

Chapter 3 - Homework and Classwork

3. Rewrite the BNF of Example 3.4 to give + precedence over * and force + to be right associative.

$\langle \text{assign} \rangle \Rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle$

$\langle \text{id} \rangle \Rightarrow A \mid B \mid C$

$\langle \text{expr} \rangle \Rightarrow \langle \text{expr} \rangle * \langle \text{term} \rangle$
 $\mid \langle \text{term} \rangle$

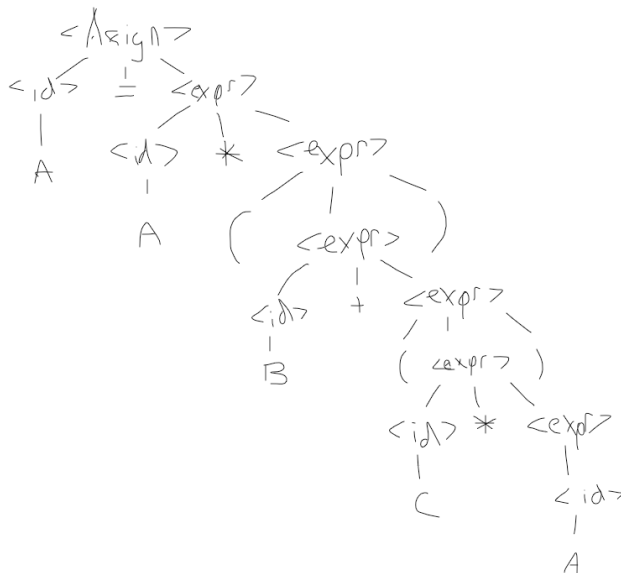
$\langle \text{term} \rangle \Rightarrow \langle \text{factor} \rangle + \langle \text{term} \rangle$
 $\mid \langle \text{factor} \rangle$

$\langle \text{factor} \rangle \Rightarrow (\langle \text{expr} \rangle)$
 $\mid \langle \text{id} \rangle$

6. Using the grammar in Example 3.2, show a parse tree and a leftmost derivation for each of the following statements:

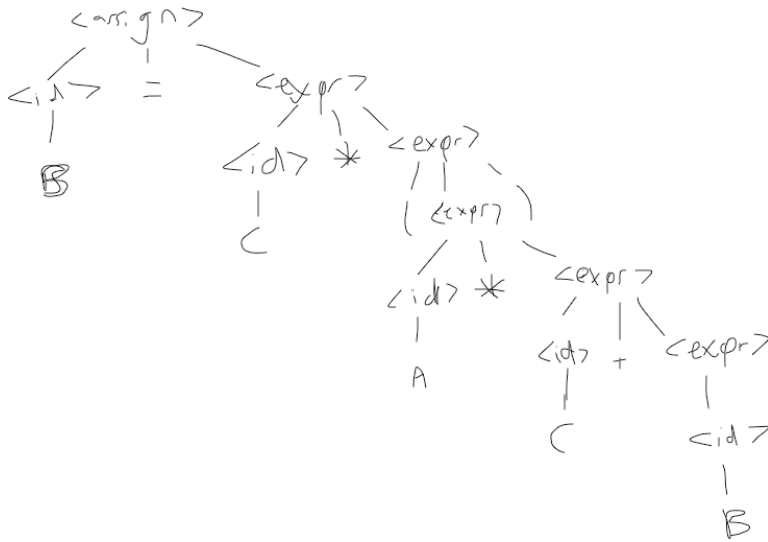
A. $A = A * (B + (C * A))$

$\langle \text{assign} \rangle \Rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle$
 $\Rightarrow A = \langle \text{expr} \rangle$
 $\Rightarrow A = \langle \text{id} \rangle * \langle \text{expr} \rangle$
 $\Rightarrow A = A * \langle \text{expr} \rangle$
 $\Rightarrow A = A * (\langle \text{expr} \rangle)$
 $\Rightarrow A = A * (\langle \text{id} \rangle + \langle \text{expr} \rangle)$
 $\Rightarrow A = A * (B + \langle \text{expr} \rangle)$
 $\Rightarrow A = A * (B + (\langle \text{id} \rangle * \langle \text{expr} \rangle))$
 $\Rightarrow A = A * (B + (C * \langle \text{expr} \rangle))$
 $\Rightarrow A = A * (B + (C * \langle \text{id} \rangle))$
 $\Rightarrow A = A * (B + (C * A))$



