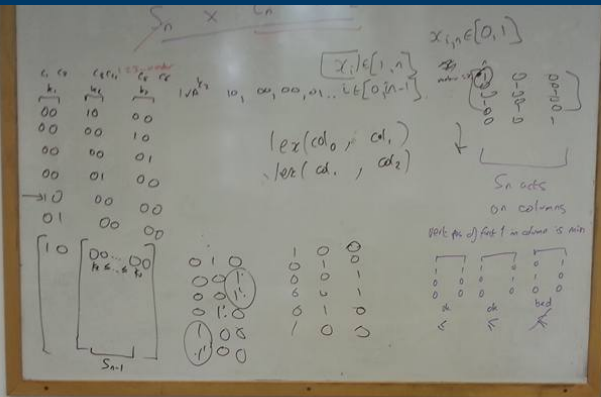




# Enumeration of (unique reduced alternating) knot diagrams

Ciaran McCreesh, Alice Miller, Patrick Prosser,  
**Craig Reilly**, James Trimble

University  
of Glasgow

# What is a knot?

- A *knot* is an embedding of the circle in  $\mathbb{R}^3$ .

# What is a knot?

- A *knot* is an embedding of the circle in  $\mathbb{R}^3$ .
- An intuitive way to think about this is to consider a knot as a knotted piece of string with the ends glued together.

# Drawing knots on paper

- A function  $f : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  where  $f(x, y, z) = f(x, y)$ , is called a *projection map*, and the image of a knot  $K$  under  $f$  is called the *projection* of  $K$ .
- Such a projection is often referred to as the *shadow* of  $K$ .

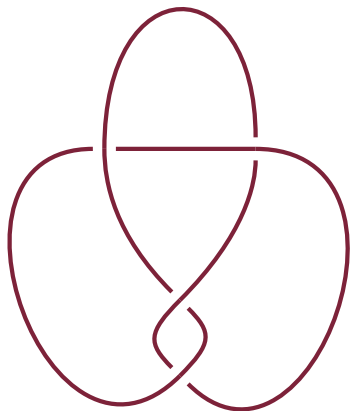
# Drawing knots on paper

- Information regarding the orientation of arcs at crossings is given by leaving gaps in a knot's shadow..

# Drawing knots on paper



# Drawing knots on paper





# Representations of knots

- Knot diagrams are really just 4-valent planar graphs.
  - The vertices in the graph correspond to the crossings in the knot diagram.
  - The arcs between vertices correspond to arcs between crossings in the knot diagram.
  - The arcs are decorated with their orientation at their source and target crossings.
- Other data structures familiar to computer scientists can be used, linked lists of crossings were popular in the 1950's.

# Representations of knots

- The representations used by topologists are typically also used for representing knots in a computer.
- Examples are Dowker-Thistlethwait codes (DT codes), **Gauss codes**, braid representatives, Conway notation, and many more.

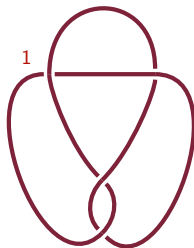
# Representing knots with Gauss codes

- The strategy for representing a given knot (with  $n$  crossings) by a Gauss code is as follows.
  - 1 Label the crossings with the numbers 1 to  $n$ .
  - 2 Pick a point on the knot.
  - 3 Pick a direction and walk around the knot, writing out a list of the numbers you come to (with a negative sign indicating that a crossing was visited on an under strand). Stop when each number appears twice once with each sign.



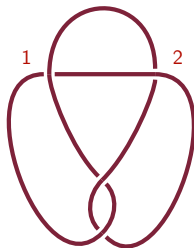
# Representing knots with Gauss codes

- The strategy for representing a given knot (with  $n$  crossings) by a Gauss code is as follows.
  - 1 Label the crossings with the numbers 1 to  $n$ .
  - 2 Pick a point on the knot.
  - 3 Pick a direction and walk around the knot, writing out a list of the numbers you come to (with a negative sign indicating that a crossing was visited on an under strand). Stop when each number appears twice once with each sign.



