

9. Give an example of a family of intervals  $A_n, n = 1, 2, \dots$ , such that  $A_{n+1} \subset A_n$  and  $\bigcap_{n=1}^{\infty} A_n = \emptyset$ .

Consider  $A_n = [n, \infty)$ .

$A_{n+1} \subset A_n$  for all  $n$ .

Proof:

Since  $n + 1 > n$  and the interval is unbounded above,  $[n + 1, \infty) \subset [n, \infty)$ . Therefore,  $A_{n+1} \subset A_n$ .

$\bigcap_{n=1}^{\infty} A_n$  is the limit as  $n \rightarrow \infty$ . But this set does not contain a real number, so  $\bigcap_{n=1}^{\infty} A_n = \emptyset$ .