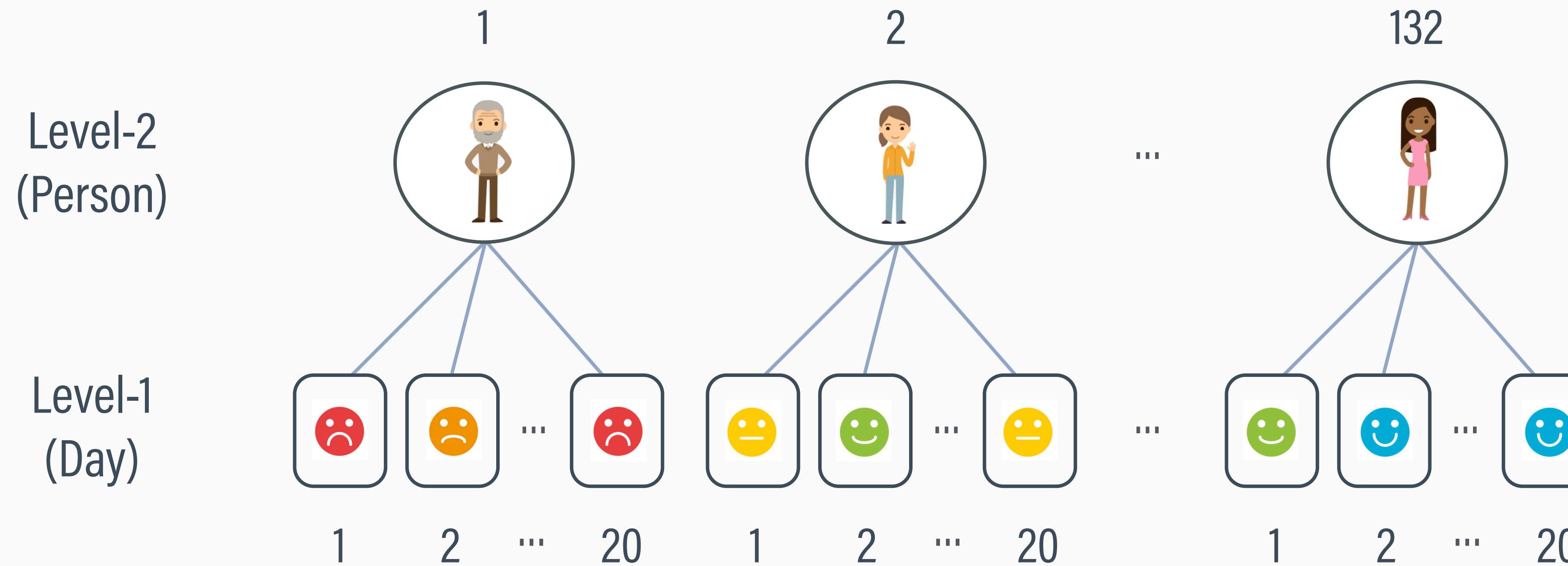


MODULE 4

RANDOM INTERCEPT MODELS WITH LEVEL-1 AND LEVEL-2 PREDICTORS

DAILY DIARY APPLICATION

- $n_j = 20$ daily positive affect and sleep assessments nested within $J = 132$ chronic pain patients ($N = 2680$ data records)



VARIABLE INFORMATION

- = predictor measured at level-2
- = predictor measured at level-1
- = outcome measured at level-1

paffet = sleep + pain + sex + stress

| Variable | Definition | Level | Scale |
|-----------|---------------------------|-------|----------------------|
| PosAffect | Positive affect composite | 1 | Numeric (0 to 10) |
| SleepQual | Sleep quality rating | 1 | Ordinal (0 to 9) |
| Pain | Pain severity composite | 1 | Numeric (0 to 10) |
| Female | Sex dummy code | 2 | 0 = Male, 1 = Female |
| Stress | Stress composite score | 2 | Numeric (0 to 5) |

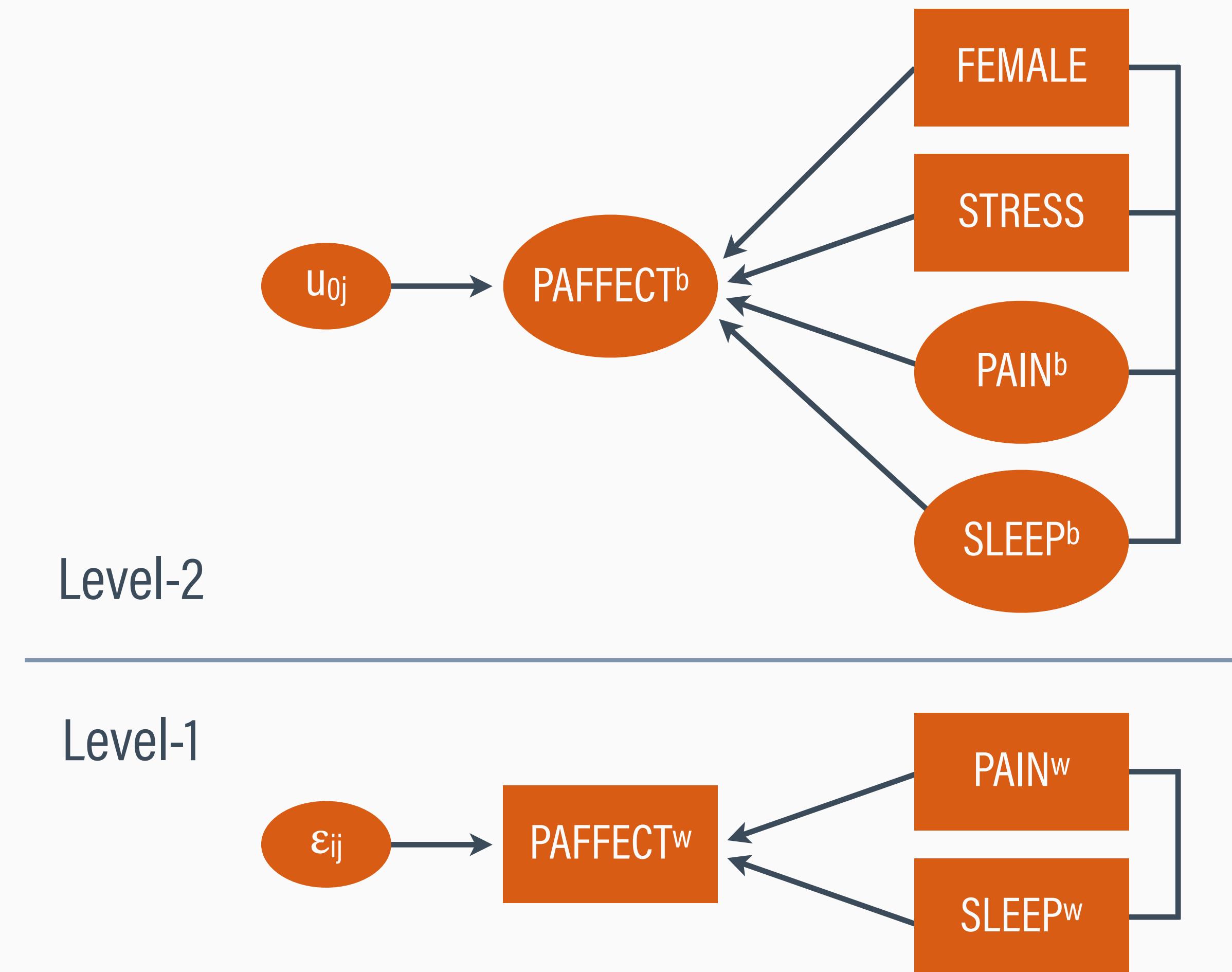
DATA STRUCTURE

- Stacked (long) data format where each level-2 unit (person) has one row per level-1 (daily) observation
- The i subscript indexes level-1 observations, and j indexes level-2 units
- Variables measured at level-2 repeat across all rows within a cluster

| Row | i | j | $AFFECT_{ij}$ | $SLEEP_{ij}$ | $FEMALE_j$ | $STRESS_j$ | |
|------|-----|-----|---------------|--------------|------------|------------|------------|
| 1 | 1 | 1 | 7.3 | 5.6 | 1 | 3.0 | Person 1 |
| 2 | 2 | 1 | 2.5 | 4.3 | 1 | 3.0 | |
| ... | ... | 1 | ... | ... | ... | ... | |
| 20 | 20 | 1 | 6.3 | 7.3 | 1 | 3.0 | |
| 21 | 1 | 2 | 4.0 | 3.9 | 0.0 | 2.3 | Person 2 |
| 22 | 2 | 2 | 4.0 | 7.1 | 0.0 | 2.3 | |
| ... | ... | 2 | ... | ... | ... | ... | |
| 40 | 20 | 2 | 4.4 | 3.5 | 0.0 | 2.3 | |
| ... | ... | ... | ... | ... | ... | ... | |
| 2621 | 1 | 132 | 3.3 | 5.4 | 1.0 | 4.1 | Person 132 |
| 2622 | 2 | 132 | 4.8 | 3.5 | 1.0 | 4.1 | |
| ... | ... | 132 | ... | ... | ... | ... | |
| 2640 | 20 | 132 | 4.8 | 7.9 | 1.0 | 4.1 | |

PATH DIAGRAM AND MODEL FEATURES

- Level-1 variables have two sources of variation: average values and daily fluctuations around those means
- Variables correlate within level only (e.g., sleep relates to stress via its means)
- Statistical control occurs within level



MODELING STEPS

- Estimate the ICCs of all level-1 variables
- Add level-1 variables
- Add level-2 variables

OUTLINE

- 1 Modeling Step 1: Estimate ICCs
- 2 Modeling Step 2: Add Within-Cluster Predictors
- 3 Rights and Sterba Effect Sizes
- 4 Modeling Step 3: Add Between-Cluster Predictors
- 5 Latent Variable Specification

OUTLINE

- 1 Modeling Step 1: Estimate ICCs
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ESTIMATING INTRACLASS CORRELATIONS

- Estimate intraclass correlations (ICCs) before model fitting to assess how much variance lies within and between clusters
- Some predictors have little or no level-2 variation, in which case they are already “pure” within-cluster variables
- Disaggregating a level-1 predictor and modeling its means is only warranted when there is sufficient level-2 variation

BLIMP STUDIO SCRIPT 4.1

DATA: PainDiary.dat;

VARIABLES: Person Day PosAffect NegAffect Pain WorkGoal LifeGoal SleepQual Female Education
Employment MarStatus NumDiagnose ActivityLevel PainAccept Catastrophize Stress Anxiety;

CLUSTERID: Person;

MODEL: { PosAffect SleepQual Pain } ~ intercept | intercept; # empty model for all variables in {}

BURN: 10000;

ITERATIONS: 10000;

SEED: 90291;

RBLIMP SCRIPT 4 (MODEL 1)

```
model1 <- rblimp(  
  data = PainDiary,  
  clustered = 'Person',  
  model = '{ PosAffect SleepQual Pain } ~ intercept | intercept;',  
  seed = 90291,  
  burn = 10000,  
  iter = 10000)  
  
output(model1)
```

