The Fibonacci Double Index Formula

(a) Use the Fibonacci addition formula to prove the Fibonacci double index formulas

$$F_{2n-1} = F_{n-1}^2 + F_n^2,$$

$$F_{2n} = F_n\left(F_{n-1} + F_{n+1}\right).$$

(b) Show that

$$F_{2n} = L_n F_n$$

Solution

(a) Taking $F_{n+m} = F_{n-1}F_m + F_n F_{m+1}$ with $m \to n-1$ gives the result.

(b) $m \to n$ gives the result.