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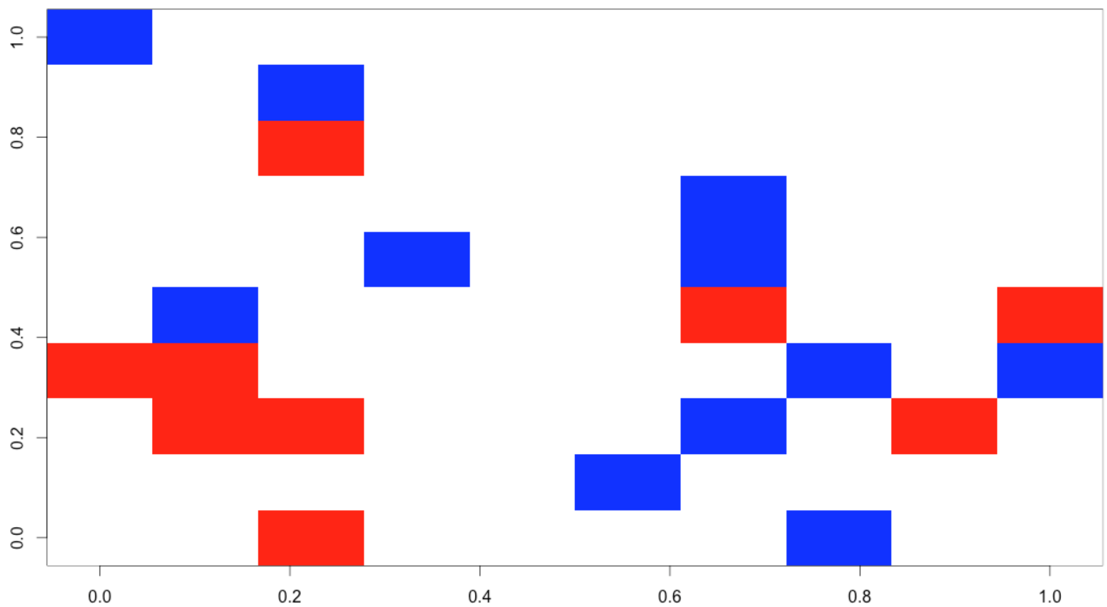
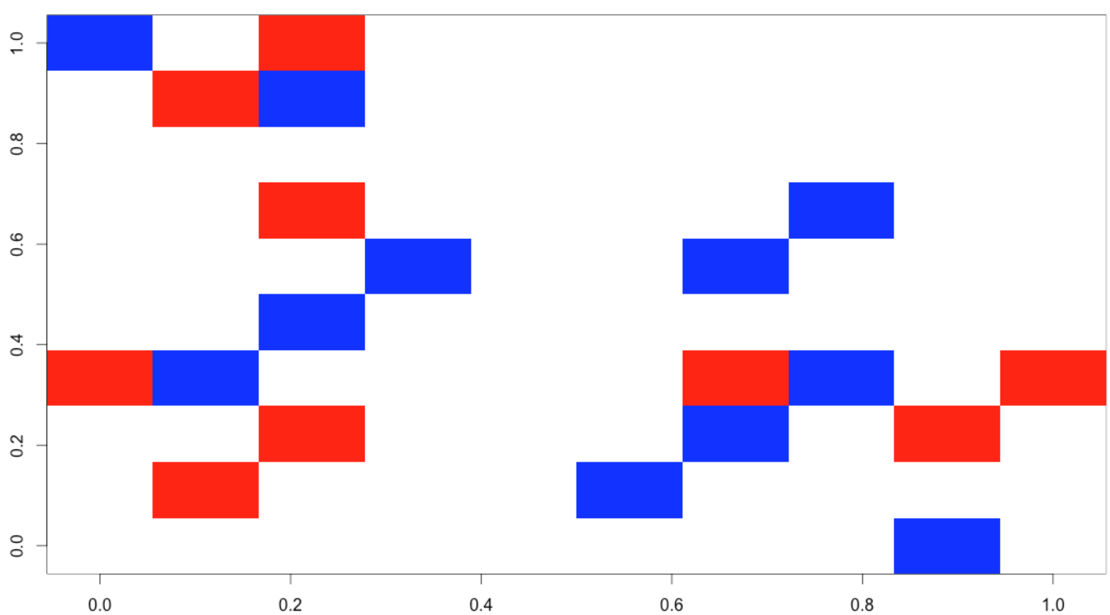
BML Simulation Study

For what values of p, the density of the grid, did you find free flowing traffic and traffic jams? Did you find any cases of a mixture of jams and free flowing traffic?

