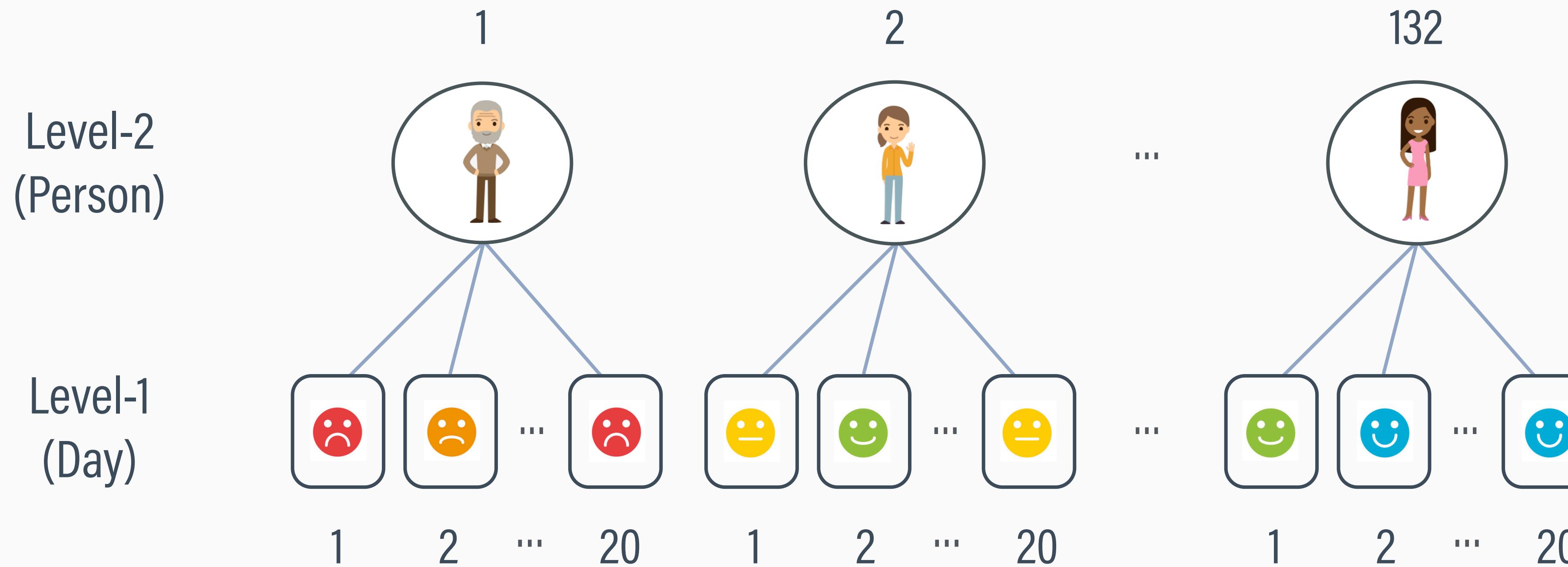


# MODULE 4

## RANDOM INTERCEPT MODELS WITH LEVEL-1 AND LEVEL-2 PREDICTORS

# DAILY DIARY APPLICATION

- $n_j = 20$  daily positive affect and sleep assessments nested within  $J = 132$  chronic pain patients ( $N = 2680$  data records)



# VARIABLE INFORMATION

---

- = predictor measured at level-2
- = predictor measured at level-1
- = outcome measured at level-1

paffect = sleep + pain + sex + stress

Variable	Definition	Level	Scale
PosAffect	Positive affect composite	1	Numeric (0 to 10)
SleepQual	Sleep quality rating	1	Ordinal (0 to 9)
Pain	Pain severity composite	1	Numeric (0 to 10)
Female	Sex dummy code	2	0 = Male, 1 = Female
Stress	Stress composite score	2	Numeric (0 to 5)

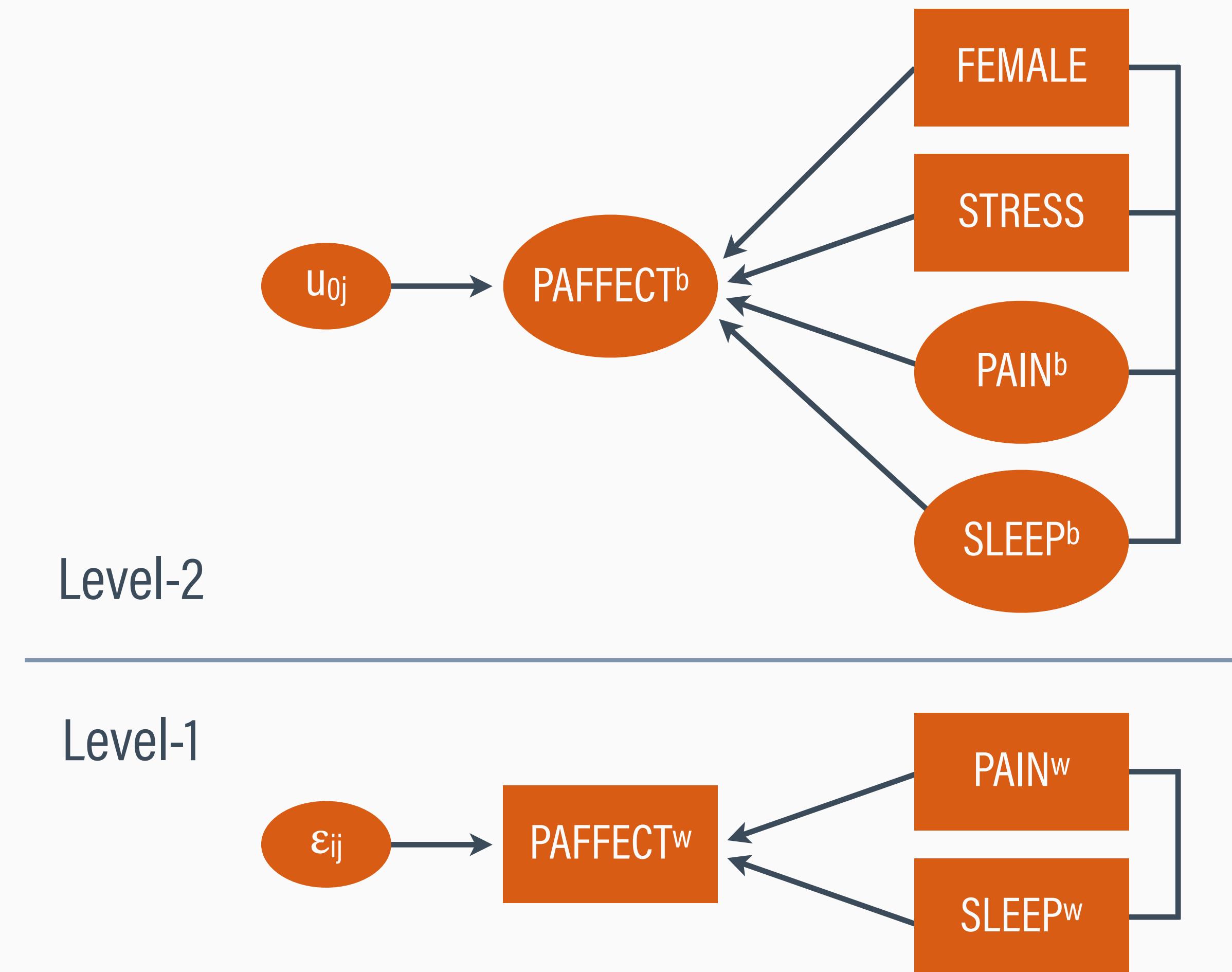
# DATA STRUCTURE

- Stacked (long) data format where each level-2 unit (person) has one row per level-1 (daily) observation
- The  $i$  subscript indexes level-1 observations, and  $j$  indexes level-2 units
- Variables measured at level-2 repeat across all rows within a cluster

Row	$i$	$j$	$AFFECT_{ij}$	$SLEEP_{ij}$	$FEMALE_j$	$STRESS_j$	
1	1	1	7.3	5.6	1	3.0	Person 1
2	2	1	2.5	4.3	1	3.0	
...	...	1	...	...	...	...	
20	20	1	6.3	7.3	1	3.0	
21	1	2	4.0	3.9	0.0	2.3	Person 2
22	2	2	4.0	7.1	0.0	2.3	
...	...	2	...	...	...	...	
40	20	2	4.4	3.5	0.0	2.3	
...	...	...	...	...	...	...	
2621	1	132	3.3	5.4	1.0	4.1	Person 132
2622	2	132	4.8	3.5	1.0	4.1	
...	...	132	...	...	...	...	
2640	20	132	4.8	7.9	1.0	4.1	

# PATH DIAGRAM AND MODEL FEATURES

- Level-1 variables have two sources of variation: average values and daily fluctuations around those means
- Variables correlate within level only (e.g., sleep relates to stress via its means)
- Statistical control occurs within level



# MODELING STEPS

---

- Estimate the ICCs of all level-1 variables
- Add level-1 variables
- Add level-2 variables

# OUTLINE

- 1 Modeling Step 1: Estimate ICCs
- 2 Modeling Step 2: Add Within-Cluster Predictors
- 3 Rights and Sterba Effect Sizes
- 4 Modeling Step 3: Add Between-Cluster Predictors
- 5 Latent Variable Specification

# OUTLINE

- 1 Modeling Step 1: Estimate ICCs
- 2 Modeling Step 2: Add Within-Cluster Predictors
- 3 Rights and Sterba Effect Sizes
- 4 Modeling Step 3: Add Between-Cluster Predictors
- 5 Latent Variable Specification

# ESTIMATING INTRACLASS CORRELATIONS

---

- Estimate intraclass correlations (ICCs) before model fitting to assess how much variance lies within and between clusters
- Some predictors have little or no level-2 variation, in which case they are already “pure” within-cluster variables
- Disaggregating a level-1 predictor and modeling its means is only warranted when there is sufficient level-2 variation

# BLIMP STUDIO SCRIPT 4.1

---

**DATA:** PainDiary.dat;

**VARIABLES:** Person Day PosAffect NegAffect Pain WorkGoal LifeGoal SleepQual Female Education  
Employment MarStatus NumDiagnose ActivityLevel PainAccept Catastrophize Stress Anxiety;

