

MODULE 5

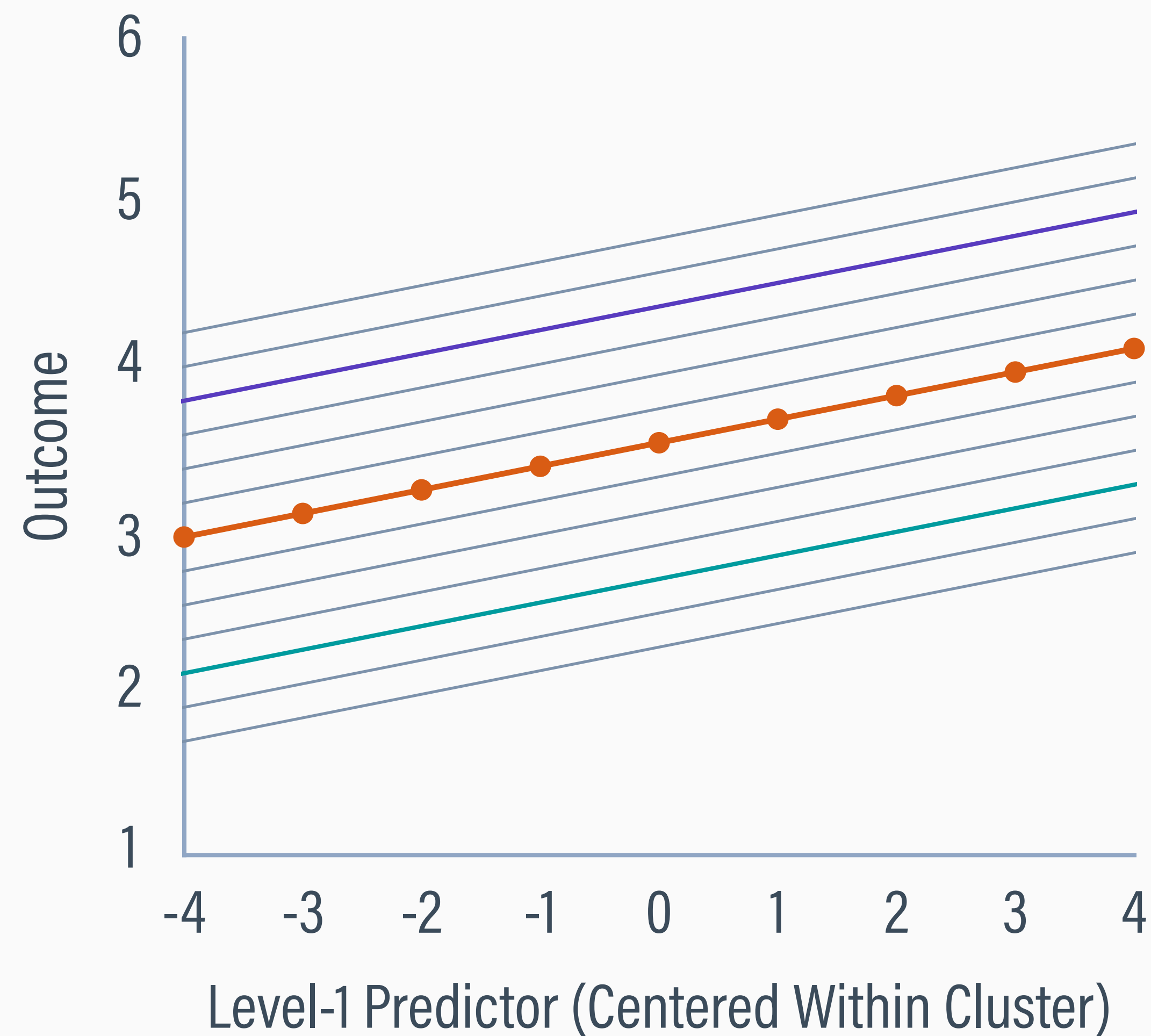
RANDOM COEFFICIENT MODELS

RANDOM SLOPE COEFFICIENTS

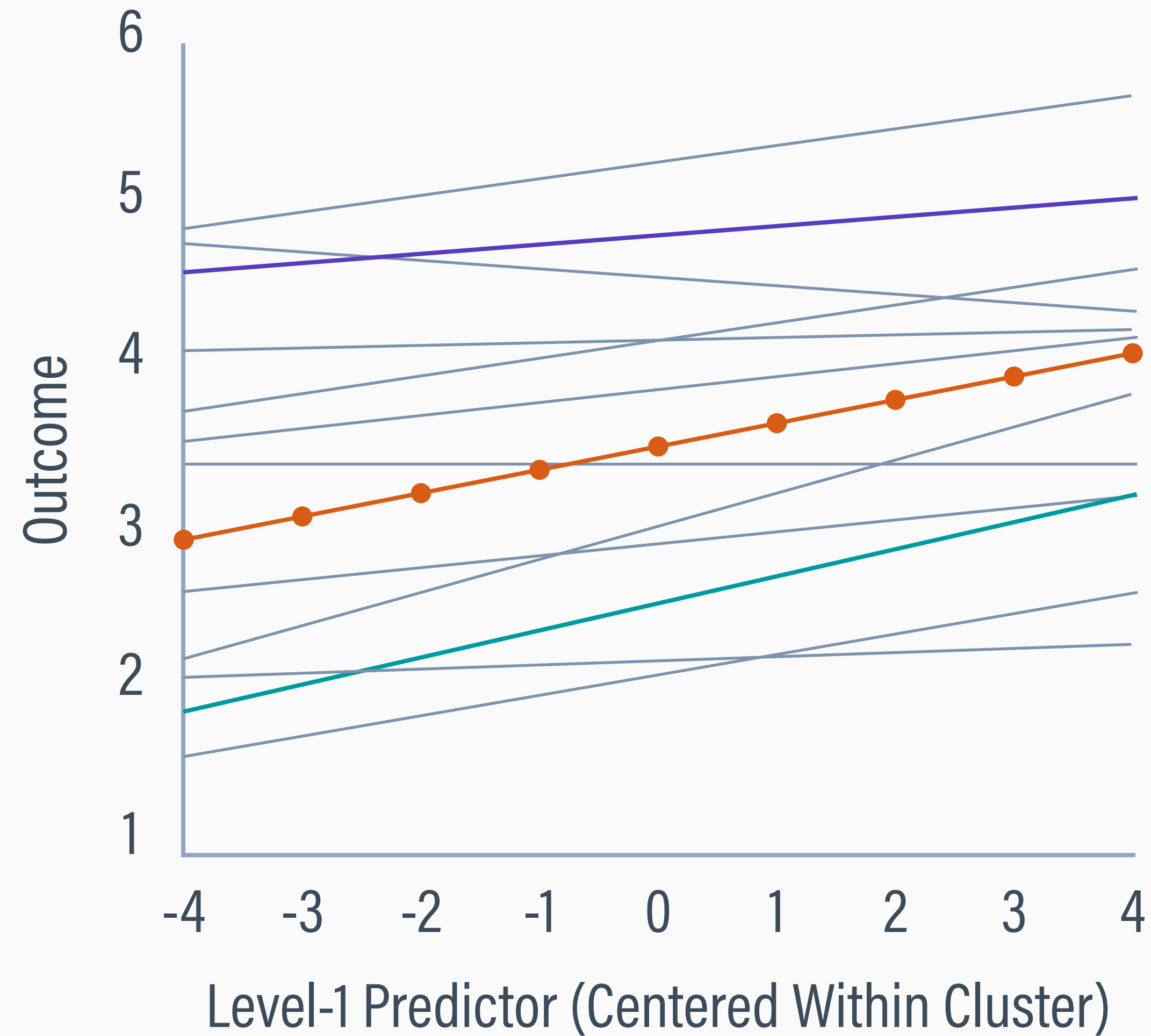
- Each cluster contains multiple observations, effectively forming its own dataset and allowing for cluster-specific slopes
- Random slope (random coefficient) models allow the effect of a level-1 predictor to vary across level-2 units
- The influence of a level-1 predictor on the outcome may be stronger in some clusters than in others
- Level-2 predictors cannot have cluster-varying slopes because they have only one observed value per cluster

MODELING OVERVIEW

Random Intercept Model

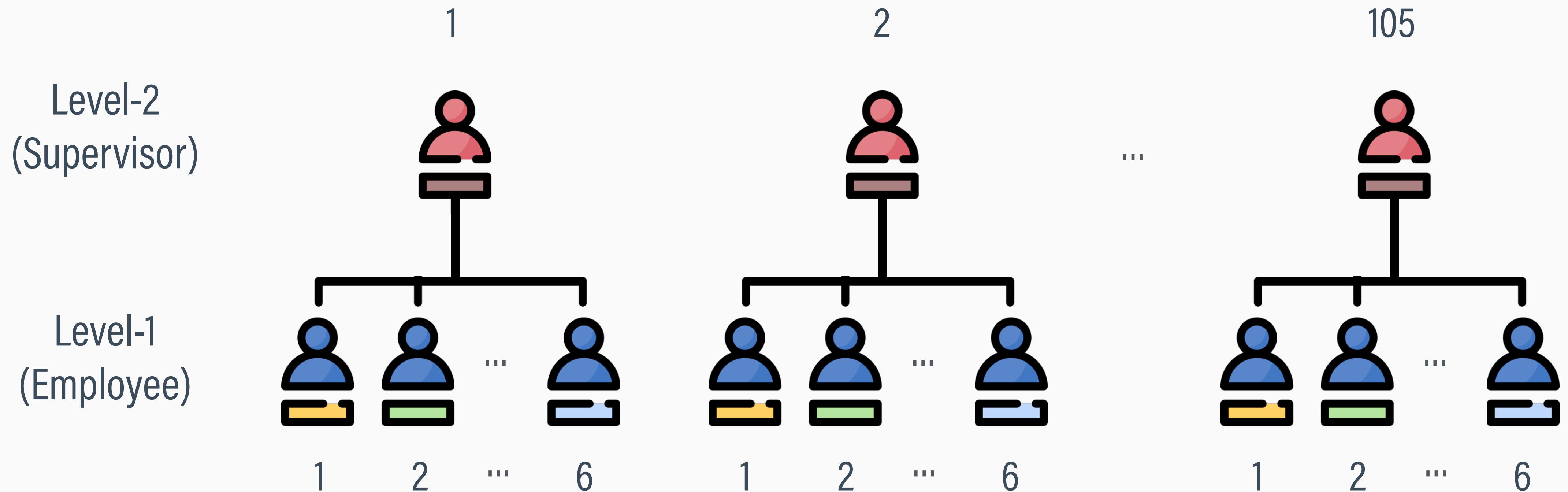


Random Slope Model



ORGANIZATIONAL APPLICATION

- $n_j = 6$ employees at level-1 nested within $J = 105$ teams or workgroups at level-2 ($N = 630$ data records in total)



VARIABLE INFORMATION

- = predictor measured at level-2
- = predictor measured at level-1
- = outcome measured at level-1

empower = lmx + male + climate

Variable	Definition	Level	Scale
Team	Team-level (level-2) identifier	2	Integers (1 to 105)
Empower	Employee empowerment	1	Numeric (14 to 42)
LMX	Leader–member exchange (supervisor–supervisee relationship quality)	1	Numeric (0 to 17)
Male	Sex dummy code	1	Female = 0, Male = 1
Climate	Leadership climate	2	Numeric (10 to 33)

DATA STRUCTURE

- Stacked (long) data format where each level-2 unit (supervisor) has one row per level-1 (supervisee) observation
- The i subscript indexes level-1 observations, and j indexes level-2 units
- Variables measured at level-2 repeat across all rows within a cluster

Row	i	j	JobSat_{ij}	LMX_{ij}	Climate_j	
1	1	1	1.0	5.6	7.3	Team 1
2	2	1	2.0	4.3	7.3	
...	...	1	
6	6	1	5.0	7.3	7.3	
7	1	2	7.0	3.9	4.0	Team 2
8	2	2	3.0	7.1	4.0	
...	...	2	
12	6	2	6.0	3.5	4.0	
...	
625	1	105	2.0	5.4	3.3	Team 105
626	2	105	3.0	3.5	3.3	
...	...	105		
630	6	105	1.0	7.9	3.3	

OUTLINE

1

Modeling Step 1: Estimate ICCs

2

Modeling Step 2: Random Intercept Model

3

Modeling Step 3: Random Slope Model 1

4

Deciding Whether a Random Slope Is Needed

5

Modeling Step 3: Random Slope Model 2

6

Checking MLM Assumptions

7

Latent Variable Specification

ESTIMATING INTRACLASST CORRELATIONS

- Estimate intraclass correlations (ICCs) before model fitting to assess how much variance lies within and between clusters
- Some predictors have little or no level-2 variation, in which case they are already “pure” within-cluster variables
- Disaggregating a level-1 predictor and modeling its means is only warranted when there is sufficient level-2 variation

