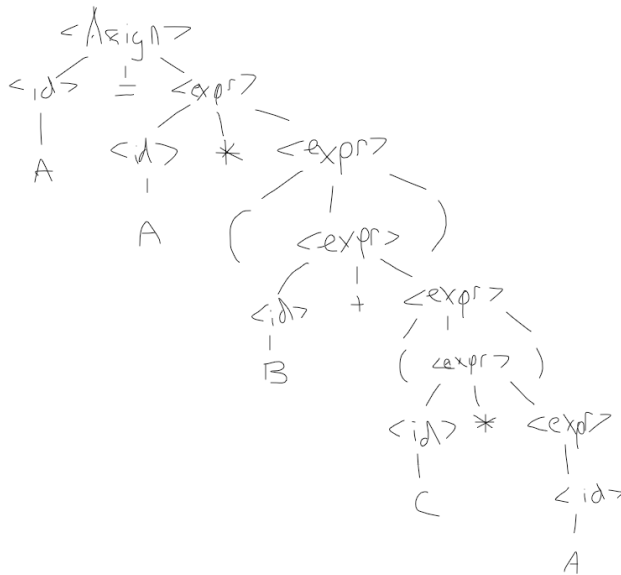


6. Using the grammar in Example 3.2, show a parse tree and a leftmost derivation for each of the following statements:

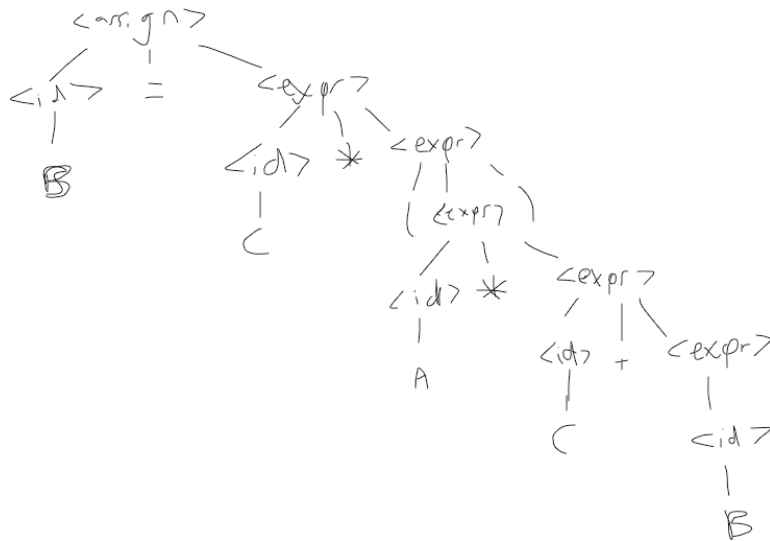
A. $A = A * (B + (C * A))$

$$\begin{aligned}
 \langle \text{assign} \rangle &\Rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle \\
 &\Rightarrow A = \langle \text{expr} \rangle \\
 &\Rightarrow A = \langle \text{id} \rangle * \langle \text{expr} \rangle \\
 &\Rightarrow A = A * \langle \text{expr} \rangle \\
 &\Rightarrow A = A * (\langle \text{expr} \rangle) \\
 &\Rightarrow A = A * (\langle \text{id} \rangle + \langle \text{expr} \rangle) \\
 &\Rightarrow A = A * (B + \langle \text{expr} \rangle) \\
 &\Rightarrow A = A * (B + (\langle \text{id} \rangle * \langle \text{expr} \rangle)) \\
 &\Rightarrow A = A * (B + (C * \langle \text{expr} \rangle)) \\
 &\Rightarrow A = A * (B + (C * \langle \text{id} \rangle)) \\
 &\Rightarrow A = A * (B + (C * A))
 \end{aligned}$$


