Densities definitions

There are mainly two definitions that we are using for densities:

- Galaxy number density:

$$n(z) = \frac{dN}{dV}(z)$$

(where the volume is comoving volume)

- Survey number density (the one used in CoLoRe):

$$m(z) = \frac{dN}{dz d\Omega}(z)$$

Effective redshift computation

Exact derivation (full pairs)

If we want to count all the possible pairs of galaxies in our survey, we want to compute the following quantity:

$$z_{\text{eff}} = \frac{1}{N^2} \int dN(z) dN(z') \frac{z+z'}{2} =$$
$$\frac{1}{N^2} \left(\int dN(z) dN(z') z/2 + \int dN(z) dN(z') z'/2 \right) =$$
$$\frac{1}{N} \int dN(z) z$$

However, this does not work in our case because our correlations are limited to the closest galaxies (200Mpc in the whole box is a really small distance).

Exact derivation (real pairs)

$$z_{\text{eff}} = \frac{1}{N_{\text{norm}}} \int_0^{r_{\max}} dN(\vec{r}) \int_{|\vec{r}-\vec{r'}|<200Mpc} dN(\vec{r'}) \frac{z(r)+z'(r)}{2}$$

This could be done computationally but not trivial (maybe not possible?) to get an analitical solution. The value of N_{norm} will be the integral without redshift factors.

Approximate derivation

We can consider that the number of pairs is simply a local variable defined as:

$$\text{number of pairs} \simeq \text{number of galaxies}^2$$

Therefore only one integral over all the space is needed to compute the effective redshfit.:

$$z_{\text{eff}} = \frac{1}{N} \int dN^2(z) z$$

Include distances in the derivation

Up to now I haven't included any distances, and all the definitions have been done in terms of differential of number of galaxies. Using the relations mentioned on the top of this page:

$$\frac{dN^2}{dV} = n^2(z)$$

$$\frac{dN^2}{dV d\Omega} = m^2(z)$$

The units of these two values are:

$$[n^2(z)] = \text{Mpc}^{-3}$$

$$[m^2(z)] = (\text{redshift} \cdot \text{solid angle})^{-1}$$

This basically states that these quantities scale as volume. It might seem wrong because the real number of pairs should scale as volume^2. But we are using pairs as a local variable so it should scale as volume^1.

Therefore the effective redshift computed from the two densities are:

$$z_{\text{eff}} = \frac{1}{V} \int n^2(z) \cdot z \cdot dV$$

$$z_{\text{eff}} = \frac{1}{\Delta z} \int m^2(z) \cdot z \cdot dz d\Omega$$

This two expressions are completely equivalent if $m^2(z)$ and $n^2(z)$ transform (has the same units) as $m(z)$ and $n(z)$.

This second expression is the one I have been using in my computations, and therefore I think they are fine. But there are issues with the densities used in simulations, since I used survey number density (m) constant instead of galaxy number density (n) constant (as it should be).