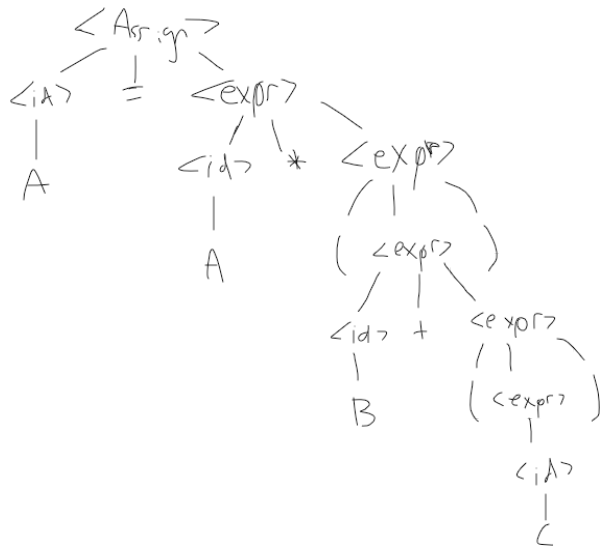
B. $B = C * (A * C + B)$

$\langle \text{assign} \rangle \Rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle$
 $\Rightarrow B = \langle \text{expr} \rangle$
 $\Rightarrow B = \langle \text{id} \rangle * \langle \text{expr} \rangle$
 $\Rightarrow B = C * \langle \text{expr} \rangle$
 $\Rightarrow B = C * (\langle \text{expr} \rangle)$
 $\Rightarrow B = C * (\langle \text{id} \rangle * \langle \text{expr} \rangle)$
 $\Rightarrow B = C * (A * \langle \text{expr} \rangle)$
 $\Rightarrow B = C * (A * \langle \text{id} \rangle + \langle \text{expr} \rangle)$
 $\Rightarrow B = C * (A * C + \langle \text{expr} \rangle)$
 $\Rightarrow B = C * (A * C + \langle \text{id} \rangle)$
 $\Rightarrow B = C * (A * C + B)$



C. $A = A * (B + (C))$

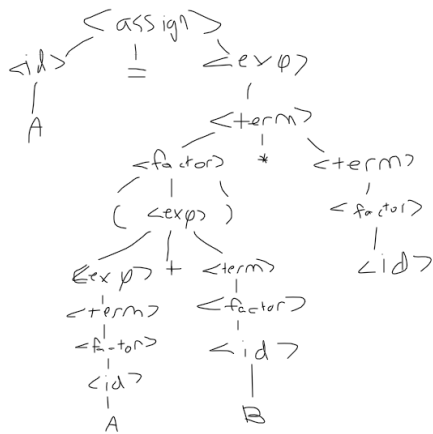
$\langle \text{assign} \rangle \Rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle$
 $\Rightarrow A = \langle \text{expr} \rangle$
 $\Rightarrow A = \langle \text{id} \rangle * \langle \text{expr} \rangle$
 $\Rightarrow A = A * \langle \text{expr} \rangle$
 $\Rightarrow A = A * (\langle \text{expr} \rangle)$
 $\Rightarrow A = A * (\langle \text{id} \rangle + \langle \text{expr} \rangle)$
 $\Rightarrow A = A * (B + \langle \text{expr} \rangle)$
 $\Rightarrow A = A * (B + (\langle \text{expr} \rangle))$
 $\Rightarrow A = A * (B + (\langle \text{id} \rangle))$
 $\Rightarrow A = A * (B + (C))$



