This is looking at the photon emission from a plasma due to Bremsstrahlung source between two limits.

```
ZEff2 = 4 * .076 + (1 - .076) (* Effective Z^2 of a plasma with primordial abundance *)
```

1.228

$$P[\omega\_] := \frac{16}{3}\sqrt{\frac{2\pi}{3}}\ \alpha\, r_e{}^2\, c\, \hbar\, Z_e{}^2\, n^2 \sqrt{\frac{m_e\, c^2}{k_b\, T}}\ \mathrm{Exp}\left[-\frac{\hbar\, \omega}{k_b\, T}\right]$$

(* Power emitted per unit volume per frequency interval. *)

```
Ptot = Simplify[Integrate[P[ω], {ω, 0, ∞}], {T > 0, ℏ > 0, kb > 0}]
```

$$\frac{16}{3}\, c\, n^2 \sqrt{\frac{2\pi}{3}}\ \alpha \sqrt{c^2\, T\, k_b\, m_e}\ r_e^2\, Z_e^2$$

This is the total power emitted per unit volume, integrated over all frequencies. This verifies the $n^2\sqrt{T}$ dependence for the total power.

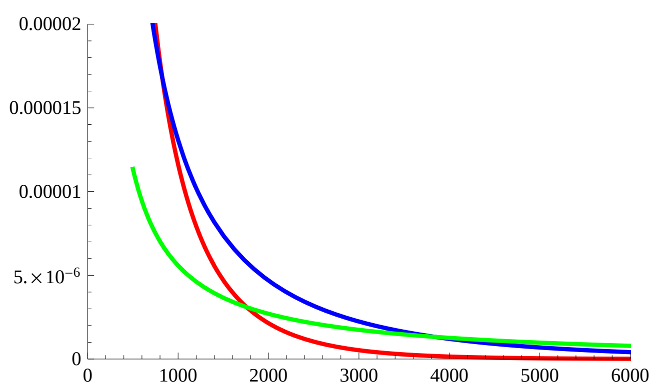
However, Chandra data is in photons / cm^2 sec, so we need to calculate the number of photons emitted per unit volume per second. This necessitates dividing by the energy per photon.

$$Nph[En\_] := \frac{16}{3}\sqrt{\frac{2\pi}{3}}\ \alpha\, r_e{}^2\, c\ Z_e{}^2\, n^2 \sqrt{\frac{m_e\, c^2}{k_b\, T}}\ \frac{\mathrm{Exp}\left[-\frac{En}{k_b\, T}\right]}{En}$$

(* Number of photons emitted per unit volume per energy interval. *)

$$\mathrm{Plot}\left[\left\{\frac{\mathrm{Exp}\left[\frac{-En}{T}\right]}{En\sqrt{T}} /.\ T \to 1000,\ \frac{\mathrm{Exp}\left[\frac{-En}{T}\right]}{En\sqrt{T}} /.\ T \to 3000,\ \frac{\mathrm{Exp}\left[\frac{-En}{T}\right]}{En\sqrt{T}} /.\ T \to 30\,000\right\},\right.$$

```
{En, 500, 6000}, PlotStyle → {{Red, Thick}, {Blue, Thick}, {Green, Thick}},
PlotRange → {{0, 6000}, {0, 2 * 10^-5}}]
```

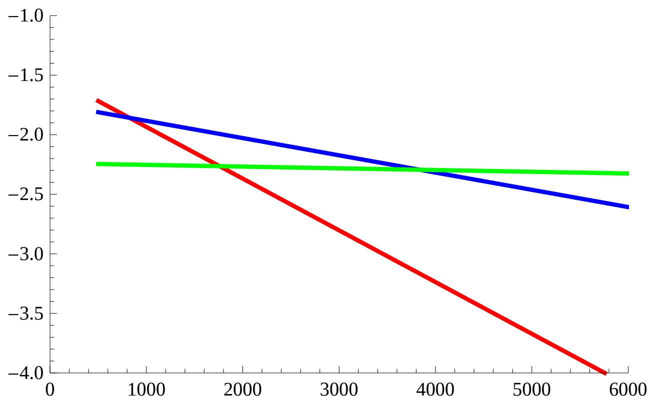


These look roughly like the spectra from Chandra. The semilog slope of Nph(E) *E should give me the temperature.

$$\texttt{Plot}\Big[\Big\{\texttt{Log}\Big[10,\ \frac{\texttt{Exp}\big[\frac{-\texttt{En}}{\texttt{T}}\big]}{\sqrt{\texttt{T}}}\Big]\ /.\ \texttt{T}\to 1000,$$

$$\texttt{Log}\Big[10,\ \frac{\texttt{Exp}\big[\frac{-\texttt{En}}{\texttt{T}}\big]}{\sqrt{\texttt{T}}}\Big]\ /.\ \texttt{T}\to 3000,\ \texttt{Log}\Big[10,\ \frac{\texttt{Exp}\big[\frac{-\texttt{En}}{\texttt{T}}\big]}{\sqrt{\texttt{T}}}\Big]\ /.\ \texttt{T}\to 30\,000\Big\},\ \{\texttt{En},\ 500,\ 6000\},$$

$$\texttt{PlotStyle}\to\{\{\texttt{Red, Thick}\},\{\texttt{Blue, Thick}\},\{\texttt{Green, Thick}\}\},\ \texttt{PlotRange}\to\{\{0,\ 6000\},\{-4,\ -1\}\}\Big]$$



$$\texttt{Integrate}\Big[\frac{\texttt{Exp}[-\texttt{En}]}{\texttt{En}},\ \texttt{En}\Big]$$

```
ExpIntegralEi[-En]
```

