LogNormal RSD model

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Previous model (Kaiser)

$$(1+\delta^s_{LN}) = \frac{(1+\delta_{LN})}{(1-y)} \simeq 1+\delta_{LN}+y \qquad y \equiv \frac{-\partial_z V_z}{H(z)}$$

$$\langle \delta^s_{LN}\delta^s_{LN}\rangle = \langle \delta_{LN}\delta_{LN}\rangle + 2\langle \delta_{LN}\, y\rangle + \langle yy\rangle$$

New model (assumes $|y|$ small, but not δ_{LN})

$$(1+\delta^s_{LN})_{new} = \frac{(1+\delta_{LN})}{(1-y)} \simeq 1+\delta_{LN}+y+\varepsilon \qquad \varepsilon \equiv \delta_{LN}\cdot y$$

$$\langle \delta^s_{LN}\delta^s_{LN}\rangle_{new} = \langle \delta^s_{LN}\delta^s_{LN}\rangle_{old} + 2\langle \delta_{LN}\varepsilon\rangle + 2\langle y\varepsilon\rangle + \langle \varepsilon\varepsilon\rangle$$

Test correlations $(b_A = 0,\ b_B \neq 0)$

(1) $\langle \delta^s_A \delta^s_A\rangle = \langle yy\rangle$

(2) $\langle \delta^s_A \delta_B\rangle = \langle y\,\delta_B\rangle$

(3) $\langle \delta^s_A \delta^s_B\rangle = \langle yy\rangle + \langle y\delta_B\rangle + \langle y\varepsilon_B\rangle$

(4) $\langle \delta_B \delta_B\rangle = \langle \delta_B\delta_B\rangle$

(5) $\langle \delta_B \delta^s_B\rangle = \langle \delta_B\delta_B\rangle + \langle \delta_B y\rangle + \langle \delta_B\varepsilon_B\rangle$

(6) $\langle \delta^s_B \delta^s_B\rangle = \langle \delta_B\delta_B\rangle + \langle yy\rangle + \langle \varepsilon_B\varepsilon_B\rangle$
$+2\left[\langle \delta_B y\rangle + \langle \delta_B\varepsilon_B\rangle + \langle y\varepsilon_B\rangle\right]$

Test correlations $\quad (b_A = 0 \,,\, b_B \neq 0)$

(1) $\langle \delta_A^s \delta_A^s \rangle = \langle y y \rangle$

(2) $\langle \delta_A^s \delta_B \rangle = \langle y \delta_B \rangle$

(3) $\langle \delta_A^s \delta_B^s \rangle = \langle y y \rangle + \langle y \delta_B \rangle + \langle y \varepsilon_B \rangle$

(4) $\langle \delta_B \delta_B \rangle = \langle \delta_B \delta_B \rangle$

(5) $\langle \delta_B \delta_B^s \rangle = \langle \delta_B \delta_B \rangle + \langle \delta_B y \rangle + \langle \delta_B \varepsilon_B \rangle$

(6) $\langle \delta_B^s \delta_B^s \rangle = \langle \delta_B \delta_B \rangle + \langle y y \rangle + \langle \varepsilon_B \varepsilon_B \rangle + 2\left[\langle \delta_B y \rangle + \langle \delta_B \varepsilon_B \rangle + \langle y \varepsilon_B \rangle \right]$

$\langle y \varepsilon_B \rangle = (3) - (1) - (2)$ Term (11)

$\langle \delta_B \varepsilon_B \rangle = (5) - (2) - (4)$ Term (12)

$$\langle \varepsilon_B \varepsilon_B \rangle = (6) - 2\left[(5)-(2)-(4)\right] - 2\left[(3)-(1)-(2)\right] - 2(2) - (1) - (4)$$

$$= (6) - 2(5) - 2(3) + 2(2) + (4) + (1)$$ Term (13)

How can we test this model?

We can define a new sample with inverse RSD

$$\delta_B^{-s} = \delta_B - y - \varepsilon_B \qquad \varepsilon_B \equiv \langle \delta_B y \rangle$$

New test correlations

(7) $\langle \delta_A^{s} \delta_B^{-s} \rangle = \langle y \delta_B \rangle - \langle y y \rangle - \langle y \varepsilon_B \rangle$

(8) $\langle \delta_B \delta_B^{-s} \rangle = \langle \delta_B \delta_B \rangle - \langle \delta_B y \rangle - \langle \delta_B \varepsilon_B \rangle$

(9) $\langle \delta_B^{s} \delta_B^{-s} \rangle = \langle \delta_B \delta_B \rangle - \langle y y \rangle - \langle \varepsilon_B \varepsilon_B \rangle - 2 \langle y \varepsilon_B \rangle$

(10) $\langle \delta_B^{-s} \delta_B^{-s} \rangle = \langle \delta_B \delta_B \rangle + \langle y y \rangle + \langle \varepsilon_B \varepsilon_B \rangle - 2 \langle \delta_B y \rangle - 2 \langle \delta_B \varepsilon_B \rangle + 2 \langle y \varepsilon_B \rangle$

$\rightarrow$ (3) + (7) = 2 [(1) + (2)]

$\rightarrow$ (5) + (8) = 2 (4)

$\rightarrow$ (6) + (9) = 2 (5)

$\rightarrow$ (9) + (10) = 2 (8)

(2) $\langle \delta_A^{s} \delta_B \rangle = \langle y \delta_B \rangle$

(1) $\langle \delta_A^{s} \delta_A^{s} \rangle = \langle y y \rangle$

(3) $\langle \delta_A^{s} \delta_B^{s} \rangle = \langle y y \rangle + \langle y \delta_B \rangle + \langle y \varepsilon_B \rangle$

(4) $\langle \delta_B \delta_B \rangle = \langle \delta_B \delta_B \rangle$

(5) $\langle \delta_B \delta_B^{s} \rangle = \langle \delta_B \delta_B \rangle + \langle \delta_B y \rangle + \langle \delta_B \varepsilon_B \rangle$

(6) $\langle \delta_B^{s} \delta_B^{s} \rangle = \langle \delta_B \delta_B \rangle + \langle y y \rangle + \langle \varepsilon_B \varepsilon_B \rangle + 2 \left[\langle \delta_B y \rangle + \langle \delta_B \varepsilon_B \rangle + \langle y \varepsilon_B \rangle \right]$