7. Using the grammar in Example 3.4, show a parse tree and a leftmost derivation for each of the following statements:

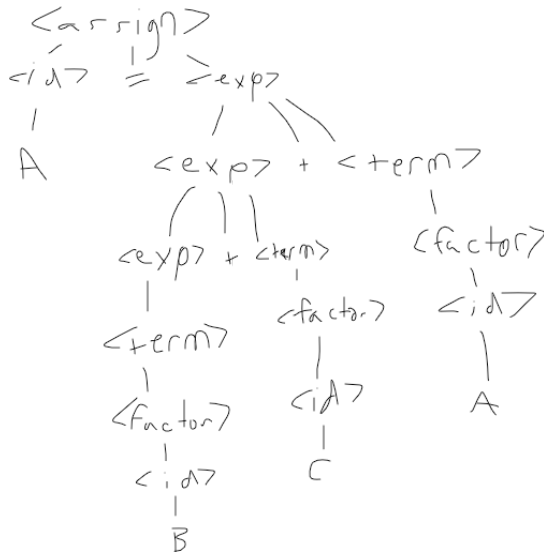
A. $A = (A + B) * C$

$\langle \text{assign} \rangle \Rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle$
 $\Rightarrow A = \langle \text{expr} \rangle$
 $\Rightarrow A = \langle \text{term} \rangle$
 $\Rightarrow A = \langle \text{factor} \rangle * \langle \text{term} \rangle$
 $\Rightarrow A = (\langle \text{expr} \rangle) * \langle \text{term} \rangle$
 $\Rightarrow A = (\langle \text{expr} \rangle + \langle \text{term} \rangle) * \langle \text{term} \rangle$
 $\Rightarrow A = (\langle \text{term} \rangle + \langle \text{term} \rangle) * \langle \text{term} \rangle$
 $\Rightarrow A = (\langle \text{factor} \rangle + \langle \text{term} \rangle) * \langle \text{term} \rangle$
 $\Rightarrow A = (\langle \text{id} \rangle + \langle \text{term} \rangle) * \langle \text{term} \rangle$
 $\Rightarrow A = (A + \langle \text{term} \rangle) * \langle \text{term} \rangle$
 $\Rightarrow A = (A + \langle \text{factor} \rangle) * \langle \text{term} \rangle$
 $\Rightarrow A = (A + \langle \text{id} \rangle) * \langle \text{term} \rangle$
 $\Rightarrow A = (A + B) * \langle \text{term} \rangle$
 $\Rightarrow A = (A + B) * \langle \text{factor} \rangle$
 $\Rightarrow A = (A + B) * \langle \text{id} \rangle$
 $\Rightarrow A = (A + B) * C$



