MODULE 2

DISAGGREGATING LEVEL-1 PREDICTORS

LEVEL-1 PREDICTORS ARE SPECIAL

- Multilevel modeling texts often recommend a model-building process that begins by adding predictors at level-1
- Level-1 predictors require several key decisions that distinguish them from predictors in standard regression
- One such issue is that level-1 predictors can exert effects at multiple levels; disaggregation separates those effects.

OUTLINE

1. Associations at Level-1 and Level-2
2. Review of Grand Mean Centering
3. Disaggregating a Level-1 Predictor by Centering Within Cluster
4. Analysis Examples
5. Using Raw Level-1 Predictors (Smushed Model)
6. Latent Variable Specification

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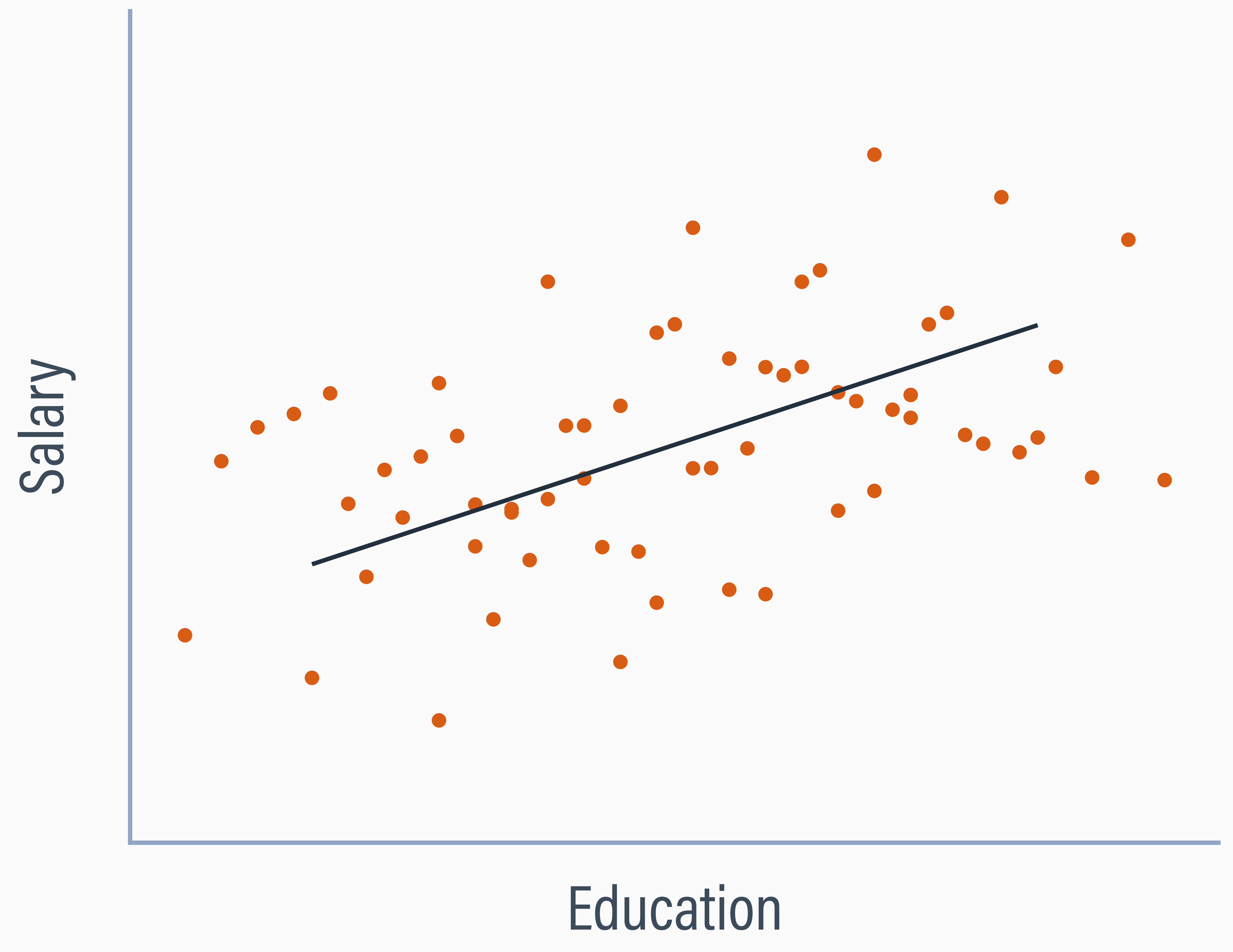
BIVARIATE SCATTERPLOT

- Bivariate scatterplot where years of education predicts salary
- As years of education increases, so too does salary



NESTED DATA SCATTERPLOT

- Multilevel data where employees are nested in industries

- Within a given industry, salary increases with years of education

LEVEL-2 CLUSTER MEANS

- The large circles are the level-2 (industry-level) averages
- There are level-2 mean differences on education (horizontal separation of dots) and salary (vertical separation of dots)

BETWEEN-CLUSTER ASSOCIATION

- The industry-level association involving the level-2 cluster means is negative
- Industries with higher mean education have lower mean wages

IMPORTANT TAKEAWAYS

- Level-1 predictors can have two sources of variation, just like outcomes measured at level-1
- A single predictor can produce level-1 and level-2 regression slopes can differ in magnitude or sign
- We isolate the effect of interest by disaggregating level-1 predictors into distinct level-1 and level-2 components

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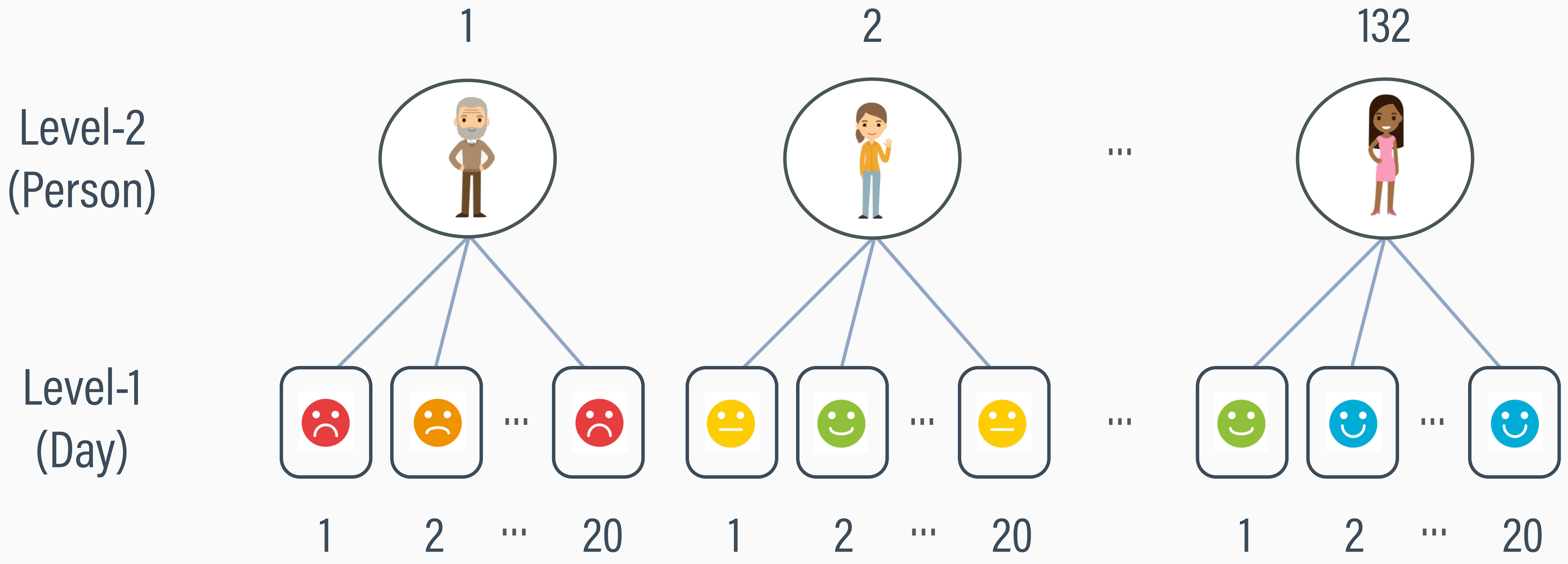
CENTERING PREDICTORS

- Centering involves subtracting a predictor's mean from its scores (we would never center an outcome)
- In single-level regression models, centering predictors at the overall grand mean is a common option
- Multilevel models offer multiple forms of centering

DAILY DIARY APPLICATION

- n_j = 20 daily positive affect and sleep assessments nested within J = 132 chronic pain patients (N = 2680 data records)

$$paffect_{ij} = \beta_{0j} + \beta_1(sleep_{ij}) + \varepsilon_{ij}$$



DATA STRUCTURE

- Data in stacked or long format
- Each level-2 unit (person) has multiple rows, one per level-1 (daily) observation
- The i subscript indexes level-1 observations, and j indexes level-2 units

Row	i	j	$PAFFECT_{ij}$	$SLEEP_{ij}$	
1	1	1	7.3	5.6	
2	2	1	2.5	4.3	
…	…	1	…	…	Person 1
20	20	1	6.3	7.3	
21	1	2	4.0	3.9	
22	2	2	4.0	7.1	
…	…	2	…	…	Person 2
40	20	2	4.4	3.5	
…	…	…	…	…	
2621	1	132	3.3	5.4	
2622	2	132	4.8	3.5	
…	…	132	…	…	Person 132
2640	20	132	4.8	7.9	

CENTERING AT THE GRAND MEAN (CGM)

- CGM subtracts the grand mean from every level-1 score

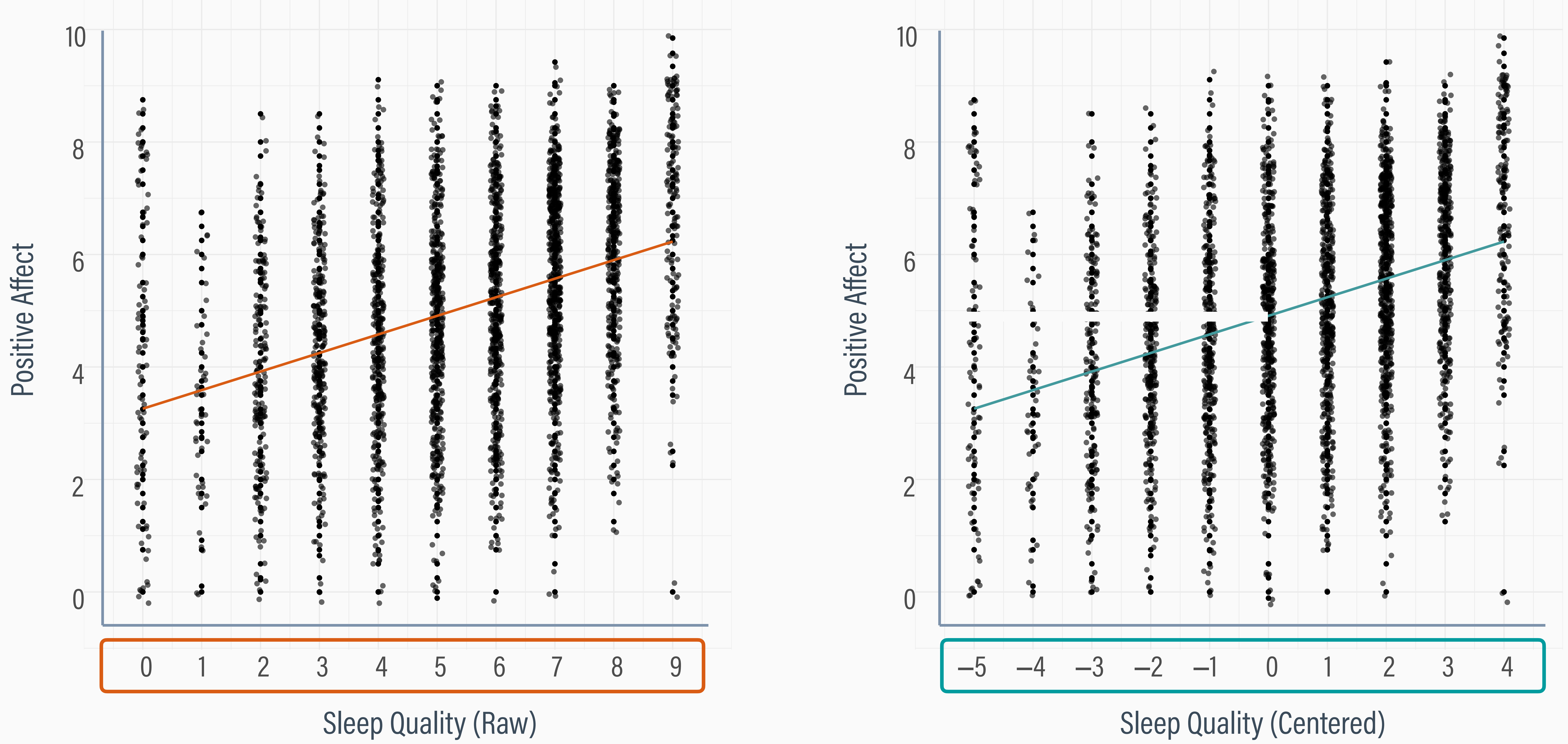
$$X_{ij}^{c} = X_{ij} - \mu_X$$

- Centering recodes scores as distances (deviations) from the center of the data

Person	Day	Sleep (X_{ij})		Sleep Mean (μ_X)		Sleep CGM (X_{ij}^{c})
1	1	1.25	-	5	=	-3.75
1	2	2.75	-	5	=	-2.25
1	3	2.00	-	5	=	-3.00
2	1	3.25	-	5	=	-1.75
2	2	5.00	-	5	=	0
2	3	6.75	-	5	=	1.75
3	1	8.75	-	5	=	3.75
3	2	7.00	-	5	=	2.00
3	3	8.25	-	5	=	3.25

VISUALIZING CENTERING

Centering recodes predictor scores on the X axis



IMPACT OF CENTERING ON THE INTERCEPT

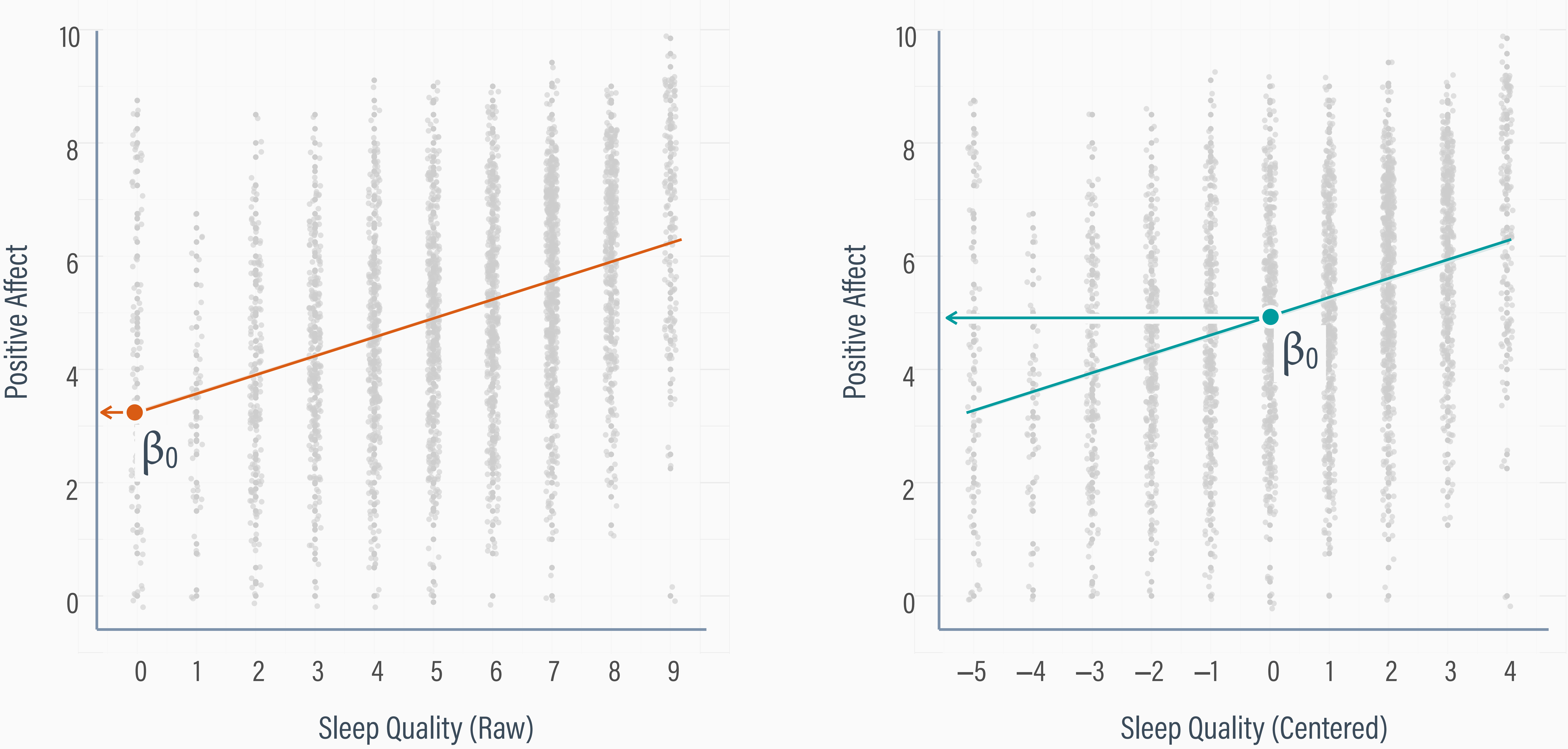
- The intercept is the expected outcome score when the predictor equals zero (often a score outside the data)

$$\widehat{paffect}_i = \beta_0 + \beta_1(sleep_i)$$
$$\widehat{paffect}_i = \beta_0 + \beta_1(0) = \beta_0$$

- Because a regression line must intersect the means of both variables, making the predictor's mean equal 0 by centering defines the intercept as the outcome variable's mean

CENTERING AND THE INTERCEPT

Centering changes the intercept but the slope remains the same!