BLIMP SCRIPT 5.1

DATA: Employee.dat;

VARIABLES: Employee Team Turnover Male Empower LMX JobSat Climate TeamPerf;

CLUSTERID: Team;

NOMINAL: Male; # invokes logistic regression model

MODEL:

{ Empower LMX Male } ~ intercept | intercept; # empty model for all variables in { }

BURN: 20000;


ITERATIONS: 10000;

SEED: 90291;

RBLIMP SCRIPT 5 (MODEL 1)

```
model1 <- rblimp(  
  data = Employee,  
  nominal = 'Male',  
  clusterid = 'Team',  
  model = '{ Empower LMX Male } ~ intercept | intercept',  
  seed = 90291,  
  burn = 10000,  
  iter = 20000)  
output(model1)
```

BLIMP OUTPUT


 = level-2 estimate

 = level-1 estimate

Outcome Variable: Empower

Parameters	Estimate	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
Variances:							
L2 : Var(Intercept)	2.552	0.811	1.268	4.413	---	---	1120.636
Residual Var.	18.067	1.122	16.076	20.457	---	---	5080.598
Coefficients:							
Intercept	28.609	0.231	28.154	29.065	15296.126	0.000	3350.180
Standard Deviations:							
L2 : SD(Intercept)	1.598	0.250	1.126	2.101	---	---	1061.977
Residual SD	4.251	0.131	4.010	4.523	---	---	5088.976
Proportion Variance Explained							
by Coefficients	0.000	0.000	0.000	0.000	---	---	nan
by Level-2 Random Intercepts	0.124	0.036	0.064	0.200	---	---	1109.611
by Level-1 Residual Variation	0.876	0.036	0.800	0.936	---	---	1109.611

BLIMP OUTPUT


 = level-2 estimate

 = level-1 estimate

Outcome Variable: LMX

Parameters	Estimate	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
Variances:							
L2 : Var(Intercept)	1.313	0.391	0.668	2.185	---	---	1038.524
Residual Var.	7.906	0.494	7.014	8.948	---	---	4320.947
Coefficients:							
Intercept	9.594	0.160	9.283	9.910	3602.019	0.000	3114.055
Standard Deviations:							
L2 : SD(Intercept)	1.146	0.170	0.817	1.478	---	---	940.443
Residual SD	2.812	0.088	2.648	2.991	---	---	4350.316
Proportion Variance Explained							
by Coefficients	0.000	0.000	0.000	0.000	---	---	nan
by Level-2 Random Intercepts	0.143	0.038	0.074	0.223	---	---	1023.422
by Level-1 Residual Variation	0.857	0.038	0.777	0.926	---	---	1023.422

BLIMP OUTPUT

 = level-2 estimate

 = level-1 estimate

Outcome Variable: Male

Parameters	Estimate	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
Male.1							
Variances:							
L2 : Var(Intercept)	0.094	0.102	0.004	0.372	---	---	145.999
Coefficients:							
Intercept	-0.066	0.087	-0.237	0.108	0.575	0.448	7164.489
Standard Deviations:							
L2 : SD(Intercept)	0.307	0.146	0.063	0.610	---	---	112.963
Odds Ratio:							
Intercept	0.936	0.082	0.789	1.114	---	---	7166.122
Proportion Variance Explained							
by Coefficients	0.000	0.000	0.000	0.000	---	---	nan
by Level-2 Random Intercepts	0.028	0.028	0.001	0.102	---	---	140.269
by Level-1 Residual Variation	0.972	0.028	0.898	0.999	---	---	140.269



The ICC for sex is $\sim .03$. Sex is dummy coded such that female = 0 and male = 1. In small groups of two or three, discuss the meaning of the team-level gender averages (random intercepts). What does it mean for gender to have a non-zero ICC?

SUMMARY

- Both the outcome and LMX predictor have salient level-2 variation in the team-specific means ($ICC_{\text{empower}} = .12$ and $ICC_{\text{lmx}} = .14$)
- This feature indicates the need for MLM (independence violation) with a disaggregated predictor
- The level-1 sex dummy code has very little level-2 variation in its team-specific means ($ICC_{\text{male}} = .03$)
- Disaggregating sex is unnecessary because it already functions primarily as a within-team predictor

OUTLINE

1

Modeling Step 1: Estimate ICCs

2

Modeling Step 2: Random Intercept Model

3

Modeling Step 3: Random Slope Model 1

4

Deciding Whether a Random Slope Is Needed

5

Modeling Step 3: Random Slope Model 2

6

Checking MLM Assumptions

7

Latent Variable Specification

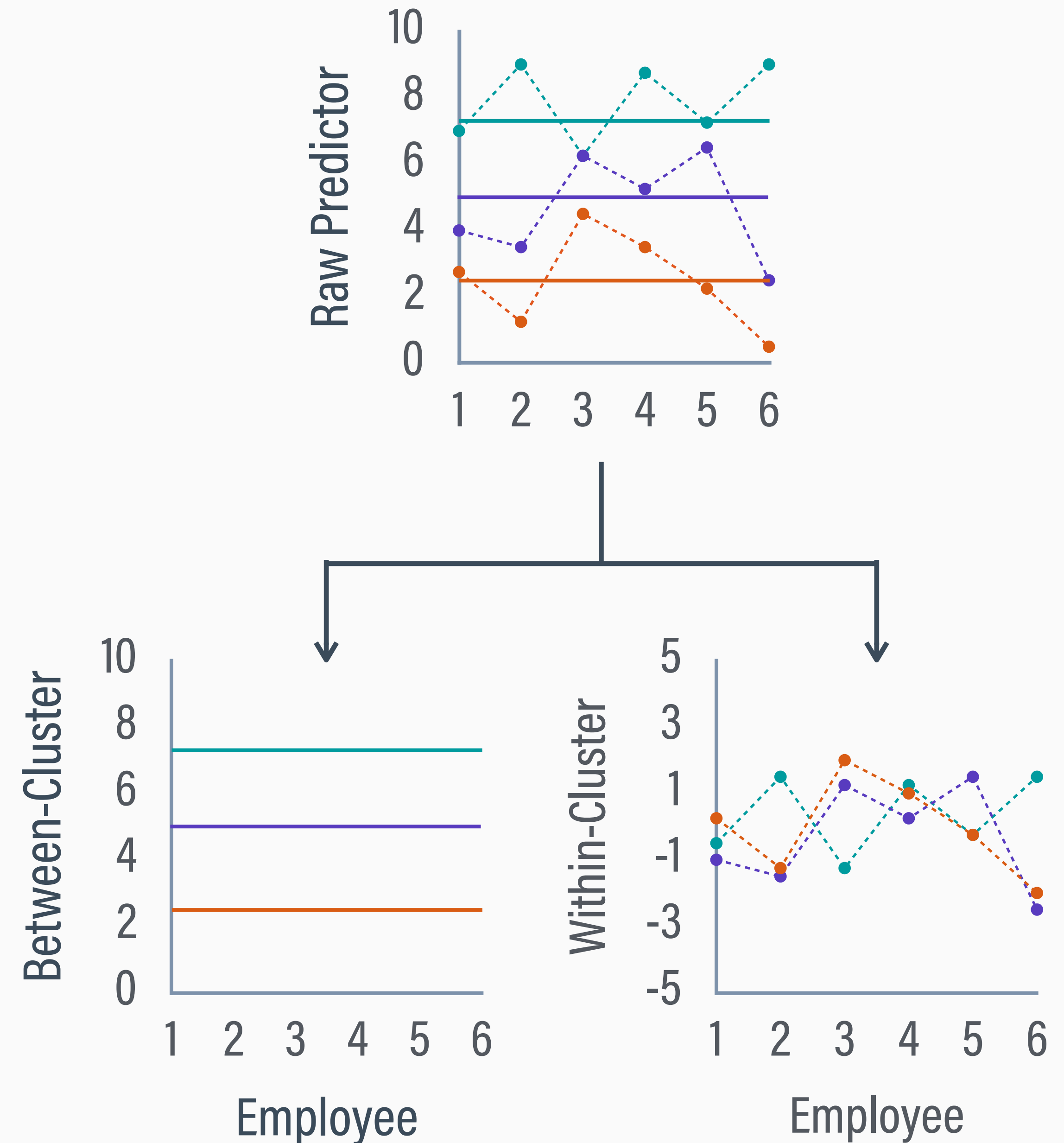
DISAGGREGATED PREDICTOR

- Disaggregation centers each person's score around its level-2 team mean

$$\text{Imx}_j^b = \mu_{j(\text{Imx})}$$

$$\text{Imx}_{ij}^w = \text{Imx}_{ij} - \mu_{j(\text{Imx})}$$

- Imx^w contains only within-team (level-1) variation, and Imx^b reflects only team-level (level-2) mean differences



WITHIN-CLUSTER (LEVEL-1) MODEL

- Empowerment score for employee i in team j is the sum of a level-2 mean (β_{0j}), fixed effects due to within-team predictors (β_1 and β_2), and a within-team residual (ε_{ij})

$$\text{empower}_{ij} = \beta_{0j} + \beta_1(\text{lm}x_{ij}^w) + \beta_2(\text{male}_{ij}) + \varepsilon_{ij}$$

- Assumption: residuals are normal with constant variation across all employees (level-1 units) and teams (level-2 units)

$$\varepsilon_{ij} \sim N(0, \sigma_\varepsilon^2)$$

BETWEEN-CLUSTER (LEVEL-2) MODEL

- ◉ The empowerment mean for team j (β_{0j}) is the sum of the grand mean (γ_{00}), fixed effects due to team-average LMX and climate, and a between-team residual (u_{0j})

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{lm}x_j^b) + \gamma_{02}(\text{climate}_j) + u_{0j}$$

$$\beta_1 = \gamma_{10}$$

$$\beta_2 = \gamma_{20}$$

- ◉ Assumption: random intercept residuals are normal with constant variation across teams (level-2 units)

$$u_{0j} \sim N(0, \sigma_u^2)$$



In small groups of two or three, perform the substitution demonstrated in the Module 4 slides to get the combined-model notation from the Raudenbush and Bryk (2002) book.

$$\begin{aligned} \text{empower}_{ij} = & \gamma_{00} + \gamma_{10}(\text{lm}x_{ij}^w) + \gamma_{20}(\text{male}_{ij}) \\ & + \gamma_{01}(\text{lm}x_j^b) + \gamma_{02}(\text{climate}_j) + u_{0j} + \varepsilon_{ij} \end{aligned}$$

COMMON NOTATIONAL SYSTEMS

Combined-model equation (Raudenbush & Bryk, 2002)