**CLUSTERID:** Person;

**MODEL:** { PosAffect SleepQual Pain } ~ intercept | intercept; # empty model for all variables in {}

**BURN:** 10000;

**ITERATIONS:** 10000;

**SEED:** 90291;

# RBLIMP SCRIPT 4 (MODEL 1)

---

```
model1 <- rblimp(  
  data = PainDiary,  
  clustered = 'Person',  
  model = '{ PosAffect SleepQual Pain } ~ intercept | intercept;',  
  seed = 90291,  
  burn = 10000,  
  iter = 10000)  
  
output(model1)
```

# BLIMP OUTPUT

□ = level-2 estimate

□ = level-1 estimate

Outcome Variable: PosAffect

Parameters	Estimate	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
<hr/>							
Variances:							
L2 : Var(Intercept)	<b>2.529</b>	0.333	1.987	3.271	---	---	8414.025
Residual Var.	<b>1.395</b>	0.039	1.323	1.474	---	---	9257.666
<hr/>							
Coefficients:							
Intercept	<b>5.034</b>	0.147	4.749	5.316	1177.150	0.000	124.250
<hr/>							
Standard Deviations:							
L2 : SD(Intercept)	<b>1.590</b>	0.103	1.410	1.809	---	---	8415.412
Residual SD	<b>1.181</b>	0.016	1.150	1.214	---	---	9254.467
<hr/>							
Proportion Variance Explained							
by Coefficients	<b>0.000</b>	0.000	0.000	0.000	---	---	nan
by Level-2 Random Intercepts	<b>0.644</b>	0.030	0.585	0.703	---	---	8658.787
by Level-1 Residual Variation	<b>0.356</b>	0.030	0.297	0.415	---	---	8658.787

# BLIMP OUTPUT

= level-2 estimate

= level-1 estimate

Outcome Variable: SleepQual

Parameters	Estimate	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
<hr/>							
Variances:							
L2 : Var(Intercept)	<b>1.919</b>	0.262	1.490	2.508	---	---	6730.439
Residual Var.	<b>3.001</b>	0.084	2.845	3.172	---	---	9268.567
<hr/>							
Coefficients:							
Intercept	<b>5.416</b>	0.130	5.168	5.671	1729.526	0.000	400.385
<hr/>							
Standard Deviations:							
L2 : SD(Intercept)	<b>1.385</b>	0.093	1.221	1.584	---	---	6767.067
Residual SD	<b>1.732</b>	0.024	1.687	1.781	---	---	9265.589
<hr/>							
Proportion Variance Explained							
by Coefficients	<b>0.000</b>	0.000	0.000	0.000	---	---	nan
by Level-2 Random Intercepts	<b>0.390</b>	0.032	0.330	0.457	---	---	6737.692
by Level-1 Residual Variation	<b>0.610</b>	0.032	0.543	0.670	---	---	6737.692

# BLIMP OUTPUT

□ = level-2 estimate

□ = level-1 estimate

Outcome Variable: Pain

Parameters	Estimate	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
<hr/>							
Variances:							
L2 : Var(Intercept)	<b>2.805</b>	0.375	2.189	3.654	---	---	7747.670
Residual Var.	<b>2.246</b>	0.063	2.125	2.373	---	---	8881.762
<hr/>							
Coefficients:							
Intercept	<b>3.533</b>	0.152	3.238	3.823	536.962	0.000	236.236
<hr/>							
Standard Deviations:							
L2 : SD(Intercept)	<b>1.675</b>	0.110	1.480	1.912	---	---	7692.880
Residual SD	<b>1.499</b>	0.021	1.458	1.541	---	---	8884.094
<hr/>							
Proportion Variance Explained							
by Coefficients	<b>0.000</b>	0.000	0.000	0.000	---	---	nan
by Level-2 Random Intercepts	<b>0.556</b>	0.033	0.492	0.622	---	---	7537.167
by Level-1 Residual Variation	<b>0.444</b>	0.033	0.378	0.508	---	---	7537.167

# SUMMARY

---

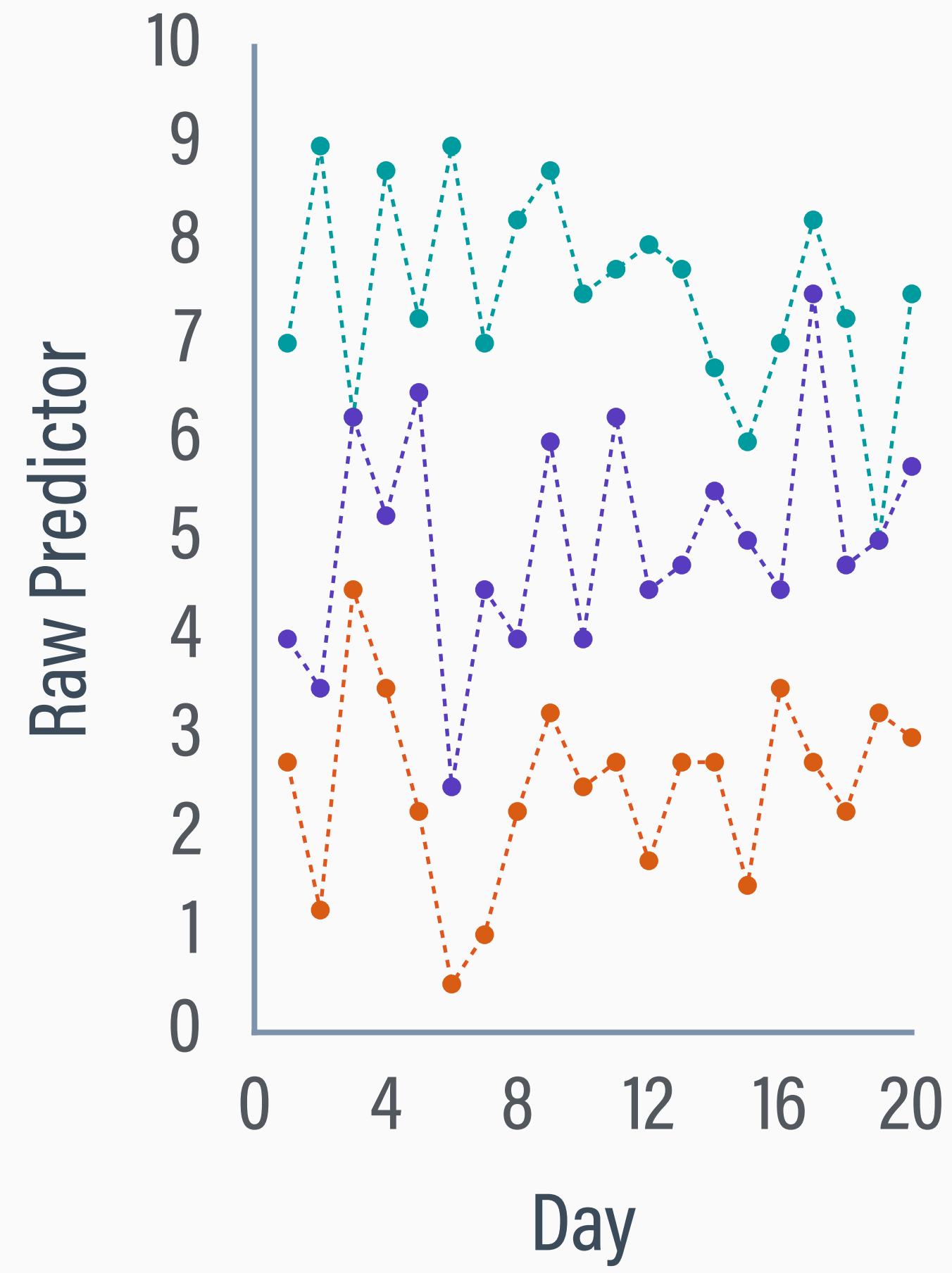
- The outcome has substantial between-person variation in the person-specific means ( $ICC_{affect} = .64$ )
- This feature indicates the need for MLM (independence violation)
- Both level-1 predictors have a substantial amount of between-person variation in their means ( $ICC_{sleep} = .39$  and  $ICC_{pain} = .55$ )
- High predictor ICCs motivate disaggregation, as there could be distinct within- and between-person effects

# OUTLINE

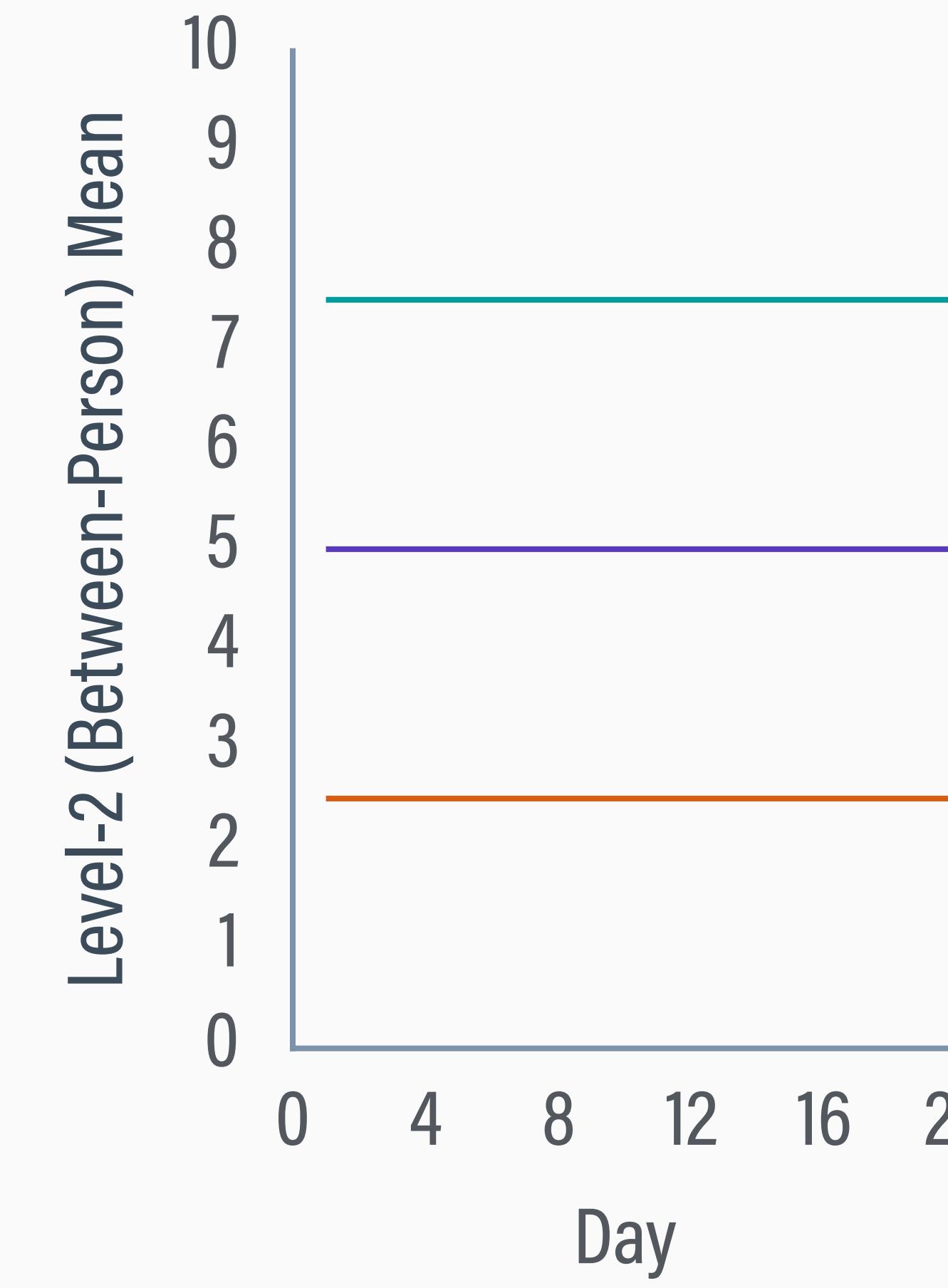
- 1 Modeling Step 1: Estimate ICCs
- 2 Modeling Step 2: Add Within-Cluster Predictors
- 3 Rights and Sterba Effect Sizes
- 4 Modeling Step 3: Add Between-Cluster Predictors
- 5 Latent Variable Specification