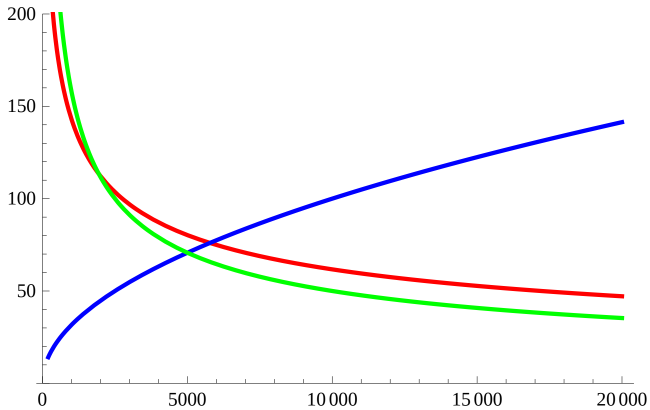
```
FullSimplify[ExpIntegralEi[-x] == -ExpIntegralE[1, x], x > 0]
```

```
True
```

$$\texttt{Ntot} = \frac{16}{3}\sqrt{\frac{2\pi}{3}}\ \alpha\, r_e^2\, c\, Z_e^2\, n^2\sqrt{\frac{m_e c^2}{k_b\, T}}\ \Big(\texttt{ExpIntegralE}\Big[1,\ \frac{\texttt{Emin}}{k_b\, T}\Big]-\texttt{ExpIntegralE}\Big[1,\ \frac{\texttt{Emax}}{k_b\, T}\Big]\Big);$$

The exponential integral diverges at E=0, necessitating cutting it off. When we do this, we see that the temperature dependence is opposite to $\sqrt{T}$.

```
Plot[{(ExpIntegralE[1, Emin/T] - ExpIntegralE[1, Emax/T]) Sqrt[511000/T] /.
   {Emax → 2 * 10^6, Emin → 1}, Sqrt[T], 5000/Sqrt[T]}, {T, 200, 20000},
 PlotStyle → {{Red, Thick}, {Blue, Thick}, {Green, Thick}}, PlotRange → {0, 200}]
```



This shows the total number of photons/Volume/time as a function of T in eV, integrated over most of the spectrum. It decreases rather than increasing with T.

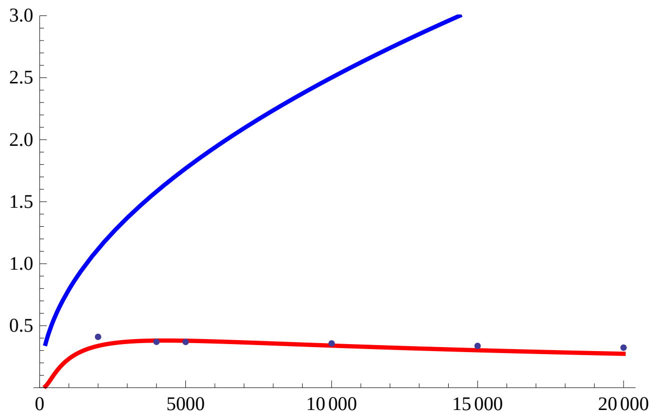
In[11]:=
```
PreFactor = 16/3 Sqrt[2 π/3] α re^2 c Ze^2 NeNi * 10^14 /.
   {Ze → 1.23, NeNi → 1.23, α → 1/137, re → 2.82 * 10^-13, c → 3.0 * 10^10} // Simplify
```

Out[11]= 0.0250116

In[12]:=
```
ApecList =
  {{2000, .410}, {4000, .370}, {5000, .369}, {10000, .357}, {15000, .337}, {20000, .323}};
```

In[13]:=
```
Show[Plot[{PreFactor * (ExpIntegralE[1, Emin/T] - ExpIntegralE[1, Emax/T]) Sqrt[511000/T] /.
    {Emax → 6000 * (1.3), Emin → 500 (1.3)}, PreFactor * √T}, {T, 200, 20000},
   PlotStyle → {{Red, Thick}, {Blue, Thick}}, PlotRange → {0, 3}], ListPlot[ApecList]]
```

Out[13]=



This shows the total number of photons/Volume/time as a function of T in eV, for realistic energy limits and plasma temperatures. It is relatively flat with T. It agrees relatively well with the apec code, including with the full normalization