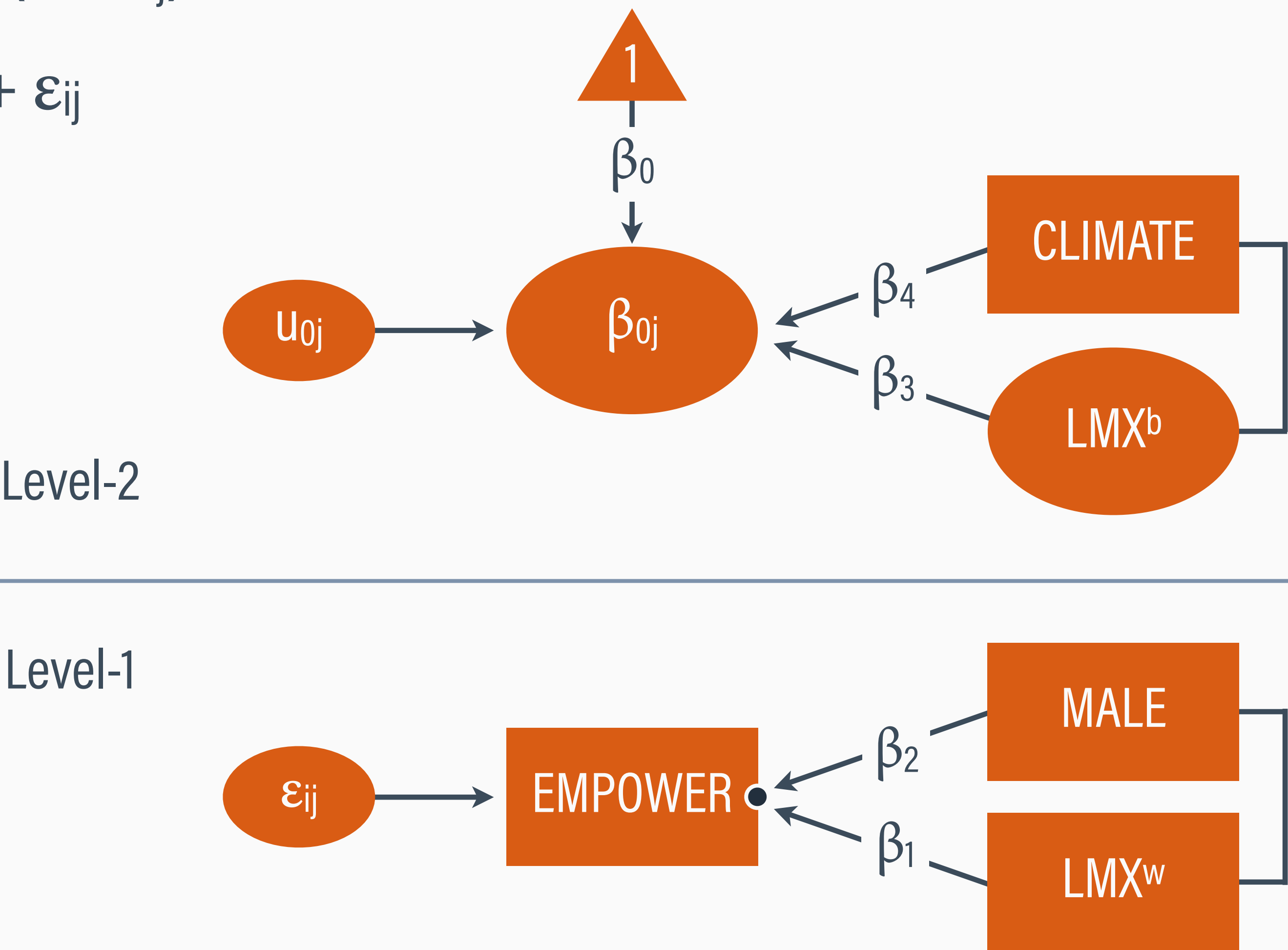
$$\text{empower}_{ij} = \gamma_{00} + \gamma_{10}(\text{lm}x_{ij}^w) + \gamma_{20}(\text{male}_{ij}) \\ + \gamma_{01}(\text{lm}x_j^b) + \gamma_{02}(\text{climate}_j) + u_{0j} + \varepsilon_{ij}$$

Standard(ish) regression notation (Scott, Shrout, & Weinberg, 2013)

$$\text{empower}_{ij} = \beta_0 + \beta_1(\text{lm}x_{ij}^w) + \beta_2(\text{male}_{ij}) \\ + \beta_3(\text{lm}x_j^b) + \beta_4(\text{climate}_j) + u_{0j} + \varepsilon_{ij}$$

PATH DIAGRAM

$$\text{empower}_{ij} = (\beta_0 + u_{0j}) + \beta_1(\text{lmx}_{ij}^w) + \beta_2(\text{male}_{ij}) + \beta_3(\text{lmx}_j^b) + \beta_4(\text{climate}_j) + \varepsilon_{ij}$$



• = random intercept (β_{0j})

BLIMP SCRIPT 5.2

DATA: Employee.dat;

VARIABLES: Employee Team Turnover Male Empower LMX JobSat Climate TeamPerf;

NOMINAL: Male; # invokes categorical variable model

CLUSTERID: Team;

CENTER: groupmean = LMX; grandmean = LMX.mean Climate;

MODEL: Empower ~ intercept LMX Male LMX.mean Climate | intercept;

BURN: 10000;

ITERATIONS: 10000;

SEED: 90291;

RBLIMP SCRIPT 5 (MODEL 2)

```
model2 <- rblimp(  
  data = Employee,  
  nominal = 'Male',  
  clusterid = 'Team',  
  center = 'groupmean = LMX; grandmean = LMX.mean Climate',  
  model = 'Empower ~ intercept LMX Male LMX.mean Climate | intercept',  
  seed = 90291,  
  burn = 10000,  
  iter = 10000)  
output(model2)
```


BLIMP OUTPUT

- = level-2 estimate
- = level-1 estimate
- = combined estimate

Outcome Variable: Empower

Grand Mean Centered: Climate LMX.mean[Team]

Group Mean Centered: LMX

Parameters	Estimate	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff

Variances:							
L2 : Var(Intercept)	0.890	0.562	0.097	2.225	---	---	202.846
Residual Var.	13.531	0.838	12.010	15.270	---	---	2690.721
Coefficients:							
Intercept	27.689	0.267	27.170	28.216	10771.431	0.000	2528.639
LMX	0.673	0.056	0.563	0.782	143.910	0.000	4166.700
Male.1	1.894	0.305	1.297	2.494	38.680	0.000	8259.863
LMX.mean[Team]	-0.251	0.311	-0.964	0.268	0.777	0.378	362.172
Climate	0.282	0.052	0.183	0.385	29.835	0.000	1313.674
Standard Deviations:							
L2 : SD(Intercept)	0.943	0.300	0.312	1.492	---	---	166.959
Residual SD	3.678	0.113	3.466	3.908	---	---	2688.662

...

FIXED EFFECT INTERPRETATIONS

- ◉ $\beta_0 = 27.69$ is the empowerment mean for the male = 0 group (because all numeric predictors are centered)
- ◉ $\beta_1 = 0.67$ is the expected empowerment difference between two LMX scores from the same team that differ by one point, controlling for sex
- ◉ $\beta_2 = 1.89$ is the expected empowerment difference for males (relative to females), controlling for within-team LMX

FIXED EFFECTS, CONTINUED

- $\beta_3 = -0.25$ is the expected empowerment difference between two teams whose average LMX ratings differ by one point, controlling for team-level climate
- $\beta_4 = 0.28$ is the expected empowerment difference between two teams whose climate ratings differ by one point, controlling for team-average LMX

RANDOM EFFECT INTERPRETATIONS

- $u_{0j} = \beta_{0j} - (\beta_0 + \beta_3(LMX_j^b) + \beta_4(climate_j))$
- $var(u_{0j}) = 0.89$ is the average squared distance between the level-2 empowerment means and their predicted values
- $sd(u_{0j}) = 0.94$ is the average distance between the level-2 empowerment means and their predicted values
- $\varepsilon_{ij} = empower_{ij} - (\beta_{0j} + \beta_1(lmx_{ij}^w) + \beta_2(male_{ij}))$
- $var(\varepsilon_{ij}) = 13.53$ is the average squared distance between the level-1 empowerment scores and their predicted values
- $sd(\varepsilon_{ij}) = 3.68$ is the average distance between the level-1 empowerment scores and their predicted values

BLIMP OUTPUT, CONTINUED

 = level-2 estimate

 = level-1 estimate

 = combined estimate

Outcome Variable: Empower

Grand Mean Centered: Climate LMX.mean[Team]

Group Mean Centered: LMX

Parameters	Estimate	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff

...							
Standardized Coefficients:							
LMX	0.415	0.031	0.351	0.474	174.148	0.000	1976.422
Male.1	0.208	0.032	0.143	0.270	40.965	0.000	8210.609
LMX.mean[Team]	-0.058	0.063	-0.181	0.068	0.830	0.362	425.266
Climate	0.248	0.044	0.163	0.334	31.957	0.000	1311.770
Proportion Variance Explained							
by Coefficients	0.305	0.032	0.241	0.369	---	---	1224.888
by Level-2 Random Intercepts	0.043	0.026	0.005	0.104	---	---	199.719
by Level-1 Residual Variation	0.649	0.034	0.583	0.716	---	---	850.733

EFFECT SIZE INTERPRETATIONS

- $R^2_{\text{(predictors)}} = .31$ is the proportion of the total variation explained by all predictors
- $R^2_{\text{(residual-between)}} = .04$ is the proportion of the total variation attributable to the between-cluster residuals (the u_{0j} terms)
- $R^2_{\text{(residual-within)}} = .65$ is the proportion of the total variation attributable to the within-cluster residuals (the ε_{ij} terms)