# DISAGGREGATION REVIEW

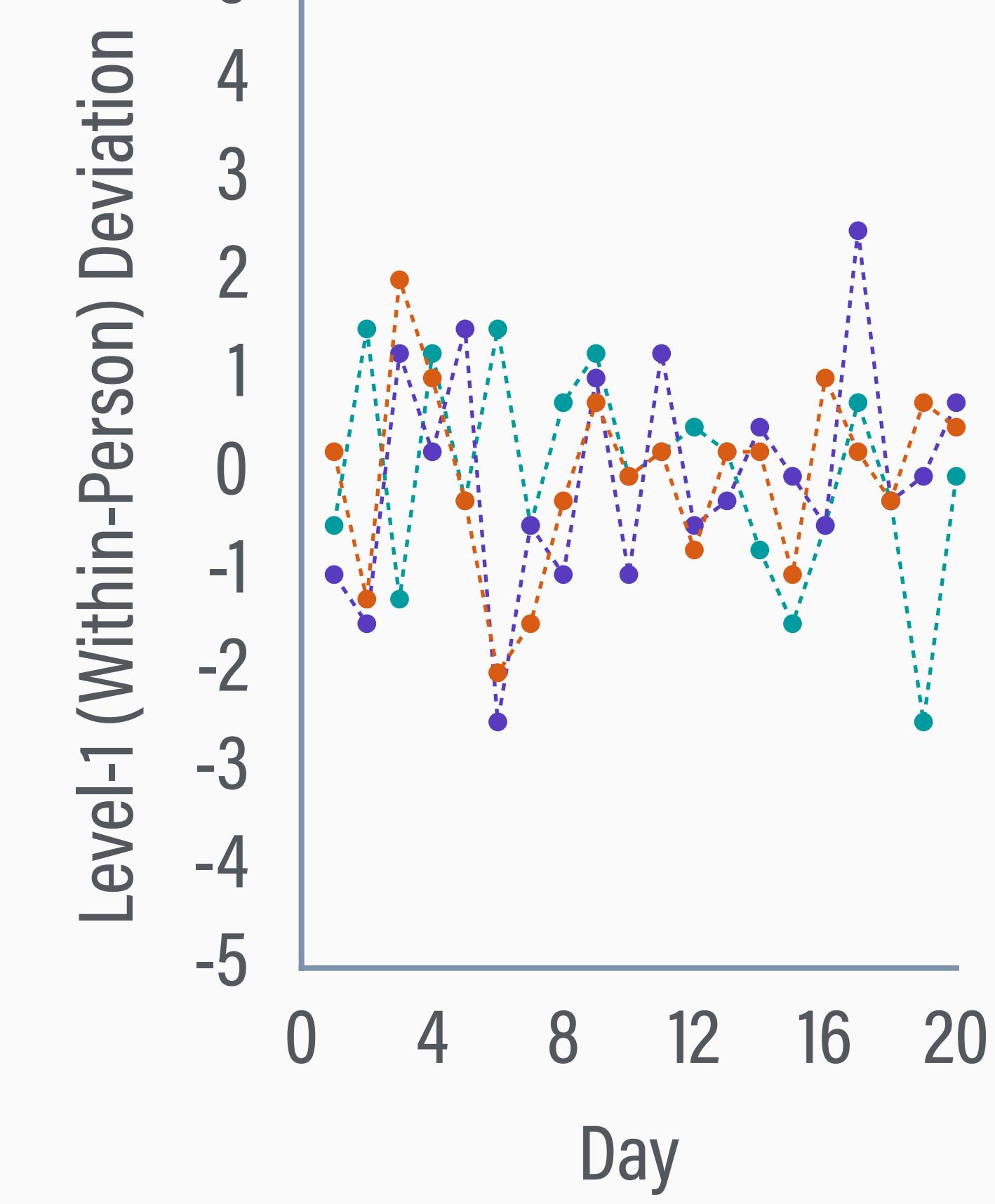
Raw Predictor = Level-2 (Between-Person) Mean + Level-1 (Within-Person) Residual



||



+



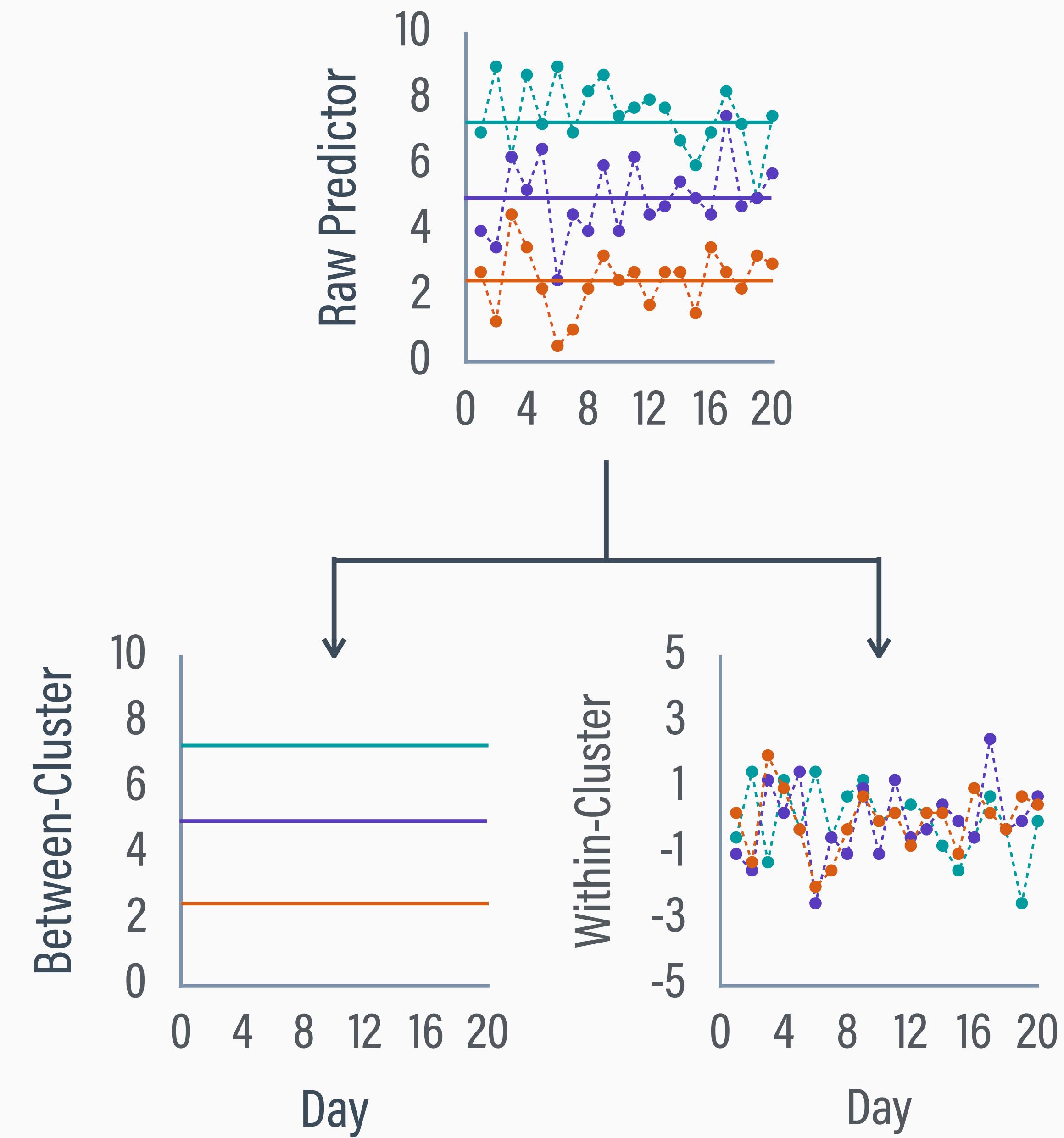
# CENTERING WITHIN CLUSTER (CWC)

- Disaggregation centers level-1 scores around their cluster's level-2 mean

$$\text{pain}_{ij}^w = \text{pain}_{ij} - \mu_j(\text{pain}) = \text{pain}_{ij} - \text{pain}_j^b$$

$$\text{sleep}_{ij}^w = \text{sleep}_{ij} - \mu_j(\text{sleep}) = \text{sleep}_{ij} - \text{sleep}_j^b$$

- $\text{pain}^w$  and  $\text{sleep}^w$  are “pure” within-cluster predictors that contain only intraindividual (level-1) variation



