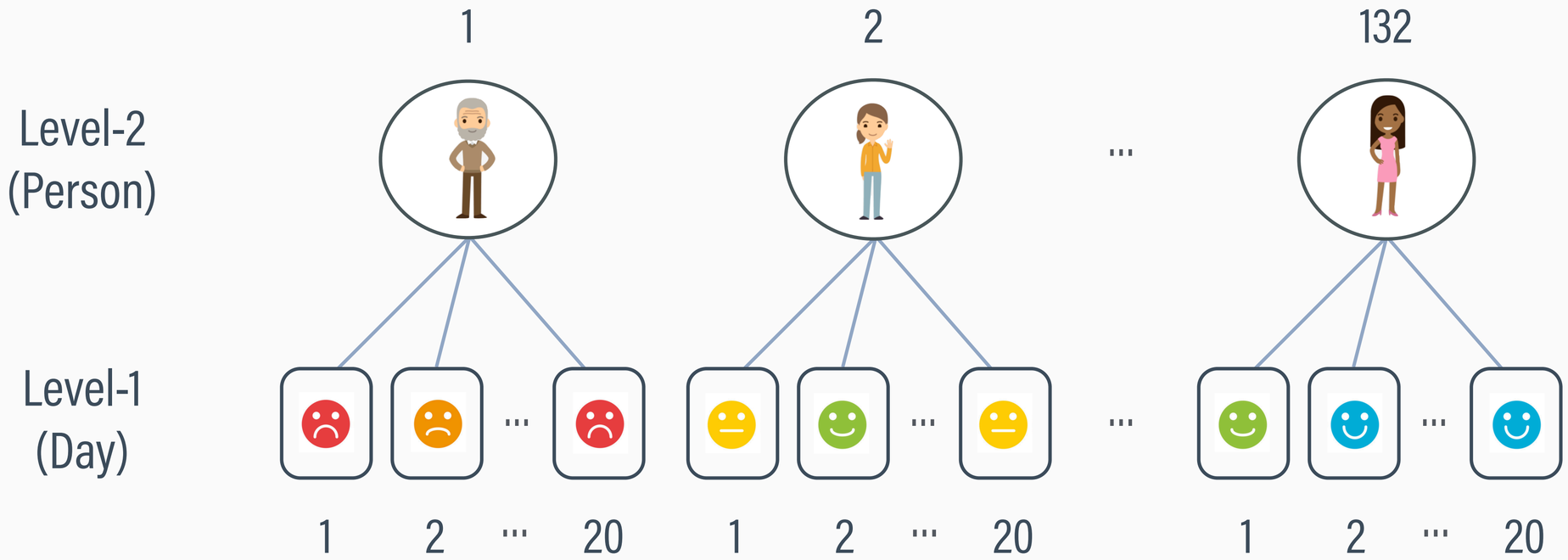
# MODULE 4

# RANDOM INTERCEPT MODELS WITH LEVEL-1 AND LEVEL-2 PREDICTORS

# DAILY DIARY APPLICATION

- $n_j = 20$ daily positive affect and sleep assessments nested within J = 132 chronic pain patients (N = 2680 data records)



# VARIABLE INFORMATION

= predictor measured at level-2

= predictor measured at level-1

= outcome measured at level-1

paffect = sleep + pain + sex + stress

| Variable | Definition | Level | Scale |
|---|---|---|---|
| PosAffect | Positive affect composite | 1 | Numeric (0 to 10) |
| SleepQual | Sleep quality rating | 1 | Ordinal (0 to 9) |
| Pain | Pain severity composite | 1 | Numeric (0 to 10) |
| Female | Sex dummy code | 2 | 0 = Male, 1 = Female |
| Stress | Stress composite score | 2 | Numeric (0 to 5) |

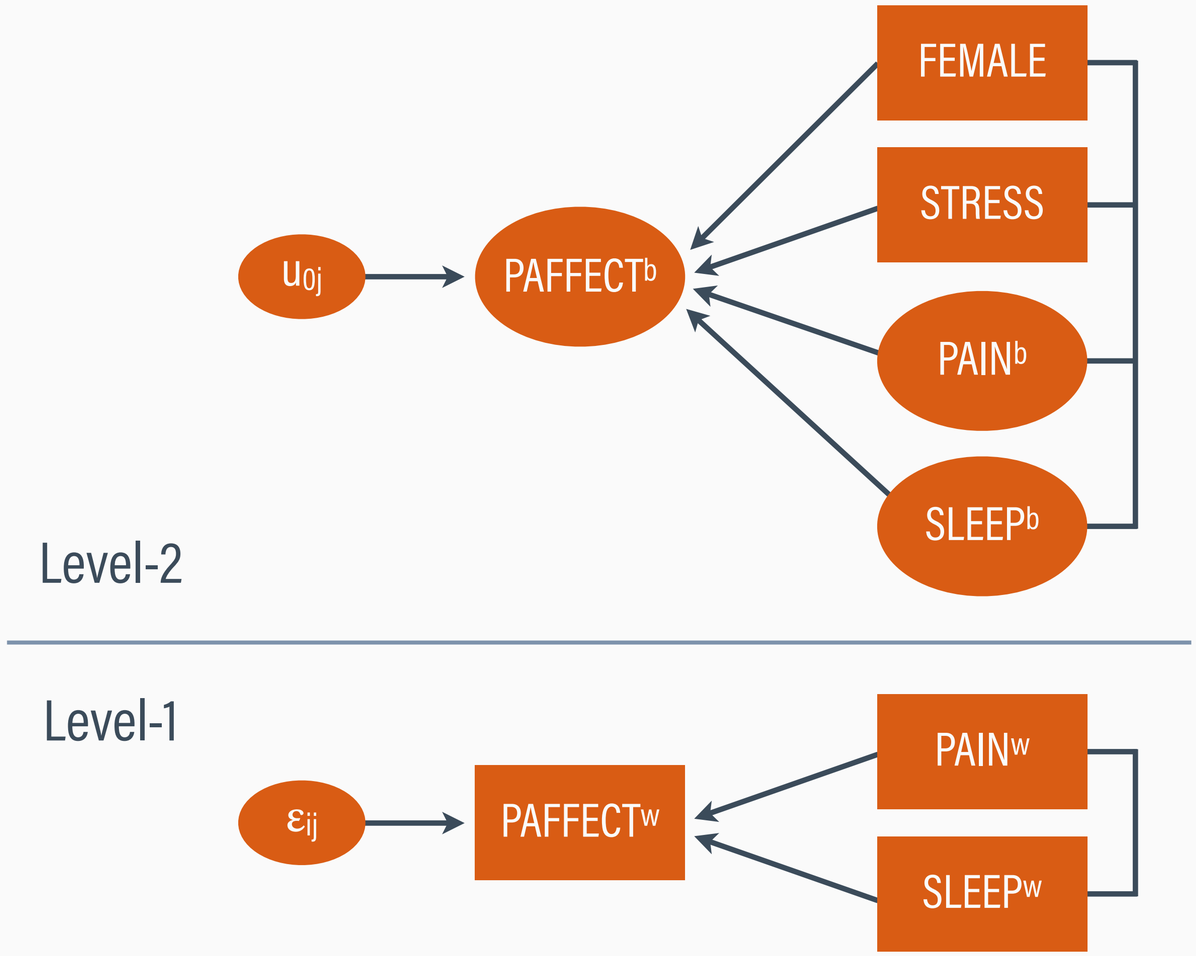
# DATA STRUCTURE

- Stacked (long) data format where each level-2 unit (person) has one row per level-1 (daily) observation
- The i subscript indexes level-1 observations, and j indexes level-2 units
- Variables measured at level-2 repeat across all rows within a cluster

| Row | i | j | $AFFECT_{ij}$ | $SLEEP_{ij}$ | $FEMALE_j$ | $STRESS_j$ | |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 7.3 | 5.6 | 1 | 3.0 | Person 1 |
| 2 | 2 | 1 | 2.5 | 4.3 | 1 | 3.0 | |
| ... | ... | 1 | ... | ... | ... | ... | |
| 20 | 20 | 1 | 6.3 | 7.3 | 1 | 3.0 | |
| 21 | 1 | 2 | 4.0 | 3.9 | 0.0 | 2.3 | Person 2 |
| 22 | 2 | 2 | 4.0 | 7.1 | 0.0 | 2.3 | |
| ... | ... | 2 | ... | ... | ... | ... | |
| 40 | 20 | 2 | 4.4 | 3.5 | 0.0 | 2.3 | |
| ... | ... | ... | ... | ... | ... | ... | |
| 2621 | 1 | 132 | 3.3 | 5.4 | 1.0 | 4.1 | Person 132 |
| 2622 | 2 | 132 | 4.8 | 3.5 | 1.0 | 4.1 | |
| ... | ... | 132 | ... | ... | ... | ... | |
| 2640 | 20 | 132 | 4.8 | 7.9 | 1.0 | 4.1 | |