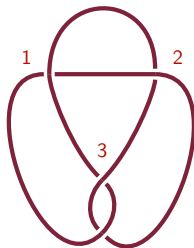
# Representing knots with Gauss codes

- The strategy for representing a given knot (with  $n$  crossings) by a Gauss code is as follows.
  - 1 Label the crossings with the numbers 1 to  $n$ .
  - 2 Pick a point on the knot.
  - 3 Pick a direction and walk around the knot, writing out a list of the numbers you come to (with a negative sign indicating that a crossing was visited on an under strand). Stop when each number appears twice once with each sign.



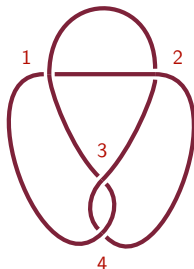
# Representing knots with Gauss codes

- The strategy for representing a given knot (with  $n$  crossings) by a Gauss code is as follows.
  - 1 Label the crossings with the numbers 1 to  $n$ .
  - 2 Pick a point on the knot.
  - 3 Pick a direction and walk around the knot, writing out a list of the numbers you come to (with a negative sign indicating that a crossing was visited on an under strand). Stop when each number appears twice once with each sign.



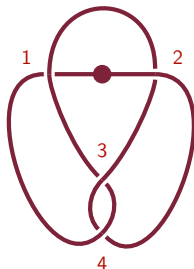
# Representing knots with Gauss codes

- The strategy for representing a given knot (with  $n$  crossings) by a Gauss code is as follows.
  - 1 Label the crossings with the numbers 1 to  $n$ .
  - 2 Pick a point on the knot.
  - 3 Pick a direction and walk around the knot, writing out a list of the numbers you come to (with a negative sign indicating that a crossing was visited on an under strand). Stop when each number appears twice once with each sign.



# Representing knots with Gauss codes

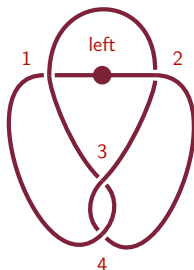
- The strategy for representing a given knot (with  $n$  crossings) by a Gauss code is as follows.
  - 1 Label the crossings with the numbers 1 to  $n$ .
  - 2 Pick a point on the knot.
  - 3 Pick a direction and walk around the knot, writing out a list of the numbers you come to (with a negative sign indicating that a crossing was visited on an under strand). Stop when each number appears twice once with each sign.





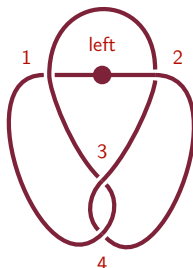
# Representing knots with Gauss codes

- The strategy for representing a given knot (with  $n$  crossings) by a Gauss code is as follows.
  - 1 Label the crossings with the numbers 1 to  $n$ .
  - 2 Pick a point on the knot.
  - 3 Pick a direction and walk around the knot, writing out a list of the numbers you come to (with a negative sign indicating that a crossing was visited on an under strand). Stop when each number appears twice once with each sign.



# Representing knots with Gauss codes

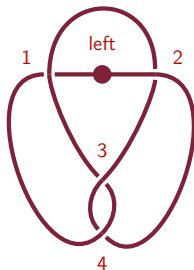
- The strategy for representing a given knot (with  $n$  crossings) by a Gauss code is as follows.
  - 1 Label the crossings with the numbers 1 to  $n$ .
  - 2 Pick a point on the knot.
  - 3 Pick a direction and walk around the knot, writing out a list of the numbers you come to (with a negative sign indicating that a crossing was visited on an under strand). Stop when each number appears twice once with each sign.



-1

# Representing knots with Gauss codes

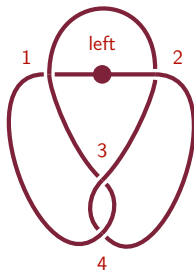
- The strategy for representing a given knot (with  $n$  crossings) by a Gauss code is as follows.
  - 1 Label the crossings with the numbers 1 to  $n$ .
  - 2 Pick a point on the knot.
  - 3 Pick a direction and walk around the knot, writing out a list of the numbers you come to (with a negative sign indicating that a crossing was visited on an under strand). Stop when each number appears twice once with each sign.



