

MODULE 2

DISAGGREGATING LEVEL-1 PREDICTORS

LEVEL-1 PREDICTORS ARE SPECIAL

- Multilevel modeling texts often recommend a model-building process that begins by adding predictors at level-1
- Level-1 predictors require several key decisions that distinguish them from predictors in standard regression
- One such issue is that level-1 predictors can exert effects at multiple levels; disaggregation separates those effects.

OUTLINE

1

Associations at Level-1 and Level-2

2

Review of Grand Mean Centering

3

Disaggregating a Level-1 Predictor by Centering Within Cluster

4

Analysis Examples

5

Using Raw Level-1 Predictors (Smushed Model)

6

Latent Variable Specification

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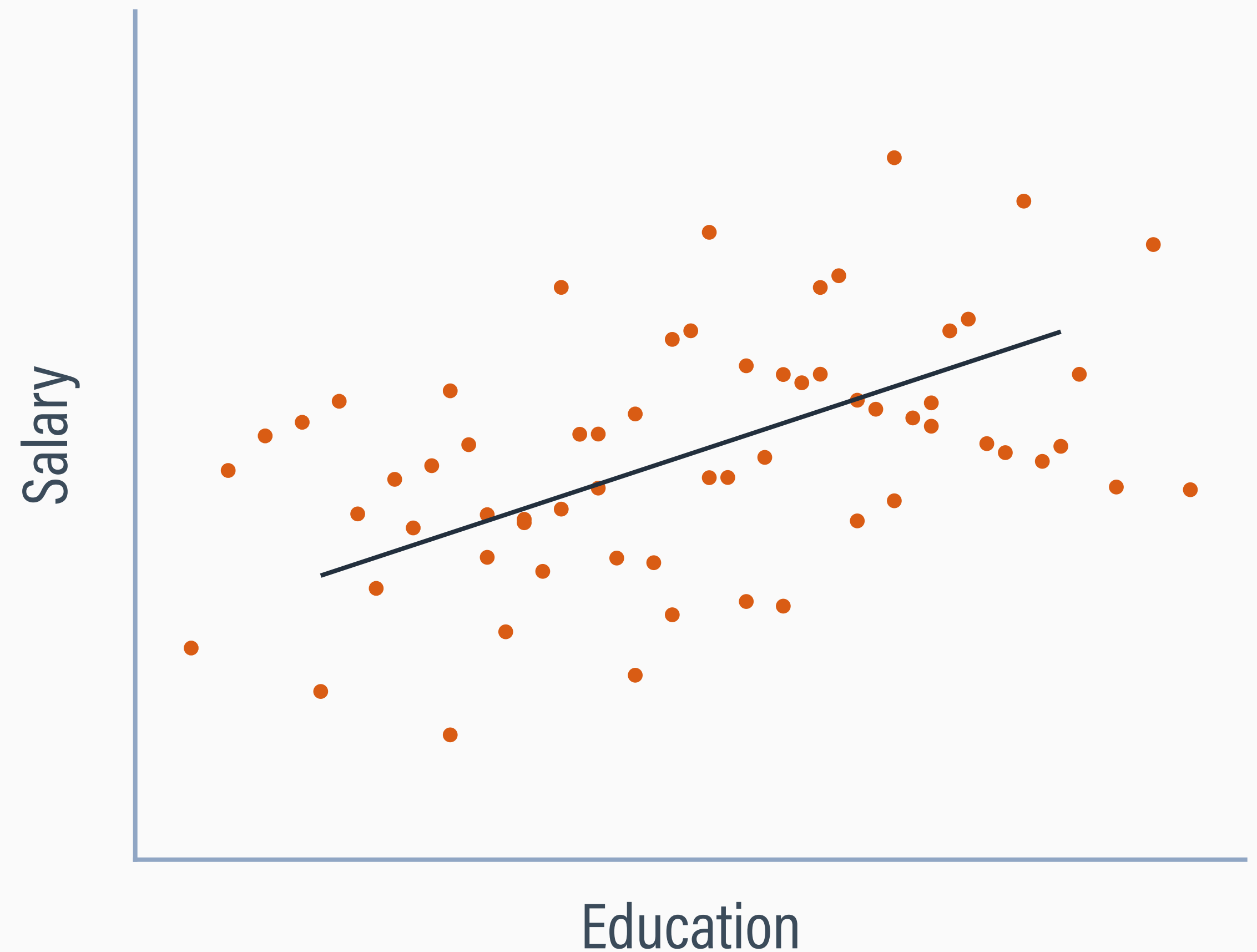
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Latent Variable Specification

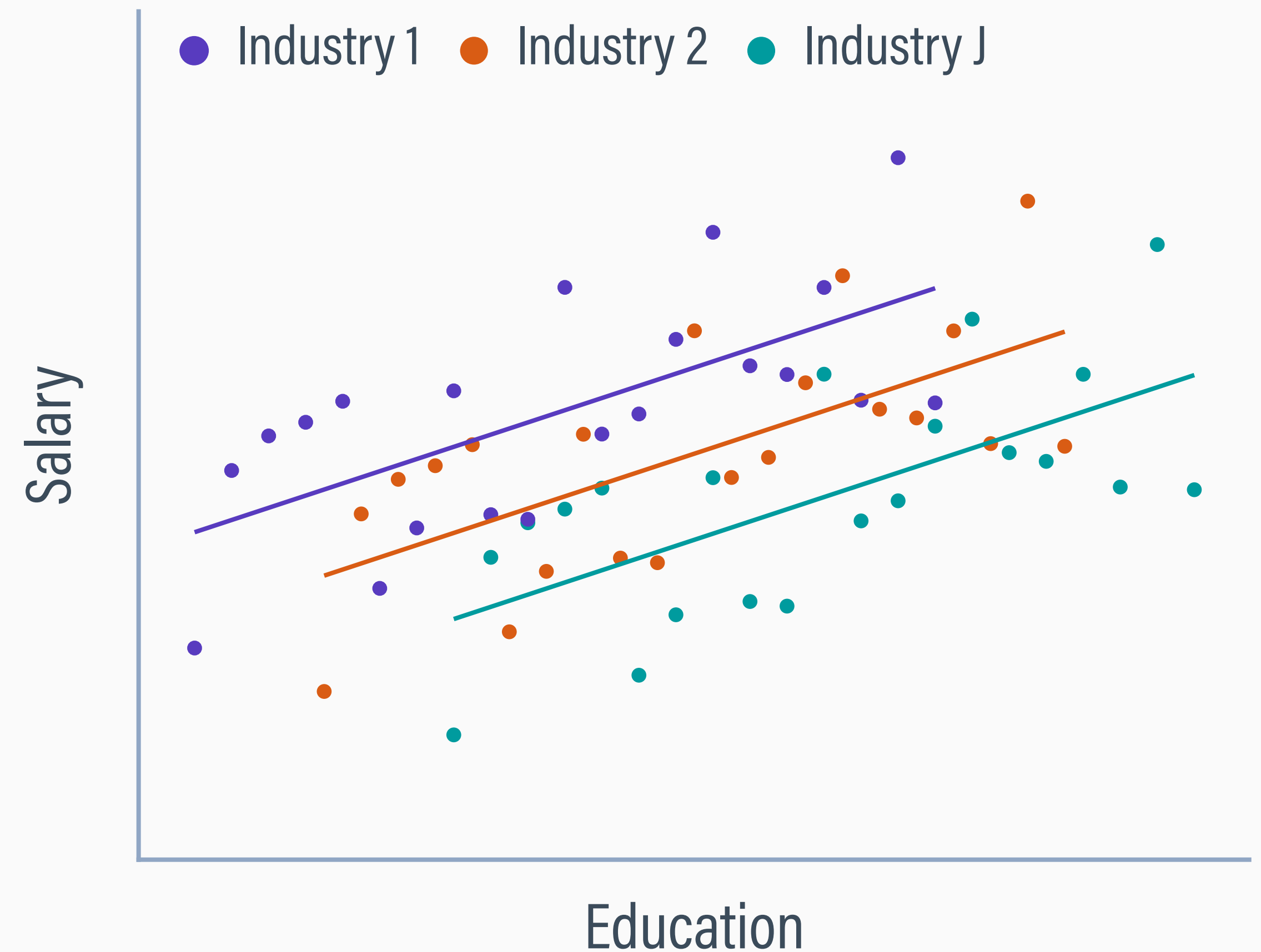
BIVARIATE SCATTERPLOT

- Bivariate scatterplot where years of education predicts salary
- As years of education increases, so too does salary



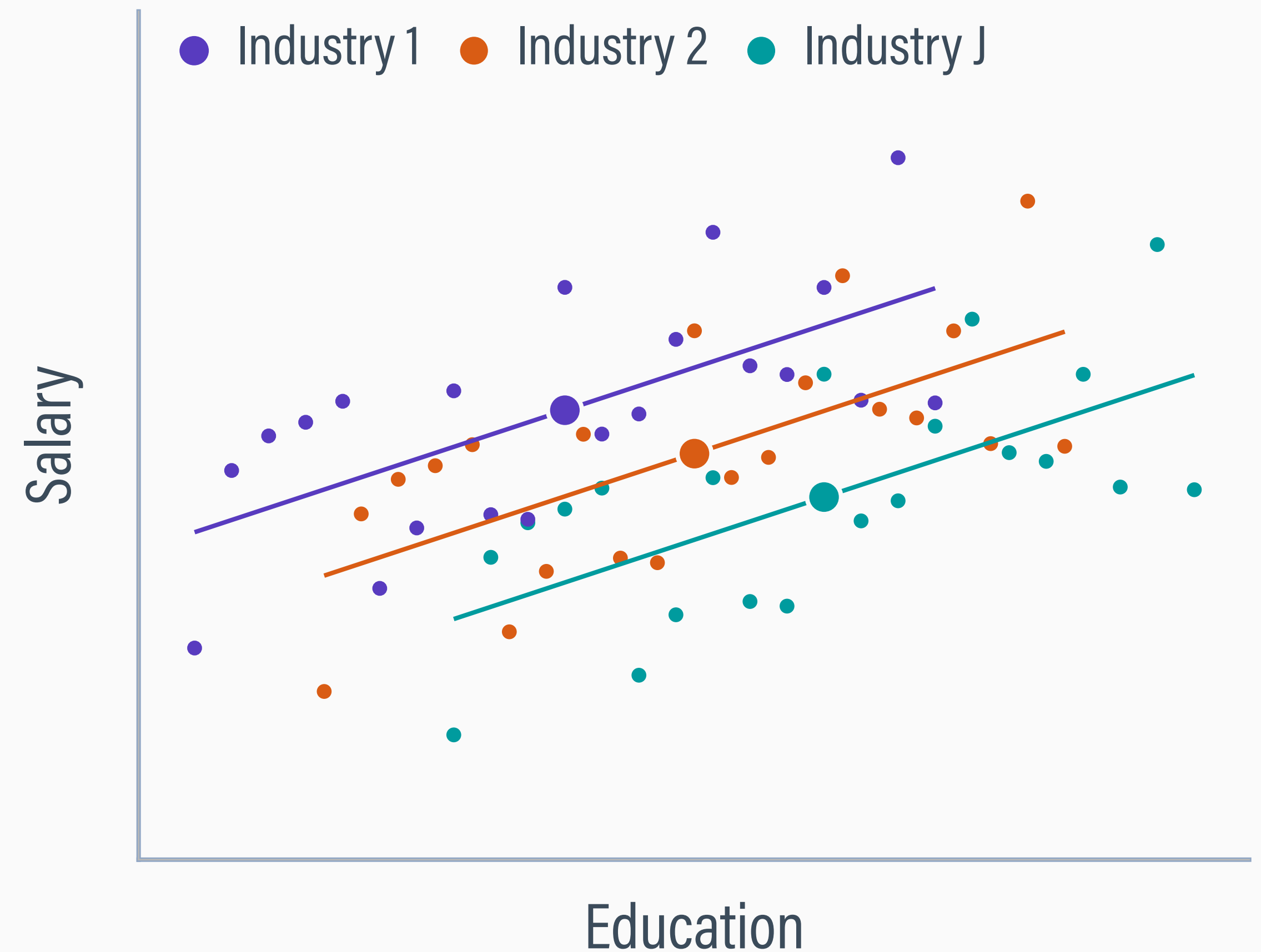
NESTED DATA SCATTERPLOT

- Multilevel data where employees are nested in industries
- Within a given industry, salary increases with years of education



LEVEL-2 CLUSTER MEANS

- The large circles are the level-2 (industry-level) averages
- There are level-2 mean differences on education (horizontal separation of dots) and salary (vertical separation of dots)



BETWEEN-CLUSTER ASSOCIATION

- The industry-level association involving the level-2 cluster means is negative
- Industries with higher mean education have lower mean wages



IMPORTANT TAKEAWAYS

- Level-1 predictors can have two sources of variation, just like outcomes measured at level-1
- A single predictor can produce level-1 and level-2 regression slopes can differ in magnitude or sign
- We isolate the effect of interest by disaggregating level-1 predictors into distinct level-1 and level-2 components

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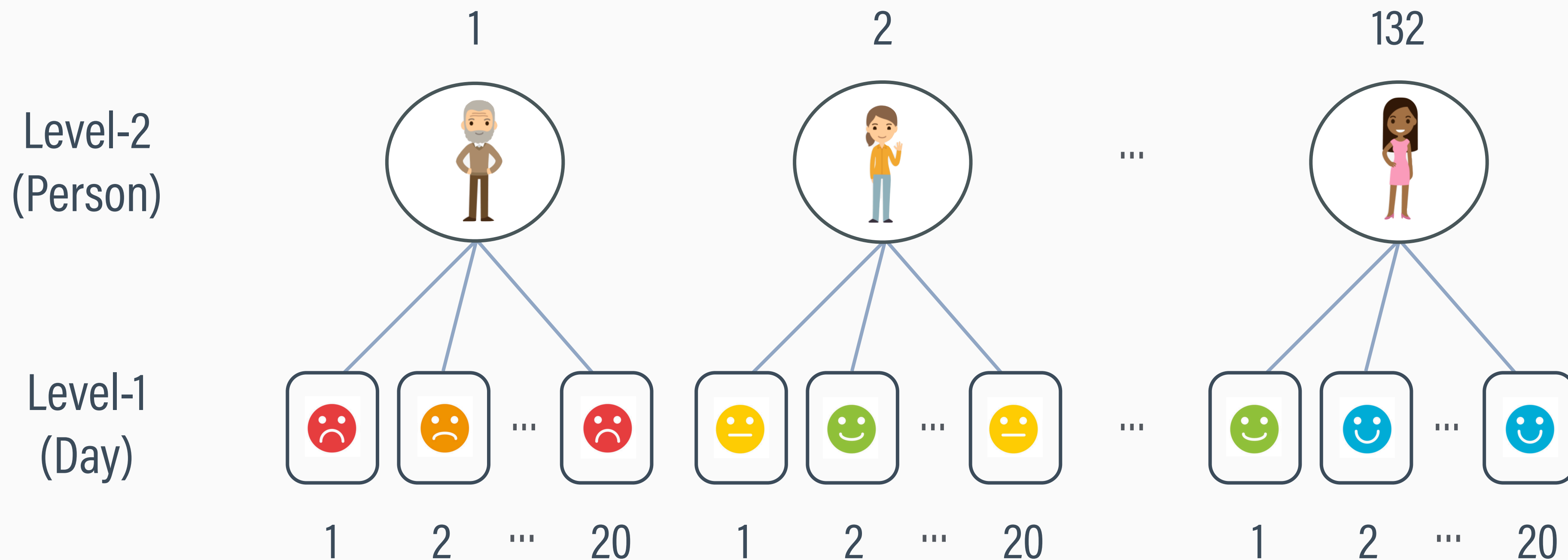
CENTERING PREDICTORS

- Centering involves subtracting a predictor's mean from its scores (we would never center an outcome)
- In single-level regression models, centering predictors at the overall grand mean is a common option
- Multilevel models offer multiple forms of centering

DAILY DIARY APPLICATION

- $n_j = 20$ daily positive affect and sleep assessments nested within $J = 132$ chronic pain patients ($N = 2680$ data records)

$$\text{paffected}_{ij} = \beta_{0j} + \beta_1(\text{sleep}_{ij}) + \varepsilon_{ij}$$



DATA STRUCTURE

- Data in stacked or long format
- Each level-2 unit (person) has multiple rows, one per level-1 (daily) observation
- The i subscript indexes level-1 observations, and j indexes level-2 units

Row	i	j	$PAFFECT_{ij}$	$SLEEP_{ij}$	
1	1	1	7.3	5.6	Person 1
2	2	1	2.5	4.3	
...	...	1	
20	20	1	6.3	7.3	
21	1	2	4.0	3.9	Person 2
22	2	2	4.0	7.1	
...	...	2	
40	20	2	4.4	3.5	
...	
2621	1	132	3.3	5.4	Person 132
2622	2	132	4.8	3.5	
...	...	132	
2640	20	132	4.8	7.9	

CENTERING AT THE GRAND MEAN (CGM)

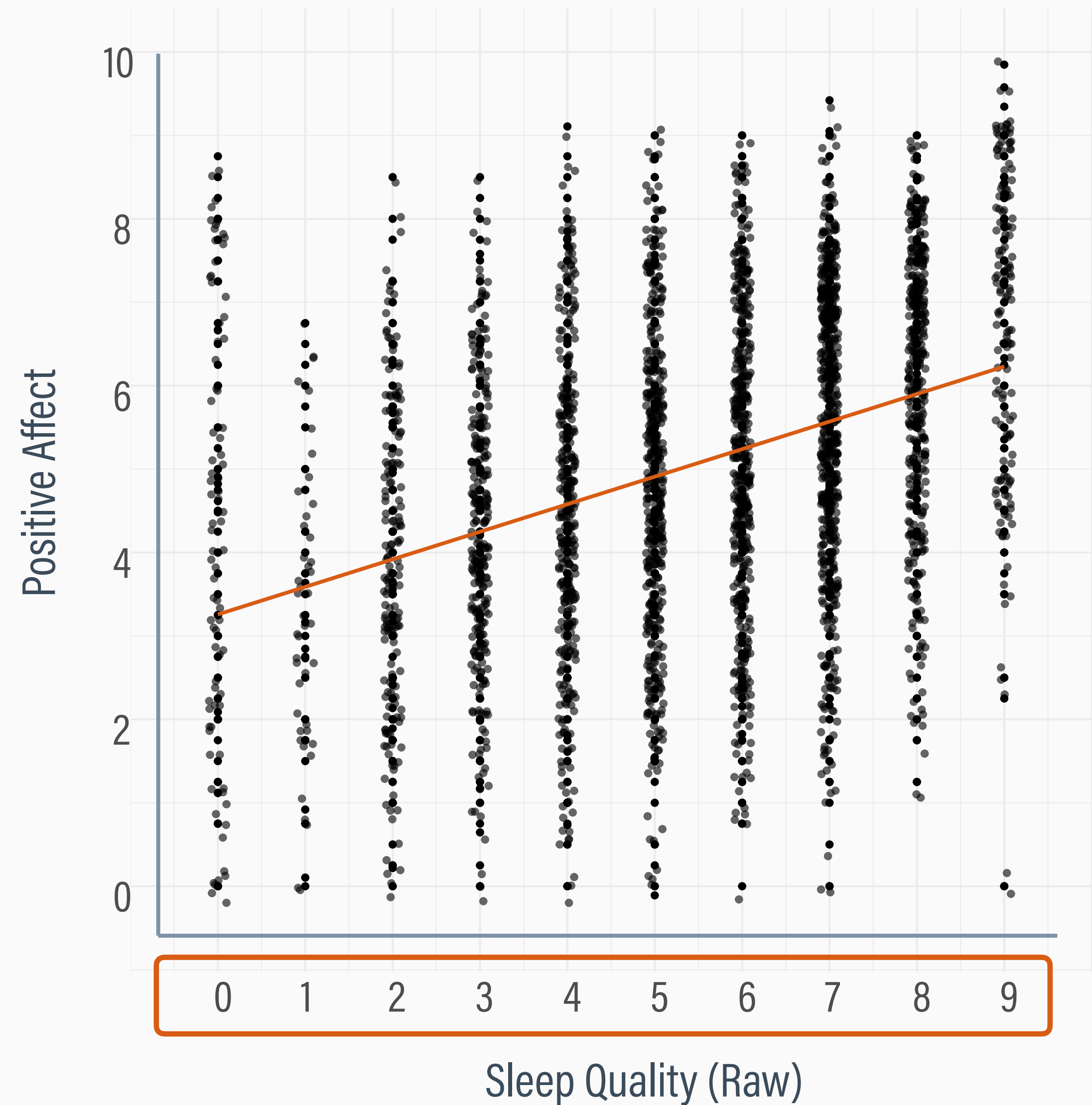
- CGM subtracts the grand mean from every level-1 score

$$X_{ij}^c = X_{ij} - \mu_X$$

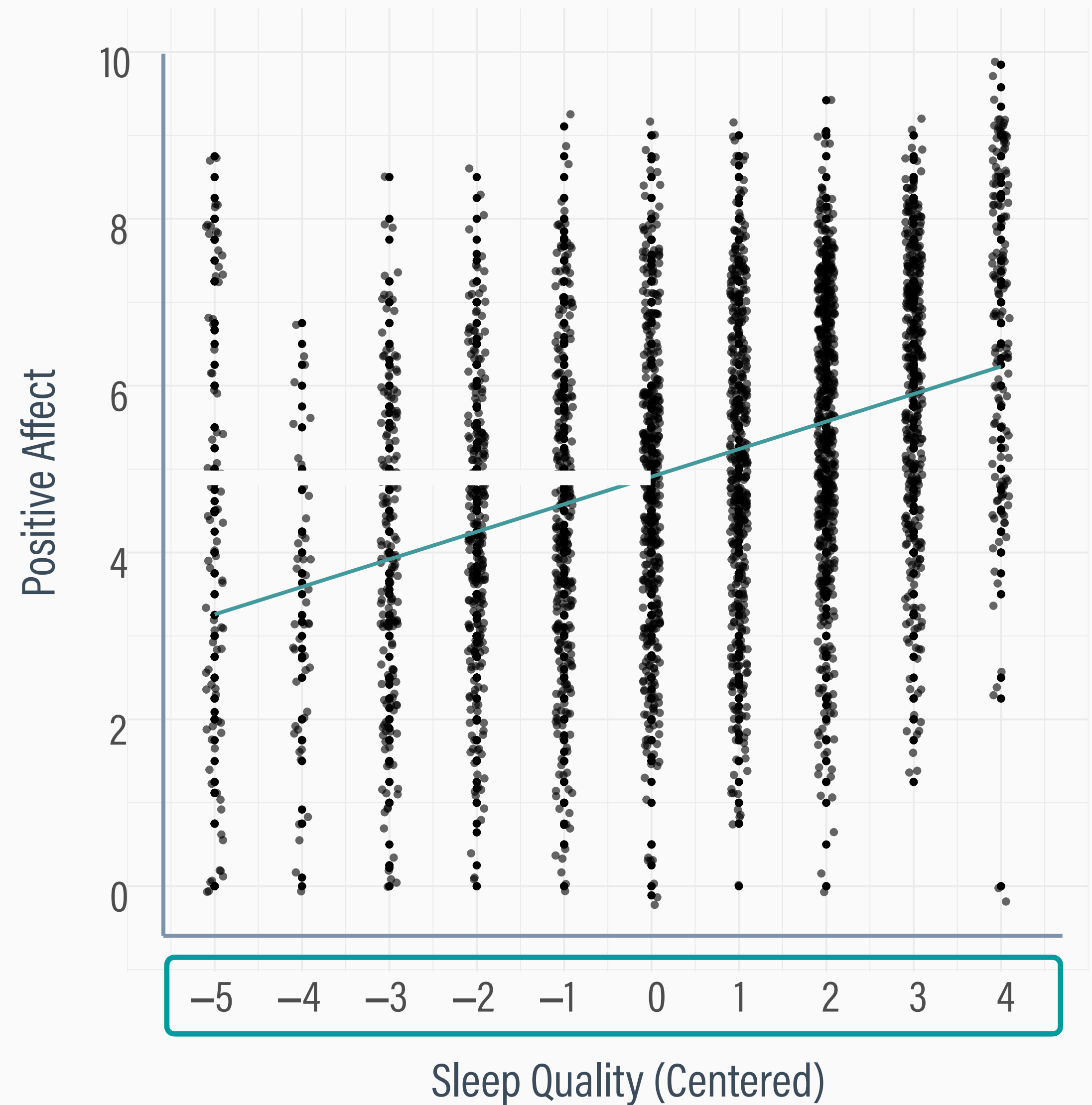
- Centering recodes scores as distances (deviations) from the center of the data

