \left(\sum_{t=1}^N x^t \right)^2 = N \sum_{t=1}^N x^t r^t - \sum_{t=1}^N x^t \sum_{t=1}^N r^t \quad (9)$$

Factor out w_1 from the L.H.S of (9) and divide and we have:

$$w_1 = \frac{N \sum_{t=1}^N x^t r^t - \sum_{t=1}^N x^t \sum_{t=1}^N r^t}{N \sum_{t=1}^N (x^t)^2 - \left(\sum_{t=1}^N x^t \right)^2} \quad (10)$$

And rearranging (6) to solve for w_0 :

$$w_0 = \bar{r}^t - w_1 \bar{x}^t \quad (11)$$

The solutions above for w_0 and w_1 are the optimal values.

1.2 (ii)

This time we let $SSE = \frac{1}{N} \sum_{t=1}^N (r^t - (v_2(x^t)^2 + v_1x^t + v_0))^2$ and we want to minimize the SSE by setting the derivative with respect to v_0 and v_1 and v_2 to zero:

$$\frac{\partial SSE}{\partial v_0} = \frac{1}{N} \sum_{t=1}^N 2(r^t - v_2(x^t)^2 - v_1x^t - v_0)(-1) = 0 \quad (12)$$

$$\frac{\partial SSE}{\partial v_1} = \frac{1}{N} \sum_{t=1}^N 2(r^t - v_2(x^t)^2 - v_1x^t - v_0)(-x) = 0 \quad (13)$$

$$\frac{\partial SSE}{\partial v_2} = \frac{1}{N} \sum_{t=1}^N 2(r^t - v_2(x^t)^2 - v_1x^t - v_0)(-x^2) = 0 \quad (14)$$

We apply the same techniques as for the previous problem, pull the 2 out, multiple by $\frac{N}{2}$ and distribute the summations:

$$Nv_0 + v_1 \sum_{t=1}^N x^t + v_2 \sum_{t=1}^N (x^t)^2 = \sum_{t=1}^N r^t \quad (15)$$

$$v_0 \sum_{t=1}^N x^t + v_1 \sum_{t=1}^N (x^t)^2 + v_2 \sum_{t=1}^N (x^t)^3 = \sum_{t=1}^N x^t r^t \quad (16)$$

$$v_0 \sum_{t=1}^N (x^t)^2 + v_1 \sum_{t=1}^N (x^t)^3 + v_2 \sum_{t=1}^N (x^t)^4 = \sum_{t=1}^N (x^t)^2 r^t \quad (17)$$

We get a linear system of three equations and three unknowns. We build a matrix from the linear system:

$$\begin{bmatrix} N & \sum_{t=1}^N x^t & \sum_{t=1}^N (x^t)^2 \\ \sum_{t=1}^N x^t & \sum_{t=1}^N (x^t)^2 & \sum_{t=1}^N (x^t)^3 \\ \sum_{t=1}^N (x^t)^2 & \sum_{t=1}^N (x^t)^3 & \sum_{t=1}^N (x^t)^4 \end{bmatrix} \begin{bmatrix} v_0 \\ v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \sum_{t=1}^N r^t \\ \sum_{t=1}^N x^t r^t \\ \sum_{t=1}^N (x^t)^2 r^t \end{bmatrix}$$

At this point, directly solving for v_0, v_1, v_2 is best left to a computer. Given the data, the sums can be calculated, fed into the matrix and the parameters calculated by solving the system for \vec{v}

1.3 (iii)

Q: Professor Gopher claims that $E(v_2^*, v_1^*, v_0^* | Z_{train}) \leq E(w_1^*, w_0^* | Z_{train})$ is true for any Z_{train} , is Professor Gopher's claim correct?

A: Yes, Professor Gopher's claim is correct. For any given Z_{train} , adding more polynomial terms will give a better fit for the data and, hence, a lower error for the model. The difference in error will be exactly equal if the true model for Z_{train} is linear and less if the true model for Z_{train} is non-linear.

1.4 (iv)

Q: Professor Gopher claims that $E(v_2^*, v_1^*, v_0^* | Z_{test}) \leq E(w_1^*, w_0^* | Z_{test})$ is true for any Z_{train} , is Professor Gopher's claim correct?

A: Strictly speaking, Professor Gopher's claim is not true. The model is fit on Z_{train} and it is possible that the true models could differ between Z_{train} and Z_{test} . For instance, if Z_{train} 's data had a non-linear relationship and Z_{test} 's model had a linear relationship, the quadratic model could be overfit on Z_{train} and not generalize well to Z_{test} . Generally speaking this should only happen when there isn't a lot of data or there is some disproportionate representative values in one set and not the other. These issues can be rectified by gathering more data and ensuring the data is randomly split between Z_{train} and Z_{test} . Ensuring more data and randomly splitting data sets makes Professor Gopher's claims more believable.