B. $B = C * (A * C + B)$

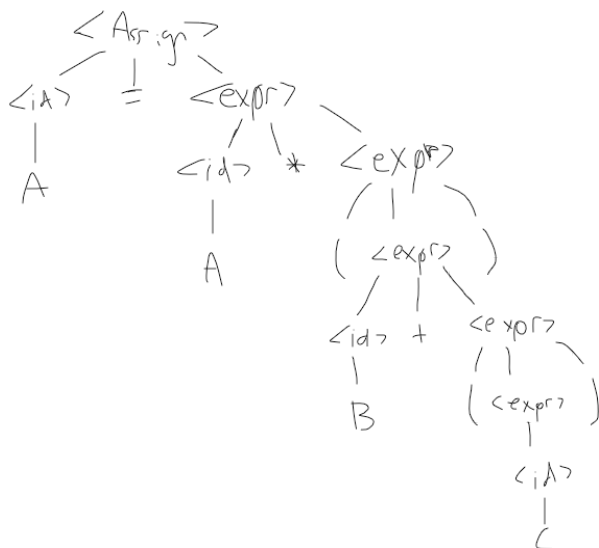
$$\begin{aligned}
 \langle \text{assign} \rangle &\Rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle \\
 &\Rightarrow B = \langle \text{expr} \rangle \\
 &\Rightarrow B = \langle \text{id} \rangle * \langle \text{expr} \rangle \\
 &\Rightarrow B = C * \langle \text{expr} \rangle \\
 &\Rightarrow B = C * (\langle \text{expr} \rangle) \\
 &\Rightarrow B = C * (\langle \text{id} \rangle * \langle \text{expr} \rangle) \\
 &\Rightarrow B = C * (A * \langle \text{expr} \rangle) \\
 &\Rightarrow B = C * (A * \langle \text{id} \rangle + \langle \text{expr} \rangle)
 \end{aligned}$$

$\Rightarrow B = C * (A * C + \langle \text{expr} \rangle)$
 $\Rightarrow B = C * (A * C + \langle \text{id} \rangle)$
 $\Rightarrow B = C * (A * C + B)$



C. $A = A * (B + (C))$

$\langle \text{assign} \rangle \Rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle$
 $\Rightarrow A = \langle \text{expr} \rangle$
 $\Rightarrow A = \langle \text{id} \rangle * \langle \text{expr} \rangle$
 $\Rightarrow A = A * \langle \text{expr} \rangle$
 $\Rightarrow A = A * (\langle \text{expr} \rangle)$
 $\Rightarrow A = A * (\langle \text{id} \rangle + \langle \text{expr} \rangle)$
 $\Rightarrow A = A * (B + \langle \text{expr} \rangle)$
 $\Rightarrow A = A * (B + (\langle \text{expr} \rangle))$
 $\Rightarrow A = A * (B + (\langle \text{id} \rangle))$
 $\Rightarrow A = A * (B + (C))$



3. Rewrite the BNF of Example 3.4 to give + precedence over * and force + to be right associative.

```

<assign> => <id> = <expr>
<id> => A | B | C
<expr> => <expr> * <term>
          | <term>
<term> => <factor> + <term>
          | <factor>
<factor> => ( <expr> )
          | <id>

```

Class Activity/Homework - 1/23/23

11. Consider the following grammar:

- $\langle S \rangle \rightarrow \langle A \rangle a \langle B \rangle b$
- $\langle A \rangle \rightarrow \langle A \rangle b \mid b$
- $\langle B \rangle \rightarrow a \langle B \rangle \mid a$

Which of the following sentences are in the language generated by this grammar?

- | | |
|---|--|
| <p>A. baab</p> <p>$\langle a \rangle a \langle b \rangle b$</p> <p>$ba \langle b \rangle b$</p> <p>$ba \langle b \rangle b$</p> <p>baab</p> | <p>in the language</p> |
| <p>B. bbbab</p> | <p>not in the language. End up with $bbba \langle B \rangle b$.</p> |
| <p>C. bbaaaaa</p> | <p>not in the language. Has to end in b.</p> |
| <p>D. bbaab</p> | <p>in the language</p> |

$\langle a \rangle a \langle b \rangle b$
 $\langle a \rangle ba \langle b \rangle b$
 $bba \langle b \rangle b$
 bbaab

12. Consider the following grammar:

- $\langle S \rangle \rightarrow a \langle S \rangle c \langle B \rangle \mid \langle A \rangle \mid b$
- $\langle A \rangle \rightarrow c \langle A \rangle \mid c$
- $\langle B \rangle \rightarrow d \mid \langle A \rangle$

Which of the following sentences are in the language generated by this grammar?