# WITHIN-CLUSTER (LEVEL-1) MODEL

---

- Affect observation  $i$  for person  $j$  is the sum of a level-2 affect mean ( $\beta_{0j}$ ), fixed effects due to within-person sleep and pain variation ( $\beta_1$  and  $\beta_2$ ), and a within-person residual ( $\varepsilon_{ij}$ )

$$\text{pffect}_{ij} = \beta_{0j} + \beta_1(\text{sleep}_{ij}^W) + \beta_2(\text{pain}_{ij}^W) + \varepsilon_{ij}$$

- Assumption: residuals are normal with constant variation across all days (level-1 units) and persons (level-2 units)

$$\varepsilon_{ij} \sim N(0, \sigma_\varepsilon^2)$$

## BETWEEN-CLUSTER (LEVEL-2) MODEL

---

- Slopes are constant across level-2 units, but the affect mean for person  $j$  ( $\beta_{0j}$ ) is the sum of the grand mean ( $\gamma_{00}$ ) and a between-person residual ( $u_{0j}$ )

$$\beta_{0j} = \gamma_{00} + u_{0j}$$

$$\beta_1 = \gamma_{10}$$

$$\beta_2 = \gamma_{20}$$

- Assumption: random intercept residuals are normal with constant variation across persons (level-2 units)

$$u_{0j} \sim N(0, \sigma_u^2)$$

# DECODING THE SUBSCRIPTS

---

The second subscript numbers the coefficients on the right side starting at 0 (the intercept)

$$\beta_{0j} = \gamma_{00} + u_{0j}$$

$$\beta_{1j} = \gamma_{10}$$

$$\beta_{2j} = \gamma_{20}$$

The first subscript tells which level-2 outcome variable these terms are attached to ( $\gamma_{00}$  and  $u_{0j}$  belong in  $\beta_{0j}$ 's equation,  $\gamma_{10}$  is attached to  $\beta_1$ )

# COMBINED-MODEL NOTATION

- Level-specific regression equations can be reduced into a single combined-model equation (Raudenbush & Bryk, 2002)
- Replace the  $\beta_{0j}$ ,  $\beta_1$ , and  $\beta_2$  terms in the level-1 equation with their level-2 equations

$$\begin{array}{c} \beta_2 = \gamma_{20} \\ \beta_1 = \gamma_{10} \\ \beta_{0j} = \gamma_{00} + u_{0j} \\ \downarrow \quad \downarrow \\ \text{pffect}_{ij} = \beta_{0j} + \beta_1(\text{sleep}_{ij}^w) + \beta_2(\text{pain}_{ij}^w) + \varepsilon_{ij} \\ \downarrow \\ \text{pffect}_{ij} = \gamma_{00} + \gamma_{10}(\text{sleep}_{ij}^w) \\ + \gamma_{20}(\text{pain}_{ij}^w) + u_{0j} + \varepsilon_{ij} \end{array}$$

# COMMON NOTATIONAL SYSTEMS

Combined-model equation (Raudenbush & Bryk, 2002)

$$\text{paffect}_{ij} = \gamma_{00} + \gamma_{10}(\text{sleep}_{ij}^w) + \gamma_{20}(\text{pain}_{ij}^w) + u_{0j} + \varepsilon_{ij}$$

Standard(ish) regression notation (Scott, Shrout, & Weinberg, 2013)

$$\text{paffect}_{ij} = \beta_0 + \beta_1(\text{sleep}_{ij}^w) + \beta_2(\text{pain}_{ij}^w) + u_{0j} + \varepsilon_{ij}$$

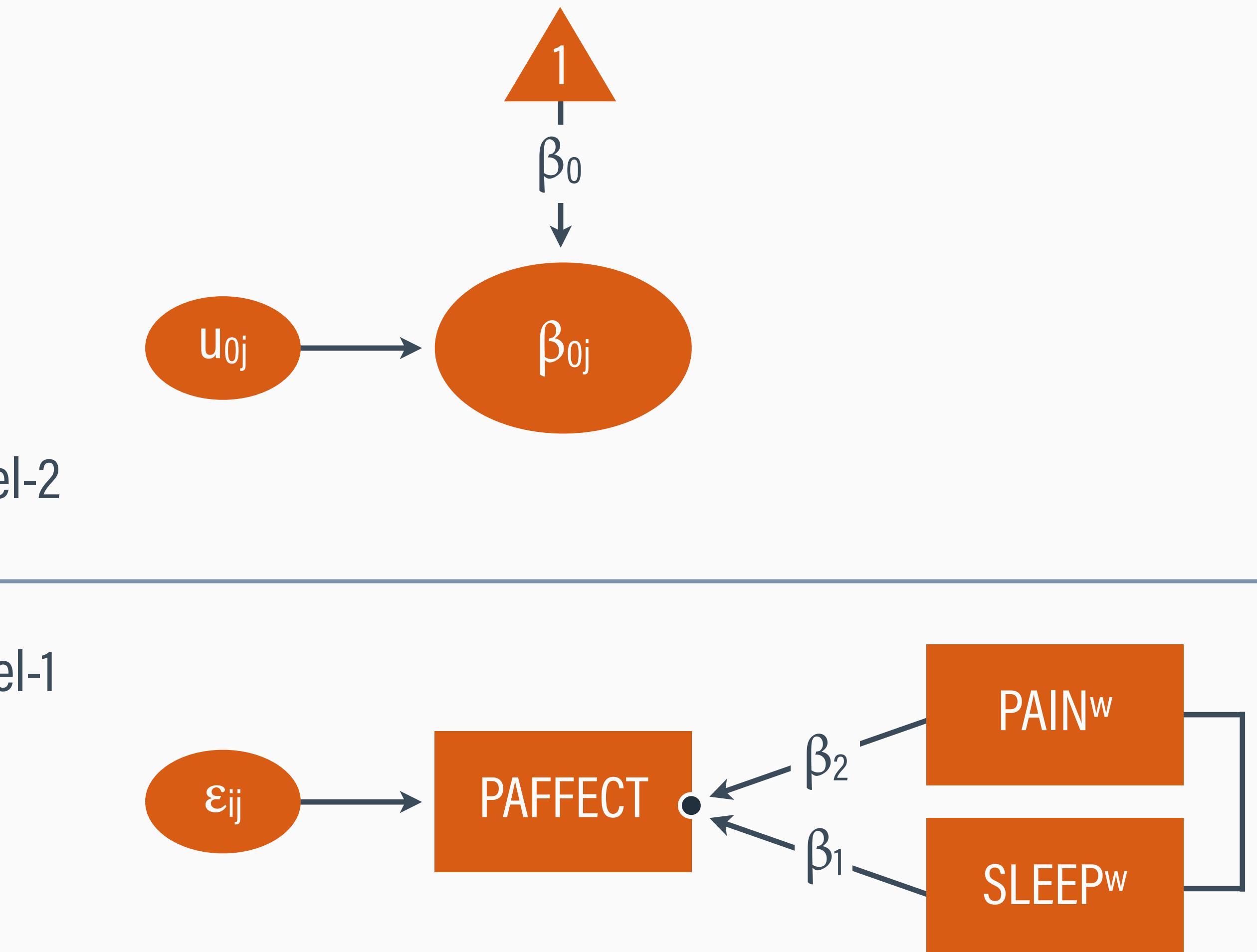
Linear mixed model cluster-level matrix equation

$$\mathbf{y}_j = \mathbf{X}_j\boldsymbol{\beta} + \mathbf{Z}_j\mathbf{u}_j + \boldsymbol{\varepsilon}_j = \begin{pmatrix} \text{posaffect}_{1j} \\ \text{posaffect}_{2j} \\ \dots \\ \text{posaffect}_{nj} \end{pmatrix} = \begin{pmatrix} 1 & \text{sleep}_{1j}^w & \text{pain}_{1j}^w \\ 1 & \text{sleep}_{2j}^w & \text{pain}_{2j}^w \\ \dots & \dots & \dots \\ 1 & \text{sleep}_{nj}^w & \text{pain}_{nj}^w \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ \dots \\ 1 \end{pmatrix} \mathbf{u}_{0j} + \begin{pmatrix} \varepsilon_{1j} \\ \varepsilon_{2j} \\ \dots \\ \varepsilon_{nj} \end{pmatrix}$$

# PATH DIAGRAM

---

$$paffect_{ij} = \beta_0 + \beta_1(sleep_{ij}^w) + \beta_2(pain_{ij}^w) + u_{0j} + \varepsilon_{ij}$$



# BLIMP STUDIO SCRIPT 4.2

---

**DATA:** PainDiary.dat;

**VARIABLES:** Person Day PosAffect NegAffect Pain WorkGoal LifeGoal SleepQual Female Education  
Employment MarStatus NumDiagnose ActivityLevel PainAccept Catastrophize Stress Anxiety;