ANALYSIS COMPARISON

```
Outcome Variable:  paffect

Grand Mean Centered: sleep

Parameters                          Estimate
                                ------------
Variances:
  L2 : Var(Intercept)                  2.168
  Residual Var.                        1.306

Coefficients:
  Intercept                            5.038
  SleepQual                            0.181

Proportion Variance Explained
  by Coefficients                      0.044
  by Level-2 Random Intercepts         0.597
  by Level-1 Residual Variation        0.359

                                ------------
```

β_0 = paffect grand mean

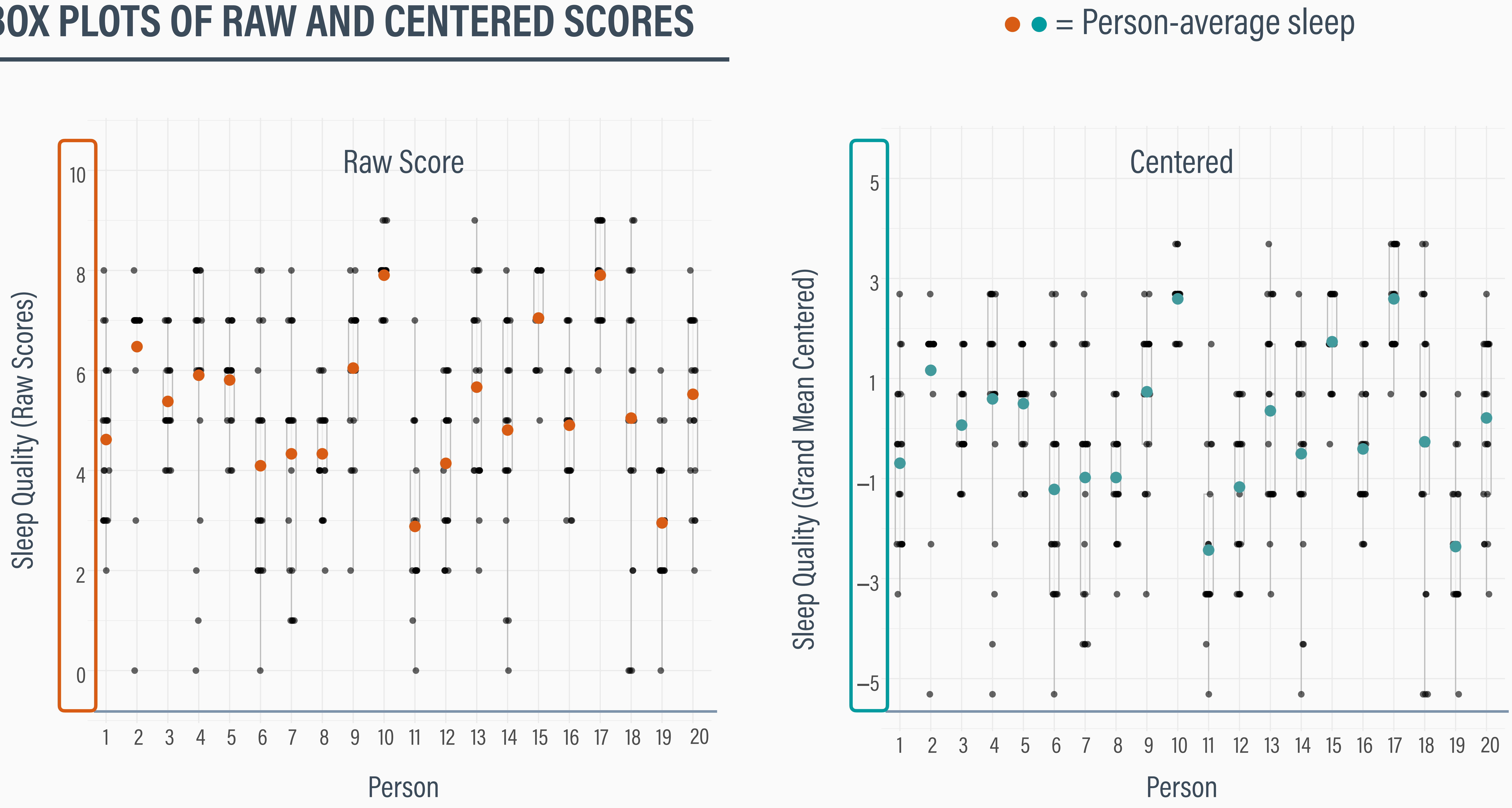
```
Outcome Variable:  paffect

Parameters                          Estimate
                                ------------
Variances:
  L2 : Var(Intercept)                  2.171
  Residual Var.                        1.307

Coefficients:
  Intercept                            4.047
  SleepQual                            0.181

Proportion Variance Explained
  by Coefficients                      0.043
  by Level-2 Random Intercepts         0.597
  by Level-1 Residual Variation        0.359

                                ------------
```

$\beta_0 = \widehat{\text{paffect}}$ for sleep = 0

GRAND MEAN CENTERING IN MLM

- Level-1 predictors can have two sources of variation, just like outcomes measured at level-1
- People differ in their average sleep quality, and daily observations fluctuate around each person’s own mean
- How does grand mean centering affect sources of variation?

BOX PLOTS OF RAW AND CENTERED SCORES

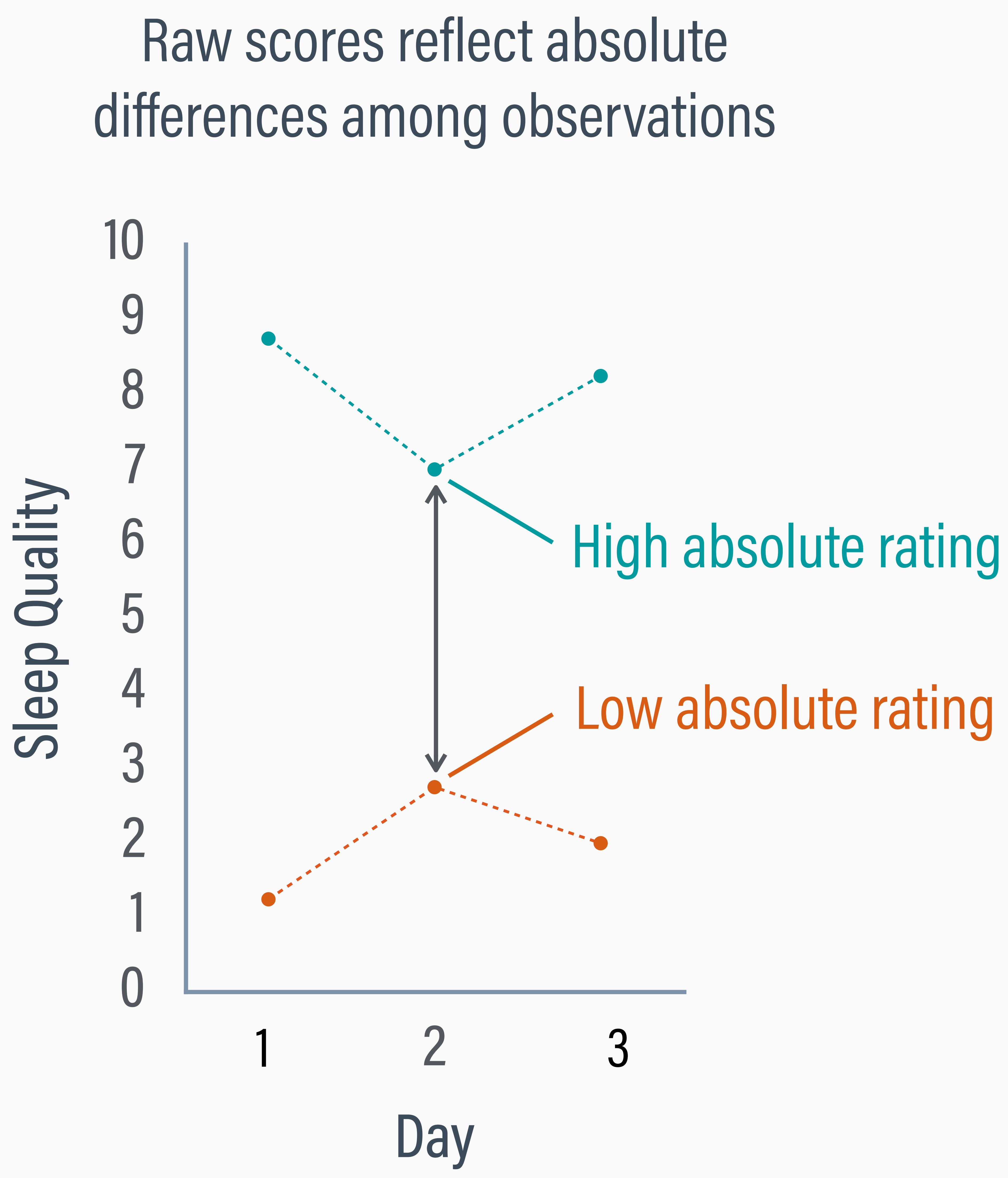


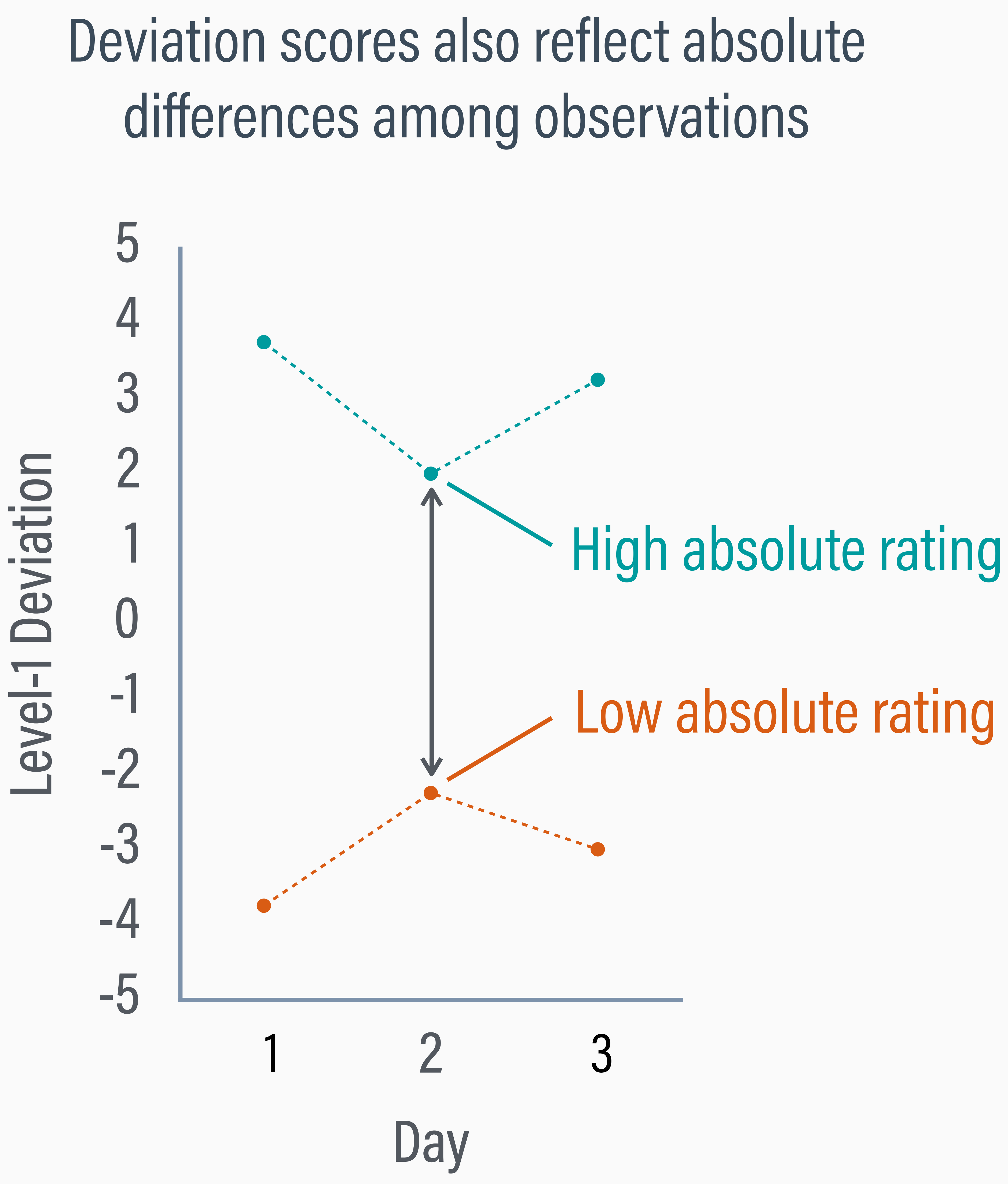
The intraclass correlation for daily sleep quality is ICC = .39. In small groups of two or three, compare the two plots, then discuss whether and how centering affects level-2 variation in the person means.

GRAND MEAN CENTERING SUMMARY

- Grand mean centering is a recoding operation that maintains the rank ordering and relative magnitude of the raw scores
- Centered scores retain all between-cluster variation of the original variable (ICCs are the same as the raw scores)
- Centering defines the intercept as the outcome's grand mean

CGM RETAINS BOTH SOURCES OF VARIATION





OUTLINE

1 Associations at Level-1 and Level-2

2 Review of Grand Mean Centering

3 Disaggregating a Level-1 Predictor by Centering Within Cluster

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6 Latent Variable Specification

DISAGGREGATION

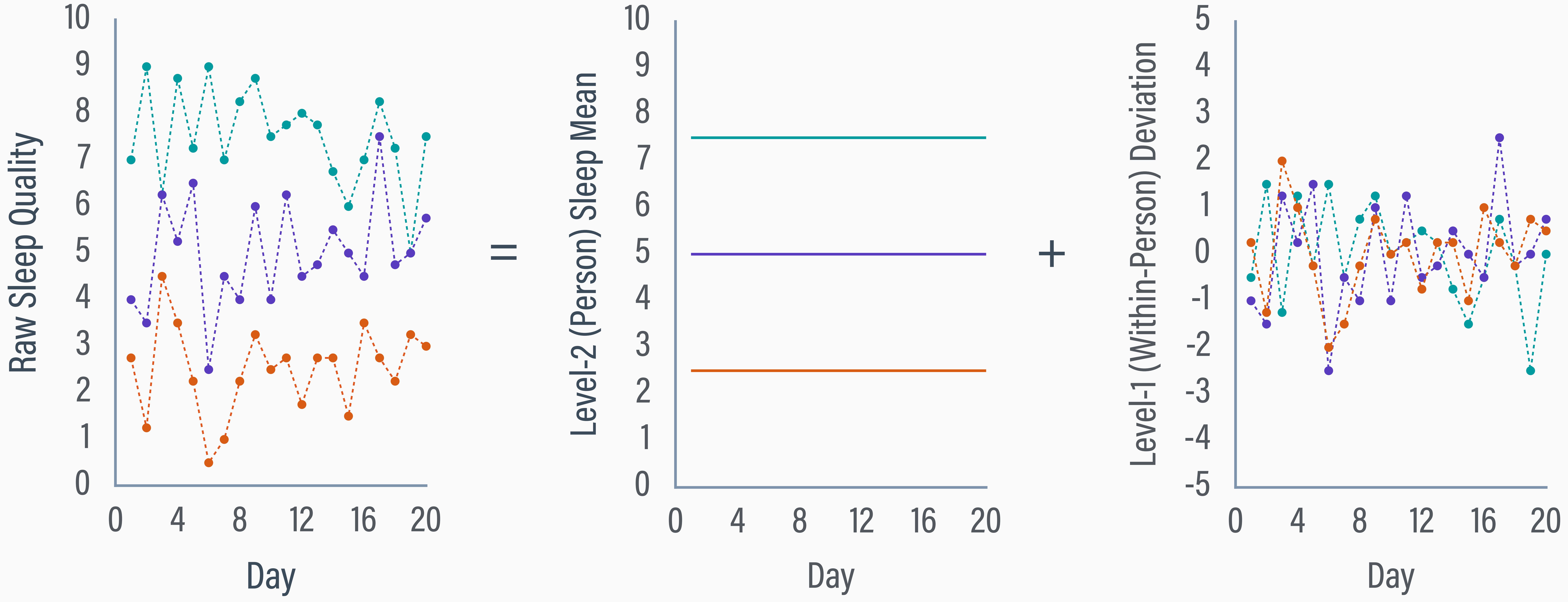
- The literature generally recommends disaggregating level-1 predictor variables into two distinct, uncorrelated variables

$$X_{ij} = X_j^b + X_{ij}^w = \text{cluster mean} + \text{within-cluster deviation}$$

- The level-2 cluster means (sleep averages) define a between-cluster variable (X_j^b), and deviations around the level-2 means define a pure within-cluster variable (X_{ij}^w)

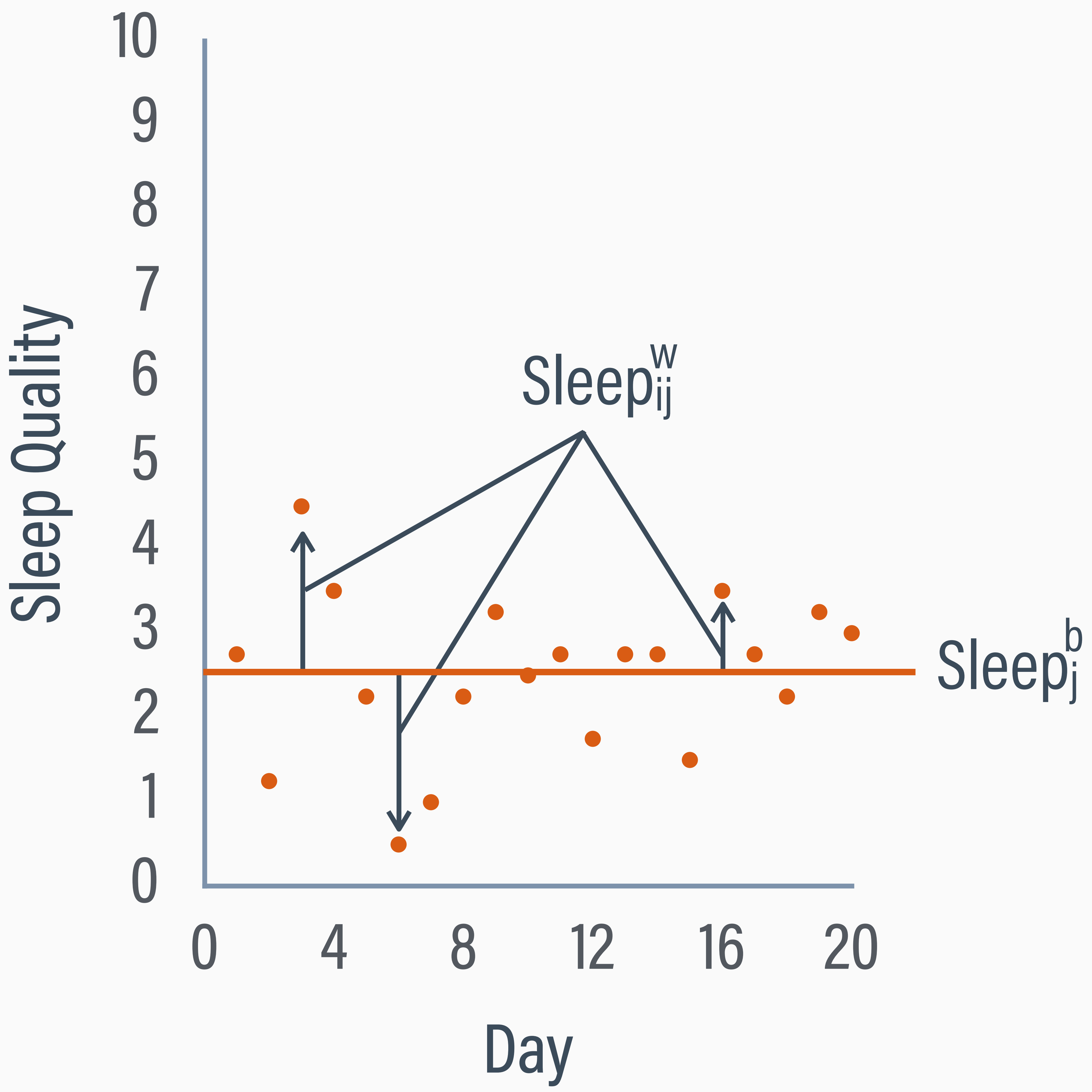
DISAGGREGATION GRAPHIC

Score = Level-2 (Between-Cluster) Mean + Level-1 (Within-Cluster) Residual



DISAGGREGATION EXAMPLE 1

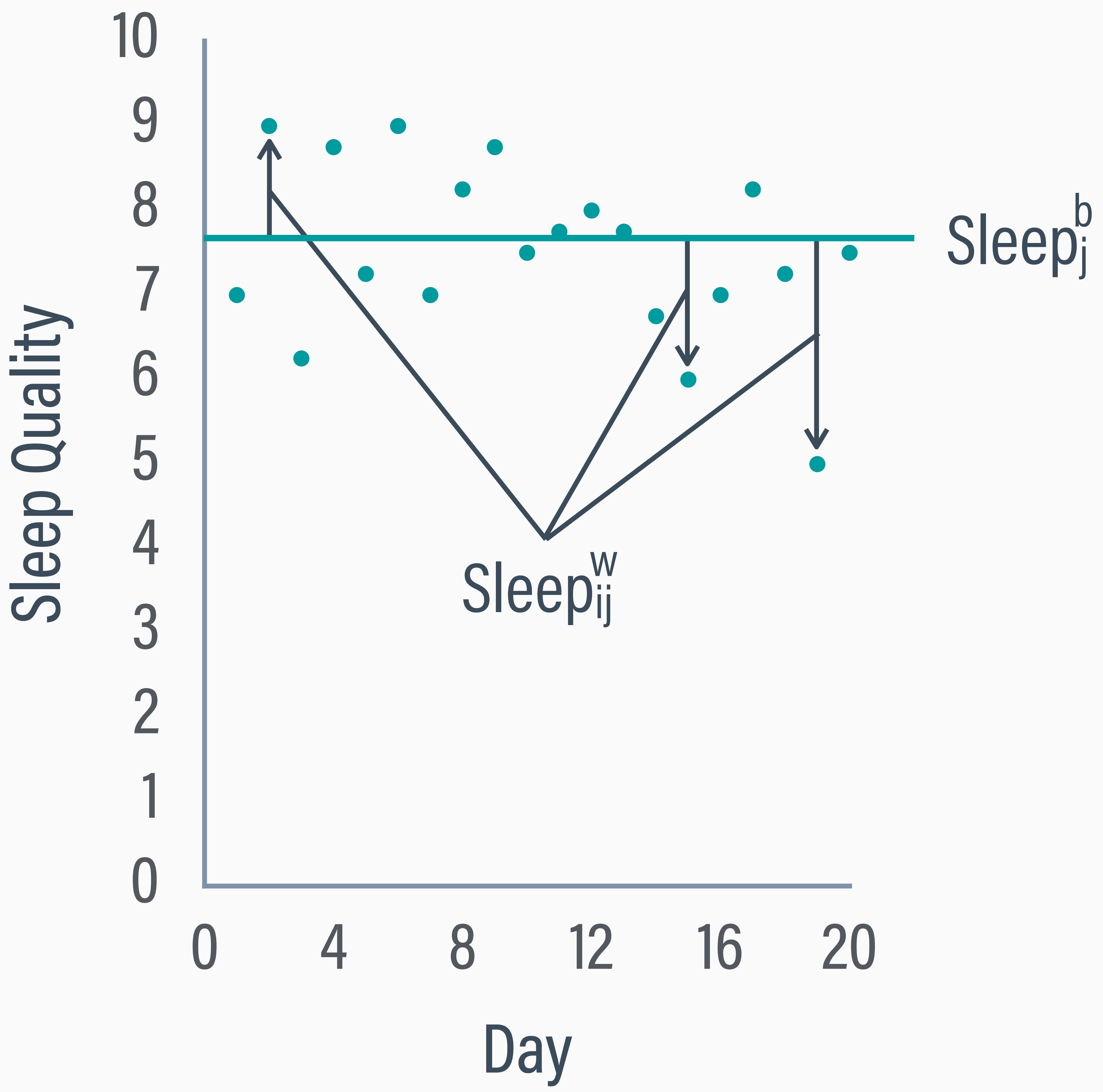
Score = Level-2 (Between-Cluster) Mean + Level-1 (Within-Cluster) Residual