Person	Day	Sleep (X_{ij})		Sleep Mean (μ_X)		Sleep CGM (X_{ij}^c)
1	1	1.25	–	5	=	–3.75
1	2	2.75	–	5	=	–2.25
1	3	2.00	–	5	=	–3.00
2	1	3.25	–	5	=	–1.75
2	2	5.00	–	5	=	0
2	3	6.75	–	5	=	1.75
3	1	8.75	–	5	=	3.75
3	2	7.00	–	5	=	2.00
3	3	8.25	–	5	=	3.25

VISUALIZING CENTERING



Centering recodes predictor scores on the X axis



IMPACT OF CENTERING ON THE INTERCEPT

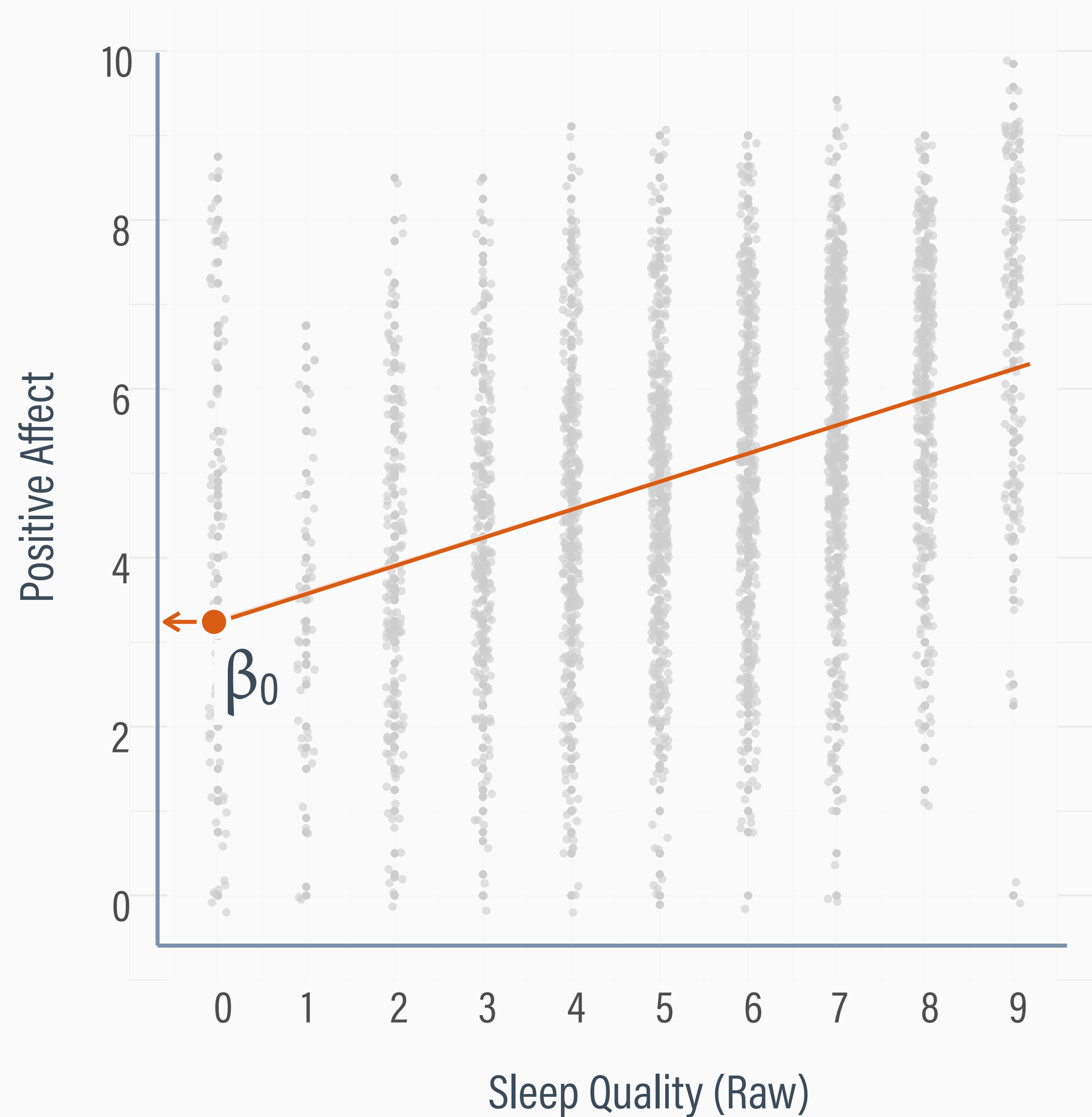
- The intercept is the expected outcome score when the predictor equals zero (often a score outside the data)

$$\hat{p}_{\text{affect}_i} = \beta_0 + \beta_1(\text{sleep}_i)$$

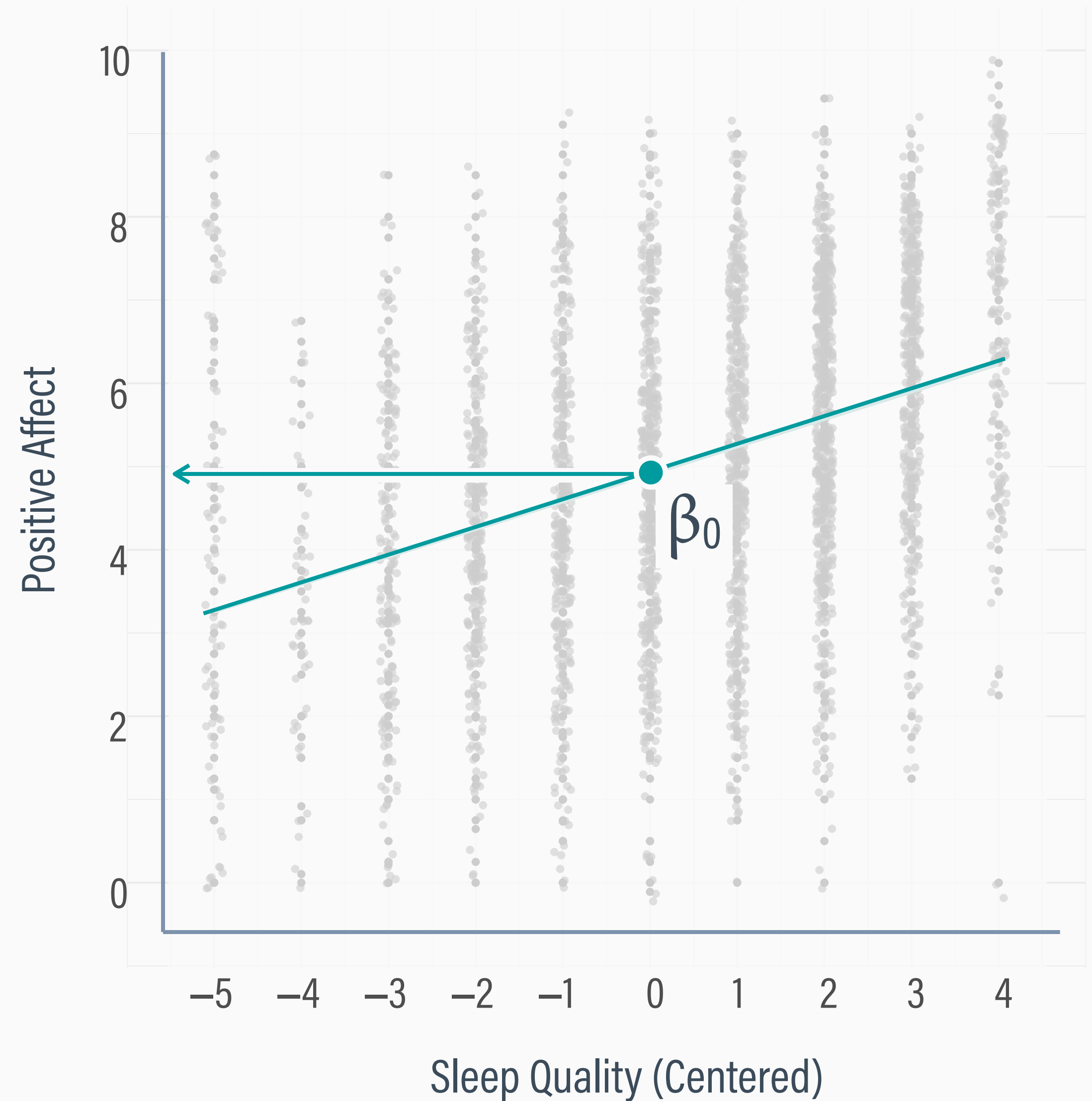
$$\hat{p}_{\text{affect}_i} = \beta_0 + \beta_1(0) = \beta_0$$

- Because a regression line must intersect the means of both variables, making the predictor's mean equal 0 by centering defines the intercept as the outcome variable's mean

CENTERING AND THE INTERCEPT



Centering changes the intercept
but the slope remains the same!



ANALYSIS COMPARISON

Outcome Variable: paffect

Grand Mean Centered: sleep

Parameters	Estimate

Variances:	
L2 : Var(Intercept)	2.168
Residual Var.	1.306
Coefficients: $\beta_0 = \text{paffect grand mean}$	
Intercept	5.038
SleepQual	0.181
Proportion Variance Explained	
by Coefficients	0.044
by Level-2 Random Intercepts	0.597
by Level-1 Residual Variation	0.359

Outcome Variable: paffect

Parameters	Estimate

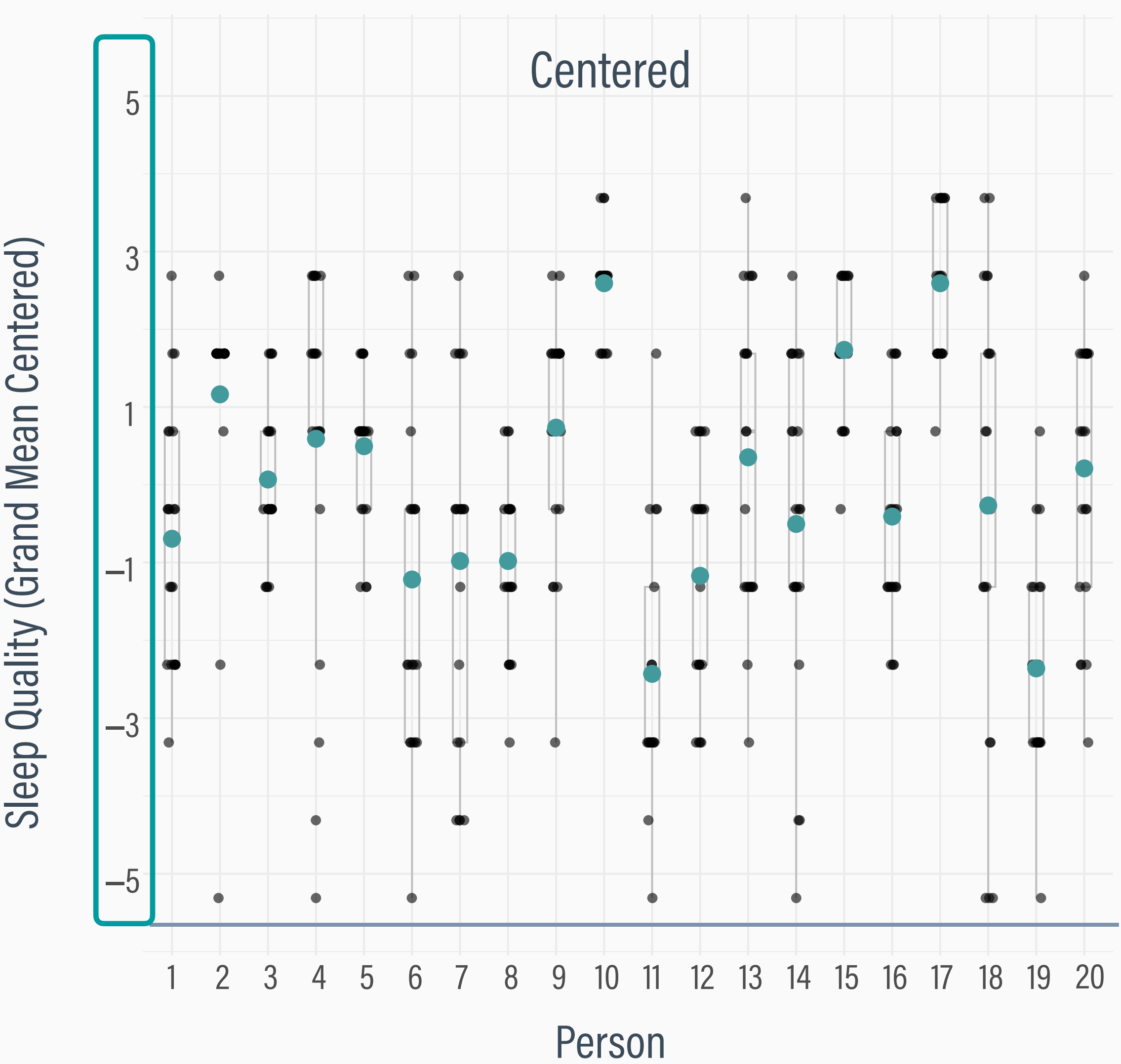
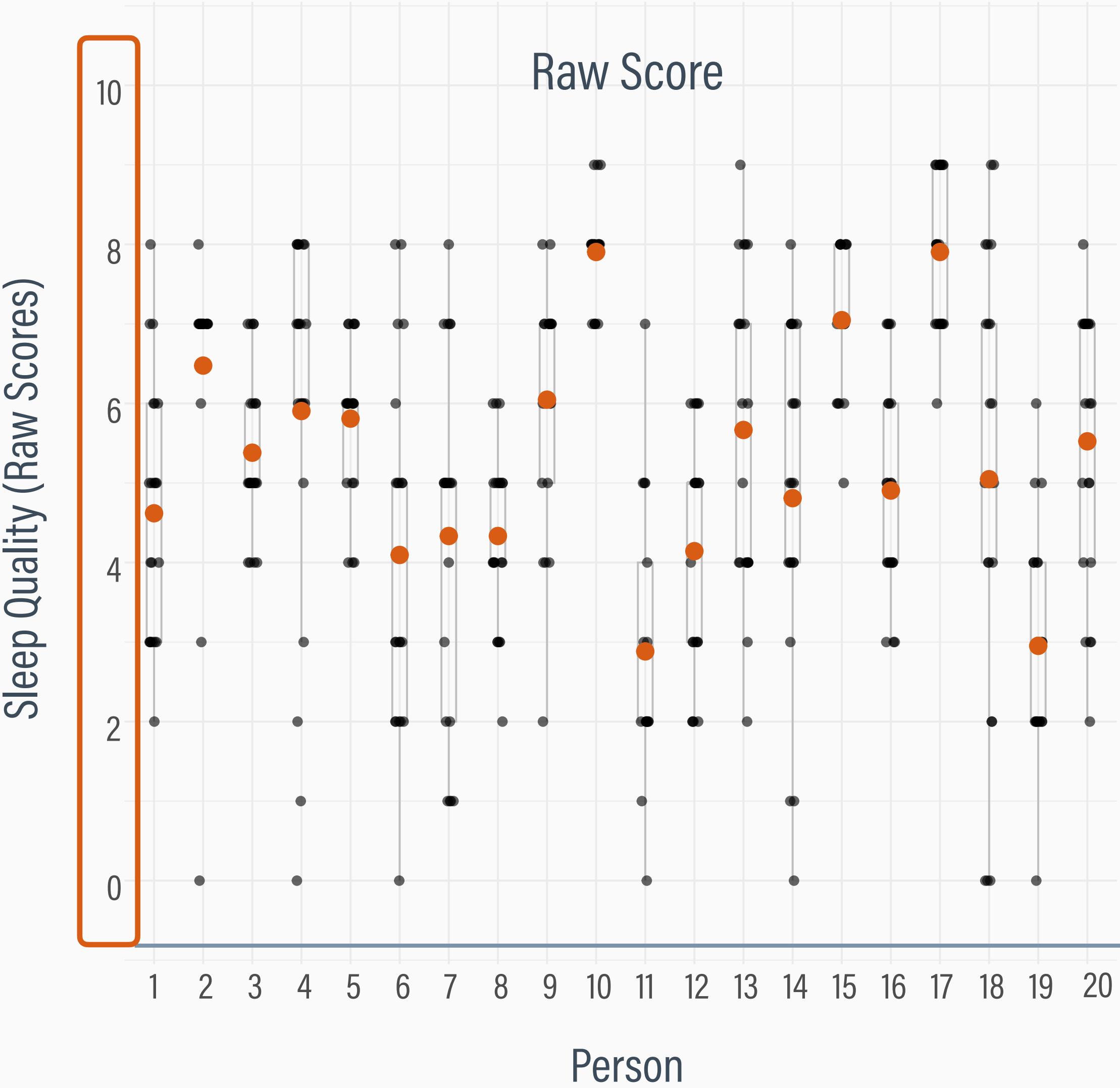
Variances:	
L2 : Var(Intercept)	2.171
Residual Var.	1.307
Coefficients: $\beta_0 = \text{paffect}^\wedge \text{ for sleep} = 0$	
Intercept	4.047
SleepQual	0.181
Proportion Variance Explained	
by Coefficients	0.043
by Level-2 Random Intercepts	0.597
by Level-1 Residual Variation	0.359

GRAND MEAN CENTERING IN MLM

- Level-1 predictors can have two sources of variation, just like outcomes measured at level-1
- People differ in their average sleep quality, and daily observations fluctuate around each person's own mean
- How does grand mean centering affect sources of variation?

BOX PLOTS OF RAW AND CENTERED SCORES

● ● = Person-average sleep





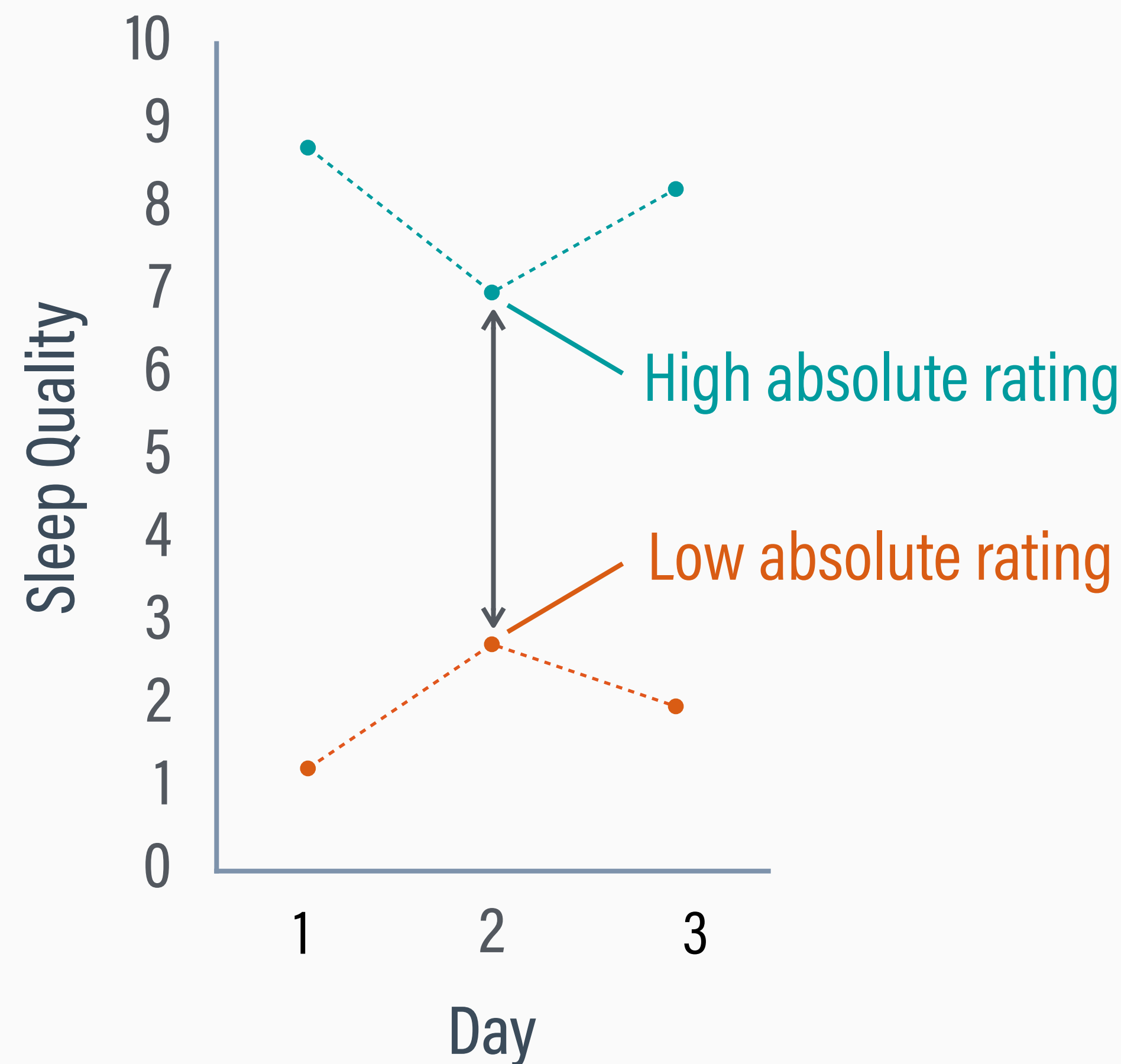
The intraclass correlation for daily sleep quality is $ICC = .39$.
In small groups of two or three, compare the two plots, then discuss whether and how centering affects level-2 variation in the person means.