A. abcd

$a\langle s\rangle c\langle b\rangle | \langle a\rangle | b$

$abc\langle b\rangle$

$abcd$

Works

B. acccbd

$a\langle s\rangle c\langle b\rangle | \langle a\rangle | b$

$a\langle a\rangle c\langle b\rangle$

$a\langle a\rangle cc\langle b\rangle$

$acc\langle b\rangle$

Does not work. Can't get "b" from $\langle b\rangle$

C. acccbcc

$a\langle s\rangle c\langle b\rangle | \langle a\rangle | b$

$a\langle a\rangle c\langle b\rangle$

$a\langle a\rangle cc\langle b\rangle$

$acc\langle b\rangle$

Does not work. Won't be able to add in a "b" before cc at the end.

D. acd

$a\langle s\rangle c\langle b\rangle | \langle a\rangle | b$

$a\langle s\rangle c\langle b\rangle$

Does not work. Can't have "a" in front without having 4 letters

E. accc

$a\langle s\rangle c\langle b\rangle | \langle a\rangle | b$

$a\langle a\rangle c\langle b\rangle$

$acc\langle b\rangle$

$acc\langle a\rangle$

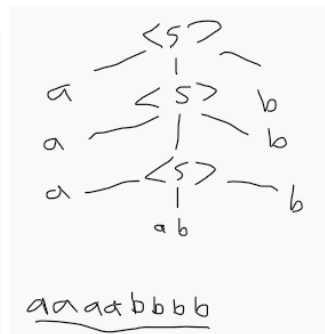
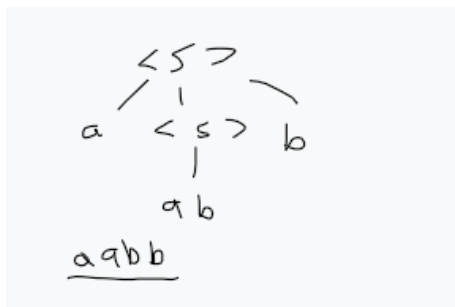
$accc$

Works

13. Write a grammar for the language consisting of strings that have n copies of the letter a followed by the same number of copies of the letter b, where $n > 0$. For example, the strings ab, aaaabbbb, and aaaaaaabbabbbb are in the language but a, abb, ba, and aaabb are not.

$S \Rightarrow a\langle S\rangle b \mid ab$

14. Draw parse trees for the sentences aabb and aaaabbbb, as derived from the grammar of Problem 13.



16. Convert the BNF of Example 3.3 to EBNF.

Original from 3.3:

$\langle \text{assign} \rangle \Rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle$
 $\langle \text{id} \rangle \Rightarrow A \mid B \mid C$
 $\langle \text{expr} \rangle \Rightarrow \langle \text{expr} \rangle + \langle \text{expr} \rangle$
 $\quad \mid \langle \text{expr} \rangle * \langle \text{expr} \rangle$
 $\quad \mid (\langle \text{expr} \rangle)$
 $\quad \mid \langle \text{id} \rangle$

Converted to EBNF:

$\langle \text{assign} \rangle \Rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle$

$\langle \text{id} \rangle \Rightarrow A \mid B \mid C$

$\langle \text{expr} \rangle \Rightarrow \langle \text{expr} \rangle (+ \mid *) \langle \text{expr} \rangle$

