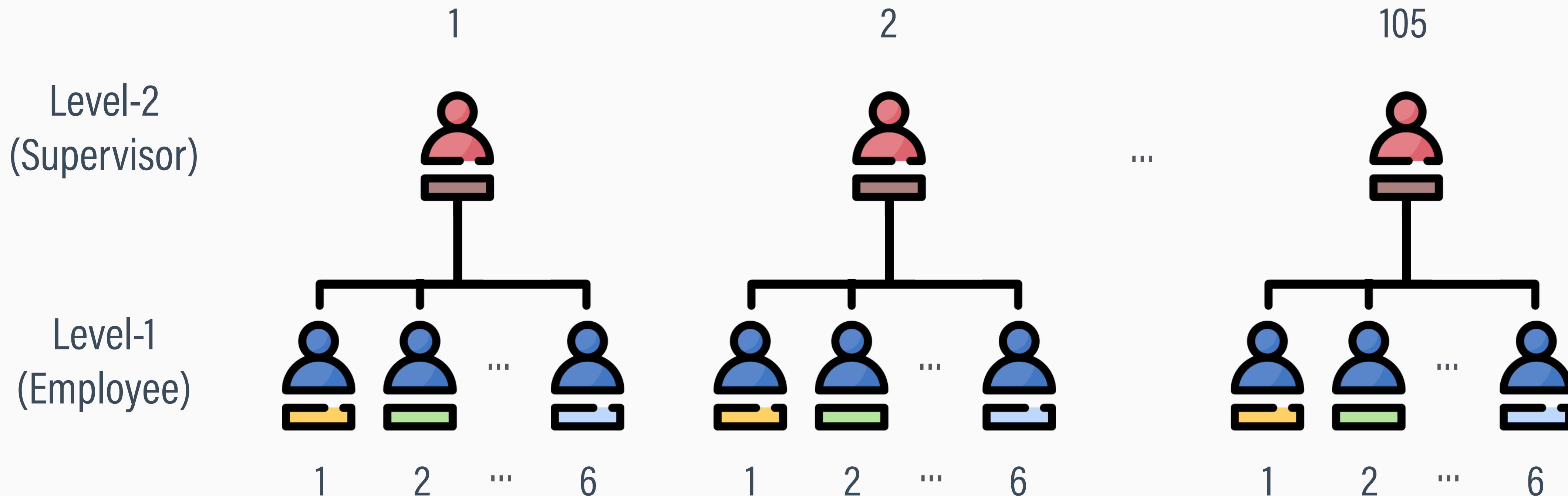


# **MODULE 5**

## **RANDOM COEFFICIENT MODELS**

# ORGANIZATIONAL APPLICATION

- $n_j = 6$  employees at level-1 nested within  $J = 105$  teams or workgroups at level-2 ( $N = 630$  data records in total)



# VARIABLE INFORMATION

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Variable	Definition	Level	Scale
Team	Team-level (level-2) identifier	2	Integers (1 to 105)
JobSat	Job satisfaction rating	1	Ordinal (1 to 7)
LMX	Leader-member exchange (supervisor-supervisee relationship quality)	1	Numeric (0 to 17)
Empower	Employee empowerment	1	Numeric (14 to 42)
Male	Male dummy code	1	Female = 0, Male = 1

# DATA STRUCTURE

- Stacked (long) data format where each level-2 unit (supervisor) has one row per level-1 (supervisee) observation
- The  $i$  subscript indexes level-1 observations, and  $j$  indexes level-2 units
- Variables measured at level-2 repeat across all rows within a cluster

Row	i	j	JobSat <sub>ij</sub>	LMX <sub>ij</sub>	Climate <sub>ij</sub>	
1	1	1	1.0	5.6	7.3	Team 1
2	2	1	2.0	4.3	7.3	
...	...	1	...	...	...	
6	6	1	5.0	7.3	7.3	
7	1	2	7.0	3.9	4.0	Team 2
8	2	2	3.0	7.1	4.0	
...	...	2	...	...	...	
12	6	2	6.0	3.5	4.0	
...	...	...	...	...	...	
625	1	105	2.0	5.4	3.3	Team 105
626	2	105	3.0	3.5	3.3	
...	...	105	...	...	...	
630	6	105	1.0	7.9	3.3	

# RANDOM SLOPE COEFFICIENTS

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- Thus far, we have fit random intercept models where the outcome mean (intercept) varied across clusters
- Random slope or random coefficient models allow the influence of a predictor to vary across level-2 units
- Essentially, random slopes are akin to interaction effect where the predictor's influence varies across level-2 groups

# CONCEPTUAL MODEL

---

-  Measured at level-2 (team)
-  Measured at level-1 (person)

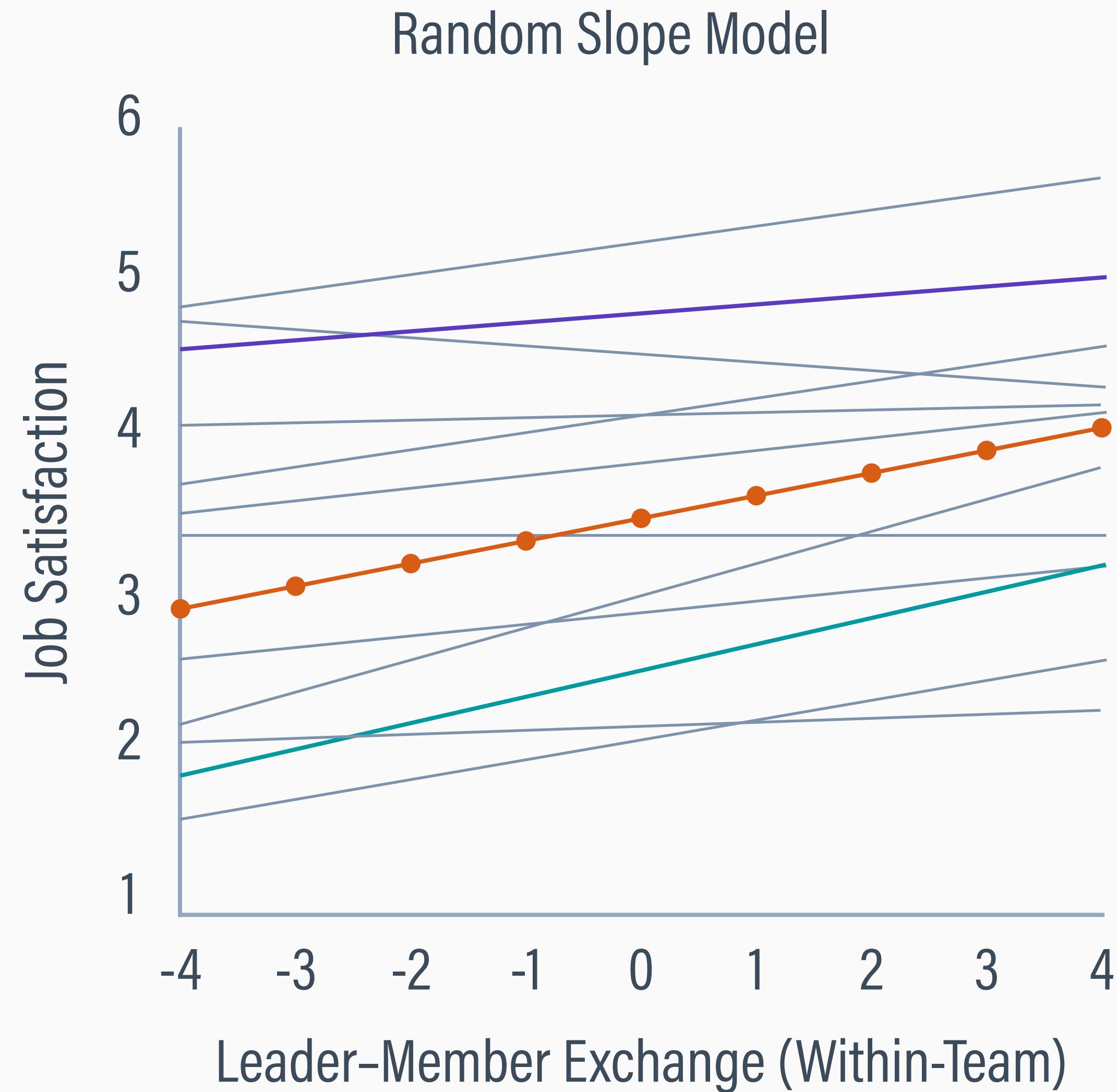
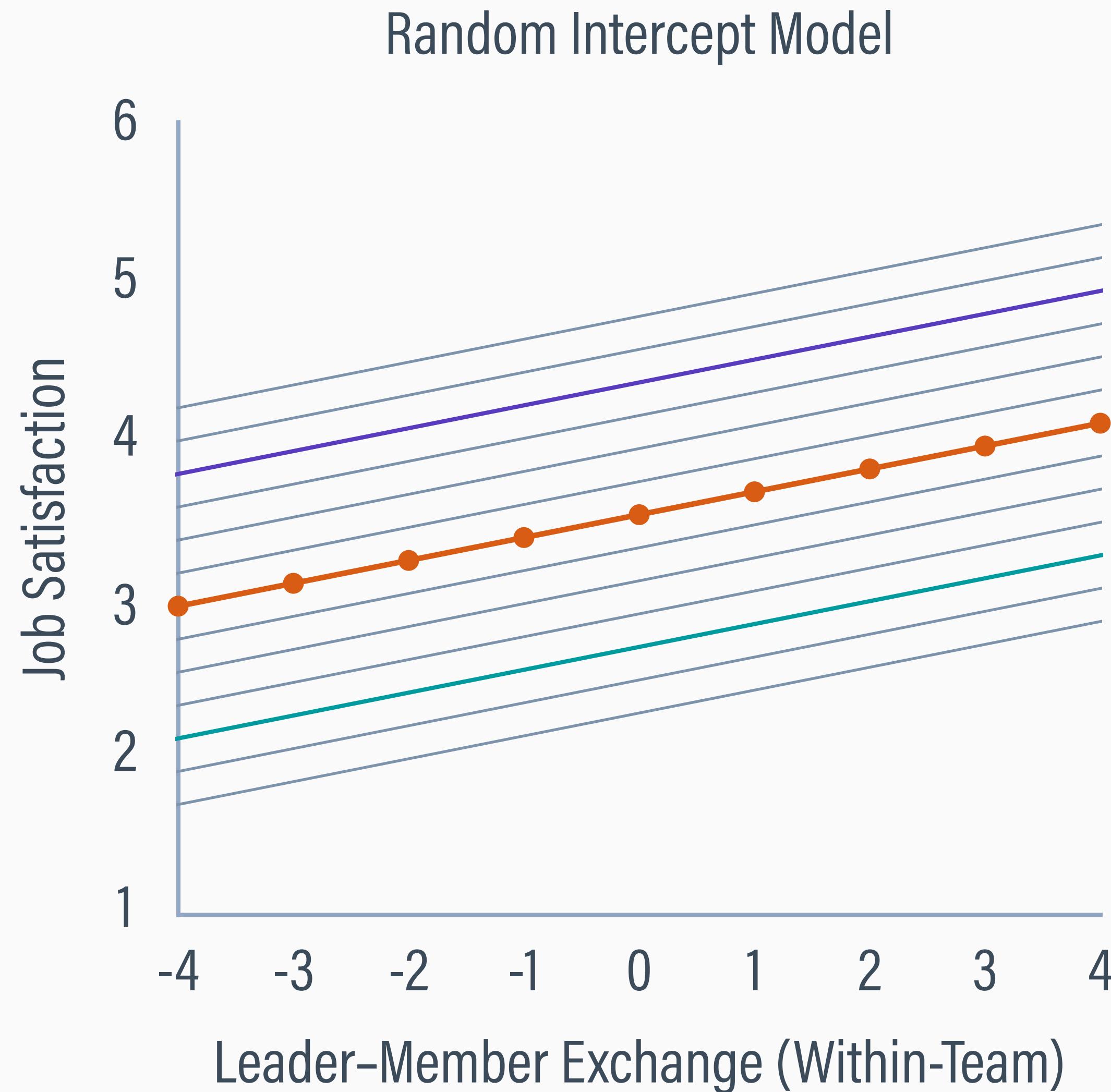
- The conceptual model includes three level-1 predictors

$$\text{jobsat} = \text{mean} + \text{lmx} + \text{empowerment} + \text{gender}$$

- The level-1 predictors may require disaggregation
- The effects of the level-1 predictors may vary across level-2 units (slope coefficients could be “random”)

# MODELING OVERVIEW

---



# MODEL-BUILDING STEPS

---

- The methods literature often recommends a multistep model-building procedure that begins at level-1

1) Estimate ICCs

2) Add predictors measured at level-1

3) Add random coefficients for level-1 predictors

4a) Add cluster means of level-1 predictors

4b) Add predictors measured at level-2

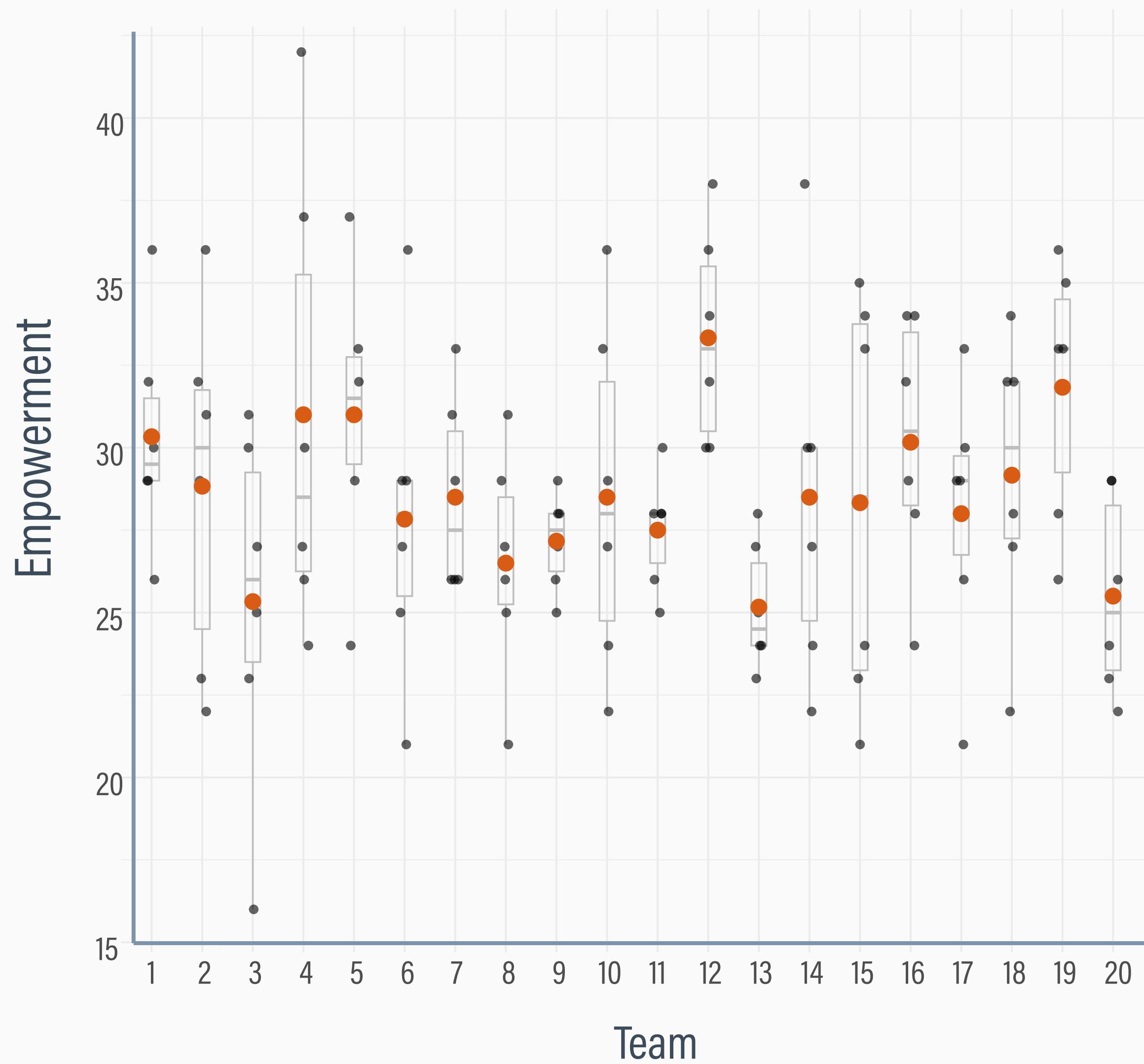
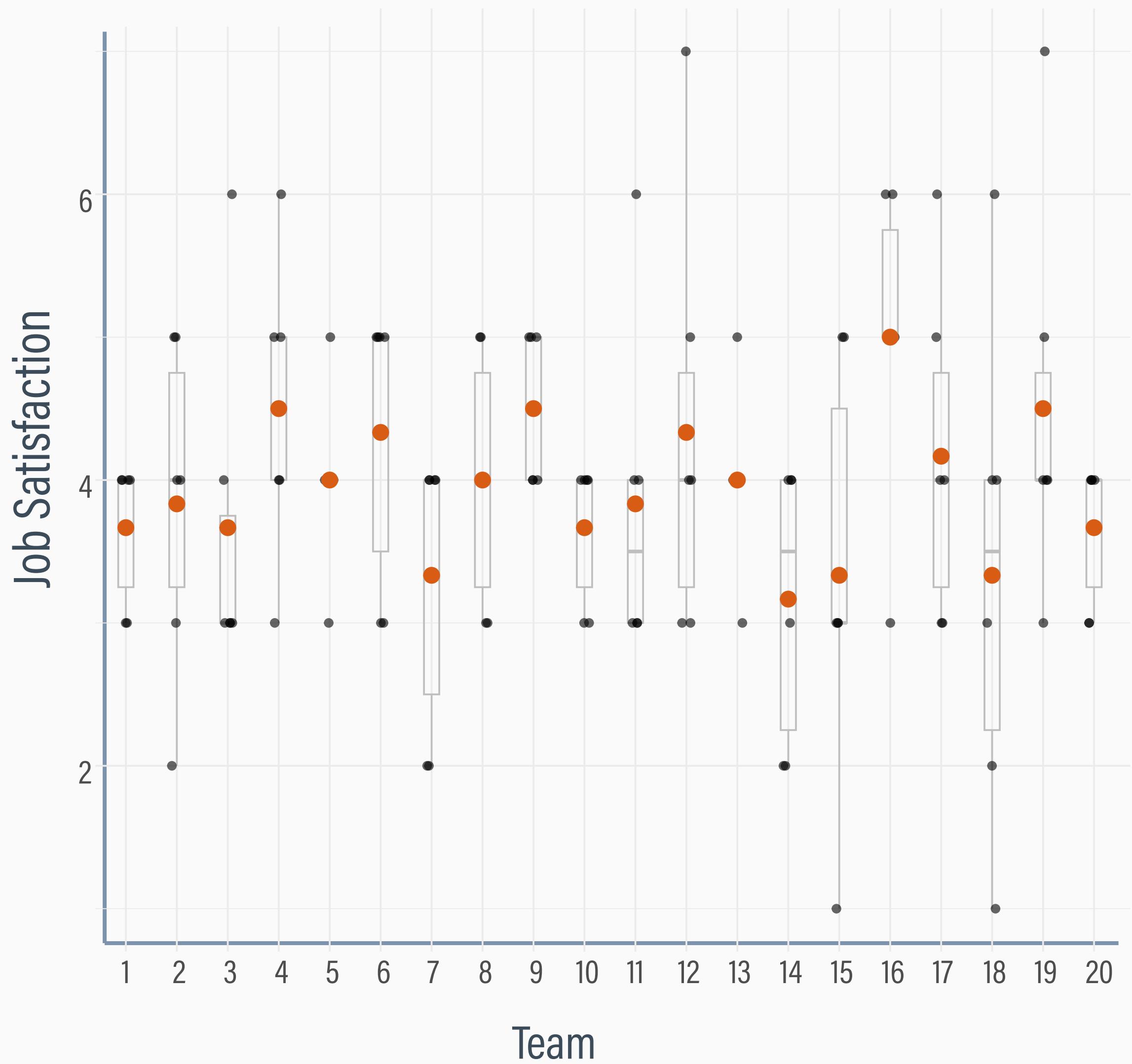
} May be combined into a single step