GRAND MEAN CENTERING SUMMARY

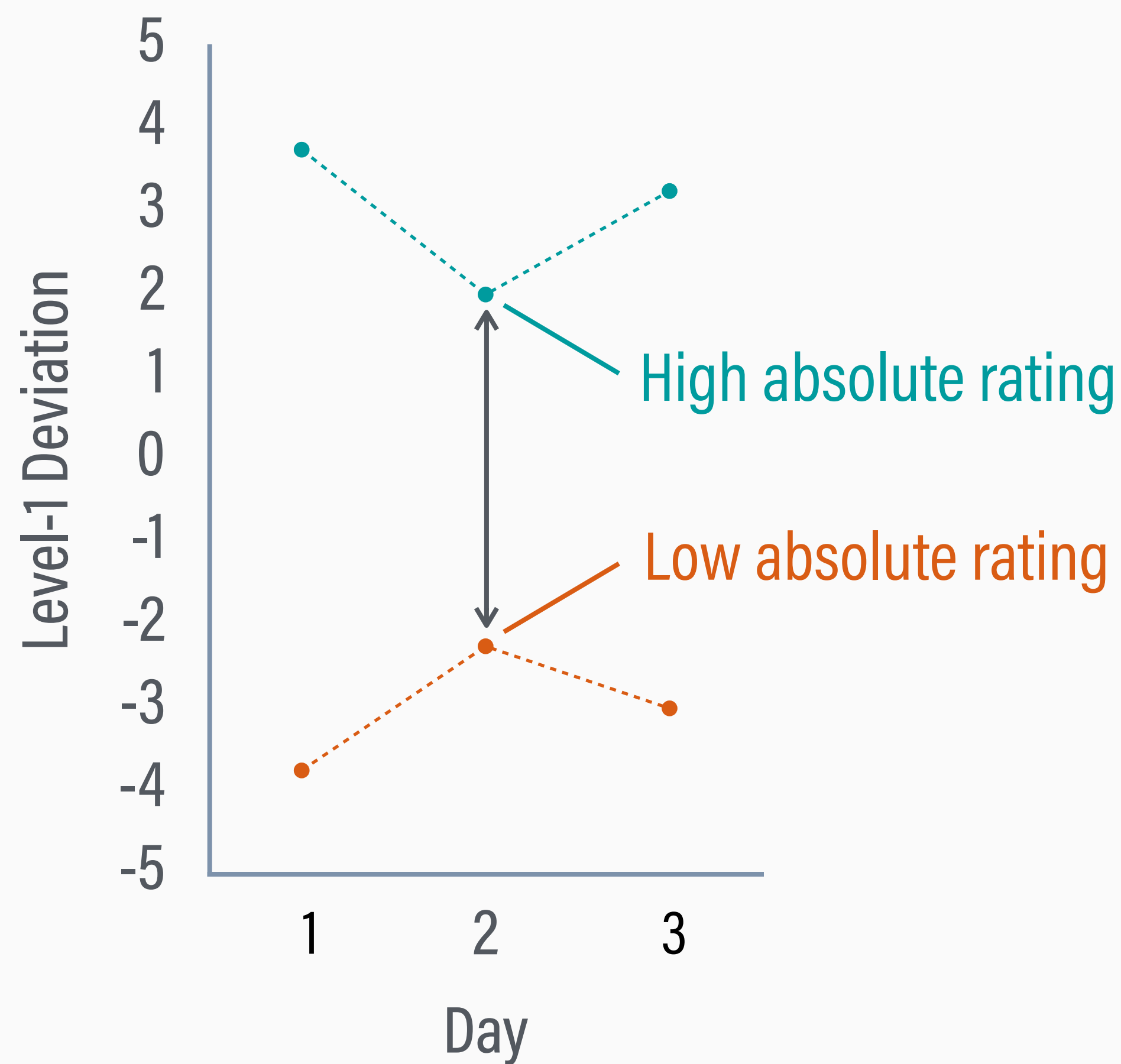
- Grand mean centering is a recoding operation that maintains the rank ordering and relative magnitude of the raw scores
- Centered scores retain all between-cluster variation of the original variable (ICCs are the same as the raw scores)
- Centering defines the intercept as the outcome's grand mean

CGM RETAINS BOTH SOURCES OF VARIATION

Raw scores reflect absolute differences among observations



Deviation scores also reflect absolute differences among observations



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DISAGGREGATION

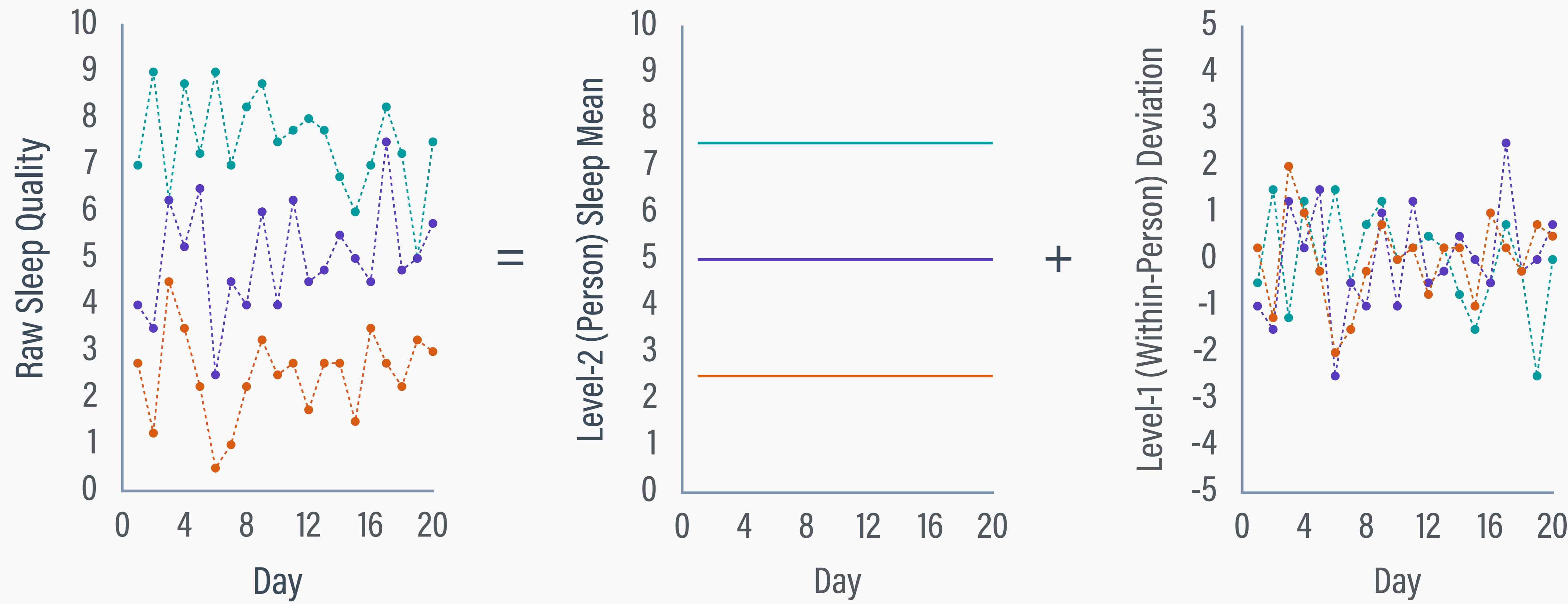
- The literature generally recommends disaggregating level-1 predictor variables into two distinct, uncorrelated variables

$$X_{ij} = X_j^b + X_{ij}^w = \text{cluster mean} + \text{within-cluster deviation}$$

- The level-2 cluster means (sleep averages) define a between-cluster variable (X_j^b), and deviations around the level-2 means define a pure within-cluster variable (X_{ij}^w)

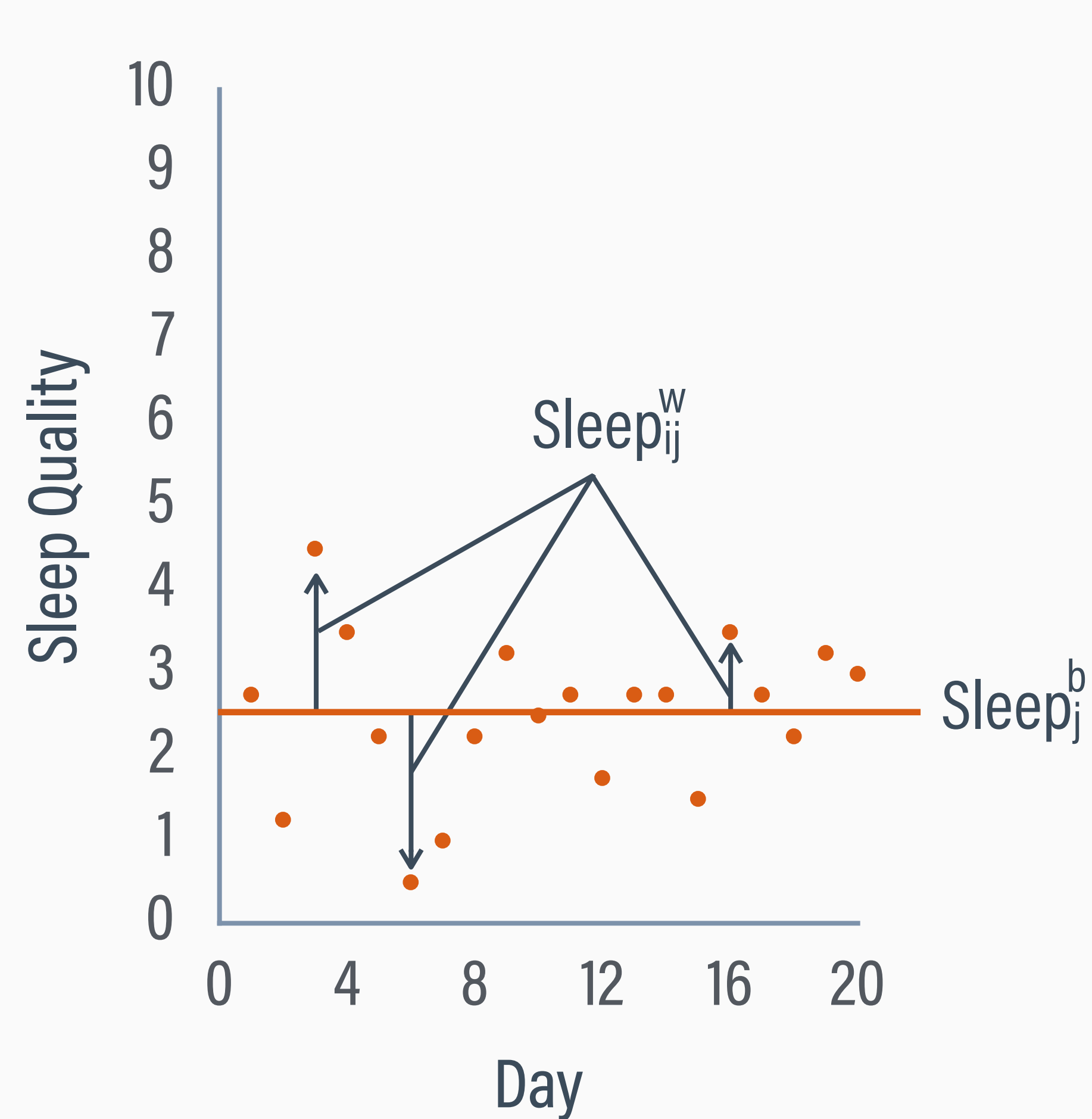
DISAGGREGATION GRAPHIC

Score = Level-2 (Between-Cluster) Mean + Level-1 (Within-Cluster) Residual



DISAGGREGATION EXAMPLE 1

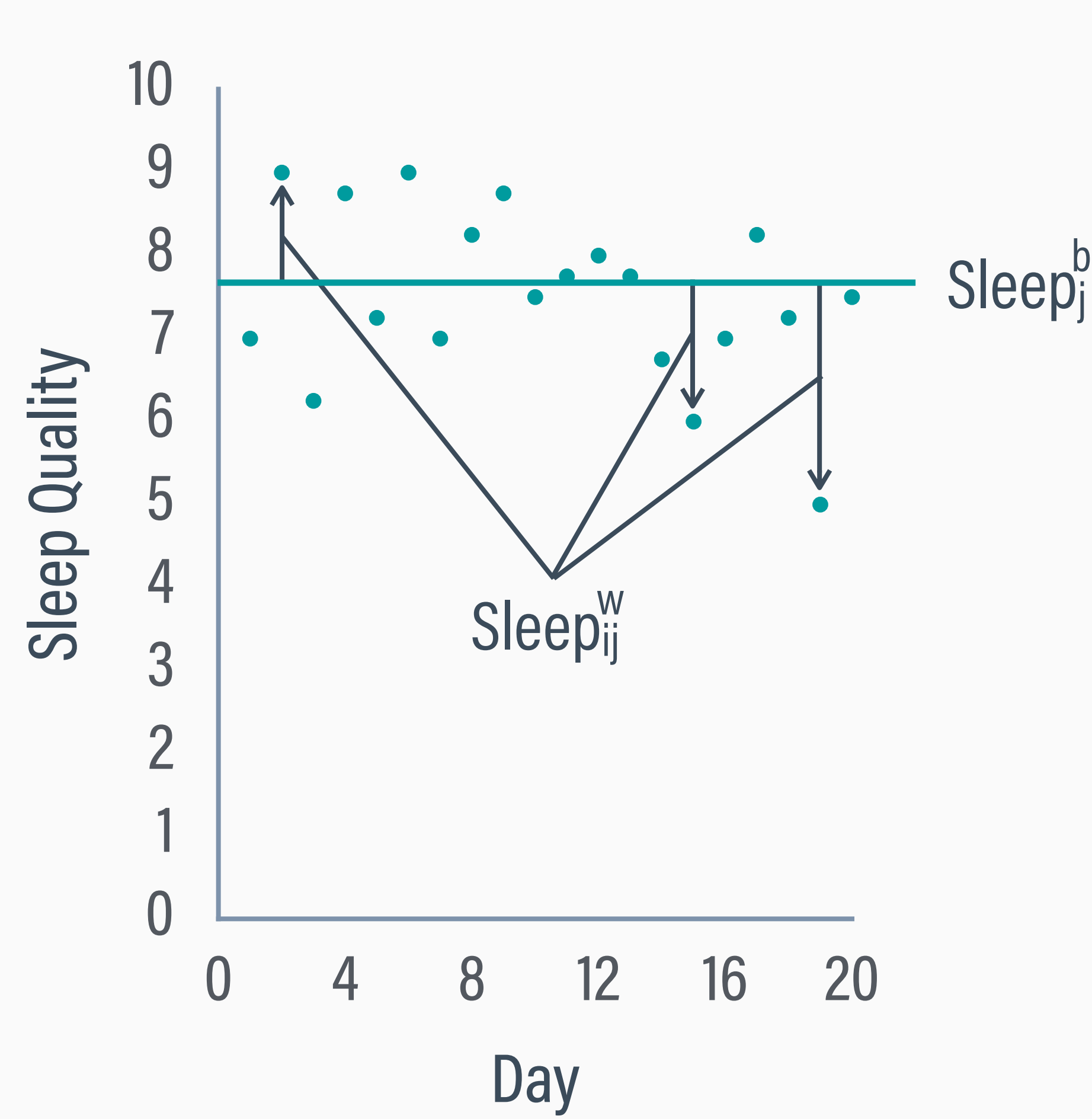
Score = Level-2 (Between-Cluster) Mean + Level-1 (Within-Cluster) Residual



Person	Day	Sleep _{ij}		Sleep _j ^b		Sleep _{ij} ^w
1	1	2.75	=	2.5	+	0.25
1	2	1.25	=	2.5	+	-1.25
1	3	4.50	=	2.5	+	2.00
1	=	...	+	...
1	18	2.25	=	2.5	+	-0.25
1	19	3.25	=	2.5	+	0.75
1	20	3.00	=	2.5	+	0.50

DISAGGREGATION EXAMPLE 2

Score = Level-2 (Between-Cluster) Mean + Level-1 (Within-Cluster) Residual



Person	Day	Sleep _{ij}		Sleep _j ^b		Sleep _{ij} ^w
3	1	7.00	=	7.5	+	-0.50
3	2	9.00	=	7.5	+	1.50
3	3	6.25	=	7.5	+	-1.25
3	=	...	+	...
3	18	7.25	=	7.5	+	-0.25
3	19	5.00	=	7.5	+	-2.50
3	20	7.50	=	7.5	+	0

CENTERING WITHIN CLUSTER (CWC)

- CWC subtracts out each person’s own level-2 mean from their level-1 scores

$$X_{ij}^w = X_{ij} - \mu_{X_j} = X_{ij} - X_j^b$$

- CWC deviation scores are “pure” measures of within-cluster variation
- Also called **group mean centering**

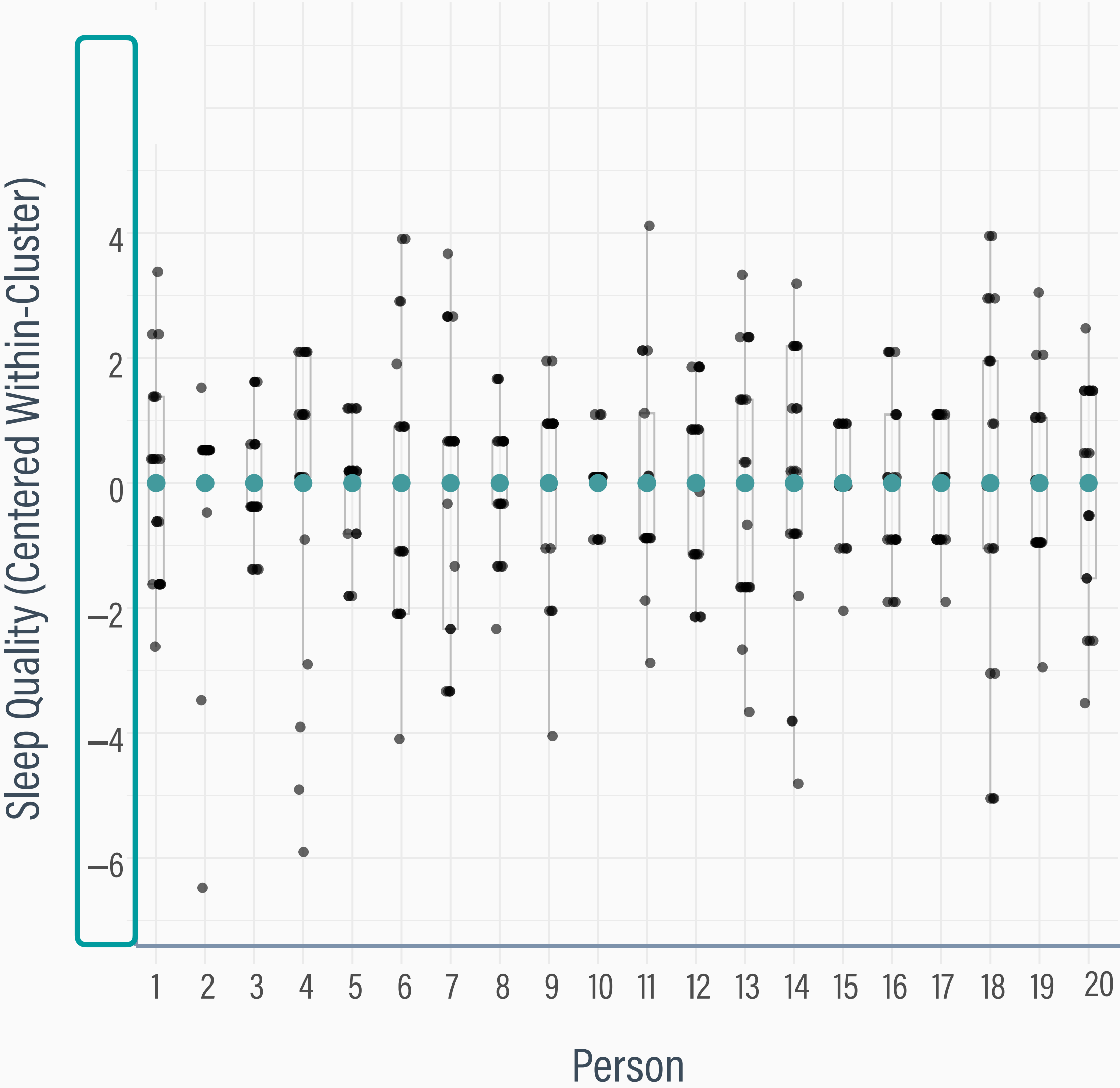
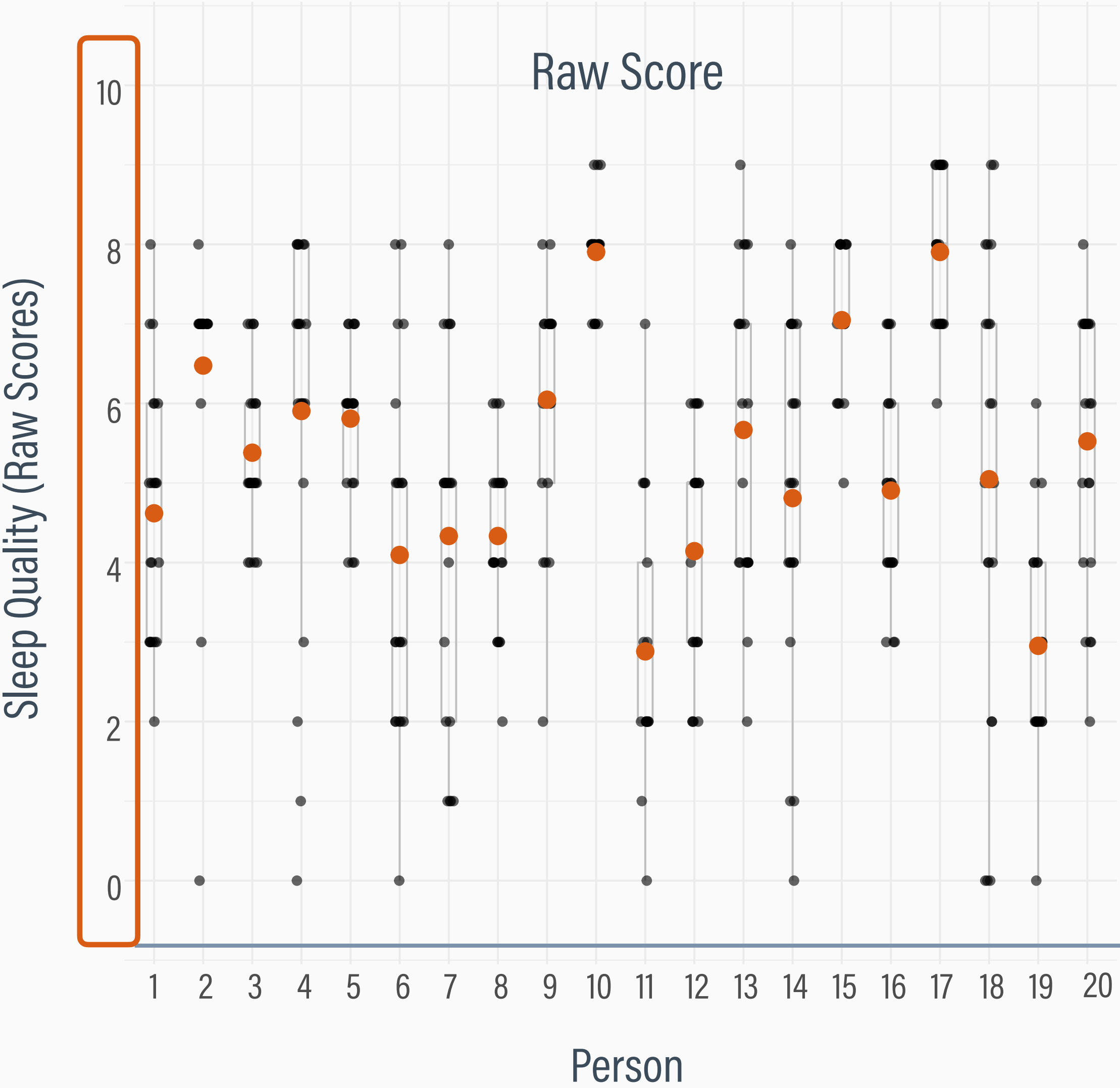
Person	Day	Sleep (X_{ij})		Sleep Mean (X_j^b)		Sleep CWC (X_{ij}^w)
1	1	1.25	–	2	=	–0.75
1	2	2.75	–	2	=	0.75
1	3	2.00	–	2	=	0
2	1	8.75	–	8	=	0.75
2	2	7	–	8	=	–1.00
2	3	8.25	–	8	=	0.25

