University of Nottingham
School of Mathematical Sciences

MATH4063/G14SCC — Scientific Computing and C++

Deadline: 7th January 2022, 5:00pm (GMT) — **Coursework 2 – Solution Template**

Your solutions to the assessed coursework may be submitted using this template. Please cut and paste the output from your codes into the correct parts of this file and include your plots and responses to the questions where suggested. Once this template has been completed, you must then create a pdf file for submission. Under Windows or Mac you can use Texmaker + a LaTeX compiler; from the Windows Virtual Desktop this may be accessed as follows:

`Start > UoN Application > (UoN) Texmaker 5`

Open this file under `File`*; to build the pdf file, click the arrow next to* `Quick Build`*; this will then generate the file* `coursework2_submission.pdf`*.*

You may use an alternative document processing system, such as Word, to produce a pdf file containing your results, plots and answers. However, if you do, you must format your answers in the same way as suggested below.

A single zip or tar file containing the file `coursework2_submission.pdf` *and all the files in the requested folders in the checklists below should be submitted on Moodle. Note that all parameters and values should be set within your codes: do NOT use inputs such as those obtained with* `std::cin` *or from the command line.*

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Question 1

* There are 31 datas below and time gap is 1. The file ForwardEulerSolver.dat saves all 10001 datas. I select 30 h_k in the chart and Eh.dat saves all 990 datas.

```
%%%%%%%%%%%%%%%%%% Output of Driver.cpp %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
t          v          x
0        1.5          0
1  0.0948626   0.997524
2   -1.48658   0.141096
3  -0.305134  -0.977567
4    1.44342  -0.279369
5   0.509301   0.938051
6   -1.37138   0.412053
7  -0.703278  -0.879767
8    1.27191  -0.536493
9   0.883185   0.803882
10   -1.14698   0.650199
11   -1.04542  -0.711914
12   0.999113  -0.750897
13    1.18674   0.605702
14  -0.831252   0.836571
15   -1.30432  -0.487372
16   0.646761  -0.905508
17     1.3958   0.359292
18   -0.44933   0.956329
19   -1.45936  -0.224022
20   0.242909  -0.988016
21    1.49372  0.0842711
22 -0.0316284   0.999936
23   -1.49819  0.0571662
24  -0.180285   -0.99185
25    1.47269   -0.19746
26   0.388592    0.96392
27   -1.41772   0.333803
28  -0.589124  -0.916705
29     1.3344  -0.463467
30   0.777869   0.851149

   h         log(h)      E(h)
0.000100  -9.210340  0.000112
0.000103  -9.176784  0.000116
0.000107  -9.142062  0.000120
0.000111  -9.106090  0.000125
0.000115  -9.068777  0.000129
0.000120  -9.030017  0.000134
0.000125  -8.989694  0.000140
0.000130  -8.947676  0.000146
0.000136  -8.903815  0.000152
0.000142  -8.857942  0.000160
0.000149  -8.809863  0.000168
```

0.000157	-8.759355	0.000176
0.000166	-8.706159	0.000186
0.000175	-8.649974	0.000197
0.000186	-8.590444	0.000209
0.000198	-8.527144	0.000222
0.000212	-8.459564	0.000238
0.000228	-8.387085	0.000256
0.000246	-8.308938	0.000276
0.000268	-8.224164	0.000301
0.000294	-8.131531	0.000330
0.000326	-8.029433	0.000366
0.000365	-7.915713	0.000410
0.000415	-7.787382	0.000466
0.000481	-7.640123	0.000540
0.000571	-7.467371	0.000641
0.000704	-7.258412	0.000791
0.000917	-6.993933	0.001030
0.001316	-6.633318	0.001478
0.002326	-6.063785	0.002613

%%%

Figure 1 is the picture of x(t) using ForwardEulerSolver method.

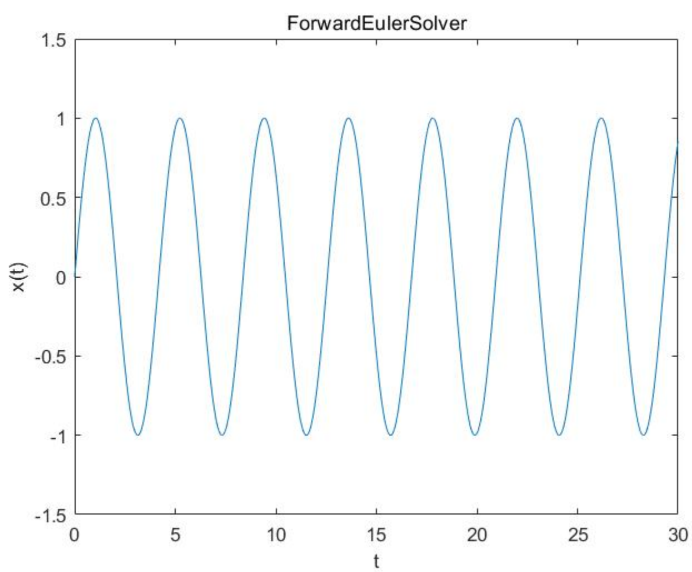


Figure 1: ForwardEulerSolver

The result that estimating relationship between log(h) and log(E(h)) by using OLS method is

$$log(E(h)) = log(h) + 0.1187 \tag{1}$$

then,

$$E(h) = exp(log(h) + 0.1187) \tag{2}$$

Consequently, E(h)=O(h)

