

10. Give an example of a family of intervals $A_n, n = 1, 2, \dots$, such that $A_{n+1} \subset A_n$ and $\bigcap_{n=1}^{\infty} A_n$ consists of a single real number.

Consider the set of intervals A_n , where each interval is given by $(0, \frac{n}{n^2}]$.

$A_{n+1} \subset A_n$ for all n .

Proof ($A_{n+1} \subset A_n$):

$A_{n+1} \subset A_n$ is equivalent to $[0, \frac{1}{n+1}] \subset [0, \frac{1}{n}]$.

For every element x of $[0, \frac{1}{n+1}]$, $0 \leq x \leq \frac{1}{n+1}$ and for every element y of $[0, \frac{1}{n}]$, $0 \leq y \leq \frac{1}{n}$.

$\frac{1}{n+1} < \frac{1}{n}$, so every element of A_{n+1} is also an element of A_n .

It follows that A_{n+1} is a subset of A_n for all n .

Proof ($\bigcap_{n=1}^{\infty} A_n$ consists of a single real number):

$\bigcap_{n=1}^{\infty} A_n$ is the limit as $n \rightarrow \infty$ (which is the intersection of all A_n).

When $n \rightarrow \infty$, $[0, \frac{1}{n}] = [0, 0]$, which is a set with a single real number (0).