BLIMP OUTPUT

 = level-2 estimate

 = level-1 estimate

Outcome Variable: PosAffect

| Parameters | Estimate | StdDev | 2.5% | 97.5% | ChiSq | PValue | N_Eff |
|-------------------------------|--------------|--------|-------|-------|----------|--------|----------|
| <hr/> | | | | | | | |
| Variances: | | | | | | | |
| L2 : Var(Intercept) | 2.529 | 0.333 | 1.987 | 3.271 | --- | --- | 8414.025 |
| Residual Var. | 1.395 | 0.039 | 1.323 | 1.474 | --- | --- | 9257.666 |
| <hr/> | | | | | | | |
| Coefficients: | | | | | | | |
| Intercept | 5.034 | 0.147 | 4.749 | 5.316 | 1177.150 | 0.000 | 124.250 |
| <hr/> | | | | | | | |
| Standard Deviations: | | | | | | | |
| L2 : SD(Intercept) | 1.590 | 0.103 | 1.410 | 1.809 | --- | --- | 8415.412 |
| Residual SD | 1.181 | 0.016 | 1.150 | 1.214 | --- | --- | 9254.467 |
| <hr/> | | | | | | | |
| Proportion Variance Explained | | | | | | | |
| by Coefficients | 0.000 | 0.000 | 0.000 | 0.000 | --- | --- | nan |
| by Level-2 Random Intercepts | 0.644 | 0.030 | 0.585 | 0.703 | --- | --- | 8658.787 |
| by Level-1 Residual Variation | 0.356 | 0.030 | 0.297 | 0.415 | --- | --- | 8658.787 |

BLIMP OUTPUT

□ = level-2 estimate

□ = level-1 estimate

Outcome Variable: SleepQual

| Parameters | Estimate | StdDev | 2.5% | 97.5% | ChiSq | PValue | N_Eff |
|-------------------------------|--------------|--------|-------|-------|----------|--------|----------|
| <hr/> | | | | | | | |
| Variances: | | | | | | | |
| L2 : Var(Intercept) | 1.919 | 0.262 | 1.490 | 2.508 | --- | --- | 6730.439 |
| Residual Var. | 3.001 | 0.084 | 2.845 | 3.172 | --- | --- | 9268.567 |
| <hr/> | | | | | | | |
| Coefficients: | | | | | | | |
| Intercept | 5.416 | 0.130 | 5.168 | 5.671 | 1729.526 | 0.000 | 400.385 |
| <hr/> | | | | | | | |
| Standard Deviations: | | | | | | | |
| L2 : SD(Intercept) | 1.385 | 0.093 | 1.221 | 1.584 | --- | --- | 6767.067 |
| Residual SD | 1.732 | 0.024 | 1.687 | 1.781 | --- | --- | 9265.589 |
| <hr/> | | | | | | | |
| Proportion Variance Explained | | | | | | | |
| by Coefficients | 0.000 | 0.000 | 0.000 | 0.000 | --- | --- | nan |
| by Level-2 Random Intercepts | 0.390 | 0.032 | 0.330 | 0.457 | --- | --- | 6737.692 |
| by Level-1 Residual Variation | 0.610 | 0.032 | 0.543 | 0.670 | --- | --- | 6737.692 |

BLIMP OUTPUT

□ = level-2 estimate

□ = level-1 estimate

Outcome Variable: Pain

| Parameters | Estimate | StdDev | 2.5% | 97.5% | ChiSq | PValue | N_Eff |
|-------------------------------|--------------|--------|-------|-------|---------|--------|----------|
| <hr/> | | | | | | | |
| Variances: | | | | | | | |
| L2 : Var(Intercept) | 2.805 | 0.375 | 2.189 | 3.654 | --- | --- | 7747.670 |
| Residual Var. | 2.246 | 0.063 | 2.125 | 2.373 | --- | --- | 8881.762 |
| <hr/> | | | | | | | |
| Coefficients: | | | | | | | |
| Intercept | 3.533 | 0.152 | 3.238 | 3.823 | 536.962 | 0.000 | 236.236 |
| <hr/> | | | | | | | |
| Standard Deviations: | | | | | | | |
| L2 : SD(Intercept) | 1.675 | 0.110 | 1.480 | 1.912 | --- | --- | 7692.880 |
| Residual SD | 1.499 | 0.021 | 1.458 | 1.541 | --- | --- | 8884.094 |
| <hr/> | | | | | | | |
| Proportion Variance Explained | | | | | | | |
| by Coefficients | 0.000 | 0.000 | 0.000 | 0.000 | --- | --- | nan |
| by Level-2 Random Intercepts | 0.556 | 0.033 | 0.492 | 0.622 | --- | --- | 7537.167 |
| by Level-1 Residual Variation | 0.444 | 0.033 | 0.378 | 0.508 | --- | --- | 7537.167 |

SUMMARY

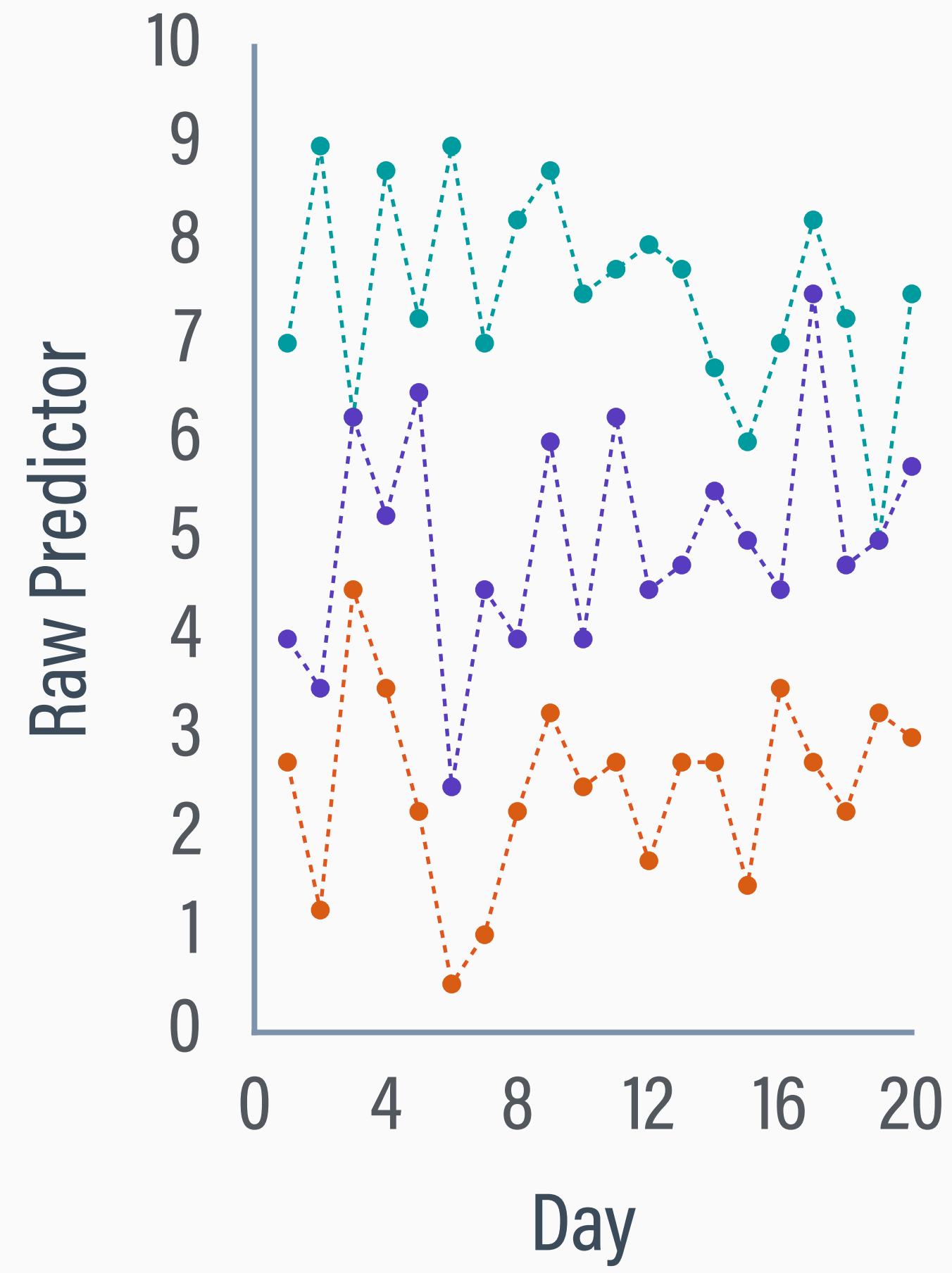
- The outcome has substantial between-person variation in the person-specific means ($ICC_{affect} = .64$)
- This feature indicates the need for MLM (independence violation)
- Both level-1 predictors have a substantial amount of between-person variation in their means ($ICC_{sleep} = .39$ and $ICC_{pain} = .55$)
- High predictor ICCs motivate disaggregation, as there could be distinct within- and between-person effects

OUTLINE

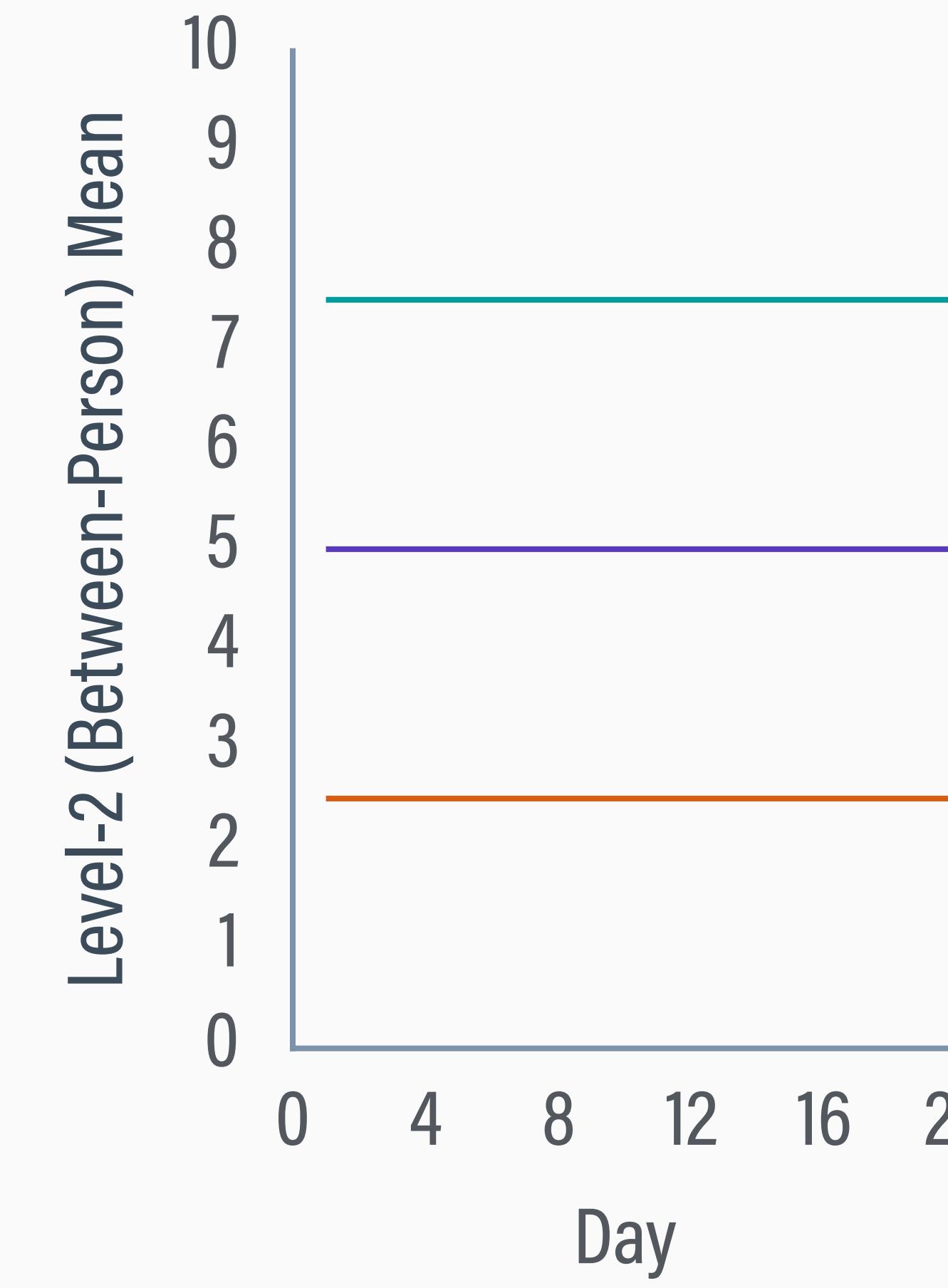
- 1 Modeling Step 1: Estimate ICCs
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DISAGGREGATION REVIEW