**CLUSTERID:** Person;

**CENTER:** groupmean = SleepQual Pain; # cwc with level-2 latent cluster means

**MODEL:** PosAffect ~ intercept SleepQual Pain | intercept;

**BURN:** 10000;

**ITER:** 10000;

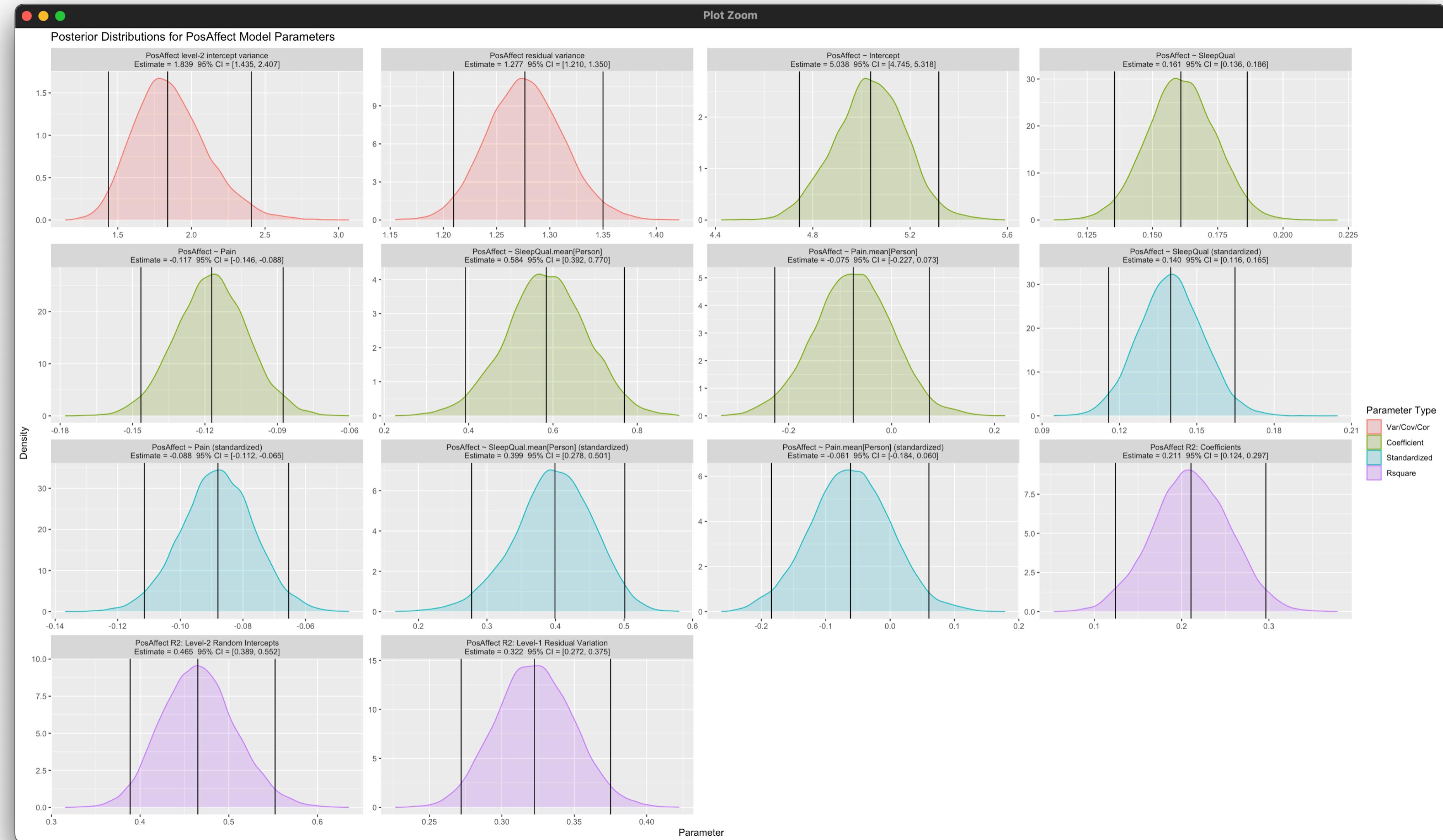
**SEED:** 90291;

# RBLIMP SCRIPT 4 (MODEL 2)

---

```
model2 <- rblimp(  
  data = PainDiary,  
  clusterid = 'Person',  
  center = 'groupmean = SleepQual Pain',  
  model = 'PosAffect ~ intercept SleepQual Pain | intercept',  
  seed = 90291,  
  burn = 10000,  
  iter = 10000)  
  
output(model2)  
plot_posterior(model2, 'PosAffect')
```

# PARAMETER PLOTS (RBLIMP ONLY)



# BLIMP OUTPUT

= level-2 estimate

= level-1 estimate

Outcome Variable: PosAffect

Group Mean Centered: Pain SleepQual

Parameters	Estimate	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
<hr/>							
Variances:							
L2 : Var(Intercept)	<b>2.496</b>	0.325	1.953	3.218	---	---	8178.774
Residual Var.	<b>1.276</b>	0.036	1.210	1.350	---	---	9009.151
<hr/>							
Coefficients:							
Intercept	<b>5.010</b>	0.147	4.742	5.312	1169.499	0.000	105.520
SleepQual	<b>0.162</b>	0.013	0.136	0.187	156.661	0.000	8279.469
Pain	<b>-0.117</b>	0.015	-0.147	-0.088	60.912	0.000	8510.442
<hr/>							
Standard Deviations:							
L2 : SD(Intercept)	<b>1.580</b>	0.102	1.397	1.794	---	---	8123.400
Residual SD	<b>1.130</b>	0.016	1.100	1.162	---	---	9006.557
<hr/>							
Standardized Coefficients:							
SleepQual	<b>0.142</b>	0.013	0.118	0.167	126.552	0.000	7959.671
Pain	<b>-0.089</b>	0.012	-0.113	-0.066	55.997	0.000	8548.133
<hr/>							
Proportion Variance Explained							
by Coefficients	0.031	0.005	0.023	0.041	---	---	8140.114
by Level-2 Random Intercepts	<b>0.641</b>	0.030	0.582	0.698	---	---	8075.546
by Level-1 Residual Variation	<b>0.328</b>	0.027	0.276	0.382	---	---	8119.500

# FIXED EFFECT INTERPRETATIONS

---

- $p\text{affect}_{ij} = \beta_0 + \beta_1(\text{sleep}_{ij}^W) + \beta_2(\text{pain}_{ij}^W) + u_{0j} + \varepsilon_{ij}$
- $\beta_0 = 5.01$  is the positive affect grand mean (because all predictors are centered)
- $\beta_1 = 0.16$  is the expected affect difference between two daily sleep scores from the same person that differ by one point, controlling for within-person pain
- $\beta_2 = -0.12$  is the expected affect difference between two daily pain scores from the same person that differ by one point, controlling for within-person sleep

# RANDOM EFFECT INTERPRETATIONS

---

- $u_{0j} = \beta_{0j} - \beta_0$
- $\text{var}(u_{0j}) = 2.50$  is the average squared distance between the level-2 affect means and the grand mean
- $\text{sd}(u_{0j}) = 1.58$  is the average distance between the level-2 affect means and the grand mean
- $\varepsilon_{ij} = p\text{affect}_{ij} - (\beta_{0j} + \beta_1(\text{sleep}_{ij}^W) + \beta_2(\text{pain}_{ij}^W))$
- $\text{var}(\varepsilon_{ij}) = 1.28$  is the average squared distance between the level-1 affect observations and their predicted values
- $\text{sd}(\varepsilon_{ij}) = 1.13$  is the average distance between the level-1 affect observations and their predicted values

# OUTLINE

- 1 Modeling Step 1: Estimate ICCs
- 2 Modeling Step 2: Add Within-Cluster Predictors
- 3 Rights and Sterba Effect Sizes
- 4 Modeling Step 3: Add Between-Cluster Predictors
- 5 Latent Variable Specification

# MULTILEVEL EFFECT SIZES

---

- Numerous effect sizes exist for multilevel models
- Rights and Sterba (2019, Psychological Methods) provides a unifying framework that subsumes different approaches
- They assign a proportion variance explained ( $R^2$ ) to each source of variation in the multilevel model

# PREDICTED OUTCOME VARIATION

---

- The model-predicted total variance is the sum of (a) variance explained by predictors at both levels, (b) residual random-intercept variation, and (c) residual within-cluster variation

$$\sigma_y^2 = \beta^\top \Sigma_x \beta + \sigma_u^2 + \sigma_\varepsilon^2$$

total variation = explained by predictors + level-2 residuals + level-1 residuals

# VARIANCE EXPLAINED MEASURES

Fixed effects  
of predictors

$$R^2_{\text{predictors}} = \frac{\beta^T \Sigma_X \beta}{\sigma_Y^2}$$

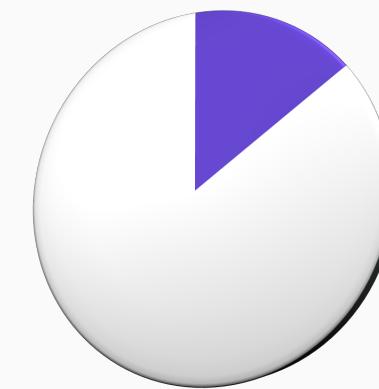
Level-2 random  
intercept residuals

$$R^2_{\text{residual-between}} = \frac{\sigma_{u_0}^2}{\sigma_Y^2}$$

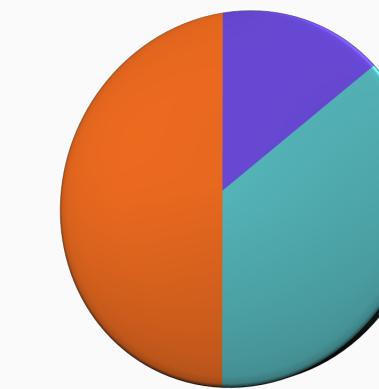
Level-1 within-  
cluster residuals

$$R^2_{\text{residual-within}} = \frac{\sigma_\varepsilon^2}{\sigma_Y^2}$$

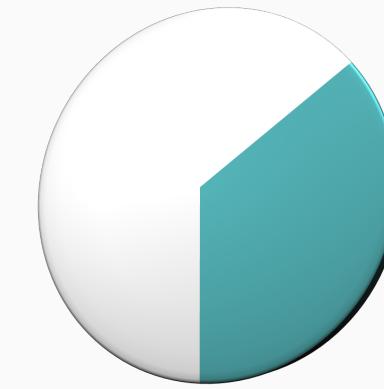
Explained ÷ Total



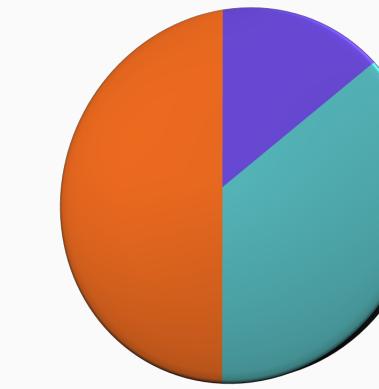
÷



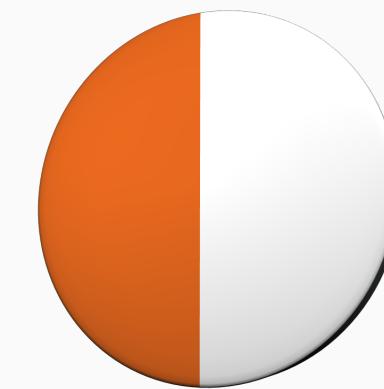
÷



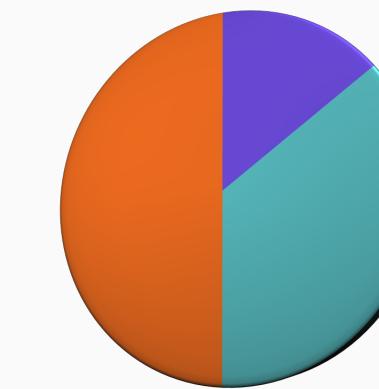
÷



÷



÷



# BLIMP OUTPUT

- = level-2 estimate
- = level-1 estimate

## Outcome Variable: PosAffect

# Group Mean Centered: Pain SleepQual

Parameters	Estimate	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
<hr/>							
Variances:							
L2 : Var(Intercept)	2.496	0.325	1.953	3.218	---	---	8178.774
Residual Var.	1.276	0.036	1.210	1.350	---	---	9009.151
<hr/>							
Coefficients:							
Intercept	5.010	0.147	4.742	5.312	1169.499	0.000	105.520
SleepQual	0.162	0.013	0.136	0.187	156.661	0.000	8279.469
Pain	-0.117	0.015	-0.147	-0.088	60.912	0.000	8510.442
<hr/>							
...							
<hr/>							
Proportion Variance Explained							
by Coefficients	<b>0.031</b>	0.005	0.023	0.041	---	---	8140.114
by Level-2 Random Intercepts	<b>0.641</b>	0.030	0.582	0.698	---	---	8075.546
by Level-1 Residual Variation	<b>0.328</b>	0.027	0.276	0.382	---	---	8119.500