OUTLINE

1

Modeling Step 1: Estimate ICCs

2

Modeling Step 2: Random Intercept Model

3

Modeling Step 3: Random Slope Model 1

4

Deciding Whether a Random Slope Is Needed

5

Modeling Step 3: Random Slope Model 2

6

Checking MLM Assumptions

7

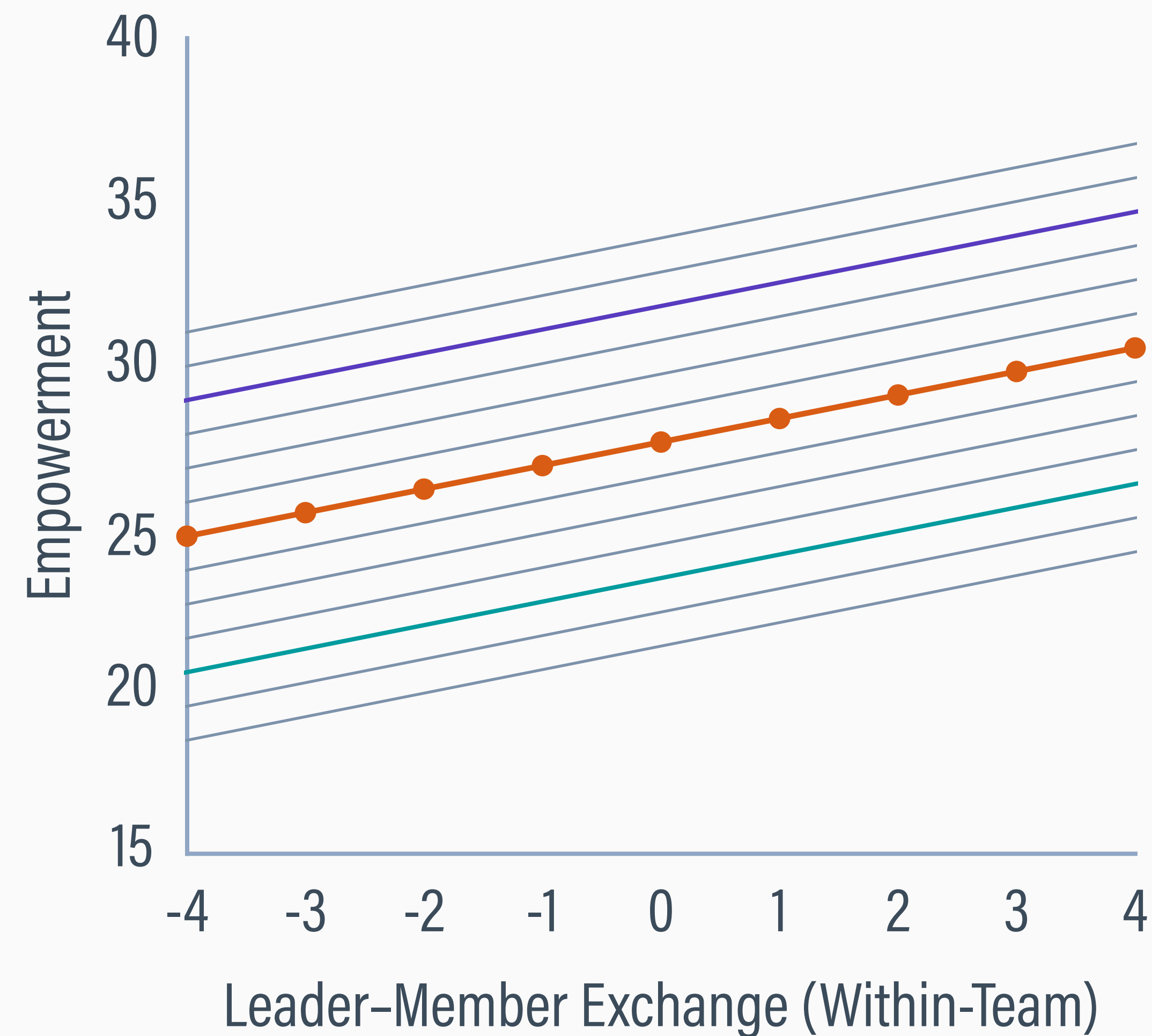
Latent Variable Specification

RANDOM SLOPE COEFFICIENTS

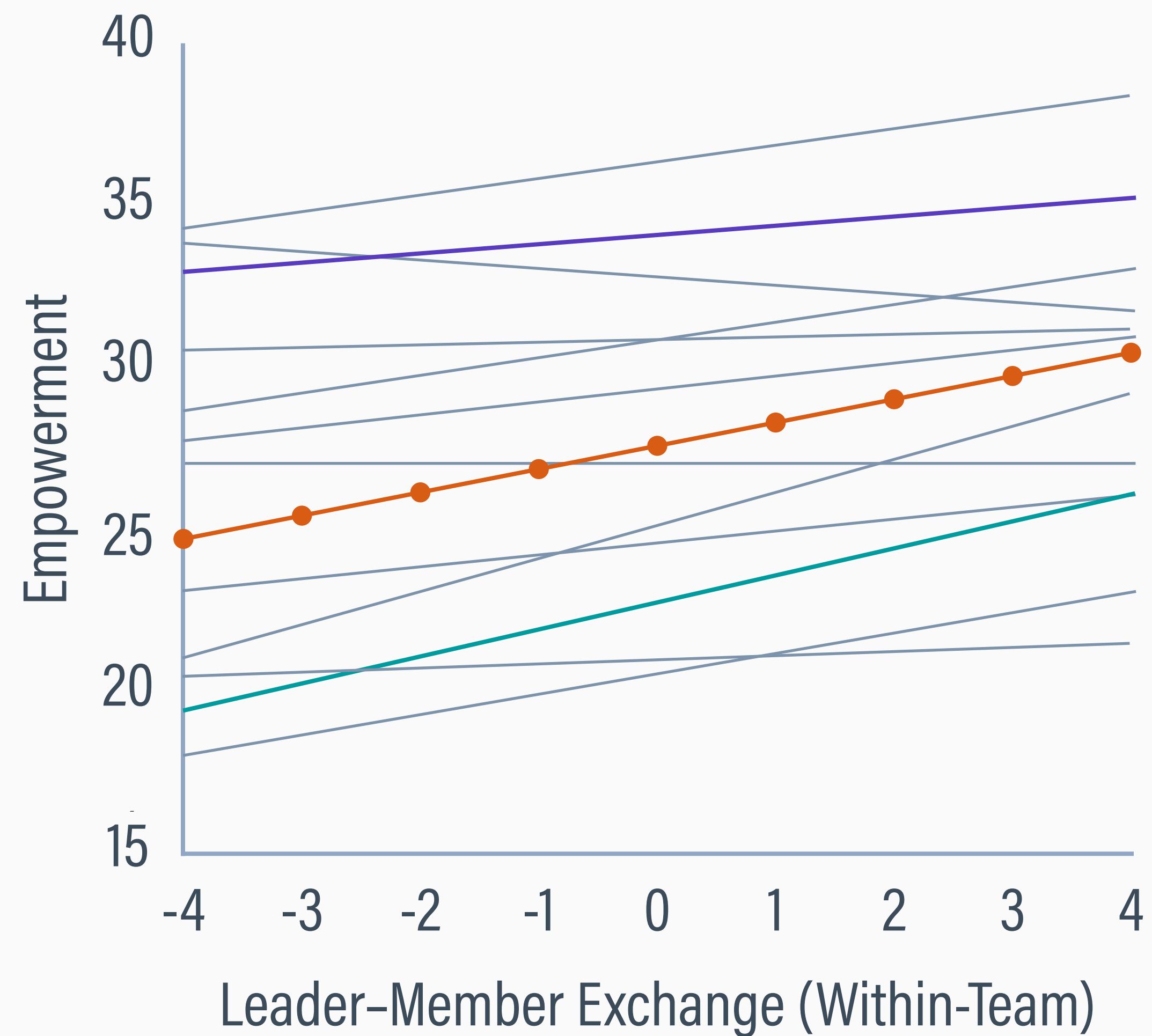
- Random slope or random coefficient models allow the influence of a level-1 predictor to vary across level-2 units
- The influence of LMX or sex could vary across teams
- e.g., In some teams, increases in supervisor–supervisee relationship quality (LMX) may strongly influence empowerment, whereas in others the link may be weaker

MODEL COMPARISON

Random Intercept Model



Random Slope Model



WITHIN-CLUSTER (LEVEL-1) MODEL

- Both the team-specific empowerment mean (β_{0j}) and the leader-member exchange slope vary across clusters (β_{1j}), but the sex slope (β_2) is constant

$$\text{empower}_{ij} = \beta_{0j} + \beta_{1j}(\text{lm}x_{ij}^w) + \beta_2(\text{male}_{ij}) + \varepsilon_{ij}$$

- Assumption: residuals are normal with constant variation across all employees (level-1 units) and teams (level-2 units)

$$\varepsilon_{ij} \sim N(0, \sigma_\varepsilon^2)$$

BETWEEN-CLUSTER (LEVEL-2) MODEL

- The empowerment mean for team j (β_{0j}) is the sum of the grand mean (γ_{00}), fixed effects (γ_{01} and γ_{02}), and a between-team residual (u_{0j})
- The LMX slope for team j (β_{1j}) is the sum of the mean slope (γ_{10}) and a team-level residual (u_{1j})
- Assumption: random intercept and slope residuals are bivariate normal *and correlated*

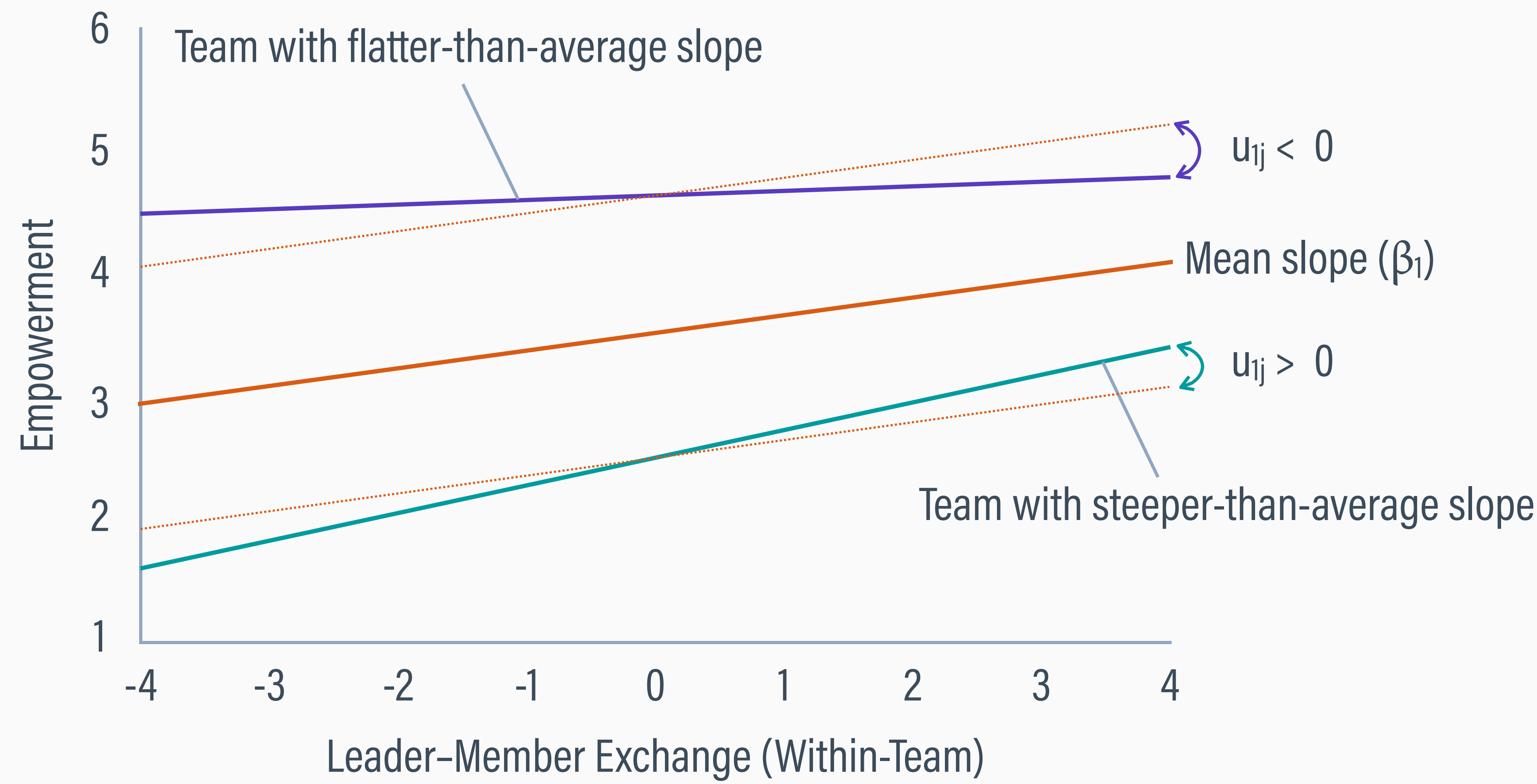
$$\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{lmx}_j^b) + \gamma_{02}(\text{climate}_j) + u_{0j}$$

$$\beta_{1j} = \gamma_{10} + u_{1j}$$

$$\beta_{2j} = \gamma_{20}$$

$$\begin{pmatrix} u_{0j} \\ u_{1j} \end{pmatrix} \sim N(0, \Sigma_u) \quad \Sigma_u = \begin{pmatrix} \sigma_{u0}^2 & \sigma_{u0u1} \\ \sigma_{u1u0} & \sigma_{u1}^2 \end{pmatrix}$$

SLOPE RESIDUAL GRAPHIC



DECODING THE SUBSCRIPTS

The second subscript numbers the coefficients on the right side starting at 0 (the intercept)

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{lmx}_j) + \gamma_{02}(\text{climate}_j) + u_{0j}$$

$$\beta_{1j} = \gamma_{10} + u_{1j}$$

$$\beta_{2j} = \gamma_{20}$$

The first subscript tells which level-2 outcome variable these terms are attached to (γ_{00} and u_{0j} belong in β_{0j} 's equation, γ_{10} is attached to β_{1j})

COMBINED-MODEL EQUATION

Substituting the right sides
of the level-2 equations ...

into their coefficients
from the level-1 equation

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{lm}x_j^b) + \gamma_{02}(\text{climate}_j) + u_{0j}$$

$$\beta_1 = \gamma_{10} + u_{1j}$$

$$\beta_2 = \gamma_{20}$$

$$\text{jobsat}_{ij} = \beta_{0j} + \beta_{1j}(\text{lm}x_{ij}^w) + \beta_2(\text{male}_{ij}) + \varepsilon_{ij}$$

gives the combined-model regression equation (Raudenbush & Bryk, 2002)

Leading 0 subscript conveys team-level effects

$$\text{empower}_{ij} = (\gamma_{00} + u_{0j}) + \gamma_{01}(\text{lm}x_j^b) + \gamma_{02}(\text{climate}_j) + (\gamma_{10} + u_{1j})(\text{lm}x_{ij}^w) + \gamma_{20}(\text{male}_{ij}) + \varepsilon_{ij}$$

Non-zero subscript conveys within-team effects

COMMON NOTATIONAL SYSTEMS

Combined-model equation (Raudenbush & Bryk, 2002)

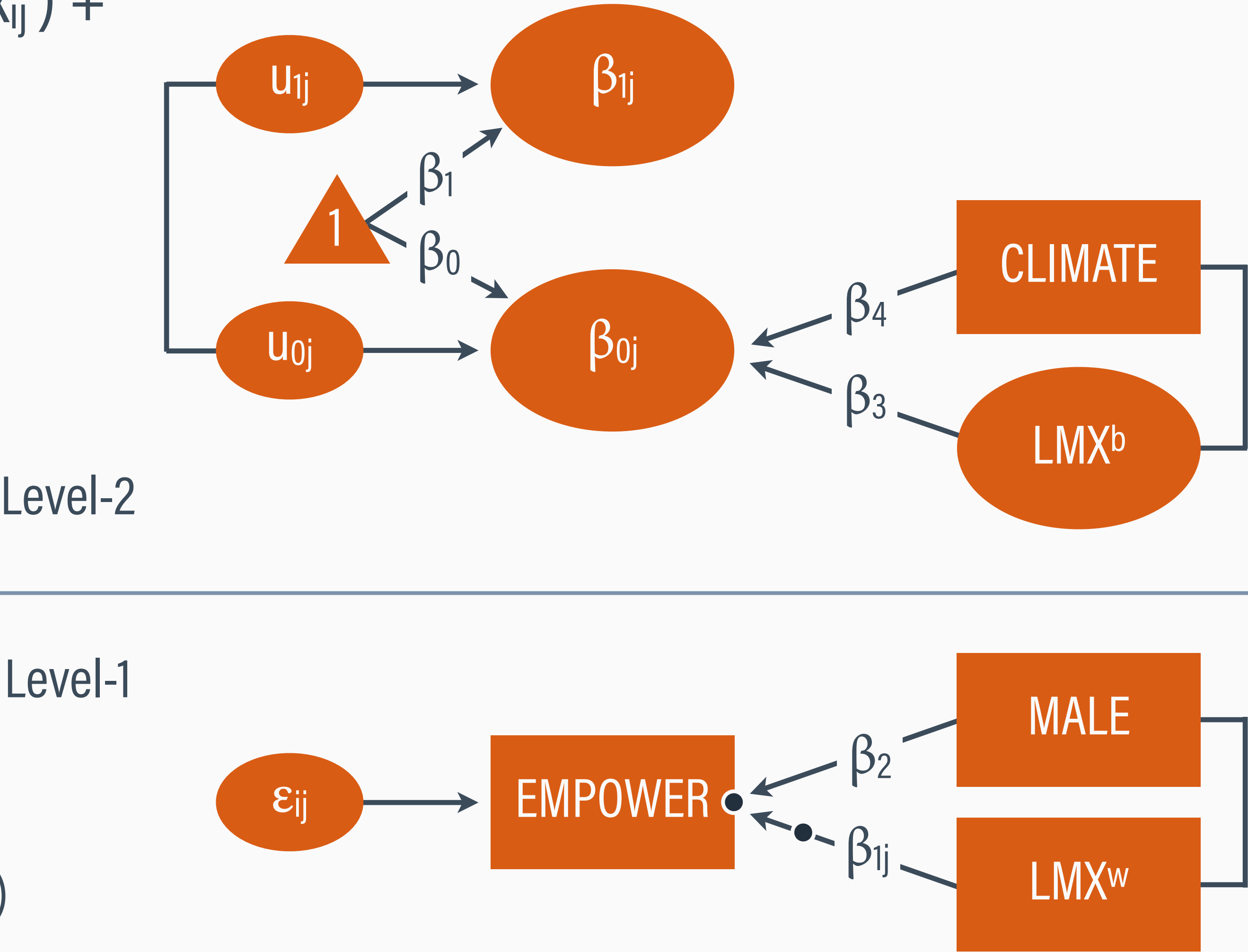
$$\text{empower}_{ij} = (\gamma_{00} + u_{0j}) + (\gamma_{10} + u_{1j})(\text{lmx}_{ij}^w) + \gamma_{20}(\text{male}_{ij}) + \gamma_{01}(\text{lmx}_j^b) + \gamma_{02}(\text{climate}_j) + \varepsilon_{ij}$$