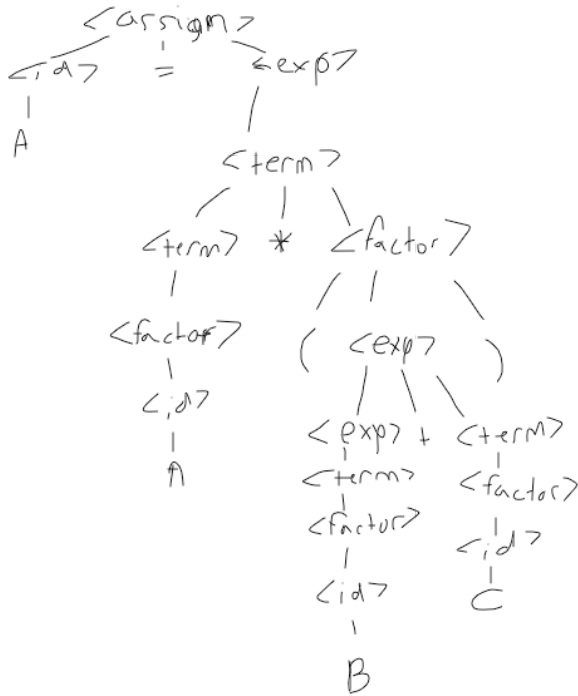
B. $A = B + C + A$

$\langle \text{assign} \rangle \Rightarrow \langle \text{id} \rangle = \langle \text{exp} \rangle$
 $\Rightarrow A = \langle \text{exp} \rangle + \langle \text{term} \rangle$
 $\Rightarrow A = \langle \text{exp} \rangle + \langle \text{term} \rangle + \langle \text{term} \rangle$
 $\Rightarrow A = \langle \text{term} \rangle + \langle \text{term} \rangle + \langle \text{term} \rangle$
 $\Rightarrow A = \langle \text{factor} \rangle + \langle \text{term} \rangle + \langle \text{term} \rangle$
 $\Rightarrow A = \langle \text{id} \rangle + \langle \text{term} \rangle + \langle \text{term} \rangle$
 $\Rightarrow A = B + \langle \text{term} \rangle + \langle \text{term} \rangle$
 $\Rightarrow A = B + \langle \text{factor} \rangle + \langle \text{term} \rangle$
 $\Rightarrow A = B + \langle \text{id} \rangle + \langle \text{term} \rangle$
 $\Rightarrow A = B + C + \langle \text{term} \rangle$
 $\Rightarrow A = B + C + \langle \text{factor} \rangle$
 $\Rightarrow A = B + C + \langle \text{id} \rangle$
 $\Rightarrow A = B + C + A$



C. $A = A * (B + C)$

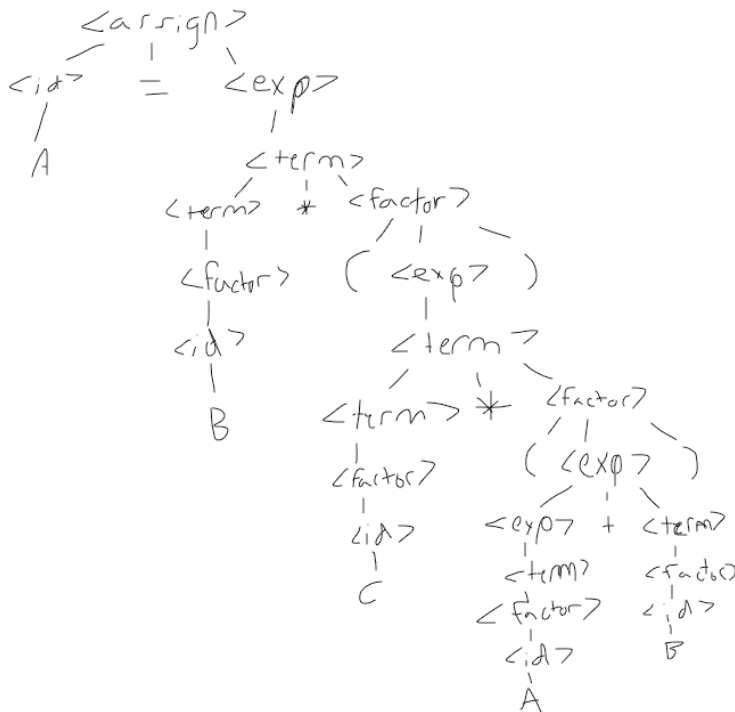
$\langle \text{assign} \rangle \Rightarrow \langle \text{id} \rangle = \langle \text{exp} \rangle$
 $\Rightarrow A = \langle \text{exp} \rangle$
 $\Rightarrow A = \langle \text{term} \rangle$
 $\Rightarrow A = \langle \text{term} \rangle * \langle \text{factor} \rangle$
 $\Rightarrow A = \langle \text{factor} \rangle * \langle \text{factor} \rangle$
 $\Rightarrow A = \langle \text{id} \rangle * \langle \text{factor} \rangle$
 $\Rightarrow A = A * \langle \text{factor} \rangle$
 $\Rightarrow A = A * (\langle \text{exp} \rangle)$
 $\Rightarrow A = A * (\langle \text{exp} \rangle + \langle \text{term} \rangle)$
 $\Rightarrow A = A * (\langle \text{term} \rangle + \langle \text{term} \rangle)$
 $\Rightarrow A = A * (\langle \text{factor} \rangle + \langle \text{term} \rangle)$
 $\Rightarrow A = A * (\langle \text{id} \rangle + \langle \text{term} \rangle)$
 $\Rightarrow A = A * (B + \langle \text{term} \rangle)$
 $\Rightarrow A = A * (B + \langle \text{factor} \rangle)$
 $\Rightarrow A = A * (B + \langle \text{id} \rangle)$
 $\Rightarrow A = A * (B + C)$



