

4. Every odd natural number is one of the forms $4n + 1$ or $4n + 3$, where n is an integer.

Proof:

Every odd number is of the form $2k + 1$ where k is an integer.

If k is even, then $k = 2n$. Substituting into $2k + 1$ gives $2(2n) + 1 = 4n + 1$.

If k is odd, then $k = 2n + 1$. Substituting into $2k + 1$ gives $2(2n + 1) + 1 = 4n + 2 + 1 = 4n + 3$.

Therefore, every odd number can be expressed as $4n + 1$ or $4n + 3$, where n is an integer.