# EFFECT SIZE INTERPRETATIONS

---

- $R^2_{(\text{predictors})} = .03$  is the proportion of the total variation explained by the within-cluster predictors
- $R^2_{(\text{residual-between})} = .64$  is the proportion of the total variation attributable to the between-cluster residuals (the  $u_{0j}$  terms)
- $R^2_{(\text{residual-within})} = .33$  is the proportion of the total variation attributable to the within-cluster residuals (the  $\varepsilon_{ij}$  terms)

# MODEL COMPARISON

Parameter	Empty Model	Level-1 Predictors (Within Only)	Level-2 Predictors (Within + Between)
Fixed intercept	5.03	5.03	5.03
Sleep (within-person)	--	0.16	0.16
Pain (within-person)	--	-0.12	-0.12
Sleep (between-person)	--	--	-0.11
Pain (between-person)	--	--	0.57
Female (between-person)	--	--	0.38
Stress (between-person)	--	--	-0.36
Residual intercept variance	2.53	2.50	1.74
Residual within-person variance	1.40	1.28	1.28
R-square predictors	0	.03	.25
R-square intercept variance	.64	.64	.43
R-square within-person variance	.36	.33	.32



The next analysis will add the level-2 (person-specific) predictors to the model. In small groups of two or three, discuss whether each effect size will change and, if so, how.

# OUTLINE

- 1 Modeling Step 1: Estimate ICCs
- 2 Modeling Step 2: Add Within-Cluster Predictors
- 3 Rights and Sterba Effect Sizes
- 4 Modeling Step 3: Add Between-Cluster Predictors
- 5 Latent Variable Specification

# WITHIN-CLUSTER (LEVEL-1) MODEL

---

- Affect observation  $i$  for person  $j$  is the sum of a level-2 affect mean ( $\beta_{0j}$ ), fixed effects due to within-person sleep and pain variation ( $\beta_1$  and  $\beta_2$ ), and a within-person residual ( $\varepsilon_{ij}$ )

$$\text{pffect}_{ij} = \beta_{0j} + \beta_1(\text{sleep}_{ij}^W) + \beta_2(\text{pain}_{ij}^W) + \varepsilon_{ij}$$

- Assumption: residuals are normal with constant variation across all days (level-1 units) and persons (level-2 units)

$$\varepsilon_{ij} \sim N(0, \sigma_\varepsilon^2)$$

## BETWEEN-CLUSTER (LEVEL-2) MODEL

---

- The affect mean for person  $j$  ( $\beta_{0j}$ ) is the sum of a mean ( $\gamma_{00}$ ), fixed effects due to the level-2 predictors ( $\gamma_{01}$  through  $\gamma_{04}$ ), and a between-person residual ( $u_{0j}$ )

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{sleep}_j^b) + \gamma_{02}(\text{pain}_j^b) + \gamma_{03}(\text{female}_j) + \gamma_{04}(\text{stress}_j) + u_{0j}$$

$$\beta_1 = \gamma_{10}$$

$$\beta_2 = \gamma_{20}$$

- Assumption: random intercept residuals are normal with constant variation across persons (level-2 units)

$$u_{0j} \sim N(0, \sigma_u^2)$$

# COMMON NOTATIONAL SYSTEMS

---

Combined-model equation (Raudenbush & Bryk, 2002)

$$\begin{aligned} \text{paffect}_{ij} = & \gamma_{00} + \gamma_{10}(\text{sleep}_{ij}^w) + \gamma_{20}(\text{pain}_{ij}^w) + \gamma_{01}(\text{sleep}_j^b) + \gamma_{02}(\text{pain}_j^b) \\ & + \gamma_{03}(\text{female}_j) + \gamma_{04}(\text{stress}_j) + u_{0j} + \varepsilon_{ij} \end{aligned}$$

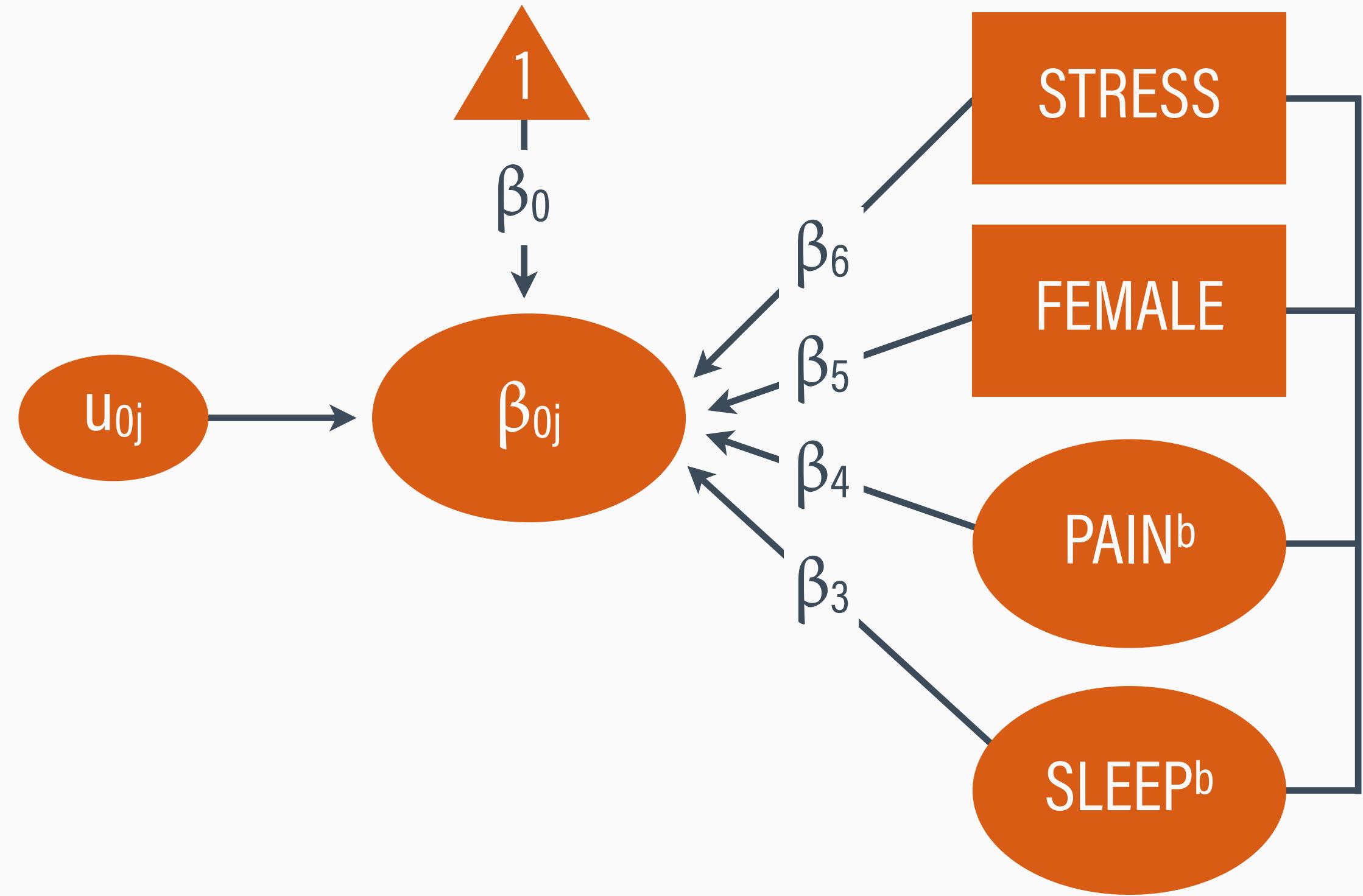
Standard(ish) regression notation (Scott, Shrout, & Weinberg, 2013)

$$\begin{aligned} \text{paffect}_{ij} = & \beta_0 + \beta_1(\text{sleep}_{ij}^w) + \beta_2(\text{pain}_{ij}^w) + \beta_3(\text{sleep}_j^b) + \beta_4(\text{pain}_j^b) \\ & + \beta_5(\text{female}_j) + \beta_6(\text{stress}_j) + u_{0j} + \varepsilon_{ij} \end{aligned}$$