2 Problem 2

2.1 (i)

$\text{tr}(A)$: 701
 $\text{tr}(A^T)$: 701
 $\text{tr}(A^T A)$: 484533
 $\text{tr}(AA^T)$: 484533

2.2 (ii)

The absolute value of $|A|$ represents the "volume" of the 5 dimensional "parallelogram" formed by the rows of A .

2.3 (iii)

Because the columns of A are clearly not linearly dependent, $\det A \neq 0$. The columns are not linearly dependent because there is no way to form a linear combination of one column from the other columns. I know this because observing the relationship between the columns I note that any column can be created based on element-wise multiplication of two other columns. For instance, column 4 can be created by multiplying the elements of columns 2 and 3. Because of this, there is no way to multiply a column by a scalar to create any other column. Note that the exception to "every column is an element-wise multiplication of two other columns" is column one, but no other column is a linear combination of that and vice-versa.

2.4 (iv)

As stated in the previous problem, because the columns of A are clearly not linearly dependent, A is full rank, with rank 5.

3 Problem 3

3.1 (i) my_cross_val()

method: LinearSVC
dataset: Boston50

Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	mean	std dev
0.1765	0.3333	0.1765	0.3137	0.4314	0.1961	0.2200	0.2800	0.1400	0.1600	0.2427	0.0889

method: LinearSVC
dataset: Boston75

Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	mean	std dev
0.2157	0.1373	0.1176	0.2549	0.1765	0.6863	0.0600	0.1600	0.1200	0.1000	0.2028	0.1699

method: LinearSVC
dataset: Digits

Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	mean	std dev
0.0611	0.0667	0.0444	0.0778	0.0222	0.0444	0.0556	0.0670	0.0559	0.0559	0.0551	0.0146

method: SVC
dataset: Boston50

Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	mean	std dev
0.2353	0.4118	0.3725	0.2745	0.3333	0.2549	0.3400	0.3400	0.2400	0.4400	0.3242	0.0680

method: SVC
dataset: Boston75

Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	mean	std dev
0.2157	0.1373	0.3137	0.3333	0.2549	0.1961	0.2200	0.2200	0.3000	0.2600	0.2451	0.0566

method: SVC
dataset: Digits

Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	mean	std dev
0.4611	0.6222	0.3833	0.4222	0.4944	0.4278	0.5389	0.4637	0.5196	0.5140	0.4847	0.0651

method: LogisticRegression

dataset: Boston50

Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	mean	std dev
0.0784	0.0784	0.0784	0.1176	0.1373	0.1961	0.1400	0.2000	0.1400	0.1200	0.1286	0.0421

method: LogisticRegression

dataset: Boston75

Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	mean	std dev
0.0588	0.0588	0.0588	0.1569	0.0980	0.0980	0.1200	0.1000	0.0800	0.1200	0.0949	0.0304

method: LogisticRegression

dataset: Digits

Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	mean	std dev
0.0278	0.0444	0.0222	0.0333	0.0389	0.0333	0.0556	0.0503	0.0279	0.0447	0.0378	0.0102

3.2 (ii) my_train_test()

method: LinearSVC

dataset: Boston50

Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	mean	std dev
0.2540	0.1349	0.2460	0.1429	0.2540	0.1508	0.2937	0.2222	0.1032	0.3492	0.2151	0.0752

method: LinearSVC

dataset: Boston75

Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	mean	std dev
0.0556	0.0714	0.5794	0.1825	0.1984	0.2222	0.5079	0.1349	0.3571	0.5079	0.2817	0.1827

method: LinearSVC

dataset: Digits

Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	mean	std dev
0.0334	0.0468	0.0445	0.0713	0.0334	0.0601	0.0624	0.0490	0.0646	0.0423	0.0508	0.0125

method: SVC

dataset: Boston50

Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	mean	std dev
0.3492	0.2698	0.3651	0.4286	0.3730	0.3730	0.4524	0.3889	0.3413	0.3175	0.3659	0.0495

method: SVC

dataset: Boston75

Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	mean	std dev
0.2222	0.2381	0.2778	0.2302	0.2222	0.2381	0.2302	0.2778	0.2619	0.1667	0.2365	0.0307

method: SVC

dataset: Digits

Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	mean	std dev
0.7082	0.4365	0.6125	0.5301	0.6481	0.6882	0.5991	0.5657	0.7082	0.4811	0.5978	0.0898

method: LogisticRegression

dataset: Boston50

Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	mean	std dev
0.1667	0.1667	0.1746	0.1746	0.1032	0.1984	0.1667	0.1349	0.1429	0.1508	0.1579	0.0250

method: LogisticRegression
dataset: Boston75

Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	mean	std dev
0.1270	0.1111	0.0873	0.0714	0.1032	0.1746	0.1032	0.0873	0.1032	0.1270	0.1095	0.0272

method: LogisticRegression
dataset: Digits

Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	mean	std dev
0.0423	0.0356	0.0535	0.0401	0.0512	0.0223	0.0267	0.0423	0.0223	0.0535	0.0390	0.0115

