Another Derivation of Binet's formula

Use the linearization formulas

$\phi^n = F_n\phi + F_{n-1}$ and $(-\varphi)^n = -F_n\varphi + F_{n-1}$

to derive Binet's formula.

$F_n(\phi + \varphi) = \phi^n - (-\varphi)^n$

Substitute $\phi + \varphi = \sqrt{5}$:

$F_n(\sqrt{5}) = \phi^n - (-\varphi)^n$

$F_n = \dfrac{\phi^n - (-\varphi)^n}{\sqrt{5}}$