TWO TYPES OF CLUSTER MEANS

- A cluster-level average (X^b) can be defined as the arithmetic mean of cluster j 's scores (**manifest group mean**) or as a random intercept (**latent group mean**)
- Arithmetic averages are biased with small N s and assume each cluster's mean uses the same number of level-1 scores
- Latent cluster means are noisy in small samples, but they are unbiased and appropriate with unequal cluster sizes

BOX PLOTS OF RAW AND CENTERED SCORES

● ● = Person-average sleep

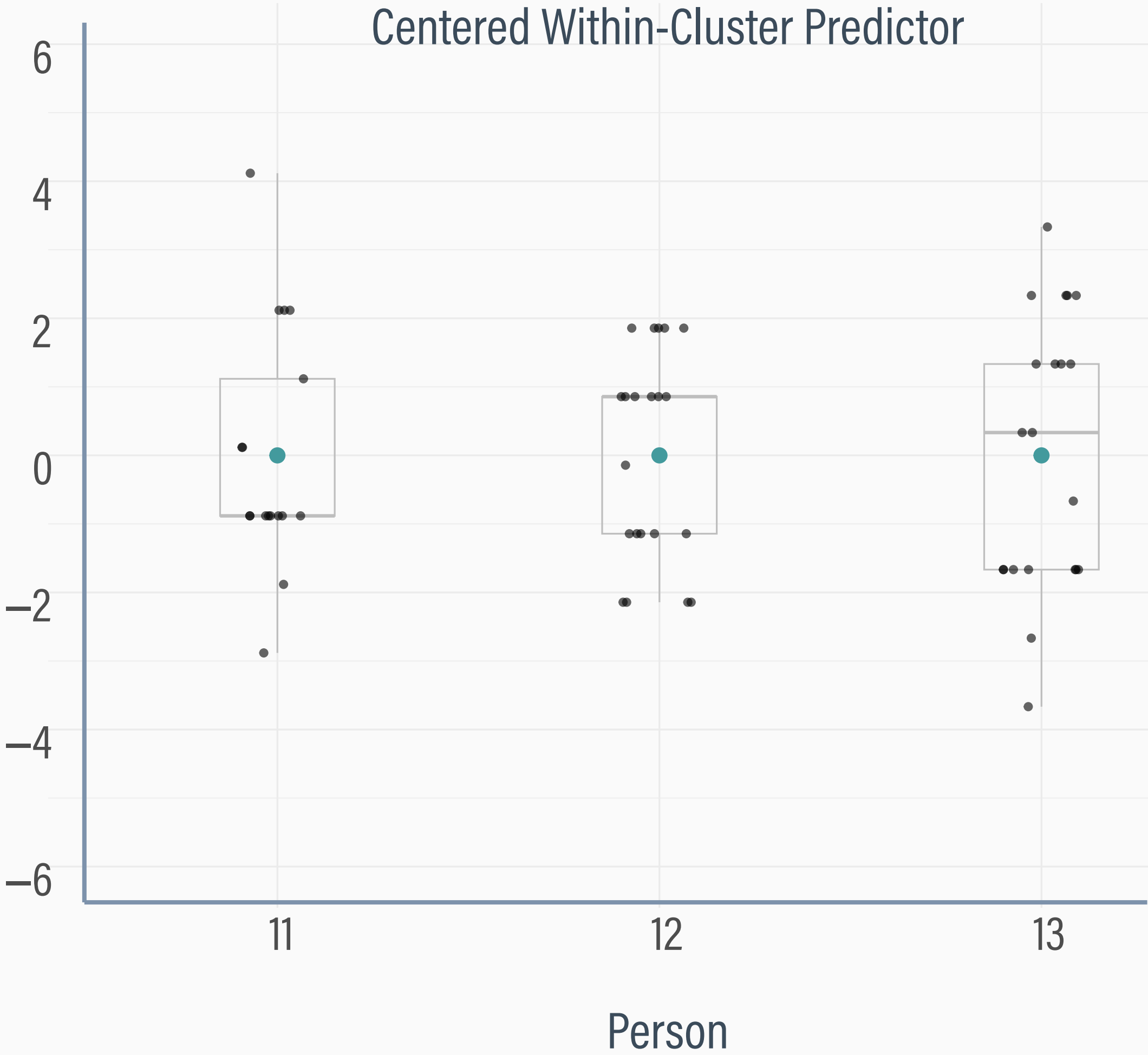
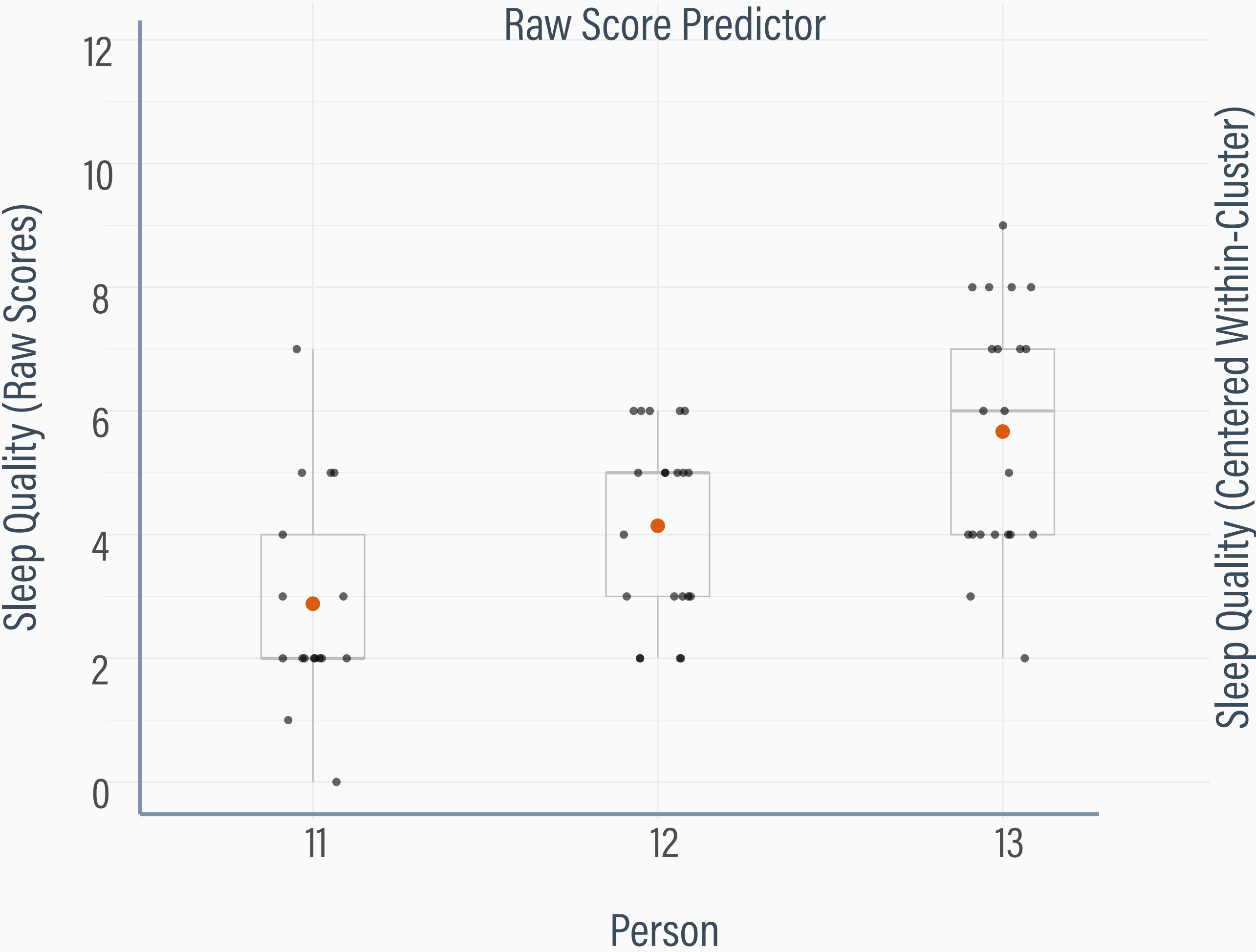




The intraclass correlation for daily sleep quality is $ICC = .39$.
In small groups of two or three, compare the two plots, then discuss whether and how centering affects level-2 variation in the person means.

BOX PLOTS FOR THREE PEOPLE

● ● = Person-average sleep





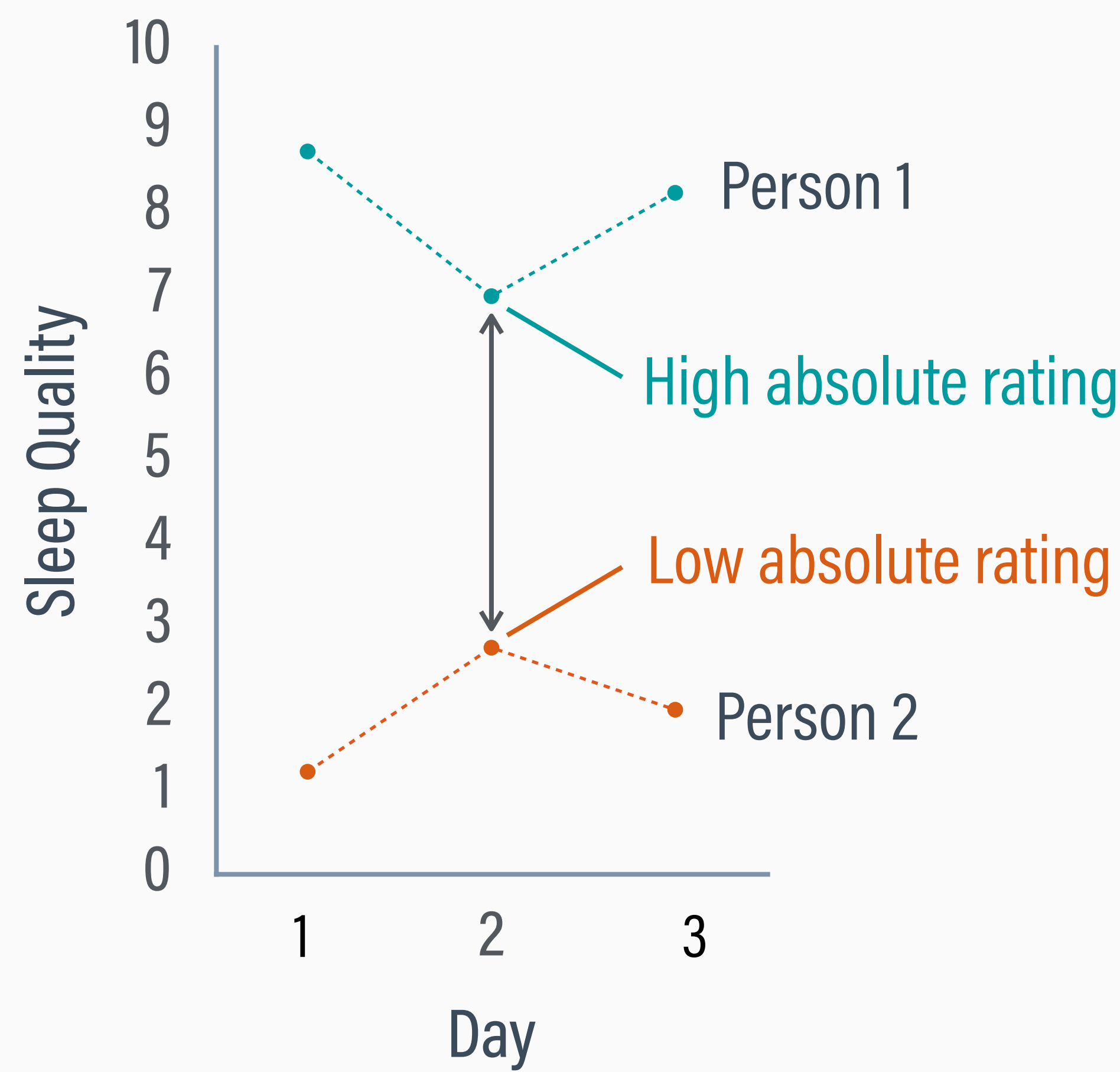
When we add a predictor to a regression model, its slope reflects the predicted outcome difference for two observations that differ by one point on the predictor. In small groups of two or three, discuss the meaning of a “one point difference” on the predictor when using raw and CWC centered scores. Use the previous box plots in your discussion.

INTERPRETING CENTERED SCORES

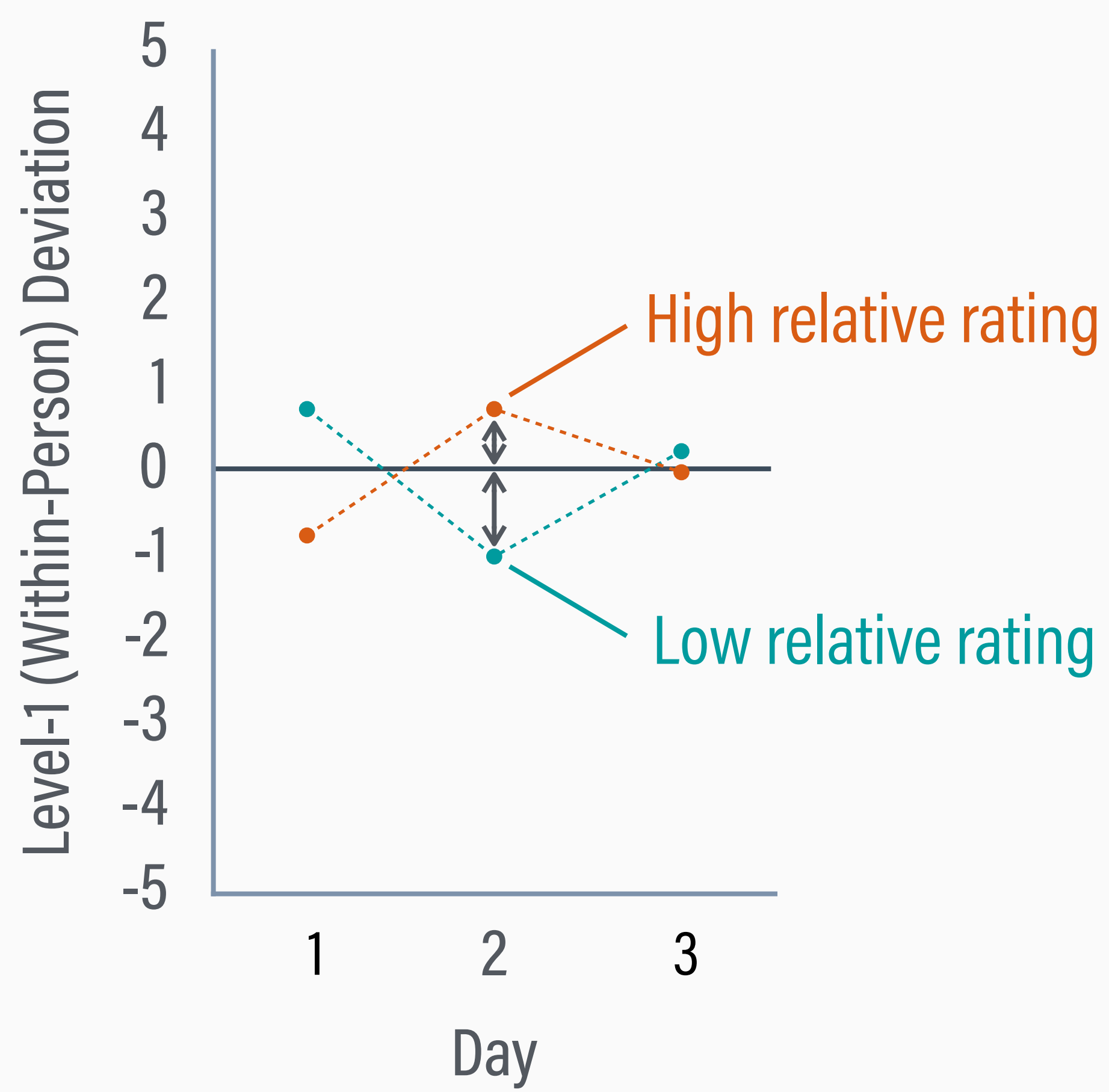
- CWC removes level-2 mean differences from the level-1 scores (after centering, all level-2 cluster means equal 0)
- CWC deviations reflect “pure” within-cluster variation among observations around their Level-2 cluster means
- The effect of a “one-point difference” on the predictor reflects a comparison between two scores from the same cluster

CWC DEVIATIONS ARE CLUSTER-SPECIFIC

Raw scores reflect absolute differences among observations



CWC scores reflect relative differences among observations from the same cluster



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IMPORTANT TAKEAWAYS

- Predictors measured at level-1 can produce level-1 and level-2 regression slopes
- We isolate the effect of interest by disaggregating level-1 predictors into distinct level-1 and level-2 components
- Depending on the goal, researchers may estimate the level-1 effect only, the level-2 effect only, or both simultaneously

ANALYSIS EXAMPLES

1

Estimate Intraclass Correlations

2

Within-Cluster (Level-1) Regression Only

3

Between-Cluster (Level-2) Regression Only

4

Within-Cluster (Level-1) and Between-Cluster (Level-2) Regression

ESTIMATING INTRACLASST CORRELATIONS

- Estimate intraclass correlations (ICCs) before model fitting to assess how much variance lies within and between clusters
- Some predictors have little or no level-2 variation, in which case they are already “pure” within-cluster variables
- Disaggregating a level-1 predictor and modeling its means is only warranted when there is sufficient level-2 variation

BLIMP STUDIO SCRIPT 2.1

DATA: PainDiary.dat;

VARIABLES: Person Day PosAffect NegAffect Pain WorkGoal LifeGoal SleepQual Female Education
Employment MarStatus NumDiagnose ActivityLevel PainAccept Catastrophize Stress Anxiety;

CLUSTERID: Person;

MODEL:

PosAffect ~ intercept | intercept; # empty model for outcome

SleepQual ~ intercept | intercept; # empty model for predictor

BURN: 10000;


ITERATIONS: 10000;

SEED: 90291;

RBLIMP SCRIPT 2 (MODEL 1)

```
model1 <- rblimp(  
  data = PainDiary,  
  clustered = 'Person',  
  model = '  
    PosAffect ~ intercept | intercept;  
    SleepQual ~ intercept | intercept',  
  seed = 90291,  
  burn = 10000,  
  iter = 10000)  
output(model1)
```

BLIMP OUTPUT


 = level-2 estimate

 = level-1 estimate

Outcome Variable: PosAffect

Parameters	Estimate	StdDev	2.5%	97.5%	ChiSq	PValue
Variances:						
L2 : Var(Intercept)	2.525	0.334	1.979	3.279	---	---
Residual Var.	1.395	0.039	1.322	1.474	---	---
Coefficients:						
Intercept	5.031	0.137	4.740	5.298	1353.904	0.000
Standard Deviations:						
L2 : SD(Intercept)	1.589	0.103	1.407	1.811	---	---
Residual SD	1.181	0.016	1.150	1.214	---	---
Proportion Variance Explained						
by Coefficients	0.000	0.000	0.000	0.000	---	---
by Level-2 Random Intercepts	0.644	0.030	0.585	0.703	---	---
by Level-1 Residual Variation	0.356	0.030	0.297	0.415	---	---

BLIMP OUTPUT

 = level-2 estimate

 = level-1 estimate

Outcome Variable: SleepQual

Parameters	Estimate	StdDev	2.5%	97.5%	ChiSq	PValue
Variances:						
L2 : Var(Intercept)	1.918	0.265	1.487	2.516	---	---
Residual Var.	3.002	0.084	2.844	3.172	---	---
Coefficients:						
Intercept	5.428	0.130	5.177	5.685	1736.935	0.000
Standard Deviations:						
L2 : SD(Intercept)	1.385	0.094	1.219	1.586	---	---
Residual SD	1.733	0.024	1.686	1.781	---	---
Proportion Variance Explained						
by Coefficients	0.000	0.000	0.000	0.000	---	---
by Level-2 Random Intercepts	0.390	0.033	0.329	0.458	---	---
by Level-1 Residual Variation	0.610	0.033	0.542	0.671	---	---



The intraclass correlation for daily sleep quality is $ICC = .39$.
In small groups of two or three, provide a brief description of the ICC aimed at a colleague with no exposure to multilevel models.