Figure2 is the picture of OLS estimation

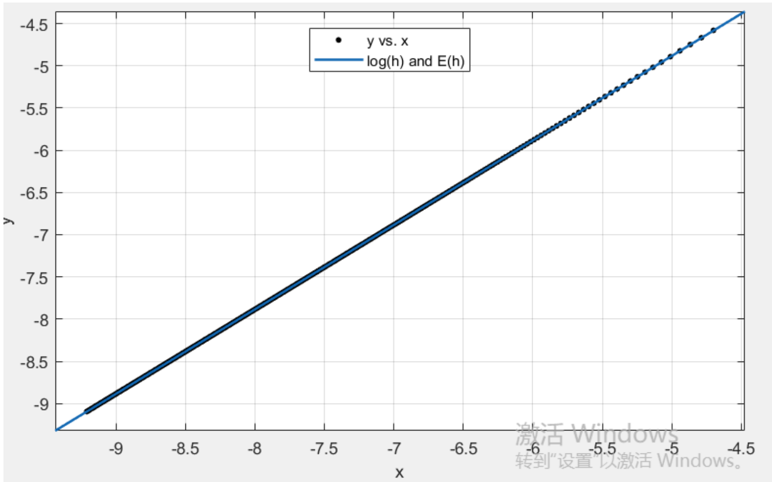


Figure 2: log(h) and log(E(h))

In ForwardeulerSolver.cpp, stepsize is from 10000 to 100, and gap is 10.($h = \frac{1}{stepsize}$)