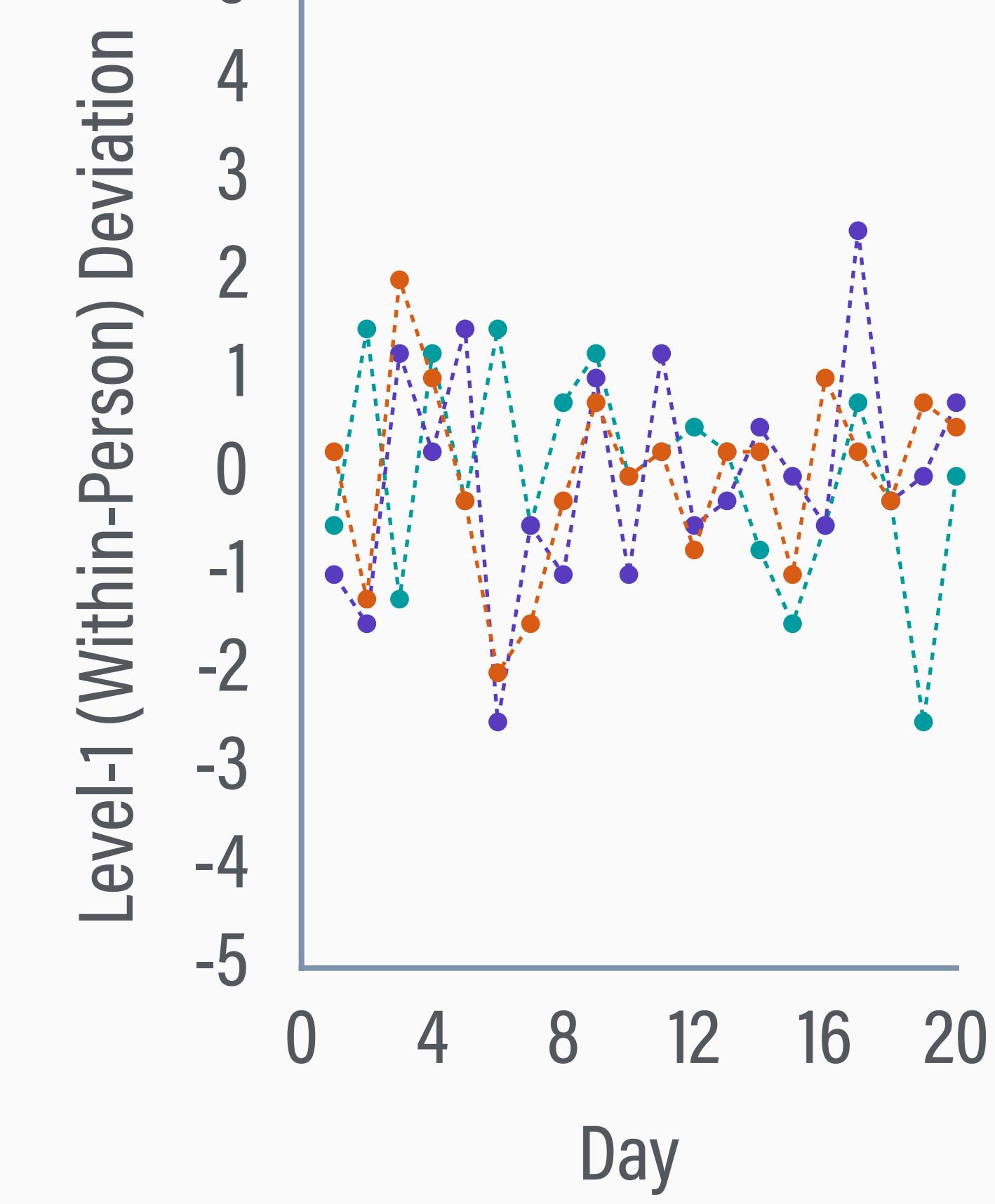
Raw Predictor = Level-2 (Between-Person) Mean + Level-1 (Within-Person) Residual



||



+



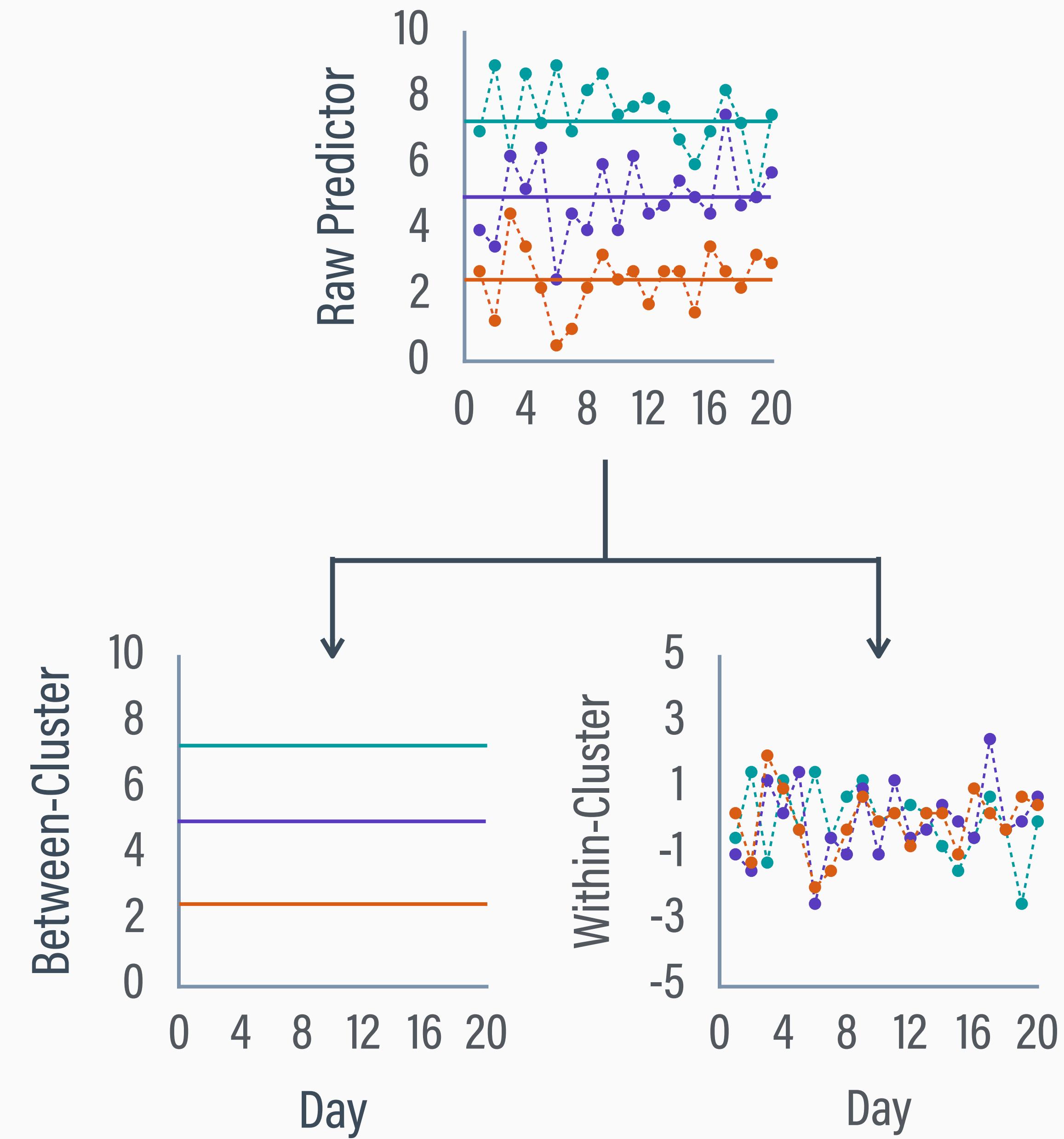
CENTERING WITHIN CLUSTER (CWC)

- Disaggregation centers level-1 scores around their cluster's level-2 mean

$$\text{pain}_{ij}^w = \text{pain}_{ij} - \mu_j(\text{pain}) = \text{pain}_{ij} - \text{pain}_j^b$$

$$\text{sleep}_{ij}^w = \text{sleep}_{ij} - \mu_j(\text{sleep}) = \text{sleep}_{ij} - \text{sleep}_j^b$$

- pain^w and sleep^w are “pure” within-cluster predictors that contain only intraindividual (level-1) variation



WITHIN-CLUSTER (LEVEL-1) MODEL

- Affect observation i for person j is the sum of a level-2 affect mean (β_{0j}), fixed effects due to within-person sleep and pain variation (β_1 and β_2), and a within-person residual (ε_{ij})

$$\text{pffect}_{ij} = \beta_{0j} + \beta_1(\text{sleep}_{ij}^W) + \beta_2(\text{pain}_{ij}^W) + \varepsilon_{ij}$$

- Assumption: residuals are normal with constant variation across all days (level-1 units) and persons (level-2 units)

$$\varepsilon_{ij} \sim N(0, \sigma_\varepsilon^2)$$

BETWEEN-CLUSTER (LEVEL-2) MODEL

- Slopes are constant across level-2 units, but the affect mean for person j (β_{0j}) is the sum of the grand mean (γ_{00}) and a between-person residual (u_{0j})

$$\beta_{0j} = \gamma_{00} + u_{0j}$$

$$\beta_1 = \gamma_{10}$$

$$\beta_2 = \gamma_{20}$$

- Assumption: random intercept residuals are normal with constant variation across persons (level-2 units)

$$u_{0j} \sim N(0, \sigma_u^2)$$

DECODING THE SUBSCRIPTS

The second subscript numbers the coefficients on the right side starting at 0 (the intercept)

$$\beta_{0j} = \gamma_{00} + u_{0j}$$

$$\beta_{1j} = \gamma_{10}$$

$$\beta_{2j} = \gamma_{20}$$

The first subscript tells which level-2 outcome variable these terms are attached to (γ_{00} and u_{0j} belong in β_{0j} 's equation, γ_{10} is attached to β_1)