SUMMARY

- Both variables have substantial variation at level-2 ($ICC_{p\text{affect}} = .64$ and $ICC_{\text{sleep}} = .39$)
- Two distinct associations are estimable
- Person-level average sleep quality can predict average positive mood (level 2), and daily deviations from one's own sleep average can predict daily affect (level 1)

ANALYSIS EXAMPLES

1

Estimate Intraclass Correlations

2

Within-Cluster (Level-1) Regression Only

3

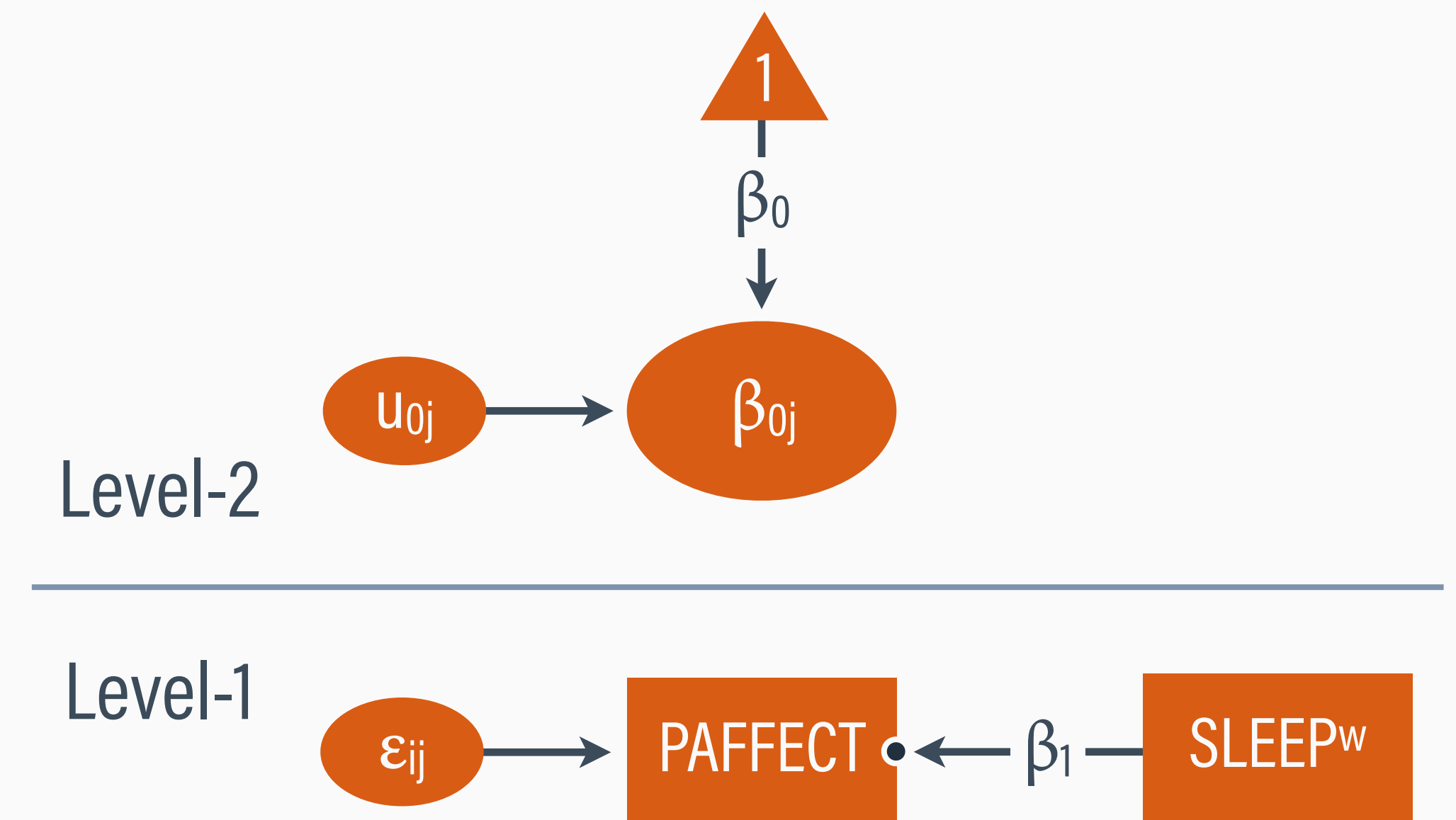
Between-Cluster (Level-2) Regression Only

4

Within-Cluster (Level-1) and Between-Cluster (Level-2) Regression

ANALYSIS OVERVIEW

- The within-cluster association captures how deviations from a person's average sleep rating relates to their daily positive affect
- This reflects within-person sleep differences that do not depend on one's average sleep
- Person-level mean differences are unmodeled



• = random intercept (β_{0j})

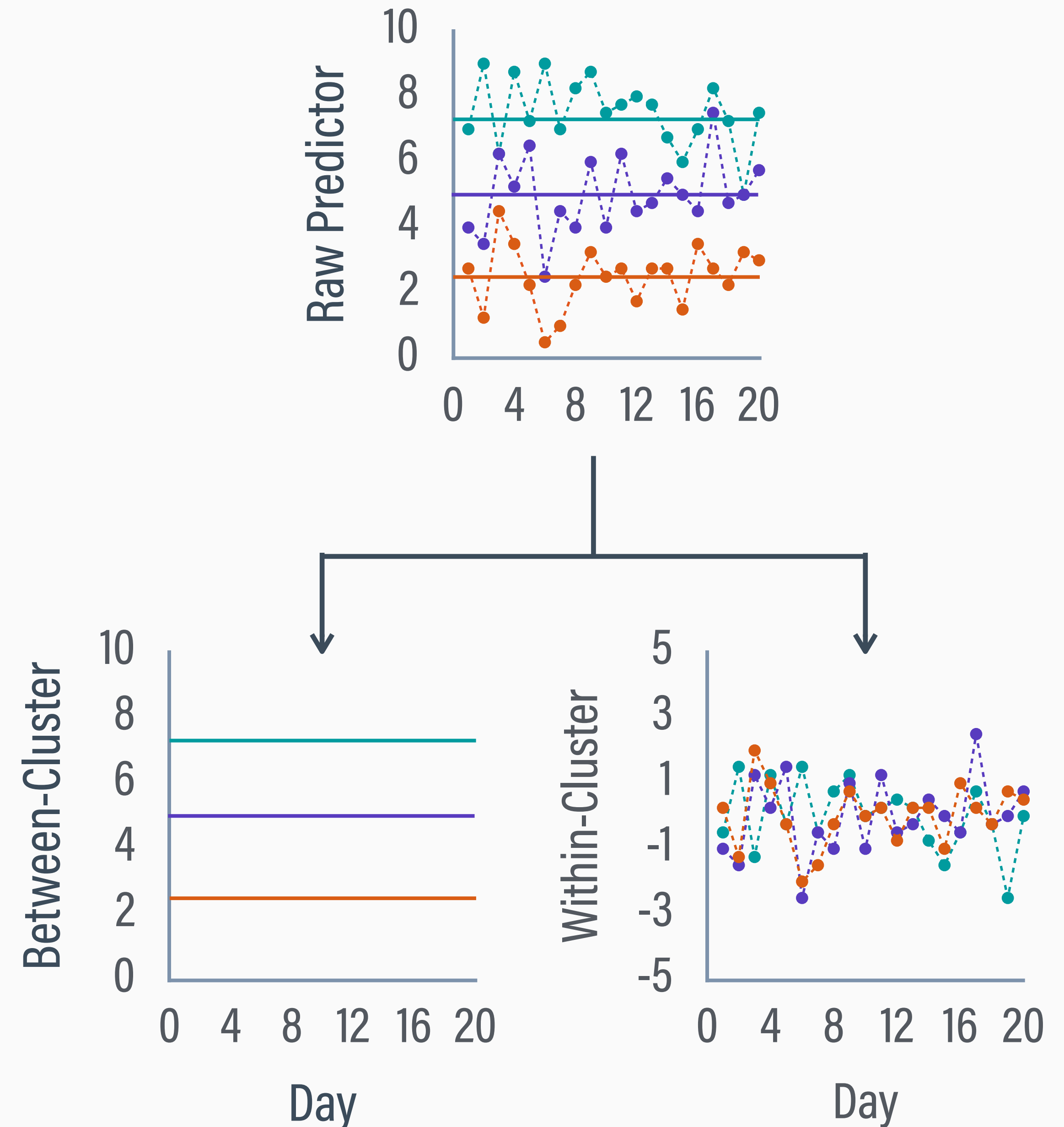
DISAGGREGATED PREDICTOR

- Disaggregation centers each daily score around its level-2 person mean

$$\text{sleep}_j^b = \mu_{j(\text{sleep})}$$

$$\text{sleep}_{ij}^w = \text{sleep}_{ij} - \mu_{j(\text{sleep})}$$

- sleep^w contains only intraindividual (level-1) variation, and sleep^b reflects only person (level-2) mean differences



WITHIN-CLUSTER (LEVEL-1) MODEL

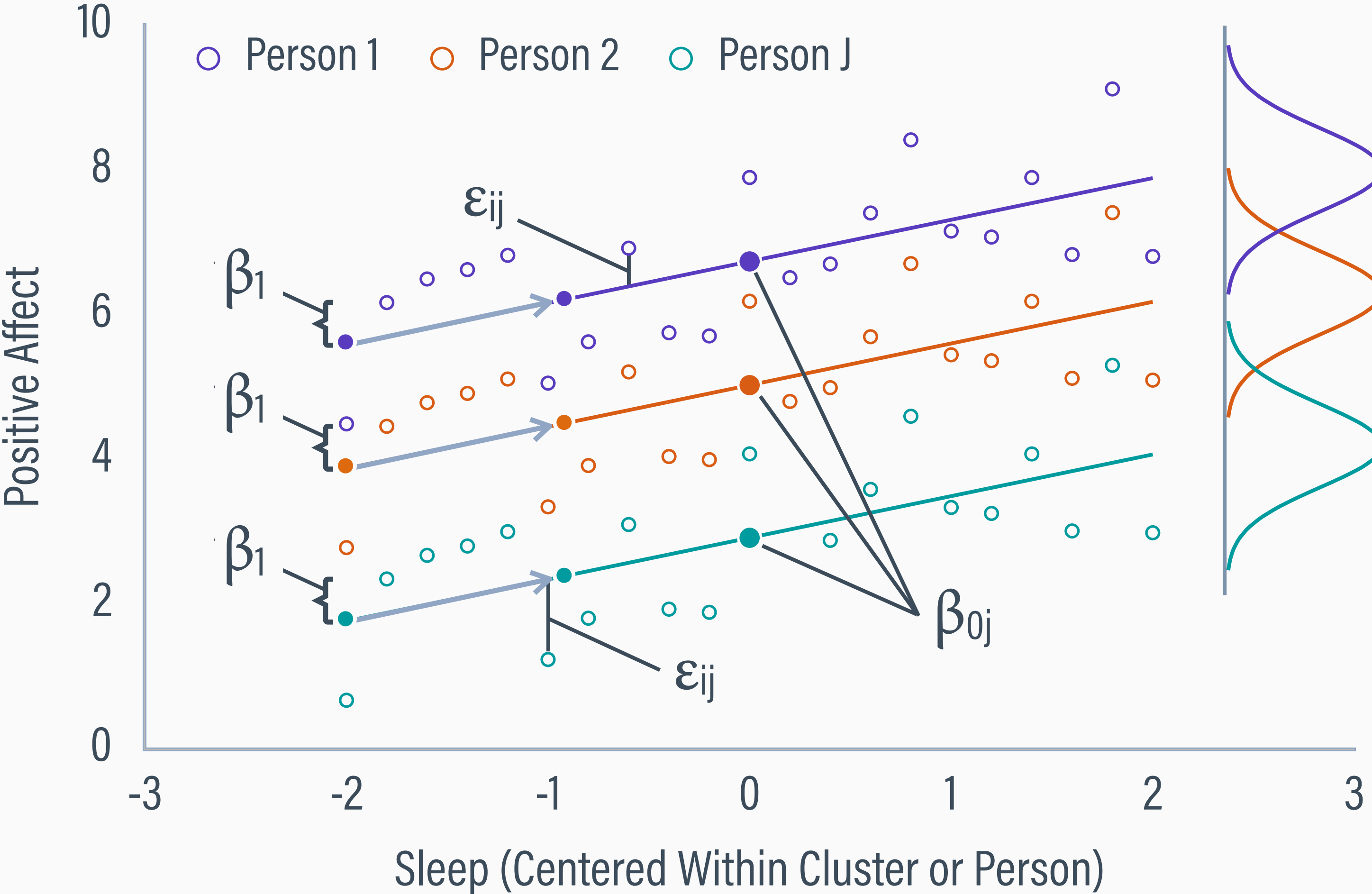
- Affect observation i for person j is the sum of a person's level-2 affect mean (β_{0j}), a fixed effect due to within-person sleep variation (β_1), and a within-person residual (ε_{ij})

$$p_{\text{affect}_{ij}} = \beta_{0j} + \beta_1(\text{sleep}_{ij}^w) + \varepsilon_{ij}$$

- Assumption: residuals are normal with constant variation across all days (level-1 units) and persons (level-2 units)

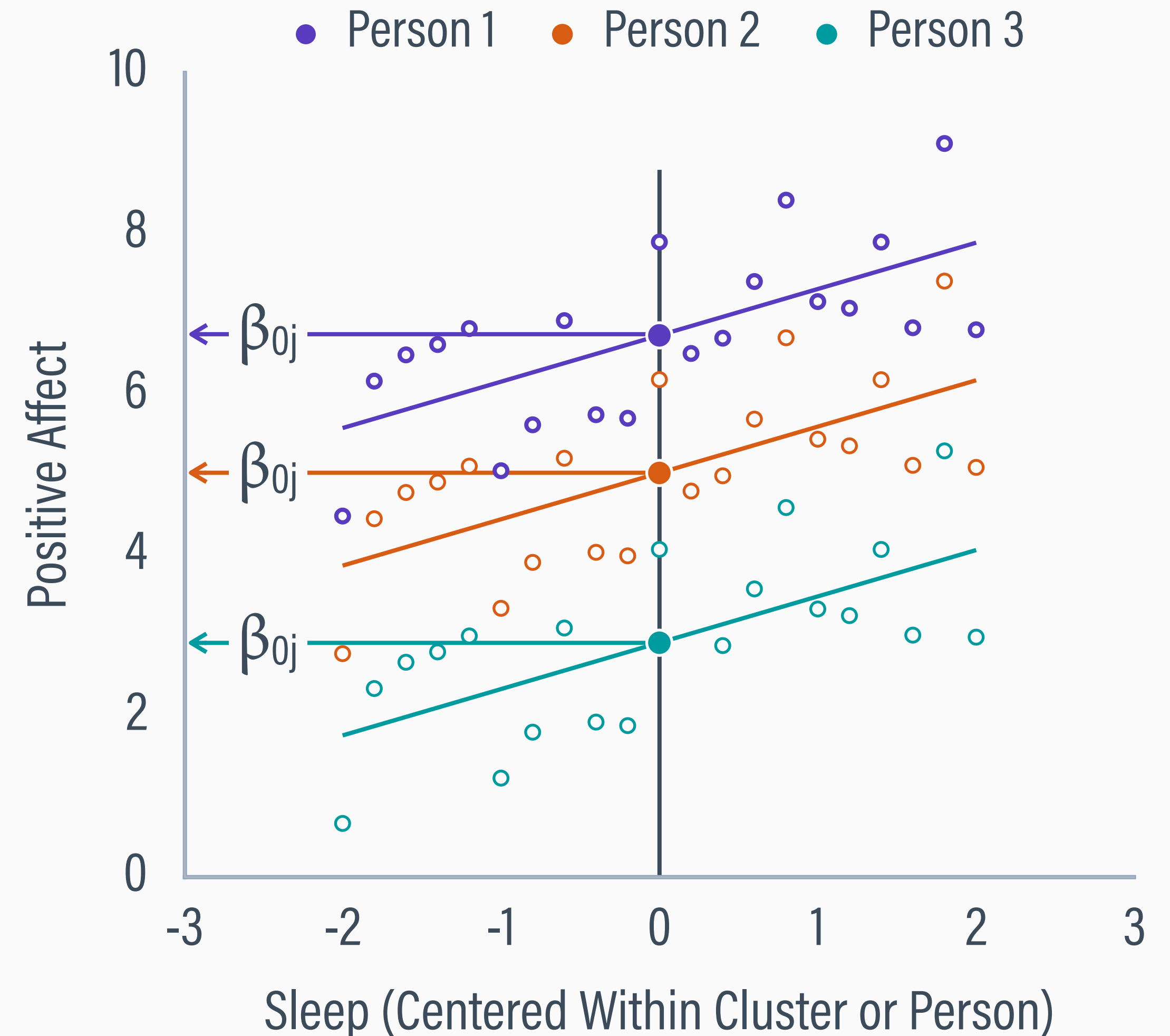
$$\varepsilon_{ij} \sim N(0, \sigma_{\varepsilon}^2)$$

LEVEL-1 MODEL GRAPHIC



RANDOM INTERCEPTS

- In regression, the intercept is the expected outcome score when the predictor equals zero
- Mean centering defines the intercept as the dependent variable's mean
- CWC centers within each cluster, so each β_{0j} is the outcome mean for cluster j



BETWEEN-CLUSTER (LEVEL-2) MODEL

- ◉ The affect mean for person j (β_{0j}) is the sum of the grand mean (γ_{00}) and a between-person residual (u_{0j})
- ◉ The level-1 slope (β_1) equals the grand mean slope (γ_{10})

$$\beta_{0j} = \gamma_{00} + u_{0j}$$

$$\beta_1 = \gamma_{10}$$

- ◉ Random intercept residuals are normal with constant variation across persons (level-2 units)

$$u_{0j} \sim N(0, \sigma_u^2)$$

DECODING THE SUBSCRIPTS

$$\begin{array}{l} \beta_{0j} = \gamma_{00} + u_{0j} \\ \beta_{1j} = \gamma_{10} \end{array}$$

The diagram illustrates the meaning of subscripts in the equations. A black line connects the second subscript '0' in the first equation to the text 'The second subscript numbers the coefficients on the right side starting at 0 (the intercept)'. An orange line connects the first subscript '0' in both equations to the text 'The first subscript tells which level-2 outcome variable these terms are attached to (gamma_00 and u_0j belong in beta_0j's equation, gamma_10 is attached to beta_1)'.

The second subscript numbers the coefficients on the right side starting at 0 (the intercept)

The first subscript tells which level-2 outcome variable these terms are attached to (γ_{00} and u_{0j} belong in β_{0j} 's equation, γ_{10} is attached to β_{1j})

COMBINED-MODEL NOTATION

- Level-specific regression equations can be reduced into a single combined-model equation (Raudenbush & Bryk, 2002)
- Replace the β_{0j} and β_1 terms in the level-1 equation with their level-2 equations

The diagram illustrates the process of combining level-1 and level-2 equations. At the top, two level-2 equations are shown: $\beta_1 = \gamma_{10}$ and $\beta_{0j} = \gamma_{00} + u_{0j}$. Arrows from these equations point to the level-1 equation $\text{pafect}_{ij} = \beta_{0j} + \beta_1(\text{sleep}_{ij}^w) + \varepsilon_{ij}$. Below this, the level-1 equation is rewritten by substituting the level-2 equations, resulting in the combined model equation: $\text{pafect}_{ij} = (\gamma_{00} + u_{0j}) + \gamma_{10}(\text{sleep}_{ij}^w) + \varepsilon_{ij}$, which is then simplified to $= \gamma_{00} + \gamma_{10}(\text{sleep}_{ij}^w) + u_{0j} + \varepsilon_{ij}$.

$$\begin{aligned}\beta_1 &= \gamma_{10} \\ \beta_{0j} &= \gamma_{00} + u_{0j} \\ \text{pafect}_{ij} &= \beta_{0j} + \beta_1(\text{sleep}_{ij}^w) + \varepsilon_{ij} \\ \text{pafect}_{ij} &= (\gamma_{00} + u_{0j}) + \gamma_{10}(\text{sleep}_{ij}^w) + \varepsilon_{ij} \\ &= \gamma_{00} + \gamma_{10}(\text{sleep}_{ij}^w) + u_{0j} + \varepsilon_{ij}\end{aligned}$$

COMMON NOTATIONAL SYSTEMS

Combined-model equation (Raudenbush & Bryk, 2002)

$$\text{pffect}_{ij} = \gamma_{00} + \gamma_{10}(\text{sleep}_{ij}^w) + u_{0j} + \varepsilon_{ij}$$

Standard(ish) regression notation (Scott, Shrout, & Weinberg, 2013)

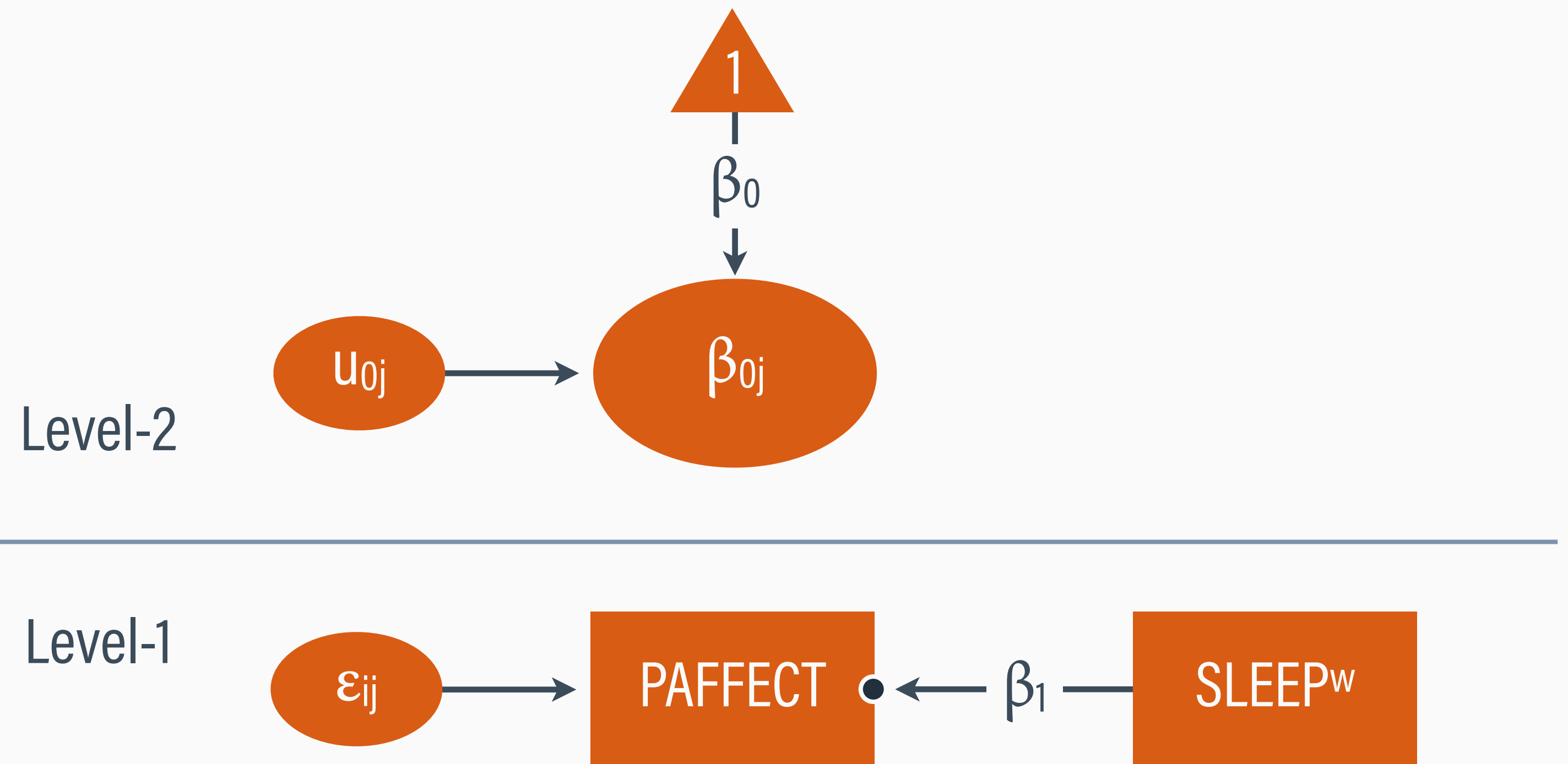
$$\text{pffect}_{ij} = \beta_0 + \beta_1(\text{sleep}_{ij}^w) + u_{0j} + \varepsilon_{ij}$$

Linear mixed model cluster-level matrix equation

$$\mathbf{y}_j = \mathbf{X}_j \boldsymbol{\beta} + \mathbf{Z}_j \mathbf{u}_j + \boldsymbol{\varepsilon}_j = \begin{pmatrix} \text{posaffect}_{1j} \\ \text{posaffect}_{2j} \\ \dots \\ \text{posaffect}_{nj} \end{pmatrix} = \begin{pmatrix} 1 & \text{sleep}_{1j}^w \\ 1 & \text{sleep}_{2j}^w \\ \dots & \dots \\ 1 & \text{sleep}_{nj}^w \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ \dots \\ 1 \end{pmatrix} u_{0j} + \begin{pmatrix} \varepsilon_{1j} \\ \varepsilon_{2j} \\ \dots \\ \varepsilon_{nj} \end{pmatrix}$$

PATH DIAGRAM

$$\text{paffected}_{ij} = \beta_0 + \beta_1(\text{sleep}_{ij}^w) + u_{0j} + \varepsilon_{ij}$$



• = random intercept (β_{0j})

BLIMP STUDIO SCRIPT 2.2

DATA: PainDiary.dat;

VARIABLES: Person Day PosAffect NegAffect Pain WorkGoal LifeGoal SleepQual Female Education
Employment MarStatus NumDiagnose ActivityLevel PainAccept Catastrophize Stress Anxiety;

CLUSTERID: Person;

CENTER: groupmean = SleepQual; # cwc with level-2 latent cluster means

MODEL: PosAffect ~ intercept SleepQual | intercept;

BURN: 10000;


ITER: 10000;

SEED: 90291;

RBLIMP SCRIPT 2 (MODEL 2)

```
model2 <- rblimp(  
  data = PainDiary,  
  clusterid = 'Person',  
  center = 'groupmean = SleepQual',  
  model = 'PosAffect ~ intercept SleepQual | intercept',  
  seed = 90291,  
  burn = 10000,  
  iter = 10000)  
output(model2)
```