# PATH DIAGRAM AND MODEL FEATURES

- Level-1 variables have two sources of variation: average values and daily fluctuations around those means
- Variables correlate within level only (e.g., sleep relates to stress via its means)
- Statistical control occurs within level



# MODELING STEPS

- Estimate the ICCs of all level-1 variables
- Add level-1 variables
- Add level-2 variables

# OUTLINE

1. Modeling Step 1: Estimate ICCs
2. Modeling Step 2: Add Within-Cluster Predictors
3. Rights and Sterba Effect Sizes
4. Modeling Step 3: Add Between-Cluster Predictors
5. Latent Variable Specification

# OUTLINE

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# ESTIMATING INTRACLASS CORRELATIONS

- Estimate intraclass correlations (ICCs) before model fitting to assess how much variance lies within and between clusters
- Some predictors have little or no level-2 variation, in which case they are already “pure” within-cluster variables
- Disaggregating a level-1 predictor and modeling its means is only warranted when there is sufficient level-2 variation

# BLIMP STUDIO SCRIPT 4.1

```
DATA: PainDiary.dat;
VARIABLES: Person Day PosAffect NegAffect Pain WorkGoal LifeGoal SleepQual Female Education
  Employment MarStatus NumDiagnose ActivityLevel PainAccept Catastrophize Stress Anxiety;
CLUSTERID: Person;
MODEL: { PosAffect SleepQual Pain } ~ intercept | intercept;    # empty model for all variables in { }
BURN: 10000;
ITERATIONS: 10000;
SEED: 90291;
```

# RBLIMP SCRIPT 4 (MODEL 1)

```
model1 <- rblimp(
        data = PainDiary,
        clustered = 'Person',
        model = '{ PosAffect SleepQual Pain } ~ intercept | intercept;',
        seed = 90291,
        burn = 10000,
        iter = 10000)
output(model1)
```

# BLIMP OUTPUT

= level-2 estimate

= level-1 estimate

Outcome Variable: PosAffect

| Parameters | Estimate | StdDev | 2.5% | 97.5% | ChiSq | PValue | N_Eff |
|---|---|---|---|---|---|---|---|
| Variances: | | | | | | | |
| L2 : Var(Intercept) | **2.529** | 0.333 | 1.987 | 3.271 | --- | --- | 8414.025 |
| Residual Var. | **1.395** | 0.039 | 1.323 | 1.474 | --- | --- | 9257.666 |
| Coefficients: | | | | | | | |
| Intercept | **5.034** | 0.147 | 4.749 | 5.316 | 1177.150 | 0.000 | 124.250 |
| Standard Deviations: | | | | | | | |
| L2 : SD(Intercept) | **1.590** | 0.103 | 1.410 | 1.809 | --- | --- | 8415.412 |
| Residual SD | **1.181** | 0.016 | 1.150 | 1.214 | --- | --- | 9254.467 |
| Proportion Variance Explained | | | | | | | |
| by Coefficients | 0.000 | 0.000 | 0.000 | 0.000 | --- | --- | nan |
| by Level-2 Random Intercepts | **0.644** | 0.030 | 0.585 | 0.703 | --- | --- | 8658.787 |
| by Level-1 Residual Variation | **0.356** | 0.030 | 0.297 | 0.415 | --- | --- | 8658.787 |

# BLIMP OUTPUT

= level-2 estimate

= level-1 estimate

Outcome Variable: SleepQual

| Parameters | Estimate | StdDev | 2.5% | 97.5% | ChiSq | PValue | N_Eff |
|---|---|---|---|---|---|---|---|
| Variances: | | | | | | | |
| L2 : Var(Intercept) | **1.919** | 0.262 | 1.490 | 2.508 | --- | --- | 6730.439 |
| Residual Var. | **3.001** | 0.084 | 2.845 | 3.172 | --- | --- | 9268.567 |
| Coefficients: | | | | | | | |
| Intercept | **5.416** | 0.130 | 5.168 | 5.671 | 1729.526 | 0.000 | 400.385 |
| Standard Deviations: | | | | | | | |
| L2 : SD(Intercept) | **1.385** | 0.093 | 1.221 | 1.584 | --- | --- | 6767.067 |
| Residual SD | **1.732** | 0.024 | 1.687 | 1.781 | --- | --- | 9265.589 |
| Proportion Variance Explained | | | | | | | |
| by Coefficients | 0.000 | 0.000 | 0.000 | 0.000 | --- | --- | nan |
| by Level-2 Random Intercepts | **0.390** | 0.032 | 0.330 | 0.457 | --- | --- | 6737.692 |
| by Level-1 Residual Variation | **0.610** | 0.032 | 0.543 | 0.670 | --- | --- | 6737.692 |

# BLIMP OUTPUT

= level-2 estimate

= level-1 estimate

Outcome Variable: Pain

| Parameters | Estimate | StdDev | 2.5% | 97.5% | ChiSq | PValue | N_Eff |
|---|---|---|---|---|---|---|---|
| Variances: | | | | | | | |
| L2 : Var(Intercept) | **2.805** | 0.375 | 2.189 | 3.654 | --- | --- | 7747.670 |
| Residual Var. | **2.246** | 0.063 | 2.125 | 2.373 | --- | --- | 8881.762 |
| Coefficients: | | | | | | | |
| Intercept | **3.533** | 0.152 | 3.238 | 3.823 | 536.962 | 0.000 | 236.236 |
| Standard Deviations: | | | | | | | |
| L2 : SD(Intercept) | **1.675** | 0.110 | 1.480 | 1.912 | --- | --- | 7692.880 |
| Residual SD | **1.499** | 0.021 | 1.458 | 1.541 | --- | --- | 8884.094 |
| Proportion Variance Explained | | | | | | | |
| by Coefficients | 0.000 | 0.000 | 0.000 | 0.000 | --- | --- | nan |
| by Level-2 Random Intercepts | **0.556** | 0.033 | 0.492 | 0.622 | --- | --- | 7537.167 |
| by Level-1 Residual Variation | **0.444** | 0.033 | 0.378 | 0.508 | --- | --- | 7537.167 |

# SUMMARY

- The outcome has substantial between-person variation in the person-specific means ($ICC_{affect} = .64$)
- This feature indicates the need for MLM (independence violation)
- Both level-1 predictors have a substantial amount of between-person variation in their means ($ICC_{sleep} = .39$ and $ICC_{pain} = .55$)
- High predictor ICCs motivate disaggregation, as there could be distinct within- and between-person effects

# OUTLINE

1. Modeling Step 1: Estimate ICCs
2. Modeling Step 2: Add Within-Cluster Predictors
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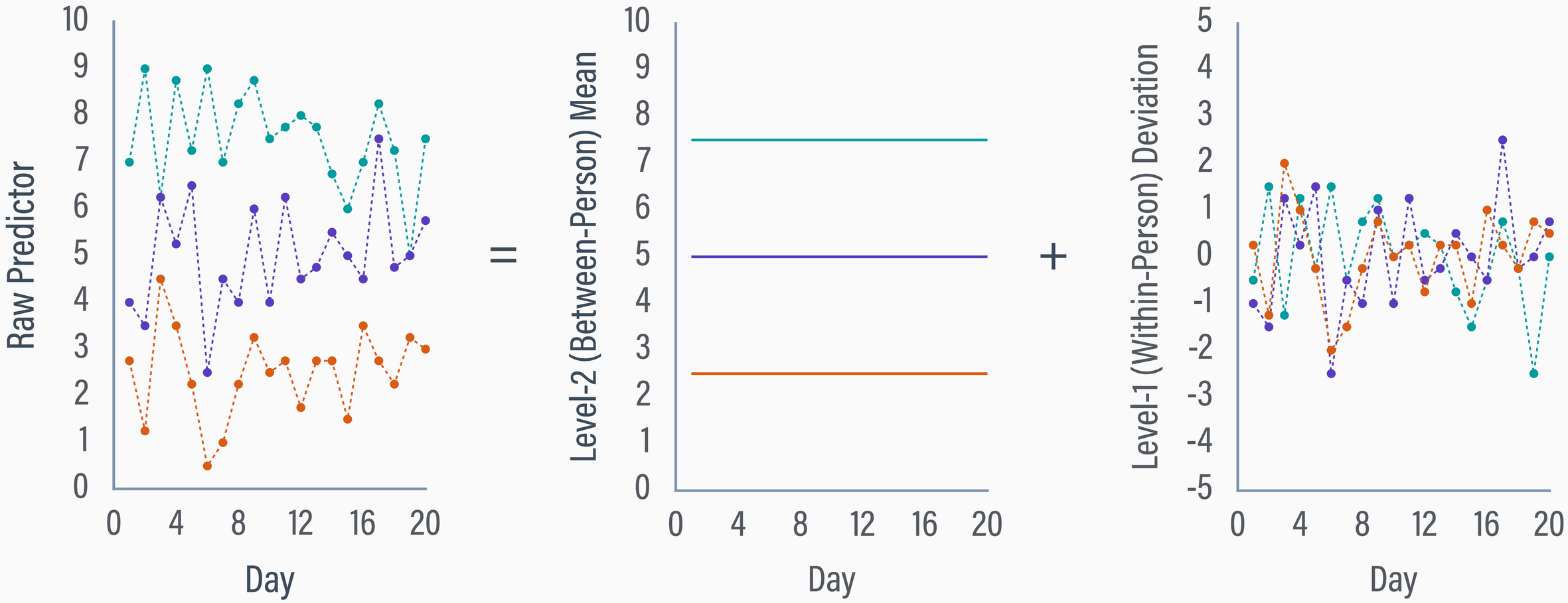
# DISAGGREGATION REVIEW