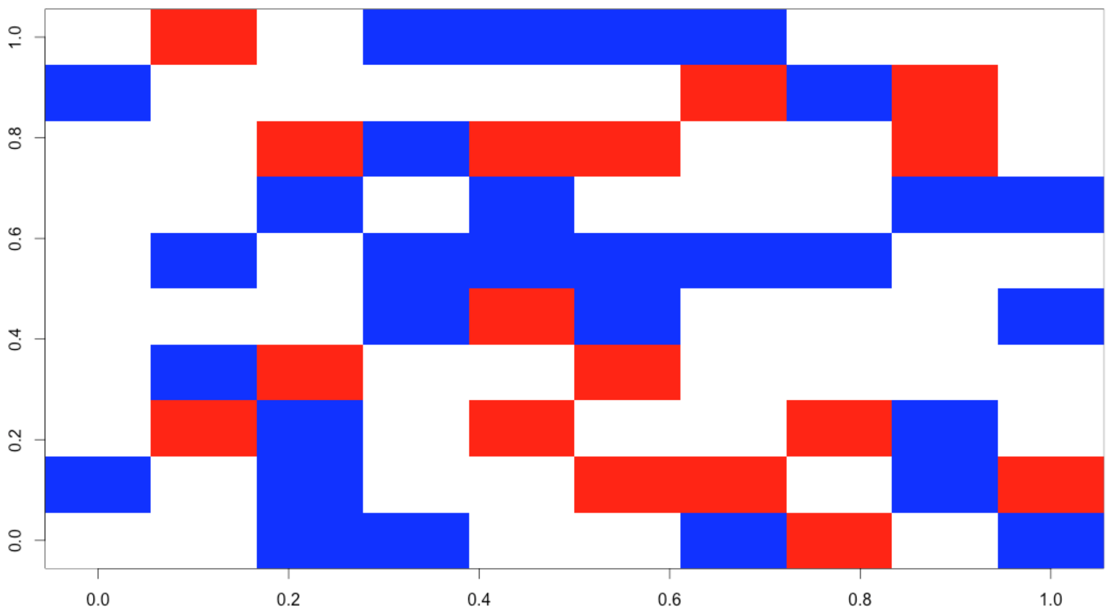
We first approach this point from a visual perspective.
We perfomed the simulation study for three different grid densities (.2,.6 and .9) and two different matrix sizes (10x10, and 100x100) to visualize the results of our analysis.
As the images below show, a really low desnity. Such as .2, will rearely produce a gridlock situation in both matrix sizes. However, we can see that once we increase the density of the grid to .6, both the 10x10and 100x100 have undergone gridlock for that specific trial, signaling that the exact grid density at which the matrix begins encountering gridlock is less than said .6. With the same thread of thought, we can observe how a grid with .9 of density barely has any free spaces for the cars to move; this inevitable produces gridlock in most cases, like we will see moving forward.
Then again, it is important to note that this visual approach allowed us to quickly identity that the density range for both matrices to reach a gridlock state is between .2 and .6, without any further computations or analysis. In the following sections of this statistical analysis we present enough information as to be able to reduce this range significantly.