$\mid (\langle \text{expr} \rangle)$

$\mid \langle \text{id} \rangle$

Class Activity - 1/25/23

23. Compute the weakest precondition for each of the following assignment statements and postconditions:

A. $a = 2 * (b - 1) - 1 \{a > 0\}$

$\{a > 0\} = 2 * (b - 1) - 1$

$0 < 2 * (b - 1) - 1$

$2b - 3 > 0$

$2b > 3$

$b > 3/2$

B. $b = (c + 10) / 3 \{b > 6\}$

$(c + 10) / 3 > 6$

$c + 10 > 18$

$c > 8$

C. $a = a + 2 * b - 1 \{a > 1\}$

$a + 2 * b - 1 > 1$

$2 * b > 2 - a$

$b > 1 - a / 2$

D. $x = 2 * y + x - 1 \{x > 11\}$

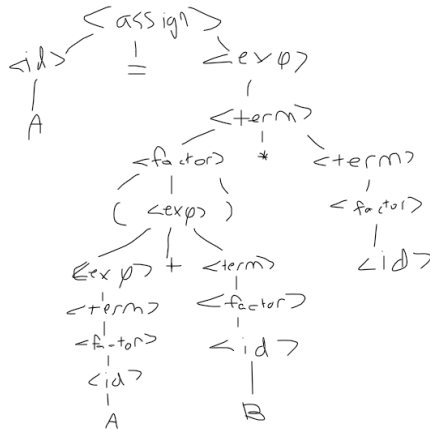
$2 * y + x - 1 > 11$

$2 * y + x > 12$

7. Using the grammar in Example 3.4, show a parse tree and a leftmost derivation for each of the following statements:

A. $A = (A + B) * C$

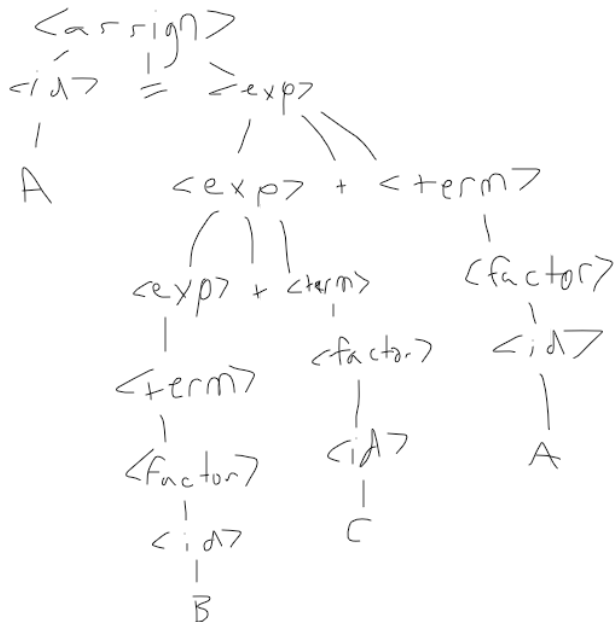
$\langle \text{assign} \rangle \Rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle$
 $\Rightarrow A = \langle \text{expr} \rangle$
 $\Rightarrow A = \langle \text{term} \rangle$
 $\Rightarrow A = \langle \text{factor} \rangle * \langle \text{term} \rangle$
 $\Rightarrow A = (\langle \text{expr} \rangle) * \langle \text{term} \rangle$
 $\Rightarrow A = (\langle \text{expr} \rangle + \langle \text{term} \rangle) * \langle \text{term} \rangle$
 $\Rightarrow A = (\langle \text{term} \rangle + \langle \text{term} \rangle) * \langle \text{term} \rangle$
 $\Rightarrow A = (\langle \text{factor} \rangle + \langle \text{term} \rangle) * \langle \text{term} \rangle$
 $\Rightarrow A = (\langle \text{id} \rangle + \langle \text{term} \rangle) * \langle \text{term} \rangle$
 $\Rightarrow A = (A + \langle \text{term} \rangle) * \langle \text{term} \rangle$
 $\Rightarrow A = (A + \langle \text{factor} \rangle) * \langle \text{term} \rangle$
 $\Rightarrow A = (A + \langle \text{id} \rangle) * \langle \text{term} \rangle$
 $\Rightarrow A = (A + B) * \langle \text{term} \rangle$
 $\Rightarrow A = (A + B) * \langle \text{factor} \rangle$
 $\Rightarrow A = (A + B) * \langle \text{id} \rangle$
 $\Rightarrow A = (A + B) * C$



B. $A = B + C + A$

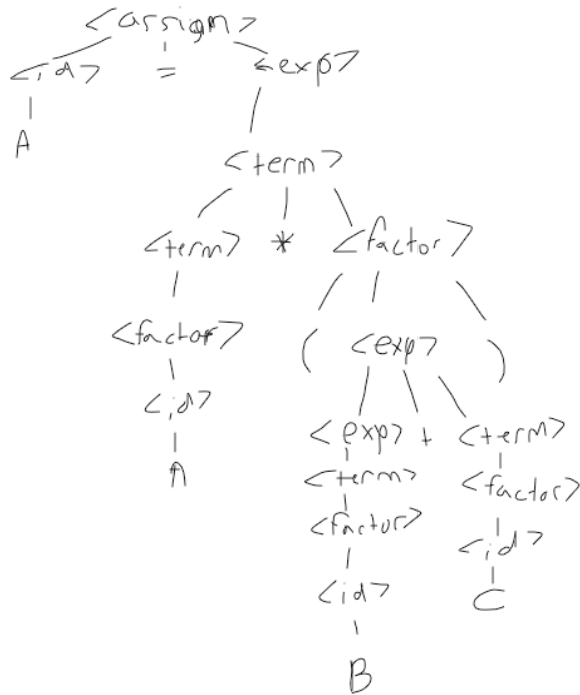
$\langle \text{assign} \rangle \Rightarrow \langle \text{id} \rangle = \langle \text{exp} \rangle$
 $\Rightarrow A = \langle \text{exp} \rangle + \langle \text{term} \rangle$
 $\Rightarrow A = \langle \text{exp} \rangle + \langle \text{term} \rangle + \langle \text{term} \rangle$
 $\Rightarrow A = \langle \text{term} \rangle + \langle \text{term} \rangle + \langle \text{term} \rangle$
 $\Rightarrow A = \langle \text{factor} \rangle + \langle \text{term} \rangle + \langle \text{term} \rangle$
 $\Rightarrow A = \langle \text{id} \rangle + \langle \text{term} \rangle + \langle \text{term} \rangle$
 $\Rightarrow A = B + \langle \text{term} \rangle + \langle \text{term} \rangle$
 $\Rightarrow A = B + \langle \text{factor} \rangle + \langle \text{term} \rangle$
 $\Rightarrow A = B + \langle \text{id} \rangle + \langle \text{term} \rangle$
 $\Rightarrow A = B + C + \langle \text{term} \rangle$