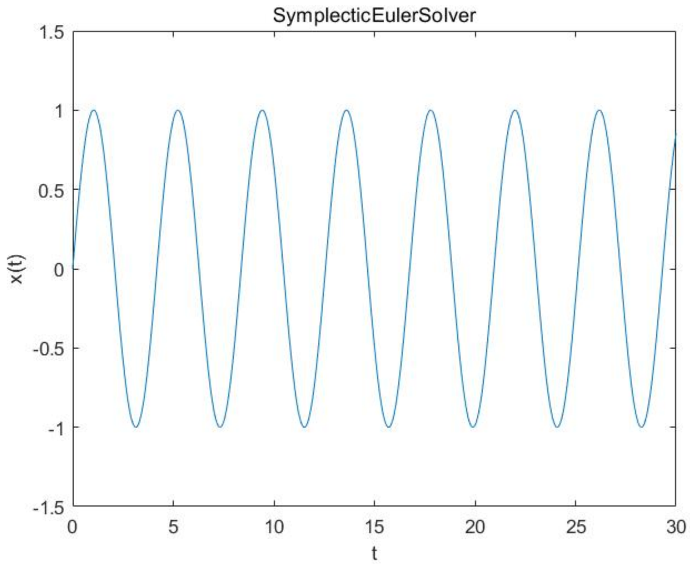
Question 2

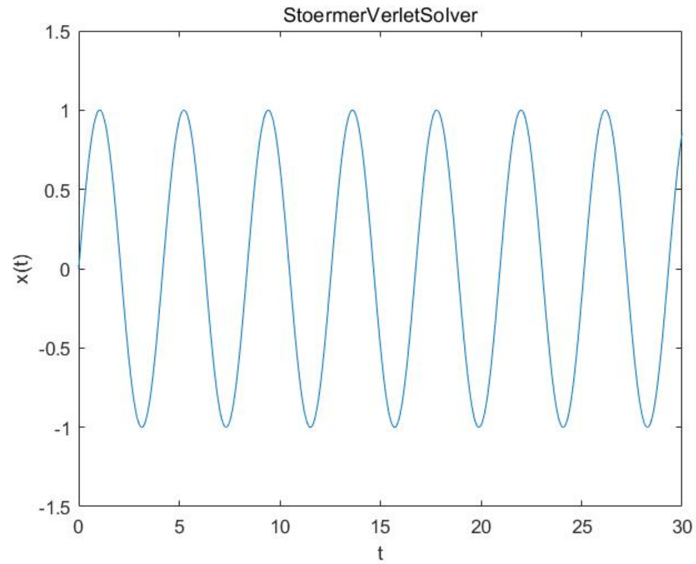
* There are 31 datas below and time gap is 1. The file SymplecticEulerSolver.dat and StoermerVerletSolver.dat saves all 10001 datas. I select 27 h_k in the chart.

```
%%%%%%%%%%%%%%%%%% Output of Driver.cpp %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
SymplecticEulerSolver:                    StoermerVerletSolver
t          v          x                   t          v          x
0        1.5          0                   0        1.5          0
1    0.117307   0.997524                  1    0.106085   0.997524
2    -1.48341   0.141096                  2    -1.48499   0.141096
3   -0.327129  -0.977567                  3   -0.316132  -0.977567
4     1.43714  -0.279369                  4     1.44028  -0.279369
5    0.530407   0.938051                  5    0.519854   0.938051
6    -1.36211   0.412053                  6    -1.36675   0.412053
7   -0.723073  -0.879767                  7   -0.713175  -0.879767
8     1.25984  -0.536493                  8     1.26587  -0.536493
9    0.901272   0.803882                  9    0.892228   0.803882
10    -1.13235   0.650199                10    -1.13967   0.650199
11    -1.06144  -0.711914                11    -1.05343  -0.711914
12    0.982218  -0.750897                12    0.990665  -0.750897
13     1.20037   0.605702                13     1.19356   0.605702
14    -0.81243   0.836571                14   -0.821841   0.836571
15    -1.31529  -0.487372                15     -1.3098  -0.487372
16    0.626387  -0.905508                16    0.636574  -0.905508
17     1.40389   0.359292                17     1.39984   0.359292
18   -0.427813   0.956329                18   -0.438571   0.956329
19     -1.4644  -0.224022                19    -1.46188  -0.224022
20    0.220679  -0.988016                20    0.231794  -0.988016
21     1.49561  0.0842711                21     1.49466  0.0842711
22 -0.00912988   0.999936                22 -0.0203792   0.999936
23     -1.4969  0.0571662                23    -1.49755  0.0571662
24   -0.202602   -0.99185                24   -0.191443   -0.99185
25     1.46825   -0.19746                25     1.47047   -0.19746
26     0.41028    0.96392                26    0.399436    0.96392
27    -1.41021   0.333803                27    -1.41397   0.333803
28    -0.60975  -0.916705                28   -0.599437  -0.916705
29     1.32397  -0.463467                29     1.32918  -0.463467
30     0.79702   0.851149                30    0.787444   0.851149

E(h) of SymplecticEulerSolver:
     h       log(h)     E(h)
1.00E-04 -9.21E+00 3.04E-09
1.03E-04 -9.18E+00 3.25E-09
1.07E-04 -9.14E+00 3.48E-09
1.11E-04 -9.11E+00 3.74E-09
1.15E-04 -9.07E+00 4.03E-09