4 Problem 4

method: LinearSVC
dataset: X1

Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	mean	std dev
0.0611	0.0778	0.0778	0.0889	0.1056	0.0722	0.1056	0.1006	0.1173	0.0615	0.0868	0.0187

method: LinearSVC
dataset: X2

Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	mean	std dev
0.0056	0.0222	0.0111	0.0167	0.0167	0.0111	0.0056	0.0112	0.0000	0.0056	0.0106	0.0063

method: LinearSVC
dataset: X3

Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	mean	std dev
0.0111	0.0667	0.0444	0.0389	0.0500	0.0444	0.0722	0.0168	0.0503	0.0838	0.0479	0.0216

method: SVC
dataset: X1

Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	mean	std dev
0.9500	0.9389	0.9222	0.9278	0.9222	0.9111	0.9167	0.9218	0.9106	0.9162	0.9237	0.0118

method: SVC
dataset: X2

Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	mean	std dev
0.9222	0.9278	0.9167	0.9333	0.9500	0.9556	0.9278	0.9330	0.9274	0.9385	0.9332	0.0114

method: SVC
dataset: X3

Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	mean	std dev
0.9611	0.9111	0.9167	0.9222	0.9222	0.9222	0.9278	0.9330	0.9218	0.9274	0.9265	0.0129

method: LogisticRegression
dataset: X1

Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	mean	std dev
0.0500	0.0389	0.0556	0.0556	0.0556	0.0444	0.0444	0.0503	0.0726	0.0279	0.0495	0.0113

method: LogisticRegression
dataset: X2

Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	mean	std dev
0.0111	0.0056	0.0056	0.0167	0.0056	0.0278	0.0222	0.0000	0.0056	0.0000	0.0100	0.0089

method: LogisticRegression
dataset: X3

Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	mean	std dev
0.0778	0.0333	0.0500	0.0444	0.0667	0.0500	0.0389	0.1061	0.0559	0.0559	0.0579	0.0202

===== ORIGINAL DATA =====

method: LinearSVC
dataset: X1

Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	mean	std dev
0.0615	0.0950	0.0838	0.0950	0.0670	0.1006	0.0782	0.0726	0.0615	0.0726	0.0788	0.0136

method: LinearSVC
dataset: X2

Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	mean	std dev
0.0168	0.0168	0.0112	0.0112	0.0056	0.0112	0.0056	0.0056	0.0000	0.0112	0.0095	0.0050

method: LinearSVC
dataset: X3

Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	mean	std dev
0.0615	0.0559	0.0503	0.0559	0.0503	0.0503	0.0670	0.0335	0.0559	0.0559	0.0536	0.0084

method: SVC
dataset: X1

Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	mean	std dev
0.8994	0.9106	0.9385	0.9218	0.9497	0.9553	0.9274	0.9274	0.9497	0.9050	0.9285	0.0187

method: SVC
dataset: X2

Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	mean	std dev
0.9330	0.9274	0.9274	0.9274	0.9441	0.9274	0.9218	0.9050	0.9330	0.9330	0.9279	0.0095

method: SVC
dataset: X3

Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	mean	std dev
0.9274	0.9385	0.9162	0.9050	0.9106	0.9162	0.9385	0.9441	0.9218	0.9274	0.9246	0.0123

method: LogisticRegression
dataset: X1

Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	mean	std dev
0.0894	0.0391	0.0615	0.0503	0.1061	0.0782	0.0559	0.0670	0.0726	0.0670	0.0687	0.0184

method: LogisticRegression
dataset: X2

Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	mean	std dev
0.0056	0.0056	0.0223	0.0056	0.0112	0.0000	0.0112	0.0335	0.0112	0.0056	0.0112	0.0093

method: LogisticRegression
dataset: X3

Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Fold 6	Fold 7	Fold 8	Fold 9	Fold 10	mean	std dev
0.0615	0.0559	0.0279	0.0335	0.0335	0.0670	0.0391	0.0782	0.0447	0.0447	0.0486	0.0156