# OUTLINE

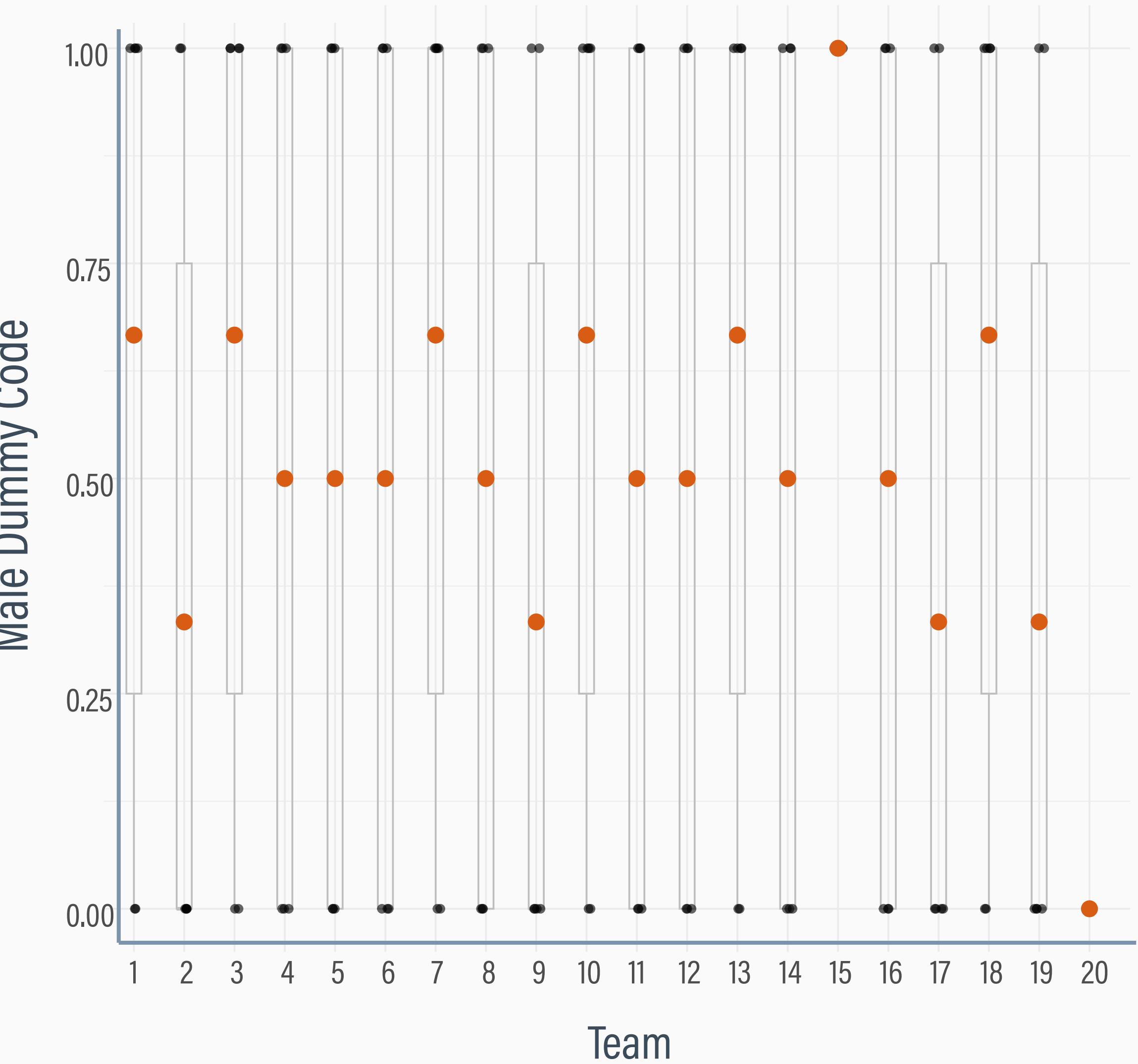
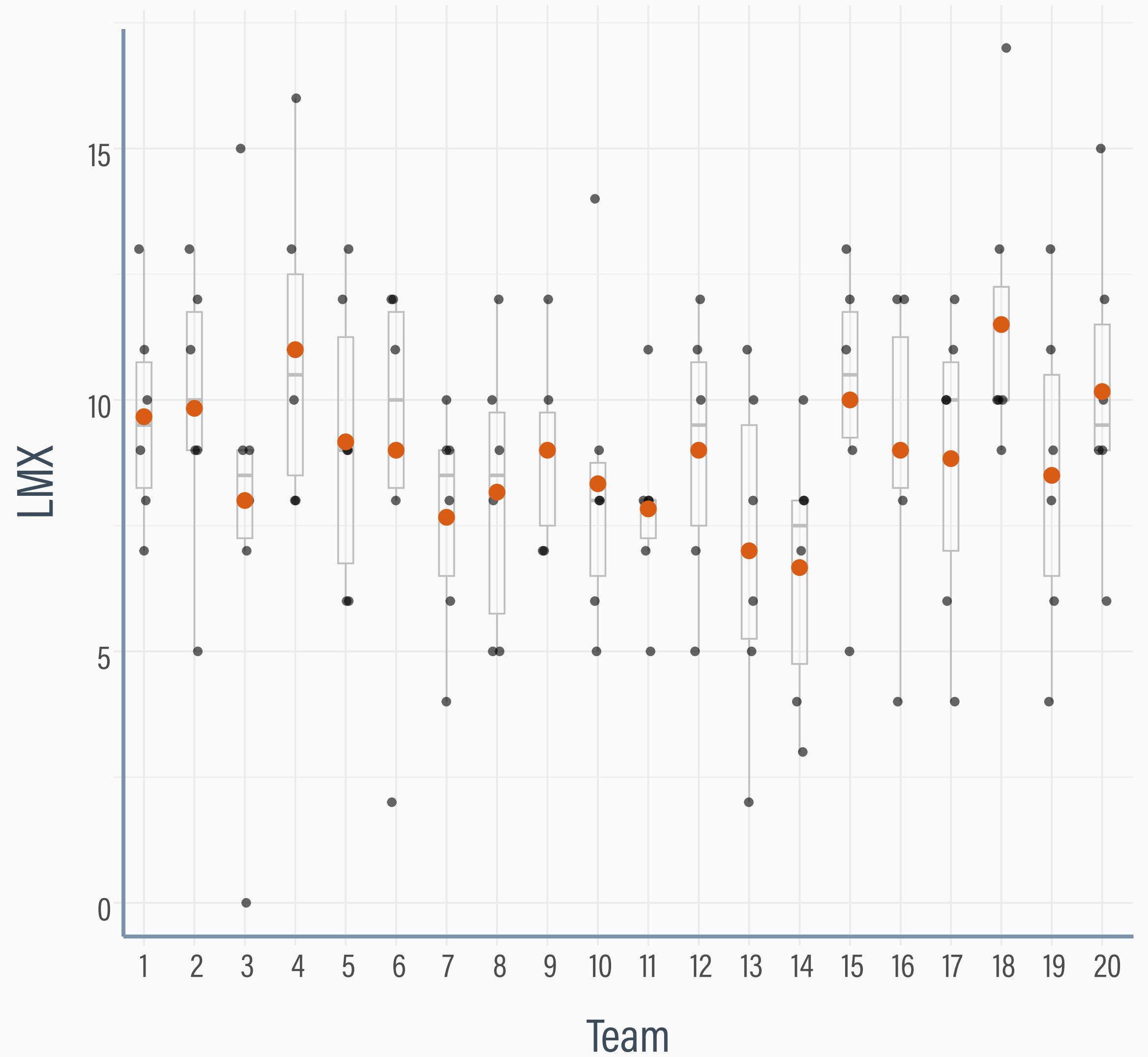
- 1 Estimate ICCs
- 2 Add Within-Cluster Predictors
- 3 Strategy 1: Evaluate Random Slopes One at a Time
- 4 Strategy 2: Add All Random Slopes at Once
- 5 Add Between-Cluster Predictors
- 6 Latent Variable Specification

# BOX PLOTS

---



# BOX PLOTS, CONTINUED



# BLIMP SCRIPT 5.1

---

**DATA:** EmployeeSatisfaction.dat;

**VARIABLES:** Employee Team Turnover Male Empower LMX JobSat Climate TeamPerf;

**CLUSTERID:** Team;

**NOMINAL:** Male; # invokes logistic regression model

**MODEL:**

{ JobSat LMX Empower Male } ~ intercept | intercept; # { } applies the same model all all variables

**BURN:** 20000;

**ITERATIONS:** 10000;

**SEED:** 90291;

# RBLIMP SCRIPT 5 (MODEL 1)

---

```
model1 <- rblimp(  
  data = Employee,  
  nominal = 'Male',  
  clusterid = 'Team',  
  model = '{ JobSat LMX Empower Male } ~ intercept | intercept',  
  seed = 90291,  
  burn = 10000,  
  iter = 20000)  
  
output(model1)
```

# BLIMP OUTPUT

 = level-2 estimate

 = level-1 estimate

# Outcome Variable: Empower

Parameters	Median	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
<hr/>							
Variances:							
L2 : Var(Intercept)	2.562	0.826	1.254	4.458	---	---	2191.061
Residual Var.	18.098	1.127	16.067	20.441	---	---	9768.133
<hr/>							
Coefficients:							
Intercept	28.613	0.232	28.160	29.070	15190.266	0.000	7213.627
<hr/>							
Proportion Variance Explained							
by Coefficients	0.000	0.000	0.000	0.000	---	---	nan
by Level-2 Random Intercepts	0.124	0.036	0.062	0.203	---	---	2133.810
by Level-1 Residual Variation	0.876	0.036	0.797	0.938	---	---	2133.810

# BLIMP OUTPUT

 = level-2 estimate

 = level-1 estimate

# Outcome Variable: JobSat

Parameters	Median	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
<hr/>							
Variances:							
L2 : Var(Intercept)	<b>0.190</b>	0.064	0.082	0.332	---	---	1568.277
Residual Var.	<b>1.416</b>	0.089	1.254	1.606	---	---	7093.805
<hr/>							
Coefficients:							
Intercept	<b>3.990</b>	0.064	3.866	4.116	3927.745	0.000	7631.814
<hr/>							
Proportion Variance Explained							
by Coefficients	0.000	0.000	0.000	0.000	---	---	nan
by Level-2 Random Intercepts	<b>0.118</b>	0.037	0.052	0.195	---	---	1500.756
by Level-1 Residual Variation	<b>0.882</b>	0.037	0.805	0.948	---	---	1500.756

# BLIMP OUTPUT

 = level-2 estimate

 = level-1 estimate

## Outcome Variable: LMX

Parameters	Median	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
<hr/>							
Variances:							
L2 : Var(Intercept)	1.329	0.389	0.685	2.221	---	---	2411.923
Residual Var.	7.904	0.493	7.013	8.938	---	---	10542.820
<hr/>							
Coefficients:							
Intercept	9.594	0.160	9.278	9.909	3578.424	0.000	6155.384
<hr/>							
Proportion Variance Explained							
by Coefficients	0.000	0.000	0.000	0.000	---	---	nan
by Level-2 Random Intercepts	0.144	0.038	0.077	0.226	---	---	2363.170
by Level-1 Residual Variation	0.856	0.038	0.774	0.923	---	---	2363.170

# BLIMP OUTPUT

 = level-2 estimate

 = level-1 estimate

Outcome Variable: Male

Parameters	Median	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
<hr/>							
Male.1							
Variances:							
L2 : Var(Intercept)	<b>0.107</b>	0.099	0.006	0.373	---	---	212.741
Coefficients:							
Intercept	<b>-0.064</b>	0.088	-0.239	0.108	0.537	0.464	13628.593
Odds Ratio:							
Intercept	0.938	0.083	0.787	1.114	---	---	13657.539
Proportion Variance Explained							
by Coefficients	0.000	0.000	0.000	0.000	---	---	nan
by Level-2 Random Intercepts	<b>0.032</b>	0.027	0.002	0.102	---	---	199.871
by Level-1 Residual Variation	<b>0.968</b>	0.027	0.898	0.998	---	---	199.871

---

# ICC SUMMARY

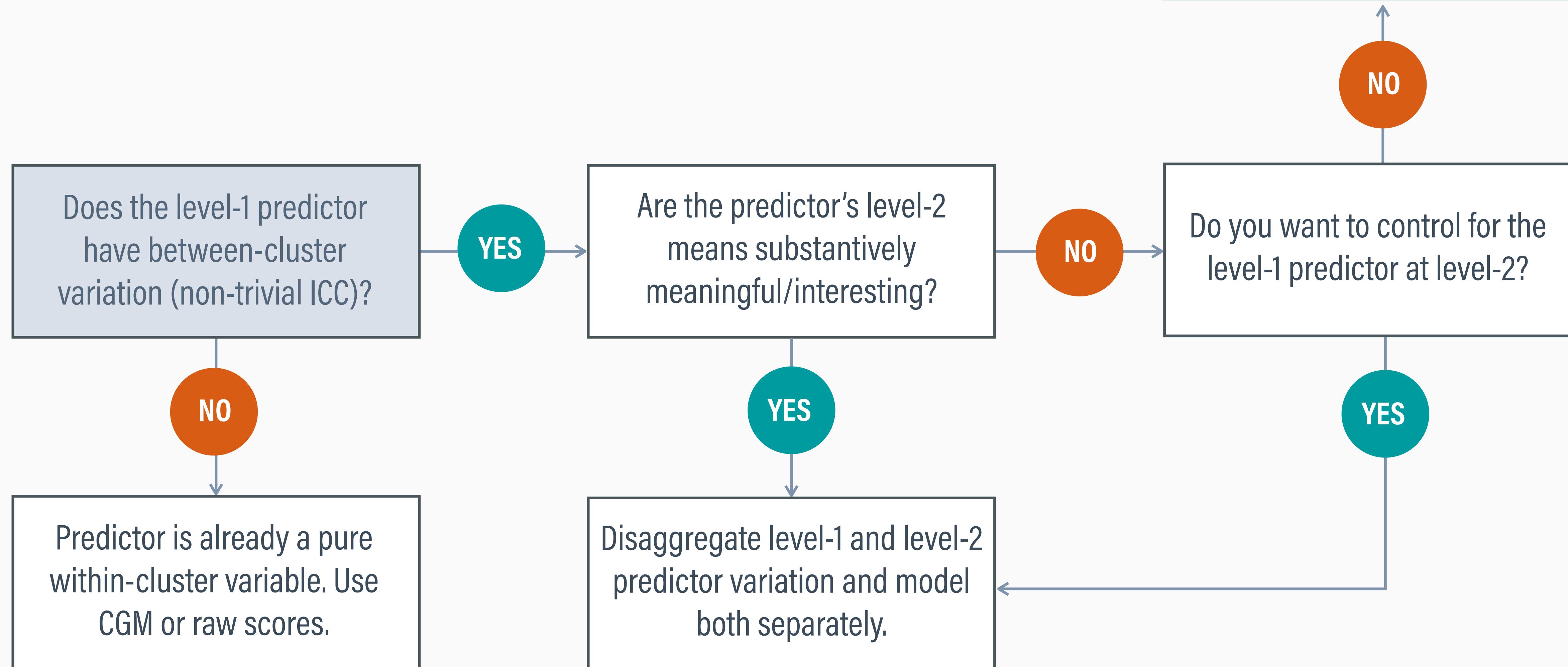
---

- The outcome has nontrivial between-person variation in the team-specific means ( $ICC_{\text{jobsat}} = .12$ ), so we need MLM
- Two level-1 predictors (LMX and empowerment) have  $> 10\%$  team-level mean variation ( $ICC_{\text{lmx}} = .14$  and  $ICC_{\text{empower}} = .12$ )
- Gender variation is comprised of 3% team-level mean variation
- The predictor ICCs suggest the need for disaggregation (there could be unique associations at level-1 and level-2)



The ICC for gender is .03. Gender is dummy coded such that female = 0 and male = 1. In small groups of two or three, discuss the meaning of the team-level gender averages (random intercepts). What does it mean for gender to have a non-zero ICC?

# DISAGGREGATION DECISION TREE



# SUMMARY, CONTINUED

---

- The gender dummy code has a relatively low ICC at .03
- Because the within-team sample sizes are small ( $n_j = 6$ ), latent group means will be very noisy
- Estimating latent means from such a small amount of variation can cause convergence issues, and invoking a categorical variable model likely exacerbates the issue
- This variable is a good candidate for grand mean centering

# OUTLINE

- 1 Estimate ICCs
- 2 Add Within-Cluster Predictors
- 3 Strategy 1: Evaluate Random Slopes One at a Time
- 4 Strategy 2: Add All Random Slopes at Once
- 5 Add Between-Cluster Predictors
- 6 Latent Variable Specification

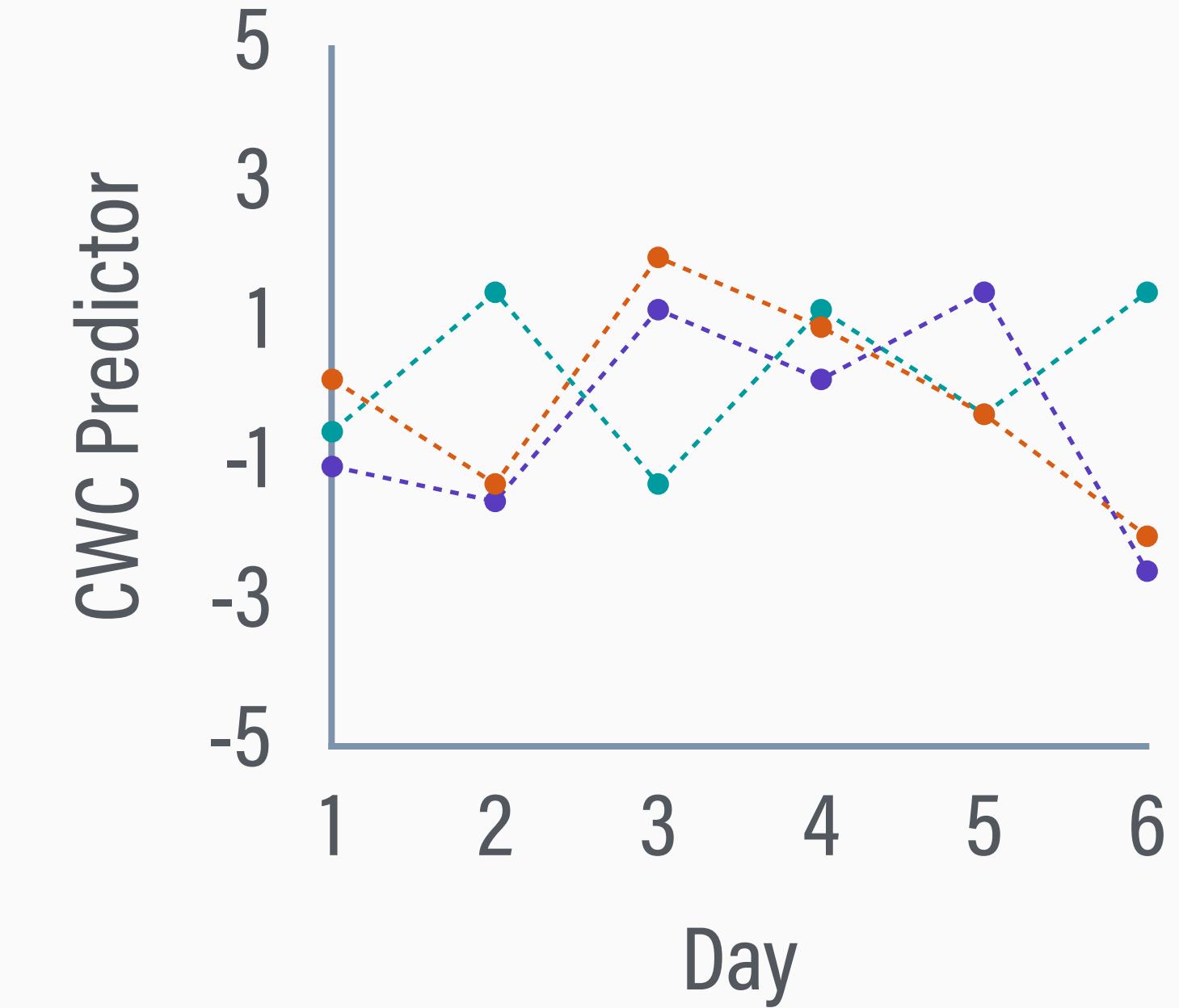
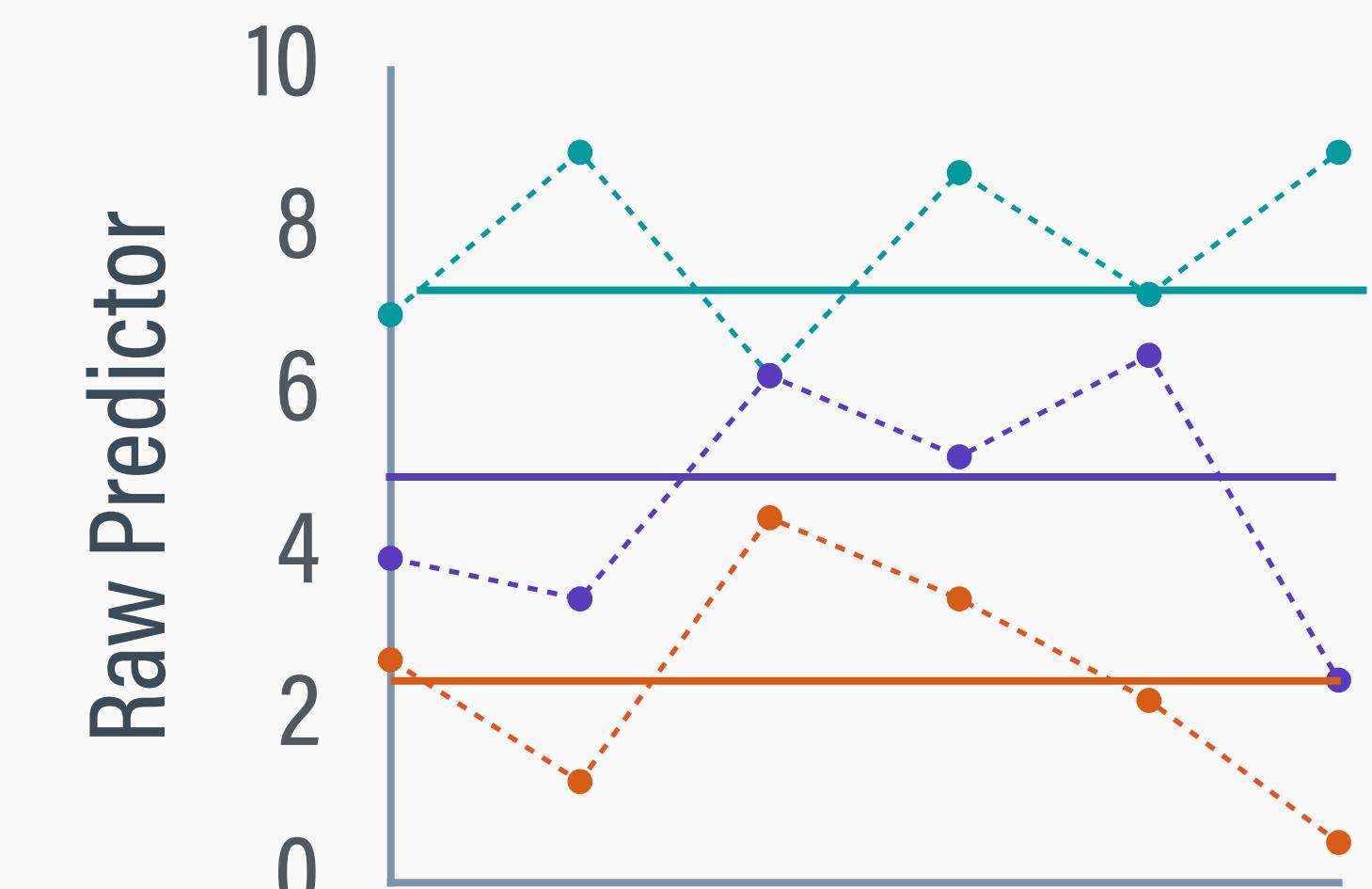
# CENTERING WITHIN CLUSTER (CWC)

- Pure within-cluster predictors are created by deviating (centering) each person's score at its level-2 team mean

$$lmx_{ij}^w = lmx_{ij} - \mu_j(lmx)$$

$$\text{empower}_{ij}^w = \text{empower}_{ij} - \mu_j(\text{empower})$$

- The resulting deviation scores contain only within-team (level-1) variation



# CENTERING BINARY PREDICTORS

---

- Binary predictors can also possess cluster-level mean variation, so centering considerations are identical
- It is absolutely appropriate to disaggregate dummy codes into separate within- and between-cluster predictors
- The ICC, not the variable's metric, informs disaggregation