Here we present our visual results, showing an image of the initial setup of the grid, and the output of the simulation that was run to fully understand the simulation's impact on the case with standard Square Lattices (10x10,5050,100x100):

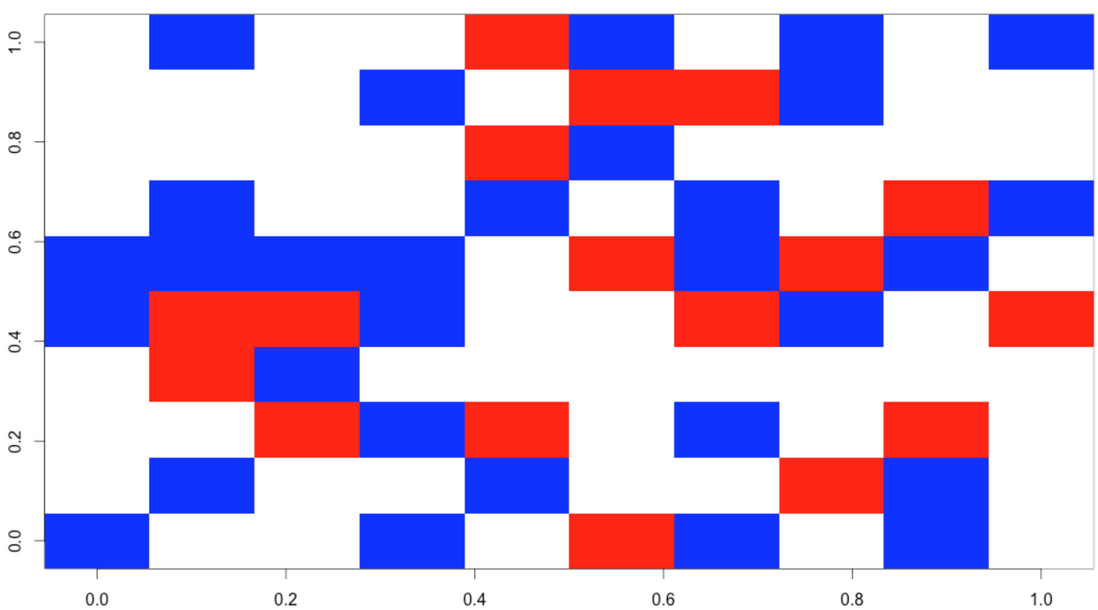
10x10 Lattice with p=.2 :Setup



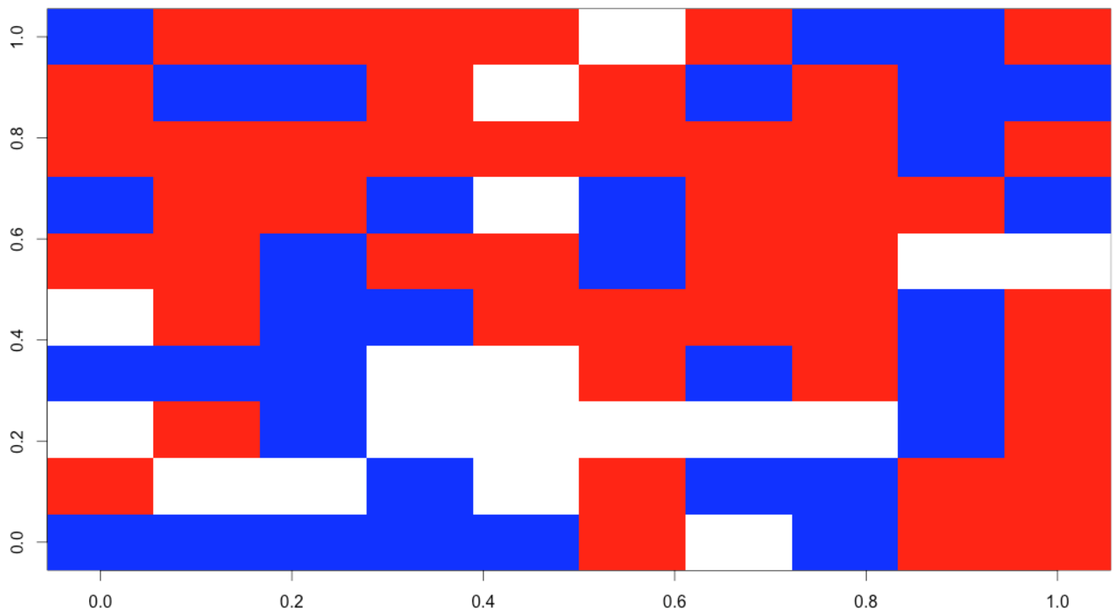