Person	Day	$Sleep_{ij}$		$Sleep^{b}_{j}$		$Sleep^{w}_{ij}$
1	1	2.75	=	2.5	+	0.25
1	2	1.25	=	2.5	+	−1.25
1	3	4.50	=	2.5	+	2.00
1	...	...	=	...	+	...
1	18	2.25	=	2.5	+	−0.25
1	19	3.25	=	2.5	+	0.75
1	20	3.00	=	2.5	+	0.50

DISAGGREGATION EXAMPLE 2

Score = Level-2 (Between-Cluster) Mean + Level-1 (Within-Cluster) Residual



Person	Day	$Sleep_{ij}$		$Sleep_j^b$		$Sleep_{ij}^w$
3	1	7.00	=	7.5	+	−0.50
3	2	9.00	=	7.5	+	1.50
3	3	6.25	=	7.5	+	−1.25
3	...	...	=	...	+	...
3	18	7.25	=	7.5	+	−0.25
3	19	5.00	=	7.5	+	−2.50
3	20	7.50	=	7.5	+	0

CENTERING WITHIN CLUSTER (CWC)

- CWC subtracts out each person's own level-2 mean from their level-1 scores

$$X_{ij}^{w} = X_{ij} - \mu_{Xj} = X_{ij} - X_{j}^{b}$$

- CWC deviation scores are "pure" measures of within-cluster variation
- Also called group mean centering

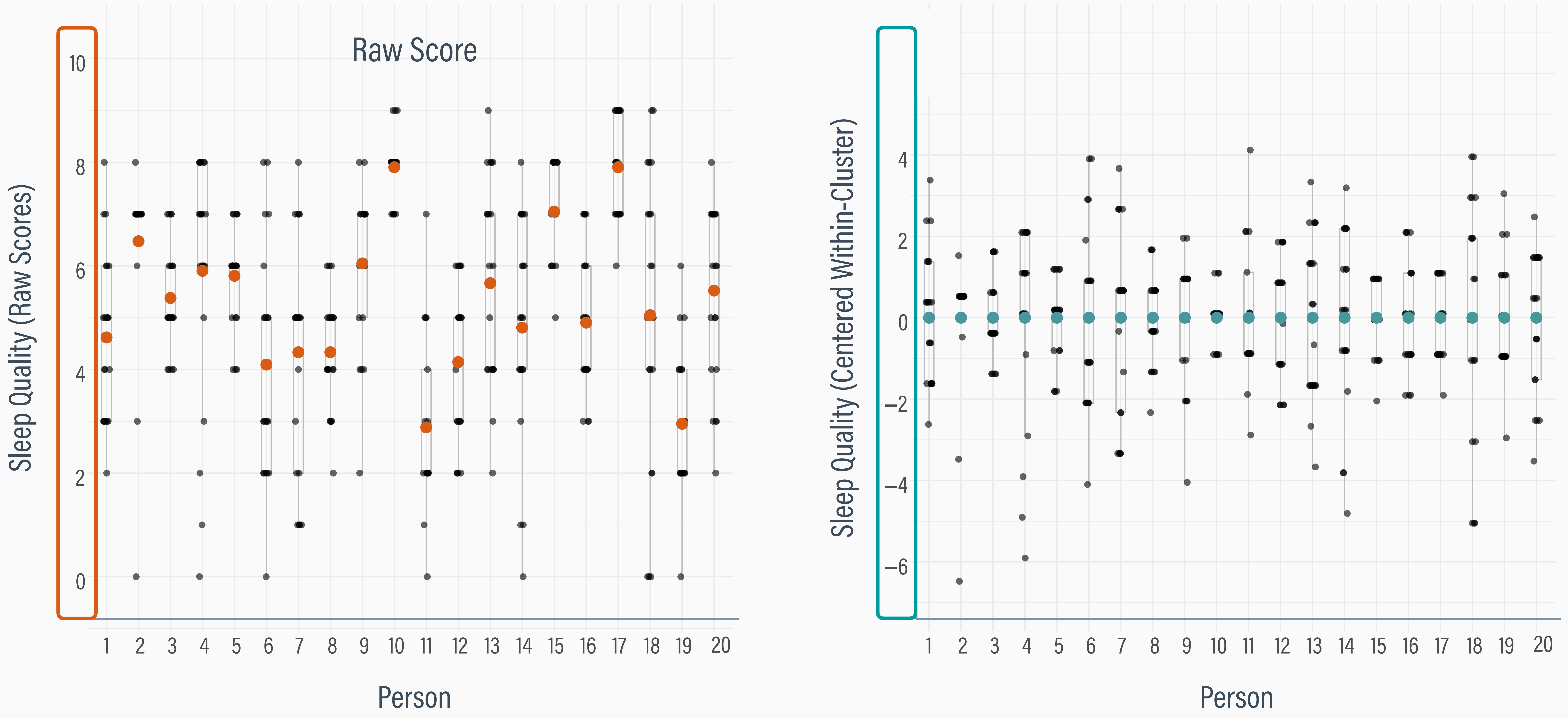
Person	Day	Sleep (X_{ij})		Sleep Mean (X_{j}^{b})		Sleep CWC (X_{ij}^{w})
1	1	1.25	–	2	=	–0.75
1	2	2.75	–	2	=	0.75
1	3	2.00	–	2	=	0
2	1	8.75	–	8	=	0.75
2	2	7	–	8	=	–1.00
2	3	8.25	–	8	=	0.25

TWO TYPES OF CLUSTER MEANS

- A cluster-level average (X^b) can be defined as the arithmetic mean of cluster j's scores (manifest group mean) or as a random intercept (latent group mean)
- Arithmetic averages are biased with small Ns and assume each cluster's mean uses the same number of level-1 scores
- Latent cluster means are noisy in small samples, but they are unbiased and appropriate with unequal cluster sizes

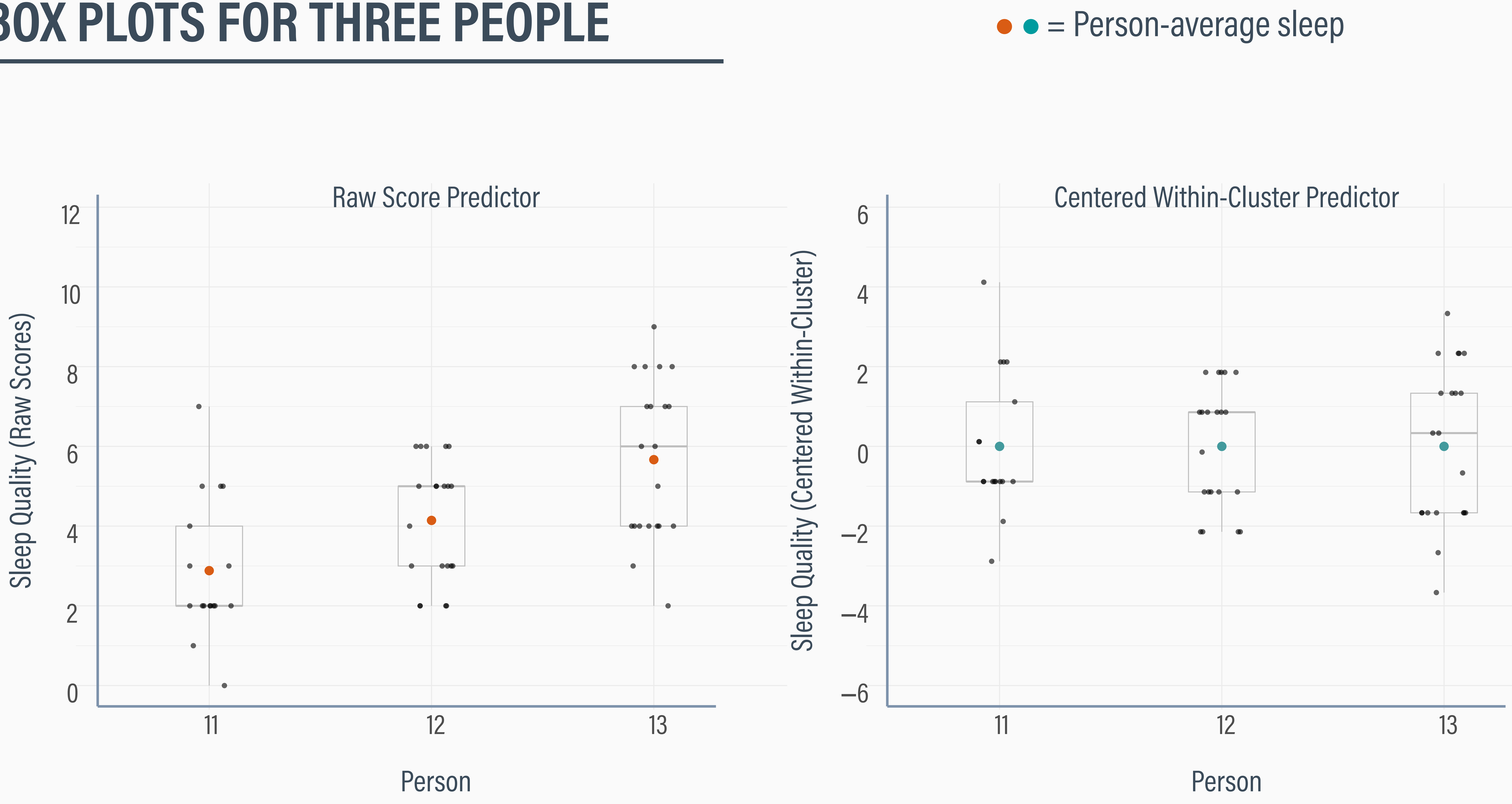
BOX PLOTS OF RAW AND CENTERED SCORES

= Person-average sleep



The intraclass correlation for daily sleep quality is ICC = .39. In small groups of two or three, compare the two plots, then discuss whether and how centering affects level-2 variation in the person means.

BOX PLOTS FOR THREE PEOPLE



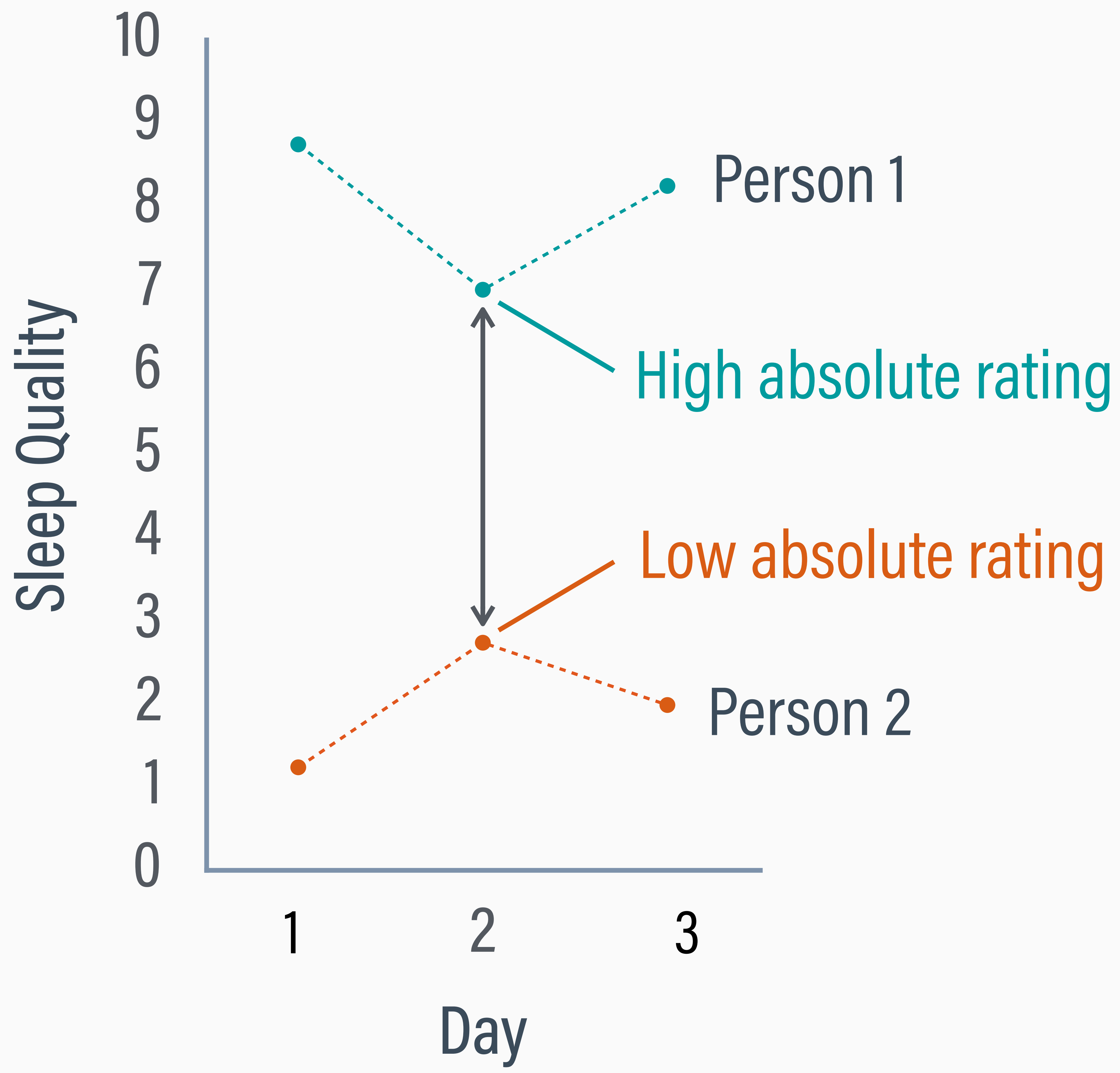
When we add a predictor to a regression model, it’s slope reflects the predicted outcome difference for two observations that differ by one point on the predictor. In small groups of two or three, discuss the meaning of a “one point difference” on the predictor when using raw and CWC centered scores. Use the previous box plots in your discussion.

INTERPRETING CENTERED SCORES

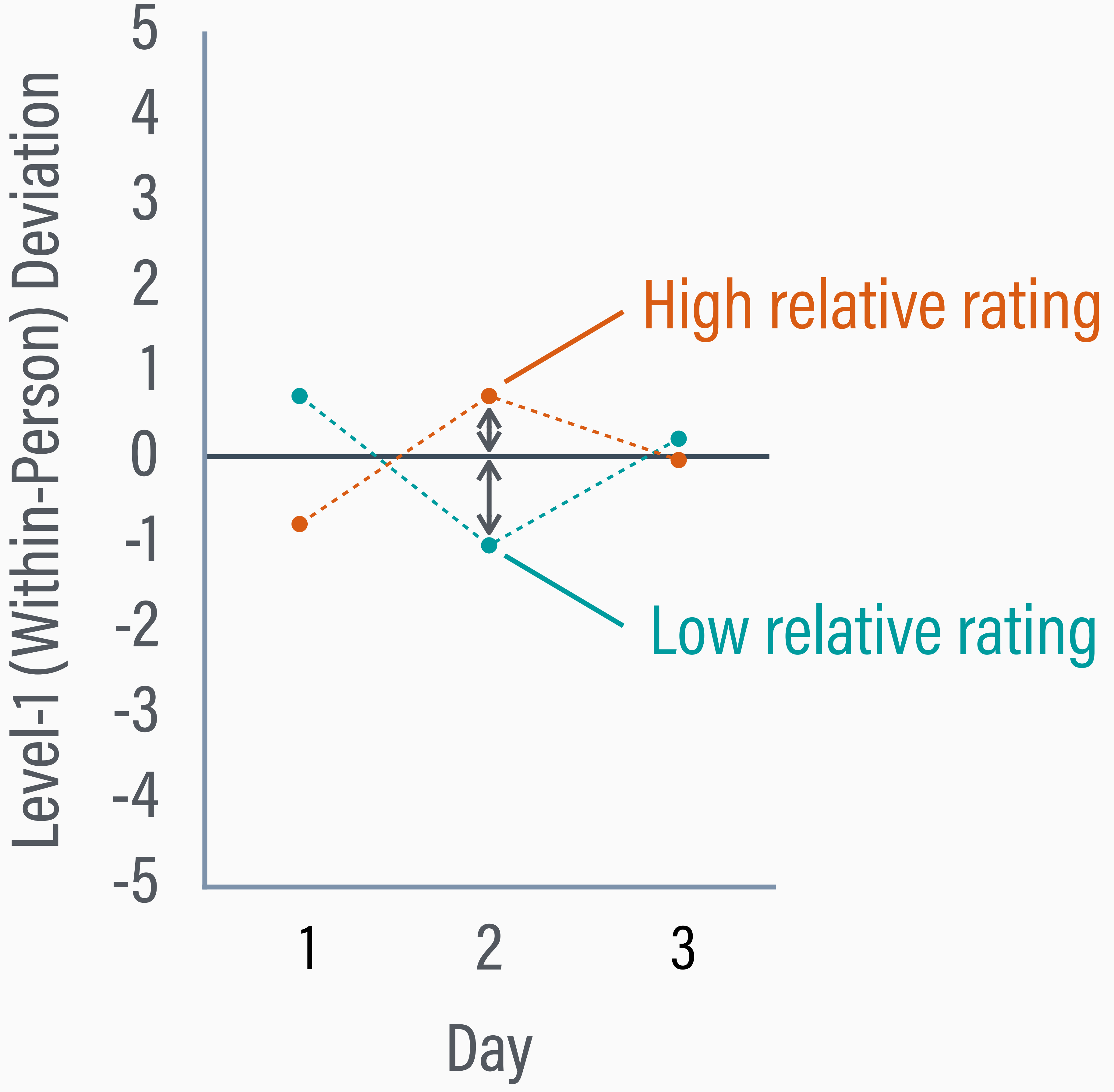
- CWC removes level-2 mean differences from the level-1 scores (after centering, all level-2 cluster means equal 0)
- CWC deviations reflect “pure” within-cluster variation among observations around their Level-2 cluster means
- The effect of a “one-point difference” on the predictor reflects a comparison between two scores from the same cluster

CWC DEVIATIONS ARE CLUSTER-SPECIFIC

Raw scores reflect absolute differences among observations



CWC scores reflect relative differences among observations from the same cluster



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IMPORTANT TAKEAWAYS

- Predictors measured at level-1 can produce level-1 and level-2 regression slopes
- We isolate the effect of interest by disaggregating level-1 predictors into distinct level-1 and level-2 components
- Depending on the goal, researchers may estimate the level-1 effect only, the level-2 effect only, or both simultaneously

ANALYSIS EXAMPLES

1. Estimate Intraclass Correlations
2. Within-Cluster (Level-1) Regression Only
3. Between-Cluster (Level-2) Regression Only
4. Within-Cluster (Level-1) and Between-Cluster (Level-2) Regression