Raw Predictor = Level-2 (Between-Person) Mean + Level-1 (Within-Person) Residual



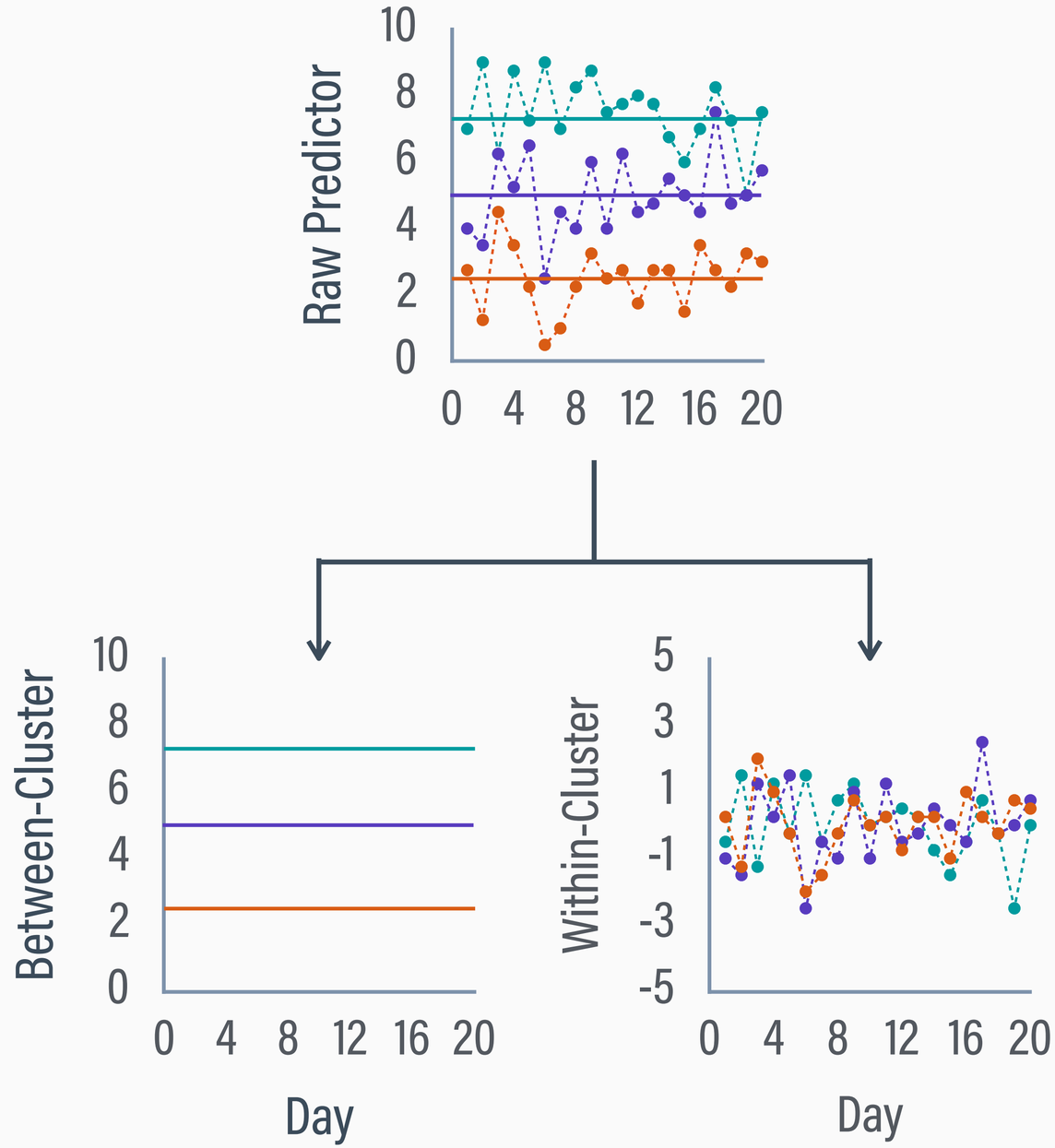
# CENTERING WITHIN CLUSTER (CWC)

- Disaggregation centers level-1 scores around their cluster's level-2 mean

$$pain_{ij}^{w} = pain_{ij} - \mu_{j(pain)} = pain_{ij} - pain_{j}^{b}$$

$$sleep_{ij}^{w} = sleep_{ij} - \mu_{j(sleep)} = sleep_{ij} - sleep_{j}^{b}$$

- $pain^{w}$ and $sleep^{w}$ are "pure" within-cluster predictors that contain only intraindividual (level-1) variation



# WITHIN-CLUSTER (LEVEL-1) MODEL

- Affect observation i for person j is the sum of a level-2 affect mean ($\beta_{0j}$), fixed effects due to within-person sleep and pain variation ($\beta_1$ and $\beta_2$), and a within-person residual ($\varepsilon_{ij}$)

$$paffect_{ij} = \beta_{0j} + \beta_1(sleep^{w}_{ij}) + \beta_2(pain^{w}_{ij}) + \varepsilon_{ij}$$

- Assumption: residuals are normal with constant variation across all days (level-1 units) and persons (level-2 units)

$$\varepsilon_{ij} \sim N(0,\sigma^2_{\varepsilon})$$

# BETWEEN-CLUSTER (LEVEL-2) MODEL

- Slopes are constant across level-2 units, but the affect mean for person j ($\beta_{0j}$) is the sum of the grand mean ($\gamma_{00}$) and a between-person residual ($u_{0j}$)

$$\beta_{0j} = \gamma_{00} + u_{0j}$$

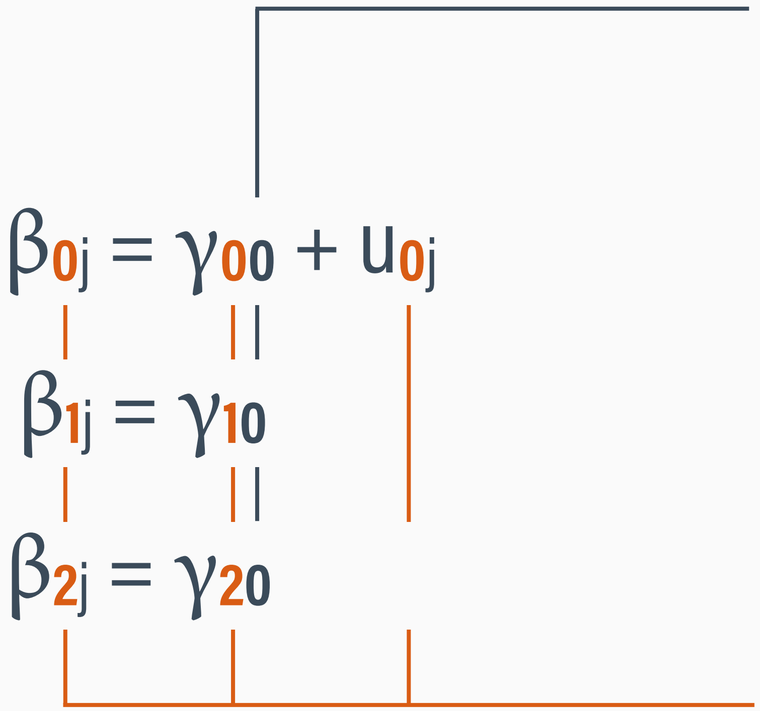
$$\beta_{1} = \gamma_{10}$$

$$\beta_{2} = \gamma_{20}$$

- Assumption: random intercept residuals are normal with constant variation across persons (level-2 units)

$$u_{0j} \sim N(0,\sigma_u^2)$$

# DECODING THE SUBSCRIPTS




The second subscript numbers the coefficients on the right side starting at 0 (the intercept)

The first subscript tells which level-2 outcome variable these terms are attached to ($\gamma_{00}$ and $u_{0j}$ belong in $\beta_{0j}$'s equation, $\gamma_{10}$ is attached to $\beta_1$)

# COMBINED-MODEL NOTATION

- Level-specific regression equations can be reduced into a single combined-model equation (Raudenbush & Bryk, 2002)
- Replace the $\beta_{0j}$, $\beta_1$, and $\beta_2$ terms in the level-1 equation with their level-2 equations

$$\beta_2 = \gamma_{20}$$

$$\beta_1 = \gamma_{10}$$

$$\beta_{0j} = \gamma_{00} + u_{0j}$$

$$\text{paffect}_{ij} = \beta_{0j} + \beta_1(\text{sleep}^{w}_{ij}) + \beta_2(\text{pain}^{w}_{ij}) + \varepsilon_{ij}$$

$$\text{paffect}_{ij} = \gamma_{00} + \gamma_{10}(\text{sleep}^{w}_{ij}) + \gamma_{20}(\text{pain}^{w}_{ij}) + u_{0j} + \varepsilon_{ij}$$