BLIMP OUTPUT

 = level-2 estimate

 = level-1 estimate

Outcome Variable: PosAffect

Group Mean Centered: SleepQual

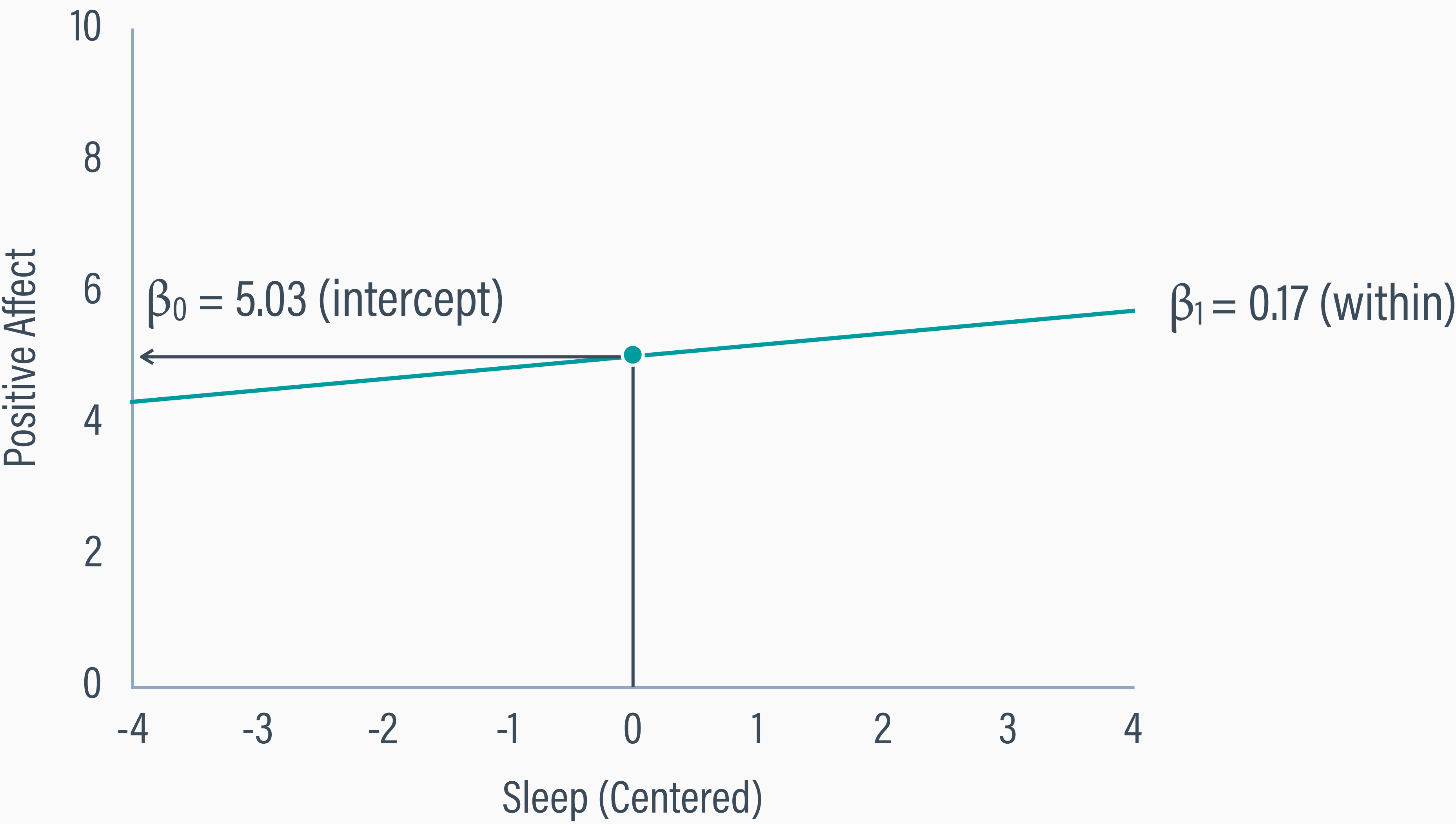
Parameters	Estimate	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff

Variances:							
L2 : Var(Intercept)	2.498	0.330	1.959	3.250	---	---	14878.523
Residual Var.	1.306	0.037	1.236	1.381	---	---	18435.058
Coefficients:							
Intercept	5.027	0.145	4.727	5.313	1197.971	0.000	247.755
SleepQual	0.173	0.013	0.148	0.199	173.667	0.000	18290.653
Standard Deviations:							
L2 : SD(Intercept)	1.580	0.103	1.399	1.802	---	---	8518.049
Residual SD	1.143	0.016	1.112	1.175	---	---	9208.015
Standardized Coefficients:							
SleepQual	0.152	0.013	0.127	0.178	136.430	0.000	17563.197
Proportion Variance Explained							
by Coefficients	0.023	0.004	0.016	0.032	---	---	17699.727
by Level-2 Random Intercepts	0.641	0.030	0.582	0.701	---	---	15523.864
by Level-1 Residual Variation	0.335	0.028	0.280	0.391	---	---	15314.734

FIXED EFFECT INTERPRETATIONS

- $\beta_0 = 5.03$ is the positive affect grand mean (because both predictors are centered)
- $\beta_1 = 0.17$ is the expected affect difference between two daily sleep scores from the same person that differ by one point

LEVEL-1 REGRESSION

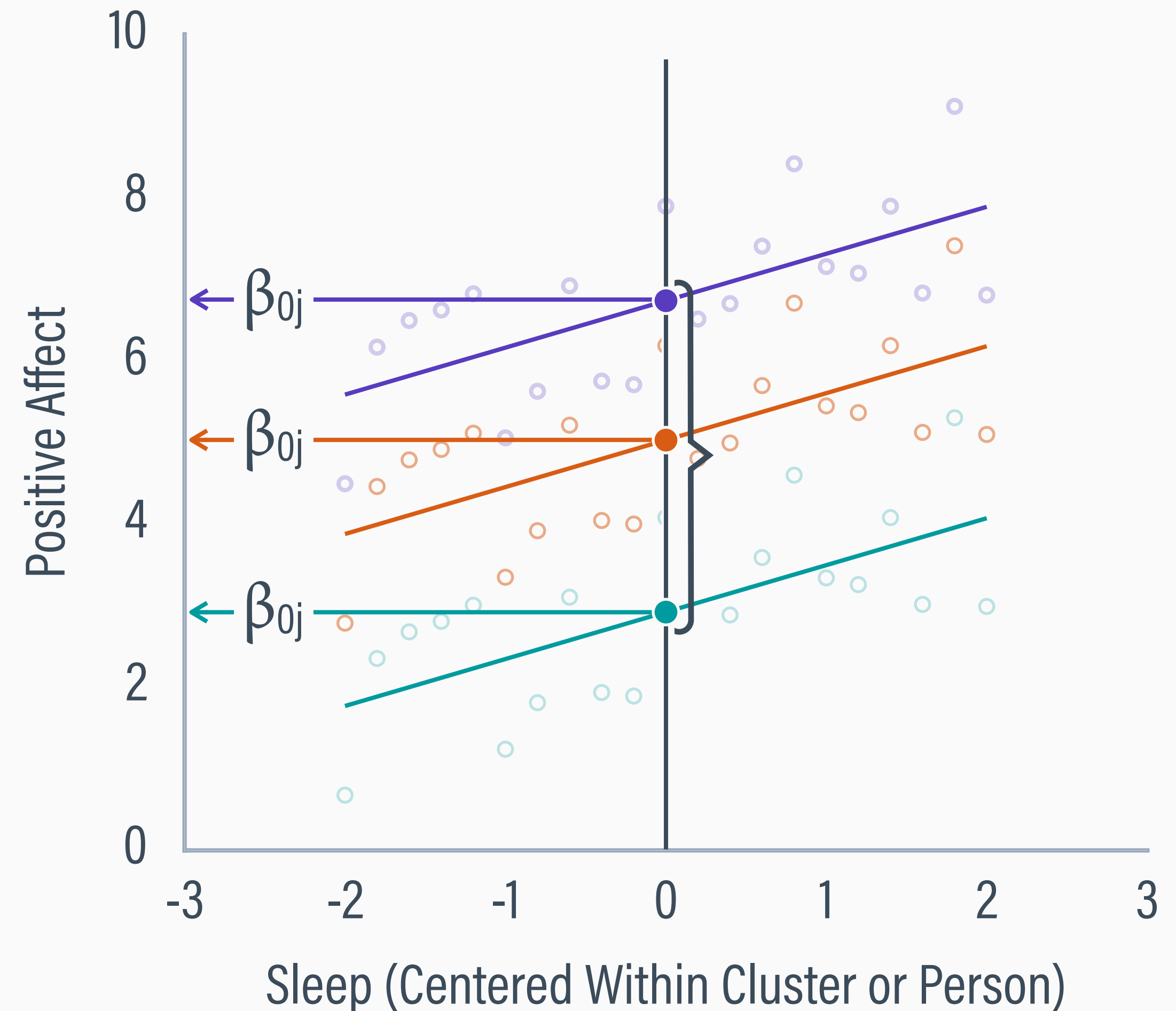


RANDOM EFFECT INTERPRETATIONS

- $u_{0j} = \beta_{0j} - \beta_0$
- $\text{var}(u_{0j}) = 2.50$ is the average squared distance between the level-2 affect means and the grand mean
- $\text{sd}(u_{0j}) = 1.58$ is the average distance between the level-2 affect means and the grand mean
- $\varepsilon_{ij} = \text{p affect}_{ij} - (\beta_{0j} + \beta_1(\text{sleep}_{ij}^w))$
- $\text{var}(\varepsilon_{ij}) = 1.31$ is the average squared distance between the level-1 affect observations and their predicted values
- $\text{sd}(\varepsilon_{ij}) = 1.14$ is the average distance between the level-1 affect observations and their predicted values

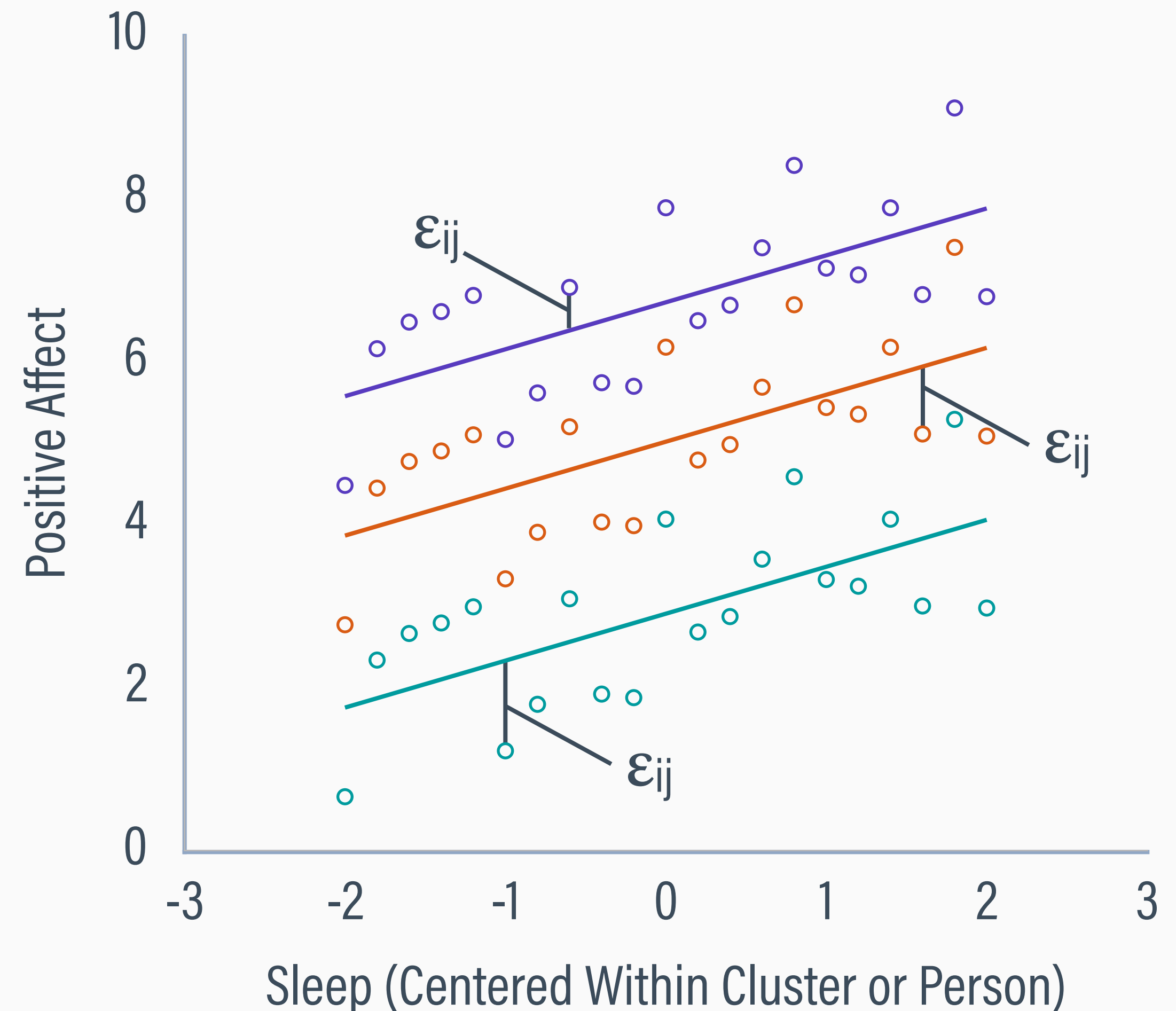
BETWEEN-CLUSTER VARIANCE

- The between-cluster variance $\text{var}(u_{0j})$ quantifies differences among person-specific means
- Visually, the random intercept variance is the vertical separation of the regression lines at sleep = 0



WITHIN-CLUSTER VARIANCE

- The within-cluster variance quantifies distances from a person's scores to their own regression
- Visually, the variance is the vertical separation of observations from the person-specific regression lines



MODEL COMPARISON

Parameter	Empty Model	Within Only	Between Only	Within + Between	Smushed (B = W)
Fixed intercept	5.03	5.02	5.05	5.03	5.04
Sleep (within-person)	--	0.17	--	0.17	0.18
Sleep (between-person)	--	--	0.61	0.61	0.18
Residual intercept variance	2.53	2.53	1.81	1.84	2.17
Residual within-person variance	1.40	1.31	1.40	1.31	1.31



The random intercept variance (variation among person means) did not change after adding a pure within-cluster (level-1) predictor, but the within-cluster variation was reduced. In small groups of two or three, discuss why you think these sources of variation behaved the way they did.

ANALYSIS EXAMPLES

1

Estimate Intraclass Correlations

2

Within-Cluster (Level-1) Regression Only

3

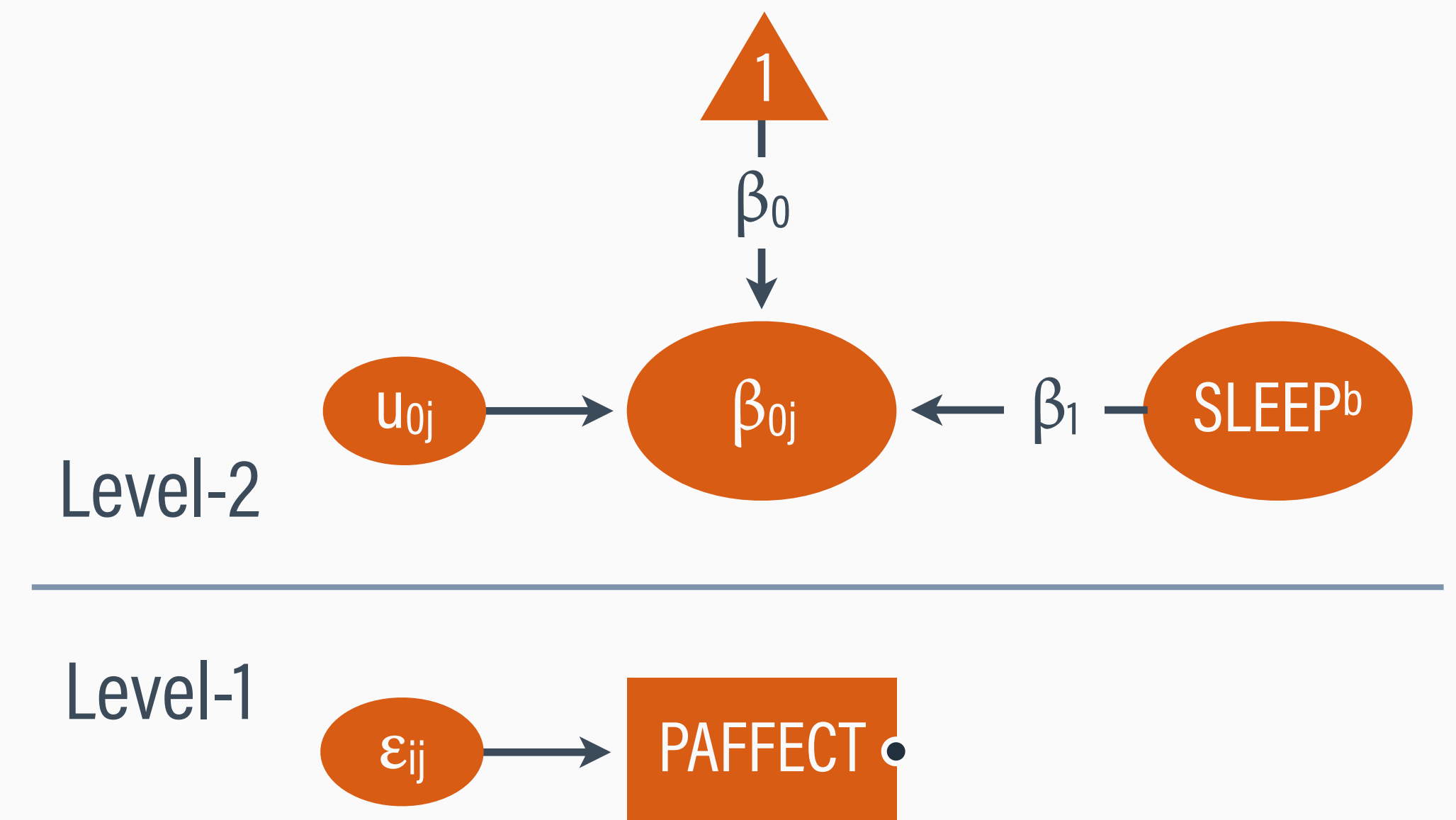
Between-Cluster (Level-2) Regression Only

4

Within-Cluster (Level-1) and Between-Cluster (Level-2) Regression

ANALYSIS OVERVIEW

- The between-cluster association captures how a person's average sleep quality relates to their average positive affect
- This reflects stable between-person differences rather than day-to-day fluctuations
- Within-person fluctuations are unmodeled



• = random intercept (β_{0j})

WITHIN-CLUSTER (LEVEL-1) MODEL

- Affect observation i for person j is the sum of their own level-2 affect mean (β_{0j}) and a within-person residual (ε_{ij})

$$p_{\text{affect}_{ij}} = \beta_{0j} + \varepsilon_{ij}$$

- Assumption: residuals are normal with constant variation across all days (level-1 units) and persons (level-2 units)

$$\varepsilon_{ij} \sim N(0, \sigma_{\varepsilon}^2)$$

BETWEEN-CLUSTER (LEVEL-2) MODEL

- The affect mean for person j (β_{0j}) is the sum of the grand mean (γ_{00}), a fixed effect due to person-average sleep (γ_{01}), and a between-person residual (u_{0j})

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{sleep}_j^b) + u_{0j}$$

- Random intercept residuals are normal with constant variation across persons (level-2 units)

$$u_{0j} \sim N(0, \sigma_u^2)$$

COMBINED-MODEL NOTATION

- Level-specific regression equations can be reduced into a single combined-model equation (Raudenbush & Bryk, 2002)
- Replace the β_{0j} and β_1 terms in the level-1 equation with their level-2 equations

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{sleep}_j^b) + u_{0j}$$



$$\text{paffect}_{ij} = \beta_{0j} + \varepsilon_{ij}$$



$$\text{paffect}_{ij} = \gamma_{00} + \gamma_{01}(\text{sleep}_j^b) + u_{0j} + \varepsilon_{ij}$$

COMMON NOTATIONAL SYSTEMS

Combined-model equation (Raudenbush & Bryk, 2002)

$$\text{pffect}_{ij} = \gamma_{00} + \gamma_{01}(\text{sleep}_j^b) + u_{0j} + \varepsilon_{ij}$$

Standard(ish) regression notation (Scott, Shrout, & Weinberg, 2013)

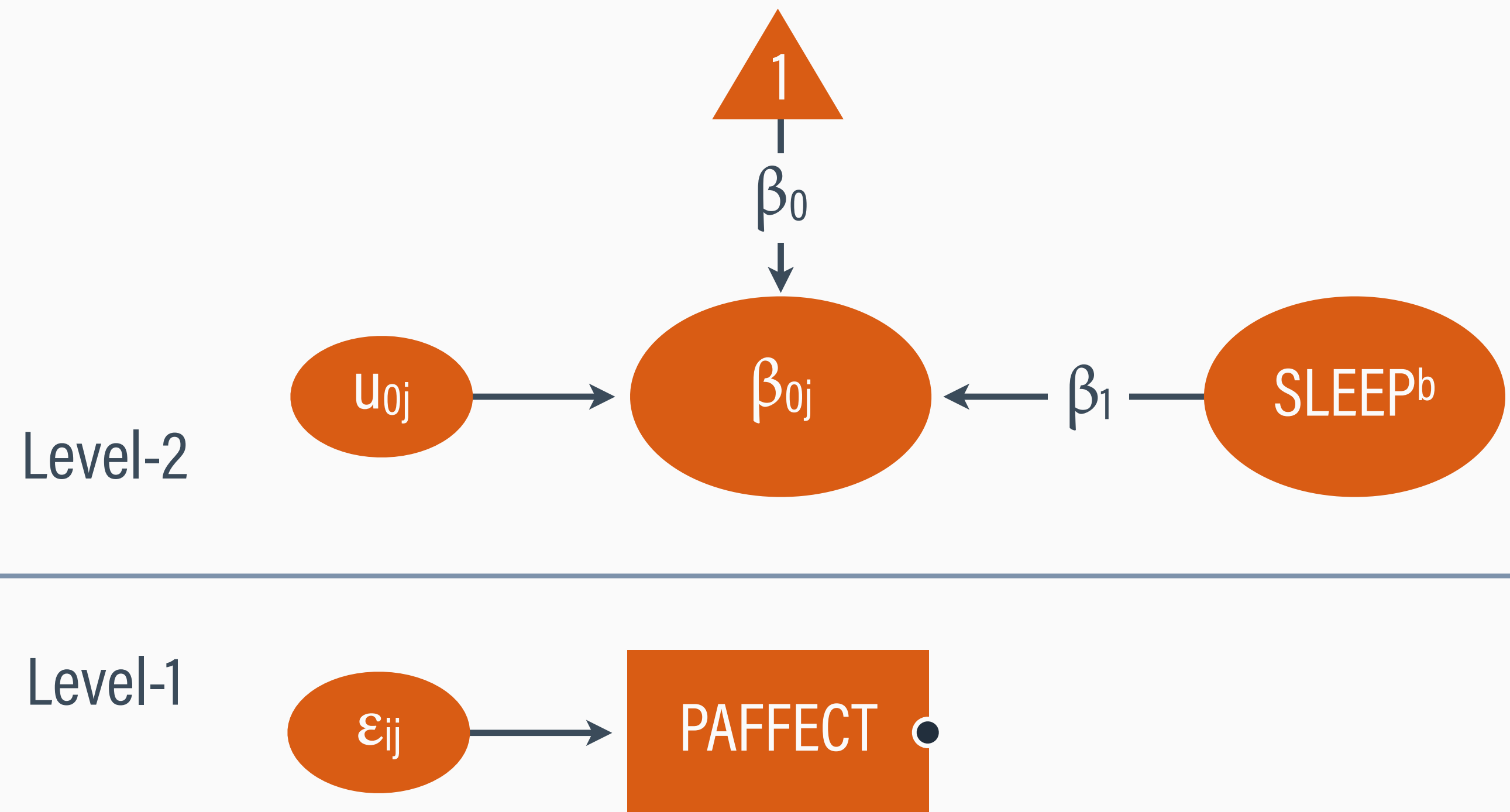
$$\text{pffect}_{ij} = \beta_0 + \beta_1(\text{sleep}_j^b) + u_{0j} + \varepsilon_{ij}$$

Linear mixed model cluster-level matrix equation

$$\mathbf{y}_j = \mathbf{X}_j \boldsymbol{\beta} + \mathbf{Z}_j u_j + \boldsymbol{\varepsilon}_j = \begin{pmatrix} \text{posaffect}_{1j} \\ \text{posaffect}_{2j} \\ \dots \\ \text{posaffect}_{nj} \end{pmatrix} = \begin{pmatrix} 1 & \text{sleep}_{1j}^b \\ 1 & \text{sleep}_{2j}^b \\ \dots & \dots \\ 1 & \text{sleep}_{nj}^b \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ \dots \\ 1 \end{pmatrix} u_{0j} + \begin{pmatrix} \varepsilon_{1j} \\ \varepsilon_{2j} \\ \dots \\ \varepsilon_{nj} \end{pmatrix}$$

PATH DIAGRAM

$$\text{pffect}_{ij} = \beta_0 + \beta_1(\text{sleep}_j^b) + u_{0j} + \varepsilon_{ij}$$



• = random intercept (β_{0j})



This model uses a person's average sleep to predict their average affect. Using the empty model as a starting point, in small groups of two or three, make predictions about what will happen to the between-person (mean-level) and within-person (daily-level) variation. Will each value increase, decrease, or stay the same?