COMBINED-MODEL NOTATION

- Level-specific regression equations can be reduced into a single combined-model equation (Raudenbush & Bryk, 2002)
- Replace the β_{0j} , β_1 , and β_2 terms in the level-1 equation with their level-2 equations

$$\begin{array}{c} \beta_2 = \gamma_{20} \\ \beta_1 = \gamma_{10} \\ \beta_{0j} = \gamma_{00} + u_{0j} \\ \downarrow \quad \downarrow \\ \text{pffect}_{ij} = \beta_{0j} + \beta_1(\text{sleep}_{ij}^w) + \beta_2(\text{pain}_{ij}^w) + \varepsilon_{ij} \\ \downarrow \\ \text{pffect}_{ij} = \gamma_{00} + \gamma_{10}(\text{sleep}_{ij}^w) \\ + \gamma_{20}(\text{pain}_{ij}^w) + u_{0j} + \varepsilon_{ij} \end{array}$$

COMMON NOTATIONAL SYSTEMS

Combined-model equation (Raudenbush & Bryk, 2002)

$$\text{paffect}_{ij} = \gamma_{00} + \gamma_{10}(\text{sleep}_{ij}^w) + \gamma_{20}(\text{pain}_{ij}^w) + u_{0j} + \varepsilon_{ij}$$

Standard(ish) regression notation (Scott, Shrout, & Weinberg, 2013)

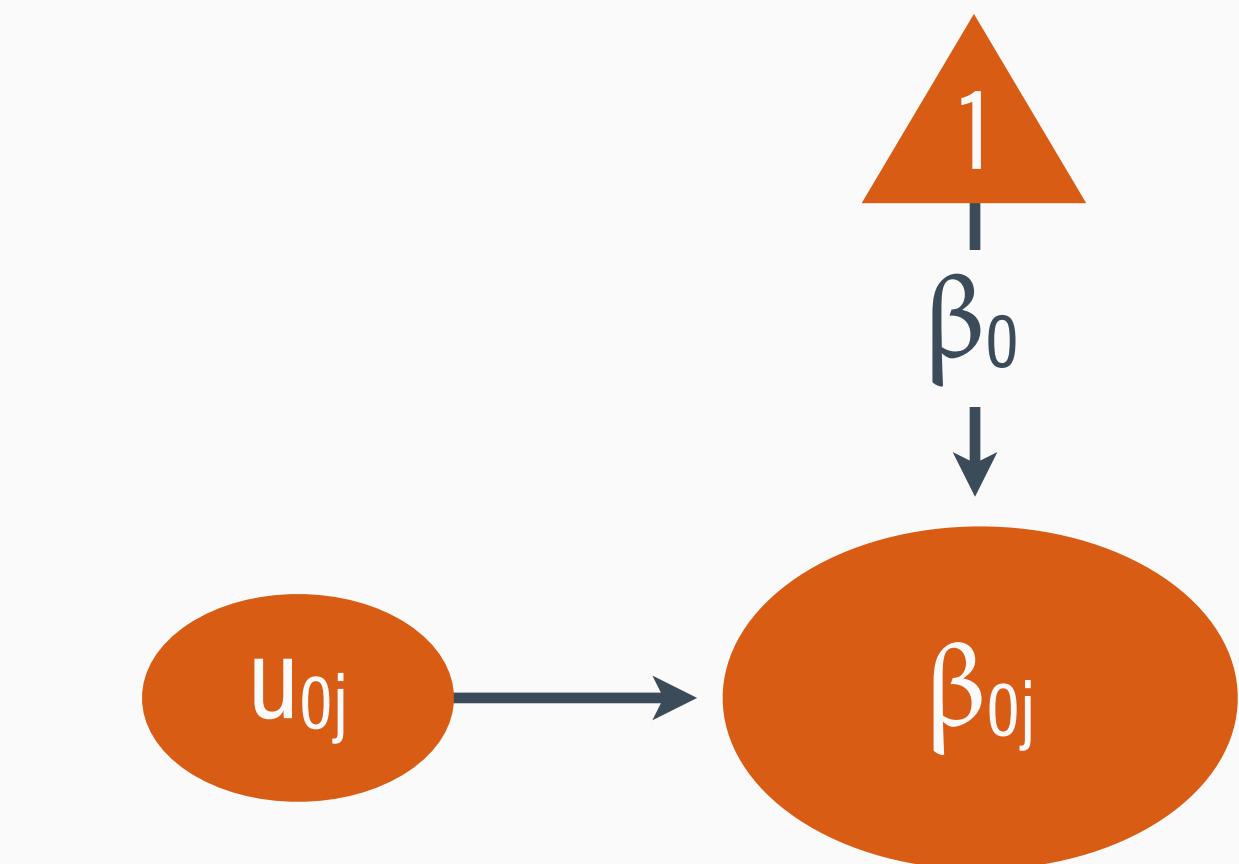
$$\text{paffect}_{ij} = \beta_0 + \beta_1(\text{sleep}_{ij}^w) + \beta_2(\text{pain}_{ij}^w) + u_{0j} + \varepsilon_{ij}$$

Linear mixed model cluster-level matrix equation

$$\mathbf{y}_j = \mathbf{X}_j\boldsymbol{\beta} + \mathbf{Z}_j\mathbf{u}_j + \boldsymbol{\varepsilon}_j = \begin{pmatrix} \text{posaffect}_{1j} \\ \text{posaffect}_{2j} \\ \dots \\ \text{posaffect}_{nj} \end{pmatrix} = \begin{pmatrix} 1 & \text{sleep}_{1j}^w & \text{pain}_{1j}^w \\ 1 & \text{sleep}_{2j}^w & \text{pain}_{2j}^w \\ \dots & \dots & \dots \\ 1 & \text{sleep}_{nj}^w & \text{pain}_{nj}^w \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ \dots \\ 1 \end{pmatrix} \mathbf{u}_{0j} + \begin{pmatrix} \varepsilon_{1j} \\ \varepsilon_{2j} \\ \dots \\ \varepsilon_{nj} \end{pmatrix}$$

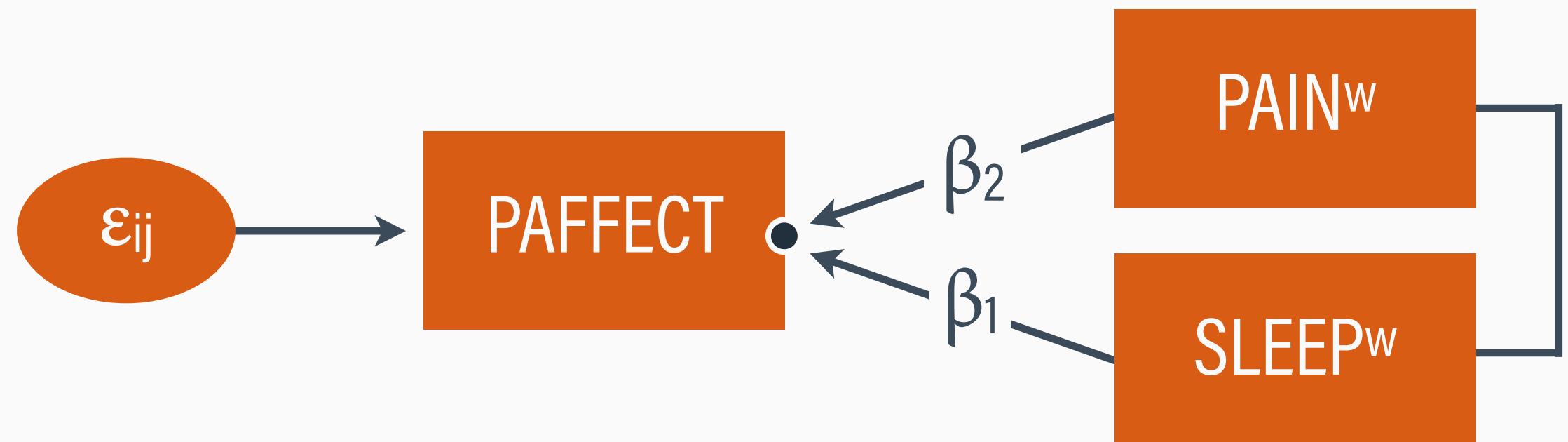
PATH DIAGRAM

$$paffect_{ij} = \beta_0 + \beta_1(sleep_{ij}^w) + \beta_2(pain_{ij}^w) + u_{0j} + \varepsilon_{ij}$$



Level-2

Level-1



• = random intercept (β_{0j})

BLIMP STUDIO SCRIPT 4.2

DATA: PainDiary.dat;

VARIABLES: Person Day PosAffect NegAffect Pain WorkGoal LifeGoal SleepQual Female Education
Employment MarStatus NumDiagnose ActivityLevel PainAccept Catastrophize Stress Anxiety;

CLUSTERID: Person;

CENTER: groupmean = SleepQual Pain; # cwc with level-2 latent cluster means

MODEL: PosAffect ~ intercept SleepQual Pain | intercept;

BURN: 10000;

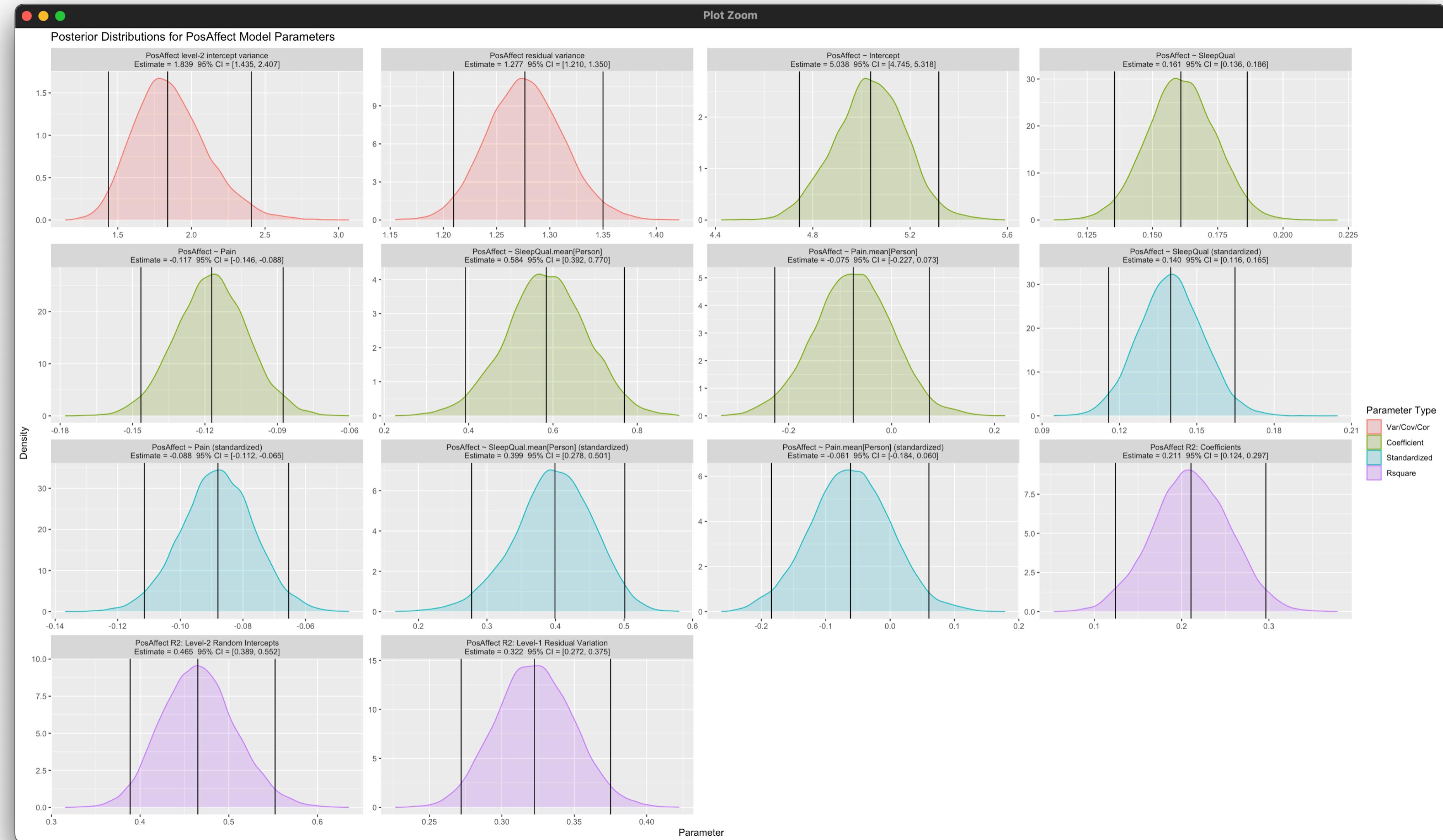
ITER: 10000;