ESTIMATING INTRACLASS CORRELATIONS

- Estimate intraclass correlations (ICCs) before model fitting to assess how much variance lies within and between clusters
- Some predictors have little or no level-2 variation, in which case they are already “pure” within-cluster variables
- Disaggregating a level-1 predictor and modeling its means is only warranted when there is sufficient level-2 variation

BLIMP STUDIO SCRIPT 2.1

```
DATA: PainDiary.dat;
VARIABLES: Person Day PosAffect NegAffect Pain WorkGoal LifeGoal SleepQual Female Education
  Employment MarStatus NumDiagnose ActivityLevel PainAccept Catastrophize Stress Anxiety;
CLUSTERID: Person;
MODEL:
PosAffect ~ intercept | intercept;   # empty model for outcome
SleepQual ~ intercept | intercept;   # empty model for predictor
BURN: 10000;
ITERATIONS: 10000;
SEED: 90291;
```

RBLIMP SCRIPT 2 (MODEL 1)

```
model1 <- rblimp(
        data = PainDiary,
        clustered = 'Person',
        model = '
           PosAffect ~ intercept | intercept;
           SleepQual ~ intercept | intercept',
        seed = 90291,
        burn = 10000,
        iter = 10000)
output(model1)
```

BLIMP OUTPUT

= level-2 estimate

= level-1 estimate

```
Outcome Variable:  PosAffect

Parameters                         Estimate     StdDev       2.5%      97.5%      ChiSq     PValue
                                  ------------------------------------------------------------------
Variances:
  L2 : Var(Intercept)                 2.525      0.334      1.979      3.279        ---        ---
  Residual Var.                       1.395      0.039      1.322      1.474        ---        ---

Coefficients:
  Intercept                           5.031      0.137      4.740      5.298   1353.904      0.000

Standard Deviations:
  L2 : SD(Intercept)                  1.589      0.103      1.407      1.811        ---        ---
  Residual SD                         1.181      0.016      1.150      1.214        ---        ---

Proportion Variance Explained
  by Coefficients                     0.000      0.000      0.000      0.000        ---        ---
  by Level-2 Random Intercepts        0.644      0.030      0.585      0.703        ---        ---
  by Level-1 Residual Variation       0.356      0.030      0.297      0.415        ---        ---

                                  ------------------------------------------------------------------
```

BLIMP OUTPUT

= level-2 estimate
= level-1 estimate

Outcome Variable: SleepQual

Parameters	Estimate	StdDev	2.5%	97.5%	ChiSq	PValue
Variances:						
L2 : Var(Intercept)	1.918	0.265	1.487	2.516	---	---
Residual Var.	3.002	0.084	2.844	3.172	---	---
Coefficients:						
Intercept	5.428	0.130	5.177	5.685	1736.935	0.000
Standard Deviations:						
L2 : SD(Intercept)	1.385	0.094	1.219	1.586	---	---
Residual SD	1.733	0.024	1.686	1.781	---	---
Proportion Variance Explained						
by Coefficients	0.000	0.000	0.000	0.000	---	---
by Level-2 Random Intercepts	0.390	0.033	0.329	0.458	---	---
by Level-1 Residual Variation	0.610	0.033	0.542	0.671	---	---

The intraclass correlation for daily sleep quality is ICC = .39. In small groups of two or three, provide a brief description of the ICC aimed at a colleague with no exposure to multilevel models.

SUMMARY

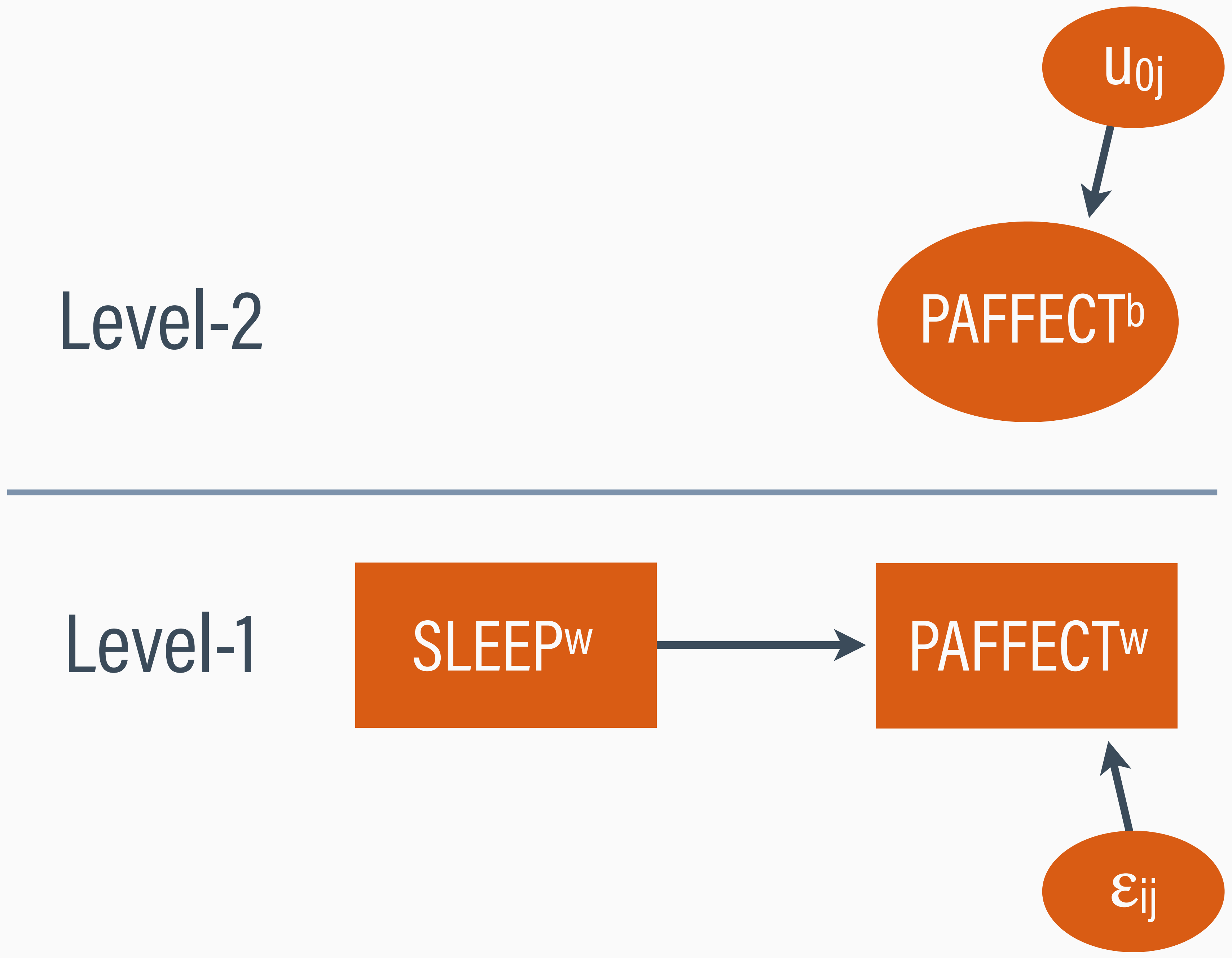
- Both variables have substantial variation at level-2 ($ICC_{paffect}$ = .64 and ICC_{sleep} = .39)
- Two distinct associations are estimable
- Person-level average sleep quality can predict average positive mood (level 2), and daily deviations from one's own sleep average can predict daily affect (level 1)

ANALYSIS EXAMPLES

1. Estimate Intraclass Correlations
2. Within-Cluster (Level-1) Regression Only
3. Between-Cluster (Level-2) Regression Only
4. Within-Cluster (Level-1) and Between-Cluster (Level-2) Regression

ANALYSIS OVERVIEW

- The within-cluster association captures how deviations from a person's average sleep rating relates to their daily positive affect
- This reflects within-person sleep differences that do not depend on one's average sleep
- Person-level mean differences are unmodeled

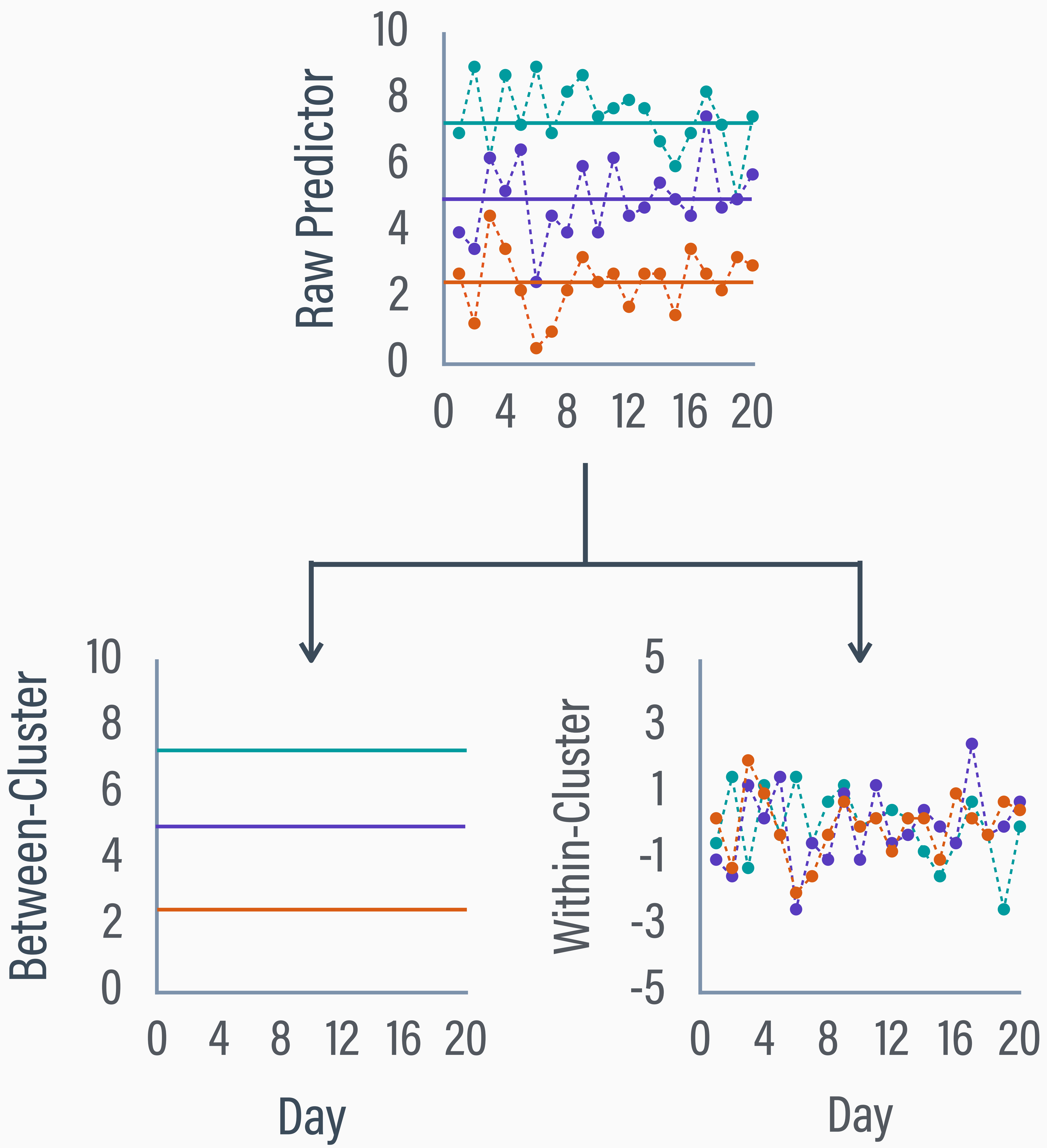


DISAGGREGATED PREDICTOR

- Disaggregation centers each daily score around its level-2 person mean

$$sleep_j^b = \mu_{j(sleep)}$$
$$sleep_{ij}^w = sleep_{ij} - \mu_{j(sleep)}$$

- $sleep^w$ contains only intraindividual (level-1) variation, and $sleep^b$ reflects only person (level-2) mean differences



WITHIN-CLUSTER (LEVEL-1) MODEL

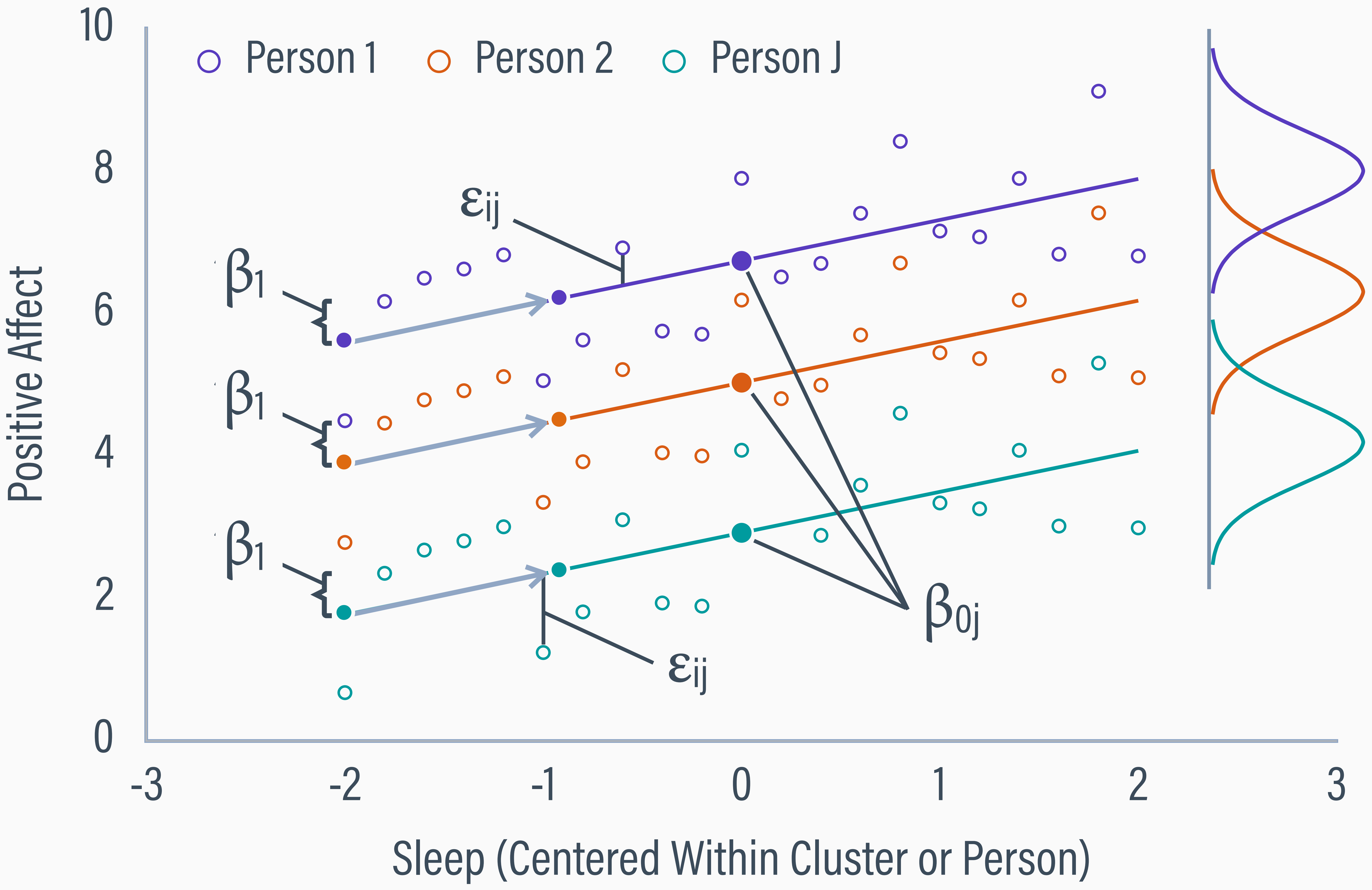
- Affect observation i for person j is the sum of a person's level-2 affect mean (β_{0j}), a fixed effect due to within-person sleep variation (β_1), and a within-person residual (ε_{ij})

$$paffect_{ij} = \beta_{0j} + \beta_1(sleep^{w}_{ij}) + \varepsilon_{ij}$$

- Assumption: residuals are normal with constant variation across all days (level-1 units) and persons (level-2 units)

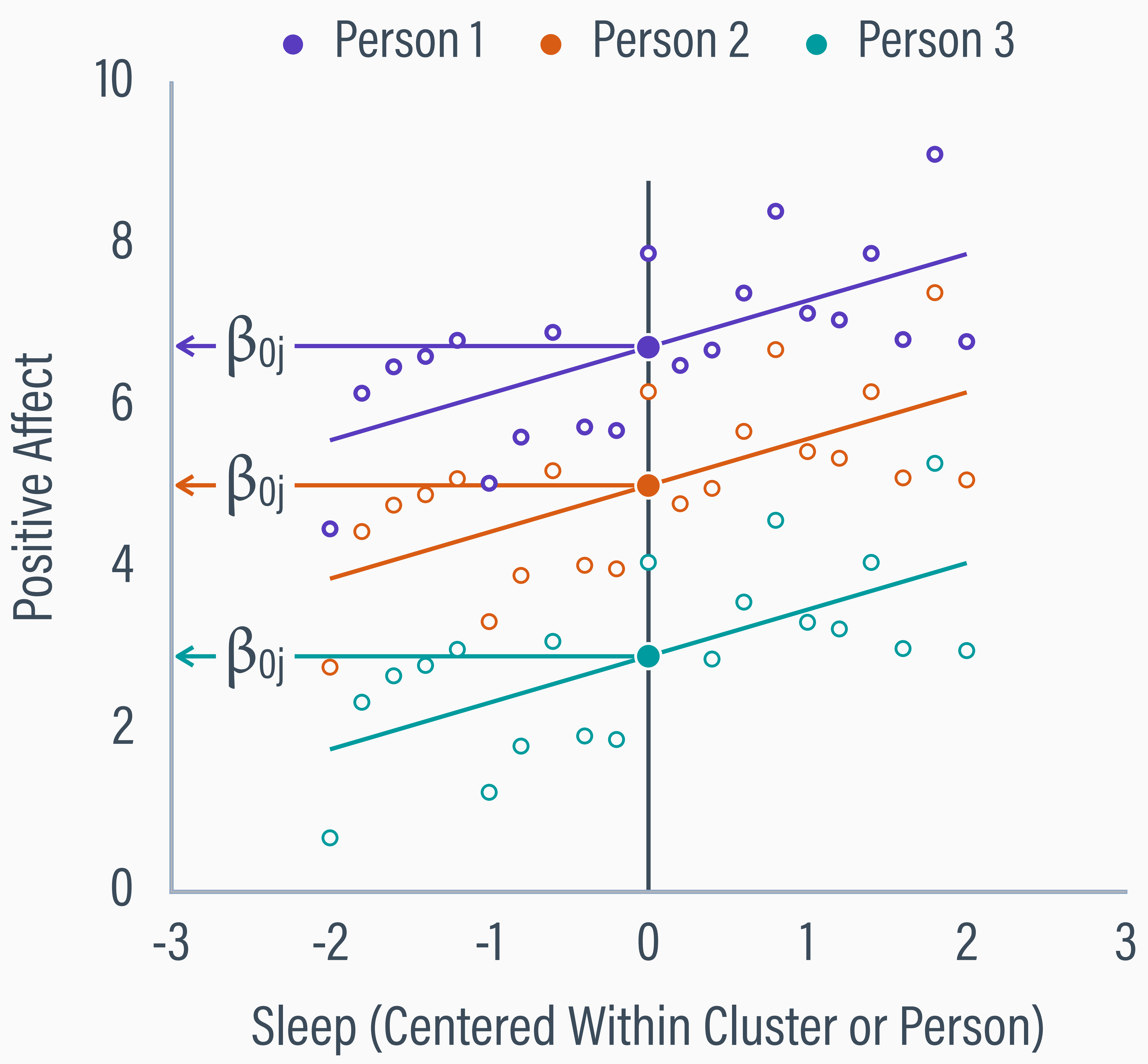
$$\varepsilon_{ij} \sim N(0,\sigma^2_{\varepsilon})$$

LEVEL-1 MODEL GRAPHIC



RANDOM INTERCEPTS

- In regression, the intercept is the expected outcome score when the predictor equals zero
- Mean centering defines the intercept as the dependent variable’s mean
- CWC centers within each cluster, so each β_{0j} is the outcome mean for cluster j



BETWEEN-CLUSTER (LEVEL-2) MODEL

- The affect mean for person j (β_{0j}) is the sum of the grand mean (γ_{00}) and a between-person residual (u_{0j})