Standard(ish) regression notation (Scott, Shrout, & Weinberg, 2013)

$$\text{empower}_{ij} = (\beta_0 + u_{0j}) + (\beta_1 + u_{1j})(\text{lmx}_{ij}^w) + \beta_2(\text{male}_{ij}) + \beta_3(\text{lmx}_j^b) + \beta_4(\text{climate}_j) + \varepsilon_{ij}$$

PATH DIAGRAM

$$\text{empower}_{ij} = (\beta_0 + u_{0j}) + (\beta_1 + u_{1j})(\text{lmx}_{ij}^w) + \beta_2(\text{male}_{ij}) + \beta_3(\text{lmx}_j^b) + \beta_4(\text{climate}_j) + \varepsilon_{ij}$$



• = random intercept (β_{0j}) and random slope (β_{1j})

BLIMP SCRIPT 5.3

DATA: Employee.dat;

VARIABLES: Employee Team Turnover Male Empower LMX JobSat Climate TeamPerf;

NOMINAL: Male; # automatic dummy coding for nominal predictors

CLUSTERID: Team;

CENTER: groupmean = LMX; grandmean = LMX.mean Climate;

MODEL: Empower ~ intercept LMX Male LMX.mean Climate | intercept LMX;

BURN: 10000;

ITERATIONS: 10000;

SEED: 90291;

RBLIMP SCRIPT 5 (MODEL 3)

```
model3 <- rblimp(  
  data = Employee,  
  nominal = 'Male',  
  clusterid = 'Team',  
  center = 'groupmean = LMX; grandmean = LMX.mean Climate',  
  model = 'Empower ~ intercept LMX Male LMX.mean Climate | intercept LMX',  
  seed = 90291,  
  burn = 10000,  
  iter = 10000)  
output(model3)
```

BLIMP OUTPUT

- = level-2 estimate
- = level-1 estimate
- = combined estimate

Outcome Variable: Empower

Grand Mean Centered: Climate LMX.mean[Team]
Group Mean Centered: LMX

Parameters	Estimate	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff

Variances:							
L2 : Var(Intercept)	1.077	0.563	0.252	2.412	---	---	197.283
L2 : Cov(LMX,Intercept)	-0.172	0.147	-0.489	0.097	---	---	324.463
L2 : Var(LMX)	0.188	0.076	0.064	0.364	---	---	390.430
Residual Var.	12.288	0.824	10.809	14.021	---	---	1868.918
Coefficients:							
Intercept	27.687	0.268	27.161	28.222	10663.961	0.000	2099.378
LMX	0.638	0.072	0.496	0.775	77.733	0.000	2189.207
Male.1	1.893	0.303	1.296	2.488	38.980	0.000	7340.383
LMX.mean[Team]	-0.154	0.297	-0.799	0.361	0.330	0.566	360.867
Climate	0.291	0.052	0.186	0.392	30.822	0.000	1256.661
Standard Deviations:							
L2 : SD(Intercept)	1.038	0.267	0.502	1.553	---	---	162.850
L2 : Cor(LMX,Intercept)	-0.419	0.307	-0.942	0.245	---	---	179.032
L2 : SD(LMX)	0.433	0.088	0.252	0.603	---	---	345.870
Residual SD	3.505	0.117	3.288	3.745	---	---	1865.490

...

FIXED EFFECT INTERPRETATIONS

- ◉ $\beta_0 = 27.69$ is the empowerment mean for the male = 0 group (because all numeric predictors are centered)
- ◉ $\beta_1 = 0.64$ is the expected empowerment difference between two LMX scores from the same team that differ by one point, controlling for sex
- ◉ $\beta_2 = 1.89$ is the expected empowerment difference for males (relative to females), controlling for within-team LMX

FIXED EFFECTS, CONTINUED

- $\beta_3 = -0.15$ is the expected empowerment difference between two teams whose average LMX ratings differ by one point, controlling for team-level climate
- $\beta_4 = 0.29$ is the expected empowerment difference between two teams whose climate ratings differ by one point, controlling for team-average LMX

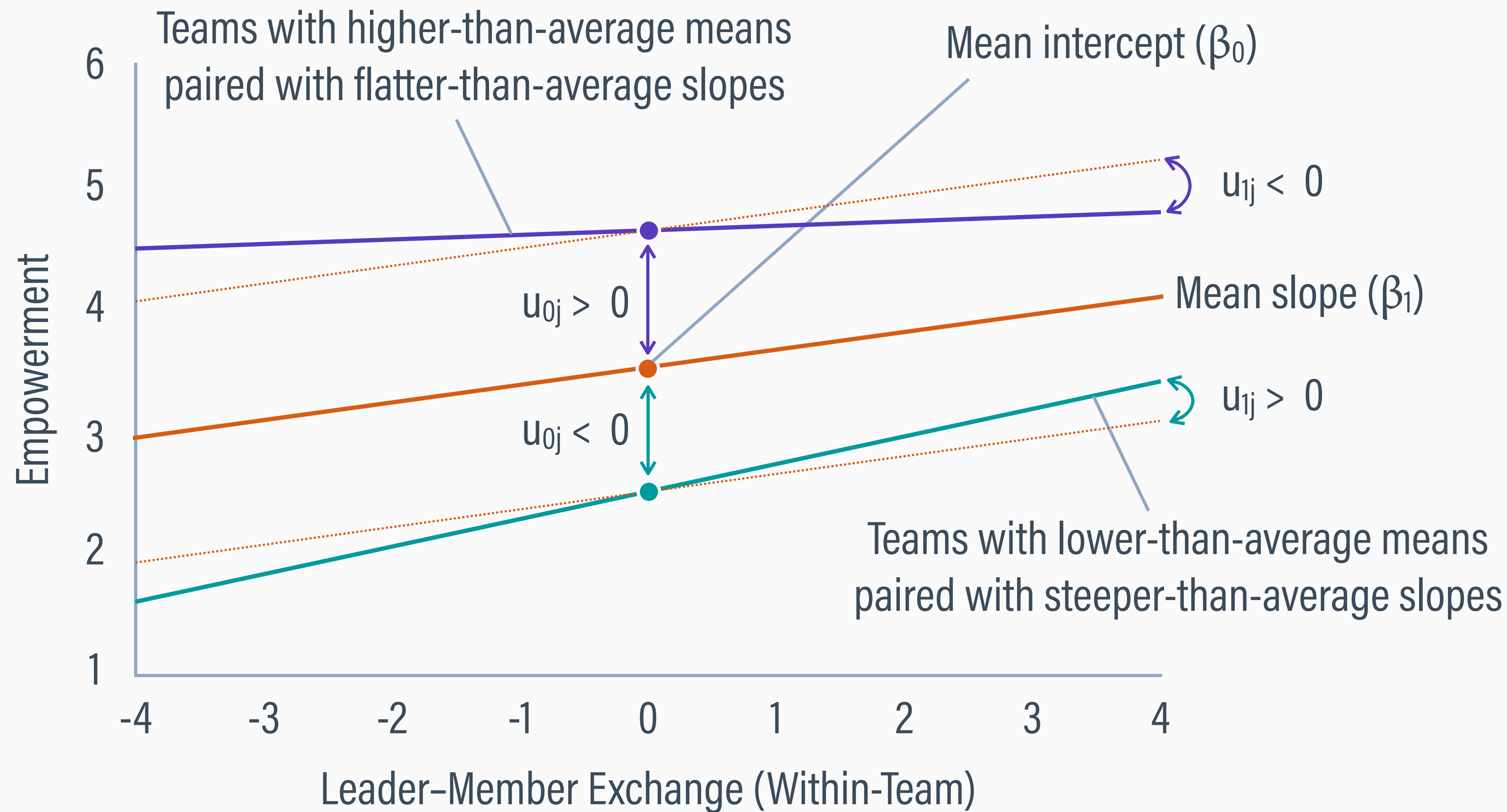
RANDOM EFFECT INTERPRETATIONS

- $u_{0j} = \beta_{0j} - (\beta_0 + \beta_3(LMX_j^b) + \beta_4(climate_j))$
- $var(u_{0j}) = 1.08$ is the average squared distance between the level-2 empowerment means and their predicted values
- $sd(u_{0j}) = 1.04$ is the average distance between the level-2 empowerment means and their predicted values
- $u_{1j} = \beta_{1j} - \beta_1$
- $var(u_{1j}) = 0.19$ is the average squared distance between the team-specific LMX slopes and the average slope
- $sd(u_{1j}) = 0.43$ is the average distance between the team-specific LMX slopes and the average slope

RANDOM EFFECT CORRELATION

- The random intercepts (team-specific means) and random slopes (team-specific LMX effects) are correlated
- The unstandardized association is $\text{cov}(u_{0j}, u_{1j}) = -0.17$, and the linear correlation is $\text{cor}(u_{0j}, u_{1j}) = -.42$
- A positive correlation implies that teams with higher-than-average means (high u_{0j}) tend to have flatter-than-average slopes (low u_{1j}) and vice versa (low u_{0j} paired with high u_{1j})

NEGATIVE CORRELATION GRAPHIC



WITHIN-CLUSTER RESIDUAL VARIANCE

- $\varepsilon_{ij} = \text{empower}_{ij} - (\beta_{0j} + \beta_1(\text{lm}x_{ij}^W) + \beta_2(\text{male}_{ij}))$
- $\text{var}(\varepsilon_{ij}) = 12.29$ is the average squared distance between the level-1 empowerment scores and their predicted values
- $\text{sd}(\varepsilon_{ij}) = 3.51$ is the average distance between the level-1 empowerment scores and their predicted values

OUTLINE

1

Modeling Step 1: Estimate ICCs

2

Modeling Step 2: Random Intercept Model

3

Modeling Step 3: Random Slope Model 1

4

Deciding Whether a Random Slope Is Needed

5

Modeling Step 3: Random Slope Model 2

6

Checking MLM Assumptions

7

Latent Variable Specification

ARE RANDOM SLOPES NEEDED?

- Ignoring important sources of random slope variation can bias the fixed effect coefficients
- However, include random slopes judiciously because overfitting impacts convergence, precision, and power
- Auditioning random slopes one predictor at a time is often an effective strategy, keeping those with support from the data

THREE IMPERFECT CRITERIA

- MCMC diagnostics (subpar values imply overfitting)
- Rights and Sterba (2019) R^2 effect size for random slopes
- Specialized χ^2 test (called a chi-bar test) for nested models involving variance parameters

PSR DIAGNOSTIC OUTPUT

Quality control check: PSRF diagnostics all < 1.05 well before the end of the burn-in period

BURN-IN POTENTIAL SCALE REDUCTION (PSR) OUTPUT:

NOTE: Split chain PSR is being used. This splits each chain's iterations to create twice as many chains.