SEED: 90291;

RBLIMP SCRIPT 4 (MODEL 2)

```
model2 <- rblimp(  
  data = PainDiary,  
  clusterid = 'Person',  
  center = 'groupmean = SleepQual Pain',  
  model = 'PosAffect ~ intercept SleepQual Pain | intercept',  
  seed = 90291,  
  burn = 10000,  
  iter = 10000)  
  
output(model2)  
plot_posterior(model2, 'PosAffect')
```

PARAMETER PLOTS (RBLIMP ONLY)



BLIMP OUTPUT

= level-2 estimate

= level-1 estimate

Outcome Variable: PosAffect

Group Mean Centered: Pain SleepQual

| Parameters | Estimate | StdDev | 2.5% | 97.5% | ChiSq | PValue | N_Eff |
|-------------------------------|---------------|--------|--------|--------|----------|--------|----------|
| <hr/> | | | | | | | |
| Variances: | | | | | | | |
| L2 : Var(Intercept) | 2.496 | 0.325 | 1.953 | 3.218 | --- | --- | 8178.774 |
| Residual Var. | 1.276 | 0.036 | 1.210 | 1.350 | --- | --- | 9009.151 |
| <hr/> | | | | | | | |
| Coefficients: | | | | | | | |
| Intercept | 5.010 | 0.147 | 4.742 | 5.312 | 1169.499 | 0.000 | 105.520 |
| SleepQual | 0.162 | 0.013 | 0.136 | 0.187 | 156.661 | 0.000 | 8279.469 |
| Pain | -0.117 | 0.015 | -0.147 | -0.088 | 60.912 | 0.000 | 8510.442 |
| <hr/> | | | | | | | |
| Standard Deviations: | | | | | | | |
| L2 : SD(Intercept) | 1.580 | 0.102 | 1.397 | 1.794 | --- | --- | 8123.400 |
| Residual SD | 1.130 | 0.016 | 1.100 | 1.162 | --- | --- | 9006.557 |
| <hr/> | | | | | | | |
| Standardized Coefficients: | | | | | | | |
| SleepQual | 0.142 | 0.013 | 0.118 | 0.167 | 126.552 | 0.000 | 7959.671 |
| Pain | -0.089 | 0.012 | -0.113 | -0.066 | 55.997 | 0.000 | 8548.133 |
| <hr/> | | | | | | | |
| Proportion Variance Explained | | | | | | | |
| by Coefficients | 0.031 | 0.005 | 0.023 | 0.041 | --- | --- | 8140.114 |
| by Level-2 Random Intercepts | 0.641 | 0.030 | 0.582 | 0.698 | --- | --- | 8075.546 |
| by Level-1 Residual Variation | 0.328 | 0.027 | 0.276 | 0.382 | --- | --- | 8119.500 |

FIXED EFFECT INTERPRETATIONS

- $p\text{affect}_{ij} = \beta_0 + \beta_1(\text{sleep}_{ij}^W) + \beta_2(\text{pain}_{ij}^W) + u_{0j} + \varepsilon_{ij}$
- $\beta_0 = 5.01$ is the positive affect grand mean (because all predictors are centered)
- $\beta_1 = 0.16$ is the expected affect difference between two daily sleep scores from the same person that differ by one point, controlling for within-person pain
- $\beta_2 = -0.12$ is the expected affect difference between two daily pain scores from the same person that differ by one point, controlling for within-person sleep

RANDOM EFFECT INTERPRETATIONS

- $u_{0j} = \beta_{0j} - \beta_0$
- $\text{var}(u_{0j}) = 2.50$ is the average squared distance between the level-2 affect means and the grand mean
- $\text{sd}(u_{0j}) = 1.58$ is the average distance between the level-2 affect means and the grand mean
- $\varepsilon_{ij} = p\text{affect}_{ij} - (\beta_{0j} + \beta_1(\text{sleep}_{ij}^W) + \beta_2(\text{pain}_{ij}^W))$
- $\text{var}(\varepsilon_{ij}) = 1.28$ is the average squared distance between the level-1 affect observations and their predicted values
- $\text{sd}(\varepsilon_{ij}) = 1.13$ is the average distance between the level-1 affect observations and their predicted values

OUTLINE

- 1 Modeling Step 1: Estimate ICCs
- 2 Modeling Step 2: Add Within-Cluster Predictors
- 3 Rights and Sterba Effect Sizes
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MULTILEVEL EFFECT SIZES

- Numerous effect sizes exist for multilevel models
- Rights and Sterba (2019, Psychological Methods) provides a unifying framework that subsumes different approaches
- They assign a proportion variance explained (R^2) to each source of variation in the multilevel model

PREDICTED OUTCOME VARIATION

- The model-predicted total variance is the sum of (a) variance explained by predictors at both levels, (b) residual random-intercept variation, and (c) residual within-cluster variation

$$\sigma_y^2 = \beta^\top \Sigma_x \beta + \sigma_u^2 + \sigma_\varepsilon^2$$

total variation = explained by predictors + level-2 residuals + level-1 residuals

VARIANCE EXPLAINED MEASURES

Fixed effects
of predictors

$$R^2_{\text{predictors}} = \frac{\beta^T \Sigma_X \beta}{\sigma_Y^2}$$

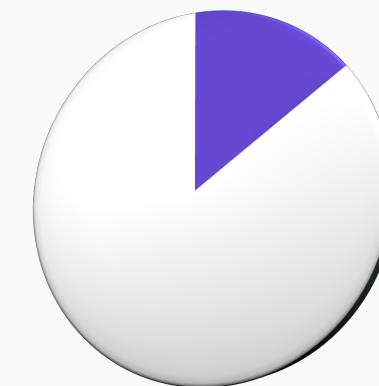
Level-2 random
intercept residuals

$$R^2_{\text{residual-between}} = \frac{\sigma_{u_0}^2}{\sigma_Y^2}$$

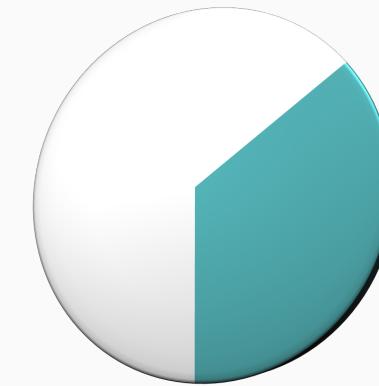
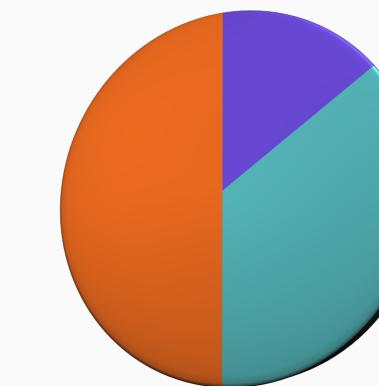
Level-1 within-
cluster residuals

$$R^2_{\text{residual-within}} = \frac{\sigma_\varepsilon^2}{\sigma_Y^2}$$

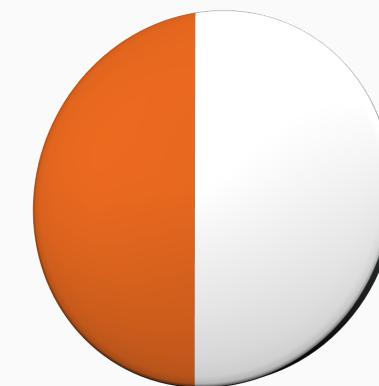
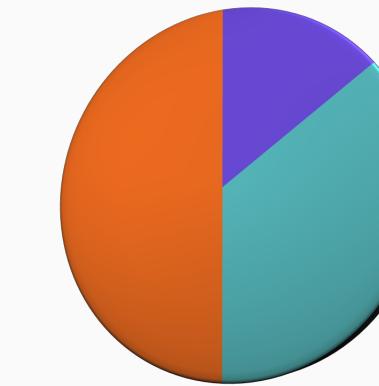
Explained ÷ Total



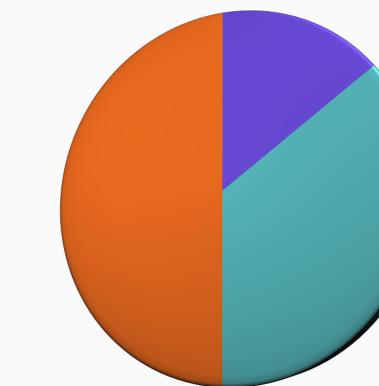
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BLIMP OUTPUT

- = level-2 estimate
- = level-1 estimate

Outcome Variable: PosAffect

Group Mean Centered: Pain SleepQual

| Parameters | Estimate | StdDev | 2.5% | 97.5% | ChiSq | PValue | N_Eff |
|-------------------------------|--------------|--------|--------|--------|----------|--------|----------|
| <hr/> | | | | | | | |
| Variances: | | | | | | | |
| L2 : Var(Intercept) | 2.496 | 0.325 | 1.953 | 3.218 | --- | --- | 8178.774 |
| Residual Var. | 1.276 | 0.036 | 1.210 | 1.350 | --- | --- | 9009.151 |
| <hr/> | | | | | | | |
| Coefficients: | | | | | | | |
| Intercept | 5.010 | 0.147 | 4.742 | 5.312 | 1169.499 | 0.000 | 105.520 |
| SleepQual | 0.162 | 0.013 | 0.136 | 0.187 | 156.661 | 0.000 | 8279.469 |
| Pain | -0.117 | 0.015 | -0.147 | -0.088 | 60.912 | 0.000 | 8510.442 |
| <hr/> | | | | | | | |
| Proportion Variance Explained | | | | | | | |
| by Coefficients | 0.031 | 0.005 | 0.023 | 0.041 | --- | --- | 8140.114 |
| by Level-2 Random Intercepts | 0.641 | 0.030 | 0.582 | 0.698 | --- | --- | 8075.546 |
| by Level-1 Residual Variation | 0.328 | 0.027 | 0.276 | 0.382 | --- | --- | 8119.500 |