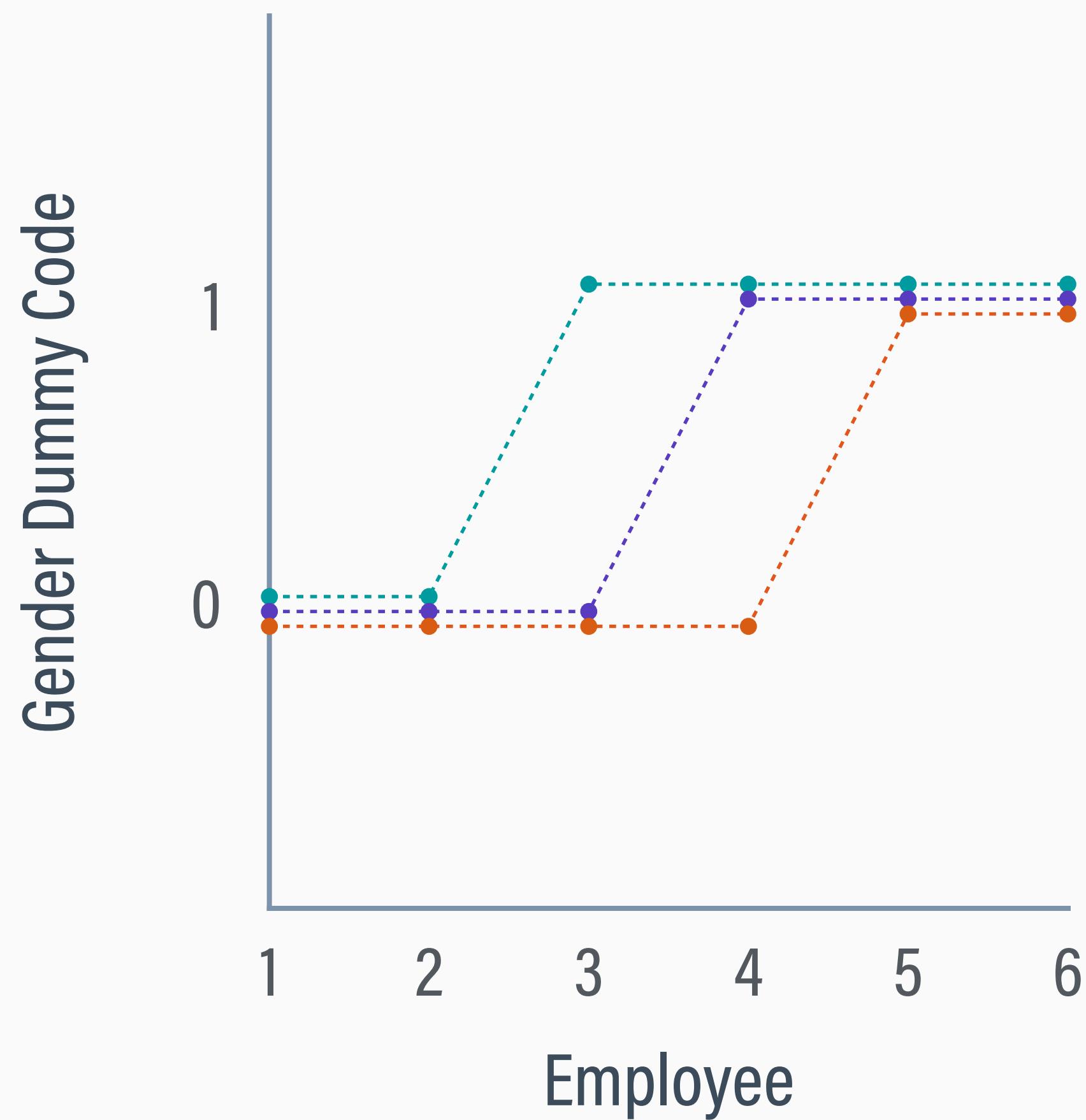
# INTERPRETING CLUSTER MEANS

- Cluster means are the proportions of males (1s) in each team (gender composition)

$$\text{male}_{ij}^w = \text{male}_{ij} - \mu_{j(\text{male})}$$

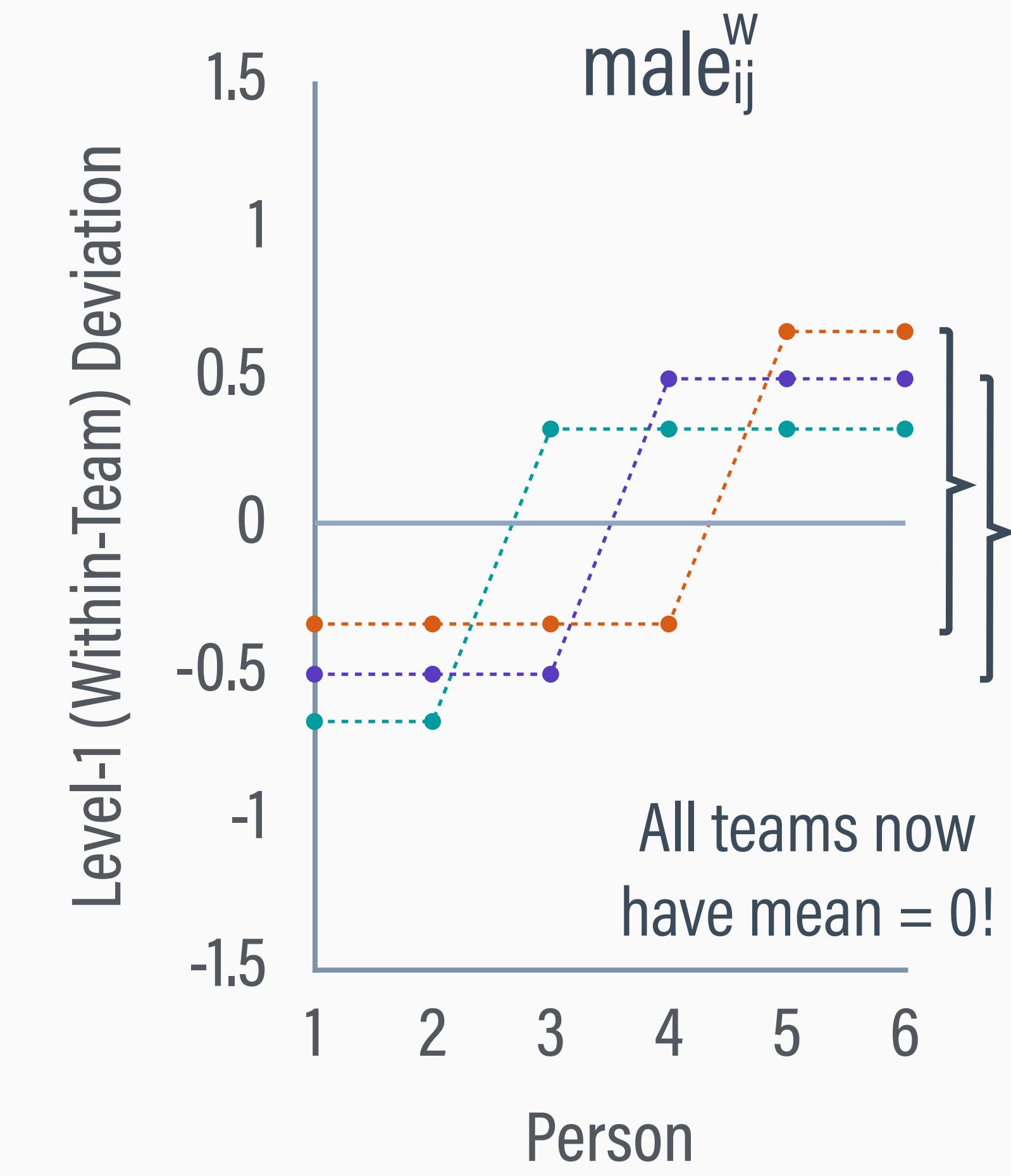
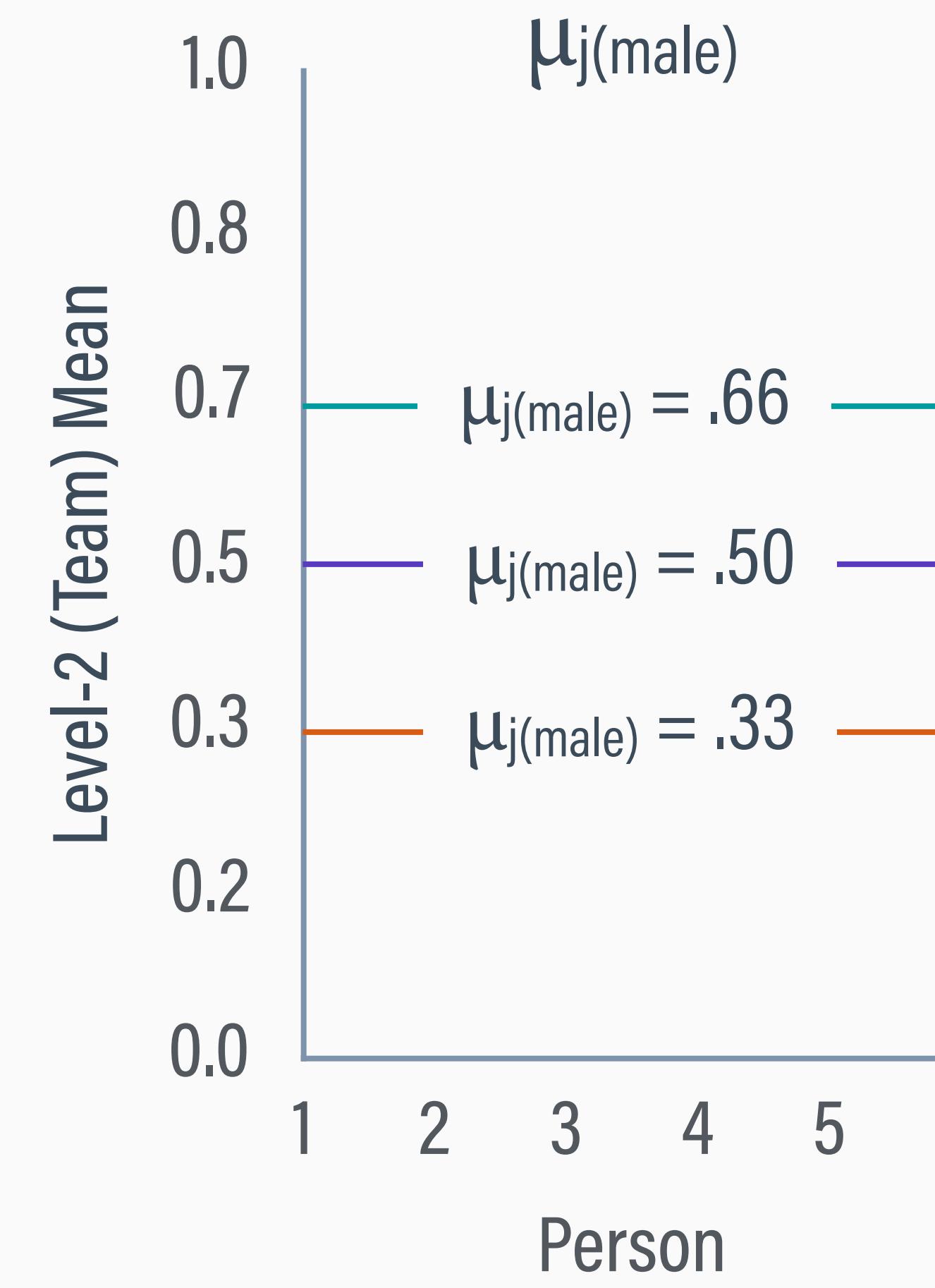
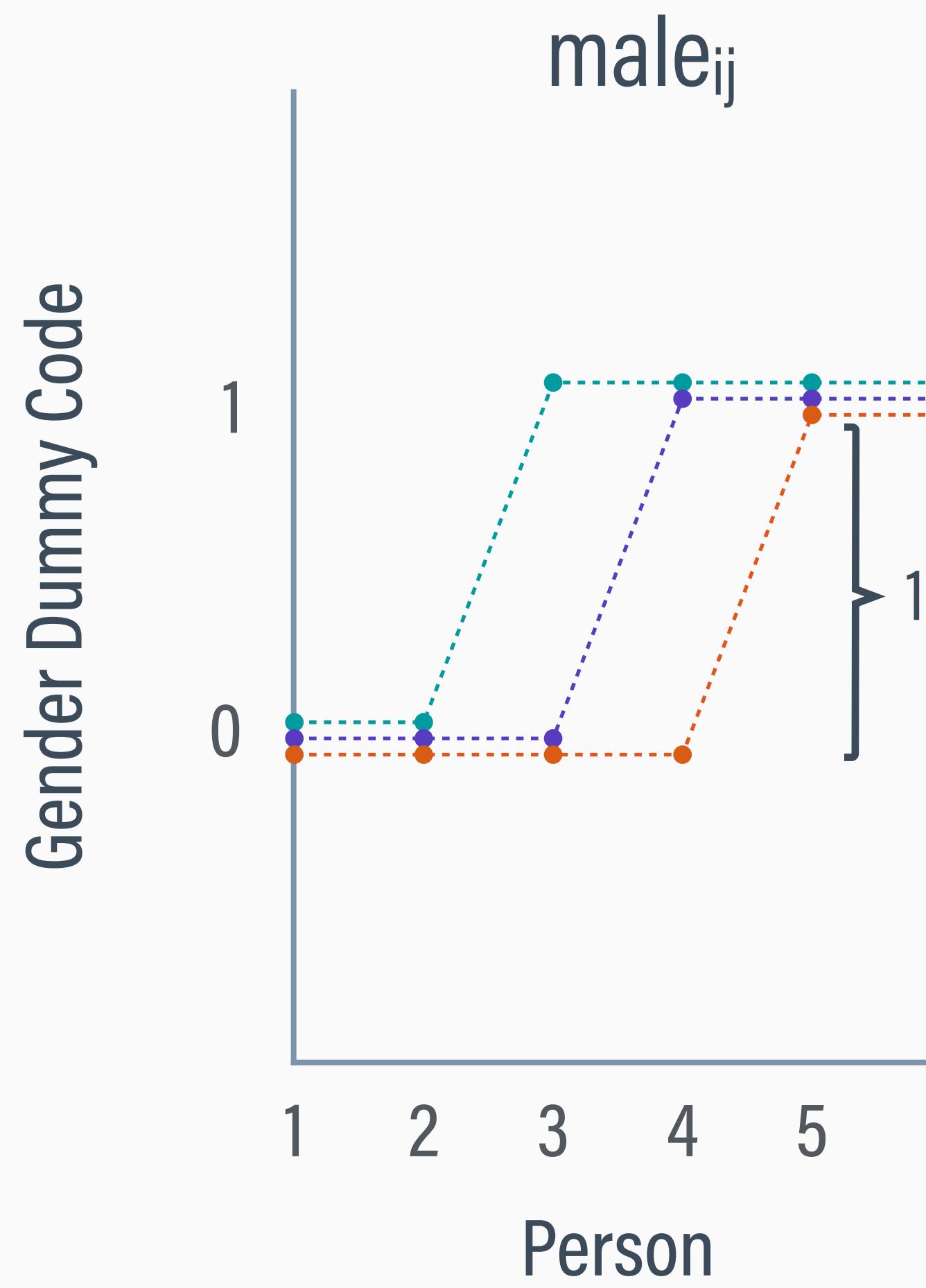
- In this example,  $\mu_{j(\text{male})} = .33, .50, \text{ and } .66$
- Centering within team removes team-level differences in gender composition

- = Team with 66% males
- = Team with 50% males
- = Team with 33% males



# WITHIN-CLUSTER CENTERING A DUMMY CODE

Level-1 Dummy Code - Level-2 Team Mean (Proportion of 1s) = Pure Level-1 (Within-Team) Code





Consider the following disaggregated regression model with disaggregated gender predictor. Provide a brief interpretation of the  $\beta_1$  and  $\beta_2$  slope coefficients.

$$\begin{aligned} \text{jobsat}_{ij} &= \beta_0 + \beta_1(\text{male}_{ij}^W) + \beta_2(\mu_{j(\text{male})}) + u_{0j} + \varepsilon_{ij} \\ &= 4 + .30(\text{male}_{ij}^W) + .60(\mu_{j(\text{male})}) + u_{0j} + \varepsilon_{ij} \end{aligned}$$

# PRACTICAL CONSIDERATIONS

---

- The bias due to “smushing” level-1 and level-2 slopes into a single slope (no disaggregation) is minimal with small ICCs
- Estimating the level-2 means (latent variables) is challenging when the ICC is small because there isn’t much variation to work with at level-2 (greater chance of convergence failures)
- Level-2 means are very noisy when the ICC is small and the number of observations per cluster is small (lower power)

# WITHIN-CLUSTER (LEVEL-1) MODEL

---

- Job satisfaction rating  $i$  for team  $j$  is the sum of a level-2 mean ( $\beta_{0j}$ ), fixed effects due to within-team predictors ( $\beta_1$  through  $\beta_3$ ), and a within-team residual ( $\varepsilon_{ij}$ )

$$\text{jobsat}_{ij} = \beta_{0j} + \beta_1(\text{Imx}_{ij}^W) + \beta_2(\text{empower}_{ij}^W) + \beta_3(\text{male}_{ij}) + \varepsilon_{ij}$$

- Assumption: residuals are normal with constant variation across all days (level-1 units) and persons (level-2 units)

$$\varepsilon_{ij} \sim N(0, \sigma_\varepsilon^2)$$

# BETWEEN-CLUSTER (LEVEL-2) MODEL

---

- Slopes are constant across level-2 units, but the job satisfaction mean for team  $j$  ( $\beta_{0j}$ ) is the sum of the grand mean ( $\gamma_{00}$ ) and a between-team residual ( $u_{0j}$ )
- Assumption: random intercept residuals are normal with constant variation across teams (level-2 units)

$$\beta_{0j} = \gamma_{00} + u_{0j}$$

$$\beta_1 = \gamma_{10}$$

$$\beta_2 = \gamma_{20}$$

$$\beta_3 = \gamma_{30}$$

$$u_{0j} \sim N(0, \sigma_u^2)$$

# DECODING THE SUBSCRIPTS

---

The second subscript numbers the coefficients on the right side starting at 0 (the intercept)

$$\beta_{0j} = \gamma_{00} + u_{0j}$$

$$\beta_{1j} = \gamma_{10}$$

$$\beta_{2j} = \gamma_{20}$$

$$\beta_{3j} = \gamma_{30}$$

The first subscript tells which level-2 outcome variable these terms are attached to ( $\gamma_{00}$  and  $u_{0j}$  belong in  $\beta_{0j}$ 's equation,  $\gamma_{10}$  is attached to  $\beta_1$ )

# COMBINED-MODEL EQUATION

Substituting the right sides  
of the level-2 equations ...

into their coefficients  
from the level-1 equation

$$\beta_{0j} = \gamma_{00} + u_{0j}$$

$$\beta_1 = \gamma_{10}$$

$$\beta_2 = \gamma_{20}$$

$$\beta_3 = \gamma_{30}$$

$$\text{jobsat}_{ij} = \beta_{0j} + \beta_1(\text{lmx}_{ij}^W) + \beta_2(\text{empower}_{ij}^W) + \beta_3(\text{male}_{ij}) + \varepsilon_{ij}$$

gives the combined-model regression equation (Raudenbush & Bryk, 2002)

$$\text{jobsat}_{ij} = \gamma_{00} + u_{0j} + \gamma_{10}(\text{lmx}_{ij}^W) + \gamma_{20}(\text{empower}_{ij}^W) + \gamma_{30}(\text{male}_{ij}) + \varepsilon_{ij}$$

Leading 0 subscript conveys team-level effects

Nonzero subscript conveys within-team effects —

# COMMON NOTATIONAL SYSTEMS

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Combined-model equation (Raudenbush & Bryk, 2002)

$$\text{jobsat}_{ij} = \gamma_{00} + \gamma_{10}(\text{Imx}_{ij}^W) + \gamma_{20}(\text{empower}_{ij}^W) + \gamma_{30}(\text{male}_{ij}) + u_{0j} + \varepsilon_{ij}$$

Standard(ish) regression notation (Scott, Shrout, & Weinberg, 2013)

$$\text{jobsat}_{ij} = \beta_0 + \beta_1(\text{Imx}_{ij}^W) + \beta_2(\text{empower}_{ij}^W) + \beta_3(\text{male}_{ij}) + u_{0j} + \varepsilon_{ij}$$

# BLIMP SCRIPT 5.2

---

**DATA:** EmployeeSatisfaction.dat;

**VARIABLES:** Employee Team Turnover Male Empower LMX JobSat Climate TeamPerf;

**NOMINAL:** Male; # invokes categorical variable model and automatic dummy coding

**CLUSTERID:** Team;

**CENTER:** groupmean = LMX Empower; grandmean = Male; # centering gender is cosmetic

**MODEL:** JobSat ~ intercept LMX Empower Male | intercept;

**BURN:** 20000;

**ITERATIONS:** 10000;

**SEED:** 90291;

# RBLIMP SCRIPT 5 (MODEL 2)

---

```
model2 <- rblimp(  
  data = Employee,  
  nominal = 'Male',  
  clusterid = 'Team',  
  center = 'groupmean = LMX Empower; grandmean = Male',  
  model = 'JobSat ~ intercept LMX Empower Male | intercept',  
  seed = 90291,  
  burn = 10000,  
  iter = 20000)  
  
output(model2)  
plot_posterior(model2, 'JobSat')
```

# BLIMP OUTPUT

---

Outcome Variable: JobSat

Grand Mean Centered: Male.1

Group Mean Centered: Empower LMX

 = level-2 estimate

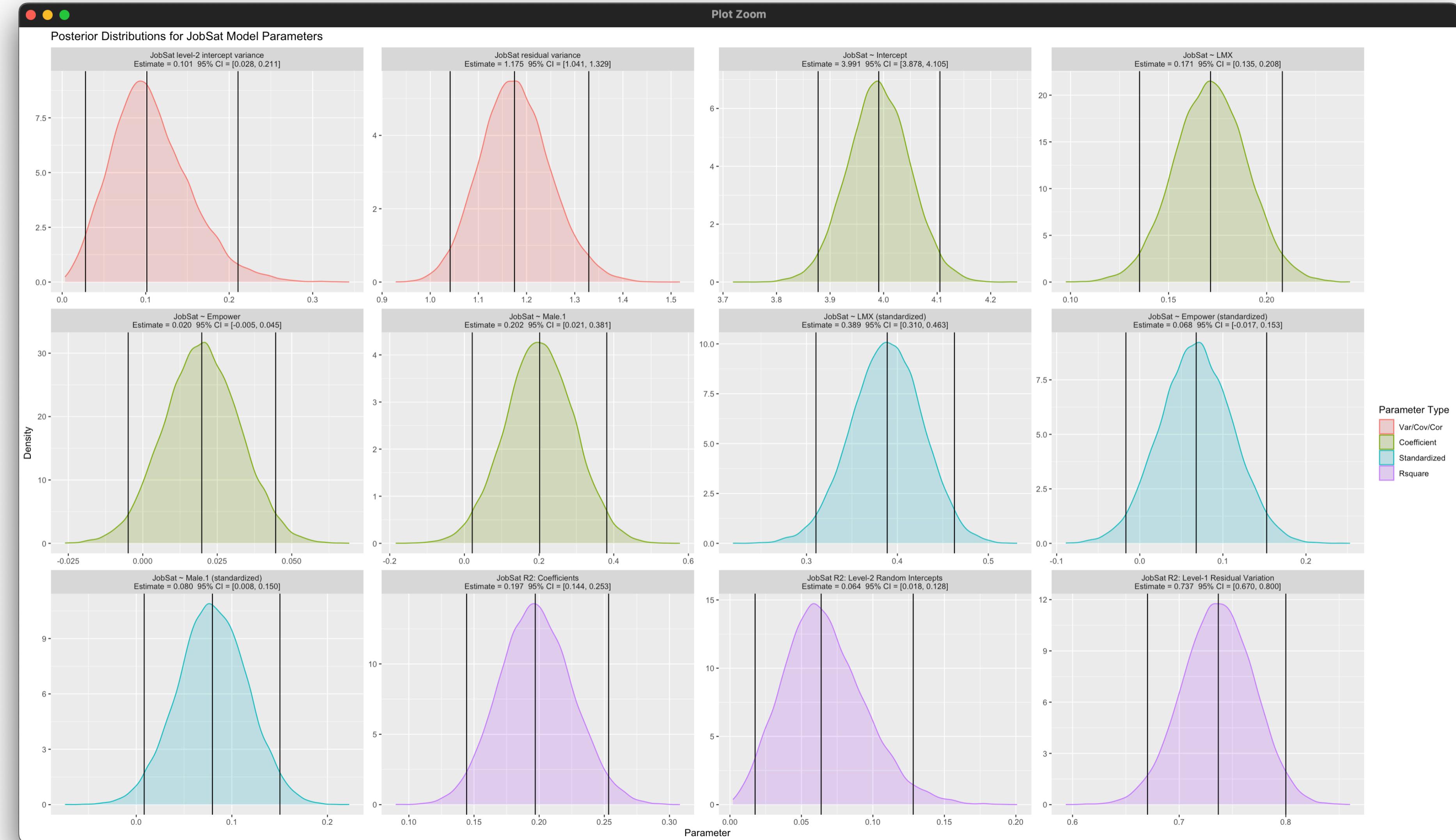
 = level-1 estimate

 = combined estimate

Parameters	Median	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
<hr/>							
Variances:							
L2 : Var(Intercept)	<b>0.101</b>	0.047	0.028	0.211	---	---	640.126
Residual Var.	<b>1.175</b>	0.073	1.041	1.329	---	---	6327.593
<hr/>							
Coefficients:							
Intercept	<b>3.991</b>	0.058	3.878	4.105	4718.233	0.000	2735.980
LMX	<b>0.171</b>	0.019	0.135	0.208	85.554	0.000	9092.221
Empower	<b>0.020</b>	0.013	-0.005	0.045	2.439	0.118	11341.607
Male.1	<b>0.202</b>	0.092	0.021	0.381	4.778	0.029	17185.784
<hr/>							
Standardized Coefficients:							
LMX	0.389	0.039	0.310	0.463	99.108	0.000	6235.514
Empower	0.068	0.043	-0.017	0.153	2.445	0.118	11091.889
Male.1	0.080	0.036	0.008	0.150	4.825	0.028	17139.893
<hr/>							
Proportion Variance Explained							
by Coefficients	<b>0.197</b>	0.028	0.144	0.253	---	---	5347.962
by Level-2 Random Intercepts	<b>0.064</b>	0.028	0.018	0.128	---	---	634.331
by Level-1 Residual Variation	<b>0.737</b>	0.033	0.670	0.800	---	---	1782.897