Comparing iterations across 2 chains	Highest PSR	Parameter #
251 to 500	2.805	10
501 to 1000	1.183	11
751 to 1500	1.090	11
1001 to 2000	1.017	5
...
2501 to 5000	1.054	11
2751 to 5500	1.018	11
3001 to 6000	1.029	11
3251 to 6500	1.020	11
3501 to 7000	1.026	11
3751 to 7500	1.007	29
4001 to 8000	1.009	10
4251 to 8500	1.027	11
4501 to 9000	1.027	11
4751 to 9500	1.018	11
5001 to 10000	1.020	11

BLIMP OUTPUT

Quality control check: All effective sample size (N_Eff) diagnostics > 100

Outcome Variable: Empower

Grand Mean Centered: Climate LMX.mean[Team]
Group Mean Centered: LMX

Parameters	Estimate	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff

Variances:							
L2 : Var(Intercept)	1.077	0.563	0.252	2.412	---	---	197.283
L2 : Cov(LMX,Intercept)	-0.172	0.147	-0.489	0.097	---	---	324.463
L2 : Var(LMX)	0.188	0.076	0.064	0.364	---	---	390.430
Residual Var.	12.288	0.824	10.809	14.021	---	---	1868.918
Coefficients:							
Intercept	27.687	0.268	27.161	28.222	10663.961	0.000	2099.378
LMX	0.638	0.072	0.496	0.775	77.733	0.000	2189.207
Male.1	1.893	0.303	1.296	2.488	38.980	0.000	7340.383
LMX.mean[Team]	-0.154	0.297	-0.799	0.361	0.330	0.566	360.867
Climate	0.291	0.052	0.186	0.392	30.822	0.000	1256.661
Standard Deviations:							
L2 : SD(Intercept)	1.038	0.267	0.502	1.553	---	---	162.850
L2 : Cor(LMX,Intercept)	-0.419	0.307	-0.942	0.245	---	---	179.032
L2 : SD(LMX)	0.433	0.088	0.252	0.603	---	---	345.870
Residual SD	3.505	0.117	3.288	3.745	---	---	1865.490

...

MULTILEVEL EFFECT SIZES

- Numerous effect sizes exist for multilevel models
- Rights and Sterba (2019, Psychological Methods) provides a unifying framework that subsumes different approaches
- They assign an R^2 effect size to the random slopes, which explain within-cluster variation by reducing residual distances to the observed scores









PREDICTED OUTCOME VARIATION

- The model-predicted total variance is the sum of (a) variance explained by predictors at both levels, (b) residual random-intercept variation, (c) within-cluster variation due to random slopes, and (d) residual within-cluster variation

$$\sigma_Y^2 = \boldsymbol{\beta}^T \boldsymbol{\Sigma}_X \boldsymbol{\beta} + \text{var}(u_{0j}) + \text{tr}(\boldsymbol{\Sigma}_u \boldsymbol{\Sigma}_X) + \sigma_\varepsilon^2$$

total variation = explained by predictors + intercept residuals + slope residuals + level-1 residuals

VARIANCE EXPLAINED MEASURES


		Explained	÷	Total
Fixed effects of predictors	$R^2_{\text{predictors}} = \frac{\boldsymbol{\beta}^T \boldsymbol{\Sigma}_X \boldsymbol{\beta}}{\sigma_Y^2}$		÷	
Level-2 random intercept residuals	$R^2_{\text{residual-between}} = \frac{\sigma_{u_0}^2}{\sigma_Y^2}$		÷	
Level-1 via random slope residuals	$R^2_{\text{slopes}} = \frac{\text{tr}(\boldsymbol{\Sigma}_u \boldsymbol{\Sigma}_X)}{\sigma_Y^2}$		÷	
Level-1 within-cluster residuals	$R^2_{\text{residual-within}} = \frac{\sigma_{\varepsilon}^2}{\sigma_Y^2}$		÷	

BLIMP OUTPUT

Outcome Variable: Empower

Grand Mean Centered: Climate LMX.mean[Team]

Group Mean Centered: LMX

 = level-2 estimate

 = level-1 estimate

 = combined estimate

Parameters	Estimate	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
...							
Standardized Coefficients:							
LMX	0.393	0.040	0.310	0.466	95.779	0.000	1718.136
Male.1	0.206	0.032	0.142	0.269	40.841	0.000	6764.759
LMX.mean[Team]	-0.034	0.060	-0.151	0.083	0.323	0.570	437.813
Climate	0.255	0.044	0.164	0.339	33.112	0.000	1199.975
Proportion Variance Explained							
by Coefficients	0.289	0.035	0.220	0.359	---	---	1349.110
by Level-2 Random Intercepts	0.051	0.025	0.012	0.109	---	---	184.323
by Level-2 Random Slopes	0.071	0.028	0.024	0.133	---	---	379.856
by Level-1 Residual Variation	0.584	0.039	0.508	0.660	---	---	872.097

EFFECT SIZE INTERPRETATIONS

- $R^2_{\text{(predictors)}} = .29$ is the proportion of the total variation explained by predictors at both levels
- $R^2_{\text{(residual-between)}} = .05$ is the proportion of the total variation attributable to the random intercept residuals (the u_{0j} terms)
- $R^2_{\text{(slopes)}} = .07$ is the proportion of the total variation attributable to the random slope residuals (the u_{1j} terms)
- $R^2_{\text{(residual-within)}} = .58$ is the proportion of the total variation attributable to the within-cluster residuals (the ε_{ij} terms)

RANDOM SLOPE R² BENCHMARKS

- Random slope R² values tend to be quite small (values > .10 appear to be uncommon)
- Across data sets from numerous MLM and longitudinal books, random slope R² values ranged from .001 to .094 (Enders, Woller, & Keller, 2023)
- The mean R² from cross-section applications was about .04, and the mean from longitudinal applications was about .05

WALD χ^2 STATISTIC

- The Wald χ^2 test equals the sum of squared standardized differences between a set of point estimates and their null values (like a squared z-statistic for multiple parameters)

$$\chi^2_{\text{MCMC}} = (\boldsymbol{\theta} - \boldsymbol{\theta}_0) \text{cov}(\boldsymbol{\theta})^{-1} (\boldsymbol{\theta} - \boldsymbol{\theta}_0)$$

- $\boldsymbol{\theta}$ contains multiple point estimates, $\boldsymbol{\theta}_0$ contains the correspond null values (typically 0s), and $\text{cov}(\boldsymbol{\theta})$ contains the variances and covariances of the estimates across MCMC cycles

SIGNIFICANCE TESTING WITH VARIANCES

- The random slope adds two parameters to the model (a variance and correlation), and both must be tested in θ
- When the true variance is zero, estimates accumulate at the boundary rather than fluctuating around it, producing a nonregular sampling distribution
- Significance testing requires a $\bar{\chi}^2$ (chi-bar) reference distribution, defined as a weighted mixture of chi-square distributions with different degrees of freedom

RBLIMP SCRIPT 5 (MODEL 3)

```
model3 <- rblimp(  
  data = Employee,  
  nominal = 'Male',  
  clusterid = 'Team',  
  center = 'groupmean = LMX; grandmean = LMX.mean Climate',  
  model = 'Empower ~ intercept LMX Male LMX.mean Climate | intercept LMX',  
  seed = 90291,  
  burn = 10000,  
  iter = 10000)  
output(model3)  
chibar_test(model = model3, DV = 'Empower', IV = 'LMX')
```


OUTPUT

Wald Chi-Square Statistic:

$\chi^2 = 6.618$

Chi-Bar Test Results:

Method	ChiSq	dfWeights	PValue	Notes
Standard chi-square	6.618	df = 2	0.037	Ignores boundary issue
Stram & Lee (1994)	6.618	(0.50, 0.25, 0.25)	0.012	Assumes independence (rho=0)
Self & Liang (1987) Case 6	6.618	(0.50, 0.12, 0.38)	0.015	Uses rho=-0.362, theta=0.382

CHI-BAR TEST INTERPRETATION

- The chi-bar test refutes the null hypothesis that the population slope variance and its covariance equal zero
- $\bar{\chi}^2 = 6.62, p = .015$
- Adding the LMX random slope improves fit

SUMMARY

- Including a random slope for LMX appears justified based on all three criteria (which won't always agree)
- MCMC diagnostics revealed no estimation issues
- The random slope R^2 effect size was salient
- The chi-bar χ^2 test was significant

OUTLINE

1

Modeling Step 1: Estimate ICCs

2

Modeling Step 2: Random Intercept Model

3

Modeling Step 3: Random Slope Model 1

4

Deciding Whether a Random Slope Is Needed

5

Modeling Step 3: Random Slope Model 2

6

Checking MLM Assumptions

7

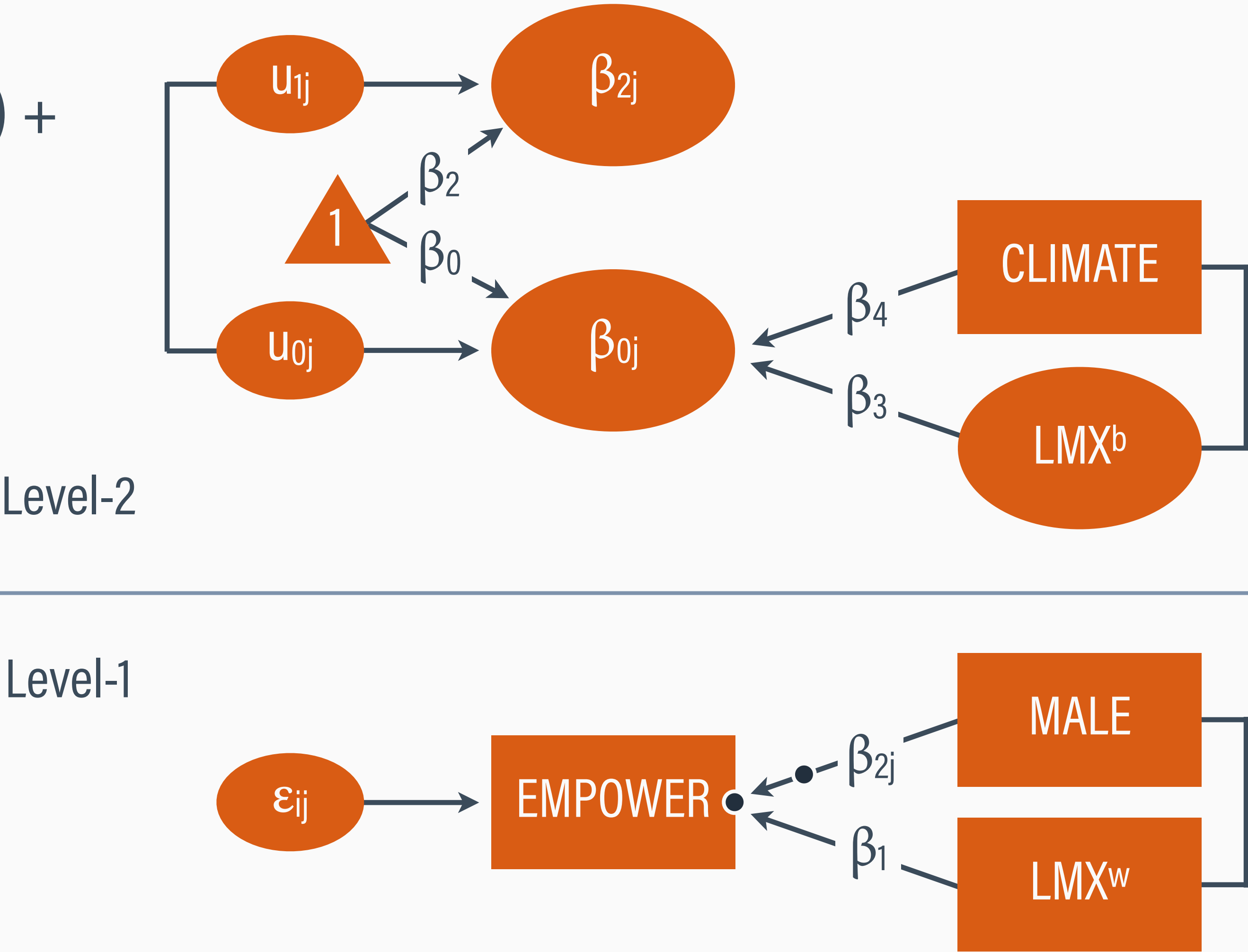
Latent Variable Specification

EVALUATING ADDITIONAL SLOPES

- ◉ Any level-1 predictor could require a random slope
- ◉ Auditioning random slopes one predictor at a time is often an effective strategy, keeping those with support from the data
- ◉ Is a random slope for the sex dummy code warranted?