EFFECT SIZE INTERPRETATIONS

- $R^2_{(\text{predictors})} = .03$ is the proportion of the total variation explained by the within-cluster predictors
- $R^2_{(\text{residual-between})} = .64$ is the proportion of the total variation attributable to the between-cluster residuals (the u_{0j} terms)
- $R^2_{(\text{residual-within})} = .33$ is the proportion of the total variation attributable to the within-cluster residuals (the ε_{ij} terms)

MODEL COMPARISON

| Parameter | Empty Model | Level-1 Predictors (Within Only) | Level-2 Predictors (Within + Between) |
|---------------------------------|-------------|-------------------------------------|--|
| Fixed intercept | 5.03 | 5.03 | 5.03 |
| Sleep (within-person) | -- | 0.16 | 0.16 |
| Pain (within-person) | -- | -0.12 | -0.12 |
| Sleep (between-person) | -- | -- | -0.11 |
| Pain (between-person) | -- | -- | 0.57 |
| Female (between-person) | -- | -- | 0.38 |
| Stress (between-person) | -- | -- | -0.36 |
| Residual intercept variance | 2.53 | 2.50 | 1.74 |
| Residual within-person variance | 1.40 | 1.28 | 1.28 |
| R-square predictors | 0 | .03 | .25 |
| R-square intercept variance | .64 | .64 | .43 |
| R-square within-person variance | .36 | .33 | .32 |



The next analysis will add the level-2 (person-specific) predictors to the model. In small groups of two or three, discuss whether each effect size will change and, if so, how.

OUTLINE

- 1 Modeling Step 1: Estimate ICCs
- 2 Modeling Step 2: Add Within-Cluster Predictors
- 3 Rights and Sterba Effect Sizes
- 4 Modeling Step 3: Add Between-Cluster Predictors
- 5 Latent Variable Specification

WITHIN-CLUSTER (LEVEL-1) MODEL

- Affect observation i for person j is the sum of a level-2 affect mean (β_{0j}), fixed effects due to within-person sleep and pain variation (β_1 and β_2), and a within-person residual (ε_{ij})

$$\text{pffect}_{ij} = \beta_{0j} + \beta_1(\text{sleep}_{ij}^W) + \beta_2(\text{pain}_{ij}^W) + \varepsilon_{ij}$$

- Assumption: residuals are normal with constant variation across all days (level-1 units) and persons (level-2 units)

$$\varepsilon_{ij} \sim N(0, \sigma_\varepsilon^2)$$

BETWEEN-CLUSTER (LEVEL-2) MODEL

- The affect mean for person j (β_{0j}) is the sum of a mean (γ_{00}), fixed effects due to the level-2 predictors (γ_{01} through γ_{04}), and a between-person residual (u_{0j})

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{sleep}_j^b) + \gamma_{02}(\text{pain}_j^b) + \gamma_{03}(\text{female}_j) + \gamma_{04}(\text{stress}_j) + u_{0j}$$

$$\beta_1 = \gamma_{10}$$

$$\beta_2 = \gamma_{20}$$

- Assumption: random intercept residuals are normal with constant variation across persons (level-2 units)

$$u_{0j} \sim N(0, \sigma_u^2)$$

COMMON NOTATIONAL SYSTEMS

Combined-model equation (Raudenbush & Bryk, 2002)

$$\begin{aligned} \text{paffect}_{ij} = & \gamma_{00} + \gamma_{10}(\text{sleep}_{ij}^w) + \gamma_{20}(\text{pain}_{ij}^w) + \gamma_{01}(\text{sleep}_j^b) + \gamma_{02}(\text{pain}_j^b) \\ & + \gamma_{03}(\text{female}_j) + \gamma_{04}(\text{stress}_j) + u_{0j} + \varepsilon_{ij} \end{aligned}$$

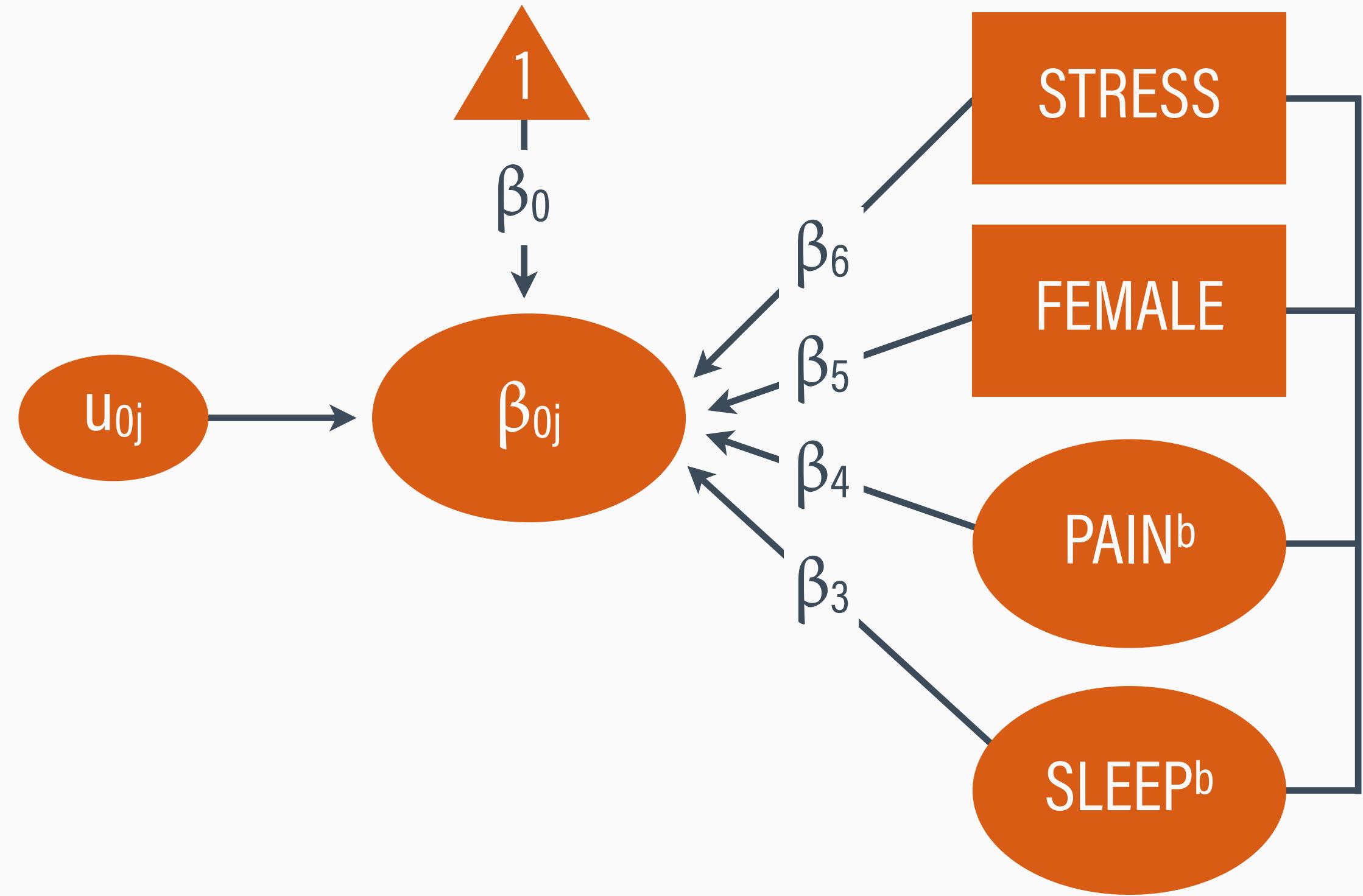
Standard(ish) regression notation (Scott, Shrout, & Weinberg, 2013)

$$\begin{aligned} \text{paffect}_{ij} = & \beta_0 + \beta_1(\text{sleep}_{ij}^w) + \beta_2(\text{pain}_{ij}^w) + \beta_3(\text{sleep}_j^b) + \beta_4(\text{pain}_j^b) \\ & + \beta_5(\text{female}_j) + \beta_6(\text{stress}_j) + u_{0j} + \varepsilon_{ij} \end{aligned}$$

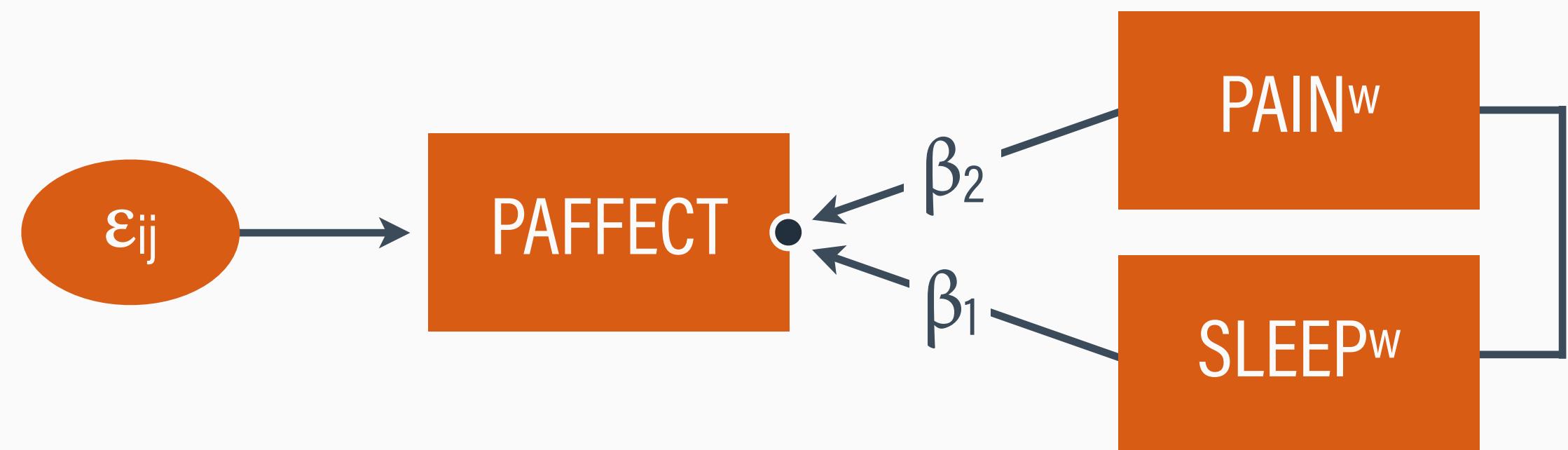
PATH DIAGRAM

$$p\text{affect}_{ij} = \beta_0 + \beta_1(\text{sleep}_{ij}^W) + \beta_2(\text{pain}_{ij}^W) + \\ \beta_3(\text{sleep}_j^b) + \beta_4(\text{pain}_j^b) + \\ \beta_5(\text{female}_j) + \beta_6(\text{stress}_j) + \\ u_{0j} + \varepsilon_{ij}$$

