

3. For any integer  $n$ ,  $n^2 + n + 1$  is odd.

TRUE.

Proof:

If  $n^2 + n + 1$  is odd, then  $n^2 + n$  is even.

So  $n(n + 1)$  is even.

$n$  can be either even or odd.

If  $n$  is odd, then  $n(n + 1)$  is even (Since an odd number multiplied by an even number is even.)

If  $n$  is even, then  $n(n + 1)$  is even (Since an even number multiplied by an even number is even.)

So  $n(n + 1)$  is even for any integer  $n$ .

This means  $n(n + 1) + 1$  is odd for all integers.

Therefore,  $n(n + 1) + 1 = n^2 + n + 1$  is odd for any integer  $n$ .