Exploring the Effectiveness of Compartmental Models to Predict Epidemiological Trends

Craig Miller (s2049878)

Supervisor: Graeme Ackland

The Coronavirus Pandemic & The Omicron Oscillation Problem

COVID-19:

- In March 2020, The Imperial College COVID-19 Response Team modelled how the pandemic could affect the UK population. They predicted that over the course of the pandemic if left completely unhindered, **COVID-19 could kill over half a million UK residents**[2].
- The study focused on strategies that could aid in lowering mortality and reduce pressure on the National Health Service (NHS).
- This report acted as a key resource for scientific advice that was supplied to the British government to aid in enforcing decisions on public guidance and social restrictions.
- This shows the importance of modelling the spread of epidemics such that we can act quickly to intervene and reduce preventable fatalities and other global impacts

Omicron Oscillation Problem:

- As identified in Fig 1, data gathered by WHO displays a clear set of **sustained oscillations in COVID-19 infections and fatalities between June 2022 and 2023**. The near constant period between waves of three months appears unnatural at first glance and requires further investigation.
- The weekly cases show a damped oscillation whereas fatalities hold consistent in their amplitudes. The decrease in amplitude of cases could likely be attributed to a decrease in testing frequency when the public were required to pay for LFTs (Free Lateral Flow Testing ended in April 2022), officially registered deaths do not have this issue and thus stay constant.
- **Finding the driving force behind these oscillations forms the fundamental aim of this project** to replicate this trend in statistics through implementation of an expanded SIRS model.