1.20E-04 -9.03E+00 4.36E-09
1.25E-04 -8.99E+00 4.72E-09
1.30E-04 -8.95E+00 5.14E-09
1.36E-04 -8.90E+00 5.61E-09
```

```
1.42E-04 -8.86E+00 6.15E-09
1.49E-04 -8.81E+00 6.77E-09
1.57E-04 -8.76E+00 7.49E-09
1.66E-04 -8.71E+00 8.33E-09
1.75E-04 -8.65E+00 9.32E-09
1.86E-04 -8.59E+00 1.05E-08
1.98E-04 -8.53E+00 1.19E-08
2.12E-04 -8.46E+00 1.36E-08
2.28E-04 -8.39E+00 1.58E-08
2.46E-04 -8.31E+00 1.84E-08
2.68E-04 -8.22E+00 2.18E-08
2.94E-04 -8.13E+00 2.63E-08
3.26E-04 -8.03E+00 3.22E-08
3.65E-04 -7.92E+00 4.05E-08
4.15E-04 -7.79E+00 5.23E-08
4.81E-04 -7.64E+00 7.02E-08
5.71E-04 -7.47E+00 9.92E-08
7.04E-04 -7.26E+00 1.51E-07

E(h) of StoermerVerletSolver:
h            log(h)     E(h)
1.00E-04 -9.21E+00 3.04E-09
1.03E-04 -9.18E+00 3.25E-09
1.07E-04 -9.14E+00 3.48E-09
1.11E-04 -9.11E+00 3.74E-09
1.15E-04 -9.07E+00 4.03E-09
1.20E-04 -9.03E+00 4.36E-09
1.25E-04 -8.99E+00 4.72E-09
1.30E-04 -8.95E+00 5.14E-09
1.36E-04 -8.90E+00 5.61E-09
1.42E-04 -8.86E+00 6.15E-09
1.49E-04 -8.81E+00 6.77E-09
1.57E-04 -8.76E+00 7.49E-09
1.66E-04 -8.71E+00 8.33E-09
1.75E-04 -8.65E+00 9.32E-09
1.86E-04 -8.59E+00 1.05E-08
1.98E-04 -8.53E+00 1.19E-08
2.12E-04 -8.46E+00 1.36E-08
2.28E-04 -8.39E+00 1.58E-08
2.46E-04 -8.31E+00 1.84E-08
2.68E-04 -8.22E+00 2.18E-08
2.94E-04 -8.13E+00 2.63E-08
3.26E-04 -8.03E+00 3.22E-08
3.65E-04 -7.92E+00 4.05E-08
4.15E-04 -7.79E+00 5.23E-08
4.81E-04 -7.64E+00 7.02E-08
5.71E-04 -7.47E+00 9.92E-08
7.04E-04 -7.26E+00 1.51E-07

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

The pictures of $\hat{x}(t)$of two methods are below





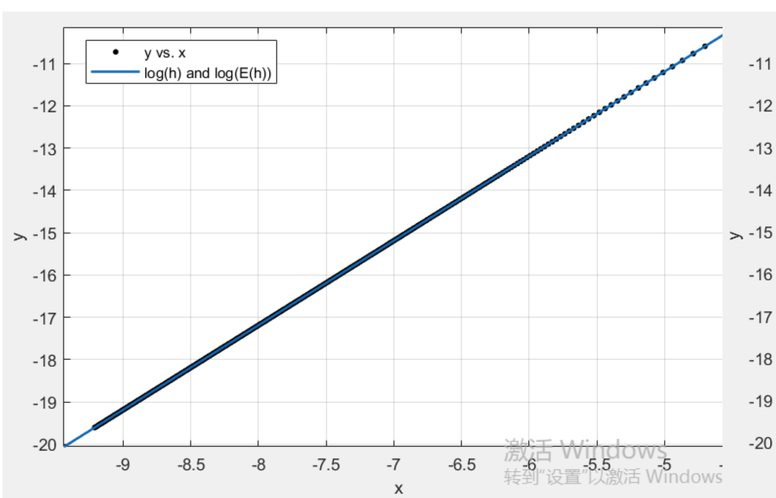
RSS is a good method to compare three ODE method. Let e1, e2 and e3 represent RSS of ForwardeulerSolver, SymplecticEulerSolver and StoermerVerletSolver respectively

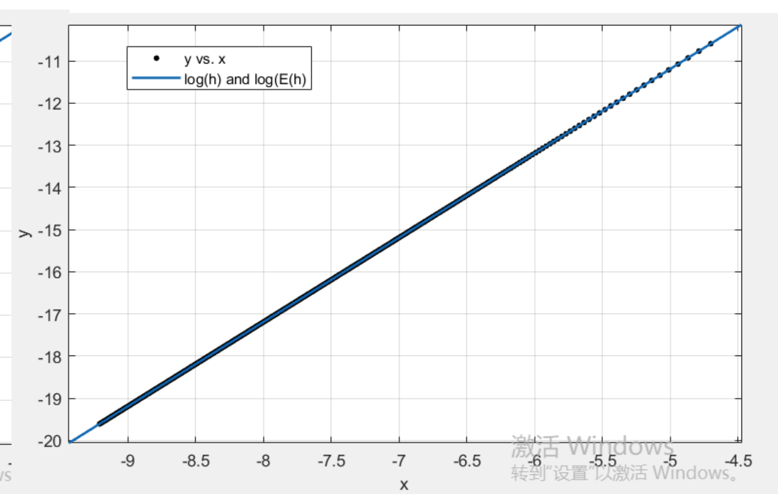
Let $\hat{\boldsymbol{x}}(\boldsymbol{t})$ be the estimate solution while $\boldsymbol{x}(\boldsymbol{t})$ is anylitic solution.

$$e = |\hat{\boldsymbol{x}}(\boldsymbol{t}) - \boldsymbol{x}(\boldsymbol{t})| \tag{3}$$

then $e1 = e2 = e3 = 0.009643847438485$

Estimating relationship between log(h) and log(E(h)) by using OLS method, and the two pictures are below, the left one is SymplecticEulerSolver and the right one is StoermerVerletSolver





We find out that the relationship of two methods are same.

$$log(E(h)) = 2log(h) - 1.191 \Rightarrow E(h) = exp(2log(h) - 1.191) = exp(-1.191)h^2 \qquad (4)$$

Consequently, E(h)=O(h) and order is 2.

Question 3

* Firstly, check whether collision happens or not and get result that it will not happen when v=(0,1018.27,0).

  I select 50 datas of location of moon, which is computed by StoermerVerletSolver method and time gap=200h=47438.6s; StoermerVerletSolver.dat saves all 10001×3 datas.