Level-2



Level-1



• = random intercept (β_{0j})

BLIMP STUDIO SCRIPT 4.3

DATA: PainDiary.dat;

VARIABLES: Person Day PosAffect NegAffect Pain WorkGoal LifeGoal SleepQual Female Education
Employment MarStatus NumDiagnose ActivityLevel PainAccept Catastrophize Stress Anxiety;

NOMINAL: Female; # automatic dummy coding for nominal predictors

CLUSTERID: Person;

CENTER:

groupmean = SleepQual Pain;

grandmean = SleepQual.mean Pain.mean Stress;

MODEL: PosAffect ~ intercept SleepQual Pain SleepQual.mean Pain.mean Female Stress | intercept;

BURN: 10000;

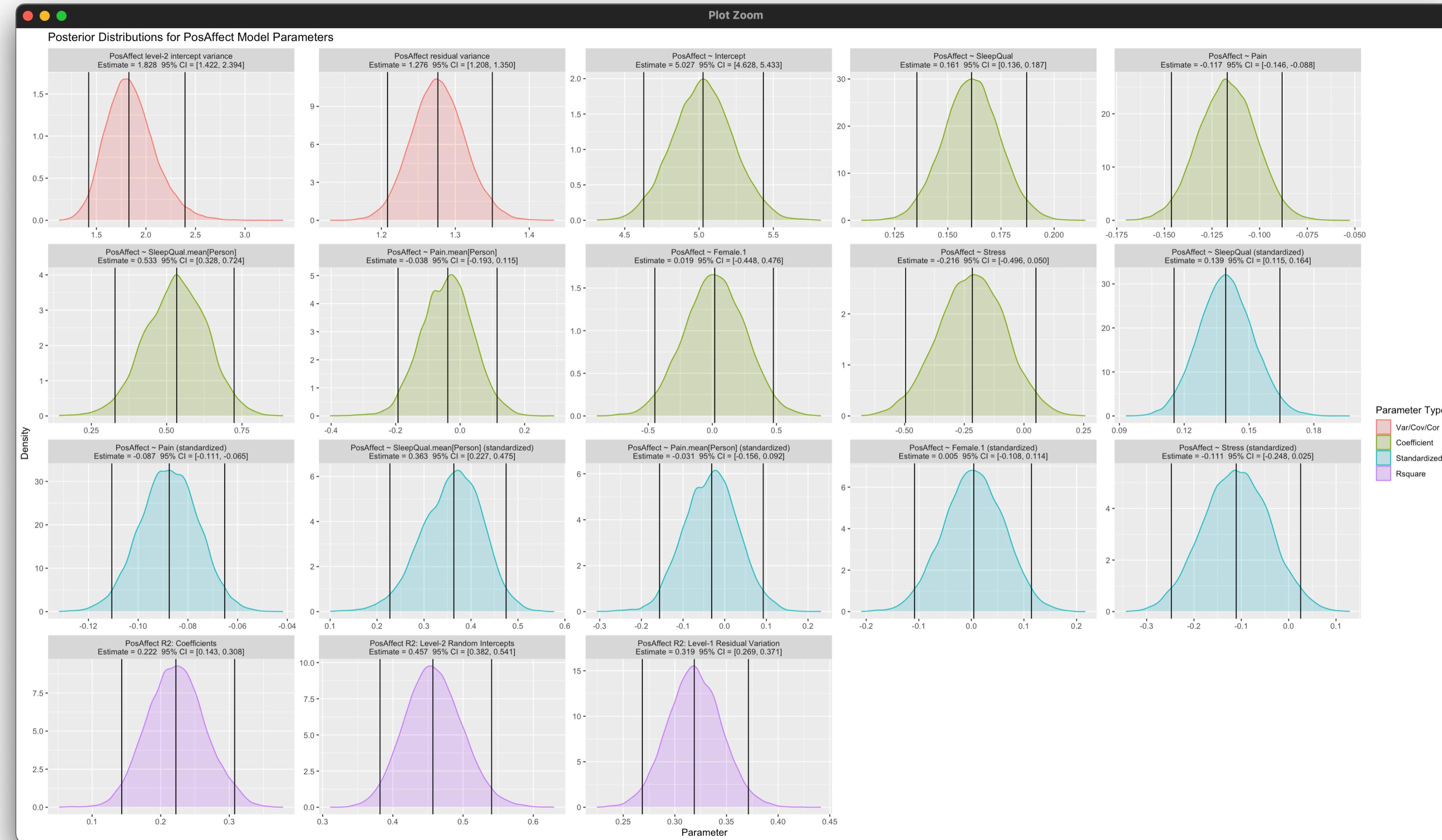
ITERATIONS: 10000;

SEED: 90291;

RBLIMP SCRIPT 4 (MODEL 3)

```
model3 <- rblimp(  
  data = PainDiary,  
  nominal = 'Female',  
  clusterid = 'Person',  
  center = 'groupmean = SleepQual Pain; grandmean = SleepQual.mean Pain.mean Stress',  
  model = 'PosAffect ~ intercept SleepQual Pain SleepQual.mean Pain.mean Female Stress | intercept',  
  seed = 90291,  
  burn = 10000,  
  iter = 10000)  
output(model3)  
plot_posterior(model3, 'PosAffect')
```

PARAMETER PLOTS (RBLIMP ONLY)



BLIMP OUTPUT

- █ = level-2 estimate
- █ = level-1 estimate
- █ = combined estimate

Outcome Variable: PosAffect

Grand Mean Centered: Pain.mean[Person] SleepQual.mean[Person] Stress

Group Mean Centered: Pain SleepQual

| Parameters | Estimate | StdDev | 2.5% | 97.5% | ChiSq | PValue | N_Eff |
|------------------------|---------------|--------|--------|--------|---------|--------|----------|
| <hr/> | | | | | | | |
| Variances: | | | | | | | |
| L2 : Var(Intercept) | 1.828 | 0.249 | 1.412 | 2.398 | --- | --- | 4385.748 |
| Residual Var. | 1.277 | 0.036 | 1.208 | 1.350 | --- | --- | 8247.203 |
| <hr/> | | | | | | | |
| Coefficients: | | | | | | | |
| Intercept | 5.041 | 0.202 | 4.641 | 5.431 | 620.928 | 0.000 | 190.993 |
| SleepQual | 0.161 | 0.013 | 0.135 | 0.187 | 150.704 | 0.000 | 9375.960 |
| Pain | -0.117 | 0.015 | -0.147 | -0.088 | 61.071 | 0.000 | 8973.437 |
| SleepQual.mean[Person] | 0.531 | 0.105 | 0.317 | 0.723 | 25.163 | 0.000 | 145.860 |
| Pain.mean[Person] | -0.039 | 0.078 | -0.187 | 0.116 | 0.247 | 0.620 | 149.240 |
| Female.1 | 0.009 | 0.228 | -0.429 | 0.466 | 0.002 | 0.967 | 194.738 |
| Stress | -0.214 | 0.136 | -0.502 | 0.041 | 2.585 | 0.108 | 176.945 |
| <hr/> | | | | | | | |
| Standard Deviations: | | | | | | | |
| L2 : SD(Intercept) | 1.352 | 0.091 | 1.188 | 1.549 | --- | --- | 4392.623 |
| Residual SD | 1.130 | 0.016 | 1.099 | 1.162 | --- | --- | 8250.812 |

...

FIXED EFFECT INTERPRETATIONS

- $\beta_0 = 5.04$ is the positive affect mean for the female = 0 group (because all numeric predictors are centered)
- $\beta_1 = 0.16$ is the expected affect difference between two daily sleep scores from the same person that differ by one point, controlling for within-person pain
- $\beta_2 = -0.12$ is the expected affect difference between two daily pain scores from the same person that differ by one point, controlling for within-person sleep

FIXED EFFECTS, CONTINUED

- $\beta_3 = 0.53$ is the expected affect difference between two people whose average sleep ratings differ by one point, controlling for average pain, sex, and stress
- $\beta_4 = -0.04$ is the expected affect difference between two people whose average pain ratings differ by one point, controlling for average sleep, sex, and stress
- $\beta_5 = 0.01$ is the expected affect difference for females (versus males), controlling for average sleep, average pain, and stress
- $\beta_6 = -0.21$ is the expected affect difference between two people whose stress scores differ by one point, controlling for average sleep, average pain, and sex

RANDOM EFFECT INTERPRETATIONS

- $u_{0j} = \beta_{0j} - (\beta_0 + \beta_3(\text{sleep}_j^b) + \beta_4(\text{pain}_j^b) + \beta_5(\text{female}_j) + \beta_6(\text{stress}_j))$
- $\text{var}(u_{0j}) = 1.83$ is the average squared distance between the level-2 affect means and the grand mean
- $\text{sd}(u_{0j}) = 1.35$ is the average distance between the level-2 affect means and the grand mean
- $\varepsilon_{ij} = \text{pffect}_{ij} - (\beta_{0j} + \beta_1(\text{sleep}_{ij}^w) + \beta_2(\text{pain}_{ij}^w))$
- $\text{var}(\varepsilon_{ij}) = 1.28$ is the average squared distance between the level-1 affect observations and their predicted values
- $\text{sd}(\varepsilon_{ij}) = 1.13$ is the average distance between the level-1 affect observations and their predicted values

BLIMP OUTPUT, CONTINUED

Outcome Variable: PosAffect

 = level-2 estimate

 = level-1 estimate

 = combined estimate

Grand Mean Centered: Pain.mean[Person] SleepQual.mean[Person] Stress

Group Mean Centered: Pain SleepQual

| Parameters | Estimate | StdDev | 2.5% | 97.5% | ChiSq | PValue | N_Eff |
|------------|----------|--------|------|-------|-------|--------|-------|
| ----- | | | | | | | |
| ... | | | | | | | |

Standardized Coefficients:

| | | | | | | | |
|------------------------|---------------|-------|--------|--------|---------|-------|----------|
| SleepQual | 0.139 | 0.013 | 0.116 | 0.165 | 124.128 | 0.000 | 2565.722 |
| Pain | -0.088 | 0.012 | -0.111 | -0.065 | 55.958 | 0.000 | 4529.435 |
| SleepQual.mean[Person] | 0.362 | 0.066 | 0.218 | 0.473 | 29.402 | 0.000 | 149.255 |
| Pain.mean[Person] | -0.032 | 0.064 | -0.150 | 0.094 | 0.249 | 0.618 | 149.922 |
| Female.1 | 0.002 | 0.055 | -0.104 | 0.112 | 0.002 | 0.966 | 194.588 |
| Stress | -0.110 | 0.068 | -0.252 | 0.021 | 2.635 | 0.105 | 176.893 |

Proportion Variance Explained

| | | | | | | | |
|-------------------------------|--------------|-------|-------|-------|-----|-----|---------|
| by Coefficients | 0.220 | 0.042 | 0.139 | 0.305 | --- | --- | 184.037 |
| by Level-2 Random Intercepts | 0.458 | 0.041 | 0.383 | 0.543 | --- | --- | 418.942 |
| by Level-1 Residual Variation | 0.320 | 0.026 | 0.270 | 0.372 | --- | --- | 496.184 |