D. $A = B * (C * (A + B))$

$\langle \text{assign} \rangle \Rightarrow \langle \text{id} \rangle = \langle \text{exp} \rangle$
 $\Rightarrow A = \langle \text{exp} \rangle$
 $\Rightarrow A = \langle \text{term} \rangle$
 $\Rightarrow A = \langle \text{term} \rangle * \langle \text{factor} \rangle$
 $\Rightarrow A = \langle \text{factor} \rangle * \langle \text{factor} \rangle$
 $\Rightarrow A = \langle \text{id} \rangle * \langle \text{factor} \rangle$

$\Rightarrow A = B * \langle \text{factor} \rangle$
 $\Rightarrow A = B * (\langle \text{exp} \rangle)$
 $\Rightarrow A = B * (\langle \text{term} \rangle)$
 $\Rightarrow A = B * (\langle \text{term} \rangle * \langle \text{factor} \rangle)$
 $\Rightarrow A = B * (\langle \text{factor} \rangle * \langle \text{factor} \rangle)$
 $\Rightarrow A = B * (\langle \text{id} \rangle * \langle \text{factor} \rangle)$
 $\Rightarrow A = B * (C * \langle \text{factor} \rangle)$
 $\Rightarrow A = B * (C * (\langle \text{exp} \rangle))$
 $\Rightarrow A = B * (C * (\langle \text{exp} \rangle + \langle \text{term} \rangle))$
 $\Rightarrow A = B * (C * (\langle \text{term} \rangle + \langle \text{term} \rangle))$
 $\Rightarrow A = B * (C * (\langle \text{factor} \rangle + \langle \text{term} \rangle))$
 $\Rightarrow A = B * (C * (\langle \text{id} \rangle + \langle \text{term} \rangle))$
 $\Rightarrow A = B * (C * (A + \langle \text{term} \rangle))$
 $\Rightarrow A = B * (C * (A + \langle \text{factor} \rangle))$
 $\Rightarrow A = B * (C * (A + \langle \text{id} \rangle))$
 $\Rightarrow A = B * (C * (A + B))$



11. Consider the following grammar:

- $\langle S \rangle \rightarrow \langle A \rangle a \langle B \rangle b$
- $\langle A \rangle \rightarrow \langle A \rangle b \mid b$
- $\langle B \rangle \rightarrow a \langle B \rangle \mid a$

Which of the following sentences are in the language generated by this grammar?

A. baab

in the language

$\langle a \rangle a \langle b \rangle b$

$ba \langle b \rangle b$

$ba \langle b \rangle b$

baab

B. bbbab

not in the language. End up with $bbba \langle B \rangle b$.

C. bbaaaaa

not in the language. Has to end in b.

D. bbaab

in the language

$\langle a \rangle a \langle b \rangle b$

$\langle a \rangle ba \langle b \rangle b$

$bba \langle b \rangle b$

bbaab

12. Consider the following grammar:

- $\langle S \rangle \rightarrow a \langle S \rangle c \langle B \rangle \mid \langle A \rangle \mid b$
- $\langle A \rangle \rightarrow c \langle A \rangle \mid c$
- $\langle B \rangle \rightarrow d \mid \langle A \rangle$

Which of the following sentences are in the language generated by this grammar?