- The level-1 slope (β_1) equals the grand mean slope (γ_{10})

$$\beta_{0j} = \gamma_{00} + u_{0j}$$
$$\beta_1 = \gamma_{10}$$

- Random intercept residuals are normal with constant variation across persons (level-2 units)

$$u_{0j} \sim N(0,\sigma_u^2)$$

DECODING THE SUBSCRIPTS

$$\beta_{0j} = \gamma_{00} + u_{0j}$$

$$\beta_{1j} = \gamma_{10}$$

The second subscript numbers the coefficients on the right side starting at 0 (the intercept)

The first subscript tells which level-2 outcome variable these terms are attached to (γ_{00} and u_{0j} belong in β_{0j}'s equation, γ_{10} is attached to β_1)

COMBINED-MODEL NOTATION

- Level-specific regression equations can be reduced into a single combined-model equation (Raudenbush & Bryk, 2002)
- Replace the β_{0j} and β_1 terms in the level-1 equation with their level-2 equations

$$\beta_1 = \gamma_{10}$$

$$\beta_{0j} = \gamma_{00} + u_{0j}$$

$$\text{paffect}_{ij} = \beta_{0j} + \beta_1(\text{sleep}^{W}_{ij}) + \varepsilon_{ij}$$

$$\begin{aligned}\text{paffect}_{ij} &= (\gamma_{00} + u_{0j}) + \gamma_{10}(\text{sleep}^{W}_{ij}) + \varepsilon_{ij} \\ &= \gamma_{00} + \gamma_{10}(\text{sleep}^{W}_{ij}) + u_{0j} + \varepsilon_{ij}\end{aligned}$$

COMMON NOTATIONAL SYSTEMS

Combined-model equation (Raudenbush & Bryk, 2002)

$$\text{paffect}_{ij} = \gamma_{00} + \gamma_{10}(\text{sleep}^{w}_{ij}) + u_{0j} + \varepsilon_{ij}$$

Standard(ish) regression notation (Scott, Shrout, & Weinberg, 2013)

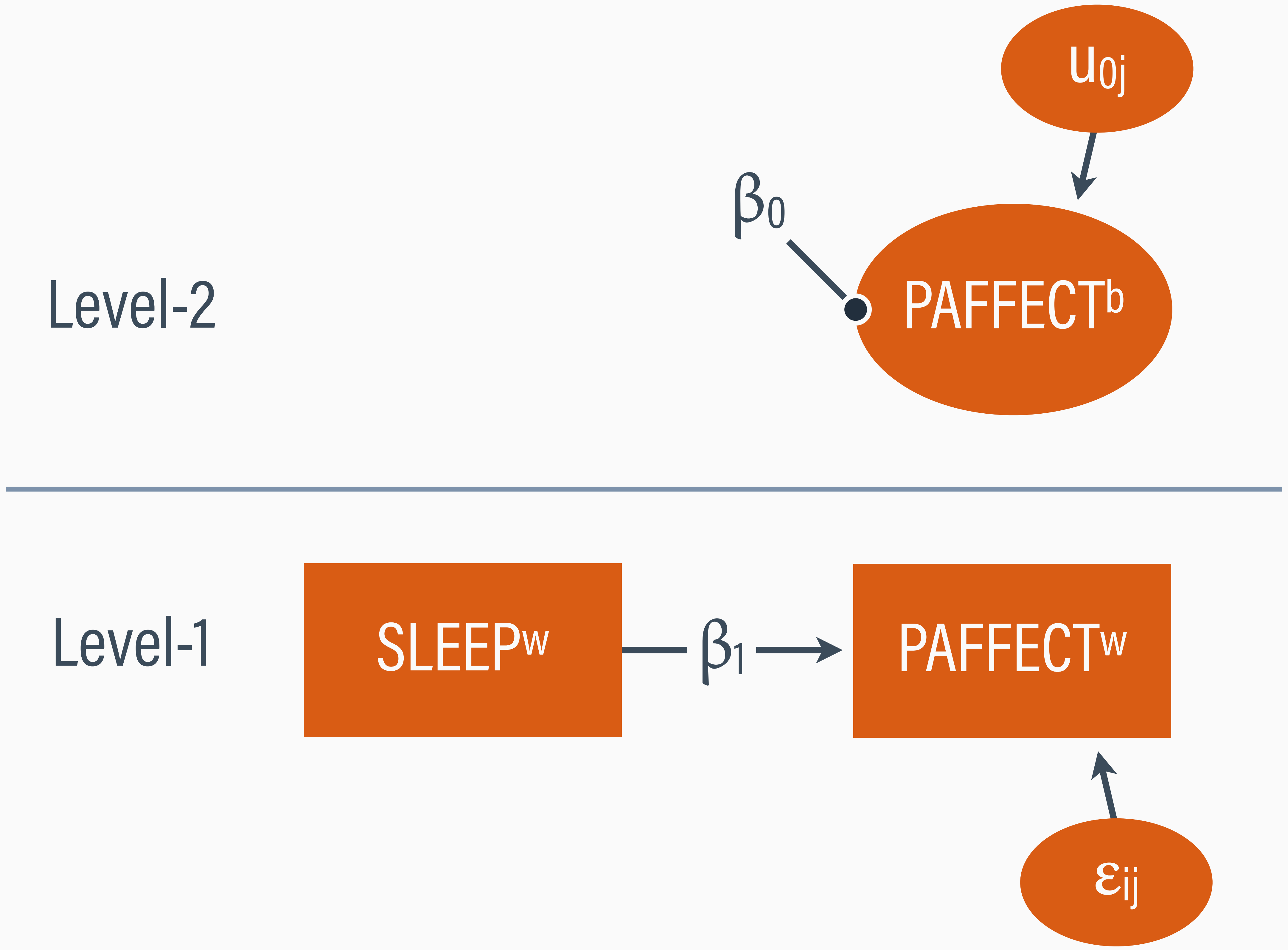
$$\text{paffect}_{ij} = \beta_0 + \beta_1(\text{sleep}^{w}_{ij}) + u_{0j} + \varepsilon_{ij}$$

Linear mixed model cluster-level matrix equation

$$\mathbf{y}_j = \mathbf{X}_j\boldsymbol{\beta} + \mathbf{Z}_j u_j + \varepsilon_j = \begin{pmatrix} \text{posaffect}_{1j} \\ \text{posaffect}_{2j} \\ \dots \\ \text{posaffect}_{nj} \end{pmatrix} = \begin{pmatrix} 1 & \text{sleep}^{w}_{1j} \\ 1 & \text{sleep}^{w}_{2j} \\ \dots & \dots \\ 1 & \text{sleep}^{w}_{nj} \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ \dots \\ 1 \end{pmatrix} u_{0j} + \begin{pmatrix} \varepsilon_{1j} \\ \varepsilon_{2j} \\ \dots \\ \varepsilon_{nj} \end{pmatrix}$$

ANALYSIS MODEL

$$paffect_{ij} = \beta_0 + \beta_1(sleep^{w}_{ij}) + u_{0j} + \varepsilon_{ij}$$



BLIMP STUDIO SCRIPT 2.2

```
DATA: PainDiary.dat;
VARIABLES: Person Day PosAffect NegAffect Pain WorkGoal LifeGoal SleepQual Female Education
  Employment MarStatus NumDiagnose ActivityLevel PainAccept Catastrophize Stress Anxiety;
CLUSTERID: Person;
CENTER: groupmean = SleepQual;    # cwc with level-2 latent cluster means
MODEL: PosAffect ~ intercept SleepQual | intercept;
BURN: 10000;
ITER: 10000;
SEED: 90291;
```

RBLIMP SCRIPT 2 (MODEL 2)

```
model2 <- rblimp(
    data = PainDiary,
    clusterid = 'Person',
    center = 'groupmean = SleepQual',
    model = 'PosAffect ~ intercept SleepQual | intercept',
    seed = 90291,
    burn = 10000,
    iter = 10000)
output(model2)
```

BLIMP OUTPUT

= level-2 estimate

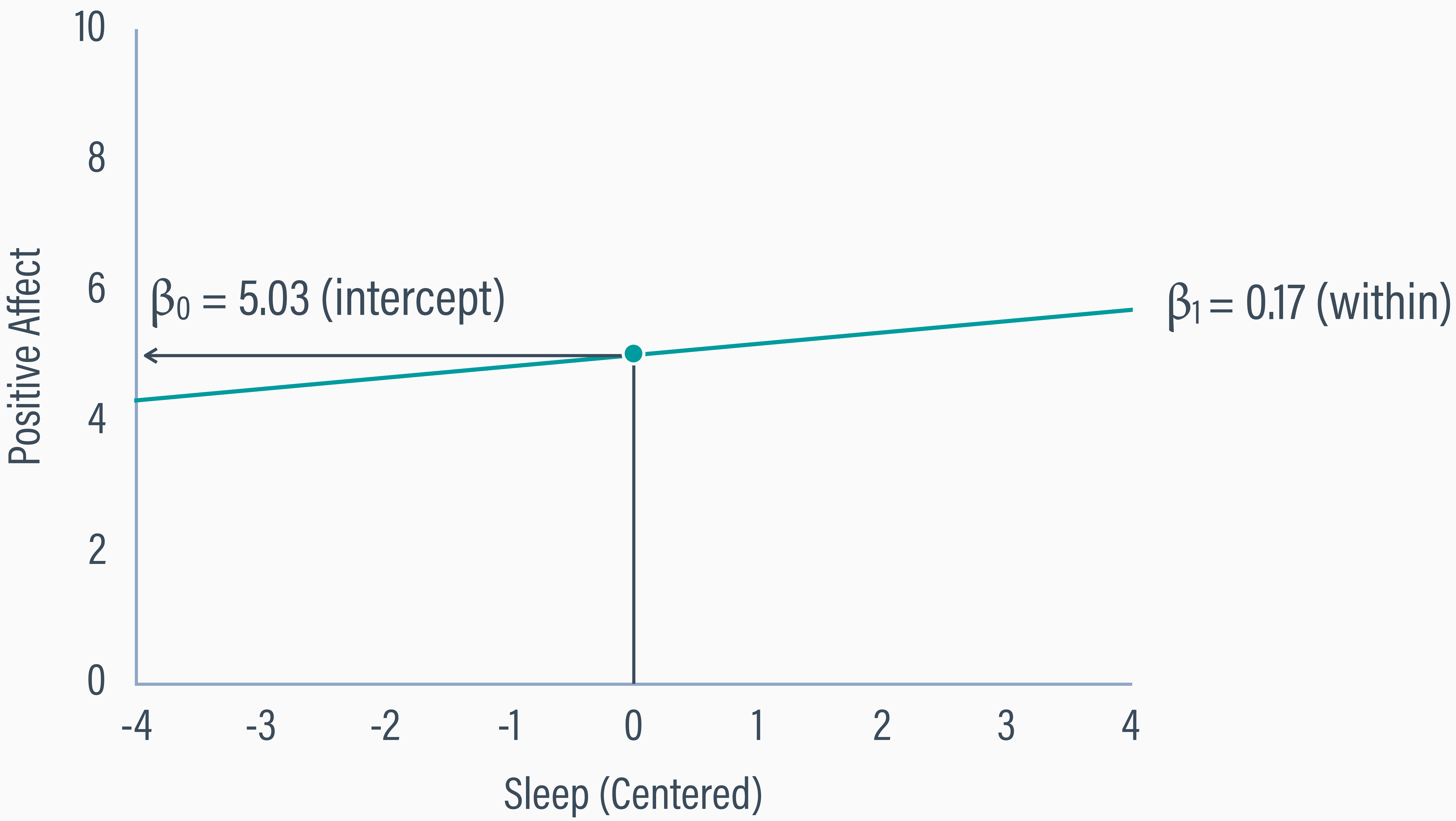
= level-1 estimate

```
Outcome Variable:  PosAffect

Group Mean Centered: SleepQual

Parameters                         Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
                               ---------------------------------------------------------------------------
Variances:
  L2 : Var(Intercept)                 2.498      0.330      1.959      3.250        ---        ---  14878.523
  Residual Var.                       1.306      0.037      1.236      1.381        ---        ---  18435.058

Coefficients:
  Intercept                           5.027      0.145      4.727      5.313   1197.971      0.000    247.755
  SleepQual                           0.173      0.013      0.148      0.199    173.667      0.000  18290.653

Standard Deviations:
  L2 : SD(Intercept)                  1.580      0.103      1.399      1.802        ---        ---   8518.049
  Residual SD                         1.143      0.016      1.112      1.175        ---        ---   9208.015

Standardized Coefficients:
  SleepQual                           0.152      0.013      0.127      0.178    136.430      0.000  17563.197

Proportion Variance Explained
  by Coefficients                     0.023      0.004      0.016      0.032        ---        ---  17699.727
  by Level-2 Random Intercepts        0.641      0.030      0.582      0.701        ---        ---  15523.864
  by Level-1 Residual Variation       0.335      0.028      0.280      0.391        ---        ---  15314.734

                               ---------------------------------------------------------------------------
```

FIXED EFFECT INTERPRETATIONS

- $\beta_0 = 5.03$ is the positive affect grand mean (because both predictors are centered)
- $\beta_1 = 0.17$ is the expected affect difference between two daily sleep scores from the same person that differ by one point

LEVEL-1 REGRESSION



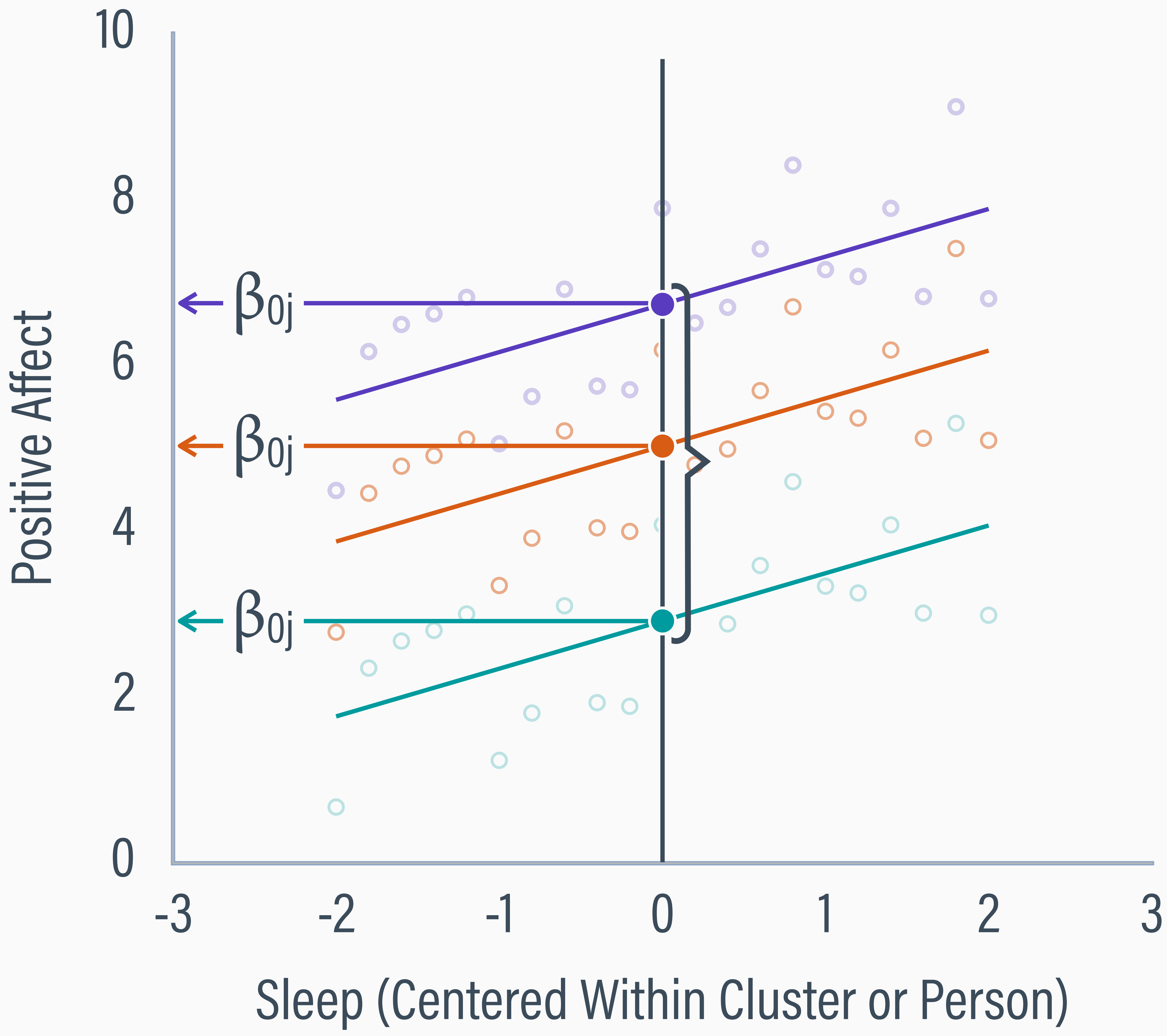
RANDOM EFFECT INTERPRETATIONS

- $u_{0j} = \beta_{0j} - \beta_0$
- var(u_{0j}) = 2.50 is the average squared distance between the level-2 affect means and the grand mean
- sd(u_{0j}) = 1.58 is the average distance between the level-2 affect means and the grand mean

- $\varepsilon_{ij} = \text{paffect}_{ij} - (\beta_{0j} + \beta_1(\text{sleep}_{ij}^{w}))$
- var(ε_{ij}) = 1.31 is the average squared distance between the level-1 affect observations and their predicted values
- sd(ε_{ij}) = 1.14 is the average distance between the level-1 affect observations and their predicted values

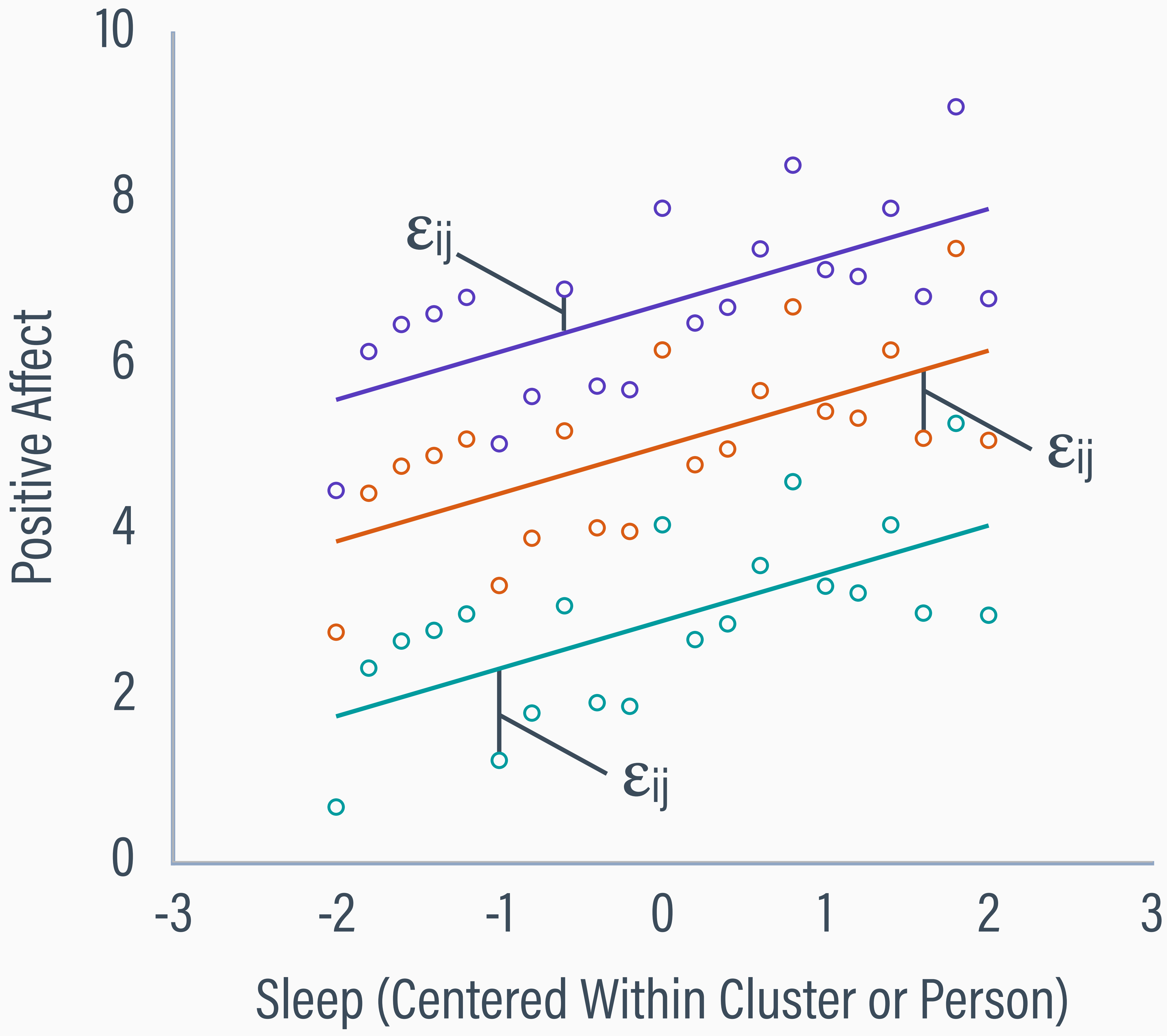
BETWEEN-CLUSTER VARIANCE

- The between-cluster variance var(u_{0j}) quantifies differences among person-specific means
- Visually, the random intercept variance is the vertical separation of the regression lines at sleep = 0



