

Sum of Lucas Numbers

Show that the sum over the first n Lucas numbers is given by

$$\sum_{i=1}^n L_i = L_{n+2} - 3.$$

Solution

The relation $L_n = L_{n+2} - L_{n+1}$ is used to form a list:

$$L_n = L_{n+2} - L_{n+1}$$

$$L_n = L_{n+1} - L_n$$

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$$L_n = L_n - L_{n-1}$$

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$$L_2 = L_4 - L_3$$

$$L_1 = L_3 - L_2$$

By adding the right side and substituting $L_2 = 3$, we obtain $\sum_{i=1}^n L_i = L_{n+2} - 3$.