```
%%%%%%%%%%%%%%%%%% Output of Driver.cpp %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
  t              x                y                   z
0              3.844e+008             0               0
47438.6        3.81369e+008   4.81781e+007            0
94877.2        3.72323e+008   9.55964e+007            0
142316         3.57406e+008   1.41507e+008            0
189754         3.36852e+008   1.85186e+008            0
237193         3.10986e+008   2.25945e+008            0
284632         2.80216e+008    2.6314e+008            0
332070         2.45026e+008   2.96185e+008            0
379509         2.05972e+008    3.2456e+008            0
426947         1.6367e+008   3.47816e+008             0
474386         1.18786e+008   3.65586e+008            0
521825         7.20294e+007   3.77591e+008            0
569263         2.41367e+007   3.83642e+008            0
616702        -2.41367e+007   3.83642e+008            0
664141        -7.20294e+007   3.77591e+008            0
711579        -1.18786e+008   3.65586e+008            0
759018        -1.6367e+008    3.47816e+008            0
806456        -2.05972e+008    3.2456e+008            0
853895        -2.45026e+008   2.96185e+008            0
901334        -2.80216e+008    2.6314e+008            0
948772        -3.10986e+008   2.25945e+008            0
996211        -3.36852e+008   1.85186e+008            0
1.04365e+006  -3.57406e+008   1.41507e+008            0
1.09109e+006  -3.72323e+008   9.55965e+007            0
1.13853e+006  -3.81369e+008   4.81782e+007            0
1.18597e+006    -3.844e+008        158.918            0
1.2334e+006   -3.81369e+008  -4.81779e+007            0
1.28084e+006  -3.72323e+008  -9.55962e+007            0
1.32828e+006  -3.57406e+008  -1.41507e+008            0
1.37572e+006  -3.36852e+008  -1.85186e+008            0
1.42316e+006  -3.10986e+008  -2.25945e+008            0
1.4706e+006   -2.80216e+008   -2.6314e+008            0
1.51804e+006  -2.45026e+008  -2.96185e+008            0
1.56547e+006  -2.05972e+008   -3.2456e+008            0
1.61291e+006   -1.6367e+008  -3.47815e+008            0
1.66035e+006  -1.18786e+008  -3.65586e+008            0
1.70779e+006  -7.20297e+007  -3.77591e+008            0
1.75523e+006   -2.4137e+007  -3.83641e+008            0
1.80267e+006   2.41364e+007  -3.83642e+008            0
1.85011e+006   7.20291e+007  -3.77591e+008            0
1.89754e+006   1.18786e+008  -3.65586e+008            0
1.94498e+006   1.63669e+008  -3.47816e+008            0
```

1.99242e+006	2.05972e+008	-3.2456e+008	0
2.03986e+006	2.45026e+008	-2.96186e+008	0
2.0873e+006	2.80215e+008	-2.6314e+008	0
2.13474e+006	3.10986e+008	-2.25945e+008	0
2.18218e+006	3.36852e+008	-1.85186e+008	0
2.22961e+006	3.57406e+008	-1.41507e+008	0
2.27705e+006	3.72323e+008	-9.55967e+007	0
2.32449e+006	3.81369e+008	-4.81784e+007	0
2.37193e+006	3.844e+008	-317.835	0

%%%

In theory, moon should revolve around earth with fixed v when initial velocity of moon is v=(0,1018.27,0), so that the distance between moon and earth should be equal to $3.844 * 10^8$ all the time. Then it is easy to compare three methods.

Let $\hat{\boldsymbol{x}}(\boldsymbol{t_k})$ be the estimated location of moon in time t_k, then let $e_k = (|\hat{\boldsymbol{x}}(\boldsymbol{t_k})| - 3.844 * 10^8)^2$

Then compute

$$e = \frac{\sum_0^{10001} e_k}{10001} \tag{5}$$

Let e1, e2 and e3 be e of ForwardeulerSolver, SymplecticEulerSolver and StoermerVerletSolver respectively, we have

$$e1 = 3.4673 \times 10^{12}, e2 = 7.2911 \times 10^9, e3 = 2.9557 \times 10^3$$

Consequently, best method is StoermerVerletSolver.

Figure 3 shows trajectories of moon based on StoermerVerletSolver.dat which saves 10001 location vectors.

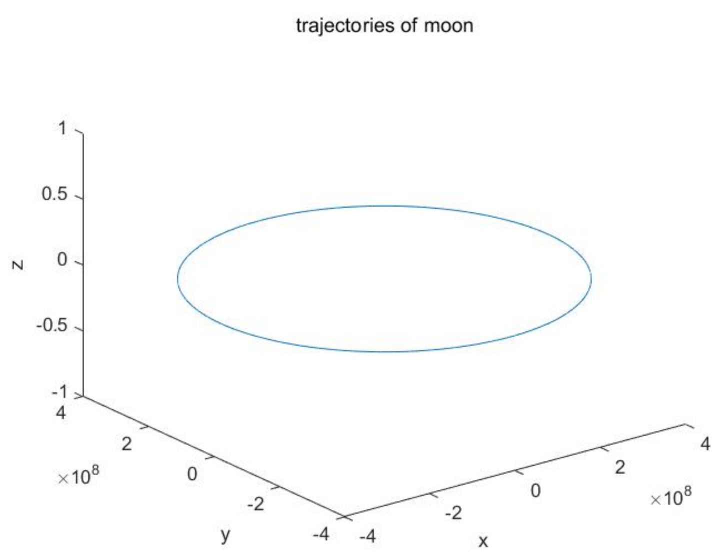


Figure 3: trajectories of moon

When initial velocity of moon v=(0,0,0), there is no doubt that the collision will happen.

I choose to define Detectcollision method in StoermerVerletSolver(Also ForwardeulerSolver and SymplecticEulerSolver) class because it also requires nearest minute it would take the moon to collide with the earth. When h is bigger than 1 minute, we need information of two time steps and more accurate ODE method(do the loop again with smaller h) between these steps.

My method to get nearest minute it would take the moon to collide with the earth is below:

1) Find out step k($1 < k < stepsize$) which is the first time that the distance of moon and earth is smaller than the sum of their own radius.

2)Back to the data of step k-1, reset h=60s(1 minute), and continue the loop.

3)Find out step k-1+j, which is the second time(include step (1)) that the distance of moon and earth is smaller than the sum of their own radius, record time t_{k-1+j}.

4)Nearest minute is $t_{k-1+j}/60$.

Then we get the nearest minute is 6979.4 min and the location of moon is $(8.009 \times 10^6, 0, 0)$when collision happens

If the two bodies are assumed that they both have zero radius, you will set the distance of two bodies is quite small when Moon collide with Earth, which causes a problem because h is not small enough that the distance will smaller than condition you set. For example, in last time step, the distance of bodies was 9e6 and now it is 10e6 because moon was accelerated between these steps and keep moving after collision because of inertia. This process is hard to find out when h is big, then the loop will be infinite.

When initial velocity of moon $v = (0, k \times 1018.27, 0), k \in [0, 2]$, the trajectories of moon is below(k=0,1,1.5,2)

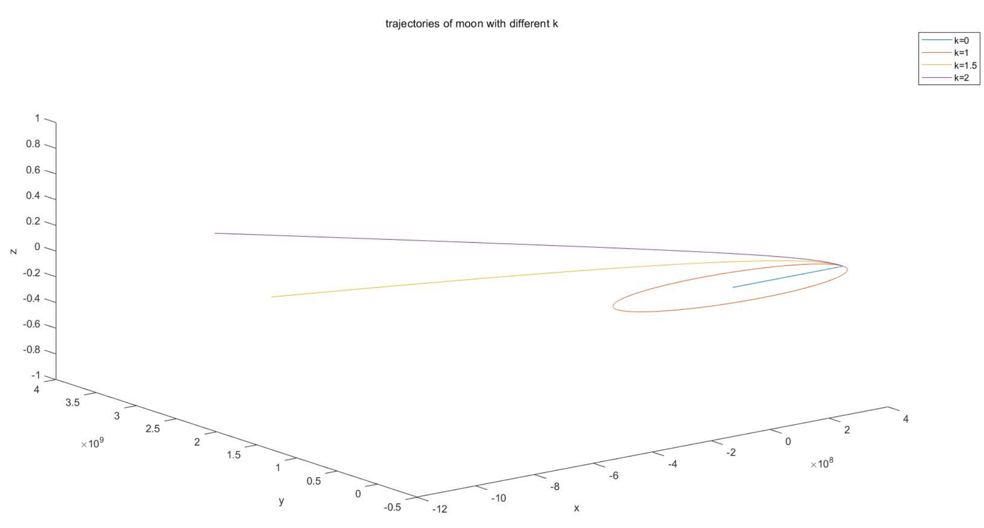


Figure 4: trajectories of moon with diferent k

Question 4

* In this section, every 3 columns represent a body in all files. For example, in earthandmoon.dat, first 3 columns is the location vector of earth, the second 3 columns is the location vector of moon.

  In Earth and Moon system, I select 21 data. $Timegap = 500h = 1.185965 \times 10^5$s

  In five bodies system, I select 21 data of Sun and Mercury. Period T=3153600s=365 days. Time gap=500h=500T/10000.

  Five bodies system includes five members who are Sun, Mercury, Venus, Earth and Mars. The chart below shows their initial state.

	mass	Own radius	Initialstate	Initialvelocity
Sun	1.9891e+030	6.966e+008	(0,0,0)	(0,0,0)
Mercury	3.3011e+023	2.44e+006	(5.791e+010,0,0)	(0,47879,0)
Venus	4.8675e+024	1.2103e+007	(0,1.08e+011,0)	(35059.8,0,0)
Earth	5.972e+024	6.378e+006	(-1.5e+011,0,0)	(0,-29749.2,0)
Mars	6.4171e+023	3.3895e+006	(0,-2.25e+011,0)	(24290.1,0,0)