WITHIN-CLUSTER VARIANCE

- The within-cluster variance quantifies distances from a person's scores to their own regression
- Visually, the variance is the vertical separation of observations from the person-specific regression lines



MODEL COMPARISON

Parameter	Empty Model	Within Only	Between Only	Within + Between	Smushed (B = W)
Fixed intercept	5.03	5.02	5.05	5.03	5.04
Sleep (within-person)	--	0.17	--	0.17	0.18
Sleep (between-person)	--	--	0.61	0.61	0.18
Residual intercept variance	2.53	2.53	1.81	1.84	2.17
Residual within-person variance	1.40	1.31	1.40	1.31	1.31

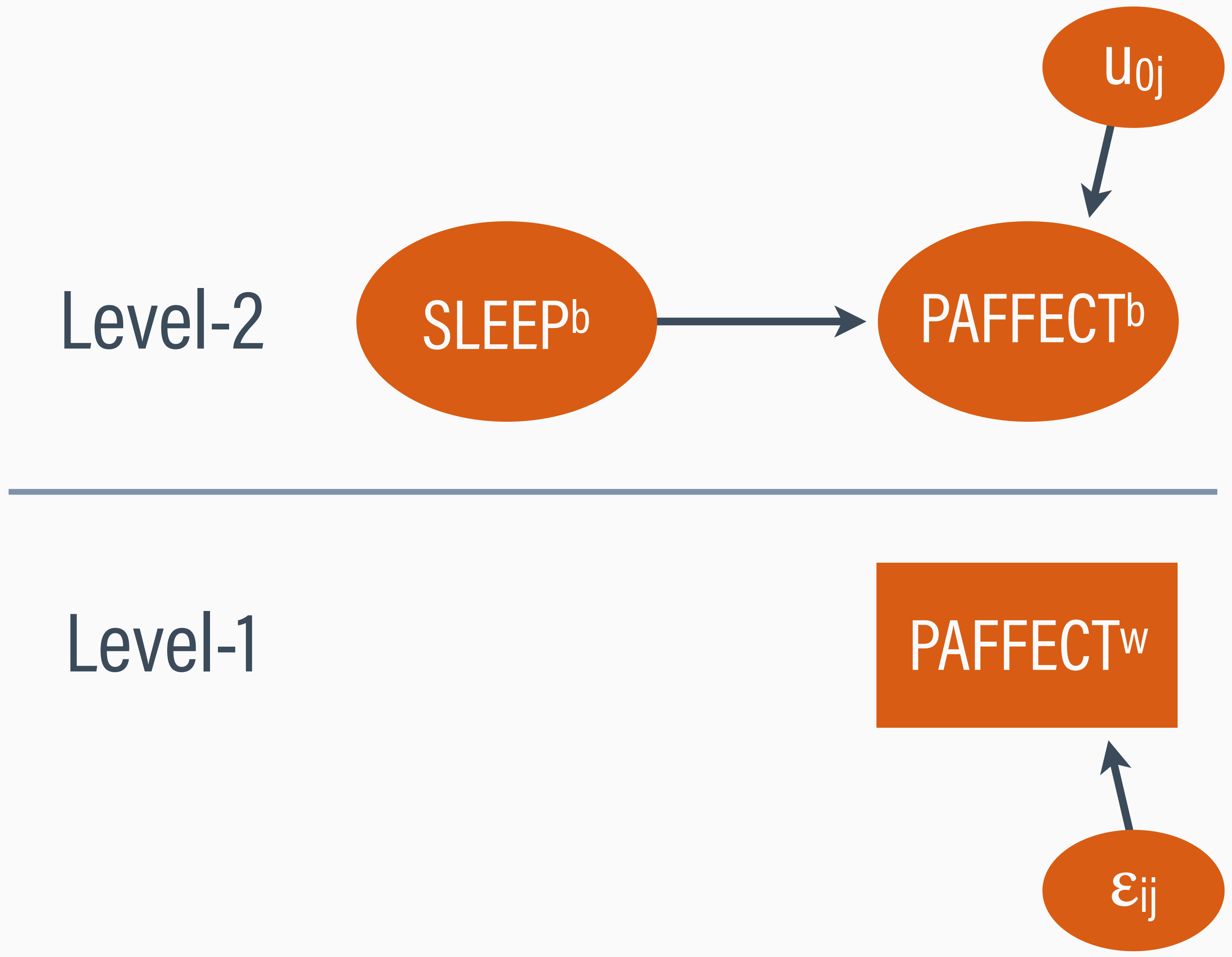
The random intercept variance (variation among person means) did not change after adding a pure within-cluster (level-1) predictor, but the within-cluster variation was reduced. In small groups of two or three, discuss why you think these sources of variation behaved the way they did.

ANALYSIS EXAMPLES

1. Estimate Intraclass Correlations
2. Within-Cluster (Level-1) Regression Only
3. Between-Cluster (Level-2) Regression Only
4. Within-Cluster (Level-1) and Between-Cluster (Level-2) Regression

ANALYSIS OVERVIEW

- The between-cluster association captures how a person's average sleep quality relates to their average positive affect
- This reflects stable between-person differences rather than day-to-day fluctuations
- Within-person fluctuations are unmodeled



WITHIN-CLUSTER (LEVEL-1) MODEL

- Affect observation i for person j is the sum of their own level-2 affect mean (β_{0j}) and a within-person residual (ε_{ij})

$$\text{paffect}_{ij} = \beta_{0j} + \varepsilon_{ij}$$

- Assumption: residuals are normal with constant variation across all days (level-1 units) and persons (level-2 units)

$$\varepsilon_{ij} \sim N(0,\sigma_{\varepsilon}^{2})$$

BETWEEN-CLUSTER (LEVEL-2) MODEL

- The affect mean for person j (β_{0j}) is the sum of the grand mean (γ_{00}), a fixed effect due to person-average sleep (γ_{01}), and a between-person residual (u_{0j})

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{sleep}_j^b) + u_{0j}$$

- Random intercept residuals are normal with constant variation across persons (level-2 units)

$$u_{0j} \sim N(0, \sigma_u^2)$$

COMBINED-MODEL NOTATION

- Level-specific regression equations can be reduced into a single combined-model equation (Raudenbush & Bryk, 2002)
- Replace the β_{0j} and β_1 terms in the level-1 equation with their level-2 equations

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{sleep}_j^b) + u_{0j}$$

$$\downarrow$$

$$\text{paffect}_{ij} = \beta_{0j} + \varepsilon_{ij}$$

$$\downarrow$$

$$\text{paffect}_{ij} = \gamma_{00} + \gamma_{01}(\text{sleep}_j^b) + u_{0j} + \varepsilon_{ij}$$

COMMON NOTATIONAL SYSTEMS

Combined-model equation (Raudenbush & Bryk, 2002)

$$\text{paffect}_{ij} = \gamma_{00} + \gamma_{01}(\text{sleep}_j^b) + u_{0j} + \varepsilon_{ij}$$

Standard(ish) regression notation (Scott, Shrout, & Weinberg, 2013)

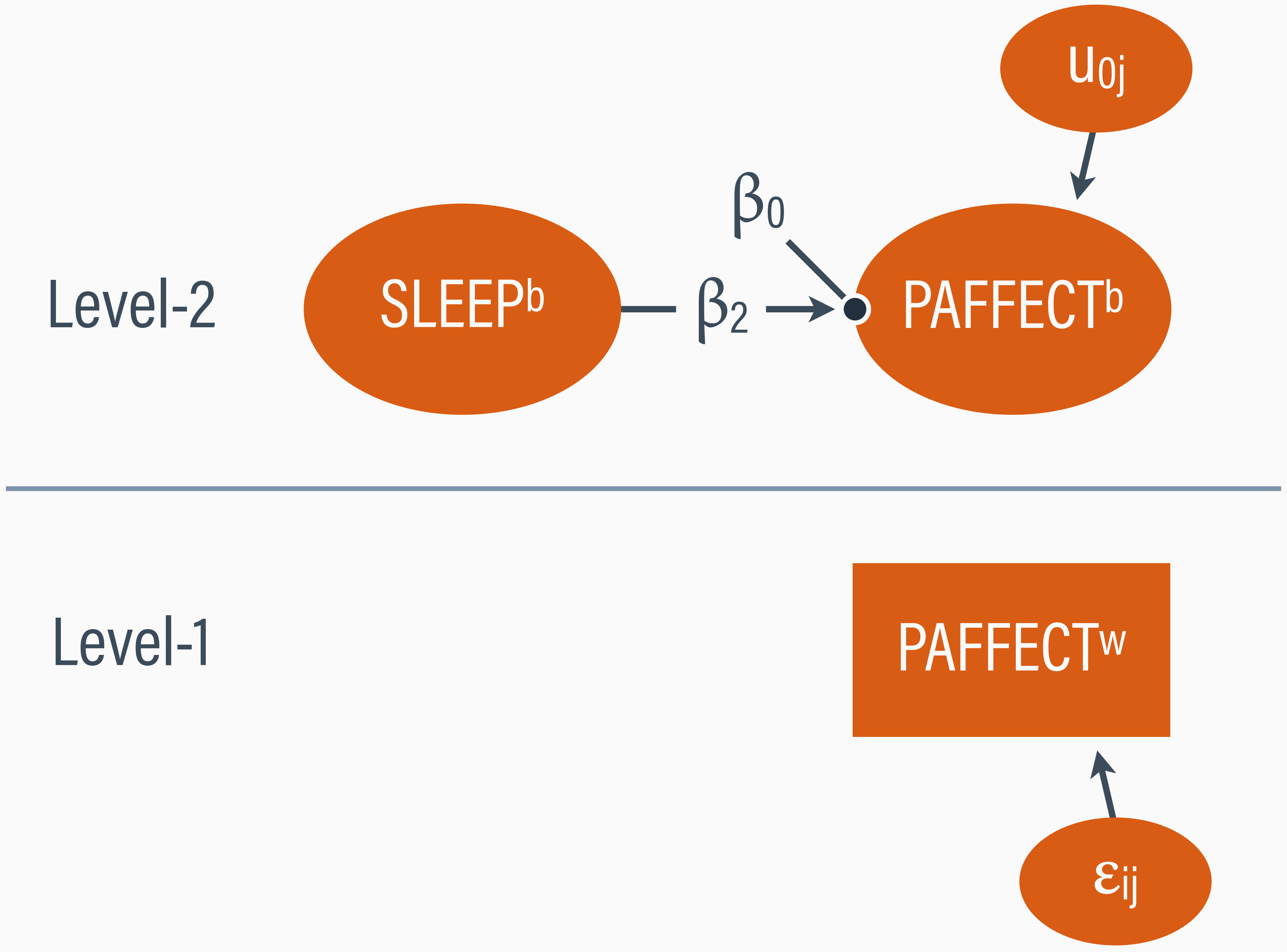
$$\text{paffect}_{ij} = \beta_0 + \beta_1(\text{sleep}_j^b) + u_{0j} + \varepsilon_{ij}$$

Linear mixed model cluster-level matrix equation

$$\mathbf{y}_j = \mathbf{X}_j\boldsymbol{\beta} + \mathbf{Z}_j u_j + \varepsilon_j = \begin{pmatrix} \text{posaffect}_{1j} \\ \text{posaffect}_{2j} \\ \dots \\ \text{posaffect}_{nj} \end{pmatrix} = \begin{pmatrix} 1 & \text{sleep}_{1j}^b \\ 1 & \text{sleep}_{2j}^b \\ \dots & \dots \\ 1 & \text{sleep}_{nj}^b \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ \dots \\ 1 \end{pmatrix} u_{0j} + \begin{pmatrix} \varepsilon_{1j} \\ \varepsilon_{2j} \\ \dots \\ \varepsilon_{nj} \end{pmatrix}$$

ANALYSIS MODEL

$$paffect_{ij} = \beta_0 + \beta_2(sleep_j^b) + u_{0j} + \varepsilon_{ij}$$



This model uses a person's average sleep to predict their average affect. Using the empty model as a starting point, in small groups of two or three, make predictions about what will happen to the between-person (mean-level) and within-person (daily-level) variation. Will each value increase, decrease, or stay the same?

BLIMP STUDIO SCRIPT 2.3

```
DATA: PainDiary.dat;
VARIABLES: Person Day PosAffect NegAffect Pain WorkGoal LifeGoal SleepQual Female Education
   Employment MarStatus NumDiagnose ActivityLevel PainAccept Catastrophize Stress Anxiety;
CLUSTERID: Person;
CENTER:  grandmean = SleepQual.mean;
MODEL: PosAffect ~ intercept SleepQual.mean | intercept;    # .mean invokes level-2 latent means
BURN: 10000;
ITERATIONS: 10000;
SEED: 90291;
```

RBLIMP SCRIPT 2 (MODEL 3)

```
model3 <- rblimp(
    data = PainDiary,
    clusterid = 'Person',
    center = 'grandmean = SleepQual.mean',
    model = 'PosAffect ~ intercept SleepQual.mean | intercept',
    seed = 90291,
    burn = 10000,
    iter = 10000)
output(model3)
```

BLIMP OUTPUT

= level-2 estimate

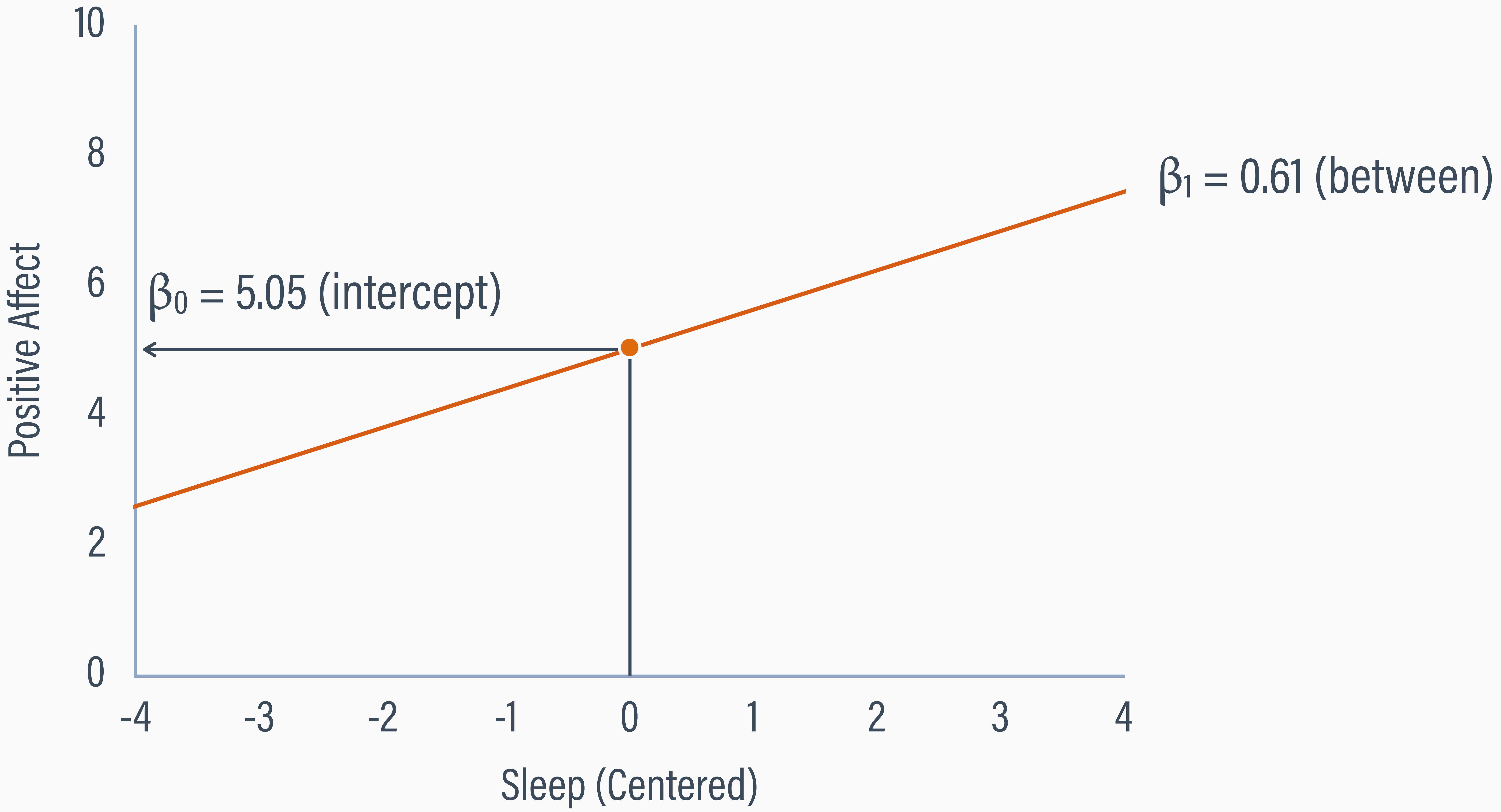
= level-1 estimate

```
Outcome Variable:  PosAffect

Parameters                      Estimate    StdDev      2.5%     97.5%      ChiSq    PValue     N_Eff
                              -------------------------------------------------------------------------
Variances:
  L2 : Var(Intercept)              1.809     0.250     1.404     2.386        ---       ---  4422.013
  Residual Var.                    1.395     0.040     1.321     1.477        ---       ---  8961.693

Coefficients:
  Intercept                        5.045     0.135     4.781     5.295   1402.647     0.000   139.527
  SleepQual.mean[Person]           0.606     0.088     0.428     0.772     46.920     0.000   114.531

Standard Deviations:
  L2 : SD(Intercept)               1.345     0.092     1.185     1.545        ---       ---  4360.602
  Residual SD                      1.181     0.017     1.149     1.215        ---       ---  8956.904

Standardized Coefficients:
  SleepQual.mean[Person]           0.419     0.052     0.304     0.508     62.779     0.000   119.344

Proportion Variance Explained
  by Coefficients                  0.175     0.043     0.093     0.258        ---       ---   119.467
  by Level-2 Random Intercepts     0.466     0.042     0.388     0.552        ---       ---   279.468
  by Level-1 Residual Variation    0.358     0.028     0.304     0.414        ---       ---   426.869

                              -------------------------------------------------------------------------
```

FIXED EFFECT INTERPRETATIONS

- $\beta_0 = 5.05$ is the positive affect grand mean (because both predictors are centered)
- $\beta_1 = 0.61$ is the expected affect difference between two individuals whose average sleep ratings differ by one point

LEVEL-2 REGRESSION



RANDOM EFFECT INTERPRETATIONS

- $u_{0j} = \beta_{0j} - (\beta_0 + \beta_2(sleep_j^b))$
- $var(u_{0j}) = 1.81$ is the average squared distance between the level-2 affect means and their predicted values
- $sd(u_{0j}) = 1.35$ is the average distance between the level-2 affect means and their predicted values

- $\varepsilon_{ij} = paffect_{ij} - \beta_{0j}$
- $var(\varepsilon_{ij}) = 1.40$ is the average squared distance between the level-1 affect observations and their level-2 means
- $sd(\varepsilon_{ij}) = 1.18$ is the average distance between the level-1 affect observations and their level-2 means