PATH DIAGRAM

$$\text{empower}_{ij} = (\beta_0 + u_{0j}) + \beta_1(\text{lmx}_{ij}^w) + (\beta_2 + u_{2j})(\text{male}_{ij}) + \beta_3(\text{lmx}_j^b) + \beta_4(\text{climate}_j) + \varepsilon_{ij}$$



• = random intercept (β_{0j}) and random slope (β_{2j})



In the previous model, the fixed effect for sex was $\beta_2 = 1.89$, representing the expected difference in empowerment for males relative to females. In small groups of two or three, discuss the meaning of a random slope for a binary predictor. Provide a conceptual explanation of this parameter that a colleague with no prior exposure to multilevel modeling could understand.

BLIMP SCRIPT 5.4

DATA: Employee.dat;

VARIABLES: Employee Team Turnover Male Empower LMX JobSat Climate TeamPerf;

NOMINAL: Male; # automatic dummy coding for nominal predictors

CLUSTERID: Team;

CENTER: groupmean = LMX; grandmean = LMX.mean Climate;

MODEL: Empower ~ intercept LMX Male LMX.mean Climate | intercept Male;

BURN: 10000;

ITERATIONS: 10000;

SEED: 90291;

RBLIMP SCRIPT 5 (MODEL 4)

```
model4 <- rblimp(  
  data = Employee,  
  nominal = 'Male',  
  clusterid = 'Team',  
  center = 'groupmean = LMX; grandmean = LMX.mean Climate',  
  model = 'Empower ~ intercept LMX Male LMX.mean Climate | intercept Male',  
  seed = 90291,  
  burn = 10000,  
  iter = 10000)  
output(model4)  
chibar_test(model = model4, DV = 'Empower', IV = 'Male')
```


PSR DIAGNOSTIC OUTPUT

Quality control check: PSRF diagnostics all < 1.05 well before the end of the burn-in period

BURN-IN POTENTIAL SCALE REDUCTION (PSR) OUTPUT:

NOTE: Split chain PSR is being used. This splits each chain's iterations to create twice as many chains.

Comparing iterations across 2 chains	Highest PSR	Parameter #
251 to 500	1.420	10
501 to 1000	1.168	11
751 to 1500	1.194	12
1001 to 2000	1.131	12
...
2501 to 5000	1.026	12
2751 to 5500	1.026	12
3001 to 6000	1.022	10
3251 to 6500	1.021	12
3501 to 7000	1.020	12
3751 to 7500	1.021	10
4001 to 8000	1.013	19
4251 to 8500	1.016	10
4501 to 9000	1.010	29
4751 to 9500	1.008	11
5001 to 10000	1.021	12

BLIMP OUTPUT

Outcome Variable: Empower

Grand Mean Centered: Climate LMX.mean[Team]

Group Mean Centered: LMX

Quality control check: One effective sample size (N_Eff) diagnostic is a bit low (< 100), suggesting that more iterations are needed

Parameters	Estimate	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff

Variances:							
L2 : Var(Intercept)	1.417	0.833	0.291	3.471	---	---	234.195
L2 : Cov(Male.1,Intercept)	-0.884	0.957	-3.275	0.376	---	---	169.972
L2 : Var(Male.1)	2.345	1.661	0.232	6.448	---	---	163.161
Residual Var.	13.006	0.848	11.458	14.778	---	---	1438.068
Coefficients:							
Intercept	27.685	0.275	27.140	28.230	10101.708	0.000	1818.877
LMX	0.676	0.057	0.564	0.787	141.650	0.000	2433.306
Male.1	1.889	0.342	1.218	2.544	30.458	0.000	5525.504
LMX.mean[Team]	-0.230	0.314	-0.953	0.302	0.645	0.422	286.900
Climate	0.281	0.052	0.179	0.384	28.725	0.000	1545.496
Standard Deviations:							
L2 : SD(Intercept)	1.191	0.339	0.540	1.863	---	---	183.668
L2 : Cor(Male.1,Intercept)	-0.553	0.376	-0.937	0.573	---	---	79.993
L2 : SD(Male.1)	1.531	0.532	0.482	2.539	---	---	127.703
Residual SD	3.606	0.117	3.385	3.844	---	---	1424.495


...

BLIMP OUTPUT

Outcome Variable: Empower

Grand Mean Centered: Climate LMX.mean[Team]

Group Mean Centered: LMX

 = level-2 estimate

 = level-1 estimate

 = combined estimate

Parameters	Estimate	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
Variances:							
L2 : Var(Intercept)	1.417	0.833	0.291	3.471	---	---	234.195
L2 : Cov(Male.1,Intercept)	-0.884	0.957	-3.275	0.376	---	---	169.972
L2 : Var(Male.1)	2.345	1.661	0.232	6.448	---	---	163.161
Residual Var.	13.006	0.848	11.458	14.778	---	---	1438.068
Coefficients:							
Intercept	27.685	0.275	27.140	28.230	10101.708	0.000	1818.877
LMX	0.676	0.057	0.564	0.787	141.650	0.000	2433.306
Male.1	1.889	0.342	1.218	2.544	30.458	0.000	5525.504
LMX.mean[Team]	-0.230	0.314	-0.953	0.302	0.645	0.422	286.900
Climate	0.281	0.052	0.179	0.384	28.725	0.000	1545.496
Standard Deviations:							
L2 : SD(Intercept)	1.191	0.339	0.540	1.863	---	---	183.668
L2 : Cor(Male.1,Intercept)	-0.553	0.376	-0.937	0.573	---	---	79.993
L2 : SD(Male.1)	1.531	0.532	0.482	2.539	---	---	127.703
Residual SD	3.606	0.117	3.385	3.844	---	---	1424.495


• • •

BLIMP OUTPUT, CONTINUED

Outcome Variable: Empower

Grand Mean Centered: Climate LMX.mean[Team]

Group Mean Centered: LMX

 = level-2 estimate

 = level-1 estimate

 = combined estimate

Parameters	Estimate	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
...							
Standardized Coefficients:							
LMX	0.414	0.032	0.350	0.473	171.102	0.000	1381.297
Male.1	0.205	0.036	0.133	0.274	32.230	0.000	5423.973
LMX.mean[Team]	-0.053	0.064	-0.178	0.075	0.672	0.412	340.039
Climate	0.245	0.044	0.158	0.330	30.727	0.000	1479.362
Proportion Variance Explained							
by Coefficients	0.300	0.032	0.238	0.364	---	---	1166.171
by Level-2 Random Intercepts	0.051	0.026	0.012	0.112	---	---	231.330
by Level-2 Random Slopes	0.028	0.019	0.003	0.075	---	---	158.762
by Level-1 Residual Variation	0.615	0.036	0.543	0.684	---	---	634.902

OUTPUT

Wald Chi-Square Statistic:

$\chi^2 = 2.482$

Chi-Bar Test Results:

Method	ChiSq	dfWeights	PValue	Notes
Standard chi-square	2.482	df = 2	0.289	Ignores boundary issue
Stram & Lee (1994)	2.482	(0.50, 0.25, 0.25)	0.101	Assumes independence (rho=0)
Self & Liang (1987) Case 6	2.482	(0.50, 0.27, 0.23)	0.098	Uses rho=-0.740, theta=0.235



In small groups of two or three, discuss whether adding a random slope for sex is warranted. Consider the MCMC diagnostics, the R^2 effect size, and the chi-bar significance test.

OUTLINE

1

Modeling Step 1: Estimate ICCs

2

Modeling Step 2: Random Intercept Model

3

Modeling Step 3: Random Slope Model 1

4

Deciding Whether a Random Slope Is Needed

5

Modeling Step 3: Random Slope Model 2

6

Checking MLM Assumptions

7

Latent Variable Specification

MLM ASSUMPTIONS

- Associations are linear
- Level-1 residuals are normal with constant variation across level-1 units (days) and level-2 units (persons)
- Level-2 random intercept residuals are normal with constant variation across level-2 units (persons)

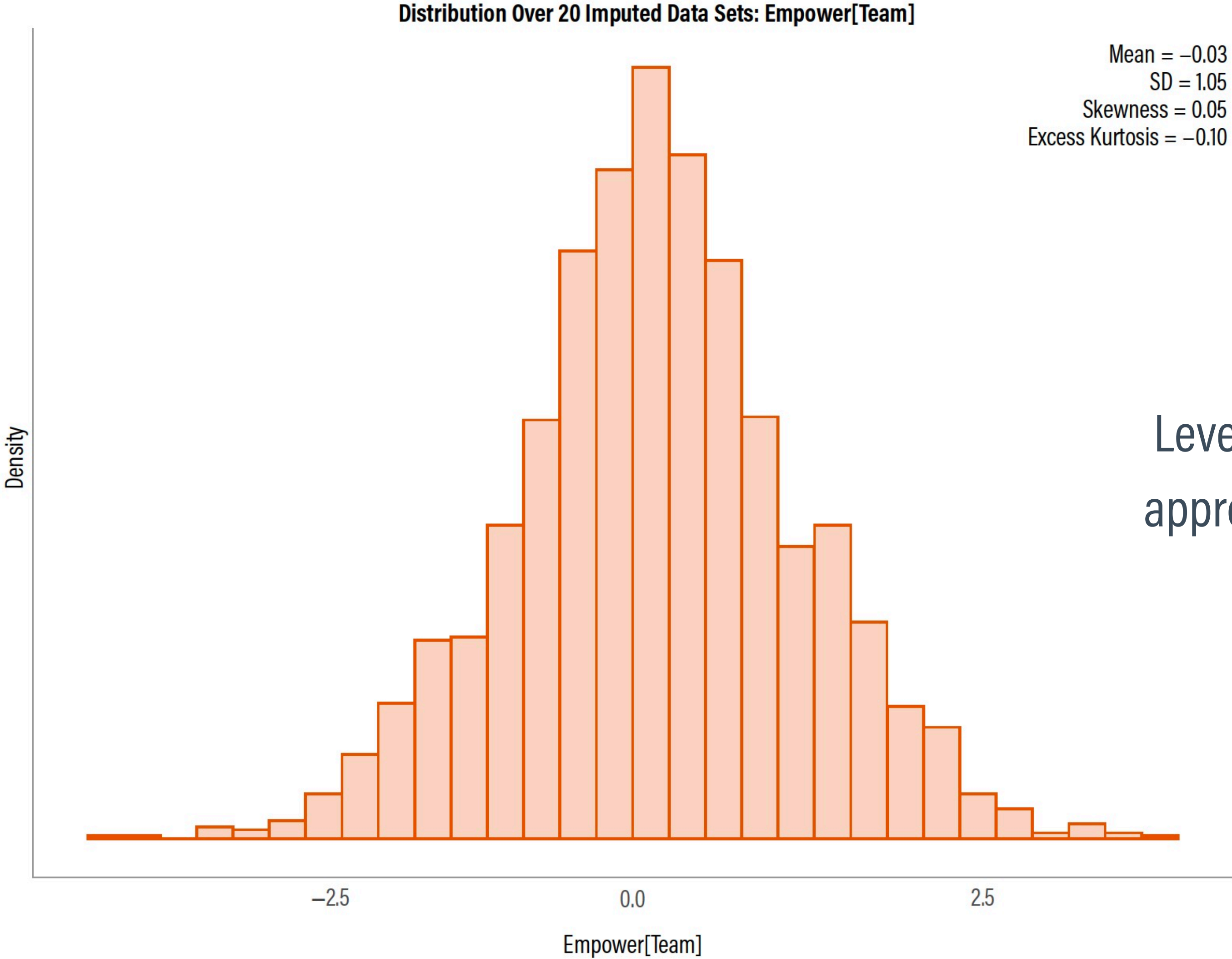
RESIDUAL DIAGNOSTICS

- Each MCMC iteration produces different parameter values—and thus different predicted values and residuals
- Residuals from different MCMC steps can be saved to create multiple datasets for plotting
- The `univariate_plot` function (imported at the top of the `rblimp` scripts) graphs residuals from multiple `rblimp` data sets
- See Module 3 slides for additional examples

RBLIMP SCRIPT 5 (MODEL 6)