# COMMON NOTATIONAL SYSTEMS

Combined-model equation (Raudenbush & Bryk, 2002)

$$\text{paffect}_{ij} = \gamma_{00} + \gamma_{10}(\text{sleep}^{w}_{ij}) + \gamma_{20}(\text{pain}^{w}_{ij}) + u_{0j} + \varepsilon_{ij}$$

Standard(ish) regression notation (Scott, Shrout, & Weinberg, 2013)

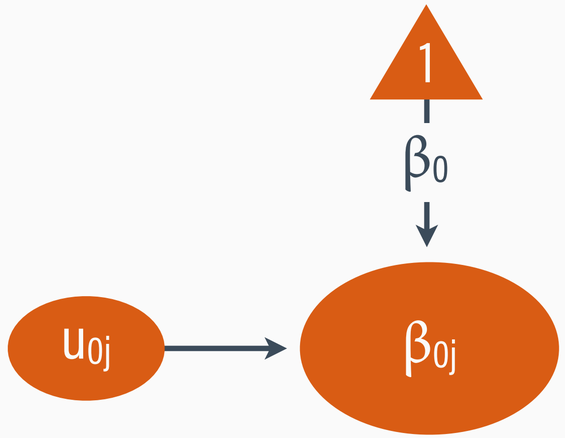
$$\text{paffect}_{ij} = \beta_0 + \beta_1(\text{sleep}^{w}_{ij}) + \beta_2(\text{pain}^{w}_{ij}) + u_{0j} + \varepsilon_{ij}$$

Linear mixed model cluster-level matrix equation

$$\mathbf{y}_j = \mathbf{X}_j\boldsymbol{\beta} + \mathbf{Z}_j u_j + \varepsilon_j = \begin{pmatrix} \text{posaffect}_{1j} \\ \text{posaffect}_{2j} \\ \dots \\ \text{posaffect}_{nj} \end{pmatrix} = \begin{pmatrix} 1 & \text{sleep}^{w}_{1j} & \text{pain}^{w}_{1j} \\ 1 & \text{sleep}^{w}_{2j} & \text{pain}^{w}_{2j} \\ \dots & \dots & \dots \\ 1 & \text{sleep}^{w}_{nj} & \text{pain}^{w}_{nj} \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ \dots \\ 1 \end{pmatrix} u_{0j} + \begin{pmatrix} \varepsilon_{1j} \\ \varepsilon_{2j} \\ \dots \\ \varepsilon_{nj} \end{pmatrix}$$

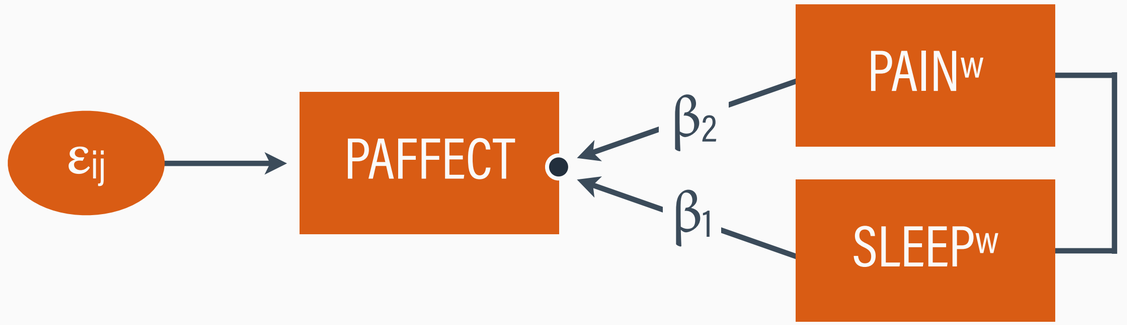
# PATH DIAGRAM

$$paffect_{ij} = \beta_0 + \beta_1(sleep^{w}_{ij}) + \beta_2(pain^{w}_{ij}) + u_{0j} + \varepsilon_{ij}$$




Level-2

Level-1




- = random intercept ($\beta_{0j}$)

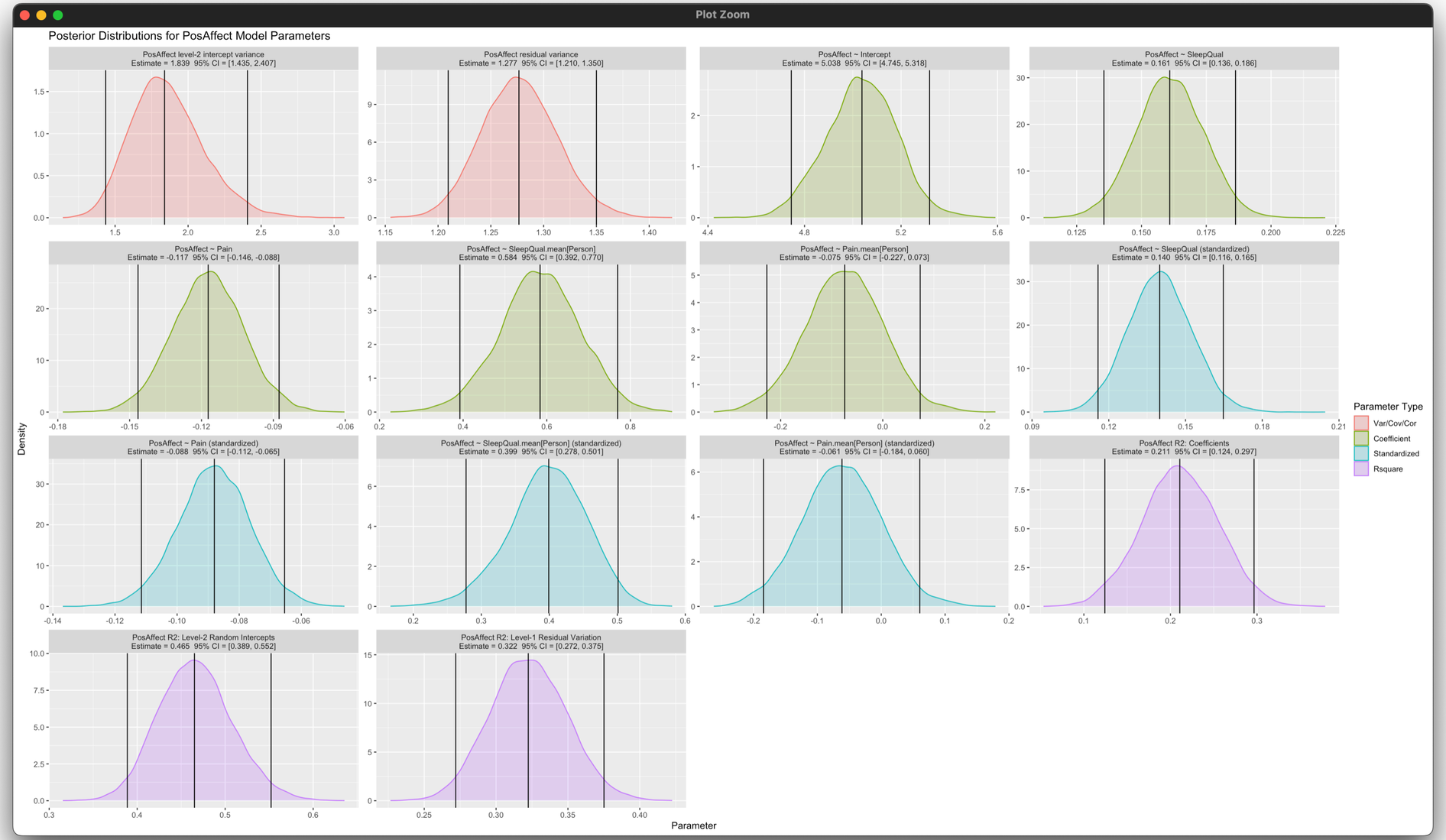
# BLIMP STUDIO SCRIPT 4.2

```
DATA: PainDiary.dat;
VARIABLES: Person Day PosAffect NegAffect Pain WorkGoal LifeGoal SleepQual  Female Education
  Employment MarStatus NumDiagnose ActivityLevel PainAccept Catastrophize Stress Anxiety;
CLUSTERID: Person;
CENTER: groupmean = SleepQual Pain;    # cwc with level-2 latent cluster means
MODEL: PosAffect ~ intercept SleepQual Pain | intercept;
BURN: 10000;
ITER: 10000;
SEED: 90291;
```

# RBLIMP SCRIPT 4 (MODEL 2)

```
model2 <- rblimp(
        data = PainDiary,
        clusterid = 'Person',
        center = 'groupmean = SleepQual Pain',
        model = 'PosAffect ~ intercept SleepQual Pain | intercept',
        seed = 90291,
        burn = 10000,
        iter = 10000)
output(model2)
plot_posterior(model2, 'PosAffect')
```