---

# PARAMETER PLOTS (RBLIMP ONLY)



# INTERPRETATIONS

---

Parameter	Est.	Interpretation
Fixed intercept	3.99	Expected job satisfaction for a supervisee with zero values on the predictor (the grand mean because the predictors are centered)
LMX (within-team)	0.17	Expected satisfaction difference between two people <i>from the same team</i> with LMX scores that differ by one point, controlling for empowerment and gender
Empower (within-team)	0.02	Expected satisfaction difference between two people <i>from the same team</i> with empowerment scores that differ by one point, controlling for LMX and gender
Gender (mostly within-team)	0.20	Expected satisfaction difference between a male and female <i>from the same team</i> , controlling for LMX and empowerment
Between-cluster variance (variance of $u_{0j}$ residuals)	0.10	Average squared distance between the job satisfaction means and the grand mean (level-2 mean differences)
Within-cluster residual variance (variance of $\epsilon_{ij}$ residuals)	1.18	Average squared distance between a supervisee's observed and predicted job satisfaction scores (residual within-cluster variation)

---

# VARIANCE EXPLAINED MEASURES

Fixed effects  
of predictors

$$R^2_{\text{predictors}} = \frac{\beta^T \Sigma_X \beta}{\sigma_Y^2}$$

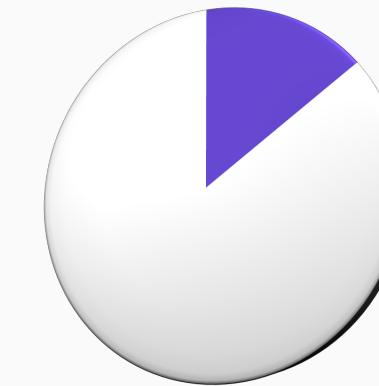
Level-2 random  
intercept residuals

$$R^2_{\text{intercepts}} = \frac{\sigma_u^2}{\sigma_Y^2}$$

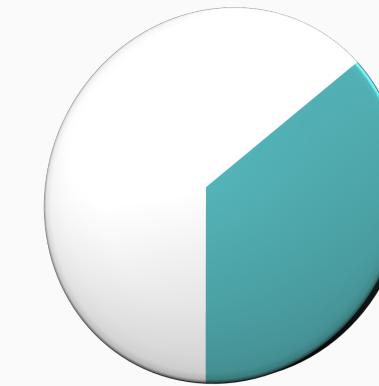
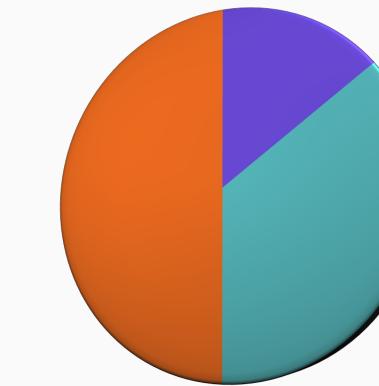
Level-1 within-  
cluster residuals

$$R^2_{\text{residual(within)}} = \frac{\sigma_u^2}{\sigma_Y^2}$$

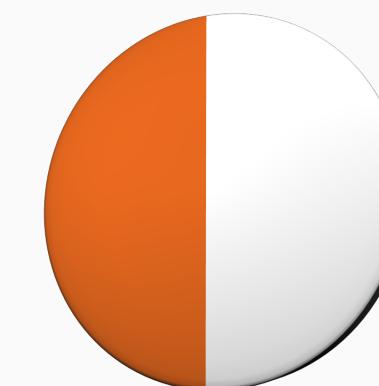
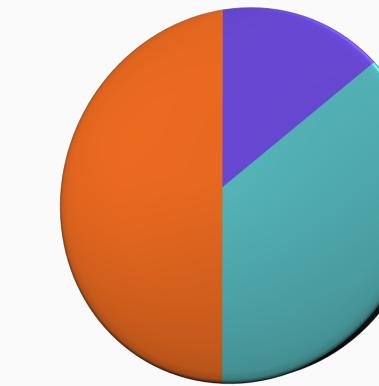
Explained ÷ Total



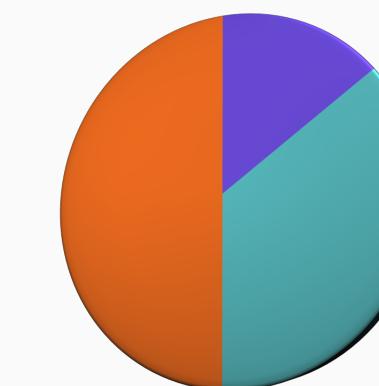
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# INTERPRETATIONS

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- Effect sizes for the level-1 and level-2 residuals are not explained variance, per se, but rather proportions that convey the size of the residual variation

R <sup>2</sup> Effect Size	Est.	Interpretation
Predictors	.20	The level-1 predictors explain 20% of the total variation in job satisfaction
Between-cluster variance (variance of $u_{0j}$ residuals)	.06	Between-team variation in the level-2 residuals (variance of $u_{0j}$ residuals = 0.10) accounts for 6% of the total variation
Within-cluster variance (variance of $\varepsilon_{ij}$ residuals)	.74	Within-team variation in the level-1 residuals (variance of $\varepsilon_{ij}$ residuals = 1.31) accounts for 74% of the total variation

# MODEL-BUILDING STEPS

---

- The methods literature often recommends a multistep model-building procedure that begins at level-1

1) Estimate ICCs

2) Add predictors measured at level-1

3) Add random coefficients for level-1 predictors

4a) Add cluster means of level-1 predictors

4b) Add predictors measured at level-2

} May be combined into a single step

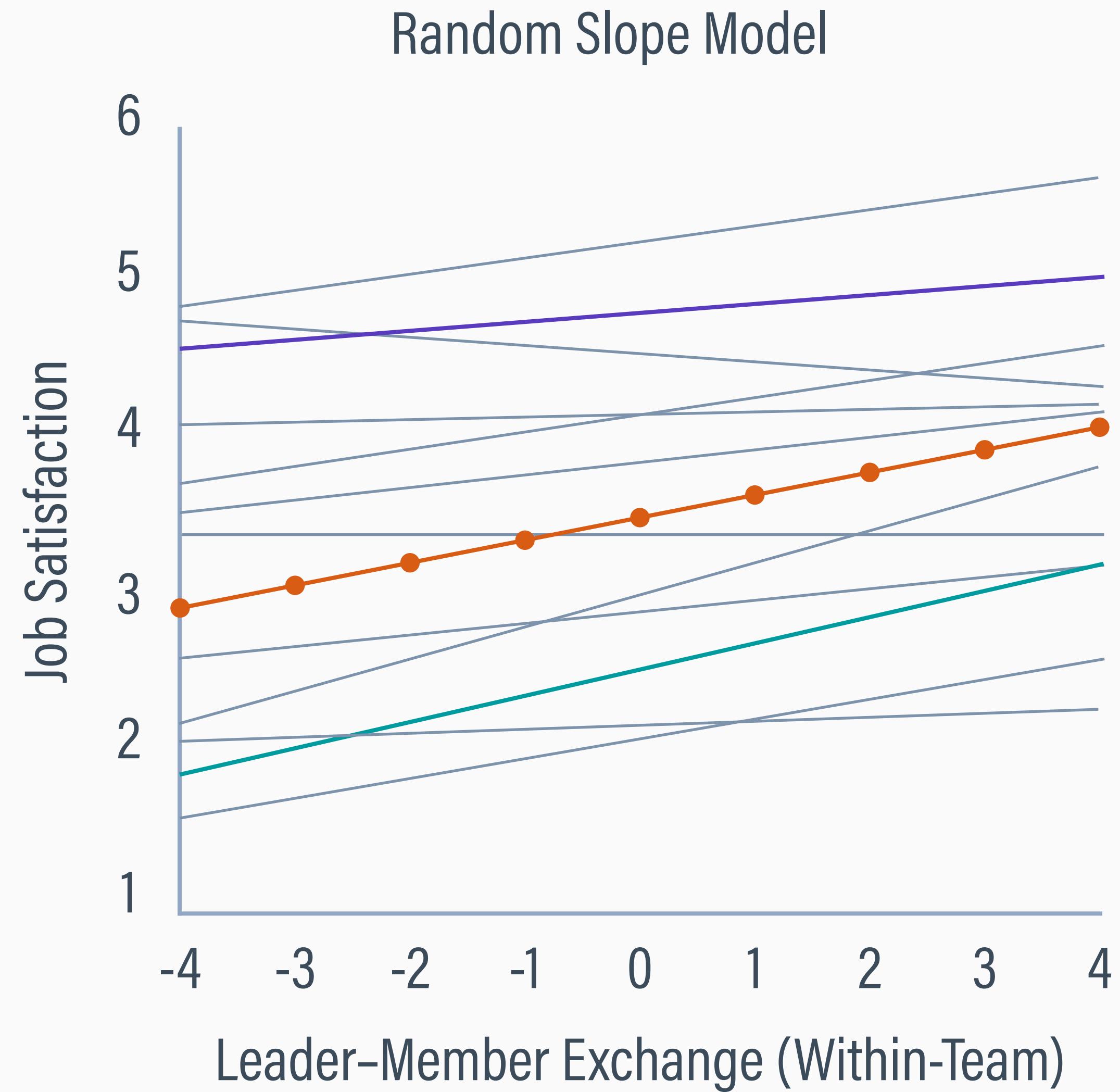
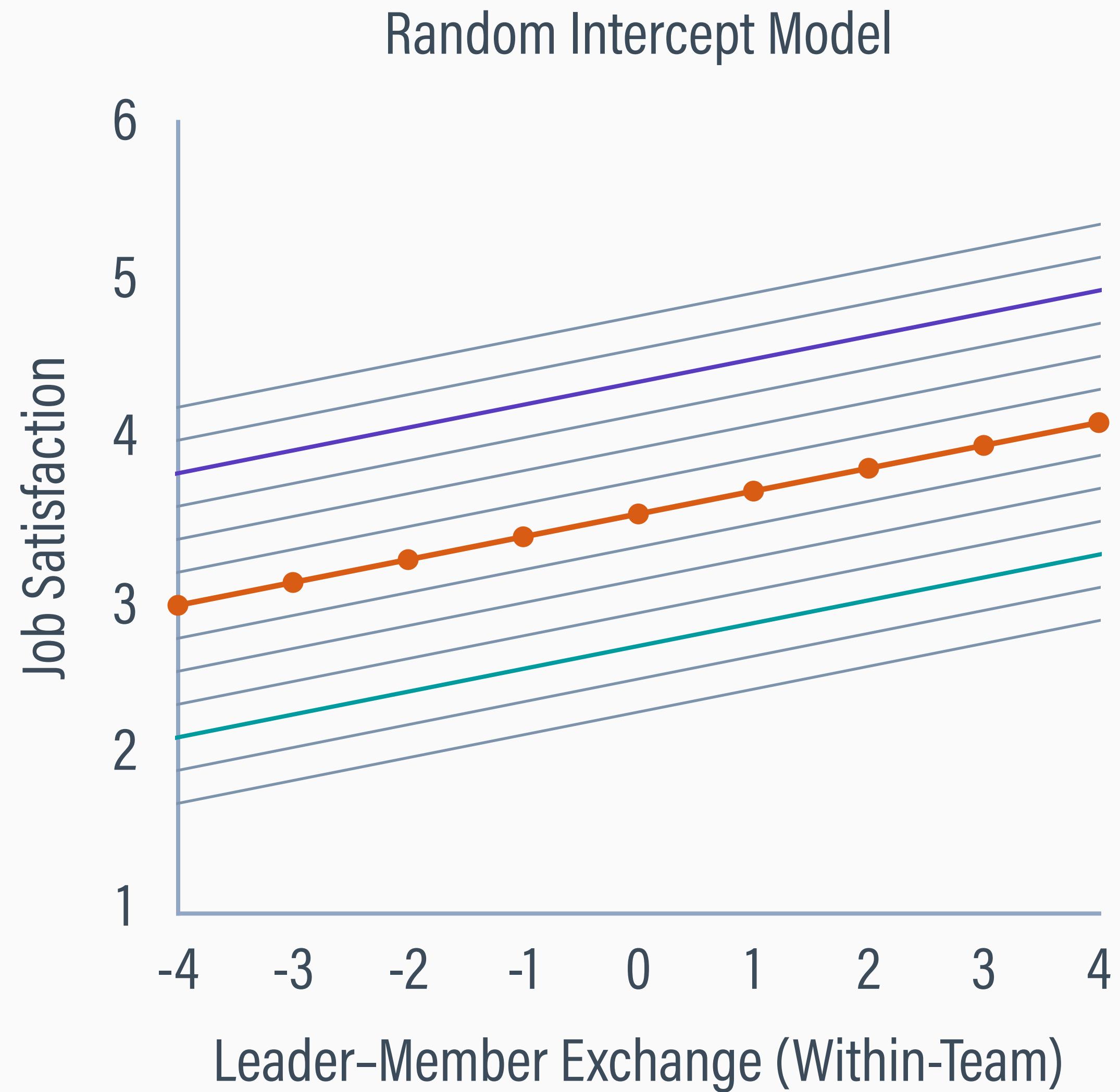
# RANDOM SLOPE COEFFICIENTS

---

- Random slope or random coefficient models allow the influence of a predictor to vary across level-2 units
- The influence of LMX, empowerment, or gender could vary across teams
- e.g., In some teams, the quality of the supervisor-supervisee relationship (LMX) may strongly influence satisfaction, whereas in others, it may not

# MODEL COMPARISON

---



# OUTLINE

- 1 Estimate ICCs
- 2 Add Within-Cluster Predictors
- 3 Strategy 1: Evaluate Random Slopes One at a Time
- 4 Strategy 2: Add All Random Slopes at Once
- 5 Add Between-Cluster Predictors
- 6 Latent Variable Specification

# PROS AND CONS

---

## Pros

- Provides insight into the univariate  $R^2$  contribution of each predictor (without regard to shared variation with others)
- Starting simple and building in complexity is more likely to produce a model that converges / has support

## Cons

- Theory may not guide the sequence
- Each step in the sequence invokes a data-dependent decision about whether to include or discard a random slope
- Different entry orders could produce different results

# DESCRIPTION OF MODEL-BUILDING

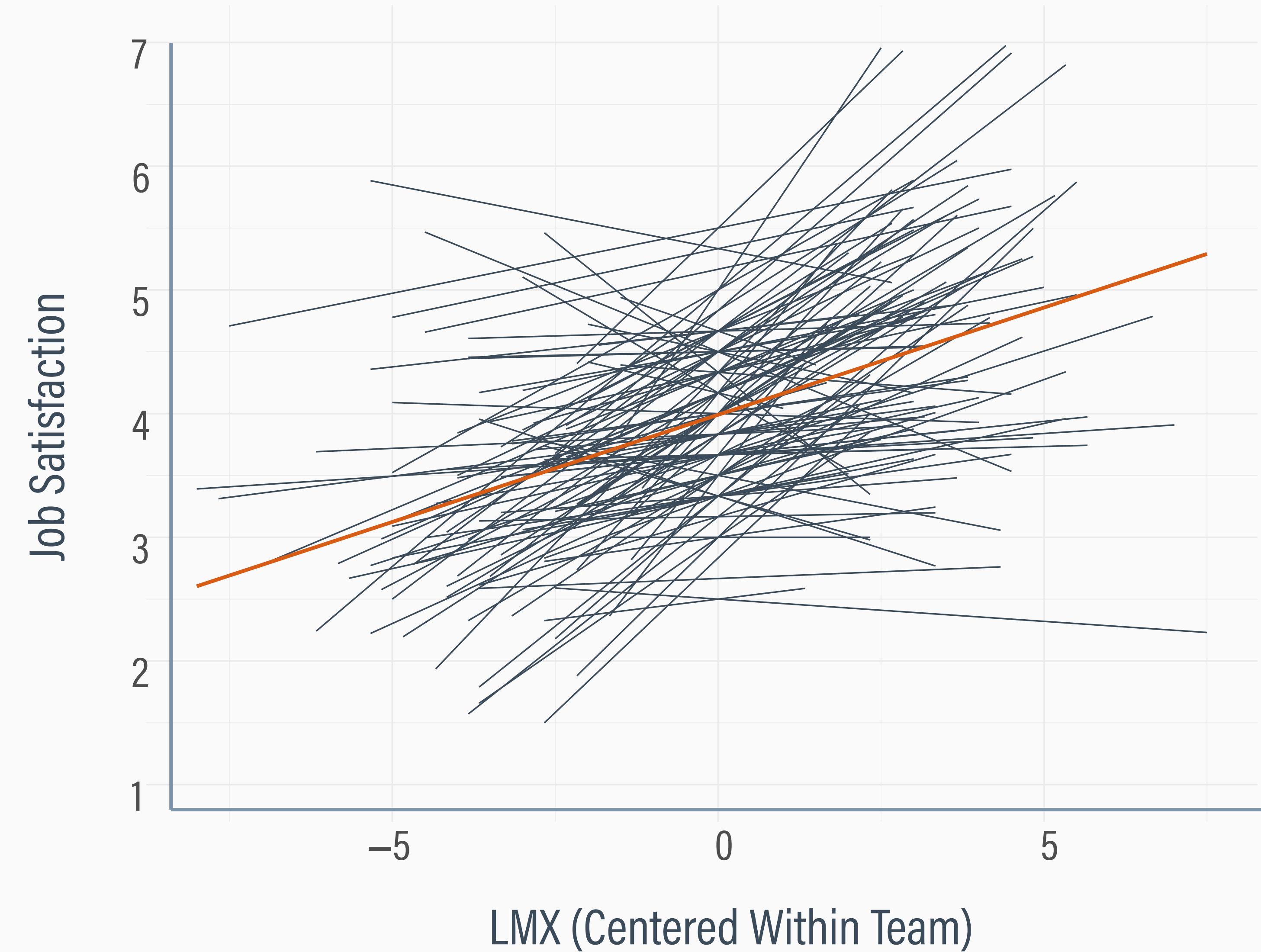
---

Description of model building steps for a paper:

To evaluate the need for random slopes, we added random coefficients for focal level-1 predictors one at a time. We did not consider random coefficients for covariates. Although this procedure was inherently exploratory, it avoids convergence problems that can occur when adding unnecessary effects to the model. For each coefficient, we considered three criteria. First, we examined MCMC diagnostics to evaluate whether there were signs of overfitting. This included potential scale reduction factor values that indicated slow convergence or small effective sample size values indicating poor support from the data. Second, we examined the Rights and Sterba (2019) R-square effect size to evaluate whether a random coefficient accounted for a meaningful amount of variation. Finally, we considered an approximate chi-bar significance test for evaluating whether the random slope and its covariance with the random intercepts were different from zero (Snijders & Bosker, 2012, p. 99). After evaluating the random slopes individually, we added coefficients with salient effects to the final model prior to adding level-2 predictors.

# TEAM-SPECIFIC LMX SLOPES

The R script for this example includes a function to make this graph.



# WITHIN-CLUSTER (LEVEL-1) MODEL

---

- Both the team-specific job satisfaction mean ( $\beta_{0j}$ ) and the leader-member exchange slope vary across clusters ( $\beta_{1j}$ ), but the empowerment and gender slopes are constant

$$\text{jobsat}_{ij} = \beta_{0j} + \beta_{1j}(\text{lmx}_{ij}^W) + \beta_2(\text{empower}_{ij}^W) + \beta_3(\text{male}_{ij}) + \varepsilon_{ij}$$

- Assumption: residuals are normal with constant variation across all persons (level-1 units) and teams (level-2 units)

$$\varepsilon_{ij} \sim N(0, \sigma_\varepsilon^2)$$

# BETWEEN-CLUSTER (LEVEL-2) MODEL

---

- The job satisfaction mean for team  $j$  ( $\beta_{0j}$ ) is the sum of the grand mean ( $\gamma_{00}$ ) and a between-team residual ( $u_{0j}$ )
- The LMX slope for team  $j$  ( $\beta_{1j}$ ) is the sum of the mean slope ( $\gamma_{10}$ ) and a team-level residual ( $u_{1j}$ )
- Assumption: random intercept and slope residuals are bivariate normal *and correlated*

$$\beta_{0j} = \gamma_{00} + u_{0j}$$

$$\beta_{1j} = \gamma_{10} + u_{1j}$$

$$\beta_2 = \gamma_{20}$$

$$\beta_3 = \gamma_{30}$$

$$\begin{pmatrix} u_{0j} \\ u_{1j} \end{pmatrix} \sim N(0, \Sigma_u) \quad \Sigma_u = \begin{pmatrix} \sigma_{u0}^2 & \sigma_{u0u1} \\ \sigma_{u0u1} & \sigma_{u1}^2 \end{pmatrix}$$