MODEL COMPARISON

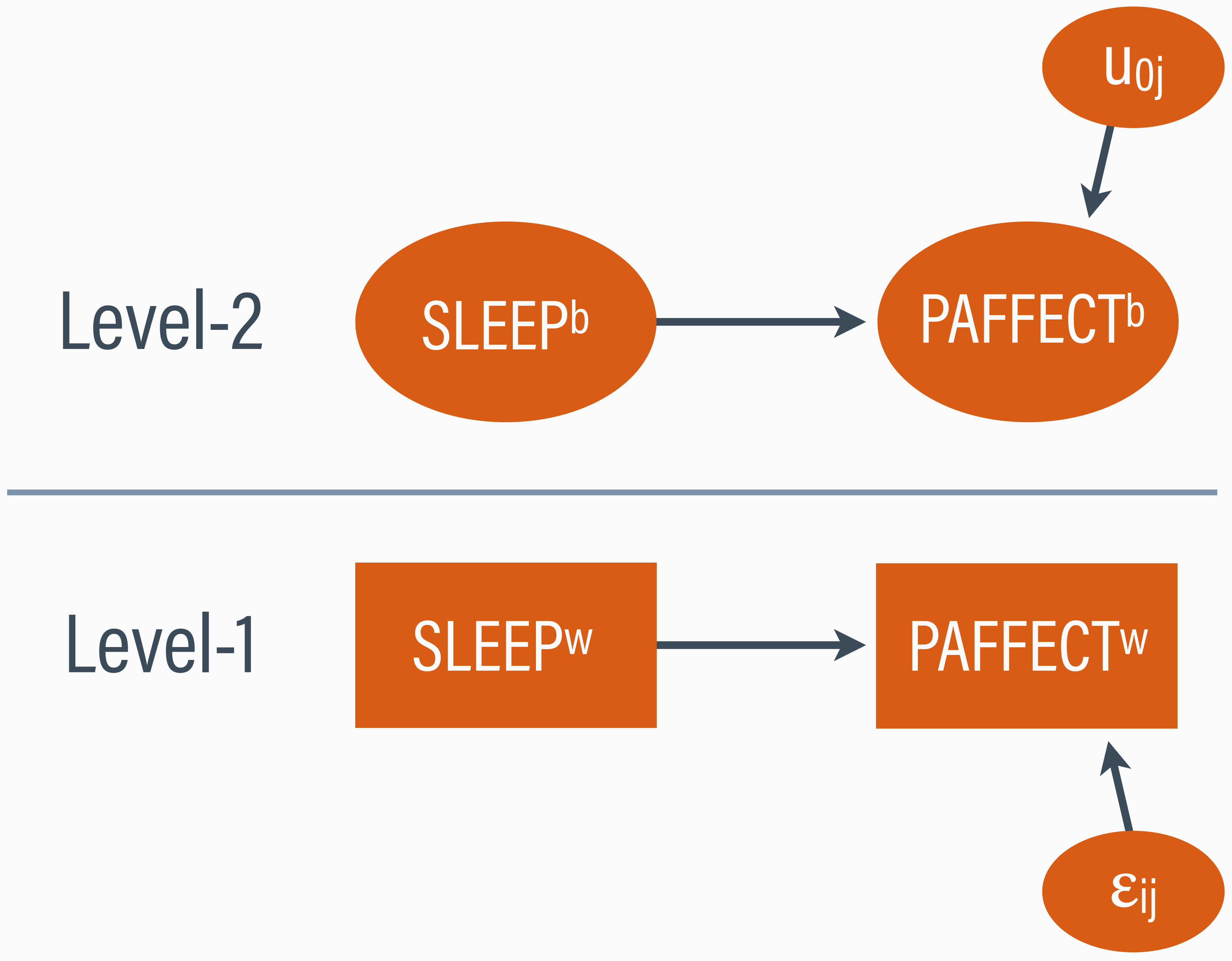
Parameter	Empty Model	Within Only	Between Only	Within + Between	Smushed (B = W)
Fixed intercept	5.03	5.02	5.05	5.03	5.04
Sleep (within-person)	--	0.17	--	0.17	0.18
Sleep (between-person)	--	--	0.61	0.61	0.18
Residual intercept variance	2.53	2.53	1.81	1.84	2.17
Residual within-person variance	1.40	1.31	1.40	1.31	1.31

ANALYSIS EXAMPLES

1. Estimate Intraclass Correlations
2. Within-Cluster (Level-1) Regression Only
3. Between-Cluster (Level-2) Regression Only
4. Within-Cluster (Level-1) and Between-Cluster (Level-2) Regression

ANALYSIS OVERVIEW

- The within-cluster association captures how deviations from a person's average sleep rating relates to their daily positive affect
- The between-cluster association captures how a person's average sleep quality relates to their average positive affect



WITHIN-CLUSTER (LEVEL-1) MODEL

- Affect observation i for person j is the sum of a person's level-2 affect mean (β_{0j}), a fixed effect due to within-person sleep variation (β_1), and a within-person residual (ε_{ij})

$$paffect_{ij} = \beta_{0j} + \beta_1(sleep_{ij}^{w}) + \varepsilon_{ij}$$

- Assumption: residuals are normal with constant variation across all days (level-1 units) and persons (level-2 units)

$$\varepsilon_{ij} \sim N(0,\sigma_\varepsilon^2)$$

BETWEEN-CLUSTER (LEVEL-2) MODEL

- The affect mean for person j (β_{0j}) is the sum of the grand mean (γ_{00}), a fixed effect due to person-average sleep (γ_{01}), and a between-person residual (u_{0j})

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{sleep}_j^b) + u_{0j}$$

$$\beta_1 = \gamma_{10}$$

- Random intercept residuals are normal with constant variation across persons (level-2 units)

$$u_{0j} \sim N(0, \sigma_u^2)$$

DECODING THE SUBSCRIPTS

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(sleep_j) + u_{0j}$$

$$\beta_{1j} = \gamma_{10}$$

The second subscript numbers the coefficients on the right side starting at 0 (the intercept)

The first subscript tells which level-2 outcome variable these terms are attached to (γ_{00} and u_{0j} belong in β_{0j}'s equation, γ_{10} is attached to β_1)

COMBINED-MODEL NOTATION

- Level-specific regression equations can be reduced into a single combined-model equation (Raudenbush & Bryk, 2002)
- Replace the β_{0j} and β_1 terms in the level-1 equation with their level-2 equations
- The γs are called fixed effects and u_{0j} is a random effect

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{sleep}_j^b) + u_{0j}$$

$$\beta_1 = \gamma_{10}$$

$$\text{paffect}_{ij} = \beta_{0j} + \beta_1(\text{sleep}_{ij}^w) + \varepsilon_{ij}$$

$$\text{paffect}_{ij} = \gamma_{00} + \gamma_{01}(\text{sleep}_j^b) + \gamma_{10}(\text{sleep}_{ij}^w) + u_{0j} + \varepsilon_{ij}$$

COMMON NOTATIONAL SYSTEMS

Combined-model equation (Raudenbush & Bryk, 2002)

$$\text{paffect}_{ij} = \gamma_{00} + \gamma_{10}(\text{sleep}^{w}_{ij}) + \gamma_{01}(\text{sleep}^{b}_{j}) + u_{0j} + \varepsilon_{ij}$$

Standard(ish) regression notation (Scott, Shrout, & Weinberg, 2013)

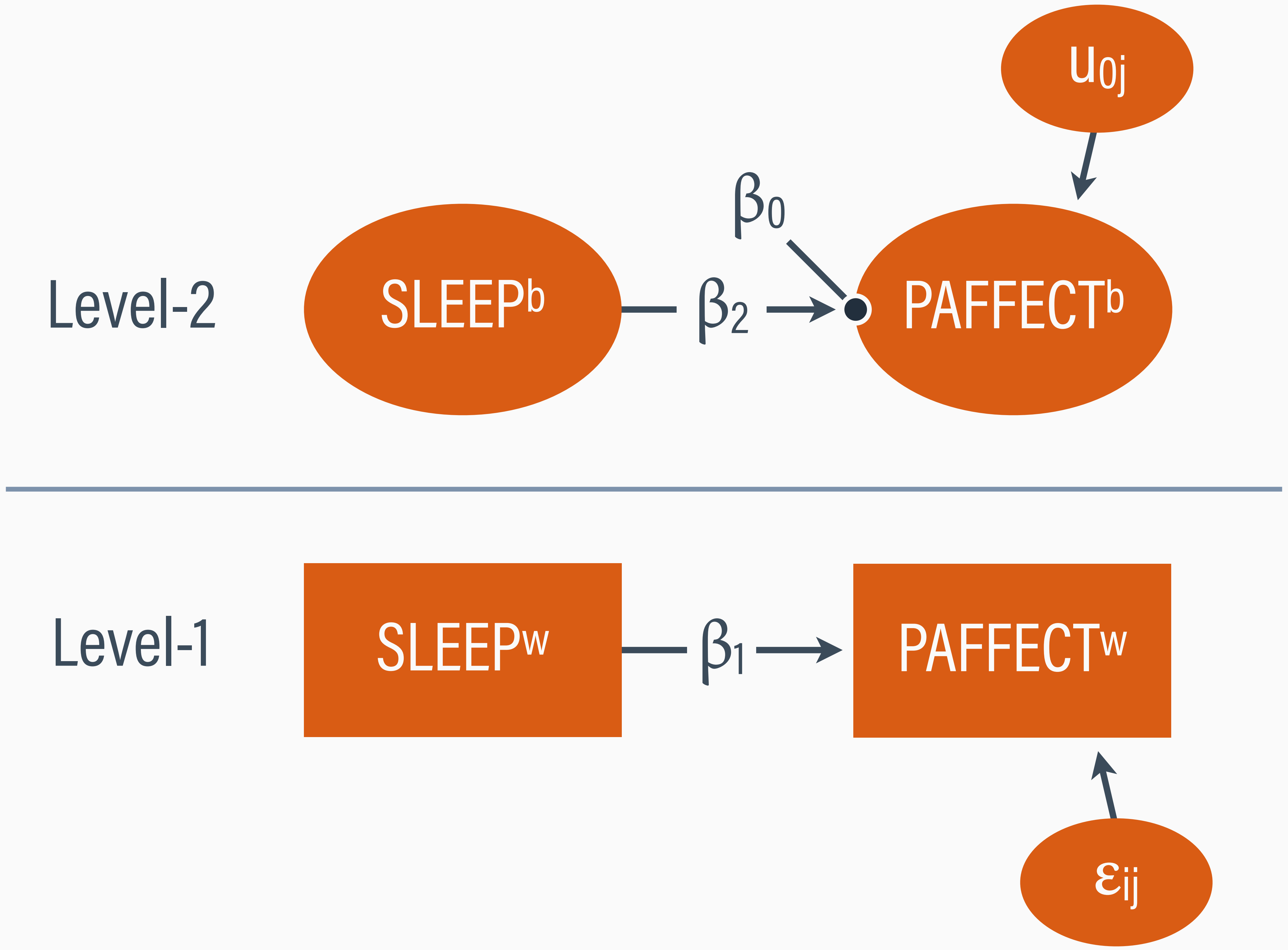
$$\text{paffect}_{ij} = \beta_0 + \beta_1(\text{sleep}^{w}_{ij}) + \beta_2(\text{sleep}^{b}_{j}) + u_{0j} + \varepsilon_{ij}$$

Linear mixed model cluster-level matrix equation

$$\mathbf{y}_j = \mathbf{X}_j\boldsymbol{\beta} + \mathbf{Z}_j u_j + \varepsilon_j = \begin{pmatrix} \text{posaffect}_{1j} \\ \text{posaffect}_{2j} \\ \dots \\ \text{posaffect}_{nj} \end{pmatrix} = \begin{pmatrix} 1 & \text{sleep}^{w}_{1j} & \text{SLEEP}^{b}_{1j} \\ 1 & \text{sleep}^{w}_{2j} & \text{SLEEP}^{b}_{2j} \\ \dots & \dots & \dots \\ 1 & \text{sleep}^{w}_{nj} & \text{SLEEP}^{b}_{nj} \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ \dots \\ 1 \end{pmatrix} u_{0j} + \begin{pmatrix} \varepsilon_{1j} \\ \varepsilon_{2j} \\ \dots \\ \varepsilon_{nj} \end{pmatrix}$$

ANALYSIS MODEL

$$paffect_{ij} = \beta_0 + \beta_1(sleep^{w}_{ij}) + \beta_2(sleep^{b}_{j}) + u_{0j} + \varepsilon_{ij}$$



BLIMP STUDIO SCRIPT 2.4

```
DATA: PainDiary.dat;
VARIABLES: Person Day PosAffect NegAffect Pain WorkGoal LifeGoal SleepQual Female Education
   Employment MarStatus NumDiagnose ActivityLevel PainAccept Catastrophize Stress Anxiety;
CLUSTERID: Person;
CENTER:
grandmean = SleepQual.mean;    # defines the fixed (average) intercept as the grand mean
groupmean = SleepQual;    # cwc with level-2 latent group means
MODEL: PosAffect ~ intercept SleepQual SleepQual.mean | intercept;    # .mean invokes latent means
BURN: 10000;
ITERATIONS: 10000;
SEED: 90291;
```

RBLIMP SCRIPT 2 (MODEL 4)

```
model4 <- rblimp(
    data = PainDiary,
    clusterid = 'Person',
    center = 'grandmean = SleepQual.mean; groupmean = SleepQual',
    model = 'PosAffect ~ intercept SleepQual SleepQual.mean | intercept',
    seed = 90291,
    burn = 10000,
    iter = 10000)
output(model4)
```

BLIMP OUTPUT

= level-2 estimate

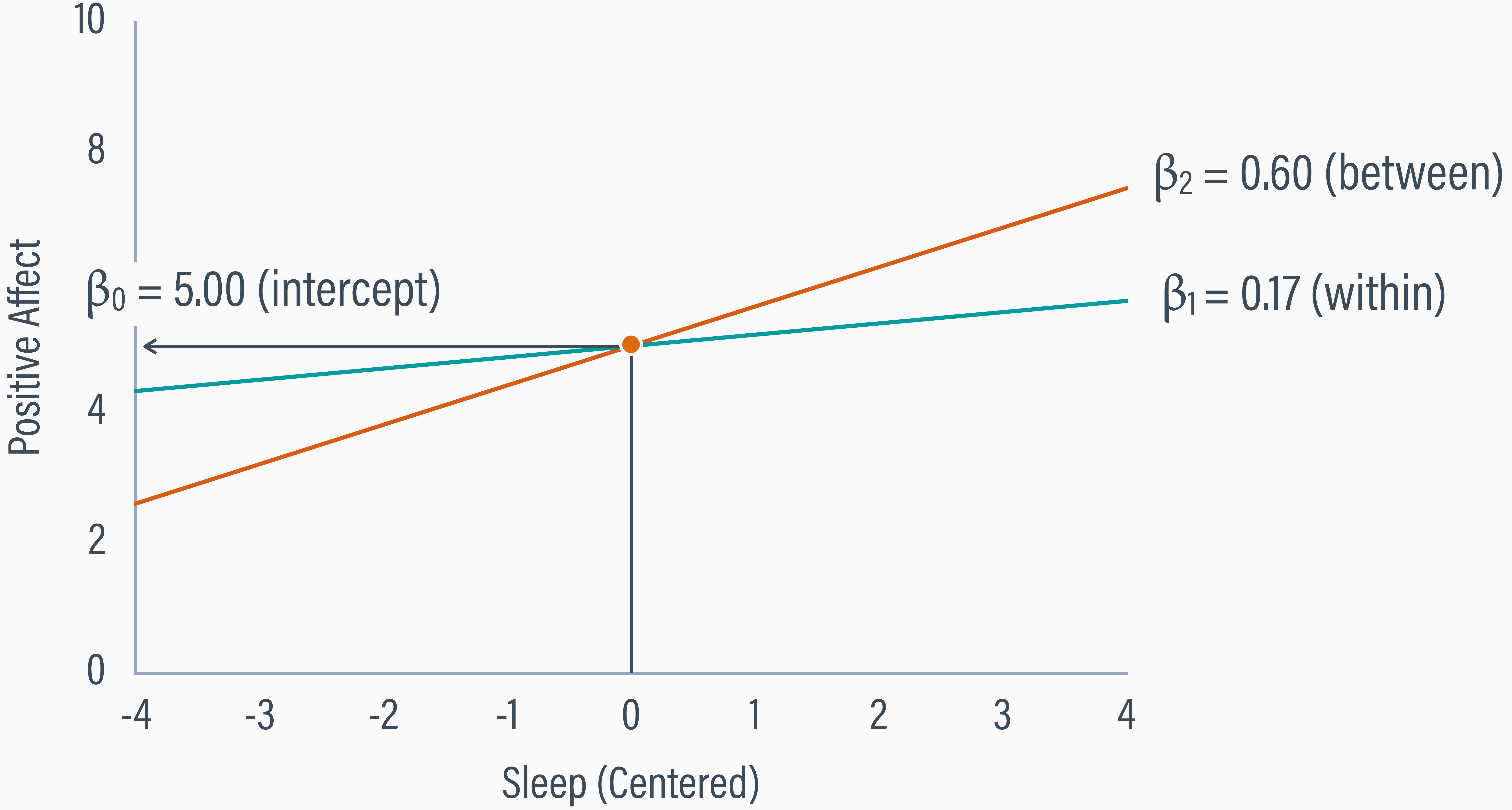
= level-1 estimate

```
Outcome Variable:  PosAffect

Group Mean Centered: SleepQual

Parameters                      Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
                                --------------------------------------------------------------------------
Variances:
  L2 : Var(Intercept)              1.839      0.246      1.436      2.399        ---        ---   5051.168
  Residual Var.                    1.307      0.037      1.238      1.380        ---        ---   8227.580

Coefficients:
  Intercept                        4.997      0.133      4.761      5.277   1423.604      0.000    154.295
  SleepQual                        0.173      0.013      0.147      0.199    174.595      0.000   9302.345
  SleepQual.mean[Person]           0.598      0.094      0.416      0.788     40.936      0.000    161.345

Standard Deviations:
  L2 : SD(Intercept)               1.356      0.090      1.198      1.549        ---        ---   5041.908
  Residual SD                      1.143      0.016      1.113      1.175        ---        ---   8223.888

Standardized Coefficients:
  SleepQual                        0.151      0.013      0.127      0.176    142.389      0.000   1996.211
  SleepQual.mean[Person]           0.412      0.055      0.297      0.513     55.809      0.000    169.087

Proportion Variance Explained
  by Coefficients                  0.193      0.044      0.112      0.286        ---        ---    175.664
  by Level-2 Random Intercepts     0.471      0.042      0.392      0.555        ---        ---    414.230
  by Level-1 Residual Variation    0.334      0.027      0.280      0.387        ---        ---    511.829

                                --------------------------------------------------------------------------
```

FIXED EFFECT INTERPRETATIONS

- $\beta_0 = 5.00$ is the positive affect grand mean (because both predictors are centered)
- $\beta_1 = 0.17$ is the expected affect difference between two daily sleep scores from the same person that differ by one point
- $\beta_2 = 0.60$ is the expected affect difference between two individuals whose average sleep ratings differ by one point

LEVEL-1 AND LEVEL-2 REGRESSIONS



RANDOM EFFECT INTERPRETATIONS

- $u_{0j} = \beta_{0j} - (\beta_0 + \beta_2(sleep_j^b))$
- $var(u_{0j}) = 1.84$ is the average squared distance between the level-2 affect means and their predicted values
- $sd(u_{0j}) = 1.36$ is the average distance between the level-2 affect means and their predicted values

- $\varepsilon_{ij} = paffect_{ij} - (\beta_{0j} + \beta_1(sleep_{ij}^w))$
- $var(\varepsilon_{ij}) = 1.31$ is the average squared distance between the level-1 affect observations and their predicted values
- $sd(\varepsilon_{ij}) = 1.14$ is the average distance between the level-1 affect observations and their predicted values

MODEL COMPARISON

Parameter	Empty Model	Within Only	Between Only	Within + Between	Smushed (B = W)
Fixed intercept	**5.03**	**5.02**	**5.05**	**5.00**	5.04
Sleep (within-person)	--	**0.17**	--	**0.17**	0.18
Sleep (between-person)	--	--	**0.61**	**0.60**	0.18
Residual intercept variance	2.53	2.53	**1.81**	**1.84**	2.17
Residual within-person variance	1.40	**1.31**	1.40	**1.31**	1.31

The within-person slope of 0.17 did not change when moving from a model with just the level-1 predictor to a model that additionally used the person-specific sleep averages as a level-2 predictor. Similarly, the between-person slope was constant at 0.61. In small groups of two or three, discuss why you think the effect of sleep at one level did not depend on whether the model included sleep at the other level.