A. abcd

$a \langle s \rangle c \langle b \rangle \mid \langle a \rangle \mid b$

$abc \langle b \rangle$

abcd

Works

B. acccbd

$a \langle s \rangle c \langle b \rangle \mid \langle a \rangle \mid b$

$a \langle a \rangle c \langle b \rangle$

$a \langle a \rangle cc \langle b \rangle$

$accc \langle b \rangle$

Does not work. Can't get "b" from $\langle b \rangle$

C. acccbcc

$a \langle s \rangle c \langle b \rangle \mid \langle a \rangle \mid b$

$a \langle a \rangle c \langle b \rangle$

$a \langle a \rangle cc \langle b \rangle$

$accc \langle b \rangle$

Does not work. Won't be able to add in a "b" before cc at the end.

D. acd

$a \langle s \rangle c \langle b \rangle \mid \langle a \rangle \mid b$

$a \langle s \rangle c \langle b \rangle$

Does not work. Can't have "a" in front without having 4 letters

E. accc

$a \langle s \rangle c \langle b \rangle \mid \langle a \rangle \mid b$

$a \langle a \rangle c \langle b \rangle$

acc

acc<a>

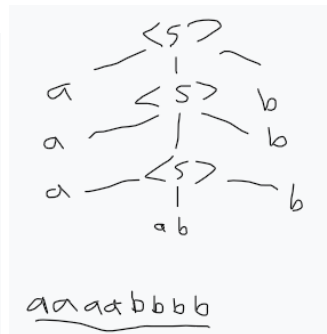
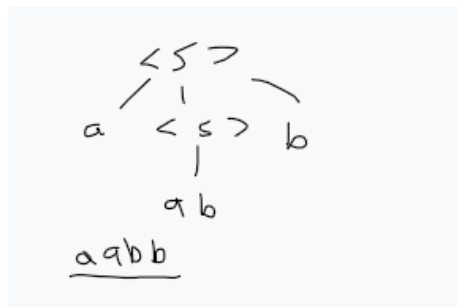
accc

Works

13. Write a grammar for the language consisting of strings that have n copies of the letter a followed by the same number of copies of the letter b, where $n > 0$. For example, the strings ab, aaaabbbb, and aaaaaaabbabbbb are in the language but a, abb, ba, and aaabb are not.

$S \Rightarrow a \langle S \rangle b \mid a b$

14. Draw parse trees for the sentences aabb and aaaabbbb, as derived from the grammar of Problem 13.



16. Convert the BNF of Example 3.3 to EBNF.

Original from 3.3:

$\langle \text{assign} \rangle \Rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle$

$\langle \text{id} \rangle \Rightarrow A \mid B \mid C$

$\langle \text{expr} \rangle \Rightarrow \langle \text{expr} \rangle + \langle \text{expr} \rangle$
 $\mid \langle \text{expr} \rangle * \langle \text{expr} \rangle$
 $\mid (\langle \text{expr} \rangle)$
 $\mid \langle \text{id} \rangle$

Converted to EBNF:

$\langle \text{assign} \rangle \Rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle$

$\langle \text{id} \rangle \Rightarrow A \mid B \mid C$

$\langle \text{expr} \rangle \Rightarrow \langle \text{expr} \rangle (+ \mid *) \langle \text{expr} \rangle$
 $\mid (\langle \text{expr} \rangle)$
 $\mid \langle \text{id} \rangle$

Class Activity - 1/25/23

23. Compute the weakest precondition for each of the following assignment statements and postconditions:

A. $a = 2 * (b - 1) - 1 \{a > 0\}$

$$\{a > 0\} = 2 * (b - 1) - 1$$

$$0 < 2 * (b - 1) - 1$$

$$2b - 3 > 0$$

$$2b > 3$$

$$b > 3/2$$

$$\mathbf{B. \quad b = (c + 10) / 3 \quad \{b > 6\}}$$

$$(c + 10) / 3 > 6$$

$$c + 10 > 18$$

$$c > 8$$

$$\mathbf{C. \quad a = a + 2 * b - 1 \quad \{a > 1\}}$$

$$a + 2 * b - 1 > 1$$

$$2 * b > 2 - a$$

$$b > 1 - a / 2$$

$$\mathbf{D. \quad x = 2 * y + x - 1 \quad \{x > 11\}}$$

$$2 * y + x - 1 > 11$$

$$2 * y + x > 12$$