# DECODING THE SUBSCRIPTS

---

The second subscript numbers the coefficients on the right side starting at 0 (the intercept)

$$\beta_{0j} = \gamma_{00} + u_{0j}$$

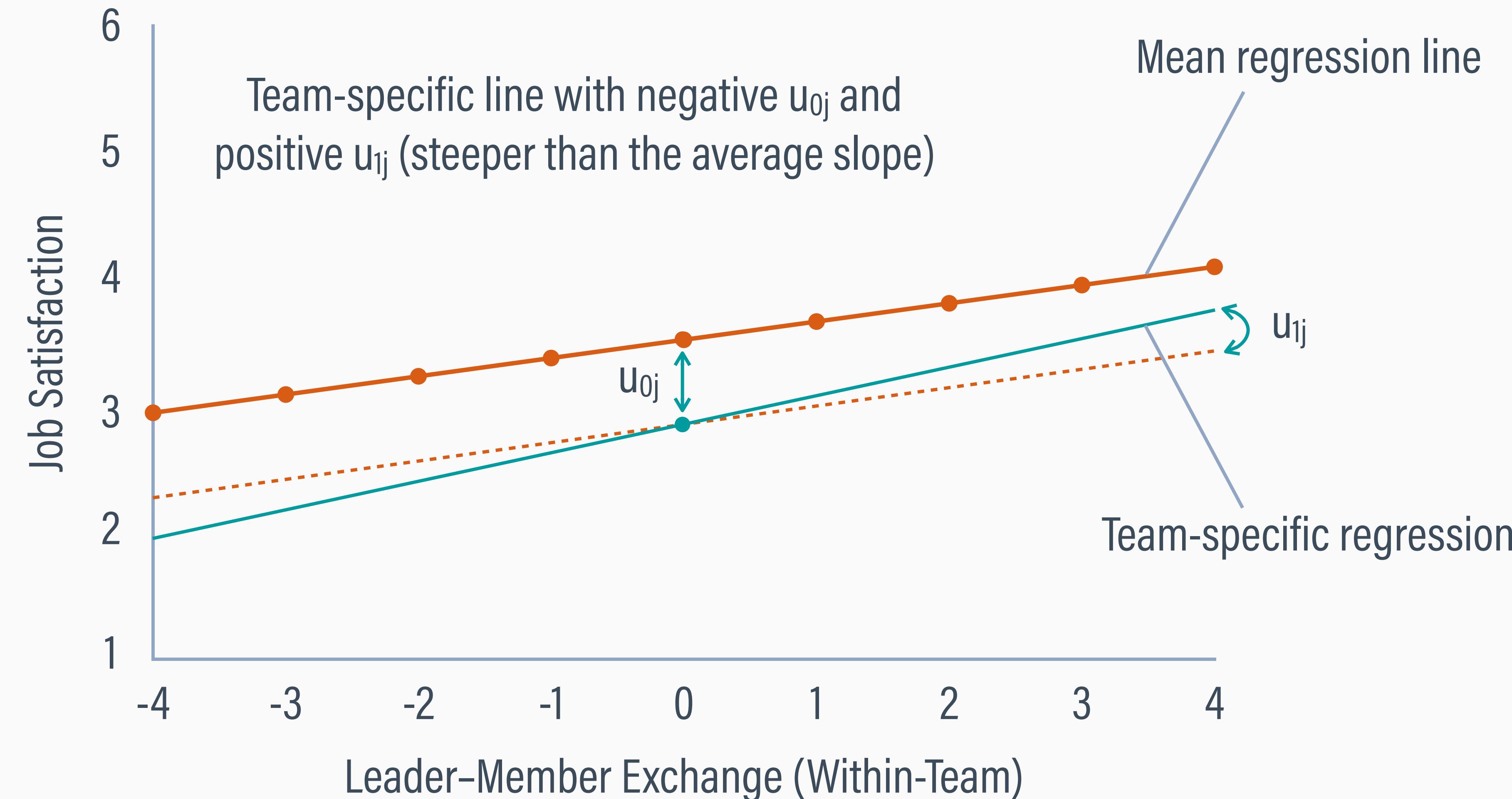
$$\beta_{1j} = \gamma_{10} + u_{1j}$$

$$\beta_{2j} = \gamma_{20}$$

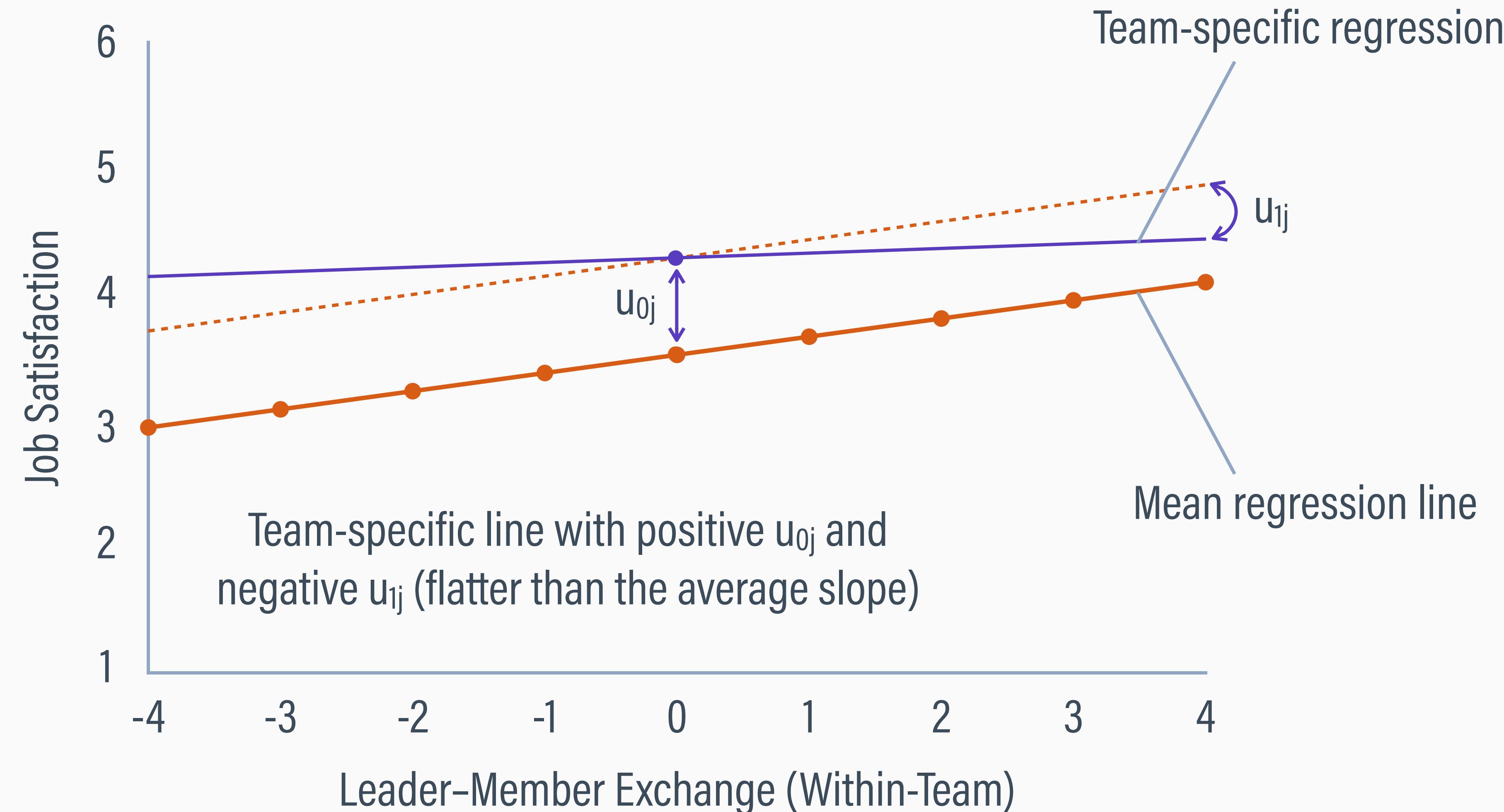
$$\beta_{3j} = \gamma_{30}$$

The first subscript tells which level-2 outcome variable these terms are attached to ( $\gamma_{00}$  and  $u_{0j}$  belong in  $\beta_{0j}$ 's equation,  $\gamma_{10}$  is attached to  $\beta_1$ )

# ILLUSTRATION: POSITIVE SLOPE RESIDUAL



# ILLUSTRATION: NEGATIVE SLOPE RESIDUAL



# COMBINED-MODEL EQUATION

Substituting the right sides  
of the level-2 equations ...

into their coefficients  
from the level-1 equation

$$\beta_{0j} = \gamma_{00} + u_{0j}$$

$$\beta_1 = \gamma_{10} + u_{1j}$$

$$\beta_2 = \gamma_{20}$$

$$\beta_3 = \gamma_{30}$$

$$\text{jobsat}_{ij} = \beta_{0j} + \beta_1(\text{Imx}_{ij}^W) + \beta_2(\text{empower}_{ij}^W) + \beta_3(\text{male}_{ij}) + \varepsilon_{ij}$$

gives the combined-model regression equation (Raudenbush & Bryk, 2002)

Leading 0 subscript conveys team-level effects

$$\text{jobsat}_{ij} = \gamma_{00} + u_{0j} + (\gamma_{10} + u_{1j})(\text{Imx}_{ij}^W) + \gamma_{20}(\text{empower}_{ij}^W) + \gamma_{30}(\text{male}_{ij}) + \varepsilon_{ij}$$

Nonzero subscript conveys within-team effects

# COMMON NOTATIONAL SYSTEMS

---

Combined-model equation (Raudenbush & Bryk, 2002)

$$\text{jobsat}_{ij} = \gamma_{00} + \gamma_{10}(\text{lmx}_{ij}^W) + \gamma_{20}(\text{empower}_{ij}^W) + \gamma_{30}(\text{male}_{ij}) + u_{0j} + (u_{1j})(\text{lmx}_{ij}^W) + \varepsilon_{ij}$$

Standard(ish) regression notation (Scott, Shrout, & Weinberg, 2013)

$$\text{jobsat}_{ij} = \beta_0 + \beta_1(\text{lmx}_{ij}^W) + \beta_2(\text{empower}_{ij}^W) + \beta_3(\text{male}_{ij}) + u_{0j} + (u_{1j})(\text{lmx}_{ij}^W) + \varepsilon_{ij}$$

# BLIMP SCRIPT 5.3

---

**DATA:** EmployeeSatisfaction.dat;

**VARIABLES:** Employee Team Turnover Male Empower LMX JobSat Climate TeamPerf;

**NOMINAL:** Male; # invokes categorical variable model

**CLUSTERID:** Team;

**CENTER:** groupmean = LMX Empower; grandmean = Male; # cwc with latent group means

**MODEL:** JobSat ~ intercept LMX Empower Male | intercept LMX;

**BURN:** 20000;

**ITERATIONS:** 10000;

**SEED:** 90291;

# RBLIMP SCRIPT 5 (MODEL 3)

---

```
model3 <- rblimp(  
  data = Employee,  
  nominal = 'Male',  
  clusterid = 'Team',  
  center = 'groupmean = LMX Empower; grandmean = Male',  
  model = 'JobSat ~ intercept LMX Empower Male | intercept LMX',  
  seed = 90291,  
  burn = 10000,  
  iter = 20000)  
  
output(model3)  
plot_posterior(model3, 'JobSat')
```

# PSR DIAGNOSTIC OUTPUT

---

Quality control check: PSR diagnostics all < 1.05 well before the end of the burn-in period

## BURN-IN POTENTIAL SCALE REDUCTION (PSR) OUTPUT:

NOTE: Split chain PSR is being used. This splits each chain's iterations to create twice as many chains.

Comparing iterations across 2 chains	Highest PSR	Parameter #
251 to 500	1.738	25
501 to 1000	1.461	24
751 to 1500	1.427	24
1001 to 2000	1.093	24
...	...	..
2501 to 5000	1.029	26
2751 to 5500	1.028	31
3001 to 6000	1.059	31
3251 to 6500	1.044	25
3501 to 7000	1.033	2
3751 to 7500	1.025	31
4001 to 8000	1.028	30
4251 to 8500	1.024	31
4501 to 9000	1.013	25
4751 to 9500	1.031	24
5001 to 10000	1.032	2

# EFFECTIVE SAMPLE SIZE DIAGNOSTIC

Outcome Variable: JobSat

Grand Mean Centered: Male.1

Group Mean Centered: Empower LMX

Quality control check: Number of effective  
MCMC samples diagnostics all > 100

Parameters	Median	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
<hr/>							
Variances:							
L2 : Var(Intercept)	0.109	0.047	0.035	0.217	---	---	1030.885
L2 : Cov(LMX, Intercept)	0.009	0.009	-0.009	0.027	---	---	418.520
L2 : Var(LMX)	0.005	0.003	0.001	0.014	---	---	309.126
Residual Var.	1.144	0.073	1.012	1.297	---	---	4630.627
<hr/>							
Coefficients:							
Intercept	3.985	0.059	3.870	4.103	4486.797	0.000	2851.341
LMX	0.172	0.020	0.133	0.211	74.258	0.000	9594.087
Empower	0.020	0.013	-0.005	0.045	2.468	0.116	10897.156
Male.1	0.199	0.094	0.012	0.380	4.437	0.035	16785.705
<hr/>							
...							
<hr/>							
Proportion Variance Explained							
by Coefficients	0.195	0.030	0.138	0.255	---	---	6812.838
by Level-2 Random Intercepts	0.068	0.028	0.022	0.130	---	---	1004.732
by Level-2 Random Slopes	0.024	0.017	0.004	0.069	---	---	305.036
by Level-1 Residual Variation	0.708	0.037	0.632	0.777	---	---	1407.830

# BLIMP OUTPUT

---

Outcome Variable: JobSat

Grand Mean Centered: Male.1

Group Mean Centered: Empower LMX

 = level-2 estimate

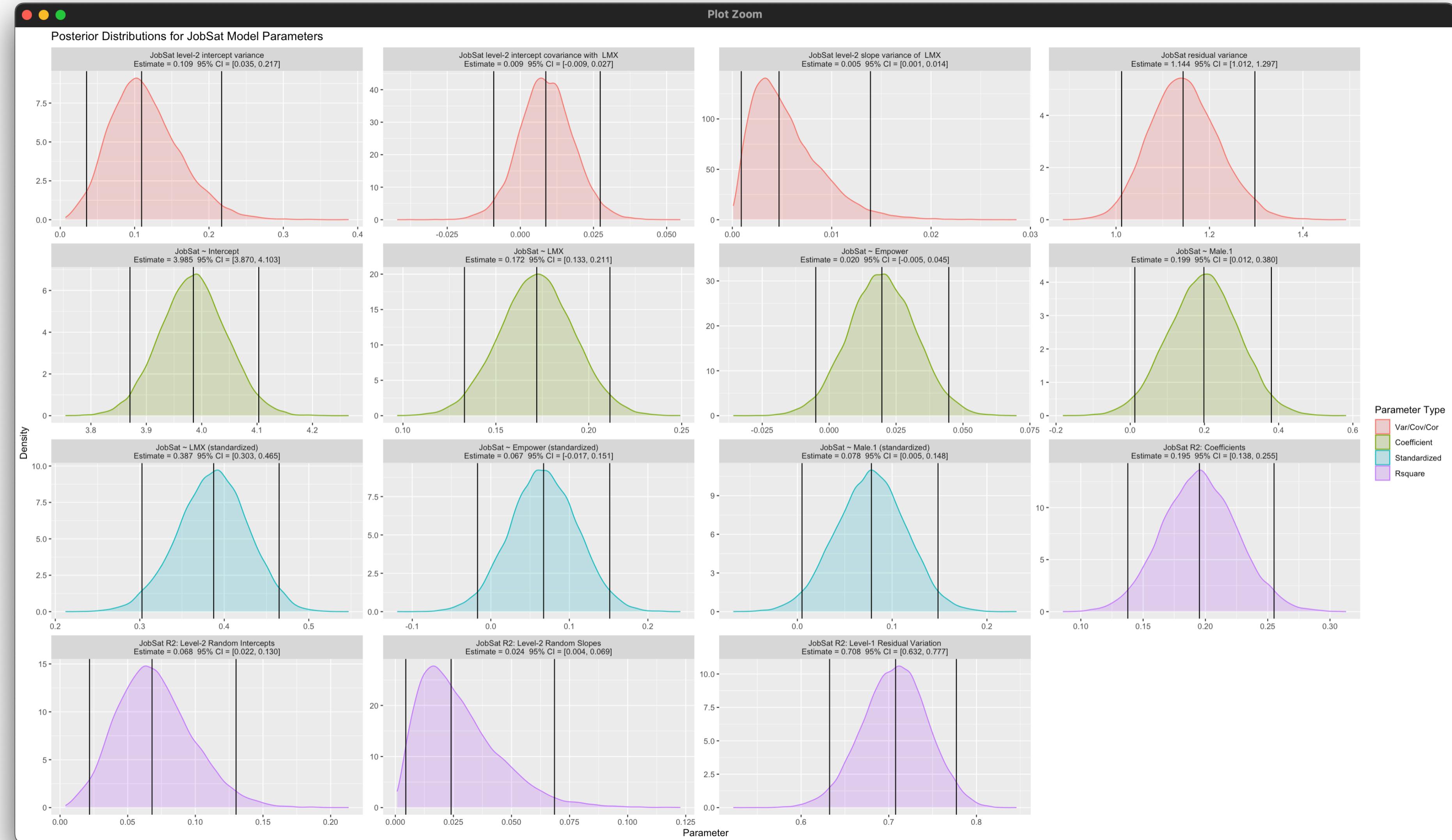
 = level-1 estimate

 = combined estimate

Parameters	Median	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
<hr/>							
Variances:							
L2 : Var(Intercept)	<b>0.109</b>	0.047	0.035	0.217	---	---	1030.885
L2 : Cov(LMX, Intercept)	<b>0.009</b>	0.009	-0.009	0.027	---	---	418.520
L2 : Var(LMX)	<b>0.005</b>	0.003	0.001	0.014	---	---	309.126
Residual Var.	<b>1.144</b>	0.073	1.012	1.297	---	---	4630.627
Coefficients:							
Intercept	<b>3.985</b>	0.059	3.870	4.103	4486.797	0.000	2851.341
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Empower	<b>0.020</b>	0.013	-0.005	0.045	2.468	0.116	10897.156
Male.1	<b>0.199</b>	0.094	0.012	0.380	4.437	0.035	16785.705
<hr/>							
...							
Proportion Variance Explained							
by Coefficients	<b>0.195</b>	0.030	0.138	0.255	---	---	6812.838
by Level-2 Random Intercepts	<b>0.068</b>	0.028	0.022	0.130	---	---	1004.732
by Level-2 Random Slopes	<b>0.024</b>	0.017	0.004	0.069	---	---	305.036
by Level-1 Residual Variation	<b>0.708</b>	0.037	0.632	0.777	---	---	1407.830

---

# PARAMETER PLOTS (RBLIMP ONLY)



# INTERPRETATIONS

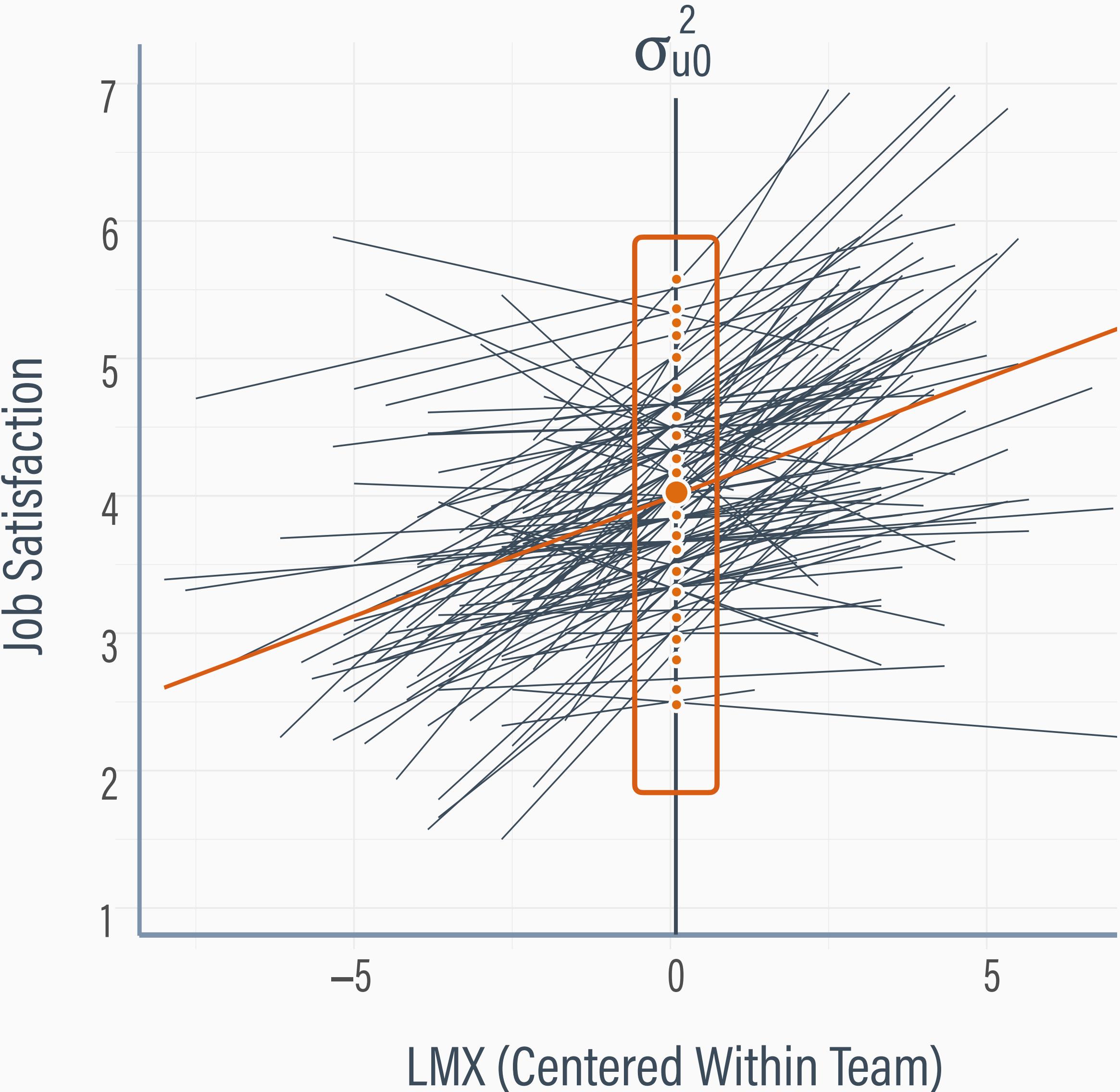
---

Parameter	Est.	Interpretation
Fixed intercept	3.99	Expected job satisfaction for a supervisee with zero values on the predictor (the grand mean because the predictors are centered)
LMX (within-team)	0.17	Expected satisfaction difference between two people <i>from the same team</i> with LMX scores that differ by one point, controlling for empowerment and gender
Empower (within-team)	0.02	Expected satisfaction difference between two people <i>from the same team</i> with empowerment scores that differ by one point, controlling for LMX and gender
Gender (mostly within-team)	0.20	Expected satisfaction difference between a male and female <i>from the same team</i> , controlling for LMX and empowerment
Between-cluster intercept variance (variance of $u_{0j}$ residuals)	0.11	Average squared distance between the job satisfaction means and the grand mean (level-2 mean differences)
Between-cluster slope variance (variance of $u_{1j}$ residuals)	0.005	Average squared distance between the team-specific slopes and the mean slope
Intercept-slope covariance (covariance of $u_{0j}$ and $u_{1j}$ residuals)	0.009	Positive association where teams with higher job satisfaction means (intercepts) also tend to have higher (more positive) LMX slopes
Within-cluster residual variance (variance of $\epsilon_{ij}$ residuals)	1.14	Average squared distance between a supervisee's observed and predicted job satisfaction scores (residual within-cluster variation)

---

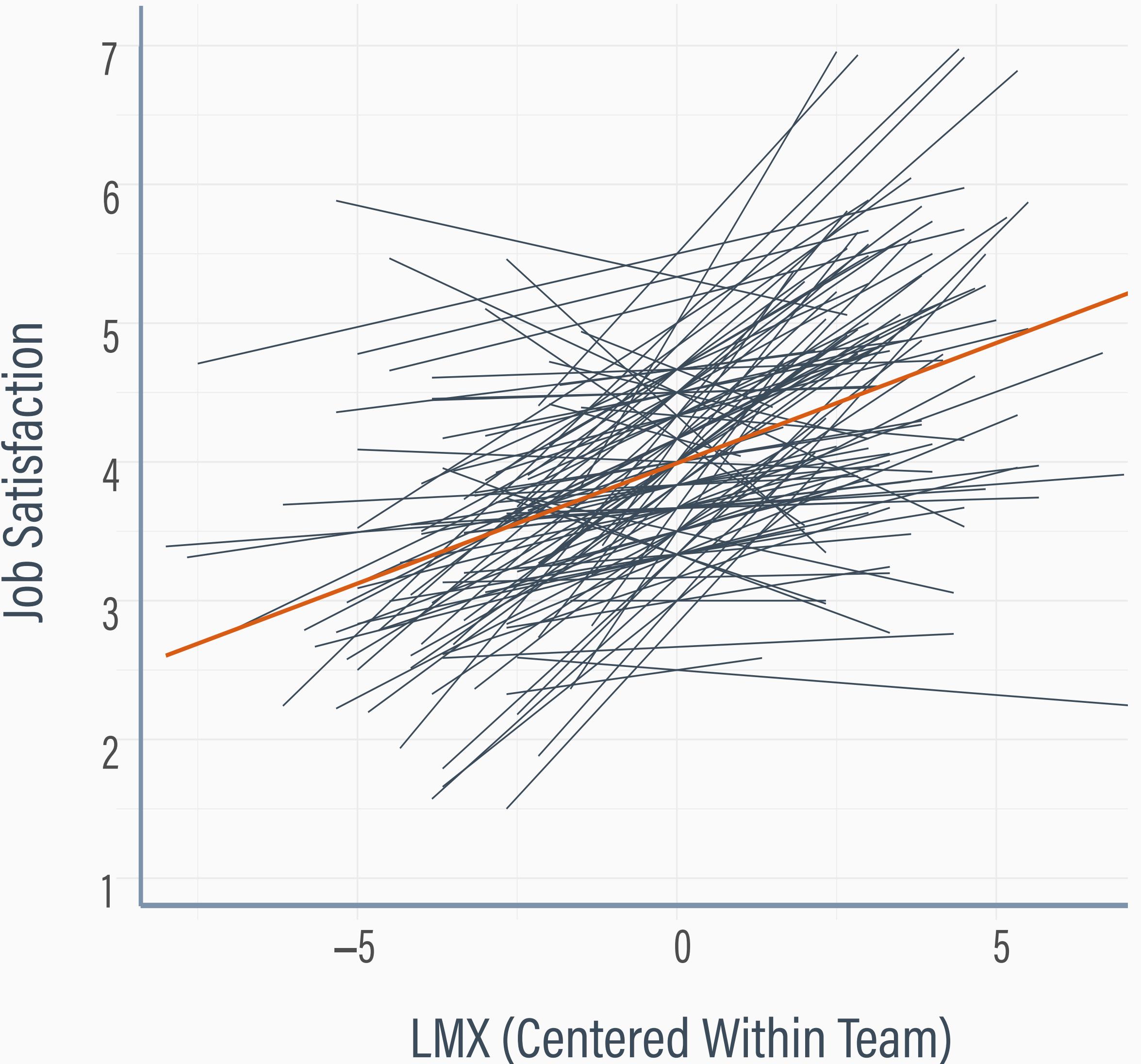
# RANDOM INTERCEPT VARIANCE

- The random intercept variance ( $\sigma_{u0}^2 = 0.11$ ) quantifies differences among team-specific intercepts or means
- Visually, the random intercept variance is the vertical separation of the lines at sleep quality = 0