# PARAMETER PLOTS (RBLIMP ONLY)



# BLIMP OUTPUT

= level-2 estimate

= level-1 estimate

Outcome Variable:  PosAffect

Group Mean Centered: Pain SleepQual

| Parameters | Estimate | StdDev | 2.5% | 97.5% | ChiSq | PValue | N_Eff |
|---|---|---|---|---|---|---|---|
| Variances: | | | | | | | |
| L2 : Var(Intercept) | **2.496** | 0.325 | 1.953 | 3.218 | --- | --- | 8178.774 |
| Residual Var. | **1.276** | 0.036 | 1.210 | 1.350 | --- | --- | 9009.151 |
| Coefficients: | | | | | | | |
| Intercept | **5.010** | 0.147 | 4.742 | 5.312 | 1169.499 | 0.000 | 105.520 |
| SleepQual | **0.162** | 0.013 | 0.136 | 0.187 | 156.661 | 0.000 | 8279.469 |
| Pain | **-0.117** | 0.015 | -0.147 | -0.088 | 60.912 | 0.000 | 8510.442 |
| Standard Deviations: | | | | | | | |
| L2 : SD(Intercept) | **1.580** | 0.102 | 1.397 | 1.794 | --- | --- | 8123.400 |
| Residual SD | **1.130** | 0.016 | 1.100 | 1.162 | --- | --- | 9006.557 |
| Standardized Coefficients: | | | | | | | |
| SleepQual | **0.142** | 0.013 | 0.118 | 0.167 | 126.552 | 0.000 | 7959.671 |
| Pain | **-0.089** | 0.012 | -0.113 | -0.066 | 55.997 | 0.000 | 8548.133 |
| Proportion Variance Explained | | | | | | | |
| by Coefficients | 0.031 | 0.005 | 0.023 | 0.041 | --- | --- | 8140.114 |
| by Level-2 Random Intercepts | **0.641** | 0.030 | 0.582 | 0.698 | --- | --- | 8075.546 |
| by Level-1 Residual Variation | **0.328** | 0.027 | 0.276 | 0.382 | --- | --- | 8119.500 |

# FIXED EFFECT INTERPRETATIONS

- $paffect_{ij} = \beta_0 + \beta_1(sleep^{w}_{ij}) + \beta_2(pain^{w}_{ij}) + u_{0j} + \varepsilon_{ij}$
- $\beta_0 = 5.01$ is the positive affect grand mean (because all predictors are centered)
- $\beta_1 = 0.16$ is the expected affect difference between two daily sleep scores from the same person that differ by one point, controlling for within-person pain
- $\beta_2 = -0.12$ is the expected affect difference between two daily pain scores from the same person that differ by one point, controlling for within-person sleep

# RANDOM EFFECT INTERPRETATIONS

- $u_{0j} = \beta_{0j} - \beta_0$
- $var(u_{0j}) = 2.50$ is the average squared distance between the level-2 affect means and the grand mean
- $sd(u_{0j}) = 1.58$ is the average distance between the level-2 affect means and the grand mean

- $\varepsilon_{ij} = paffect_{ij} - (\beta_{0j} + \beta_1(sleep^{w}_{ij}) + \beta_2(pain^{w}_{ij}))$
- $var(\varepsilon_{ij}) = 1.28$ is the average squared distance between the level-1 affect observations and their predicted values
- $sd(\varepsilon_{ij}) = 1.13$ is the average distance between the level-1 affect observations and their predicted values

# OUTLINE

1. Modeling Step 1: Estimate ICCs
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# MULTILEVEL EFFECT SIZES

- Numerous effect sizes exist for multilevel models
- Rights and Sterba (2019, Psychological Methods) provides a unifying framework that subsumes different approaches
- They assign a proportion variance explained ($R^2$) to each source of variation in the multilevel model

# PREDICTED OUTCOME VARIATION

- The model-predicted total variance is the sum of (a) variance explained by predictors at both levels, (b) residual random-intercept variation, and (c) residual within-cluster variation

$$\sigma_Y^2 = \boldsymbol{\beta}^T \boldsymbol{\Sigma}_X \boldsymbol{\beta} + \sigma_u^2 + \sigma_\varepsilon^2$$

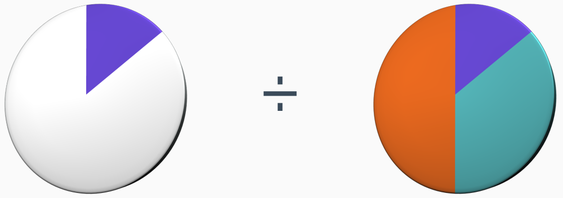
total variation = explained by predictors + level-2 residuals + level-1 residuals

# VARIANCE EXPLAINED MEASURES

Explained ÷ Total

Fixed effects of predictors

$$R^2_{\text{predictors}} = \frac{\boldsymbol{\beta}^T \boldsymbol{\Sigma}_X \boldsymbol{\beta}}{\sigma^2_Y}$$

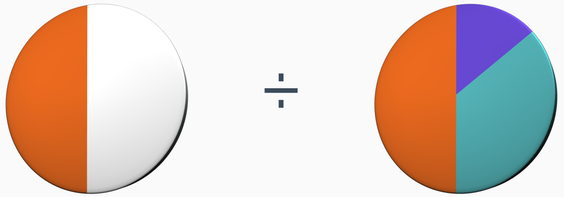


Level-2 random intercept residuals

$$R^2_{\text{residual-between}} = \frac{\sigma^2_{u_0}}{\sigma^2_Y}$$

Level-1 within-cluster residuals

$$R^2_{\text{residual-within}} = \frac{\sigma^2_\varepsilon}{\sigma^2_Y}$$



# BLIMP OUTPUT

= level-2 estimate

= level-1 estimate

Outcome Variable:  PosAffect

Group Mean Centered: Pain SleepQual

| Parameters | Estimate | StdDev | 2.5% | 97.5% | ChiSq | PValue | N_Eff |
|---|---|---|---|---|---|---|---|
| Variances: | | | | | | | |
| L2 : Var(Intercept) | 2.496 | 0.325 | 1.953 | 3.218 | --- | --- | 8178.774 |
| Residual Var. | 1.276 | 0.036 | 1.210 | 1.350 | --- | --- | 9009.151 |
| Coefficients: | | | | | | | |
| Intercept | 5.010 | 0.147 | 4.742 | 5.312 | 1169.499 | 0.000 | 105.520 |
| SleepQual | 0.162 | 0.013 | 0.136 | 0.187 | 156.661 | 0.000 | 8279.469 |
| Pain | -0.117 | 0.015 | -0.147 | -0.088 | 60.912 | 0.000 | 8510.442 |
| ... | | | | | | | |
| Proportion Variance Explained | | | | | | | |
| by Coefficients | **0.031** | 0.005 | 0.023 | 0.041 | --- | --- | 8140.114 |
| by Level-2 Random Intercepts | **0.641** | 0.030 | 0.582 | 0.698 | --- | --- | 8075.546 |
| by Level-1 Residual Variation | **0.328** | 0.027 | 0.276 | 0.382 | --- | --- | 8119.500 |

# EFFECT SIZE INTERPRETATIONS

- $R^2_{(predictors)} = .03$ is the proportion of the total variation explained by the within-cluster predictors
- $R^2_{(residual\text{-}between)} = .64$ is the proportion of the total variation attributable to the between-cluster residuals (the $u_{0j}$ terms)
- $R^2_{(residual\text{-}within)} = .33$ is the proportion of the total variation attributable to the within-cluster residuals (the $\varepsilon_{ij}$ terms)

# MODEL COMPARISON

| Parameter | Empty Model | Level-1 Predictors (Within Only) | Level-2 Predictors (Within + Between) |
|---|---|---|---|
| Fixed intercept | 5.03 | 5.03 | 5.03 |
| Sleep (within-person) | -- | 0.16 | 0.16 |
| Pain (within-person) | -- | –0.12 | –0.12 |
| Sleep (between-person) | -- | -- | –0.11 |
| Pain (between-person) | -- | -- | 0.57 |
| Female (between-person) | -- | -- | 0.38 |
| Stress (between-person) | -- | -- | –0.36 |
| Residual intercept variance | 2.53 | 2.50 | 1.74 |
| Residual within-person variance | 1.40 | 1.28 | 1.28 |
| R-square predictors | 0 | .03 | .25 |
| R-square intercept variance | .64 | .64 | .43 |
| R-square within-person variance | .36 | .33 | .32 |

The next analysis will add the level-2 (person-specific) predictors to the model. In small groups of two or three, discuss whether each effect size will change and, if so, how.