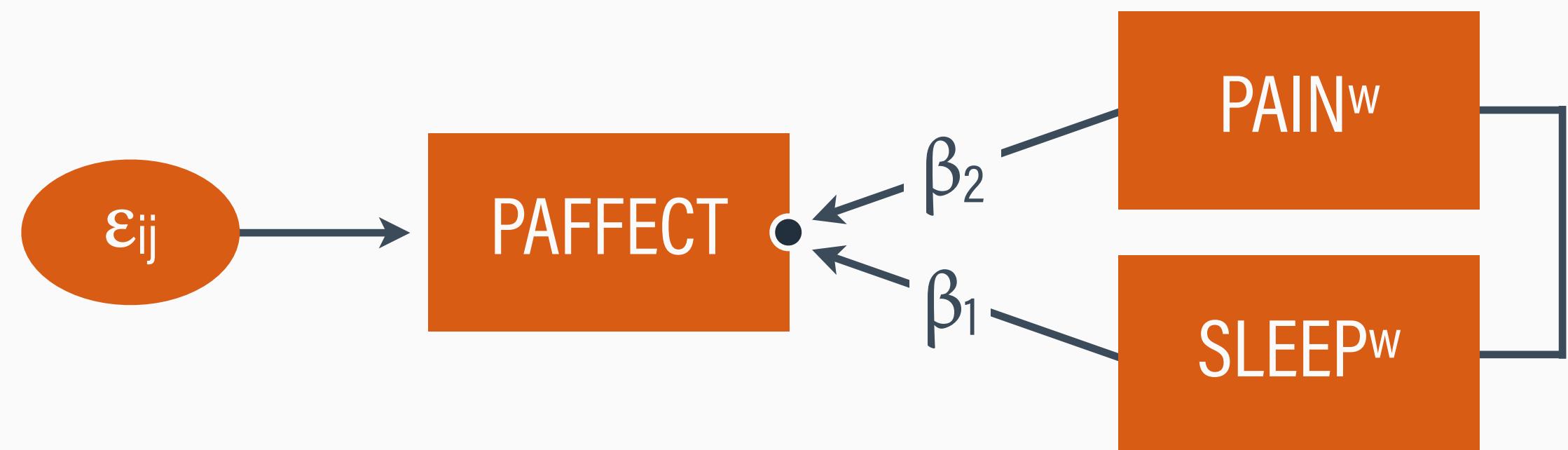
# PATH DIAGRAM

$$paffect_{ij} = \beta_0 + \beta_1(sleep_{ij}^W) + \beta_2(pain_{ij}^W) + \\ \beta_3(sleep_j^b) + \beta_4(pain_j^b) + \\ \beta_5(female_j) + \beta_6(stress_j) + \\ u_{0j} + \varepsilon_{ij}$$

Level-2



Level-1



• = random intercept ( $\beta_{0j}$ )

# BLIMP STUDIO SCRIPT 4.3

---

**DATA:** PainDiary.dat;

**VARIABLES:** Person Day PosAffect NegAffect Pain WorkGoal LifeGoal SleepQual Female Education  
Employment MarStatus NumDiagnose ActivityLevel PainAccept Catastrophize Stress Anxiety;

**NOMINAL:** Female; # automatic dummy coding for nominal predictors

**CLUSTERID:** Person;

**CENTER:**

groupmean = SleepQual Pain;

grandmean = SleepQual.mean Pain.mean Stress;

**MODEL:** PosAffect ~ intercept SleepQual Pain SleepQual.mean Pain.mean Female Stress | intercept;

**BURN:** 10000;

**ITERATIONS:** 10000;

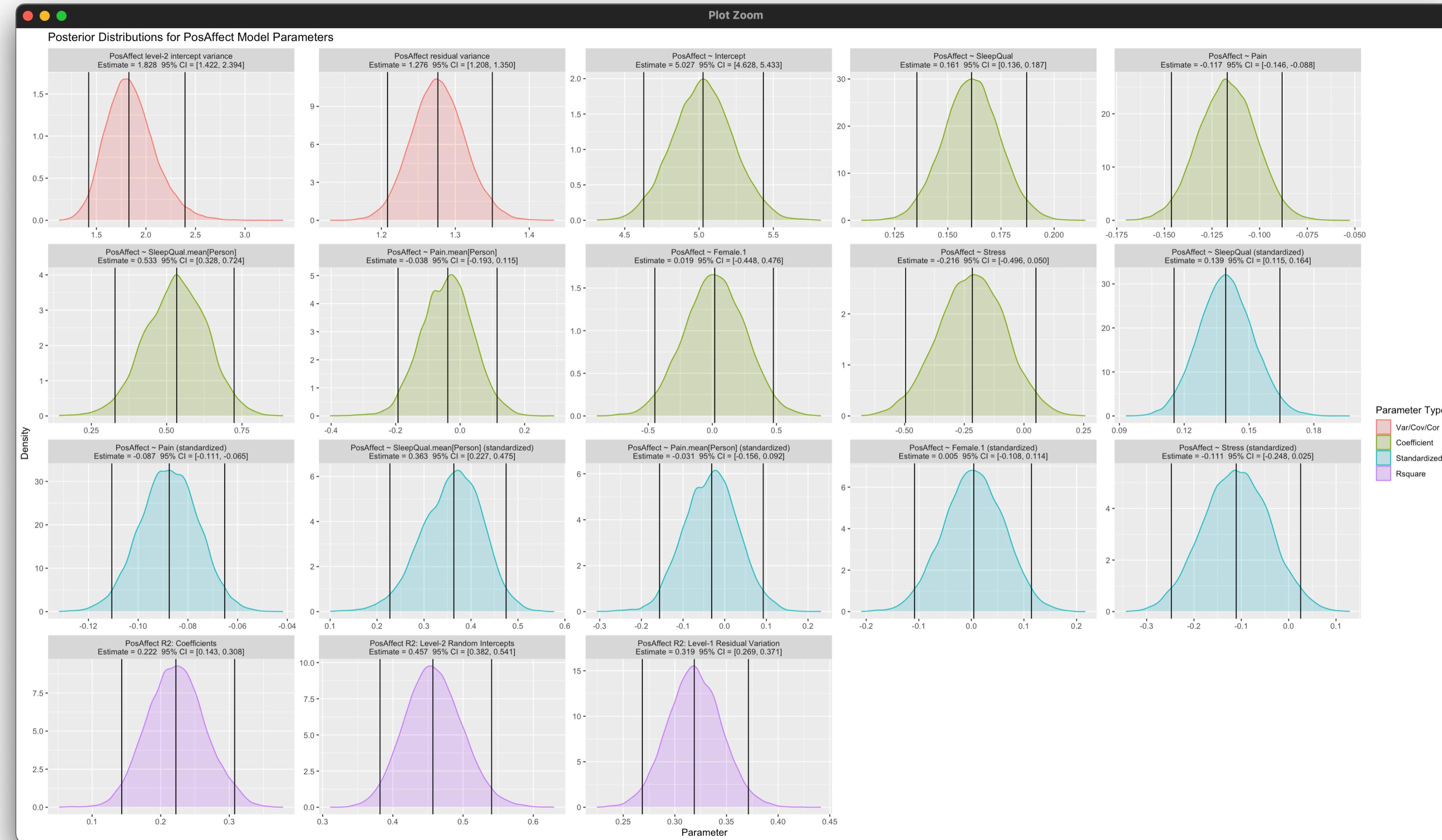
**SEED:** 90291;

# RBLIMP SCRIPT 4 (MODEL 3)

---

```
model3 <- rblimp(  
  data = PainDiary,  
  nominal = 'Female',  
  clusterid = 'Person',  
  center = 'groupmean = SleepQual Pain; grandmean = SleepQual.mean Pain.mean Stress',  
  model = 'PosAffect ~ intercept SleepQual Pain SleepQual.mean Pain.mean Female Stress | intercept',  
  seed = 90291,  
  burn = 10000,  
  iter = 10000)  
output(model3)  
plot_posterior(model3, 'PosAffect')
```

# PARAMETER PLOTS (RBLIMP ONLY)



# BLIMP OUTPUT

- █ = level-2 estimate
- █ = level-1 estimate
- █ = combined estimate

Outcome Variable: PosAffect

Grand Mean Centered: Pain.mean[Person] SleepQual.mean[Person] Stress

Group Mean Centered: Pain SleepQual

Parameters	Estimate	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
<hr/>							
Variances:							
L2 : Var(Intercept)	<b>1.828</b>	0.249	1.412	2.398	---	---	4385.748
Residual Var.	<b>1.277</b>	0.036	1.208	1.350	---	---	8247.203
<hr/>							
Coefficients:							
Intercept	<b>5.041</b>	0.202	4.641	5.431	620.928	0.000	190.993
SleepQual	<b>0.161</b>	0.013	0.135	0.187	150.704	0.000	9375.960
Pain	<b>-0.117</b>	0.015	-0.147	-0.088	61.071	0.000	8973.437
SleepQual.mean[Person]	<b>0.531</b>	0.105	0.317	0.723	25.163	0.000	145.860
Pain.mean[Person]	<b>-0.039</b>	0.078	-0.187	0.116	0.247	0.620	149.240
Female.1	<b>0.009</b>	0.228	-0.429	0.466	0.002	0.967	194.738
Stress	<b>-0.214</b>	0.136	-0.502	0.041	2.585	0.108	176.945
<hr/>							
Standard Deviations:							
L2 : SD(Intercept)	<b>1.352</b>	0.091	1.188	1.549	---	---	4392.623
Residual SD	<b>1.130</b>	0.016	1.099	1.162	---	---	8250.812

...

# FIXED EFFECT INTERPRETATIONS

---

- $\beta_0 = 5.04$  is the positive affect mean for the female = 0 group (because all numeric predictors are centered)
- $\beta_1 = 0.16$  is the expected affect difference between two daily sleep scores from the same person that differ by one point, controlling for within-person pain
- $\beta_2 = -0.12$  is the expected affect difference between two daily pain scores from the same person that differ by one point, controlling for within-person sleep

# FIXED EFFECTS, CONTINUED

---

- $\beta_3 = 0.53$  is the expected affect difference between two people whose average sleep ratings differ by one point, controlling for average pain, sex, and stress
- $\beta_4 = -0.04$  is the expected affect difference between two people whose average pain ratings differ by one point, controlling for average sleep, sex, and stress
- $\beta_5 = 0.01$  is the expected affect difference for females (versus males), controlling for average sleep, average pain, and stress
- $\beta_6 = -0.21$  is the expected affect difference between two people whose stress scores differ by one point, controlling for average sleep, average pain, and sex

# RANDOM EFFECT INTERPRETATIONS

---

- $u_{0j} = \beta_{0j} - (\beta_0 + \beta_3(\text{sleep}_j^b) + \beta_4(\text{pain}_j^b) + \beta_5(\text{female}_j) + \beta_6(\text{stress}_j))$
- $\text{var}(u_{0j}) = 1.83$  is the average squared distance between the level-2 affect means and the grand mean
- $\text{sd}(u_{0j}) = 1.35$  is the average distance between the level-2 affect means and the grand mean
- $\varepsilon_{ij} = \text{pffect}_{ij} - (\beta_{0j} + \beta_1(\text{sleep}_{ij}^w) + \beta_2(\text{pain}_{ij}^w))$
- $\text{var}(\varepsilon_{ij}) = 1.28$  is the average squared distance between the level-1 affect observations and their predicted values
- $\text{sd}(\varepsilon_{ij}) = 1.13$  is the average distance between the level-1 affect observations and their predicted values

# BLIMP OUTPUT, CONTINUED

---

Outcome Variable: PosAffect

 = level-2 estimate

 = level-1 estimate

 = combined estimate

Grand Mean Centered: Pain.mean[Person] SleepQual.mean[Person] Stress

Group Mean Centered: Pain SleepQual

Parameters	Estimate	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
<hr/>							
...							