# RANDOM SLOPE VARIANCE

- The random slope variance ( $\sigma_{u1}^2 = 0.005$ ) quantifies differences among team-specific slopes
- Visually, the random slope variance is the degree to which team-specific slopes deviate from parallel



# DO RANDOM SLOPES IMPROVE FIT?

---

- It is important to include random slopes, but do so judiciously because overfitting impacts convergence and power
- MCMC will show signs of struggle when overfitting, so checking diagnostics is especially important
- Rights and Sterba (2019) effect sizes are useful
- A specialized chi-square test (called a chi-bar test) is necessary when significance testing variances

# PREDICTED OUTCOME VARIATION

---

- Model-predicted total variance = (a) explained variation due to predictors + (b) random intercept residual variation + (c) random slope variation + (d) within-cluster residual variance

$$\text{var}(Y_{ij}) = \beta^T \Sigma_x \beta + \text{tr}(\Sigma_u \Sigma_x) + \text{var}(u_{0j}) + \text{var}(\varepsilon_{ij})$$

total = explained by predictors + slope residuals + intercept residuals + level-1 residuals

- Total variance is the denominator for R-square effect sizes

# VARIANCE EXPLAINED MEASURES

Fixed effects  
of predictors

$$R^2_{\text{predictors}} = \frac{\beta^T \Sigma_X \beta}{\sigma_Y^2}$$

Level-2 random  
intercept residuals

$$R^2_{\text{slopes}} = \frac{\text{tr}(\Sigma_u \Sigma_X)}{\sigma_Y^2}$$

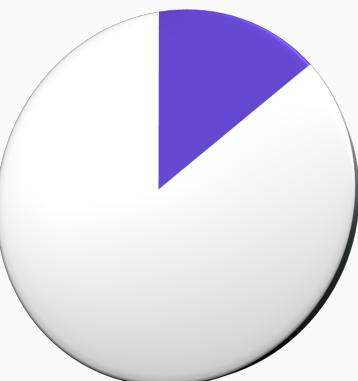
Level-2 random  
slope residuals

$$R^2_{\text{intercepts}} = \frac{\sigma_u^2}{\sigma_Y^2}$$

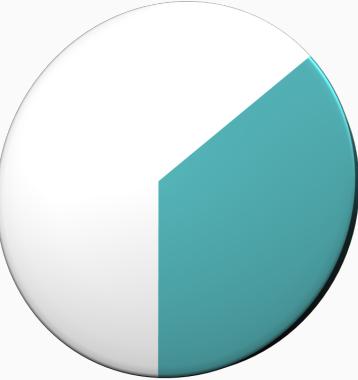
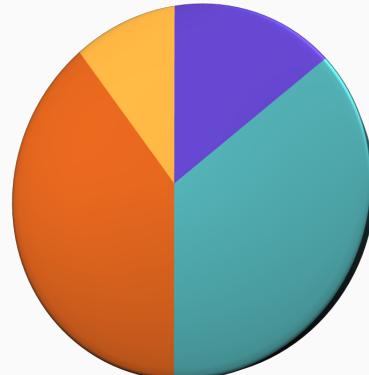
Level-1 within-  
cluster residuals

$$R^2_{\text{residual(within)}} = \frac{\sigma_\varepsilon^2}{\sigma_Y^2}$$

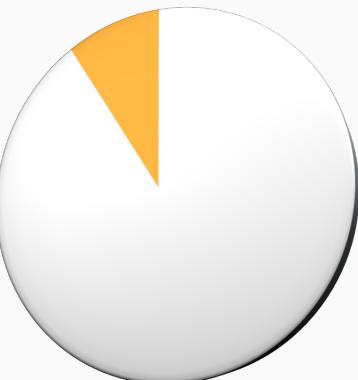
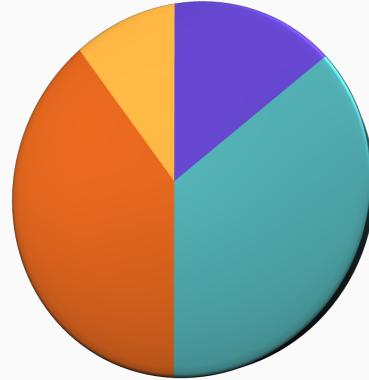
Explained ÷ Total



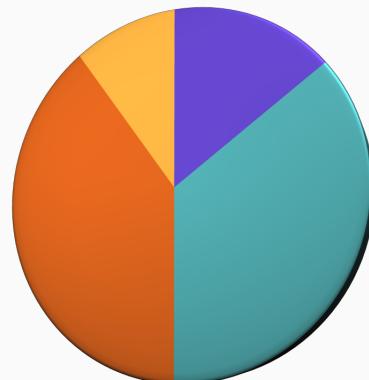
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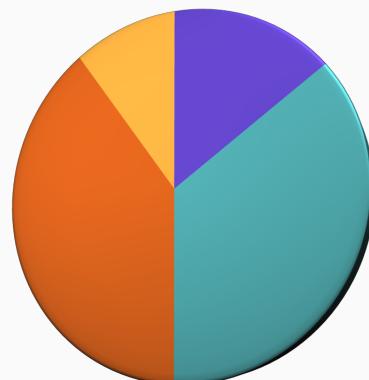
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# R<sup>2</sup> EFFECT SIZE MEASURES

---

Outcome Variable: JobSat

Grand Mean Centered: Male.1

Group Mean Centered: Empower LMX

Parameters	Median	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
<hr/>							
Variances:							
L2 : Var(Intercept)	0.109	0.047	0.035	0.217	---	---	1030.885
L2 : Cov(LMX, Intercept)	0.009	0.009	-0.009	0.027	---	---	418.520
L2 : Var(LMX)	0.005	0.003	0.001	0.014	---	---	309.126
Residual Var.	1.144	0.073	1.012	1.297	---	---	4630.627
Coefficients:							
Intercept	3.985	0.059	3.870	4.103	4486.797	0.000	2851.341
LMX	0.172	0.020	0.133	0.211	74.258	0.000	9594.087
Empower	0.020	0.013	-0.005	0.045	2.468	0.116	10897.156
Male.1	0.199	0.094	0.012	0.380	4.437	0.035	16785.705
...							
Proportion Variance Explained							
by Coefficients	0.195	0.030	0.138	0.255	---	---	6812.838
by Level-2 Random Intercepts	0.068	0.028	0.022	0.130	---	---	1004.732
by Level-2 Random Slopes	0.024	0.017	0.004	0.069	---	---	305.036
by Level-1 Residual Variation	0.708	0.037	0.632	0.777	---	---	1407.830

---

# INTERPRETATIONS

---

- Effect sizes for the level-1 and level-2 residuals are not explained variance, per se, but rather proportions that convey the size of the residual variation

R <sup>2</sup> Effect Size	Est.	Interpretation
Predictors	.20	The level-1 predictors explain 20% of the total variation in job satisfaction
Between-cluster intercept variance (variance of $u_{0j}$ residuals)	.07	Between-team variation in the level-2 residuals (variance of $u_{0j}$ residuals = 0.11) accounts for 7% of the total variation
Between-cluster slope variance (variance of $u_{1j}$ residuals)	.02	Between-team variation in the random slope residuals (variance of $u_{1j}$ residuals = 0.005) accounts for 2% of the total variation
Within-cluster variance (variance of $\varepsilon_{ij}$ residuals)	.71	Within-team variation in the level-1 residuals (variance of $\varepsilon_{ij}$ residuals = 1.14) accounts for 71% of the total variation

# RANDOM SLOPE R<sup>2</sup> BENCHMARKS

---

- Random slope R<sup>2</sup> values tend to be quite small (values > .10 appear to be uncommon)
- Across data sets from numerous MLM and longitudinal books, random slope R<sup>2</sup> values ranged from .001 to .094 (Enders, Woller, & Keller, 2023)
- The mean R<sup>2</sup> from cross-section applications was about .04, and the mean from longitudinal applications was about .05

# NESTED MODEL WALD TEST

---

- The Wald is the sum of squared standardized differences between the point estimates and the null

$$\chi^2_{\text{MCMC}} = (\boldsymbol{\theta} - \boldsymbol{\theta}_0) \text{cov}(\boldsymbol{\theta})^{-1} (\boldsymbol{\theta} - \boldsymbol{\theta}_0)$$
$$= (\text{estimates} - \text{null})(\text{covariance matrix of the estimates})^{-1}(\text{estimates} - \text{null})$$

- When evaluating random slopes,  $\boldsymbol{\theta}$  contains both the slope variance and its covariance with the intercepts (which must be zero if the random slope variance is zero)

# SIGNIFICANCE TESTING WITH VARIANCES

---

- Testing whether variances differ from zero is challenging because the null hypothesis ( $\text{variance} = 0$ ) is the lowest possible value (estimates vary on only one side of the null)
- A specialized chi-square distribution (called a chi-bar) is necessary when significance testing slope variances
- The p-value is a weighted average of p-values from chi-square distribution with different degrees of freedom

# RBLIMP SCRIPT 5 (MODEL 3)

```
model3 <- rblimp( ...  
  model = 'JobSat ~ intercept LMX Empower Male | intercept LMX',  
  ...)  
output(model3)  
chibar_test(model3, raneff = c('LMX')) # chi-bar test from mlm-functions.R
```

Wald Statistic	2.819
Number of Parameters Tested (df)	2
Probability (Chi-Bar Mixture Method)	0.169
Probability (Chi-Bar Binomial Method)	0.108
Probability (Central Chi-Square)	0.244

Use these, perhaps in tandem

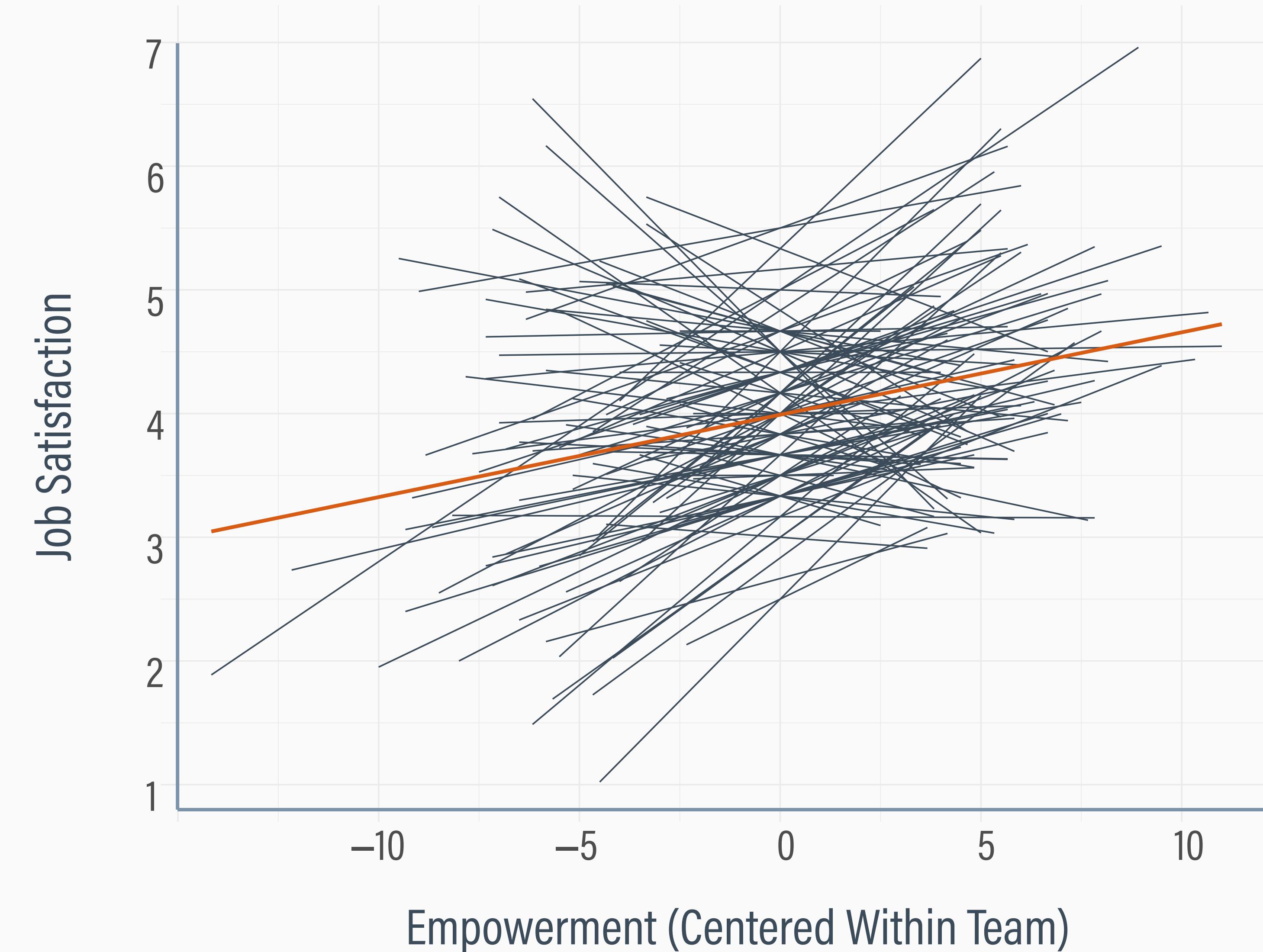
# WALD TEST INTERPRETATION

---

- The test supports the null hypothesis that the population slope variance and its covariance equal zero
- $\chi^2 = 2.81$ ,  $p = .11$  to  $.16$
- Adding the LMX random slope did not improve model fit, with the caveat that the chi-bar test is an approximation

# TEAM-SPECIFIC EMPOWERMENT SLOPES

The R script for this example includes a function to make this graph.



# WITHIN-CLUSTER (LEVEL-1) MODEL

---

- Both the team-specific job satisfaction mean ( $\beta_{0j}$ ) and the empowerment slope vary across clusters ( $\beta_{2j}$ ), but the leader-member exchange and gender slopes are constant

$$\text{jobsat}_{ij} = \beta_{0j} + \beta_1(\text{Imx}_{ij}^W) + \beta_2(\text{empower}_{ij}^W) + \beta_3(\text{male}_{ij}) + \varepsilon_{ij}$$

- Assumption: residuals are normal with constant variation across all days (level-1 units) and persons (level-2 units)

$$\varepsilon_{ij} \sim N(0, \sigma_\varepsilon^2)$$

# BETWEEN-CLUSTER (LEVEL-2) MODEL

---

- The job satisfaction mean for team  $j$  ( $\beta_{0j}$ ) is the sum of the grand mean ( $\gamma_{00}$ ) and a between-team residual ( $u_{0j}$ )
- The empowerment slope for team  $j$  ( $\beta_{2j}$ ) is the sum of the mean slope ( $\gamma_{20}$ ) and a team-level residual ( $u_{2j}$ )
- Assumption: random intercept and slope residuals are bivariate normal and correlated

$$\beta_{0j} = \gamma_{00} + u_{0j}$$

$$\beta_1 = \gamma_{10}$$

$$\beta_2 = \gamma_{20} + u_{2j}$$

$$\beta_3 = \gamma_{30}$$

$$\begin{pmatrix} u_{0j} \\ u_{2j} \end{pmatrix} \sim N(0, \Sigma_u) \quad \Sigma_u = \begin{pmatrix} \sigma_{u0}^2 & \sigma_{u0u2} \\ \sigma_{u2u0} & \sigma_{u2}^2 \end{pmatrix}$$

# COMMON NOTATIONAL SYSTEMS

---

Combined-model equation (Raudenbush & Bryk, 2002)

$$\text{jobsat}_{ij} = \gamma_{00} + \gamma_{10}(\text{lmx}_{ij}^W) + \gamma_{20}(\text{empower}_{ij}^W) + \gamma_{30}(\text{male}_{ij}) + u_{0j} + (u_{2j})(\text{empower}_{ij}^W) + \varepsilon_{ij}$$

Standard(ish) regression notation (Scott, Shrout, & Weinberg, 2013)

$$\text{jobsat}_{ij} = \beta_0 + \beta_1(\text{lmx}_{ij}^W) + \beta_2(\text{empower}_{ij}^W) + \beta_3(\text{male}_{ij}) + u_{0j} + (u_{2j})(\text{empower}_{ij}^W) + \varepsilon_{ij}$$

# BLIMP SCRIPT 5.4

---

**DATA:** EmployeeSatisfaction.dat;

**VARIABLES:** Employee Team Turnover Male Empower LMX JobSat Climate TeamPerf;

**NOMINAL:** Male; # invokes categorical variable model

**CLUSTERID:** Team;

**CENTER:** groupmean = LMX Empower; grandmean = Male; # cwc with latent group means

**MODEL:** JobSat ~ intercept LMX Empower Male | intercept Empower;

**BURN:** 20000;

**ITERATIONS:** 10000;

**SEED:** 90291;

# RBLIMP SCRIPT 5 (MODEL 4)

---

```
model4 <- rblimp(  
  data = Employee,  
  nominal = 'Male',  
  clusterid = 'Team',  
  center = 'groupmean = LMX Empower; grandmean = Male',  
  model = 'JobSat ~ intercept LMX Empower Male | intercept Empower',  
  seed = 90291,  
  burn = 10000,  
  iter = 20000)  
  
output(model4)
```

# PSR DIAGNOSTIC OUTPUT

---

Quality control check: PSR diagnostics all < 1.05 well before the end of the burn-in period

## BURN-IN POTENTIAL SCALE REDUCTION (PSR) OUTPUT:

NOTE: Split chain PSR is being used. This splits each chain's iterations to create twice as many chains.

Comparing iterations across 2 chains	Highest PSR	Parameter #
251 to 500	1.299	30
501 to 1000	1.183	30
751 to 1500	1.128	13
1001 to 2000	1.085	31
...	...	..
2501 to 5000	1.079	24
2751 to 5500	1.073	24
3001 to 6000	1.045	30
3251 to 6500	1.050	24
3501 to 7000	1.063	24
3751 to 7500	1.072	31
4001 to 8000	1.018	24
4251 to 8500	1.042	30
4501 to 9000	1.018	30
4751 to 9500	1.021	24
5001 to 10000	1.022	24

# EFFECTIVE SAMPLE SIZE DIAGNOSTIC

Outcome Variable: JobSat

Grand Mean Centered: Male.1

Group Mean Centered: Empower LMX

Quality control check: Number of effective  
MCMC samples diagnostics all > 100

Parameters	Median	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
<hr/>							
Variances:							
L2 : Var(Intercept)	0.118	0.047	0.045	0.228	---	---	899.675
L2 : Cov(Empower, Intercept)	-0.011	0.007	-0.026	0.001	---	---	535.197
L2 : Var(Empower)	0.003	0.002	0.001	0.008	---	---	335.096
Residual Var.	1.118	0.072	0.988	1.267	---	---	5029.485
<hr/>							
Coefficients:							
Intercept	4.001	0.060	3.883	4.118	4434.613	0.000	2551.627
LMX	0.173	0.019	0.137	0.210	86.221	0.000	9104.124
Empower	0.018	0.014	-0.010	0.045	1.629	0.202	8248.654
Male.1	0.202	0.093	0.019	0.387	4.721	0.030	15935.151
<hr/>							
...							
<hr/>							
Proportion Variance Explained							
by Coefficients	0.193	0.029	0.138	0.253	---	---	6754.576
by Level-2 Random Intercepts	0.073	0.027	0.028	0.135	---	---	849.843
by Level-2 Random Slopes	0.039	0.022	0.008	0.093	---	---	315.212
by Level-1 Residual Variation	0.690	0.038	0.614	0.762	---	---	1338.607