# OUTLINE

1. Modeling Step 1: Estimate ICCs
2. Modeling Step 2: Add Within-Cluster Predictors
3. Rights and Sterba Effect Sizes
4. Modeling Step 3: Add Between-Cluster Predictors
5. Latent Variable Specification

# WITHIN-CLUSTER (LEVEL-1) MODEL

- Affect observation i for person j is the sum of a level-2 affect mean ($\beta_{0j}$), fixed effects due to within-person sleep and pain variation ($\beta_1$ and $\beta_2$), and a within-person residual ($\varepsilon_{ij}$)

$$paffect_{ij} = \beta_{0j} + \beta_1(sleep^{w}_{ij}) + \beta_2(pain^{w}_{ij}) + \varepsilon_{ij}$$

- Assumption: residuals are normal with constant variation across all days (level-1 units) and persons (level-2 units)

$$\varepsilon_{ij} \sim N(0,\sigma^2_{\varepsilon})$$

# BETWEEN-CLUSTER (LEVEL-2) MODEL

- The affect mean for person j ($\beta_{0j}$) is the sum of a mean ($\gamma_{00}$), fixed effects due to the level-2 predictors ($\gamma_{01}$ through $\gamma_{04}$), and a between-person residual ($u_{0j}$)

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(sleep_j^b) + \gamma_{02}(pain_j^b) + \gamma_{03}(female_j) + \gamma_{04}(stress_j) + u_{0j}$$

$$\beta_1 = \gamma_{10}$$

$$\beta_2 = \gamma_{20}$$

- Assumption: random intercept residuals are normal with constant variation across persons (level-2 units)

$$u_{0j} \sim N(0, \sigma_u^2)$$

# COMMON NOTATIONAL SYSTEMS

Combined-model equation (Raudenbush & Bryk, 2002)

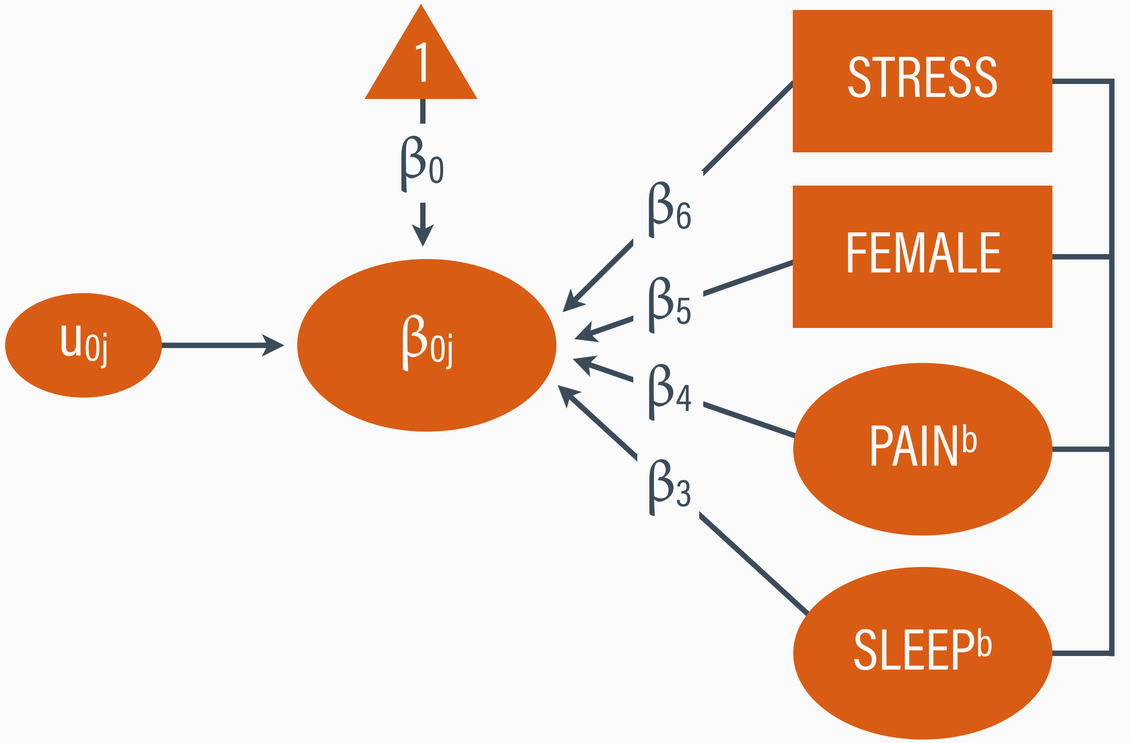
$$\text{paffect}_{ij} = \gamma_{00} + \gamma_{10}(\text{sleep}^{w}_{ij}) + \gamma_{20}(\text{pain}^{w}_{ij}) + \gamma_{01}(\text{sleep}^{b}_{j}) + \gamma_{02}(\text{pain}^{b}_{j}) + \gamma_{03}(\text{female}_{j}) + \gamma_{04}(\text{stress}_{j}) + u_{0j} + \varepsilon_{ij}$$

Standard(ish) regression notation (Scott, Shrout, & Weinberg, 2013)

$$\text{paffect}_{ij} = \beta_0 + \beta_1(\text{sleep}^{w}_{ij}) + \beta_2(\text{pain}^{w}_{ij}) + \beta_3(\text{sleep}^{b}_{j}) + \beta_4(\text{pain}^{b}_{j}) + \beta_5(\text{female}_{j}) + \beta_6(\text{stress}_{j}) + u_{0j} + \varepsilon_{ij}$$

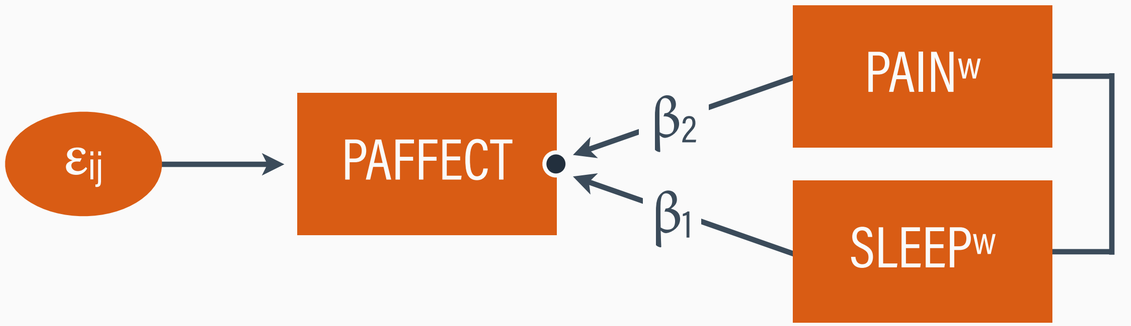
# PATH DIAGRAM

$$paffect_{ij} = \beta_0 + \beta_1(sleep^{w}_{ij}) + \beta_2(pain^{w}_{ij}) + \beta_3(sleep^{b}_{j}) + \beta_4(pain^{b}_{j}) + \beta_5(female_j) + \beta_6(stress_j) + u_{0j} + \varepsilon_{ij}$$




Level-2

Level-1




- = random intercept ($\beta_{0j}$)