$-1, 4$

# Representing knots with Gauss codes

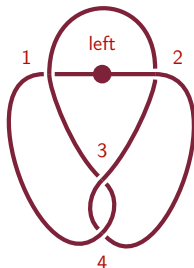
- The strategy for representing a given knot (with  $n$  crossings) by a Gauss code is as follows.
  - 1 Label the crossings with the numbers 1 to  $n$ .
  - 2 Pick a point on the knot.
  - 3 Pick a direction and walk around the knot, writing out a list of the numbers you come to (with a negative sign indicating that a crossing was visited on an under strand). Stop when each number appears twice once with each sign.



$-1, 4, -3$

# Representing knots with Gauss codes

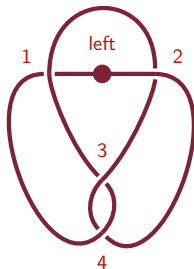
- The strategy for representing a given knot (with  $n$  crossings) by a Gauss code is as follows.
  - 1 Label the crossings with the numbers 1 to  $n$ .
  - 2 Pick a point on the knot.
  - 3 Pick a direction and walk around the knot, writing out a list of the numbers you come to (with a negative sign indicating that a crossing was visited on an under strand). Stop when each number appears twice once with each sign.



$-1, 4, -3, 1$

# Representing knots with Gauss codes

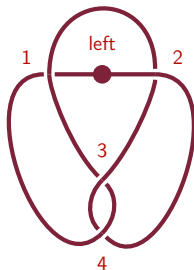
- The strategy for representing a given knot (with  $n$  crossings) by a Gauss code is as follows.
  - 1 Label the crossings with the numbers 1 to  $n$ .
  - 2 Pick a point on the knot.
  - 3 Pick a direction and walk around the knot, writing out a list of the numbers you come to (with a negative sign indicating that a crossing was visited on an under strand). Stop when each number appears twice once with each sign.



$-1, 4, -3, 1,$   
 $-2$

# Representing knots with Gauss codes

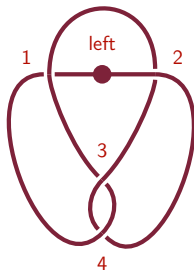
- The strategy for representing a given knot (with  $n$  crossings) by a Gauss code is as follows.
  - 1 Label the crossings with the numbers 1 to  $n$ .
  - 2 Pick a point on the knot.
  - 3 Pick a direction and walk around the knot, writing out a list of the numbers you come to (with a negative sign indicating that a crossing was visited on an under strand). Stop when each number appears twice once with each sign.



$-1, 4, -3, 1,$   
 $-2, 3$

# Representing knots with Gauss codes

- The strategy for representing a given knot (with  $n$  crossings) by a Gauss code is as follows.
  - 1 Label the crossings with the numbers 1 to  $n$ .
  - 2 Pick a point on the knot.
  - 3 Pick a direction and walk around the knot, writing out a list of the numbers you come to (with a negative sign indicating that a crossing was visited on an under strand). Stop when each number appears twice once with each sign.

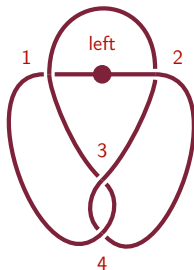


$-1, 4, -3, 1,$   
 $-2, 3, -4$



# Representing knots with Gauss codes

- The strategy for representing a given knot (with  $n$  crossings) by a Gauss code is as follows.
  - 1 Label the crossings with the numbers 1 to  $n$ .
  - 2 Pick a point on the knot.
  - 3 Pick a direction and walk around the knot, writing out a list of the numbers you come to (with a negative sign indicating that a crossing was visited on an under strand). Stop when each number appears twice once with each sign.



$-1, 4, -3, 1,$   
 $-2, 3, -4, 2$