# BLIMP OUTPUT

---

Outcome Variable: JobSat

Grand Mean Centered: Male.1

Group Mean Centered: Empower LMX

 = level-2 estimate

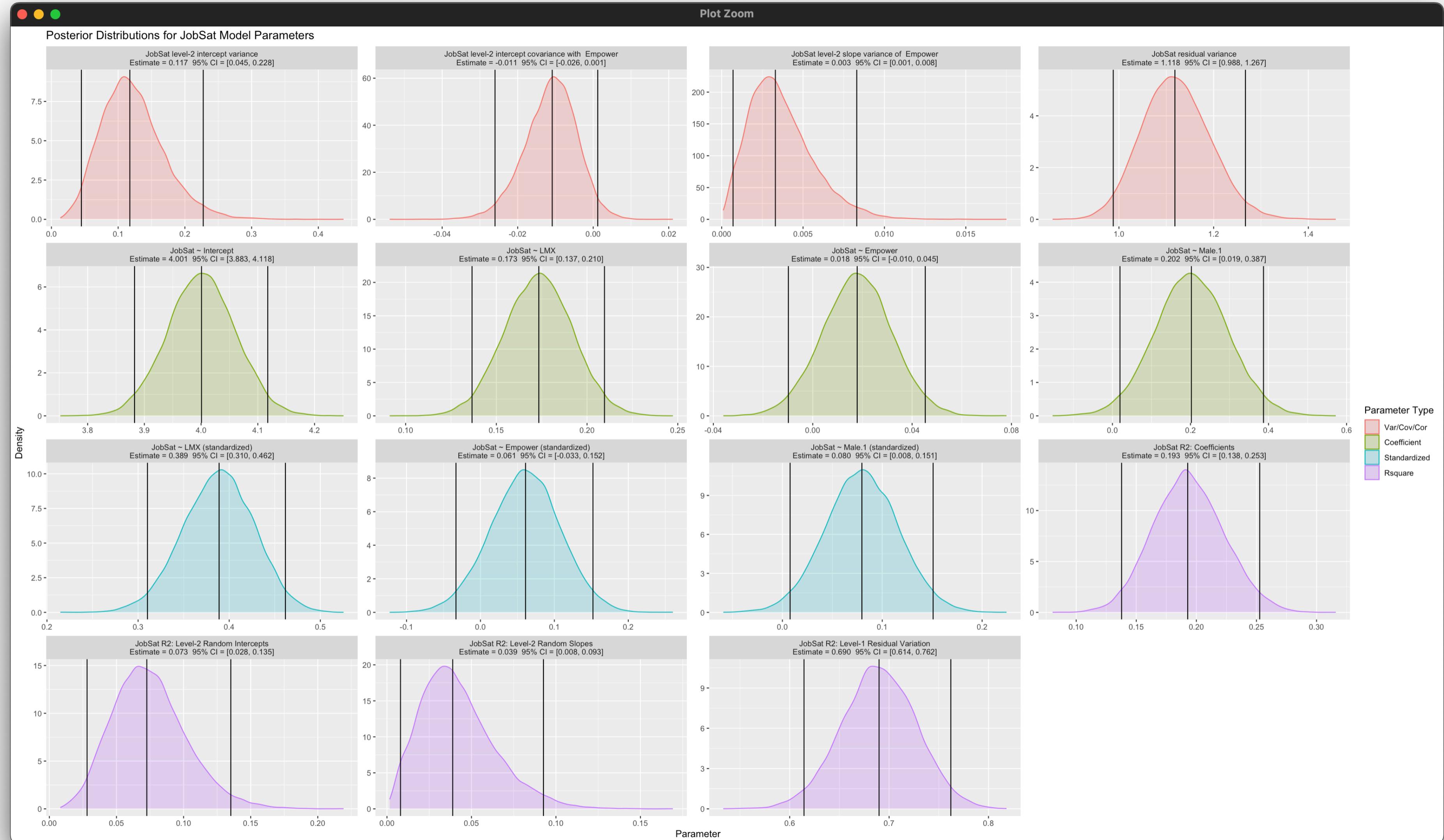
 = level-1 estimate

 = combined estimate

Parameters	Median	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
<hr/>							
Variances:							
L2 : Var(Intercept)	<b>0.118</b>	0.047	0.045	0.228	---	---	899.675
L2 : Cov(Empower, Intercept)	<b>-0.011</b>	0.007	-0.026	0.001	---	---	535.197
L2 : Var(Empower)	<b>0.003</b>	0.002	0.001	0.008	---	---	335.096
Residual Var.	<b>1.118</b>	0.072	0.988	1.267	---	---	5029.485
<hr/>							
Coefficients:							
Intercept	<b>4.001</b>	0.060	3.883	4.118	4434.613	0.000	2551.627
LMX	<b>0.173</b>	0.019	0.137	0.210	86.221	0.000	9104.124
Empower	<b>0.018</b>	0.014	-0.010	0.045	1.629	0.202	8248.654
Male.1	<b>0.202</b>	0.093	0.019	0.387	4.721	0.030	15935.151
<hr/>							
...							
Proportion Variance Explained							
by Coefficients	<b>0.193</b>	0.029	0.138	0.253	---	---	6754.576
by Level-2 Random Intercepts	<b>0.073</b>	0.027	0.028	0.135	---	---	849.843
by Level-2 Random Slopes	<b>0.039</b>	0.022	0.008	0.093	---	---	315.212
by Level-1 Residual Variation	<b>0.690</b>	0.038	0.614	0.762	---	---	1338.607

---

# PARAMETER PLOTS (RBLIMP ONLY)



# INTERPRETATIONS

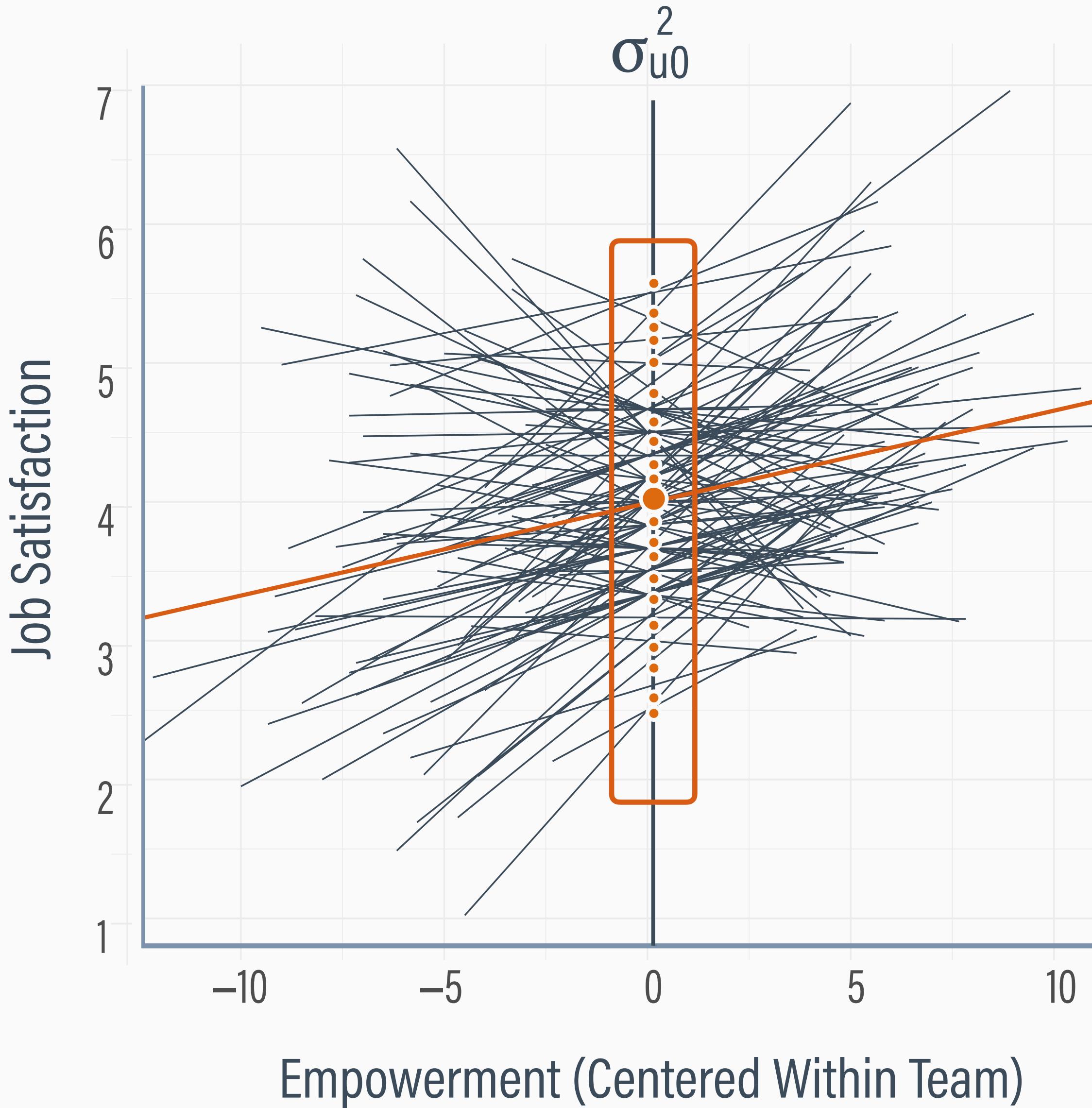
---

Parameter	Est.	Interpretation
Fixed intercept	4.01	Expected job satisfaction for a supervisee with zero values on the predictor (the grand mean because the predictors are centered)
LMX (within-team)	0.17	Expected satisfaction difference between two people <i>from the same team</i> with LMX scores that differ by one point, controlling for empowerment and gender
Empower (within-team)	0.02	Expected satisfaction difference between two people <i>from the same team</i> with empowerment scores that differ by one point, controlling for LMX and gender
Gender (mostly within-team)	0.20	Expected satisfaction difference between a male and female <i>from the same team</i> , controlling for LMX and empowerment
Between-cluster intercept variance (variance of $u_{0j}$ residuals)	0.12	Average squared distance between the job satisfaction means and the grand mean (level-2 mean differences)
Between-cluster slope variance (variance of $u_{2j}$ residuals)	0.003	Average squared distance between the team-specific slopes and the mean slope
Intercept-slope covariance (covariance of $u_{0j}$ and $u_{2j}$ residuals)	-0.01	Negative association where teams with higher job satisfaction means (intercepts) tend to have lower (less positive or perhaps negative) empowerment slopes
Within-cluster residual variance (variance of $\varepsilon_{ij}$ residuals)	1.14	Average squared distance between a supervisee's observed and predicted job satisfaction scores (residual within-cluster variation)

---

# RANDOM INTERCEPT VARIANCE

- The random intercept variance ( $\sigma_{u0}^2 = 0.12$ ) quantifies differences among team-specific intercepts or means
- Visually, the random intercept variance is the vertical separation of the lines at sleep quality = 0



# RANDOM SLOPE VARIANCE

- The random slope variance ( $\sigma_{u2}^2 = 0.003$ ) quantifies differences among team-specific slopes
- Visually, the random slope variance is the degree to which team-specific slopes deviate from parallel



# R<sup>2</sup> EFFECT SIZE MEASURES

---

Outcome Variable: JobSat

Grand Mean Centered: Male.1

Group Mean Centered: Empower LMX

Parameters	Median	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
<hr/>							
Variances:							
L2 : Var(Intercept)	0.118	0.047	0.045	0.228	---	---	899.675
L2 : Cov(Empower, Intercept)	-0.011	0.007	-0.026	0.001	---	---	535.197
L2 : Var(Empower)	0.003	0.002	0.001	0.008	---	---	335.096
Residual Var.	1.118	0.072	0.988	1.267	---	---	5029.485
Coefficients:							
Intercept	4.001	0.060	3.883	4.118	4434.613	0.000	2551.627
LMX	0.173	0.019	0.137	0.210	86.221	0.000	9104.124
Empower	0.018	0.014	-0.010	0.045	1.629	0.202	8248.654
Male.1	0.202	0.093	0.019	0.387	4.721	0.030	15935.151
...							
Proportion Variance Explained							
by Coefficients	0.193	0.029	0.138	0.253	---	---	6754.576
by Level-2 Random Intercepts	0.073	0.027	0.028	0.135	---	---	849.843
by Level-2 Random Slopes	0.039	0.022	0.008	0.093	---	---	315.212
by Level-1 Residual Variation	0.690	0.038	0.614	0.762	---	---	1338.607

---

# INTERPRETATIONS

---

- Effect sizes for the level-1 and level-2 residuals are not explained variance, per se, but rather proportions that convey the size of the residual variation

R <sup>2</sup> Effect Size	Est.	Interpretation
Predictors	.19	The level-1 predictors explain 19% of the total variation in job satisfaction
Between-cluster intercept variance (variance of $u_{0j}$ residuals)	.07	Between-team variation in the level-2 residuals (variance of $u_{0j}$ residuals = 0.12) accounts for 7% of the total variation
Between-cluster slope variance (variance of $u_{1j}$ residuals)	.04	Between-team variation in the random slope residuals (variance of $u_{2j}$ residuals = 0.003) accounts for 4% of the total variation
Within-cluster variance (variance of $\varepsilon_{ij}$ residuals)	.69	Within-team variation in the level-1 residuals (variance of $\varepsilon_{ij}$ residuals = 1.12) accounts for 69% of the total variation

# RBLIMP SCRIPT 5 (MODEL 3)

```
model4 <- rblimp( ...  
  model = 'JobSat ~ intercept LMX Empower Male | intercept Empower',  
  ...)  
output(model4)  
chibar_test(model4, raneff = c('Empower')) # chi-bar test from mlm-functions.R
```

Wald Statistic	4.437
Number of Parameters Tested (df)	2
Probability (Chi-Bar Mixture Method)	0.072
Probability (Chi-Bar Binomial Method)	0.045
Probability (Central Chi-Square)	0.109

Use these, perhaps in tandem

# WALD TEST INTERPRETATION

---

- The test is ambiguous about the null, as p-values differ across approximations
- $\chi^2 = 4.45$ ,  $p = .045$  to  $.072$
- Given the non-trivial random slope effect size ( $R^2_{\text{slopes}} = .04$ ) and quality-control diagnostics, I would include this effect



The gender (female = 0, male = 1) slope was about .20, meaning that males reported higher job satisfaction by about .20. The gender slope was constant across all teams, but we could also make its coefficient random. In small groups of two or three, discuss the meaning of a random slope for gender. Had we estimated that effect, what would it have quantified and/or how would it be interpreted?

# OUTLINE

- 1 Estimate ICCs
- 2 Add Within-Cluster Predictors
- 3 Strategy 1: Evaluate Random Slopes One at a Time
- 4 Strategy 2: Add All Random Slopes at Once
- 5 Add Between-Cluster Predictors
- 6 Latent Variable Specification

# PROS AND CONS

---

## Pros

- Reduces the data-dependent model-building decisions
- Easy to implement, fewer modeling steps

## Cons

- Convergence failures highly likely as the number of random predictors increases
- May produce overly complex model with unnecessary parameters
- No information about the incremental  $R^2$  contribution of each predictor

# WITHIN-CLUSTER (LEVEL-1) MODEL

---

- Both the team-specific job satisfaction mean ( $\beta_{0j}$ ), the leader-member exchange and empowerment slopes vary across clusters ( $\beta_{1j}$ ), but the gender slope is constant

$$\text{jobsat}_{ij} = \beta_{0j} + \beta_{1j}(\text{lmx}_{ij}^W) + \beta_{2j}(\text{empower}_{ij}^W) + \beta_3(\text{male}_{ij}) + \varepsilon_{ij}$$

- Assumption: residuals are normal with constant variation across all days (level-1 units) and persons (level-2 units)

$$\varepsilon_{ij} \sim N(0, \sigma_\varepsilon^2)$$

# BETWEEN-CLUSTER (LEVEL-2) MODEL

- The job satisfaction mean for team j ( $\beta_{0j}$ ) is the sum of the grand mean ( $\gamma_{00}$ ) and a between-team residual ( $u_{0j}$ )
- The LMX and empowerment slopes for team j ( $\beta_{1j}$  and  $\beta_{2j}$ ) are the sum of the mean slopes ( $\gamma_{10}$  and  $\gamma_{20}$ ) and team-level residuals ( $u_{1j}$  and  $u_{2j}$ )
- Assumption: random intercept and slope residuals are multivariate normal and correlated

$$\beta_{0j} = \gamma_{00} + u_{0j}$$

$$\beta_{1j} = \gamma_{10} + u_{1j}$$

$$\beta_{2j} = \gamma_{20} + u_{2j}$$

$$\beta_3 = \gamma_{30}$$

$$\begin{pmatrix} u_{0j} \\ u_{1j} \\ u_{2j} \end{pmatrix} \sim N(0, \Sigma_u) \quad \Sigma_u = \begin{pmatrix} \sigma_{u0}^2 & \sigma_{u0u1} & \sigma_{u0u2} \\ \sigma_{u1u0} & \sigma_{u1}^2 & \sigma_{u1u2} \\ \sigma_{u2u0} & \sigma_{u2u1} & \sigma_{u2}^2 \end{pmatrix}$$

# DECODING THE SUBSCRIPTS

---

The second subscript numbers the coefficients on the right side starting at 0 (the intercept)

$$\beta_{0j} = \gamma_{00} + u_{0j}$$

$$\beta_{1j} = \gamma_{10} + u_{1j}$$

$$\beta_{2j} = \gamma_{20} + u_{2j}$$

$$\beta_{3j} = \gamma_{30}$$

The first subscript tells which level-2 outcome variable these terms are attached to ( $\gamma_{00}$  and  $u_{0j}$  belong in  $\beta_{0j}$ 's equation,  $\gamma_{10}$  is attached to  $\beta_1$ )

# COMBINED-MODEL EQUATION

Substituting the right sides  
of the level-2 equations ...

into their coefficients  
from the level-1 equation

$$\beta_{0j} = \gamma_{00} + u_{0j}$$

$$\beta_1 = \gamma_{10} + u_{1j}$$

$$\beta_2 = \gamma_{20} + u_{2j}$$

$$\beta_3 = \gamma_{30}$$

$$\text{jobsat}_{ij} = \beta_{0j} + \beta_{1j}(\text{lmx}_{ij}^W) + \beta_{2j}(\text{empower}_{ij}^W) + \beta_{3j}(\text{male}_{ij}) + \varepsilon_{ij}$$

gives the combined-model regression equation (Raudenbush & Bryk, 2002)

$$\text{jobsat}_{ij} = \gamma_{00} + u_{0j} + (\gamma_{10} + u_{1j})(\text{lmx}_{ij}^W) + (\gamma_{20} + u_{2j})(\text{empower}_{ij}^W) + \gamma_{30}(\text{male}_{ij}) + \varepsilon_{ij}$$

Leading 0 subscript conveys team-level effects

Nonzero subscript conveys within-team effects

# COMMON NOTATIONAL SYSTEMS

---

Combined-model equation (Raudenbush & Bryk, 2002)

$$\begin{aligned} \text{jobsat}_{ij} = & \gamma_{00} + \gamma_{10}(\text{Imx}_{ij}^W) + \gamma_{20}(\text{empower}_{ij}^W) + \gamma_{30}(\text{male}_{ij}) \\ & + u_{0j} + (u_{1j})(\text{Imx}_{ij}^W) + (u_{2j})(\text{empower}_{ij}^W) + \varepsilon_{ij} \end{aligned}$$

Standard(ish) regression notation (Scott, Shrout, & Weinberg, 2013)

$$\begin{aligned} \text{jobsat}_{ij} = & \beta_0 + \beta_1(\text{Imx}_{ij}^W) + \beta_2(\text{empower}_{ij}^W) + \beta_3(\text{male}_{ij}) \\ & + u_{0j} + (u_{1j})(\text{Imx}_{ij}^W) + (u_{2j})(\text{empower}_{ij}^W) + \varepsilon_{ij} \end{aligned}$$

# BLIMP SCRIPT 5.5

---

**DATA:** EmployeeSatisfaction.dat;

**VARIABLES:** Employee Team Turnover Male Empower LMX JobSat Climate TeamPerf;

**NOMINAL:** Male; # invokes categorical variable model

**CLUSTERID:** Team;

**CENTER:** groupmean = LMX Empower; grandmean = Male; # cwc with latent group means

**MODEL:** JobSat ~ intercept LMX Empower Male | intercept LMX Empower;

**BURN:** 20000;

**ITERATIONS:** 10000;

**SEED:** 90291;

# RBLIMP SCRIPT 5 (MODEL 5)

---

```
model5 <- rblimp(  
  data = Employee,  
  nominal = 'Male',  
  clusterid = 'Team',  
  center = 'groupmean = LMX Empower; grandmean = Male',  
  model = 'JobSat ~ intercept LMX Empower Male | intercept LMX Empower',  
  seed = 90291,  
  burn = 10000,  
  iter = 20000)  
  
output(model5)
```

# PSR DIAGNOSTIC OUTPUT

---

Quality control check: PSR diagnostics all < 1.05 well before the end of the burn-in period

## BURN-IN POTENTIAL SCALE REDUCTION (PSR) OUTPUT:

NOTE: Split chain PSR is being used. This splits each chain's iterations to create twice as many chains.

Comparing iterations across 2 chains	Highest PSR	Parameter #
251 to 500	1.323	27
501 to 1000	1.134	33
751 to 1500	1.109	28
1001 to 2000	1.176	34
...	...	..
2501 to 5000	1.046	34
2751 to 5500	1.043	3
3001 to 6000	1.087	3
3251 to 6500	1.054	3
3501 to 7000	1.060	3
3751 to 7500	1.036	3
4001 to 8000	1.040	28
4251 to 8500	1.035	3
4501 to 9000	1.060	3
4751 to 9500	1.046	3
5001 to 10000	1.028	3

# EFFECTIVE SAMPLE SIZE DIAGNOSTIC

Outcome Variable: JobSat

Grand Mean Centered: Male.1

Group Mean Centered: Empower LMX

Quality control check: Number of effective MCMC samples diagnostics all > 100

Parameters	Median	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
<hr/>							
Variances:							
L2 : Var(Intercept)	0.126	0.048	0.051	0.238	---	---	634.442
L2 : Cov(LMX, Intercept)	0.019	0.012	-0.004	0.044	---	---	307.673
L2 : Var(LMX)	0.010	0.006	0.003	0.024	---	---	306.059
L2 : Cov(Empower, Intercept)	-0.017	0.008	-0.035	-0.002	---	---	549.230
L2 : Cov(Empower, LMX)	-0.005	0.003	-0.013	-0.001	---	---	380.005
L2 : Var(Empower)	0.006	0.003	0.002	0.013	---	---	361.246
Residual Var.	1.071	0.071	0.942	1.220	---	---	5147.409
<hr/>							
Coefficients:							
Intercept	4.000	0.060	3.881	4.118	4395.675	0.000	3081.469
LMX	0.170	0.021	0.128	0.211	64.216	0.000	6656.881
Empower	0.018	0.015	-0.012	0.048	1.373	0.241	6176.238
Male.1	0.208	0.092	0.028	0.391	5.134	0.023	12031.703
<hr/>							
...							
<hr/>							
Proportion Variance Explained							
by Coefficients	0.185	0.029	0.130	0.244	---	---	6178.264
by Level-2 Random Intercepts	0.076	0.028	0.032	0.140	---	---	583.781
by Level-2 Random Slopes	0.081	0.029	0.036	0.149	---	---	313.079
by Level-1 Residual Variation	0.652	0.039	0.573	0.728	---	---	1029.838

# BLIMP OUTPUT

Outcome Variable: JobSat

Grand Mean Centered: Male.1

Group Mean Centered: Empower LMX

 = level-2 estimate

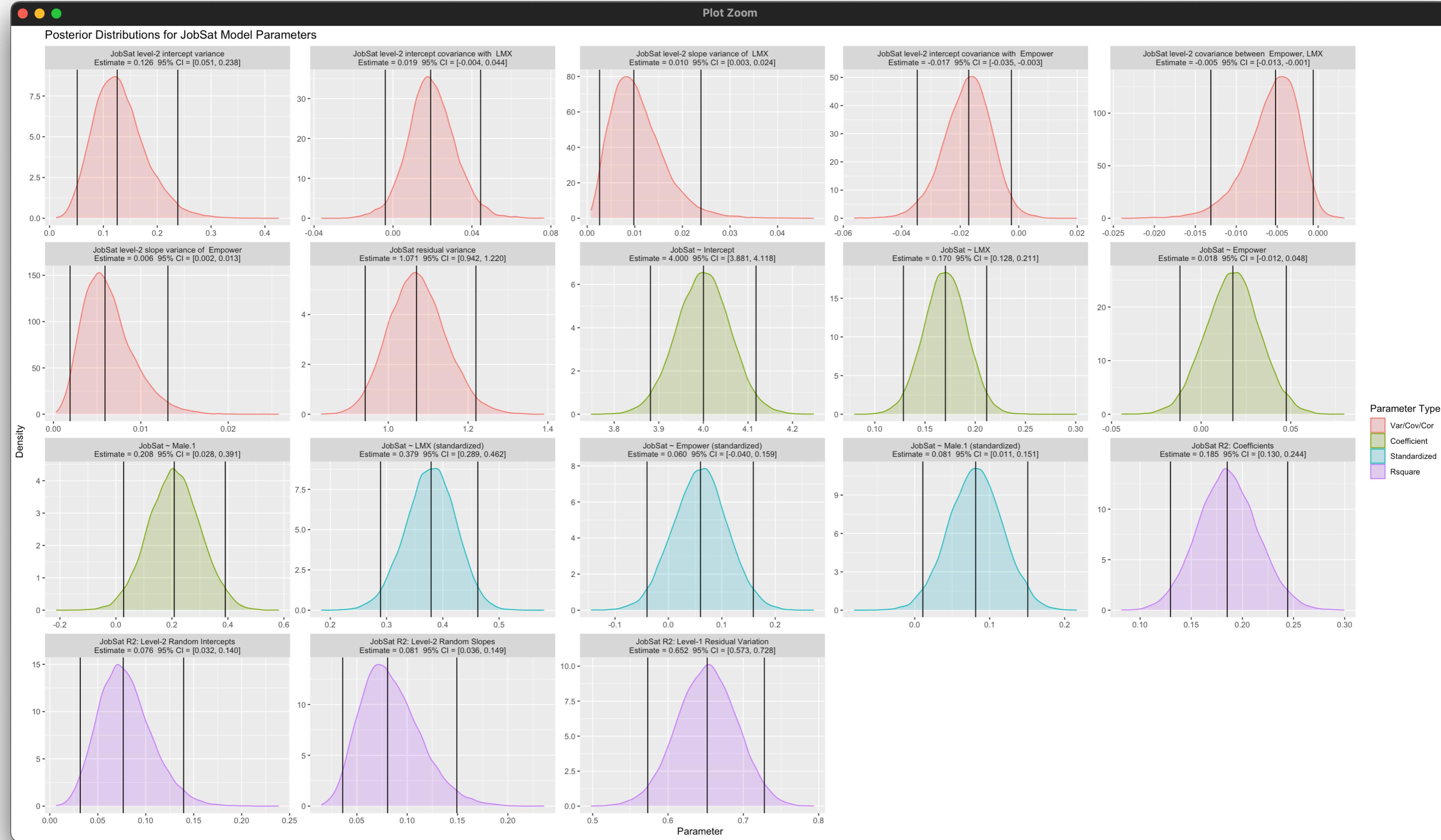
 = level-1 estimate

 = combined estimate

Parameters	Median	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
<hr/>							
Variances:							
L2 : Var(Intercept)	<b>0.126</b>	0.048	0.051	0.238	---	---	634.442
L2 : Cov(LMX, Intercept)	<b>0.019</b>	0.012	-0.004	0.044	---	---	307.673
L2 : Var(LMX)	<b>0.010</b>	0.006	0.003	0.024	---	---	306.059
L2 : Cov(Empower, Intercept)	<b>-0.017</b>	0.008	-0.035	-0.002	---	---	549.230
L2 : Cov(Empower, LMX)	<b>-0.005</b>	0.003	-0.013	-0.001	---	---	380.005
L2 : Var(Empower)	<b>0.006</b>	0.003	0.002	0.013	---	---	361.246
Residual Var.	<b>1.071</b>	0.071	0.942	1.220	---	---	5147.409
<hr/>							
Coefficients:							
Intercept	<b>4.000</b>	0.060	3.881	4.118	4395.675	0.000	3081.469
LMX	<b>0.170</b>	0.021	0.128	0.211	64.216	0.000	6656.881
Empower	<b>0.018</b>	0.015	-0.012	0.048	1.373	0.241	6176.238
Male.1	<b>0.208</b>	0.092	0.028	0.391	5.134	0.023	12031.703
<hr/>							
Proportion Variance Explained							
by Coefficients	<b>0.185</b>	0.029	0.130	0.244	---	---	6178.264
by Level-2 Random Intercepts	<b>0.076</b>	0.028	0.032	0.140	---	---	583.781
by Level-2 Random Slopes	<b>0.081</b>	0.029	0.036	0.149	---	---	313.079
by Level-1 Residual Variation	<b>0.652</b>	0.039	0.573	0.728	---	---	1029.838

...

