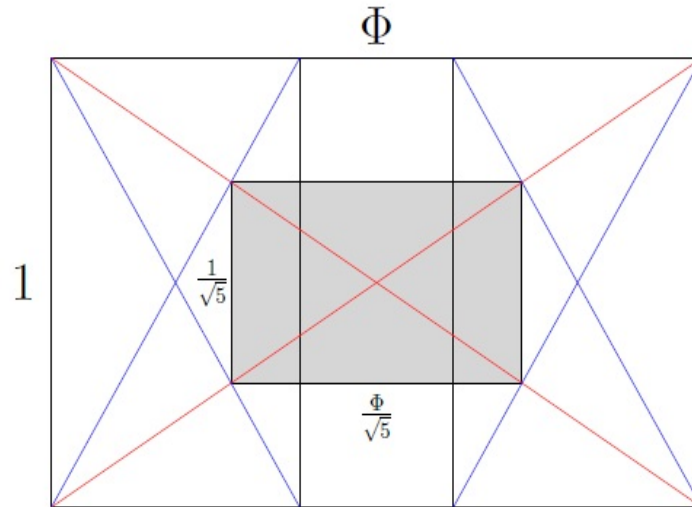# Area of the Inner Golden Rectangle

Prove that the inner golden rectangle with corners at the four eyes of God is reduced in scale from the outer golden rectangle by the factor $\sqrt{5}$.

Solution

The coordinates of the center of one of the golden spirals is given by

$$(x, y) = \left(\frac{5 + 3\sqrt{5}}{10}, \frac{5 - \sqrt{5}}{10} \right).$$

The centers of the four possible golden spirals are symmetric about $(\Phi/2, 1/2)$. The four vertices can be determined as

$$\left(\frac{\Phi}{2} + \frac{5 + \sqrt{5}}{20}, \frac{1}{2} - \frac{\sqrt{5}}{10} \right), \left(\frac{\Phi}{2} - \frac{5 + \sqrt{5}}{20}, \frac{1}{2} - \frac{\sqrt{5}}{10} \right),$$
$$\left(\frac{\Phi}{2} - \frac{5 + \sqrt{5}}{20}, \frac{1}{2} + \frac{\sqrt{5}}{10} \right), \left(\frac{\Phi}{2} + \frac{5 + \sqrt{5}}{20}, \frac{1}{2} + \frac{\sqrt{5}}{10} \right).$$

The length of the sides of the rectangle can then be calculated to be

$$L = \frac{1 + \sqrt{5}}{2\sqrt{5}}, \; W = \frac{1}{\sqrt{5}}$$

which is the golden rectangle reduced by the factor of $\sqrt{5}$.