

1. $(\exists m \in \mathbb{N})(\exists n \in \mathbb{N})(3m + 5n = 12)$

FALSE.

Proof:

Suppose $3m + 5n = 12$ for natural numbers m, n .

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$$m = \frac{12-5n}{3}$$

$$m = 4 - \frac{5n}{3}$$

The LCM of 5 and 3 is 15. So if $\frac{5n}{3}$ is an integer, then $n \geq 3$.

For $n = 3$, $m = 4 - 5$ (not in \mathbb{N}). Similarly, when $n > 3$, m is negative and thus not a natural number.

But for $n < 3$, $\frac{5n}{3}$ is not an integer, so $4 - \frac{5n}{3}$ is also not in \mathbb{N} .

This means m is not a natural number, which is a contradiction.

Therefore, it is not the case that $(\exists m \in \mathbb{N})(\exists n \in \mathbb{N})(3m + 5n = 12)$.