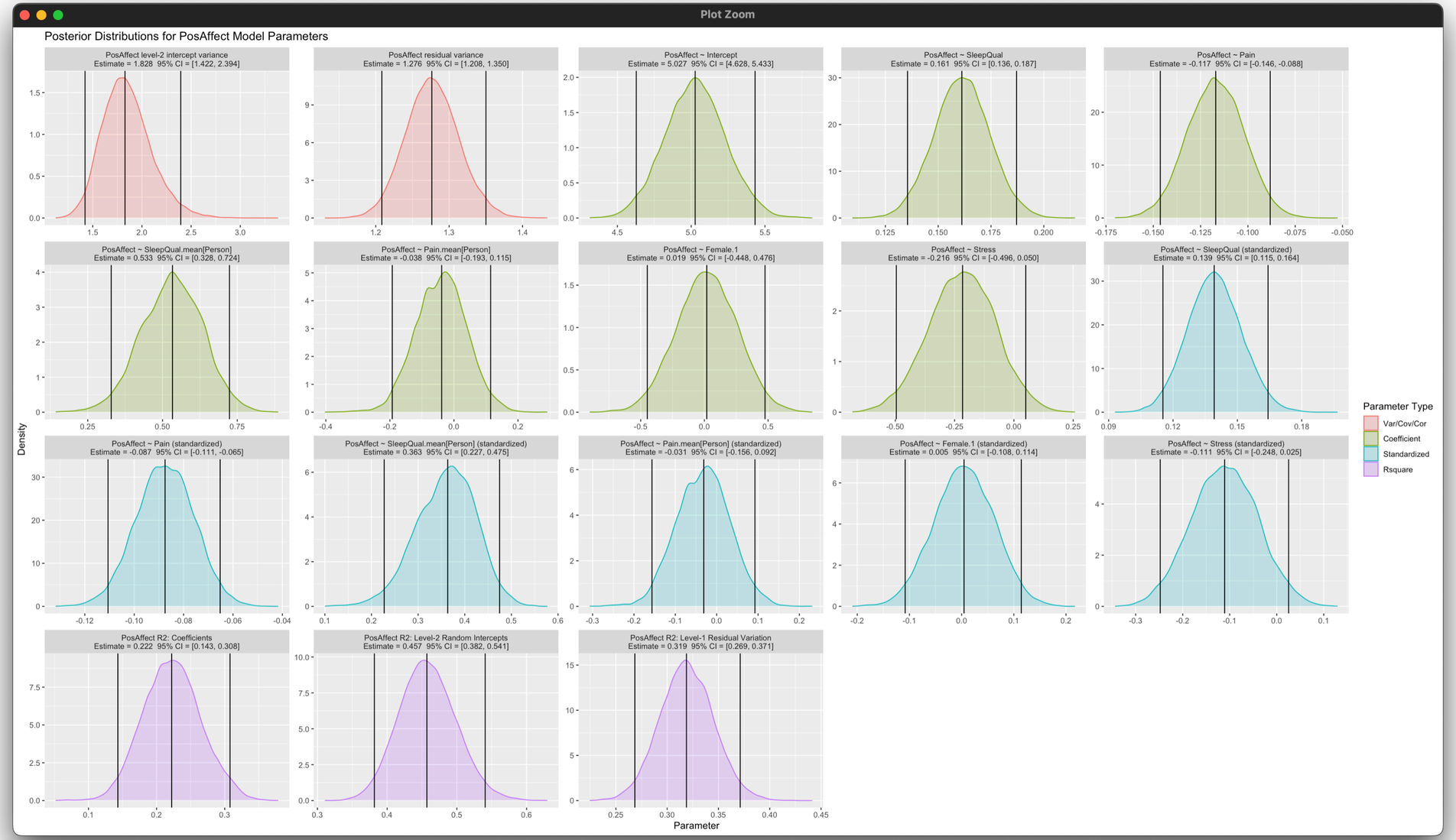
# BLIMP STUDIO SCRIPT 4.3

```
DATA: PainDiary.dat;
VARIABLES: Person Day PosAffect NegAffect Pain WorkGoal LifeGoal SleepQual  Female Education
  Employment MarStatus NumDiagnose ActivityLevel PainAccept Catastrophize Stress Anxiety;
NOMINAL: Female;     # automatic dummy coding for nominal predictors
CLUSTERID: Person;
CENTER:
groupmean = SleepQual Pain;
grandmean = SleepQual.mean Pain.mean Stress;
MODEL: PosAffect ~ intercept SleepQual Pain SleepQual.mean Pain.mean Female Stress | intercept;
BURN: 10000;
ITERATIONS: 10000;
SEED: 90291;
```

# RBLIMP SCRIPT 4 (MODEL 3)

```
model3 <- rblimp(
  data = PainDiary,
  nominal = 'Female',
  clusterid = 'Person',
  center = 'groupmean = SleepQual Pain; grandmean = SleepQual.mean Pain.mean Stress',
  model = 'PosAffect ~ intercept SleepQual Pain SleepQual.mean Pain.mean Female Stress | intercept',
  seed = 90291,
  burn = 10000,
  iter = 10000)
output(model3)
plot_posterior(model3, 'PosAffect')
```

# PARAMETER PLOTS (RBLIMP ONLY)



# BLIMP OUTPUT

= level-2 estimate

= level-1 estimate

= combined estimate

```
Outcome Variable:  PosAffect

Grand Mean Centered: Pain.mean[Person] SleepQual.mean[Person] Stress
Group Mean Centered: Pain SleepQual

Parameters                      Estimate      StdDev        2.5%       97.5%       ChiSq      PValue       N_Eff
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Variances:
  L2 : Var(Intercept)              1.828       0.249       1.412       2.398         ---         ---    4385.748
  Residual Var.                    1.277       0.036       1.208       1.350         ---         ---    8247.203

Coefficients:
  Intercept                        5.041       0.202       4.641       5.431     620.928       0.000     190.993
  SleepQual                        0.161       0.013       0.135       0.187     150.704       0.000    9375.960
  Pain                            -0.117       0.015      -0.147      -0.088      61.071       0.000    8973.437
  SleepQual.mean[Person]           0.531       0.105       0.317       0.723      25.163       0.000     145.860
  Pain.mean[Person]               -0.039       0.078      -0.187       0.116       0.247       0.620     149.240
  Female.1                         0.009       0.228      -0.429       0.466       0.002       0.967     194.738
  Stress                          -0.214       0.136      -0.502       0.041       2.585       0.108     176.945

Standard Deviations:
  L2 : SD(Intercept)               1.352       0.091       1.188       1.549         ---         ---    4392.623
  Residual SD                      1.130       0.016       1.099       1.162         ---         ---    8250.812

...

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```

# FIXED EFFECT INTERPRETATIONS

- $\beta_0 = 5.04$ is the positive affect mean for the female = 0 group (because all numeric predictors are centered)
- $\beta_1 = 0.16$ is the expected affect difference between two daily sleep scores from the same person that differ by one point, controlling for within-person pain
- $\beta_2 = -0.12$ is the expected affect difference between two daily pain scores from the same person that differ by one point, controlling for within-person sleep

# FIXED EFFECTS, CONTINUED

- $\beta_3 = 0.53$ is the expected affect difference between two people whose average sleep ratings differ by one point, controlling for average pain, sex, and stress

- $\beta_4 = -0.04$ is the expected affect difference between two people whose average pain ratings differ by one point, controlling for average sleep, sex, and stress

- $\beta_5 = 0.01$ is the expected affect difference for females (versus males), controlling for average sleep, average pain, and stress

- $\beta_6 = -0.21$ is the expected affect difference between two people whose stress scores differ by one point, controlling for average sleep, average pain, and sex

# RANDOM EFFECT INTERPRETATIONS

- $u_{0j} = \beta_{0j} - ( \beta_0 + \beta_3(sleep_j^b) + \beta_4(pain_j^b) + \beta_5(female_j) + \beta_6(stress_j) )$
- $var(u_{0j}) = 1.83$ is the average squared distance between the level-2 affect means and the grand mean
- $sd(u_{0j}) = 1.35$ is the average distance between the level-2 affect means and the grand mean
- $\varepsilon_{ij} = paffect_{ij} - (\beta_{0j} + \beta_1(sleep_{ij}^w) + \beta_2(pain_{ij}^w))$
- $var(\varepsilon_{ij}) = 1.28$ is the average squared distance between the level-1 affect observations and their predicted values
- $sd(\varepsilon_{ij}) = 1.13$ is the average distance between the level-1 affect observations and their predicted values

# BLIMP OUTPUT, CONTINUED

= level-2 estimate

= level-1 estimate

= combined estimate

```
Outcome Variable:  PosAffect

Grand Mean Centered: Pain.mean[Person] SleepQual.mean[Person] Stress
Group Mean Centered: Pain SleepQual

Parameters                          Estimate      StdDev        2.5%       97.5%       ChiSq      PValue       N_Eff
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Standardized Coefficients:
  SleepQual                            0.139       0.013       0.116       0.165     124.128       0.000    2565.722
  Pain                                -0.088       0.012      -0.111      -0.065      55.958       0.000    4529.435
  SleepQual.mean[Person]               0.362       0.066       0.218       0.473      29.402       0.000     149.255
  Pain.mean[Person]                   -0.032       0.064      -0.150       0.094       0.249       0.618     149.922
  Female.1                             0.002       0.055      -0.104       0.112       0.002       0.966     194.588
  Stress                              -0.110       0.068      -0.252       0.021       2.635       0.105     176.893

Proportion Variance Explained
  by Coefficients                      0.220       0.042       0.139       0.305         ---         ---     184.037
  by Level-2 Random Intercepts         0.458       0.041       0.383       0.543         ---         ---     418.942
  by Level-1 Residual Variation        0.320       0.026       0.270       0.372         ---         ---     496.184

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```

# EFFECT SIZE INTERPRETATIONS

- $R^2_{(predictors)} = .22$ is the proportion of the total variation explained by all predictors (was .03 in previous model)
- $R^2_{(residual\text{-}between)} = .46$ is the proportion of the total variation attributable to the between-cluster residuals (the $u_{0j}$ terms)
- $R^2_{(residual\text{-}within)} = .32$ is the proportion of the total variation attributable to the within-cluster residuals (the $\varepsilon_{ij}$ terms)

# MODEL COMPARISON

| Parameter | Empty Model | Level-1 Predictors (Within Only) | Level-2 Predictors (Within + Between) |
|---|---|---|---|
| Fixed intercept | 5.03 | 5.03 | 5.04 |
| Sleep (within-person) | -- | 0.16 | 0.16 |
| Pain (within-person) | -- | −0.12 | −0.12 |
| Sleep (between-person) | -- | -- | 0.53 |
| Pain (between-person) | -- | -- | −0.04 |
| Female (between-person) | -- | -- | 0.01 |
| Stress (between-person) | -- | -- | −0.21 |
| Residual intercept variance | 2.53 | 2.50 | 1.83 |
| Residual within-person variance | 1.40 | 1.28 | 1.28 |
| R-square predictors | 0 | .03 | .22 |
| R-square intercept variance | .64 | .64 | .46 |
| R-square within-person variance | .36 | .33 | .32 |