$\Rightarrow A = B + C + \langle \text{factor} \rangle$
 $\Rightarrow A = B + C + \langle \text{id} \rangle$
 $\Rightarrow A = B + C + A$



C. $A = A * (B + C)$

$\langle \text{assign} \rangle \Rightarrow \langle \text{id} \rangle = \langle \text{exp} \rangle$
 $\Rightarrow A = \langle \text{exp} \rangle$
 $\Rightarrow A = \langle \text{term} \rangle$
 $\Rightarrow A = \langle \text{term} \rangle * \langle \text{factor} \rangle$
 $\Rightarrow A = \langle \text{factor} \rangle * \langle \text{factor} \rangle$
 $\Rightarrow A = \langle \text{id} \rangle * \langle \text{factor} \rangle$
 $\Rightarrow A = A * \langle \text{factor} \rangle$
 $\Rightarrow A = A * (\langle \text{exp} \rangle)$
 $\Rightarrow A = A * (\langle \text{exp} \rangle + \langle \text{term} \rangle)$
 $\Rightarrow A = A * (\langle \text{term} \rangle + \langle \text{term} \rangle)$
 $\Rightarrow A = A * (\langle \text{factor} \rangle + \langle \text{term} \rangle)$
 $\Rightarrow A = A * (\langle \text{id} \rangle + \langle \text{term} \rangle)$
 $\Rightarrow A = A * (B + \langle \text{term} \rangle)$
 $\Rightarrow A = A * (B + \langle \text{factor} \rangle)$
 $\Rightarrow A = A * (B + \langle \text{id} \rangle)$
 $\Rightarrow A = A * (B + C)$



D. $A = B * (C * (A + B))$

<assign> \Rightarrow <id> = <exp>
 $\Rightarrow A = \text{<exp>}$
 $\Rightarrow A = \text{<term>}$
 $\Rightarrow A = \text{<term>} * \text{<factor>}$
 $\Rightarrow A = \text{<factor>} * \text{<factor>}$
 $\Rightarrow A = \text{<id>} * \text{<factor>}$
 $\Rightarrow A = B * \text{<factor>}$
 $\Rightarrow A = B * (\text{<exp>})$
 $\Rightarrow A = B * (\text{<term>})$
 $\Rightarrow A = B * (\text{<term>} * \text{<factor>})$
 $\Rightarrow A = B * (\text{<factor>} * \text{<factor>})$
 $\Rightarrow A = B * (\text{<id>} * \text{<factor>})$
 $\Rightarrow A = B * (C * \text{<factor>})$
 $\Rightarrow A = B * (C * (\text{<exp>}))$
 $\Rightarrow A = B * (C * (\text{<exp>} + \text{<term>}))$
 $\Rightarrow A = B * (C * (\text{<term>} + \text{<term>}))$
 $\Rightarrow A = B * (C * (\text{<factor>} + \text{<term>}))$
 $\Rightarrow A = B * (C * (\text{<id>} + \text{<term>}))$
 $\Rightarrow A = B * (C * (A + \text{<term>}))$
 $\Rightarrow A = B * (C * (A + \text{<factor>}))$
 $\Rightarrow A = B * (C * (A + \text{<id>}))$
 $\Rightarrow A = B * (C * (A + B))$