OUTLINE

SMUSHED LEVEL-1 MODEL

- Predictor variables can also be entered into a model without disaggregation (as raw scores or grand mean centered)

$$paffect_{ij} = \beta_{0j} + \beta_1(sleep_{ij}) + \varepsilon_{ij}$$

- Sometimes called a “smushed model” because it combines the level-1 and level-2 associations into a single slope

EQUALITY OF SLOPES ASSUMPTION

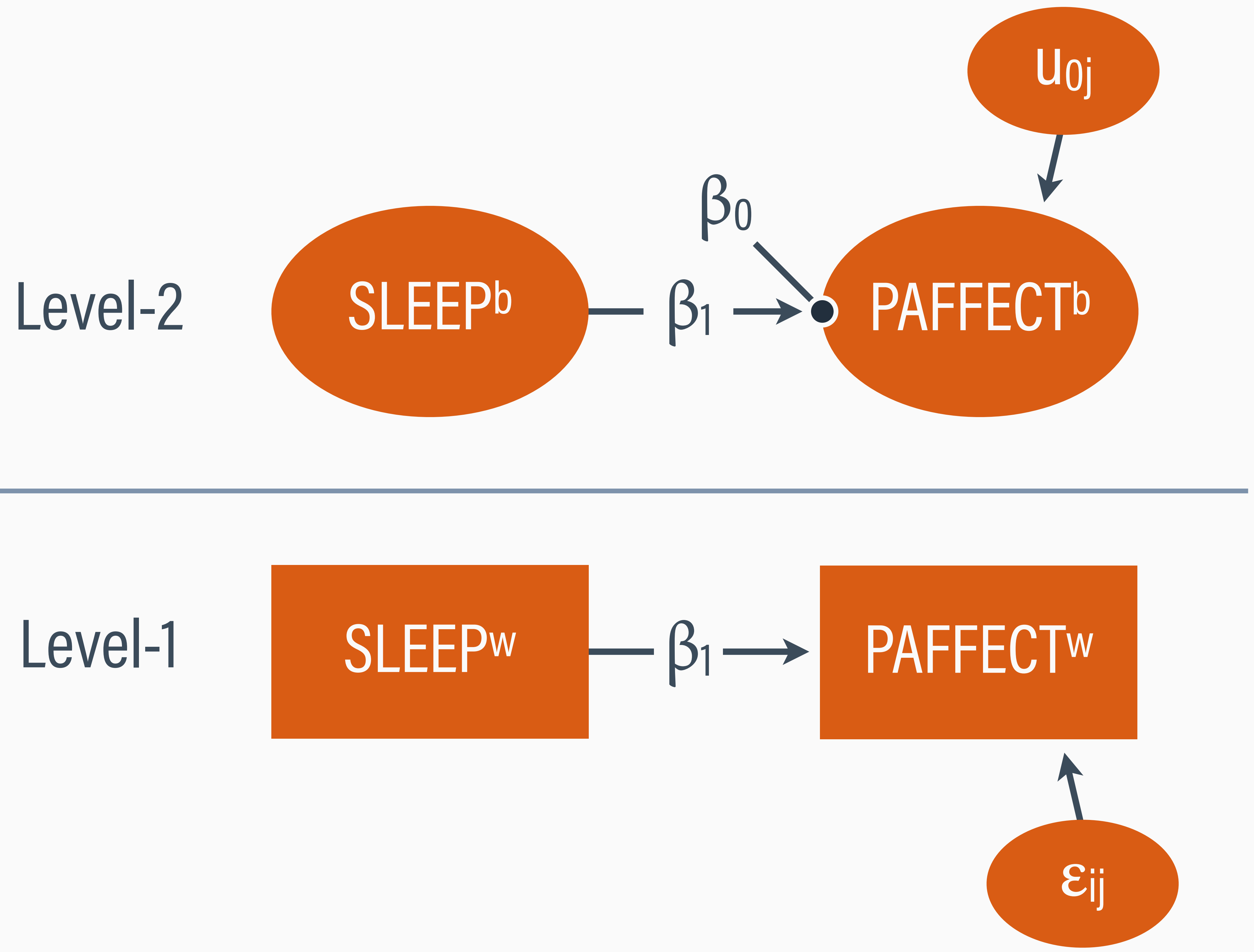
- Using an intact level-1 predictor assumes that the effect of the predictor is the same at level-1 and level-2

$$\begin{aligned} paffect_{ij} &= \beta_0 + \beta_1(sleep_{ij}) + u_{0j} + \varepsilon_{ij} && \text{\# smushed model} \\ &= \beta_0 + \beta_1(sleep_j^b + sleep_{ij}^w) + u_{0j} + \varepsilon_{ij} && \text{\# disaggregate predictor} \\ &= \beta_0 + \beta_1(sleep_j^b) + \beta_1(sleep_{ij}^w) + u_{0j} + \varepsilon_{ij} && \text{\# distribute slope} \end{aligned}$$

- A smushed model is the same as a disaggregated model where the level-1 and level-2 regression slopes are held equal

ANALYSIS MODEL

$$
\begin{aligned}
paffect_{ij} &= \beta_0 + \beta_1(sleep_{ij}) + u_{0j} + \varepsilon_{ij} \\
&= \beta_0 + \beta_1(sleep^{w}_{ij}) + \beta_1(sleep^{b}_{j}) + u_{0j} + \varepsilon_{ij}
\end{aligned}
$$



SMUSHED MODEL

```
DATA: PainDiary.dat;
VARIABLES: Person Day PosAffect NegAffect Pain WorkGoal LifeGoal SleepQual Female Education
  Employment MarStatus NumDiagnose ActivityLevel PainAccept Catastrophize Stress Anxiety;
CLUSTERID: Person;
CENTER: grandmean = SleepQual;    # grand mean centering leaves predictor intact
MODEL: PosAffect ~ intercept SleepQual | intercept;
BURN: 10000;
ITERATIONS: 10000;
SEED: 90291;
```

DISAGGREGATED MODEL WITH CONSTRAINT

```
DATA: PainDiary.dat;
VARIABLES: Person Day PosAffect NegAffect Pain WorkGoal LifeGoal SleepQual Female Education
  Employment MarStatus NumDiagnose ActivityLevel PainAccept Catastrophize Stress Anxiety;
CLUSTERID: Person;
CENTER:
groupmean = SleepQual;
grandmean = SleepQual.mean;
MODEL: PosAffect ~ intercept SleepQual@beta SleepQual.mean@beta | intercept;
BURN: 10000;                          # @beta is a label that forces slopes to be equal
ITERATIONS: 10000;
SEED: 90291;
```

MODEL COMPARISON

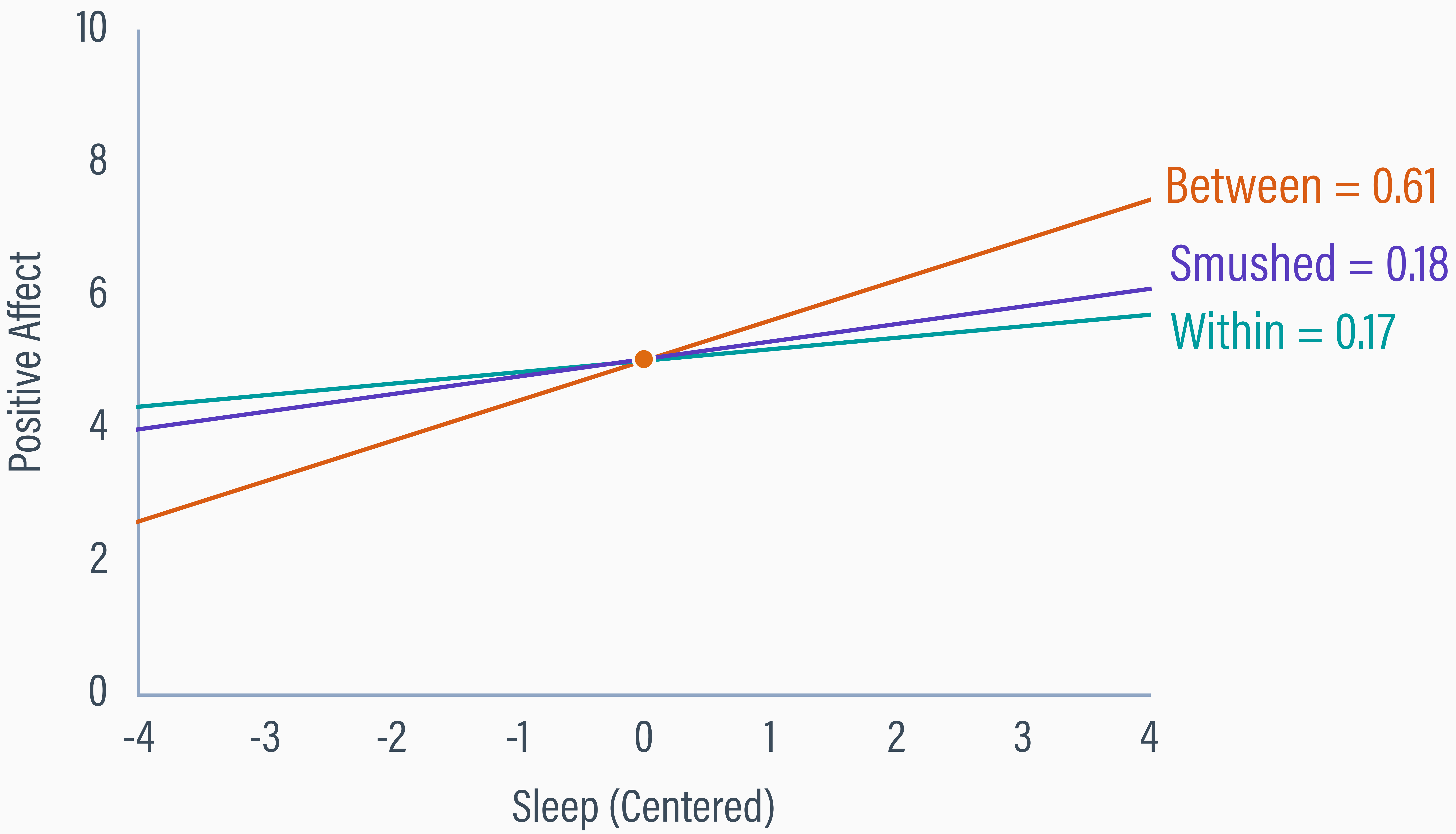
Smushed Model

```
Outcome Variable:  PosAffect

Parameters                              Estimate    StdDev
                                     ------------------------
Variances:
  L2 : Var(Intercept)                      2.168     0.286
  Residual Var.                            1.307     0.037

Coefficients:
  Intercept                                5.027     0.133
  SleepQual                                0.181     0.013

...

Proportion Variance Explained
  by Coefficients                          0.044     0.007
  by Level-2 Random Intercepts             0.597     0.032
  by Level-1 Residual Variation            0.360     0.028

                                     ------------------------
```

Disaggregated Model With Constrained Slopes

```
Outcome Variable:  PosAffect

Parameters                              Estimate    StdDev
                                     ------------------------
Variances:
  L2 : Var(Intercept)                      2.169     0.287
  Residual Var.                            1.306     0.036

Coefficients:
  Intercept                                5.040     0.136
  SleepQual                                0.180     0.013
  SleepQual.mean[Person]                   0.180     0.013

...

Proportion Variance Explained
  by Coefficients                          0.043     0.007
  by Level-2 Random Intercepts             0.597     0.032
  by Level-1 Residual Variation            0.359     0.028

                                     ------------------------
```

SLOPE COMPARISON



SMUSHED MODEL SHORTCOMINGS

- Using raw level-1 predictors combines the level-1 and level-2 associations into a single slope (appropriate only if the slopes are truly equal)
- When level-1 and level-2 slopes differ, failing to disaggregate biases the estimated effect (the combined slope is neither the within- nor the between-cluster effect)
- Failing to disaggregate can also distort the effects of other predictors in the model

WHEN TO USE A SMUSHED MODEL

- The literature broadly supports disaggregation
- The main use of the smushed model is when ICCs are very low (e.g., < .03 to .05), in which case the predictor is mostly a "pure" within-cluster variable (no threat of conflation bias)
- Many predictors have near-zero ICCs (e.g., demographic variables, stimuli in a lab study or experiment), so it is important to estimate ICCs prior to model fitting

OUTLINE

BLIMP STUDIO SCRIPT 2.5

```
DATA: PainDiary.dat;
VARIABLES: Person Day PosAffect NegAffect Pain WorkGoal LifeGoal SleepQual Female Education
  Employment MarStatus NumDiagnose ActivityLevel PainAccept Catastrophize Stress Anxiety;
CLUSTERID: Person;
LATENT: Person = beta0j;   # define level-2 intercept latent variable
CENTER: groupmean = SleepQual;
MODEL:
beta0j ~ intercept;   # level-2: β0j = γ00 + u0j
PosAffect ~ intercept@beta0j SleepQual;   # level-1: paffectij = β0j + β1(sleepij^w) + εij
BURN: 10000;
ITER: 10000;
SEED: 90291;
```

RBLIMP SCRIPT 2 (MODEL 5)

```
within_lat <- rblimp(
      data = PainDiary,
      clusterid = 'Person',
      latent = 'Person = beta0j',
      center = 'groupmean = SleepQual',
      model = '
        beta0j ~ intercept;
        PosAffect ~ intercept@beta0j SleepQual',
      seed = 90291,
      burn = 10000,
      iter = 10000)
output(within_lat)
```

LEVEL-1 OUTPUT

= level-2 estimate

= level-1 estimate

```
Outcome Variable:  PosAffect

Group Mean Centered: SleepQual

Parameters                    Estimate   StdDev     2.5%    97.5%    ChiSq   PValue     N_Eff
                             -----------------------------------------------------------------
Variances:
  Residual Var.                  1.306    0.037    1.237    1.381      ---      ---  16680.158

Coefficients:
  beta0j                       @ 1.000      ---      ---      ---      ---      ---        ---
  SleepQual                      0.173    0.013    0.148    0.199  175.710    0.000  18005.682

Standardized Coefficients:
  SleepQual                      0.254    0.018    0.218    0.289  193.670    0.000  17781.177

Proportion Variance Explained
  by Coefficients                0.065    0.009    0.047    0.084      ---      ---  17761.837
  by Residual Variation          0.935    0.009    0.916    0.953      ---      ---  17761.837

                             -----------------------------------------------------------------
```

LEVEL-2 OUTPUT

= level-2 estimate

= level-1 estimate

```
Latent Variable:   beta0j

Parameters                          Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
                                ------------------------------------------------------------------------------
Variances:
  Residual Var.                        2.502      0.330      1.961      3.245        ---        ---  17537.812

Coefficients:
  Intercept                            5.032      0.141      4.752      5.305   1270.084      0.000  18848.760

Proportion Variance Explained
  by Coefficients                      0.000      0.000      0.000      0.000        ---        ---        nan
  by Residual Variation                1.000      0.000      1.000      1.000        ---        ---        nan

                                ------------------------------------------------------------------------------
```

BLIMP STUDIO SCRIPT 2.6

```
DATA: PainDiary.dat;
VARIABLES: Person Day PosAffect NegAffect Pain WorkGoal LifeGoal SleepQual Female Education
  Employment MarStatus NumDiagnose ActivityLevel PainAccept Catastrophize Stress Anxiety;
CLUSTERID: Person;
LATENT: Person = beta0j;    # define level-2 intercept latent variable
CENTER: groupmean = SleepQual; grandmean = SleepQual.mean;
MODEL:
beta0j ~ intercept SleepQual.mean;    # level-2: β0j = γ00 + γ01(sleep_j^b) + u0j
PosAffect ~ intercept@beta0j SleepQual;    # level-1: paffect_ij = β0j + β1(sleep_ij^w) + ε_ij
BURN: 10000;
ITER: 10000;
SEED: 90291;
```

RBLIMP SCRIPT 2 (MODEL 6)

```
disagg_lat <- rblimp(
        data = PainDiary,
        clusterid = 'Person',
        latent = 'Person = beta0j',
        center = 'groupmean = SleepQual; grandmean = SleepQual.mean',
        model = '
           beta0j ~ intercept SleepQual.mean;
           PosAffect ~ intercept@beta0j SleepQual',
        seed = 90291,
        burn = 10000,
        iter = 10000)
output(disagg_lat)
```

LEVEL-1 OUTPUT

= level-2 estimate

= level-1 estimate

```
Outcome Variable:  PosAffect

Group Mean Centered: SleepQual

Parameters                 Estimate     StdDev       2.5%      97.5%       ChiSq     PValue      N_Eff
                        -----------------------------------------------------------------------------

Variances:
  Residual Var.               1.306      0.036      1.238      1.380         ---        ---  18455.553

Coefficients:
  beta0j                    @ 1.000        ---        ---        ---         ---        ---        ---
  SleepQual                   0.173      0.013      0.147      0.198     174.326      0.000  17004.230

Standardized Coefficients:
  SleepQual                   0.253      0.018      0.217      0.288     191.841      0.000  16829.613

Proportion Variance Explained
  by Coefficients             0.064      0.009      0.047      0.083         ---        ---  16802.586
  by Residual Variation       0.936      0.009      0.917      0.953         ---        ---  16802.586

                        -----------------------------------------------------------------------------
```

LEVEL-2 OUTPUT

= level-2 estimate

= level-1 estimate

```
Latent Variable:   beta0j

Grand Mean Centered: SleepQual.mean[Person]

Parameters                      Estimate     StdDev       2.5%      97.5%      ChiSq     PValue      N_Eff
                              ------------------------------------------------------------------------------
Variances:
  Residual Var.                    1.839      0.250      1.432      2.411        ---        ---  14950.945

Coefficients:
  Intercept                        5.031      0.141      4.754      5.308   1266.410      0.000   6778.361
  SleepQual.mean[Person]           0.606      0.092      0.423      0.785     42.962      0.000  12100.465

Standardized Coefficients:
  SleepQual.mean[Person]           0.522      0.064      0.382      0.631     65.880      0.000  12023.639

Proportion Variance Explained
  by Coefficients                  0.272      0.065      0.146      0.398        ---        ---  11880.090
  by Residual Variation            0.728      0.065      0.602      0.854        ---        ---  11880.090

                              ------------------------------------------------------------------------------
```

BLIMP STUDIO SCRIPT 2.7

```
DATA: PainDiary.dat;
VARIABLES: Person Day PosAffect NegAffect Pain WorkGoal LifeGoal SleepQual Female Education
  Employment MarStatus NumDiagnose ActivityLevel PainAccept Catastrophize Stress Anxiety;
CLUSTERID: Person;
LATENT: Person = beta0j;   # define level-2 intercept latent variable
CENTER: grandmean = SleepQual.mean;
MODEL:
beta0j ~ intercept SleepQual.mean;   # level-2: β0j = γ00 + γ01(sleep_j^b) + u0j
PosAffect ~ intercept@beta0j;   # level-1: paffect_ij = β0j + ε_ij
BURN: 10000;
ITER: 10000;
SEED: 90291;
```

RBLIMP SCRIPT 2 (MODEL 7)

```
between_lat <- rblimp(
        data = PainDiary,
        clusterid = 'Person',
        latent = 'Person = beta0j',
        center = 'grandmean = SleepQual.mean',
        model = '
           beta0j ~ intercept SleepQual.mean;
           PosAffect ~ intercept@beta0j',
        seed = 90291,
        burn = 10000,
        iter = 10000)
output(between_lat)
```

LEVEL-1 OUTPUT

= level-2 estimate

= level-1 estimate

Outcome Variable: PosAffect

Parameters	Estimate	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
Variances:							
Residual Var.	1.395	0.039	1.321	1.474	---	---	17839.886
Coefficients:							
beta0j	@ 1.000	---	---	---	---	---	---
Proportion Variance Explained							
by Coefficients	0.000	0.000	0.000	0.000	---	---	nan
by Residual Variation	1.000	0.000	1.000	1.000	---	---	nan

LEVEL-2 OUTPUT

= level-2 estimate

= level-1 estimate

```
Latent Variable:   beta0j

Grand Mean Centered: SleepQual.mean[Person]
```

Parameters	Estimate	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
Variances:							
Residual Var.	1.810	0.249	1.402	2.374	---	---	12507.777
Coefficients:							
Intercept	5.032	0.142	4.753	5.309	1254.555	0.000	5588.112
SleepQual.mean[Person]	0.620	0.093	0.438	0.800	44.628	0.000	10725.502
Standardized Coefficients:							
SleepQual.mean[Person]	0.533	0.063	0.394	0.641	69.950	0.000	10362.131
Proportion Variance Explained							
by Coefficients	0.284	0.065	0.155	0.410	---	---	10374.248
by Residual Variation	0.716	0.065	0.590	0.845	---	---	10374.248