Standardized Coefficients:

SleepQual	<b>0.139</b>	0.013	0.116	0.165	124.128	0.000	2565.722
Pain	<b>-0.088</b>	0.012	-0.111	-0.065	55.958	0.000	4529.435
SleepQual.mean[Person]	<b>0.362</b>	0.066	0.218	0.473	29.402	0.000	149.255
Pain.mean[Person]	<b>-0.032</b>	0.064	-0.150	0.094	0.249	0.618	149.922
Female.1	<b>0.002</b>	0.055	-0.104	0.112	0.002	0.966	194.588
Stress	<b>-0.110</b>	0.068	-0.252	0.021	2.635	0.105	176.893

Proportion Variance Explained

by Coefficients	<b>0.220</b>	0.042	0.139	0.305	---	---	184.037
by Level-2 Random Intercepts	<b>0.458</b>	0.041	0.383	0.543	---	---	418.942
by Level-1 Residual Variation	<b>0.320</b>	0.026	0.270	0.372	---	---	496.184

---

# EFFECT SIZE INTERPRETATIONS

---

- $R^2_{(\text{predictors})} = .22$  is the proportion of the total variation explained by all predictors (was  $.03$  in previous model)
- $R^2_{(\text{residual-between})} = .46$  is the proportion of the total variation attributable to the between-cluster residuals (the  $u_{0j}$  terms)
- $R^2_{(\text{residual-within})} = .32$  is the proportion of the total variation attributable to the within-cluster residuals (the  $\varepsilon_{ij}$  terms)

# MODEL COMPARISON

---

Parameter	Empty Model	Level-1 Predictors (Within Only)	Level-2 Predictors (Within + Between)
Fixed intercept	5.03	5.03	5.04
Sleep (within-person)	--	0.16	0.16
Pain (within-person)	--	-0.12	-0.12
Sleep (between-person)	--	--	0.53
Pain (between-person)	--	--	-0.04
Female (between-person)	--	--	0.01
Stress (between-person)	--	--	-0.21
Residual intercept variance	2.53	2.50	1.83
Residual within-person variance	1.40	1.28	1.28
R-square predictors	0	.03	.22
R-square intercept variance	.64	.64	.46
R-square within-person variance	.36	.33	.32

# OUTLINE

- 1 Modeling Step 1: Estimate ICCs
- 2 Modeling Step 2: Add Within-Cluster Predictors
- 3 Rights and Sterba Effect Sizes
- 4 Modeling Step 3: Add Between-Cluster Predictors
- 5 Latent Variable Specification

# BLIMP STUDIO SCRIPT 4.4

---

**DATA:** PainDiary.dat;

**VARIABLES:** Person Day PosAffect NegAffect Pain WorkGoal LifeGoal SleepQual Female Education  
Employment MarStatus NumDiagnose ActivityLevel PainAccept Catastrophize Stress Anxiety;

**CLUSTERID:** Person;

**LATENT:** Person = beta0j; # define level-2 intercept latent variable

**CENTER:** groupmean = SleepQual;

**MODEL:**

beta0j ~ intercept; # level-2 regression equation ( $\beta_{0j} = \gamma_{00} + u_{0j}$ )

PosAffect ~ intercept@beta0j SleepQual Pain; # level-1:  $p\text{affect}_{ij} = \beta_{0j} + \beta_1(\text{sleep}_{ij}^W) + \beta_2(\text{pain}_{ij}^W) + \varepsilon_{ij}$

**BURN:** 10000;

**ITER:** 20000;

**SEED:** 90291;

# RBLIMP SCRIPT 4 (MODEL 4)

---

```
model1 <- rblimp(  
  data = PainDiary,  
  clusterid = 'Person',  
  latent = 'Person = beta0j',  
  center = 'groupmean = SleepQual Pain',  
  model = '  
    beta0j ~ intercept;  
    PosAffect ~ intercept@beta0j SleepQual Pain',  
  seed = 90291,  
  burn = 10000,  
  iter = 20000)  
output(model1)
```

# LEVEL-1 OUTPUT

 = level-2 estimate

 = level-1 estimate

Outcome Variable: PosAffect

Group Mean Centered: Pain SleepQual

Parameters	Median	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
<hr/>							
Variances:							
Residual Var.	<b>1.276</b>	0.036	1.209	1.349	---	---	17487.328
<hr/>							
Coefficients:							
beta0j	@ 1.000	---	---	---	---	---	---
SleepQual	<b>0.162</b>	0.013	0.136	0.188	154.282	0.000	17775.118
Pain	<b>-0.117</b>	0.015	-0.147	-0.088	61.358	0.000	18195.499
<hr/>							
Standardized Coefficients:							
SleepQual	0.237	0.018	0.201	0.273	167.226	0.000	17168.959
Pain	-0.148	0.019	-0.185	-0.111	63.333	0.000	18197.357
<hr/>							
Proportion Variance Explained							
by Coefficients	0.087	0.010	0.067	0.108	---	---	17479.089
by Residual Variation	0.913	0.010	0.892	0.933	---	---	17479.089

# LEVEL-2 OUTPUT

■ = level-2 estimate

■ = level-1 estimate

Latent Variable: beta0j

Parameters	Median	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
<hr/>							
Variances:							
Residual Var.	<b>2.489</b>	0.330	1.946	3.237	---	---	16621.302
<hr/>							
Coefficients:							
Intercept	<b>5.034</b>	0.140	4.756	5.310	1284.262	0.000	18856.765
<hr/>							
Proportion Variance Explained							
by Coefficients	0.000	0.000	0.000	0.000	---	---	nan
by Residual Variation	1.000	0.000	1.000	1.000	---	---	nan

# BLIMP STUDIO SCRIPT 4.5

---

**DATA:** PainDiary.dat;

**VARIABLES:** Person Day PosAffect NegAffect Pain WorkGoal LifeGoal SleepQual Female Education  
Employment MarStatus NumDiagnose ActivityLevel PainAccept Catastrophize Stress Anxiety;

**NOMINAL:** Female;

**CLUSTERID:** Person;

**LATENT:** Person = beta0j; # define level-2 intercept latent variable

**CENTER:** groupmean = SleepQual; grandmean = SleepQual.mean Pain.mean Stress;

**MODEL:** # level-2:  $\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{sleep}_j^b) + \gamma_{02}(\text{pain}_j^b) + \gamma_{03}(\text{female}_j) + \gamma_{04}(\text{stress}_j) + u_{0j}$

beta0j ~ intercept SleepQual.mean Pain.mean Female Stress;

PosAffect ~ intercept@beta0j SleepQual Pain; # level-1:  $p\text{affect}_{ij} = \beta_{0j} + \beta_1(\text{sleep}_{ij}^w) + \beta_2(\text{pain}_{ij}^w) + \varepsilon_{ij}$

**BURN:** 10000;

**ITER:** 20000;

**SEED:** 90291;

# RBLIMP SCRIPT 4 (MODEL 5)

---

```
model3 <- rblimp(  
  data = PainDiary,  
  nominal = 'Female',  
  clusterid = 'Person',  
  latent = 'Person = beta0j',  
  center = 'groupmean = SleepQual Pain; grandmean = SleepQual.mean Pain.mean Stress',  
  model = '  
    beta0j ~ intercept SleepQual.mean Pain.mean Female Stress;  
    PosAffect ~ intercept@beta0j SleepQual Pain',  
  seed = 90291,  
  burn = 10000,  
  iter = 20000)  
output(model3)
```

# LEVEL-1 OUTPUT

 = level-2 estimate

 = level-1 estimate

Outcome Variable: PosAffect

Group Mean Centered: Pain SleepQual

Parameters	Median	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
<hr/>							
Variances:							
Residual Var.	<b>1.277</b>	0.036	1.208	1.349	---	---	18773.456
<hr/>							
Coefficients:							
beta0j	@ 1.000	---	---	---	---	---	---
SleepQual	<b>0.161</b>	0.013	0.136	0.187	154.038	0.000	16824.474
Pain	<b>-0.117</b>	0.015	-0.147	-0.088	60.356	0.000	18396.872
<hr/>							
Standardized Coefficients:							
SleepQual	0.236	0.018	0.200	0.272	167.294	0.000	17192.061
Pain	-0.149	0.019	-0.185	-0.112	62.454	0.000	18410.177
<hr/>							
Proportion Variance Explained							
by Coefficients	0.086	0.011	0.067	0.108	---	---	17786.052
by Residual Variation	0.914	0.011	0.892	0.933	---	---	17786.052

# LEVEL-2 OUTPUT

 = level-2 estimate

 = level-1 estimate

Latent Variable: beta0j

Grand Mean Centered: Pain.mean[Person] SleepQual.mean[Person]  
Stress

Parameters	Median	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
<hr/>							
Variances:							
Residual Var.	<b>1.835</b>	0.250	1.423	2.399	---	---	15231.617
<hr/>							
Coefficients:							
Intercept	<b>5.039</b>	0.216	4.608	5.466	541.498	0.000	8572.715
SleepQual.mean[Person]	<b>0.535</b>	0.101	0.337	0.734	27.999	0.000	11316.753
Pain.mean[Person]	<b>-0.038</b>	0.081	-0.198	0.123	0.219	0.640	14391.068
Female.1	<b>-0.008</b>	0.259	-0.512	0.500	0.001	0.972	16962.379
Stress	<b>-0.215</b>	0.132	-0.473	0.042	2.664	0.103	16321.312
<hr/>							
Standardized Coefficients:							
SleepQual.mean[Person]	0.454	0.075	0.294	0.587	35.928	0.000	11569.228
Pain.mean[Person]	-0.039	0.082	-0.198	0.123	0.221	0.638	14296.578
Female.1	-0.002	0.077	-0.153	0.149	0.001	0.971	16956.623
Stress	-0.136	0.082	-0.291	0.026	2.738	0.098	16320.026
<hr/>							
Proportion Variance Explained							
by Coefficients	0.298	0.063	0.174	0.418	---	---	12568.753
by Residual Variation	0.702	0.063	0.582	0.826	---	---	12568.753