Cartesian Plane: Point-Slope Formula for Lines
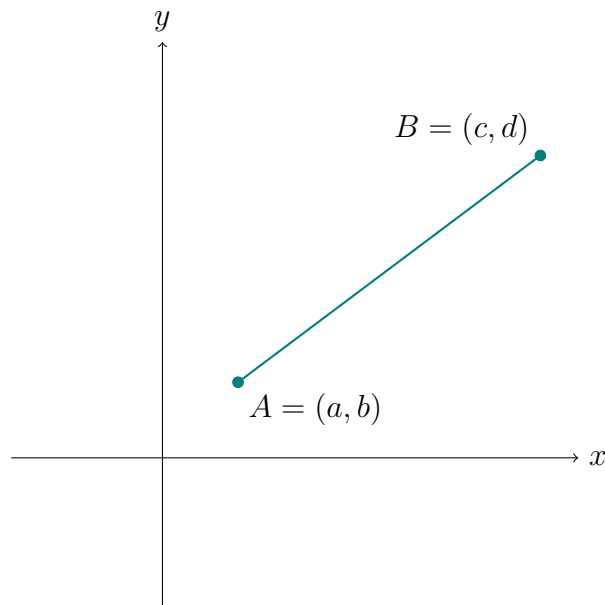
Video companion

1 Introduction

In this video: Demystify formulas for equations of lines

$$y - y_0 = m(x - x_0) \quad \text{Point-slope form}$$
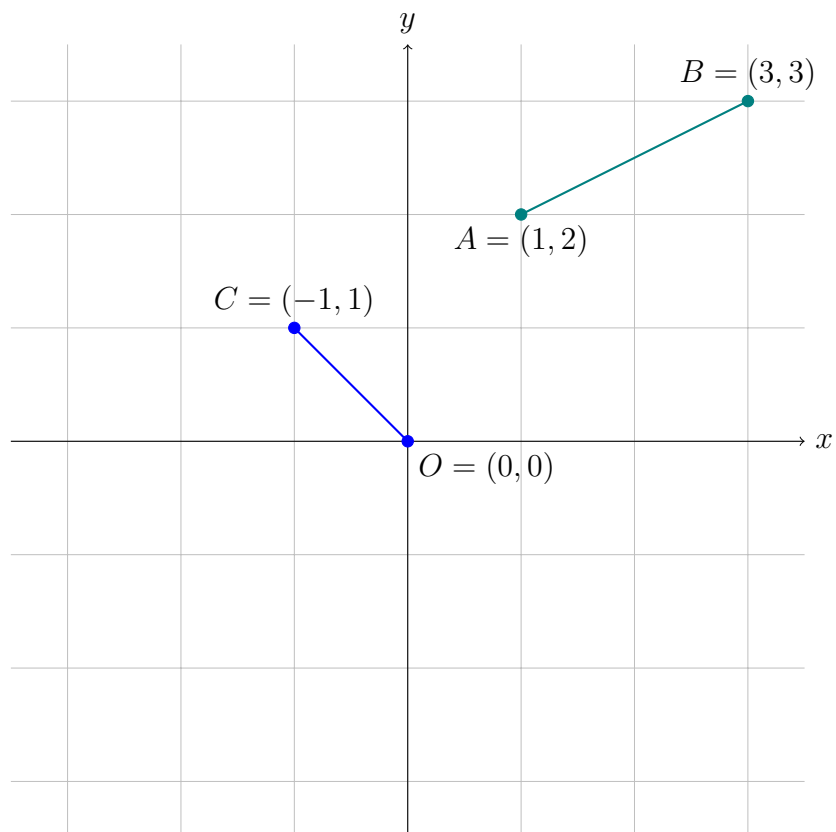$$y = mx + b \qquad\qquad \text{Slope-intercept form}$$

2 Slope of a line segment

Slope of $\overrightarrow{AB}$:

$$m = \frac{d - b}{c - a} = \frac{\text{"rise"}}{\text{"run"}}$$

3   Examples

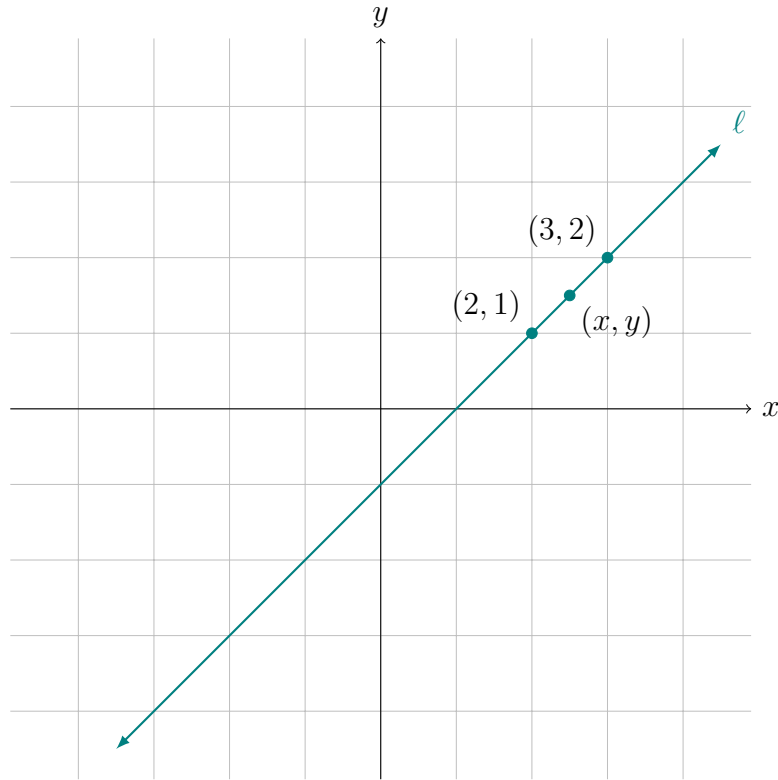Slope of $\overrightarrow{AB}$:

$$m = \frac{3-2}{3-1} = \frac{1}{2}$$

$m = \frac{1}{2}$ is a positive slope.

Slope of $\overrightarrow{CO}$:

$$m = \frac{0-1}{0-(-1)} = -1$$

$m = -1$ is a negative slope.

4   Equation of a line

For a point (x, y) to be on the line, the line segment from $(2, 1)$ to (x, y) need to have a slope of 1.

$$1 = \frac{y - 1}{x - 2}$$
$$y - 1 = 1(x - 2)$$

The line is defined by this formula:

$$\ell = \left\{ (x, y) \in \mathbb{R}^2 : y - 1 = 1(x - 2) \right\}$$

Check that $(3, 2)$ is on the line:

$$(3, 2) \in \ell \,?$$
$$2 - 1 \overset{?}{=} 1(3 - 2)$$
$$1 \overset{?}{=} 1 \quad \checkmark$$

Check if $(5, 1)$ is on the line:

$$(5, 1) \in \ell \, ?$$
$$1 - 1 \overset{?}{=} 1(5 - 2)$$
$$0 \overset{?}{=} 3 \quad \times$$

5   Point-slope formula

If a line ℓ has slope m, *and* if (x_0, y_0) is *any* point on ℓ, then ℓ has the equation

$$\boxed{y - y_0 = m(x - x_0).}$$