EFFECT SIZE INTERPRETATIONS

- $R^2_{(\text{predictors})} = .22$ is the proportion of the total variation explained by all predictors (was $.03$ in previous model)
- $R^2_{(\text{residual-between})} = .46$ is the proportion of the total variation attributable to the between-cluster residuals (the u_{0j} terms)
- $R^2_{(\text{residual-within})} = .32$ is the proportion of the total variation attributable to the within-cluster residuals (the ε_{ij} terms)

MODEL COMPARISON

| Parameter | Empty Model | Level-1 Predictors (Within Only) | Level-2 Predictors (Within + Between) |
|---------------------------------|-------------|-------------------------------------|--|
| Fixed intercept | 5.03 | 5.03 | 5.04 |
| Sleep (within-person) | -- | 0.16 | 0.16 |
| Pain (within-person) | -- | -0.12 | -0.12 |
| Sleep (between-person) | -- | -- | 0.53 |
| Pain (between-person) | -- | -- | -0.04 |
| Female (between-person) | -- | -- | 0.01 |
| Stress (between-person) | -- | -- | -0.21 |
| Residual intercept variance | 2.53 | 2.50 | 1.83 |
| Residual within-person variance | 1.40 | 1.28 | 1.28 |
| R-square predictors | 0 | .03 | .22 |
| R-square intercept variance | .64 | .64 | .46 |
| R-square within-person variance | .36 | .33 | .32 |

OUTLINE

- 1 Modeling Step 1: Estimate ICCs
- 2 Modeling Step 2: Add Within-Cluster Predictors
- 3 Rights and Sterba Effect Sizes
- 4 Modeling Step 3: Add Between-Cluster Predictors
- 5 Latent Variable Specification

BLIMP STUDIO SCRIPT 4.4

DATA: PainDiary.dat;

VARIABLES: Person Day PosAffect NegAffect Pain WorkGoal LifeGoal SleepQual Female Education
Employment MarStatus NumDiagnose ActivityLevel PainAccept Catastrophize Stress Anxiety;

CLUSTERID: Person;

LATENT: Person = beta0j; # define level-2 intercept latent variable

CENTER: groupmean = SleepQual;

MODEL:

beta0j ~ intercept; # level-2 regression equation ($\beta_{0j} = \gamma_{00} + u_{0j}$)

PosAffect ~ intercept@beta0j SleepQual Pain; # level-1: $p\text{affect}_{ij} = \beta_{0j} + \beta_1(\text{sleep}_{ij}^W) + \beta_2(\text{pain}_{ij}^W) + \varepsilon_{ij}$

BURN: 10000;

ITER: 20000;

SEED: 90291;

RBLIMP SCRIPT 4 (MODEL 4)

```
model1 <- rblimp(  
  data = PainDiary,  
  clusterid = 'Person',  
  latent = 'Person = beta0j',  
  center = 'groupmean = SleepQual Pain',  
  model = '  
    beta0j ~ intercept;  
    PosAffect ~ intercept@beta0j SleepQual Pain',  
  seed = 90291,  
  burn = 10000,  
  iter = 20000)  
output(model1)
```

LEVEL-1 OUTPUT

 = level-2 estimate

 = level-1 estimate

Outcome Variable: PosAffect

Group Mean Centered: Pain SleepQual

| Parameters | Median | StdDev | 2.5% | 97.5% | ChiSq | PValue | N_Eff |
|-------------------------------|---------------|--------|--------|--------|---------|--------|-----------|
| <hr/> | | | | | | | |
| Variances: | | | | | | | |
| Residual Var. | 1.276 | 0.036 | 1.209 | 1.349 | --- | --- | 17487.328 |
| <hr/> | | | | | | | |
| Coefficients: | | | | | | | |
| beta0j | @ 1.000 | --- | --- | --- | --- | --- | --- |
| SleepQual | 0.162 | 0.013 | 0.136 | 0.188 | 154.282 | 0.000 | 17775.118 |
| Pain | -0.117 | 0.015 | -0.147 | -0.088 | 61.358 | 0.000 | 18195.499 |
| <hr/> | | | | | | | |
| Standardized Coefficients: | | | | | | | |
| SleepQual | 0.237 | 0.018 | 0.201 | 0.273 | 167.226 | 0.000 | 17168.959 |
| Pain | -0.148 | 0.019 | -0.185 | -0.111 | 63.333 | 0.000 | 18197.357 |
| <hr/> | | | | | | | |
| Proportion Variance Explained | | | | | | | |
| by Coefficients | 0.087 | 0.010 | 0.067 | 0.108 | --- | --- | 17479.089 |
| by Residual Variation | 0.913 | 0.010 | 0.892 | 0.933 | --- | --- | 17479.089 |

LEVEL-2 OUTPUT

 = level-2 estimate

 = level-1 estimate

Latent Variable: beta0j

| Parameters | Median | StdDev | 2.5% | 97.5% | ChiSq | PValue | N_Eff |
|-------------------------------|--------------|--------|-------|-------|----------|--------|-----------|
| <hr/> | | | | | | | |
| Variances: | | | | | | | |
| Residual Var. | 2.489 | 0.330 | 1.946 | 3.237 | --- | --- | 16621.302 |
| <hr/> | | | | | | | |
| Coefficients: | | | | | | | |
| Intercept | 5.034 | 0.140 | 4.756 | 5.310 | 1284.262 | 0.000 | 18856.765 |
| <hr/> | | | | | | | |
| Proportion Variance Explained | | | | | | | |
| by Coefficients | 0.000 | 0.000 | 0.000 | 0.000 | --- | --- | nan |
| by Residual Variation | 1.000 | 0.000 | 1.000 | 1.000 | --- | --- | nan |

BLIMP STUDIO SCRIPT 4.5

DATA: PainDiary.dat;

VARIABLES: Person Day PosAffect NegAffect Pain WorkGoal LifeGoal SleepQual Female Education
Employment MarStatus NumDiagnose ActivityLevel PainAccept Catastrophize Stress Anxiety;

NOMINAL: Female;

CLUSTERID: Person;

LATENT: Person = beta0j; # define level-2 intercept latent variable

CENTER: groupmean = SleepQual; grandmean = SleepQual.mean Pain.mean Stress;

MODEL: # level-2: $\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{sleep}_j^b) + \gamma_{02}(\text{pain}_j^b) + \gamma_{03}(\text{female}_j) + \gamma_{04}(\text{stress}_j) + u_{0j}$

beta0j ~ intercept SleepQual.mean Pain.mean Female Stress;

PosAffect ~ intercept@beta0j SleepQual Pain; # level-1: $p\text{affect}_{ij} = \beta_{0j} + \beta_1(\text{sleep}_{ij}^w) + \beta_2(\text{pain}_{ij}^w) + \varepsilon_{ij}$

BURN: 10000;

ITER: 20000;

SEED: 90291;

RBLIMP SCRIPT 4 (MODEL 5)

```
model3 <- rblimp(  
  data = PainDiary,  
  nominal = 'Female',  
  clusterid = 'Person',  
  latent = 'Person = beta0j',  
  center = 'groupmean = SleepQual Pain; grandmean = SleepQual.mean Pain.mean Stress',  
  model = '  
    beta0j ~ intercept SleepQual.mean Pain.mean Female Stress;  
    PosAffect ~ intercept@beta0j SleepQual Pain',  
  seed = 90291,  
  burn = 10000,  
  iter = 20000)  
output(model3)
```

LEVEL-1 OUTPUT

 = level-2 estimate

 = level-1 estimate

Outcome Variable: PosAffect

Group Mean Centered: Pain SleepQual

| Parameters | Median | StdDev | 2.5% | 97.5% | ChiSq | PValue | N_Eff |
|-------------------------------|---------------|--------|--------|--------|---------|--------|-----------|
| <hr/> | | | | | | | |
| Variances: | | | | | | | |
| Residual Var. | 1.277 | 0.036 | 1.208 | 1.349 | --- | --- | 18773.456 |
| <hr/> | | | | | | | |
| Coefficients: | | | | | | | |
| beta0j | @ 1.000 | --- | --- | --- | --- | --- | --- |
| SleepQual | 0.161 | 0.013 | 0.136 | 0.187 | 154.038 | 0.000 | 16824.474 |
| Pain | -0.117 | 0.015 | -0.147 | -0.088 | 60.356 | 0.000 | 18396.872 |
| <hr/> | | | | | | | |
| Standardized Coefficients: | | | | | | | |
| SleepQual | 0.236 | 0.018 | 0.200 | 0.272 | 167.294 | 0.000 | 17192.061 |
| Pain | -0.149 | 0.019 | -0.185 | -0.112 | 62.454 | 0.000 | 18410.177 |
| <hr/> | | | | | | | |
| Proportion Variance Explained | | | | | | | |
| by Coefficients | 0.086 | 0.011 | 0.067 | 0.108 | --- | --- | 17786.052 |
| by Residual Variation | 0.914 | 0.011 | 0.892 | 0.933 | --- | --- | 17786.052 |

LEVEL-2 OUTPUT

 = level-2 estimate

 = level-1 estimate

Latent Variable: beta0j

Grand Mean Centered: Pain.mean[Person] SleepQual.mean[Person]
Stress

| Parameters | Median | StdDev | 2.5% | 97.5% | ChiSq | PValue | N_Eff |
|-------------------------------|---------------|--------|--------|-------|---------|--------|-----------|
| <hr/> | | | | | | | |
| Variances: | | | | | | | |
| Residual Var. | 1.835 | 0.250 | 1.423 | 2.399 | --- | --- | 15231.617 |
| <hr/> | | | | | | | |
| Coefficients: | | | | | | | |
| Intercept | 5.039 | 0.216 | 4.608 | 5.466 | 541.498 | 0.000 | 8572.715 |
| SleepQual.mean[Person] | 0.535 | 0.101 | 0.337 | 0.734 | 27.999 | 0.000 | 11316.753 |
| Pain.mean[Person] | -0.038 | 0.081 | -0.198 | 0.123 | 0.219 | 0.640 | 14391.068 |
| Female.1 | -0.008 | 0.259 | -0.512 | 0.500 | 0.001 | 0.972 | 16962.379 |
| Stress | -0.215 | 0.132 | -0.473 | 0.042 | 2.664 | 0.103 | 16321.312 |
| <hr/> | | | | | | | |
| Standardized Coefficients: | | | | | | | |
| SleepQual.mean[Person] | 0.454 | 0.075 | 0.294 | 0.587 | 35.928 | 0.000 | 11569.228 |
| Pain.mean[Person] | -0.039 | 0.082 | -0.198 | 0.123 | 0.221 | 0.638 | 14296.578 |
| Female.1 | -0.002 | 0.077 | -0.153 | 0.149 | 0.001 | 0.971 | 16956.623 |
| Stress | -0.136 | 0.082 | -0.291 | 0.026 | 2.738 | 0.098 | 16320.026 |
| <hr/> | | | | | | | |
| Proportion Variance Explained | | | | | | | |
| by Coefficients | 0.298 | 0.063 | 0.174 | 0.418 | --- | --- | 12568.753 |
| by Residual Variation | 0.702 | 0.063 | 0.582 | 0.826 | --- | --- | 12568.753 |