```
%%%%%%%%%%%%%%%%%%% Output of Driver.cpp %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Earth                          Moon
6.230E+04 2.863E+07 0.000E+00 3.801E+08 8.620E+07 0.000E+00
5.086E+05 2.871E+07 0.000E+00 3.438E+08 2.006E+08 0.000E+00
1.370E+06 2.899E+07 0.000E+00 2.737E+08 2.983E+08 0.000E+00
2.563E+06 2.960E+07 0.000E+00 1.767E+08 3.694E+08 0.000E+00
3.969E+06 3.063E+07 0.000E+00 6.233E+07 4.066E+08 0.000E+00
5.448E+06 3.212E+07 0.000E+00 -5.791E+07 4.058E+08 0.000E+00
6.847E+06 3.409E+07 0.000E+00 -1.717E+08 3.668E+08 0.000E+00
8.019E+06 3.647E+07 0.000E+00 -2.670E+08 2.935E+08 0.000E+00
8.840E+06 3.919E+07 0.000E+00 -3.336E+08 1.934E+08 0.000E+00
9.218E+06 4.210E+07 0.000E+00 -3.643E+08 7.721E+07 0.000E+00
9.114E+06 4.506E+07 0.000E+00 -3.558E+08 -4.270E+07 0.000E+00
8.539E+06 4.790E+07 0.000E+00 -3.089E+08 -1.534E+08 0.000E+00
7.555E+06 5.049E+07 0.000E+00 -2.288E+08 -2.430E+08 0.000E+00
6.269E+06 5.270E+07 0.000E+00 -1.241E+08 -3.022E+08 0.000E+00
4.818E+06 5.447E+07 0.000E+00 -6.075E+06 -3.250E+08 0.000E+00
3.353E+06 5.576E+07 0.000E+00 1.131E+08 -3.092E+08 0.000E+00
2.023E+06 5.660E+07 0.000E+00 2.213E+08 -2.567E+08 0.000E+00
9.608E+05 5.706E+07 0.000E+00 3.077E+08 -1.730E+08 0.000E+00
2.704E+05 5.723E+07 0.000E+00 3.639E+08 -6.668E+07 0.000E+00
1.846E+04 5.726E+07 0.000E+00 3.844E+08 5.183E+07 0.000E+00
2.292E+05 5.728E+07 0.000E+00 3.672E+08 1.709E+08 0.000E+00