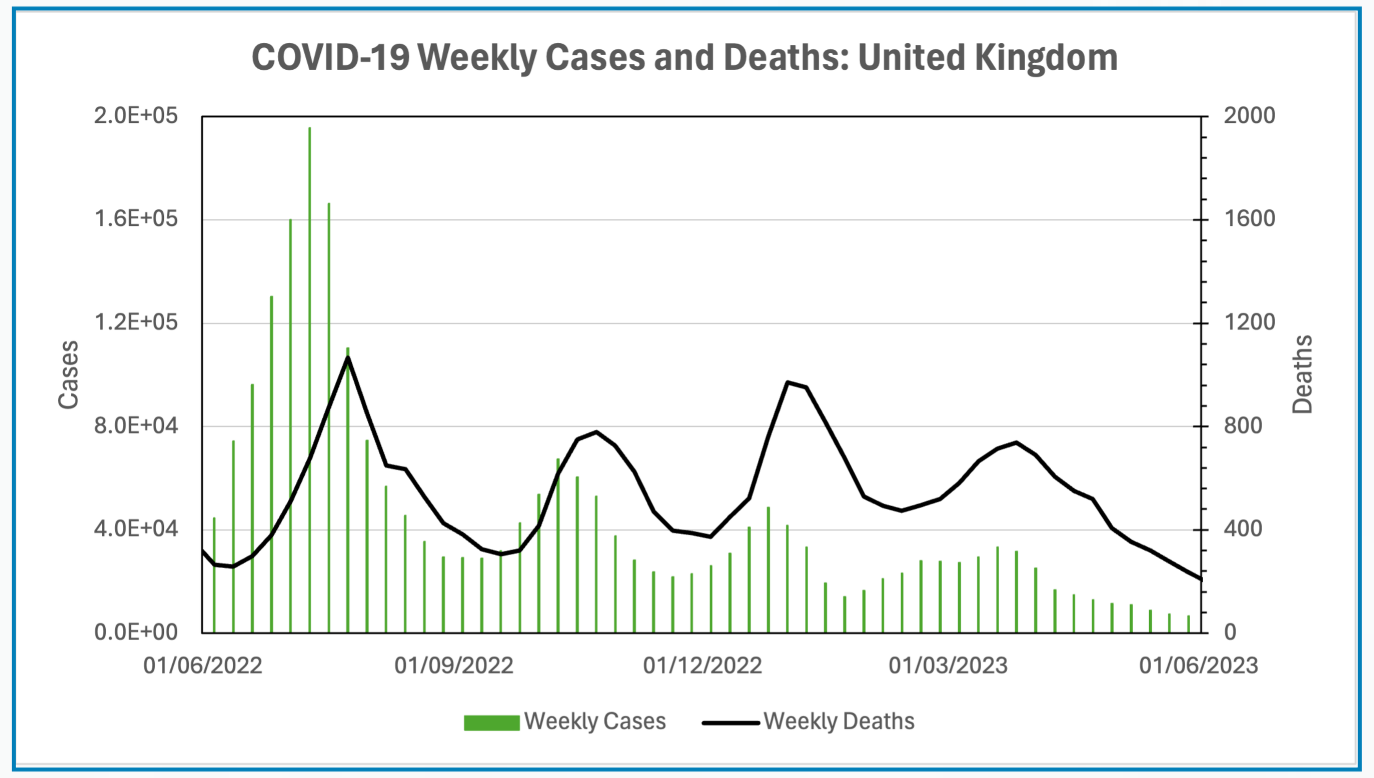

Fig 1: Weekly COVID-19 infection and fatality data among the UK population between June 2022 and June 2023. A clear oscillation pattern arises in both sets of data with death tolls peaking roughly two weeks after infections as expected (people do not pass away immediately). The period of these oscillations is 3 months. Data acquired from [1].

Compartmental Models

SIR Model:

- As seen from the study by Imperial College [2], Compartmental models are a key mechanism in predicting the effects of epidemics and other outbreaks. The foundation being the Susceptible, Infected & Resistant model, commonly known as SIR. The model is comprised of the following ODEs:

$$\frac{\partial S}{\partial t} = -\beta S I \qquad \frac{\partial I}{\partial t} = \beta S I - \gamma I \qquad \frac{\partial R}{\partial t} = \gamma I$$

- Where S, I, R each represent the fraction of the population contained within each of the three states. β & γ are the infection and recovery rates. At each time step the model updates each compartment by removing the newly infected from the susceptible compartment and adding them to the infected group then doing the same for the newly recovered people.

SEIR & SIRS:

- A key feature of compartmental models is that they can always be expanded to resemble more complex systems, The SEIR that adds an "Exposed" section effectively splitting the infection compartment in two to signify that people can be carrying the disease prior to feeling sick. The SIRS variant importantly allows individuals to lose their immunity and return to the susceptible stage over-time.

Gamma Distribution:

- The transitions in traditional SIR based models follow a negative exponential distribution, giving a maximum probability of recovery on day 0. In reality, a person will tend to be infected for a number of days before recovering thus by implementing transitions based on a Gamma distribution we can make it such that individuals remain in a state for a defined mean duration however this can be longer or shorter for different people, successfully modelling the immune responses of people in different demographics.

Expanded Model:

Applying this knowledge, the expanded model in Fig 3 was designed and implemented to be as realistic as possible to effectively reproduce the real data in Fig 1.

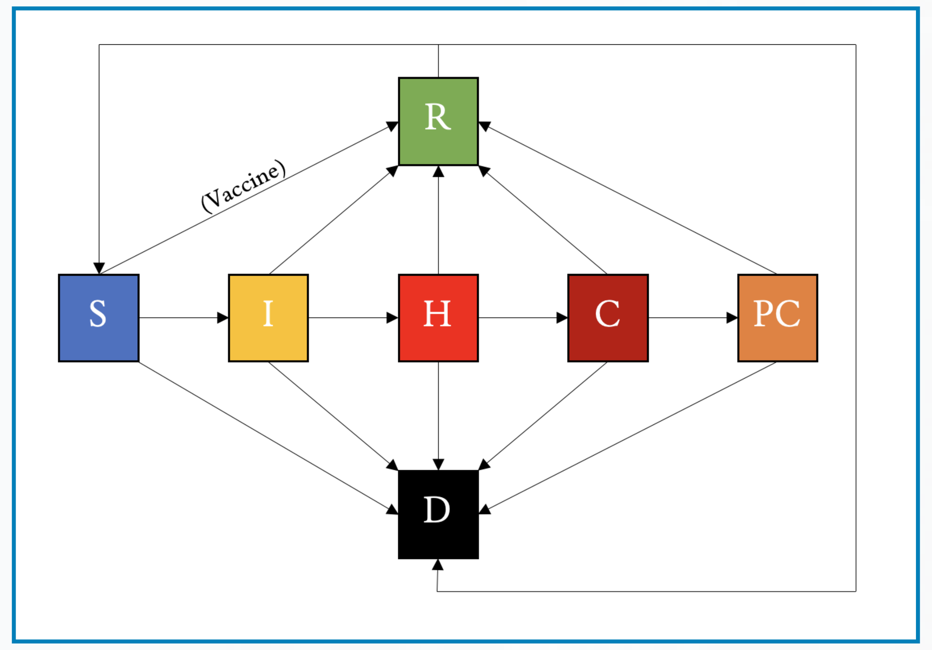

Fig 2: This Figure shows an expanded version of the SIR model, with added compartments to improve the realism and complexity. The compartments shown are as follows: S - Susceptible, I - Infected, H - Hospitalised, C - Critical Care (ICU), PC - Post Critical, R - Resistant, D - Deceased. The arrows show the possible transitions between compartments each having a unique probability.