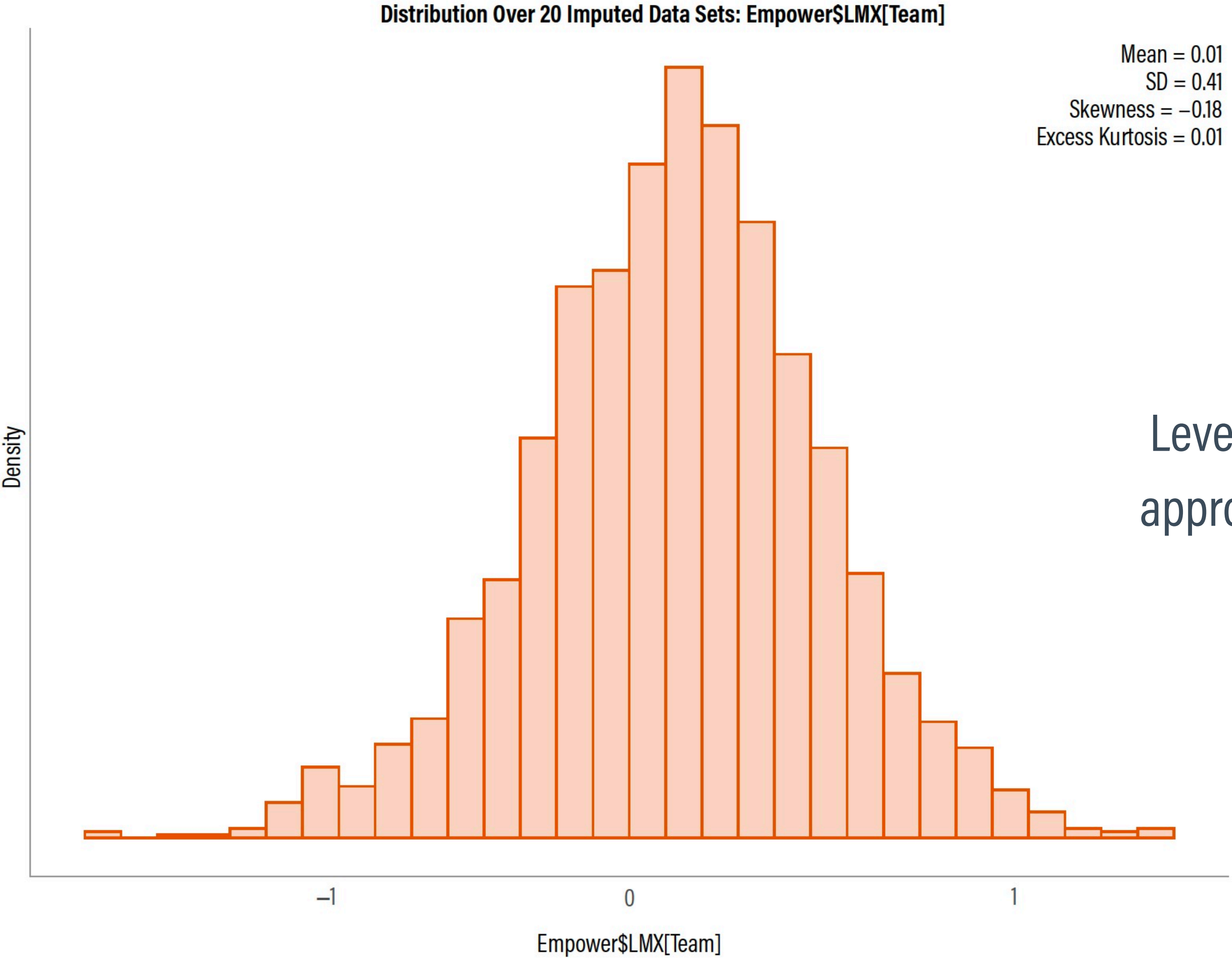
```
model6 <- rblimp(  
  data = Employee,  
  nominal = 'Male',  
  clusterid = 'Team',  
  center = 'groupmean = LMX; grandmean = LMX.mean Climate',  
  model = 'Empower ~ intercept LMX Male LMX.mean Climate | intercept LMX',  
  seed = 90291,  
  burn = 10000,  
  iter = 10000,  
  nimps = 20)  
output(model6)  
univariate_plot(vars = c('Empower[Team]', 'Empower$LMX[Team]', 'Empower.residual'),  
  model = model6, stats = T)
```

LEVEL-2 RANDOM INTERCEPT RESIDUALS



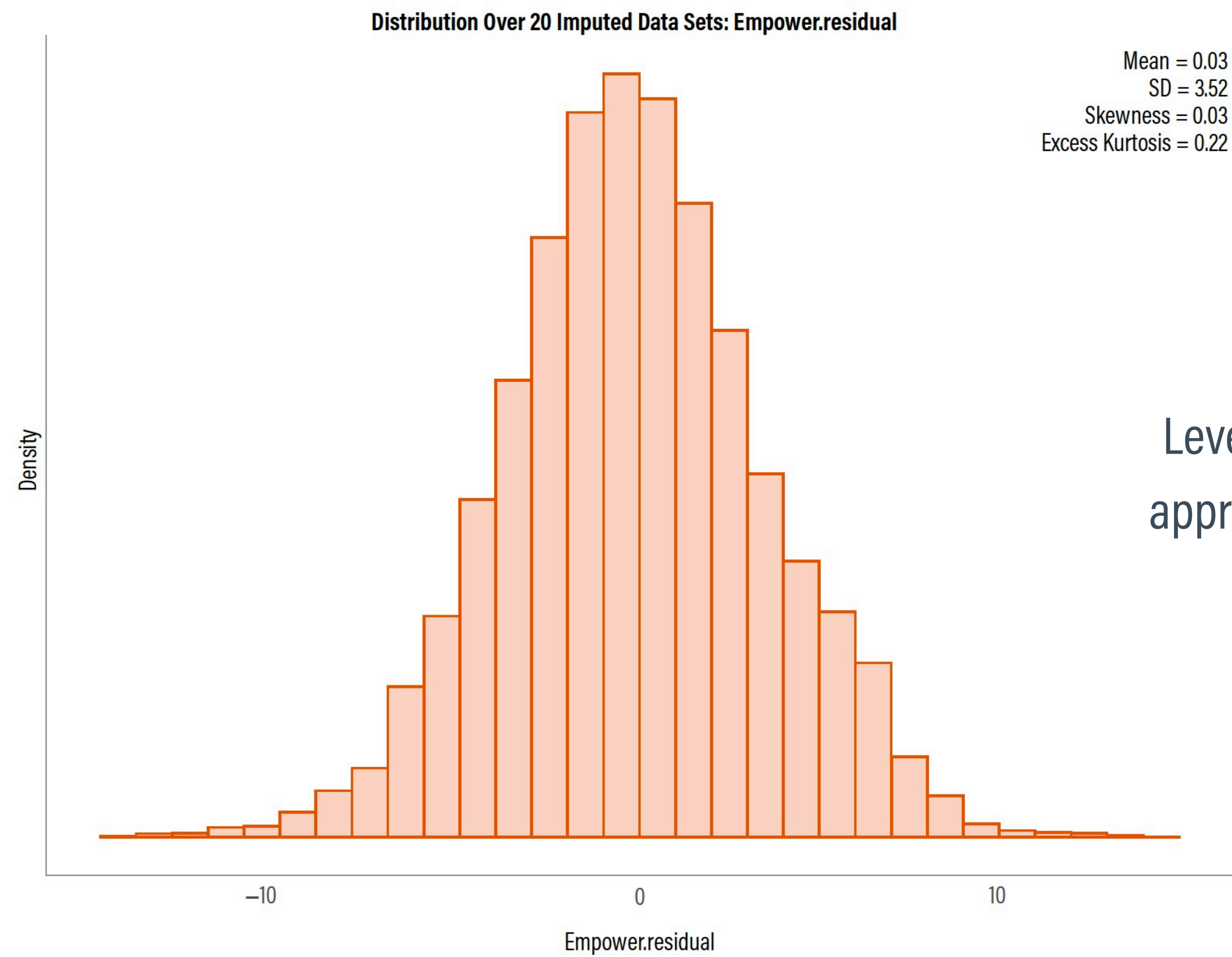
Level-2 residuals are approximately normal

LEVEL-2 RANDOM SLOPE RESIDUALS

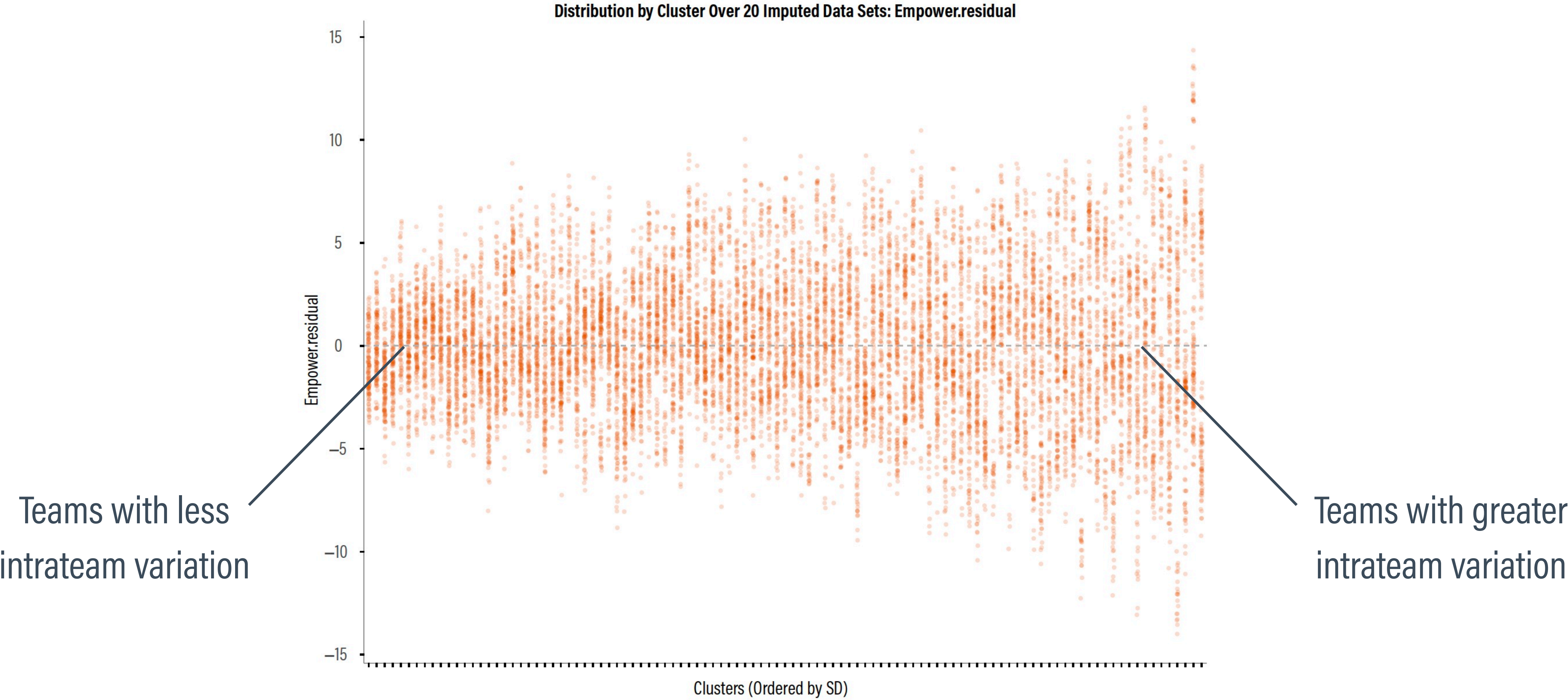


Level-2 residuals are approximately normal

LEVEL-1 RESIDUALS



LEVEL-1 RESIDUALS BY CLUSTER



OUTLINE

1

Modeling Step 1: Estimate ICCs

2

Modeling Step 2: Random Intercept Model

3

Modeling Step 3: Random Slope Model 1

4

Deciding Whether a Random Slope Is Needed

5

Modeling Step 3: Random Slope Model 2

6

Checking MLM Assumptions

7

Latent Variable Specification

BLIMP SCRIPT 5.5

DATA: Employee.dat;

VARIABLES: Employee Team Turnover Male Empower LMX JobSat Climate TeamPerf;

NOMINAL: Male;

CLUSTERID: Team;

LATENT: Team = beta0j beta1j; # define random intercept and slope

CENTER: groupmean = LMX; grandmean = LMX.mean Climate;

MODEL: beta0j ~ intercept LMX.mean Climate;

beta1j ~ intercept;

beta0j ~~ beta1j; # correlate random intercepts and slopes

Empower ~ intercept@beta0j LMX@beta1j Male;

BURN: 10000;

ITERATIONS: 10000;

SEED: 90291;

RBLIMP SCRIPT 5 (MODEL 7)

```
model7 <- rblimp(  
  data = Employee,  
  nominal = 'Male',  
  clusterid = 'Team',  
  latent = 'Team = beta0j beta1j',  
  center = 'groupmean = LMX; grandmean = LMX.mean Climate;',  
  model = '  
    beta0j ~ intercept LMX.mean Climate;  
    beta1j ~ intercept;  
    beta0j ~~ beta1j; # correlate random intercepts and slopes  
    Empower ~ intercept@beta0j LMX@beta1j Male;',  
  seed = 90291,  
  burn = 10000,  
  iter = 10000)  
output(model7)
```