Sun                            Mercury
3.166E+06 -2.282E+06 0.000E+00 3.413E+10 4.678E+10 0.000E+00
3.384E+06 -2.384E+06 0.000E+00 -3.611E+10 4.526E+10 0.000E+00
3.484E+06 -2.502E+06 0.000E+00 -5.319E+10 -2.289E+10 0.000E+00
3.469E+06 -2.630E+06 0.000E+00 8.038E+09 -5.735E+10 0.000E+00
3.378E+06 -2.758E+06 0.000E+00 5.744E+10 -7.392E+09 0.000E+00
3.267E+06 -2.862E+06 0.000E+00 2.229E+10 5.345E+10 0.000E+00
3.173E+06 -2.930E+06 0.000E+00 -4.567E+10 3.560E+10 0.000E+00
```

3.123E+06 -2.981E+06 0.000E+00 -4.640E+10 -3.465E+10 0.000E+00
3.148E+06 -3.056E+06 0.000E+00 2.118E+10 -5.390E+10 0.000E+00
3.278E+06 -3.183E+06 0.000E+00 5.758E+10 6.193E+09 0.000E+00
3.512E+06 -3.369E+06 0.000E+00 9.225E+09 5.717E+10 0.000E+00
3.813E+06 -3.608E+06 0.000E+00 -5.271E+10 2.398E+10 0.000E+00
4.131E+06 -3.897E+06 0.000E+00 -3.704E+10 -4.451E+10 0.000E+00
4.432E+06 -4.227E+06 0.000E+00 3.315E+10 -4.749E+10 0.000E+00
4.706E+06 -4.561E+06 0.000E+00 5.455E+10 1.944E+10 0.000E+00
4.939E+06 -4.844E+06 0.000E+00 -4.347E+09 5.774E+10 0.000E+00
5.119E+06 -5.031E+06 0.000E+00 -5.684E+10 1.104E+10 0.000E+00
5.245E+06 -5.113E+06 0.000E+00 -2.565E+10 -5.192E+10 0.000E+00
5.348E+06 -5.106E+06 0.000E+00 4.331E+10 -3.846E+10 0.000E+00
5.465E+06 -5.026E+06 0.000E+00 4.852E+10 3.161E+10 0.000E+00

%%%

Figure 5 is trajectories of Earth and Moon. Figure 6 is trajectories of five bodies.

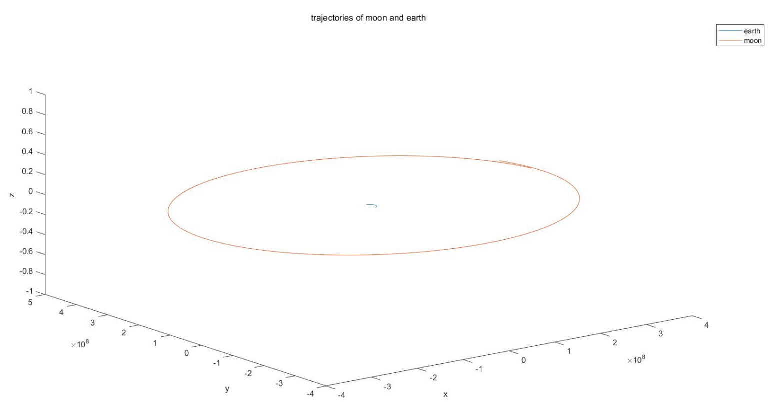


Figure 5: trajectories of Earth and Moon

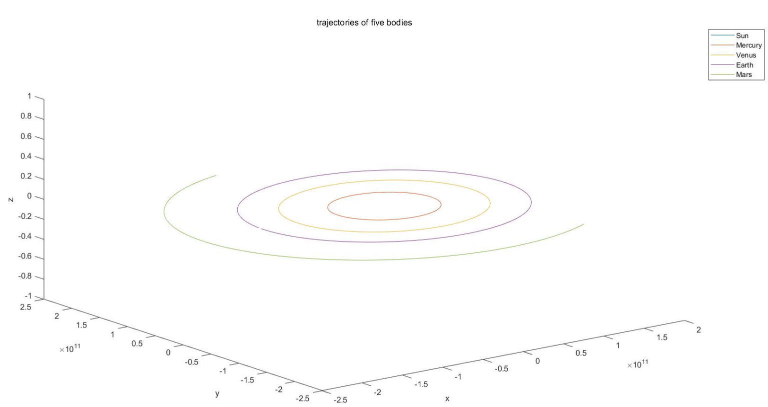


Figure 6: trajectories of five bodies

In N(N=2,4,8,16,32,64) bodies system, I choose N times Earth and Moon system.

For example, When N=2k(k=1,2,4,8,16,32), there are k systems that Earth k is on(0,0+1.5(k-1)e012,0) and Moon k is on (3.844e008,0+1.5(k-1)e012,0) and every moon has initial velocity (0,1018.27,0).

Compute CPU time I get