BLIMP STUDIO SCRIPT 2.3

DATA: PainDiary.dat;

VARIABLES: Person Day PosAffect NegAffect Pain WorkGoal LifeGoal SleepQual Female Education
Employment MarStatus NumDiagnose ActivityLevel PainAccept Catastrophize Stress Anxiety;

CLUSTERID: Person;

CENTER: grandmean = SleepQual.mean;

MODEL: PosAffect ~ intercept SleepQual.mean | intercept; # .mean invokes level-2 latent means

BURN: 10000;


ITERATIONS: 10000;

SEED: 90291;

RBLIMP SCRIPT 2 (MODEL 3)

```
model3 <- rblimp(  
  data = PainDiary,  
  clusterid = 'Person',  
  center = 'grandmean = SleepQual.mean',  
  model = 'PosAffect ~ intercept SleepQual.mean | intercept',  
  seed = 90291,  
  burn = 10000,  
  iter = 10000)  
output(model3)
```

BLIMP OUTPUT

 = level-2 estimate

 = level-1 estimate

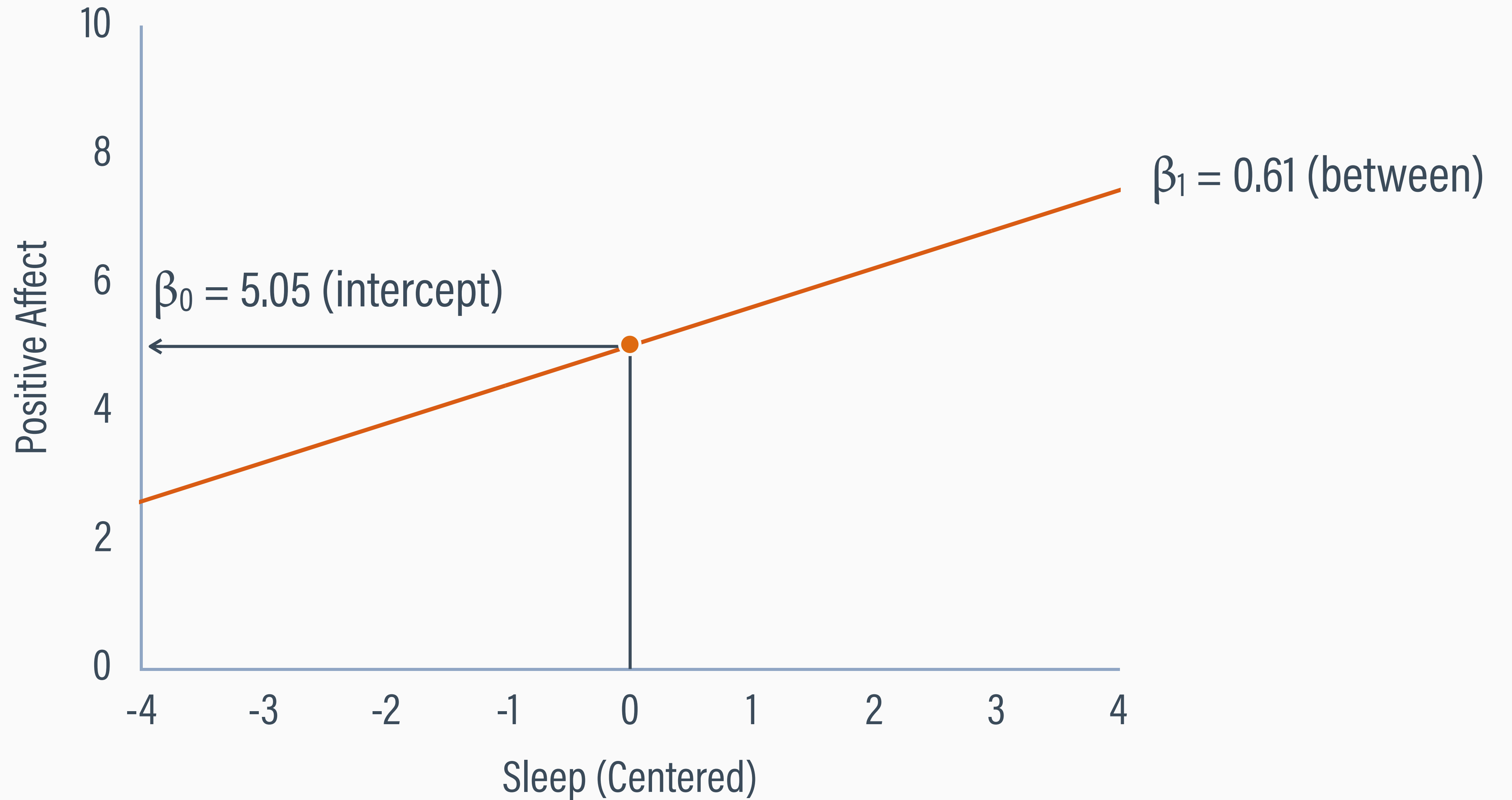
Outcome Variable: PosAffect

Parameters	Estimate	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
Variances:							
L2 : Var(Intercept)	1.809	0.250	1.404	2.386	---	---	4422.013
Residual Var.	1.395	0.040	1.321	1.477	---	---	8961.693
Coefficients:							
Intercept	5.045	0.135	4.781	5.295	1402.647	0.000	139.527
SleepQual.mean[Person]	0.606	0.088	0.428	0.772	46.920	0.000	114.531
Standard Deviations:							
L2 : SD(Intercept)	1.345	0.092	1.185	1.545	---	---	4360.602
Residual SD	1.181	0.017	1.149	1.215	---	---	8956.904
Standardized Coefficients:							
SleepQual.mean[Person]	0.419	0.052	0.304	0.508	62.779	0.000	119.344
Proportion Variance Explained							
by Coefficients	0.175	0.043	0.093	0.258	---	---	119.467
by Level-2 Random Intercepts	0.466	0.042	0.388	0.552	---	---	279.468
by Level-1 Residual Variation	0.358	0.028	0.304	0.414	---	---	426.869

FIXED EFFECT INTERPRETATIONS

- $\beta_0 = 5.05$ is the positive affect grand mean (because both predictors are centered)
- $\beta_1 = 0.61$ is the expected affect difference between two individuals whose average sleep ratings differ by one point

LEVEL-2 REGRESSION



RANDOM EFFECT INTERPRETATIONS

- $u_{0j} = \beta_{0j} - (\beta_0 + \beta_2(\text{sleep}_j^b))$
- $\text{var}(u_{0j}) = 1.81$ is the average squared distance between the level-2 affect means and their predicted values
- $\text{sd}(u_{0j}) = 1.35$ is the average distance between the level-2 affect means and their predicted values
- $\varepsilon_{ij} = \text{p affect}_{ij} - \beta_{0j}$
- $\text{var}(\varepsilon_{ij}) = 1.40$ is the average squared distance between the level-1 affect observations and their level-2 means
- $\text{sd}(\varepsilon_{ij}) = 1.18$ is the average distance between the level-1 affect observations and their level-2 means

MODEL COMPARISON

Parameter	Empty Model	Within Only	Between Only	Within + Between	Smushed (B = W)
Fixed intercept	5.03	5.02	5.05	5.03	5.04
Sleep (within-person)	--	0.17	--	0.17	0.18
Sleep (between-person)	--	--	0.61	0.61	0.18
Residual intercept variance	2.53	2.53	1.81	1.84	2.17
Residual within-person variance	1.40	1.31	1.40	1.31	1.31

ANALYSIS EXAMPLES

1

Estimate Intraclass Correlations

2

Within-Cluster (Level-1) Regression Only

3

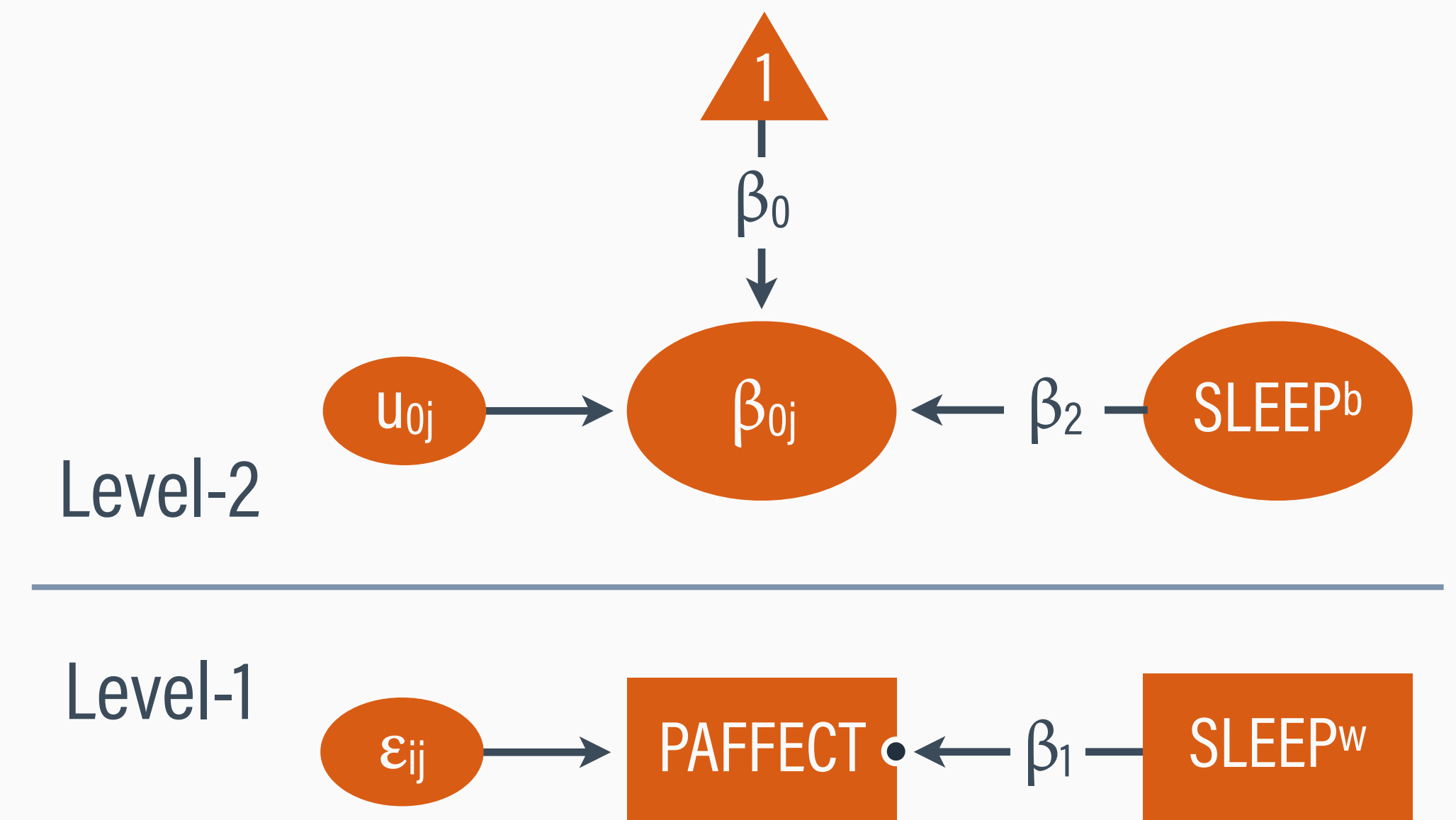
Between-Cluster (Level-2) Regression Only

4

Within-Cluster (Level-1) and Between-Cluster (Level-2) Regression

ANALYSIS OVERVIEW

- The within-cluster association captures how deviations from a person's average sleep rating relates to their daily positive affect
- The between-cluster association captures how a person's average sleep quality relates to their average positive affect



• = random intercept (β_{0j})

WITHIN-CLUSTER (LEVEL-1) MODEL

- Affect observation i for person j is the sum of a person's level-2 affect mean (β_{0j}), a fixed effect due to within-person sleep variation (β_1), and a within-person residual (ε_{ij})

$$p_{\text{affect}_{ij}} = \beta_{0j} + \beta_1(\text{sleep}_{ij}^W) + \varepsilon_{ij}$$

- Assumption: residuals are normal with constant variation across all days (level-1 units) and persons (level-2 units)

$$\varepsilon_{ij} \sim N(0, \sigma_\varepsilon^2)$$

BETWEEN-CLUSTER (LEVEL-2) MODEL

- The affect mean for person j (β_{0j}) is the sum of the grand mean (γ_{00}), a fixed effect due to person-average sleep (γ_{01}), and a between-person residual (u_{0j})

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{sleep}_j^b) + u_{0j}$$

$$\beta_1 = \gamma_{10}$$

- Random intercept residuals are normal with constant variation across persons (level-2 units)

$$u_{0j} \sim N(0, \sigma_u^2)$$

DECODING THE SUBSCRIPTS

The diagram illustrates the relationship between the coefficients in the equations for β_{0j} and β_{1j} . The equation $\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{sleep}_j) + u_{0j}$ is shown with a horizontal line above it. The equation $\beta_{1j} = \gamma_{10}$ is shown below it. Vertical lines connect the subscripts to their respective terms: a line from the first subscript '0' of β_{0j} to the first subscript '0' of γ_{00} and the first subscript '0' of u_{0j} ; a line from the second subscript 'j' of β_{0j} to the second subscript 'j' of sleep_j ; a line from the first subscript '1' of β_{1j} to the first subscript '1' of γ_{10} ; and a line from the second subscript 'j' of β_{1j} to the second subscript 'j' of γ_{10} . A horizontal line connects the bottom of these four vertical lines.

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{sleep}_j) + u_{0j}$$
$$\beta_{1j} = \gamma_{10}$$

The second subscript numbers the coefficients on the right side starting at 0 (the intercept)

The first subscript tells which level-2 outcome variable these terms are attached to (γ_{00} and u_{0j} belong in β_{0j} 's equation, γ_{10} is attached to β_{1j})

COMBINED-MODEL NOTATION

- Level-specific regression equations can be reduced into a single combined-model equation (Raudenbush & Bryk, 2002)
- Replace the β_{0j} and β_1 terms in the level-1 equation with their level-2 equations
- The γ s are called fixed effects and u_{0j} is a random effect

$$\begin{aligned} \beta_{0j} &= \gamma_{00} + \gamma_{01}(\text{sleep}_j^b) + u_{0j} \\ &\quad \downarrow \quad \beta_1 = \gamma_{10} \quad \downarrow \\ \text{paffected}_{ij} &= \beta_{0j} + \beta_1(\text{sleep}_{ij}^w) + \varepsilon_{ij} \\ &\quad \downarrow \\ \text{paffected}_{ij} &= \gamma_{00} + \gamma_{01}(\text{sleep}_j^b) \\ &\quad + \gamma_{10}(\text{sleep}_{ij}^w) + u_{0j} + \varepsilon_{ij} \end{aligned}$$

COMMON NOTATIONAL SYSTEMS

Combined-model equation (Raudenbush & Bryk, 2002)

$$\text{pffect}_{ij} = \gamma_{00} + \gamma_{10}(\text{sleep}_{ij}^w) + \gamma_{01}(\text{sleep}_j^b) + u_{0j} + \varepsilon_{ij}$$

Standard(ish) regression notation (Scott, Shrout, & Weinberg, 2013)

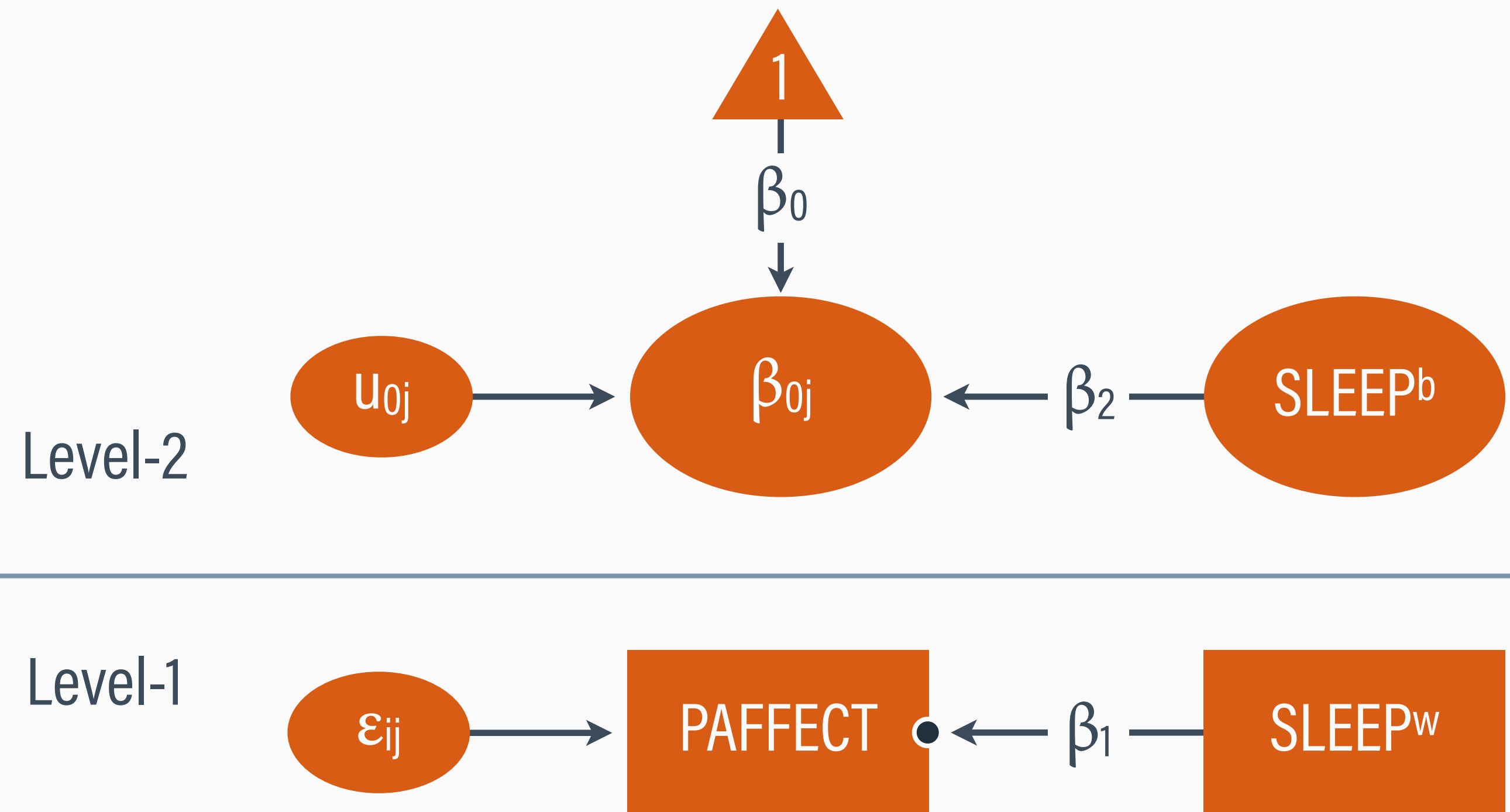
$$\text{pffect}_{ij} = \beta_0 + \beta_1(\text{sleep}_{ij}^w) + \beta_2(\text{sleep}_j^b) + u_{0j} + \varepsilon_{ij}$$

Linear mixed model cluster-level matrix equation

$$\mathbf{y}_j = \mathbf{X}_j \boldsymbol{\beta} + \mathbf{Z}_j u_j + \boldsymbol{\varepsilon}_j = \begin{pmatrix} \text{posaffect}_{1j} \\ \text{posaffect}_{2j} \\ \dots \\ \text{posaffect}_{nj} \end{pmatrix} = \begin{pmatrix} 1 & \text{sleep}_{1j}^w & \text{SLEEP}_{1j}^b \\ 1 & \text{sleep}_{2j}^w & \text{SLEEP}_{2j}^b \\ \dots & \dots & \dots \\ 1 & \text{sleep}_{nj}^w & \text{SLEEP}_{nj}^b \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ \dots \\ 1 \end{pmatrix} u_{0j} + \begin{pmatrix} \varepsilon_{1j} \\ \varepsilon_{2j} \\ \dots \\ \varepsilon_{nj} \end{pmatrix}$$

PATH DIAGRAM

$$\text{paffected}_{ij} = \beta_0 + \beta_1(\text{sleep}_{ij}^w) + \beta_2(\text{sleep}_j^b) + u_{0j} + \varepsilon_{ij}$$



• = random intercept (β_{0j})

BLIMP STUDIO SCRIPT 2.4

DATA: PainDiary.dat;

VARIABLES: Person Day PosAffect NegAffect Pain WorkGoal LifeGoal SleepQual Female Education
Employment MarStatus NumDiagnose ActivityLevel PainAccept Catastrophize Stress Anxiety;

CLUSTERID: Person;

CENTER:

grandmean = SleepQual.mean; # defines the fixed (average) intercept as the grand mean

groupmean = SleepQual; # cwc with level-2 latent group means

MODEL: PosAffect ~ intercept SleepQual SleepQual.mean | intercept; # .mean invokes latent means

BURN: 10000;


ITERATIONS: 10000;

SEED: 90291;

RBLIMP SCRIPT 2 (MODEL 4)

```
model4 <- rblimp(  
  data = PainDiary,  
  clusterid = 'Person',  
  center = 'grandmean = SleepQual.mean; groupmean = SleepQual',  
  model = 'PosAffect ~ intercept SleepQual SleepQual.mean | intercept',  
  seed = 90291,  
  burn = 10000,  
  iter = 10000)  
output(model4)
```