# OUTLINE

1. Modeling Step 1: Estimate ICCs
2. Modeling Step 2: Add Within-Cluster Predictors
3. Rights and Sterba Effect Sizes
4. Modeling Step 3: Add Between-Cluster Predictors
5. Latent Variable Specification

# BLIMP STUDIO SCRIPT 4.4

```
DATA: PainDiary.dat;
VARIABLES: Person Day PosAffect NegAffect Pain WorkGoal LifeGoal SleepQual Female Education
  Employment MarStatus NumDiagnose ActivityLevel PainAccept Catastrophize Stress Anxiety;
CLUSTERID: Person;
LATENT: Person = beta0j;    # define level-2 intercept latent variable
CENTER: groupmean = SleepQual;
MODEL:
beta0j ~ intercept;    # level-2 regression equation (β0j = γ00 + u0j)
PosAffect ~ intercept@beta0j SleepQual Pain;    # level-1: paffectij = β0j + β1(sleep^W_ij) + β2(pain^W_ij) + εij
BURN: 10000;
ITER: 20000;
SEED: 90291;
```

# RBLIMP SCRIPT 4 (MODEL 4)

```
model1 <- rblimp(
        data = PainDiary,
        clusterid = 'Person',
        latent = 'Person = beta0j',
        center = 'groupmean = SleepQual Pain',
        model = '
          beta0j ~ intercept;
          PosAffect ~ intercept@beta0j SleepQual Pain',
        seed = 90291,
        burn = 10000,
        iter = 20000)
output(model1)
```

# LEVEL-1 OUTPUT

= level-2 estimate

= level-1 estimate

Outcome Variable:  PosAffect

Group Mean Centered: Pain SleepQual

| Parameters | Median | StdDev | 2.5% | 97.5% | ChiSq | PValue | N_Eff |
|---|---|---|---|---|---|---|---|
| Variances: | | | | | | | |
| Residual Var. | **1.276** | 0.036 | 1.209 | 1.349 | --- | --- | 17487.328 |
| Coefficients: | | | | | | | |
| beta0j | @ 1.000 | --- | --- | --- | --- | --- | --- |
| SleepQual | **0.162** | 0.013 | 0.136 | 0.188 | 154.282 | 0.000 | 17775.118 |
| Pain | **-0.117** | 0.015 | -0.147 | -0.088 | 61.358 | 0.000 | 18195.499 |
| Standardized Coefficients: | | | | | | | |
| SleepQual | 0.237 | 0.018 | 0.201 | 0.273 | 167.226 | 0.000 | 17168.959 |
| Pain | -0.148 | 0.019 | -0.185 | -0.111 | 63.333 | 0.000 | 18197.357 |
| Proportion Variance Explained | | | | | | | |
| by Coefficients | 0.087 | 0.010 | 0.067 | 0.108 | --- | --- | 17479.089 |
| by Residual Variation | 0.913 | 0.010 | 0.892 | 0.933 | --- | --- | 17479.089 |

# LEVEL-2 OUTPUT

= level-2 estimate

= level-1 estimate

Latent Variable: beta0j

| Parameters | Median | StdDev | 2.5% | 97.5% | ChiSq | PValue | N_Eff |
|---|---|---|---|---|---|---|---|
| Variances: | | | | | | | |
| Residual Var. | **2.489** | 0.330 | 1.946 | 3.237 | --- | --- | 16621.302 |
| Coefficients: | | | | | | | |
| Intercept | **5.034** | 0.140 | 4.756 | 5.310 | 1284.262 | 0.000 | 18856.765 |
| Proportion Variance Explained | | | | | | | |
| by Coefficients | 0.000 | 0.000 | 0.000 | 0.000 | --- | --- | nan |
| by Residual Variation | 1.000 | 0.000 | 1.000 | 1.000 | --- | --- | nan |

# BLIMP STUDIO SCRIPT 4.5

```
DATA: PainDiary.dat;
VARIABLES: Person Day PosAffect NegAffect Pain WorkGoal LifeGoal SleepQual Female Education
  Employment MarStatus NumDiagnose ActivityLevel PainAccept Catastrophize Stress Anxiety;
NOMINAL: Female;
CLUSTERID: Person;
LATENT: Person = beta0j;    # define level-2 intercept latent variable
CENTER: groupmean = SleepQual; grandmean = SleepQual.mean Pain.mean Stress;
MODEL:                                  # level-2: β0j = γ00 + γ01(sleep^b_j) + γ02(pain^b_j) + γ03(female_j) + γ04(stress_j) + u0j
beta0j ~ intercept SleepQual.mean Pain.mean Female Stress;
PosAffect ~ intercept@beta0j SleepQual Pain;    # level-1: paffect_ij = β0j + β1(sleep^w_ij) + β2(pain^w_ij) + εij
BURN: 10000;
ITER: 20000;
SEED: 90291;
```

# RBLIMP SCRIPT 4 (MODEL 5)

```
model3 <- rblimp(
        data = PainDiary,
        nominal = 'Female',
        clusterid = 'Person',
        latent = 'Person = beta0j',
        center = 'groupmean = SleepQual Pain; grandmean = SleepQual.mean Pain.mean Stress',
        model = '
          beta0j ~ intercept SleepQual.mean Pain.mean Female Stress;
          PosAffect ~ intercept@beta0j SleepQual Pain',
        seed = 90291,
        burn = 10000,
        iter = 20000)
output(model3)
```

# LEVEL-1 OUTPUT

= level-2 estimate

= level-1 estimate

Outcome Variable:  PosAffect

Group Mean Centered: Pain SleepQual

| Parameters | Median | StdDev | 2.5% | 97.5% | ChiSq | PValue | N_Eff |
|---|---|---|---|---|---|---|---|
| Variances: | | | | | | | |
| Residual Var. | **1.277** | 0.036 | 1.208 | 1.349 | --- | --- | 18773.456 |
| Coefficients: | | | | | | | |
| beta0j | @ 1.000 | --- | --- | --- | --- | --- | --- |
| SleepQual | **0.161** | 0.013 | 0.136 | 0.187 | 154.038 | 0.000 | 16824.474 |
| Pain | **-0.117** | 0.015 | -0.147 | -0.088 | 60.356 | 0.000 | 18396.872 |
| Standardized Coefficients: | | | | | | | |
| SleepQual | 0.236 | 0.018 | 0.200 | 0.272 | 167.294 | 0.000 | 17192.061 |
| Pain | -0.149 | 0.019 | -0.185 | -0.112 | 62.454 | 0.000 | 18410.177 |
| Proportion Variance Explained | | | | | | | |
| by Coefficients | 0.086 | 0.011 | 0.067 | 0.108 | --- | --- | 17786.052 |
| by Residual Variation | 0.914 | 0.011 | 0.892 | 0.933 | --- | --- | 17786.052 |

# LEVEL-2 OUTPUT

= level-2 estimate

= level-1 estimate

Latent Variable: beta0j

Grand Mean Centered: Pain.mean[Person] SleepQual.mean[Person]
Stress

| Parameters | Median | StdDev | 2.5% | 97.5% | ChiSq | PValue | N_Eff |
|---|---|---|---|---|---|---|---|
| Variances: | | | | | | | |
| Residual Var. | **1.835** | 0.250 | 1.423 | 2.399 | --- | --- | 15231.617 |
| Coefficients: | | | | | | | |
| Intercept | **5.039** | 0.216 | 4.608 | 5.466 | 541.498 | 0.000 | 8572.715 |
| SleepQual.mean[Person] | **0.535** | 0.101 | 0.337 | 0.734 | 27.999 | 0.000 | 11316.753 |
| Pain.mean[Person] | **-0.038** | 0.081 | -0.198 | 0.123 | 0.219 | 0.640 | 14391.068 |
| Female.1 | **-0.008** | 0.259 | -0.512 | 0.500 | 0.001 | 0.972 | 16962.379 |
| Stress | **-0.215** | 0.132 | -0.473 | 0.042 | 2.664 | 0.103 | 16321.312 |
| Standardized Coefficients: | | | | | | | |
| SleepQual.mean[Person] | 0.454 | 0.075 | 0.294 | 0.587 | 35.928 | 0.000 | 11569.228 |
| Pain.mean[Person] | -0.039 | 0.082 | -0.198 | 0.123 | 0.221 | 0.638 | 14296.578 |
| Female.1 | -0.002 | 0.077 | -0.153 | 0.149 | 0.001 | 0.971 | 16956.623 |
| Stress | -0.136 | 0.082 | -0.291 | 0.026 | 2.738 | 0.098 | 16320.026 |
| Proportion Variance Explained | | | | | | | |
| by Coefficients | 0.298 | 0.063 | 0.174 | 0.418 | --- | --- | 12568.753 |
| by Residual Variation | 0.702 | 0.063 | 0.582 | 0.826 | --- | --- | 12568.753 |