5. For any integer n, at least one of the integers n, $n+2$, $n+4$ is divisible by 3.

   Proof (by induction):

   Let $P(n)$ be the statement that at least one of the integers n, $n+2$, $n+4$ is divisible by 3.

   Base Case: for $n = 1$, $n + 2 = 3$, which is divisible by 3. So $P(1)$ is true.

   Induction Step: let $n = k$ for some integer k. Assuming $P(k)$ is true we will show that $P(k+1)$ is true.
   We will show that if any one of k, $k+2$, or $k+4$ is divisible by 3, then at least one of $k+1$, $(k+1)+2$, or $(k+1)+4$ is divisible by 3.

   For k:
   k is divisible by 3 if $k = 3p$ for some integer p.
   $k + 1 = 3p + 1$.
   $(k + 1) + 2 = 3p + 3 = 3(p + 1)$ (which is divisible by 3).

   For $k + 2$:
   $k + 2$ is divisible by 3 if $k + 2 = 3p$ for some integer p.
   $(k + 1) + 2 = 3p + 1$.
   $(k + 1) + 2 + 2 = 3p + 3$.
   $(k + 1) + 4 = 3p + 3 = 3(p + 1)$ (which is divisible by 3).

   For $k + 4$:
   $k + 4$ is divisible by 3 if $k + 4 = 3p$ for some integer p.
   $(k + 1) + 4 = 3p + 1$.
   $(k + 1) + 4 - 4 = 3p + 1 - 4$.
   $k + 1 = 3p - 3 = 3(p - 1)$ (which is divisible by 3).

   So if any of of k, $k+2$, or $k+4$ is divisible by 3, then at least one of $k+1$, $(k+1)+2$, or $(k+1)+4$ is divisible by 3.

   Therefore, $P(k) \Rightarrow P(k + 1)$ and, by the principle of induction $P(n)$ is true for all integers n.