BLIMP OUTPUT

 = level-2 estimate

 = level-1 estimate

Outcome Variable: PosAffect

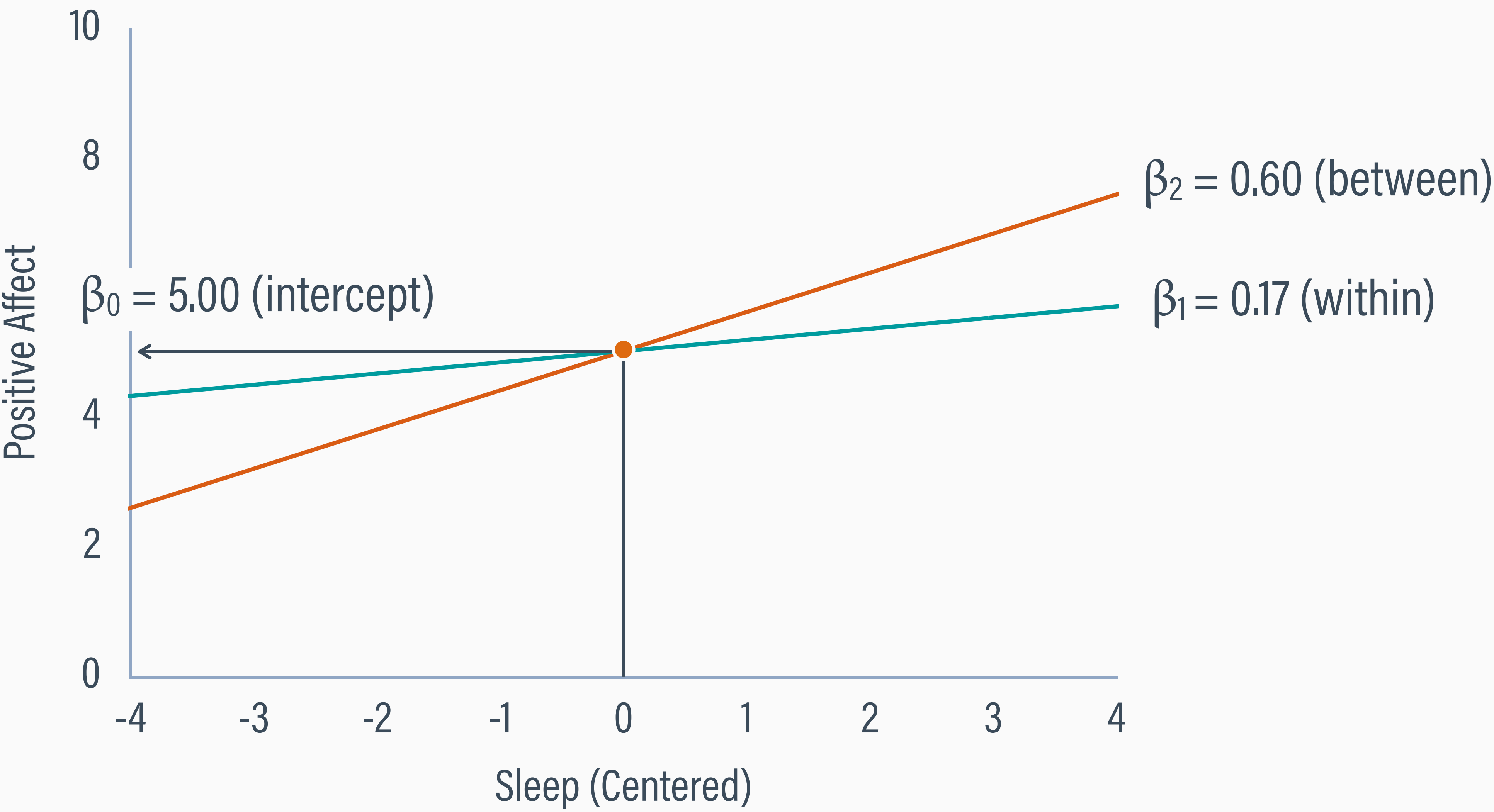
Group Mean Centered: SleepQual

Parameters	Estimate	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
Variances:							
L2 : Var(Intercept)	1.839	0.246	1.436	2.399	---	---	5051.168
Residual Var.	1.307	0.037	1.238	1.380	---	---	8227.580
Coefficients:							
Intercept	4.997	0.133	4.761	5.277	1423.604	0.000	154.295
SleepQual	0.173	0.013	0.147	0.199	174.595	0.000	9302.345
SleepQual.mean[Person]	0.598	0.094	0.416	0.788	40.936	0.000	161.345
Standard Deviations:							
L2 : SD(Intercept)	1.356	0.090	1.198	1.549	---	---	5041.908
Residual SD	1.143	0.016	1.113	1.175	---	---	8223.888
Standardized Coefficients:							
SleepQual	0.151	0.013	0.127	0.176	142.389	0.000	1996.211
SleepQual.mean[Person]	0.412	0.055	0.297	0.513	55.809	0.000	169.087
Proportion Variance Explained							
by Coefficients	0.193	0.044	0.112	0.286	---	---	175.664
by Level-2 Random Intercepts	0.471	0.042	0.392	0.555	---	---	414.230
by Level-1 Residual Variation	0.334	0.027	0.280	0.387	---	---	511.829

FIXED EFFECT INTERPRETATIONS

- $\beta_0 = 5.00$ is the positive affect grand mean (because both predictors are centered)
- $\beta_1 = 0.17$ is the expected affect difference between two daily sleep scores from the same person that differ by one point
- $\beta_2 = 0.60$ is the expected affect difference between two individuals whose average sleep ratings differ by one point

LEVEL-1 AND LEVEL-2 REGRESSIONS



RANDOM EFFECT INTERPRETATIONS

- $u_{0j} = \beta_{0j} - (\beta_0 + \beta_2(\text{sleep}_j^b))$
- $\text{var}(u_{0j}) = 1.84$ is the average squared distance between the level-2 affect means and their predicted values
- $\text{sd}(u_{0j}) = 1.36$ is the average distance between the level-2 affect means and their predicted values
- $\varepsilon_{ij} = \text{pffect}_{ij} - (\beta_{0j} + \beta_1(\text{sleep}_{ij}^w))$
- $\text{var}(\varepsilon_{ij}) = 1.31$ is the average squared distance between the level-1 affect observations and their predicted values
- $\text{sd}(\varepsilon_{ij}) = 1.14$ is the average distance between the level-1 affect observations and their predicted values

MODEL COMPARISON

Parameter	Empty Model	Within Only	Between Only	Within + Between	Smushed (B = W)
Fixed intercept	5.03	5.02	5.05	5.00	5.04
Sleep (within-person)	--	0.17	--	0.17	0.18
Sleep (between-person)	--	--	0.61	0.60	0.18
Residual intercept variance	2.53	2.53	1.81	1.84	2.17
Residual within-person variance	1.40	1.31	1.40	1.31	1.31



The within-person slope of 0.17 did not change when moving from a model with just the level-1 predictor to a model that additionally used the person-specific sleep averages as a level-2 predictor. Similarly, the between-person slope was constant at 0.61. In small groups of two or three, discuss why you think the effect of sleep at one level did not depend on whether the model included sleep at the other level.

OUTLINE

1

Associations at Level-1 and Level-2

2

Review of Grand Mean Centering

3

Disaggregating a Level-1 Predictor by Centering Within Cluster

4

Analysis Examples

5

Using Raw Level-1 Predictors (Smushed Model)

6

Latent Variable Specification

SMUSHED LEVEL-1 MODEL

- Predictor variables can also be entered into a model without disaggregation (as raw scores or grand mean centered)

$$p_{\text{affect}_{ij}} = \beta_{0j} + \beta_1(\text{sleep}_{ij}) + \varepsilon_{ij}$$

- Sometimes called a “smushed model” because it combines the level-1 and level-2 associations into a single slope

EQUALITY OF SLOPES ASSUMPTION

- Using an intact level-1 predictor assumes that the effect of the predictor is the same at level-1 and level-2

$$\begin{aligned} \text{paffected}_{ij} &= \beta_0 + \beta_1(\text{sleep}_{ij}) + u_{0j} + \varepsilon_{ij} && \# \text{ smushed model} \\ &= \beta_0 + \beta_1(\text{sleep}_j^b + \text{sleep}_{ij}^w) + u_{0j} + \varepsilon_{ij} && \# \text{ disaggregate predictor} \\ &= \beta_0 + \beta_1(\text{sleep}_j^b) + \beta_1(\text{sleep}_{ij}^w) + u_{0j} + \varepsilon_{ij} && \# \text{ distribute slope} \end{aligned}$$

- A smushed model is the same as a disaggregated model where the level-1 and level-2 regression slopes are held equal

SMUSHED MODEL

DATA: PainDiary.dat;

VARIABLES: Person Day PosAffect NegAffect Pain WorkGoal LifeGoal SleepQual Female Education
Employment MarStatus NumDiagnose ActivityLevel PainAccept Catastrophize Stress Anxiety;

CLUSTERID: Person;

CENTER: grandmean = SleepQual; # grand mean centering leaves predictor intact

MODEL: PosAffect ~ intercept SleepQual | intercept;

BURN: 10000;

ITERATIONS: 10000;

SEED: 90291;

DISAGGREGATED MODEL WITH CONSTRAINT

DATA: PainDiary.dat;

VARIABLES: Person Day PosAffect NegAffect Pain WorkGoal LifeGoal SleepQual Female Education
Employment MarStatus NumDiagnose ActivityLevel PainAccept Catastrophize Stress Anxiety;

CLUSTERID: Person;

CENTER:

groupmean = SleepQual;

grandmean = SleepQual.mean;

MODEL: PosAffect ~ intercept SleepQual@beta SleepQual.mean@beta | intercept;

BURN: 10000; # @beta is a label that forces slopes to be equal

ITERATIONS: 10000;

SEED: 90291;

MODEL COMPARISON

Smushed Model

Outcome Variable: PosAffect

Parameters	Estimate	StdDev

Variances:		
L2 : Var(Intercept)	2.168	0.286
Residual Var.	1.307	0.037
Coefficients:		
Intercept	5.027	0.133
SleepQual	0.181	0.013
...		
Proportion Variance Explained		
by Coefficients	0.044	0.007
by Level-2 Random Intercepts	0.597	0.032
by Level-1 Residual Variation	0.360	0.028

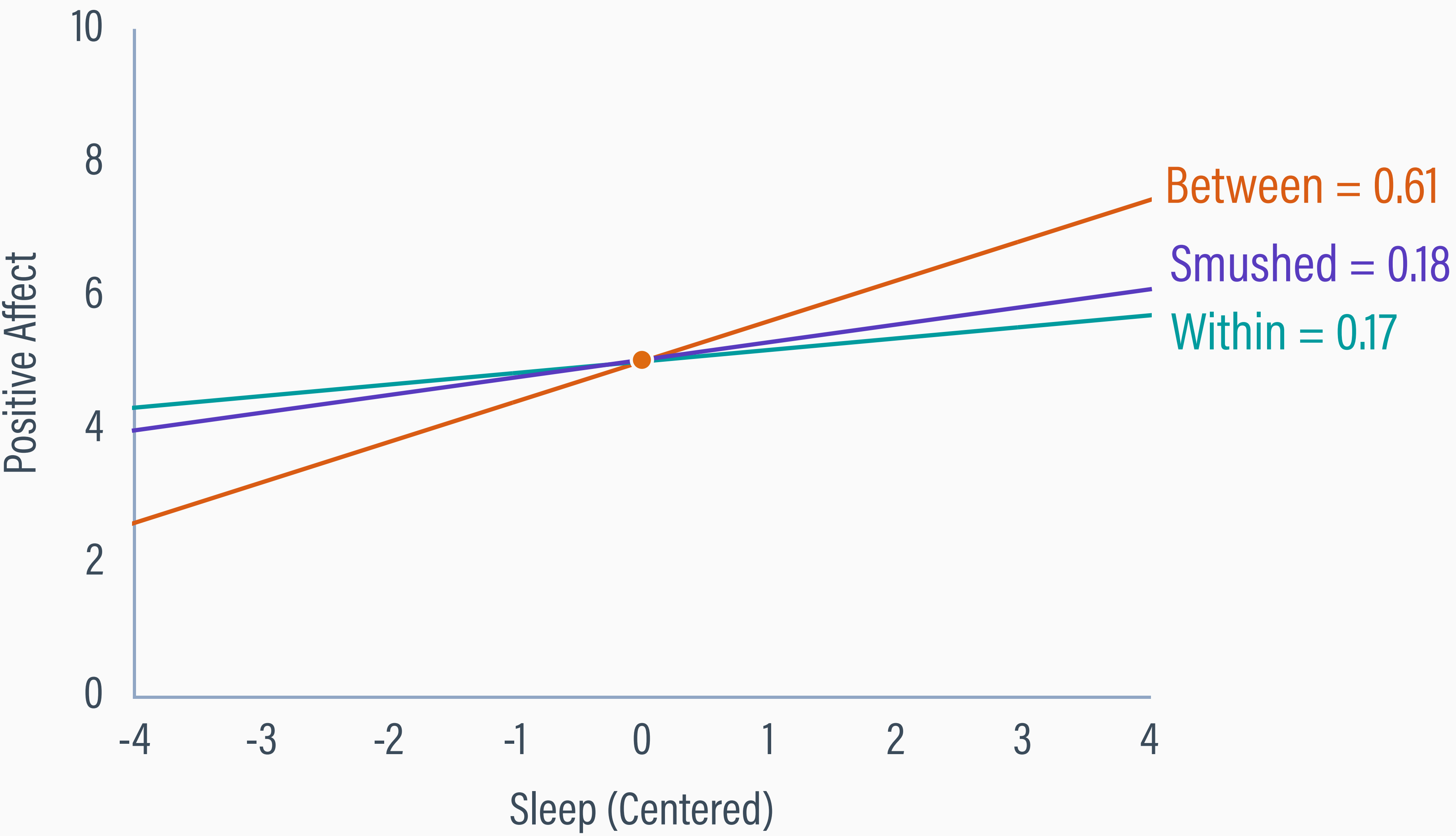
Disaggregated Model With Constrained Slopes

Outcome Variable: PosAffect

Parameters	Estimate	StdDev

Variances:		
L2 : Var(Intercept)	2.169	0.287
Residual Var.	1.306	0.036
Coefficients:		
Intercept	5.040	0.136
SleepQual	0.180	0.013
SleepQual.mean[Person]	0.180	0.013
...		
Proportion Variance Explained		
by Coefficients	0.043	0.007
by Level-2 Random Intercepts	0.597	0.032
by Level-1 Residual Variation	0.359	0.028

SLOPE COMPARISON



SMUSHED MODEL SHORTCOMINGS

- Using raw level-1 predictors combines the level-1 and level-2 associations into a single slope (appropriate only if the slopes are truly equal)
- When level-1 and level-2 slopes differ, failing to disaggregate biases the estimated effect (the combined slope is neither the within- nor the between-cluster effect)
- Failing to disaggregate can also distort the effects of other predictors in the model

WHEN TO USE A SMUSHED MODEL

- The literature broadly supports disaggregation
- The main use of the smushed model is when ICCs are very low (e.g., $< .03$ to $.05$), in which case the predictor is mostly a “pure” within-cluster variable (no threat of conflation bias)
- Many predictors have near-zero ICCs (e.g., demographic variables, stimuli in a lab study or experiment), so it is important to estimate ICCs prior to model fitting

OUTLINE

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Associations at Level-1 and Level-2

2

Review of Grand Mean Centering

3

Disaggregating a Level-1 Predictor by Centering Within Cluster

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Analysis Examples

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Using Raw Level-1 Predictors (Smushed Model)

6

Latent Variable Specification

BLIMP STUDIO SCRIPT 2.5

DATA: PainDiary.dat;

VARIABLES: Person Day PosAffect NegAffect Pain WorkGoal LifeGoal SleepQual Female Education
Employment MarStatus NumDiagnose ActivityLevel PainAccept Catastrophize Stress Anxiety;

CLUSTERID: Person;

LATENT: Person = beta0j; # define level-2 intercept latent variable

CENTER: groupmean = SleepQual;

MODEL:

beta0j ~ intercept; # level-2: $\beta_{0j} = \gamma_{00} + u_{0j}$

PosAffect ~ intercept@beta0j SleepQual; # level-1: $p_{affect_{ij}} = \beta_{0j} + \beta_1(\text{sleep}_{ij}^w) + \varepsilon_{ij}$

BURN: 10000;


ITER: 10000;

SEED: 90291;

RBLIMP SCRIPT 2 (MODEL 5)

```
within_lat <- rblimp(  
  data = PainDiary,  
  clusterid = 'Person',  
  latent = 'Person = beta0j',  
  center = 'groupmean = SleepQual',  
  model = '  
    beta0j ~ intercept;  
    PosAffect ~ intercept@beta0j SleepQual',  
  seed = 90291,  
  burn = 10000,  
  iter = 10000)  
output(within_lat)
```

LEVEL-1 OUTPUT

 = level-2 estimate


 = level-1 estimate

Outcome Variable: PosAffect

Group Mean Centered: SleepQual

Parameters	Estimate	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
Variances:							
Residual Var.	1.306	0.037	1.237	1.381	---	---	16680.158
Coefficients:							
beta0j	@ 1.000	---	---	---	---	---	---
SleepQual	0.173	0.013	0.148	0.199	175.710	0.000	18005.682
Standardized Coefficients:							
SleepQual	0.254	0.018	0.218	0.289	193.670	0.000	17781.177
Proportion Variance Explained							
by Coefficients	0.065	0.009	0.047	0.084	---	---	17761.837
by Residual Variation	0.935	0.009	0.916	0.953	---	---	17761.837

LEVEL-2 OUTPUT

 = level-2 estimate

 = level-1 estimate

Latent Variable: β_{0j}

Parameters	Estimate	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
Variances:							
Residual Var.	2.502	0.330	1.961	3.245	---	---	17537.812
Coefficients:							
Intercept	5.032	0.141	4.752	5.305	1270.084	0.000	18848.760
Proportion Variance Explained							
by Coefficients	0.000	0.000	0.000	0.000	---	---	nan
by Residual Variation	1.000	0.000	1.000	1.000	---	---	nan

BLIMP STUDIO SCRIPT 2.6

DATA: PainDiary.dat;

VARIABLES: Person Day PosAffect NegAffect Pain WorkGoal LifeGoal SleepQual Female Education
Employment MarStatus NumDiagnose ActivityLevel PainAccept Catastrophize Stress Anxiety;

CLUSTERID: Person;

LATENT: Person = beta0j; # define level-2 intercept latent variable

CENTER: groupmean = SleepQual; grandmean = SleepQual.mean;

MODEL:

beta0j ~ intercept SleepQual.mean; # level-2: $\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{sleep}_j^b) + u_{0j}$

PosAffect ~ intercept@beta0j SleepQual; # level-1: $\text{pffect}_{ij} = \beta_{0j} + \beta_1(\text{sleep}_{ij}^w) + \varepsilon_{ij}$

BURN: 10000;


ITER: 10000;

SEED: 90291;

RBLIMP SCRIPT 2 (MODEL 6)

```
disagg_lat <- rblimp(  
  data = PainDiary,  
  clusterid = 'Person',  
  latent = 'Person = beta0j',  
  center = 'groupmean = SleepQual; grandmean = SleepQual.mean',  
  model = '  
    beta0j ~ intercept SleepQual.mean;  
    PosAffect ~ intercept@beta0j SleepQual',  
  seed = 90291,  
  burn = 10000,  
  iter = 10000)  
output(disagg_lat)
```

LEVEL-1 OUTPUT

 = level-2 estimate

 = level-1 estimate


Outcome Variable: PosAffect

Group Mean Centered: SleepQual

Parameters	Estimate	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff

Variances:							
Residual Var.	1.306	0.036	1.238	1.380	---	---	18455.553
Coefficients:							
beta0j	@ 1.000	---	---	---	---	---	---
SleepQual	0.173	0.013	0.147	0.198	174.326	0.000	17004.230
Standardized Coefficients:							
SleepQual	0.253	0.018	0.217	0.288	191.841	0.000	16829.613
Proportion Variance Explained							
by Coefficients	0.064	0.009	0.047	0.083	---	---	16802.586
by Residual Variation	0.936	0.009	0.917	0.953	---	---	16802.586

LEVEL-2 OUTPUT

 = level-2 estimate

 = level-1 estimate

Latent Variable: β_{0j}

Grand Mean Centered: SleepQual.mean[Person]

Parameters	Estimate	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff

Variances:							
Residual Var.	1.839	0.250	1.432	2.411	---	---	14950.945
Coefficients:							
Intercept	5.031	0.141	4.754	5.308	1266.410	0.000	6778.361
SleepQual.mean[Person]	0.606	0.092	0.423	0.785	42.962	0.000	12100.465
Standardized Coefficients:							
SleepQual.mean[Person]	0.522	0.064	0.382	0.631	65.880	0.000	12023.639
Proportion Variance Explained							
by Coefficients	0.272	0.065	0.146	0.398	---	---	11880.090
by Residual Variation	0.728	0.065	0.602	0.854	---	---	11880.090

BLIMP STUDIO SCRIPT 2.7

DATA: PainDiary.dat;

VARIABLES: Person Day PosAffect NegAffect Pain WorkGoal LifeGoal SleepQual Female Education
Employment MarStatus NumDiagnose ActivityLevel PainAccept Catastrophize Stress Anxiety;

CLUSTERID: Person;

LATENT: Person = beta0j; # define level-2 intercept latent variable

CENTER: grandmean = SleepQual.mean;

MODEL:

beta0j ~ intercept SleepQual.mean; # level-2: $\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{sleep}_j^b) + u_{0j}$

PosAffect ~ intercept@beta0j; # level-1: $p_{\text{affect}_{ij}} = \beta_{0j} + \varepsilon_{ij}$

BURN: 10000;

ITER: 10000;

SEED: 90291;

RBLIMP SCRIPT 2 (MODEL 7)

```
between_lat <- rblimp(  
  data = PainDiary,  
  clusterid = 'Person',  
  latent = 'Person = beta0j',  
  center = 'grandmean = SleepQual.mean',  
  model = '  
    beta0j ~ intercept SleepQual.mean;  
    PosAffect ~ intercept@beta0j',  
  seed = 90291,  
  burn = 10000,  
  iter = 10000)  
output(between_lat)
```

LEVEL-1 OUTPUT

 = level-2 estimate

 = level-1 estimate

Outcome Variable: PosAffect

Parameters	Estimate	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff

Variances:							
Residual Var.	1.395	0.039	1.321	1.474	---	---	17839.886
Coefficients:							
beta0j	@ 1.000	---	---	---	---	---	---
Proportion Variance Explained							
by Coefficients	0.000	0.000	0.000	0.000	---	---	nan
by Residual Variation	1.000	0.000	1.000	1.000	---	---	nan

LEVEL-2 OUTPUT

 = level-2 estimate

 = level-1 estimate

Latent Variable: β_{0j}

Grand Mean Centered: SleepQual.mean[Person]

Parameters	Estimate	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff

Variances:							
Residual Var.	1.810	0.249	1.402	2.374	---	---	12507.777
Coefficients:							
Intercept	5.032	0.142	4.753	5.309	1254.555	0.000	5588.112
SleepQual.mean[Person]	0.620	0.093	0.438	0.800	44.628	0.000	10725.502
Standardized Coefficients:							
SleepQual.mean[Person]	0.533	0.063	0.394	0.641	69.950	0.000	10362.131
Proportion Variance Explained							
by Coefficients	0.284	0.065	0.155	0.410	---	---	10374.248
by Residual Variation	0.716	0.065	0.590	0.845	---	---	10374.248