Implementation & Results

- The main goal to reproduce the oscillatory trends in Omicron data (as displayed in Figure 1) has been carried out through the implementation of a class-based python model following the design structure laid out in Figure 2.
- To allow for further comparison, a spatial model was also carried out to contrast with the main differential equation-based version by using the concepts of cellular automata for its convenient production of natural wave patterns due to a combination of its periodic boundary conditions and temporal effects occurring for certain initial conditions (The results of which are shown in Fig 3).
- **Results**
- **Conclusions**

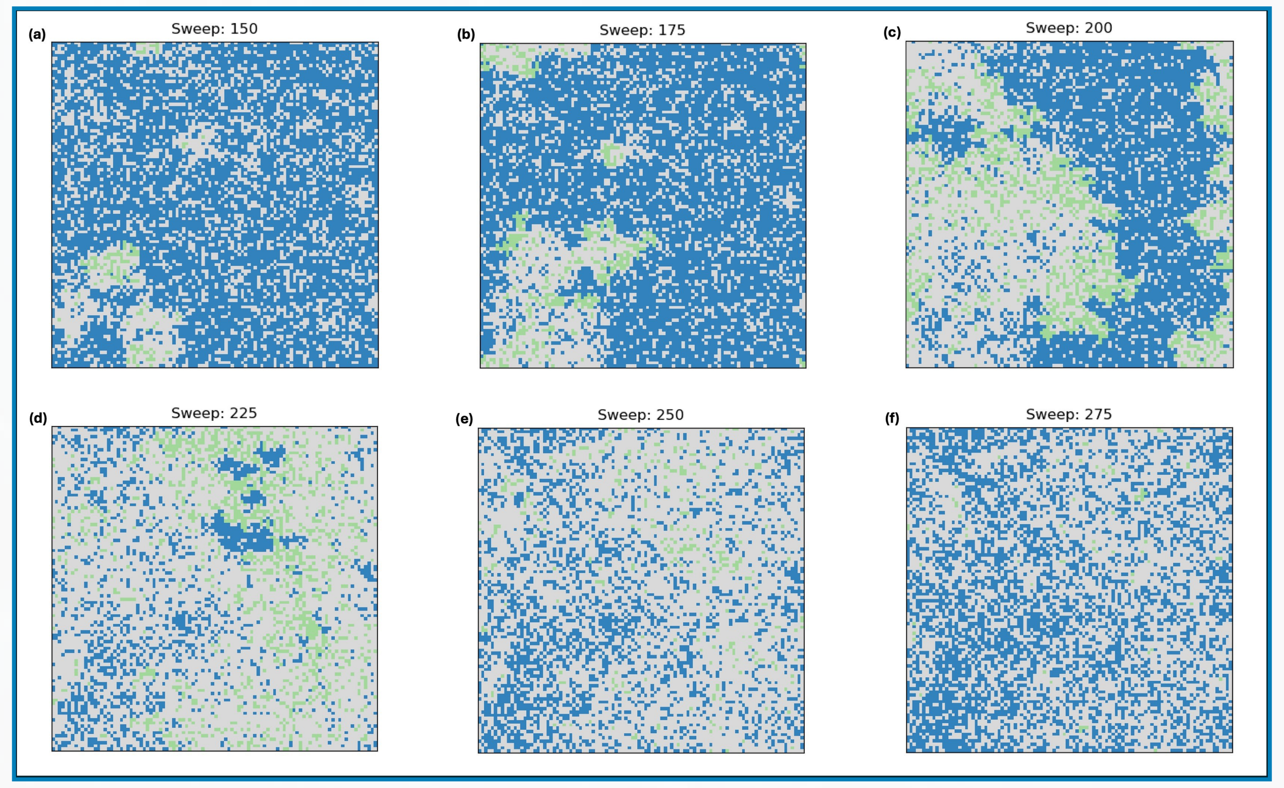

Fig 3: The plots (a)-(f) show the stages of a single wave of infection spreading through a system in a spatial SIR model. The blue, green and grey nodes represent Susceptible, Infected and Resistant individuals respectively.

References

1. **World Health Organization 2023. WHO Coronavirus (COVID-19) dashboard. Date Accessed: 15th March 2024**
2. **Neil M. Ferguson et al. Report 9: Impact of non-pharmaceutical interventions (NPIs) to reduce COVID-19 mortality and healthcare demand. *Imperial College London*, 2020**

Senior Honours Project 2023/24

Computational Physics BSc