```
%%%%%%%%%%%%%%%%%%% Output of Driver.cpp %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
N=                  CPU time
  2                  983ms
  4                  4558ms
  8                  12122ms
 16                  46470ms
 32                  181956ms
 64                  767602ms

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

The estimated relationship between N and CPU time is

$$Time = 143.8N^{2.064} \tag{6}$$

Figure 7 is curve fitting picture.

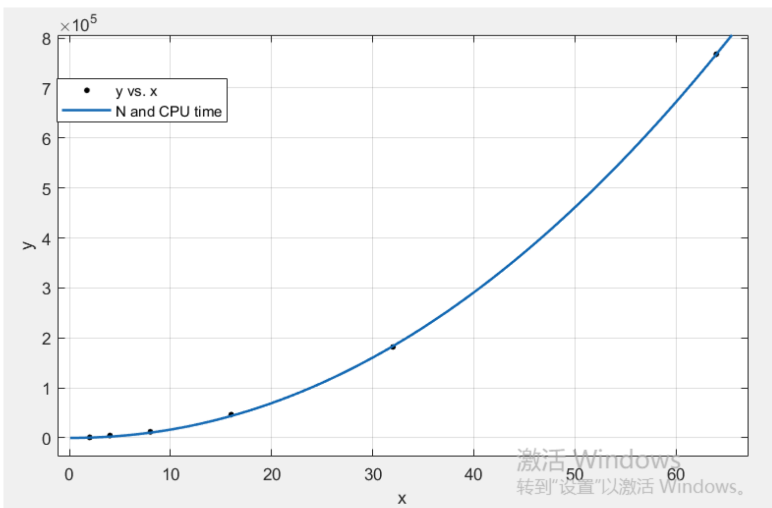


Figure 7: N and CPU time

When $N = 2^{37}$, CPU time will be $1.403087634231875 \times 10^{79}$ms

1 The model of interbank market

2 Graph neural network and prediction

2.1 SIR model

Subsections may be used. Use a clear structure in your report.
We denote the set of real numbers by , the set of integers by and the set of complex numbers by $\mathbb{C}$.
Our analysis is based on the equation $e^{\pi i} = -1$ and the relation

$$dP_{ij}(t) = f\left(w^{[ij]}, P_{ij}\right) dt + \sigma\left(P_{ij}\right) dW_{ij}(t) \tag{7}$$

$$d\beta_i(t) = b\left(a - \beta_i(t)\right) dt + \gamma\sqrt{\beta_i} dW_i(t) \tag{8}$$

which we verify in the appendix Useful consequences are