# PARAMETER PLOTS (RBLIMP ONLY)



# COEFFICIENT INTERPRETATIONS

---

Parameter	Est.	Interpretation
Fixed intercept	4.01	Expected job satisfaction for a supervisee with zero values on the predictor (the grand mean because the predictors are centered)
LMX (within-team)	0.17	Expected satisfaction difference between two people <i>from the same team</i> with LMX scores that differ by one point, controlling for empowerment and gender
Empower (within-team)	0.02	Expected satisfaction difference between two people <i>from the same team</i> with empowerment scores that differ by one point, controlling for LMX and gender
Gender (mostly within-team)	0.21	Expected satisfaction difference between a male and female from the same team, controlling for LMX and empowerment

---

# VARIANCE-COVARIANCE INTERPRETATIONS

---

Parameter	Est.	Interpretation
Between-cluster intercept variance (variance of $u_{0j}$ residuals)	0.13	Average squared distance between the job satisfaction means and the grand mean (level-2 mean differences)
Between-cluster slope variance (variance of $u_{1j}$ residuals)	0.01	Average squared distance between the team-specific LMX slopes and the mean LMX slope
Between-cluster slope variance (variance of $u_{2j}$ residuals)	0.006	Average squared distance between the team-specific empowerment slopes and the mean empowerment slope
Intercept-slope covariance (covariance of $u_{0j}$ and $u_{1j}$ residuals)	0.02	Positive association where teams with higher job satisfaction means (intercepts) tend to have higher (steeper, more positive) LMX slopes
Intercept-slope covariance (covariance of $u_{0j}$ and $u_{2j}$ residuals)	-0.02	Negative association where teams with higher job satisfaction means (intercepts) tend to have lower (less positive or perhaps negative) empowerment slopes
Slope-slope covariance (covariance of $u_{1j}$ and $u_{2j}$ residuals)	-0.005	Negative association where teams with higher (more positive) LMX slopes tend to have lower (less positive or perhaps negative) empowerment slopes
Within-cluster residual variance (variance of $\varepsilon_{ij}$ residuals)	1.07	Average squared distance between a supervisee's observed and predicted job satisfaction scores (residual within-cluster variation)

---

# R<sup>2</sup> EFFECT SIZE MEASURES

---

Outcome Variable: JobSat

Grand Mean Centered: Male.1

Group Mean Centered: Empower LMX

Collectively, the pair of random slopes has a relatively large effect size ( $R^2_{\text{slopes}} = .08$ )

Parameters	Median	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
<hr/>							
Variances:							
L2 : Var(Intercept)	<b>0.126</b>	0.048	0.051	0.238	---	---	634.442
L2 : Cov(LMX, Intercept)	0.019	0.012	-0.004	0.044	---	---	307.673
L2 : Var(LMX)	<b>0.010</b>	0.006	0.003	0.024	---	---	306.059
L2 : Cov(Empower, Intercept)	-0.017	0.008	-0.035	-0.002	---	---	549.230
L2 : Cov(Empower, LMX)	-0.005	0.003	-0.013	-0.001	---	---	380.005
L2 : Var(Empower)	<b>0.006</b>	0.003	0.002	0.013	---	---	361.246
Residual Var.	<b>1.071</b>	0.071	0.942	1.220	---	---	5147.409
<hr/>							
Coefficients:							
Intercept	4.000	0.060	3.881	4.118	4395.675	0.000	3081.469
LMX	<b>0.170</b>	0.021	0.128	0.211	64.216	0.000	6656.881
Empower	<b>0.018</b>	0.015	-0.012	0.048	1.373	0.241	6176.238
Male.1	<b>0.208</b>	0.092	0.028	0.391	5.134	0.023	12031.703
<hr/>							
Proportion Variance Explained							
by Coefficients	<b>0.185</b>	0.029	0.130	0.244	---	---	6178.264
by Level-2 Random Intercepts	<b>0.076</b>	0.028	0.032	0.140	---	---	583.781
by Level-2 Random Slopes	<b>0.081</b>	0.029	0.036	0.149	---	---	313.079
by Level-1 Residual Variation	<b>0.652</b>	0.039	0.573	0.728	---	---	1029.838

---

...

# INTERPRETATIONS

---

- Effect sizes for the level-1 and level-2 residuals are not explained variance, per se, but rather proportions that convey the size of the residual variation

R <sup>2</sup> Effect Size	Est.	Interpretation
Predictors	.19	The level-1 predictors explain 19% of the total variation in job satisfaction
Between-cluster intercept variance (variance of $u_{0j}$ residuals)	.08	Between-team variation in the level-2 residuals (variance of $u_{0j}$ residuals = 0.12) accounts for 7% of the total variation
Between-cluster slope variance (variance of $u_{1j}$ residuals)	.08	Between-team variation in the random slope residuals (variance of $u_{1j}$ and $u_{2j}$ residuals collectively) accounts for 8% of the total variation
Within-cluster variance (variance of $\varepsilon_{ij}$ residuals)	.65	Within-team variation in the level-1 residuals (variance of $\varepsilon_{ij}$ residuals = 1.07) accounts for 65% of the total variation

# RBLIMP SCRIPT 5 (MODEL 3)

```
model5 <- rblimp( ...  
  model = 'JobSat ~ intercept LMX Empower Male | intercept LMX Empower',  
  ...)  
output(model5)  
chibar_test(model5, raneff = c('LMX','Empower')) # chi-bar test from mlm-functions.R
```

Wald Statistic	9.583	
Number of Parameters Tested (df)	5	
Probability (Chi-Bar Mixture Method)	NA	
Probability (Chi-Bar Binomial Method)	0.020	Use these, perhaps in tandem
Probability (Central Chi-Square)	0.088	

# WALD TEST INTERPRETATION

---

- The test refutes the null hypothesis that all population slope variances and covariances (five parameters) equal zero
- $\chi^2 = 9.58, p = .02$
- Adding the pair of random slopes improves model fit, with the caveat that the chi-bar test is an approximation

# OUTLINE

- 1 Estimate ICCs
- 2 Add Within-Cluster Predictors
- 3 Strategy 1: Evaluate Random Slopes One at a Time
- 4 Strategy 2: Add All Random Slopes at Once
- 5 Add Between-Cluster Predictors
- 6 Latent Variable Specification

# WITHIN-CLUSTER (LEVEL-1) MODEL

---

- Both the team-specific job satisfaction mean ( $\beta_{0j}$ ) and the empowerment slope vary across clusters ( $\beta_{2j}$ ), but the leader-member exchange and gender slopes are constant

$$\text{jobsat}_{ij} = \beta_{0j} + \beta_1(\text{Imx}_{ij}^W) + \beta_2(\text{empower}_{ij}^W) + \beta_3(\text{male}_{ij}) + \varepsilon_{ij}$$

- Assumption: residuals are normal with constant variation across all days (level-1 units) and persons (level-2 units)

$$\varepsilon_{ij} \sim N(0, \sigma_\varepsilon^2)$$

# BETWEEN-CLUSTER (LEVEL-2) MODEL

---

- The job satisfaction mean for team  $j$  ( $\beta_{0j}$ ) is a function of team-average LMX and empowerment plus a between-team residual ( $u_{0j}$ )
- The empowerment slope for team  $j$  ( $\beta_{2j}$ ) is the sum of the mean slope ( $\gamma_{20}$ ) and a team-level residual ( $u_{2j}$ )
- Assumption: random intercept residuals are bivariate normal and correlated

$$\beta_{0j} = \gamma_{00} + \gamma_{01}(lmx_j^b) + \gamma_{02}(empower_j^b) + u_{0j}$$

$$\beta_1 = \gamma_{10}$$

$$\beta_2 = \gamma_{20} + u_{2j}$$

$$\beta_3 = \gamma_{30}$$

$$\begin{pmatrix} u_{0j} \\ u_{2j} \end{pmatrix} \sim N(0, \Sigma_u) \quad \Sigma_u = \begin{pmatrix} \sigma_{u0}^2 & \sigma_{u0u2} \\ \sigma_{u2u0} & \sigma_{u2}^2 \end{pmatrix}$$

# COMMON NOTATIONAL SYSTEMS

---

Combined-model equation (Raudenbush & Bryk, 2002)

$$\begin{aligned} \text{jobsat}_{ij} = & \gamma_{00} + \gamma_{10}(\text{lmx}_{ij}^w) + \gamma_{20}(\text{empower}_{ij}^w) + \gamma_{30}(\text{male}_{ij}) \\ & + \gamma_{01}(\text{lmx}_j^b) + \gamma_{02}(\text{empower}_j^b) + u_{0j} + (u_{2j})(\text{empower}_{ij}^w) + \varepsilon_{ij} \end{aligned}$$

Standard(ish) regression notation (Scott, Shrout, & Weinberg, 2013)

$$\begin{aligned} \text{jobsat}_{ij} = & \beta_0 + \beta_1(\text{lmx}_{ij}^w) + \beta_2(\text{empower}_{ij}^w) + \beta_3(\text{male}_{ij}) \\ & + \beta_4(\text{lmx}_j^b) + \beta_5(\text{empower}_j^b) + u_{0j} + (u_{2j})(\text{empower}_{ij}^w) + \varepsilon_{ij} \end{aligned}$$

# BLIMP SCRIPT 5.6

---

**DATA:** EmployeeSatisfaction.dat;

**VARIABLES:** Employee Team Turnover Male Empower LMX JobSat Climate TeamPerf;

**NOMINAL:** Male; # invokes categorical variable model

**CLUSTERID:** Team;

**CENTER:** groupmean = LMX Empower; grandmean = Male LMX.mean Empower.mean;

**MODEL:** JobSat ~ intercept LMX Empower Male LMX.mean Empower.mean | intercept Empower;

**BURN:** 10000;

**ITERATIONS:** 20000;

**SEED:** 90291;

# RBLIMP SCRIPT 5 (MODEL 6)

---

```
model6 <- rblimp(  
  data = Employee,  
  nominal = 'Male',  
  clusterid = 'Team',  
  center = 'groupmean = LMX Empower; grandmean = Male LMX.mean Empower.mean',  
  model = 'intercept LMX Empower Male LMX.mean Empower.mean | intercept Empower',  
  seed = 90291,  
  burn = 10000,  
  iter = 20000)  
  
output(model6)
```

# PSR DIAGNOSTIC OUTPUT

---

Quality control check: PSR diagnostics all < 1.05 well before the end of the burn-in period

## BURN-IN POTENTIAL SCALE REDUCTION (PSR) OUTPUT:

NOTE: Split chain PSR is being used. This splits each chain's iterations to create twice as many chains.

Comparing iterations across 2 chains	Highest PSR	Parameter #
251 to 500	1.912	29
501 to 1000	1.102	29
751 to 1500	1.387	34
1001 to 2000	1.175	34
...	...	..
2501 to 5000	1.059	35
2751 to 5500	1.053	35
3001 to 6000	1.088	35
3251 to 6500	1.048	34
3501 to 7000	1.060	34
3751 to 7500	1.068	28
4001 to 8000	1.037	10
4251 to 8500	1.029	2
4501 to 9000	1.023	1
4751 to 9500	1.032	18
5001 to 10000	1.038	3

# EFFECTIVE SAMPLE SIZE DIAGNOSTIC

Outcome Variable: JobSat

Grand Mean Centered: Empower.mean[Team] LMX.mean[Team] Male.1

Group Mean Centered: Empower LMX

Quality control check: Number of effective MCMC samples diagnostics **not** all > 100, increase the number of iterations

Parameters	Median	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
<hr/>							
Variances:							
L2 : Var(Intercept)	0.084	0.046	0.014	0.190	---	---	288.791
L2 : Cov(Empower, Intercept)	-0.009	0.007	-0.024	0.002	---	---	76.677
L2 : Var(Empower)	0.003	0.002	0.000	0.008	---	---	100.363
Residual Var.	1.117	0.072	0.987	1.270	---	---	1303.014
<hr/>							
Coefficients:							
Intercept	3.995	0.064	3.869	4.121	3885.470	0.000	2053.484
LMX	0.168	0.019	0.130	0.204	78.051	0.000	6849.160
Empower	0.013	0.014	-0.015	0.041	0.844	0.358	5740.524
Male.1	0.205	0.092	0.025	0.385	4.946	0.026	15196.444
LMX.mean[Team]	0.202	0.125	0.021	0.541	3.053	0.081	237.334
Empower.mean[Team]	0.189	0.106	0.047	0.474	3.789	0.052	203.254
<hr/>							
...							
<hr/>							
Proportion Variance Explained							
by Coefficients	0.225	0.034	0.162	0.295	---	---	1085.122
by Level-2 Random Intercepts	0.051	0.027	0.008	0.113	---	---	282.645
by Level-2 Random Slopes	0.037	0.022	0.000	0.087	---	---	96.012
by Level-1 Residual Variation	0.682	0.038	0.606	0.754	---	---	553.156

# BLIMP SCRIPT 5.6

---

**DATA:** EmployeeSatisfaction.dat;

**VARIABLES:** Employee Team Turnover Male Empower LMX JobSat Climate TeamPerf;

**NOMINAL:** Male; # invokes categorical variable model

**CLUSTERID:** Team;

**CENTER:** groupmean = LMX Empower; grandmean = Male LMX.mean Empower.mean;

**MODEL:** JobSat ~ intercept LMX Empower Male LMX.mean Empower.mean | intercept Empower;

**BURN:** 10000;

**ITERATIONS:** 30000;

**SEED:** 90291;

# RBLIMP SCRIPT 5 (MODEL 6)

---

```
model6 <- rblimp(  
  data = Employee,  
  nominal = 'Male',  
  clusterid = 'Team',  
  center = 'groupmean = LMX Empower; grandmean = Male LMX.mean Empower.mean',  
  model = 'intercept LMX Empower Male LMX.mean Empower.mean | intercept Empower',  
  seed = 90291,  
  burn = 10000,  
  iter = 30000)  
  
output(model6)
```

# BLIMP OUTPUT

---

Outcome Variable: JobSat

Grand Mean Centered: Empower.mean[Team] LMX.mean[Team] Male.1

Group Mean Centered: Empower LMX

Quality control check: Number of effective  
MCMC samples diagnostics all > 100

Parameters	Median	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
<hr/>							
Variances:							
L2 : Var(Intercept)	0.084	0.046	0.014	0.192	---	---	372.982
L2 : Cov(Empower, Intercept)	-0.009	0.007	-0.024	0.002	---	---	195.594
L2 : Var(Empower)	0.003	0.002	0.000	0.008	---	---	157.077
Residual Var.	1.116	0.072	0.987	1.270	---	---	3159.871
<hr/>							
Coefficients:							
Intercept	3.995	0.064	3.870	4.121	3908.187	0.000	3387.670
LMX	0.168	0.019	0.131	0.205	78.759	0.000	10859.531
Empower	0.013	0.014	-0.015	0.041	0.851	0.356	8238.233
Male.1	0.205	0.092	0.024	0.385	4.913	0.027	21706.381
LMX.mean[Team]	0.201	0.131	0.014	0.545	2.750	0.097	375.083
Empower.mean[Team]	0.188	0.107	0.043	0.469	3.596	0.058	344.974
<hr/>							
Proportion Variance Explained							
by Coefficients	0.225	0.034	0.162	0.295	---	---	1505.422
by Level-2 Random Intercepts	0.051	0.027	0.009	0.114	---	---	362.076
by Level-2 Random Slopes	0.037	0.022	0.001	0.087	---	---	151.893
by Level-1 Residual Variation	0.682	0.038	0.606	0.754	---	---	1343.492

# BLIMP OUTPUT

---

 = level-2 estimate

 = level-1 estimate

 = combined estimate

Outcome Variable: JobSat

Grand Mean Centered: Empower.mean[Team] LMX.mean[Team] Male.1

Group Mean Centered: Empower LMX

Parameters	Median	StdDev	2.5%	97.5%	ChiSq	PValue	N_Eff
<hr/>							
Variances:							
L2 : Var(Intercept)	<b>0.084</b>	0.046	0.014	0.192	---	---	372.982
L2 : Cov(Empower, Intercept)	<b>-0.009</b>	0.007	-0.024	0.002	---	---	195.594
L2 : Var(Empower)	<b>0.003</b>	0.002	0.000	0.008	---	---	157.077
Residual Var.	<b>1.116</b>	0.072	0.987	1.270	---	---	3159.871
<hr/>							
Coefficients:							
Intercept	<b>3.995</b>	0.064	3.870	4.121	3908.187	0.000	3387.670
LMX	<b>0.168</b>	0.019	0.131	0.205	78.759	0.000	10859.531
Empower	<b>0.013</b>	0.014	-0.015	0.041	0.851	0.356	8238.233
Male.1	<b>0.205</b>	0.092	0.024	0.385	4.913	0.027	21706.381
LMX.mean[Team]	<b>0.201</b>	0.131	0.014	0.545	2.750	0.097	375.083
Empower.mean[Team]	<b>0.188</b>	0.107	0.043	0.469	3.596	0.058	344.974
<hr/>							
Proportion Variance Explained							
by Coefficients	<b>0.225</b>	0.034	0.162	0.295	---	---	1505.422
by Level-2 Random Intercepts	<b>0.051</b>	0.027	0.009	0.114	---	---	362.076
by Level-2 Random Slopes	<b>0.037</b>	0.022	0.001	0.087	---	---	151.893
by Level-1 Residual Variation	<b>0.682</b>	0.038	0.606	0.754	---	---	1343.492

---

# INTERPRETATIONS

---

Parameter	Est.	Interpretation
Fixed intercept	4.00	Expected job satisfaction for a supervisee with zero values on the predictor (the grand mean because the predictors are centered)
LMX (within-team)	0.17	Expected satisfaction difference between two people <i>from the same team</i> with LMX scores that differ by one point, controlling for empowerment and gender
Empower (within-team)	0.01	Expected satisfaction difference between two people <i>from the same team</i> with empowerment scores that differ by one point, controlling for LMX and gender
Gender (mostly within-team)	0.21	Expected satisfaction difference between a male and female <i>from the same team</i> , controlling for LMX and empowerment
LMX (between-team)	0.20	Expected satisfaction mean difference between two teams with average LMX ratings that differ by one point, controlling for average empowerment and gender
Empower (between-team)	0.19	Expected satisfaction mean difference between two teams with average empowerment ratings that differ by one point, controlling for average LMX and gender

# INTERPRETATIONS, CONTINUED

---

Parameter	Est.	Interpretation
Between-cluster intercept variance (variance of $u_{0j}$ residuals)	0.08	Average squared distance between the job satisfaction means and their predicted values (residual level-2 mean differences)
Between-cluster slope variance (variance of $u_{2j}$ residuals)	0.003	Average squared distance between the team-specific slopes and the mean slope
Intercept-slope covariance (covariance of $u_{0j}$ and $u_{2j}$ residuals)	-0.01	Positive association where teams with higher job satisfaction means (intercepts) tend to have lower (less positive or perhaps negative) slopes
Within-cluster residual variance (variance of $\varepsilon_{ij}$ residuals)	1.12	Average squared distance between a supervisee's observed and predicted job satisfaction scores (residual within-cluster variation)

# OUTLINE

- 1 Estimate ICCs
- 2 Add Within-Cluster Predictors
- 3 Strategy 1: Evaluate Random Slopes One at a Time
- 4 Strategy 2: Add All Random Slopes at Once
- 5 Strategy 3: Add Random Slopes in a Sequence
- 6 Add Between-Cluster Predictors
- 7 Latent Variable Specification

# BLIMP SCRIPT 5.7

---

**DATA:** EmployeeSatisfaction.dat;

**VARIABLES:** Employee Team Turnover Male Empower LMX JobSat Climate TeamPerf;

**NOMINAL:** Male;

**CLUSTERID:** Team;

**LATENT:** Team = beta0j beta2j; # define random intercept and slope

**CENTER:** groupmean = LMX Empower; grandmean = Male LMX.mean Empower.mean;

**MODEL:**

beta0j ~ intercept LMX.mean Empower.mean;

beta2j ~ intercept;

beta0j ~~ beta2j; # correlate random intercepts and slopes

JobSat ~ intercept@beta0j LMX Empower@beta2j Male;

**BURN:** 20000;

**ITERATIONS:** 30000;

**SEED:** 90291;

# RBLIMP SCRIPT 5 (MODEL 7)

---

```
model7 <- rblimp(  
  data = Employee,  
  nominal = 'Male',  
  clusterid = 'Team',  
  latent = 'Team = beta0j beta2j',  
  center = 'groupmean = LMX Empower; grandmean = Male LMX.mean Empower.mean',  
  model = '  
    beta0j ~ intercept LMX.mean Empower.mean;  
    beta2j ~ intercept;  
    beta0j ~~ beta2j;  
    JobSat ~ intercept@beta0j LMX Empower@beta2j Male',  
  seed = 90291,  
  burn = 20000,  
  iter = 30000)  
output(model7)
```