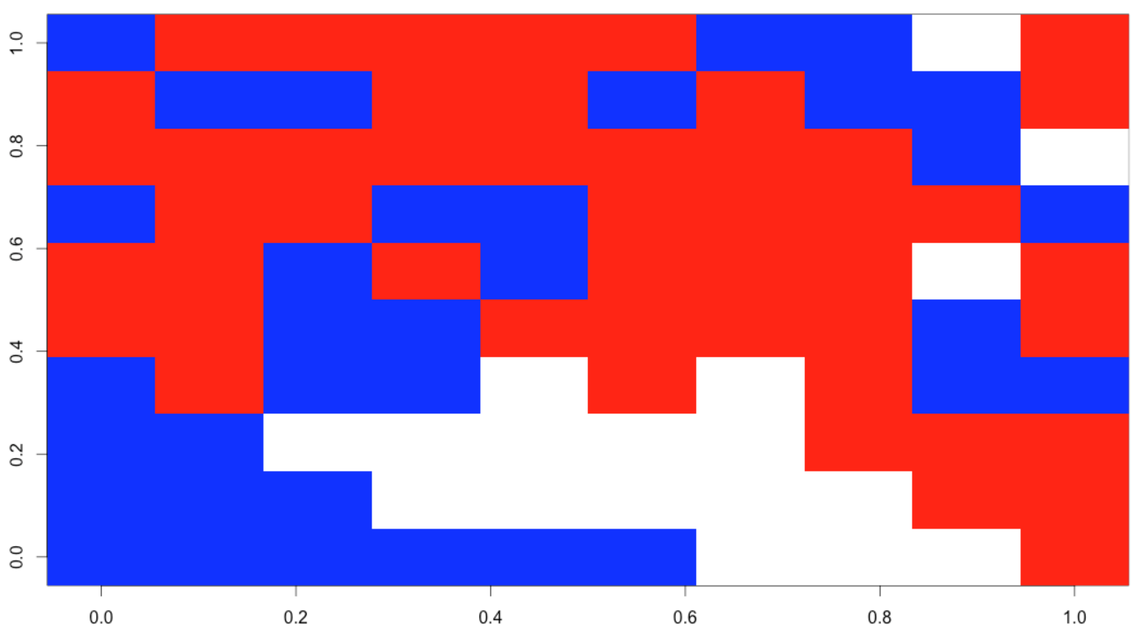
10x10 Lattice with p=.6 :Setup



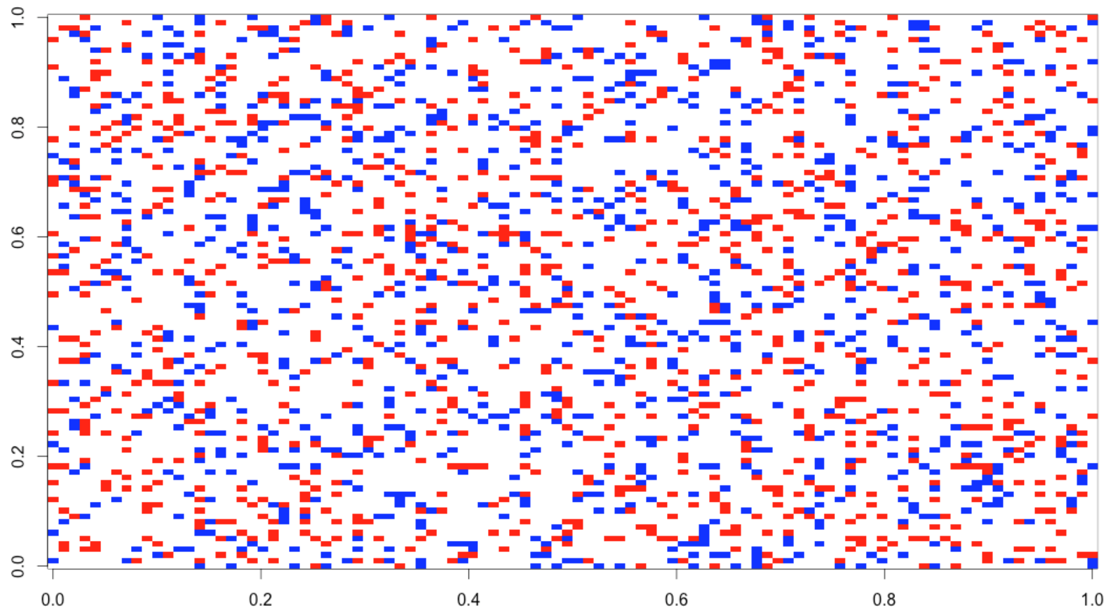


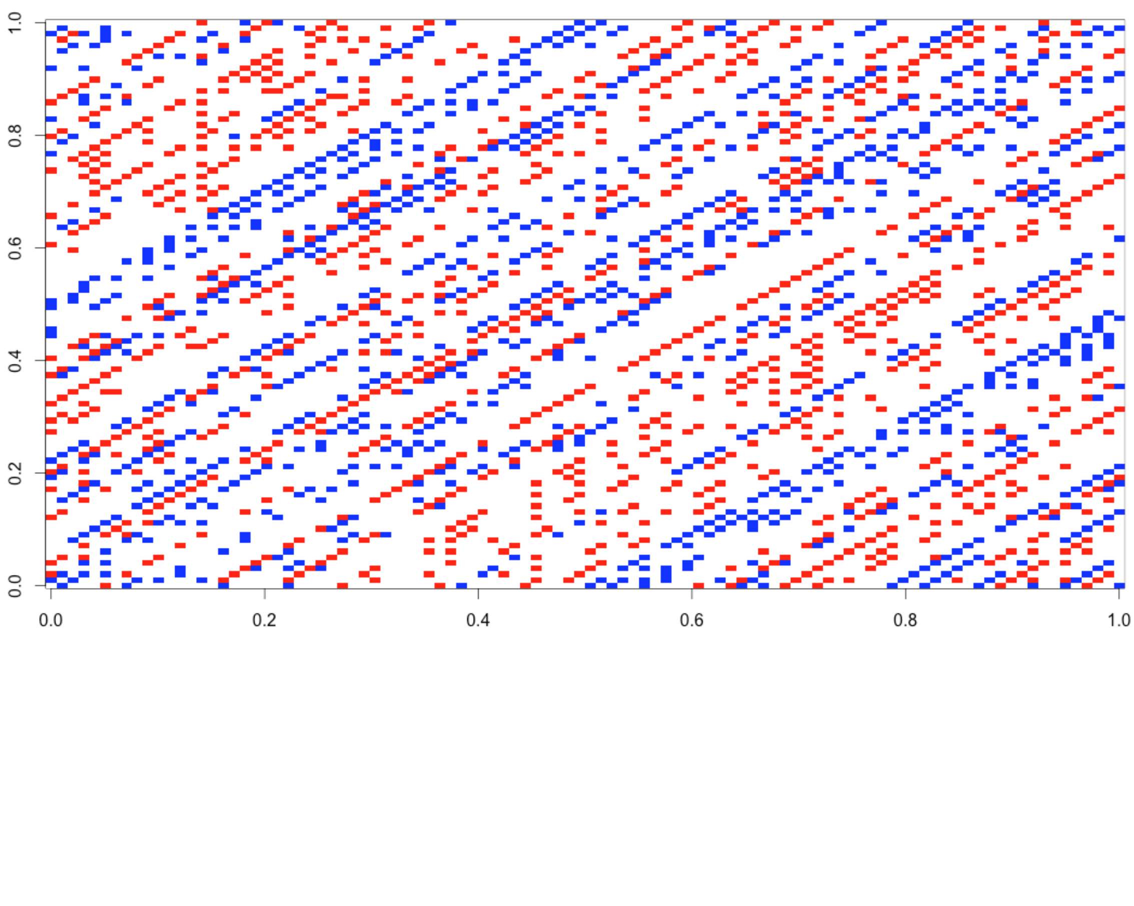
10x10 Lattice with p=.9 :Setup



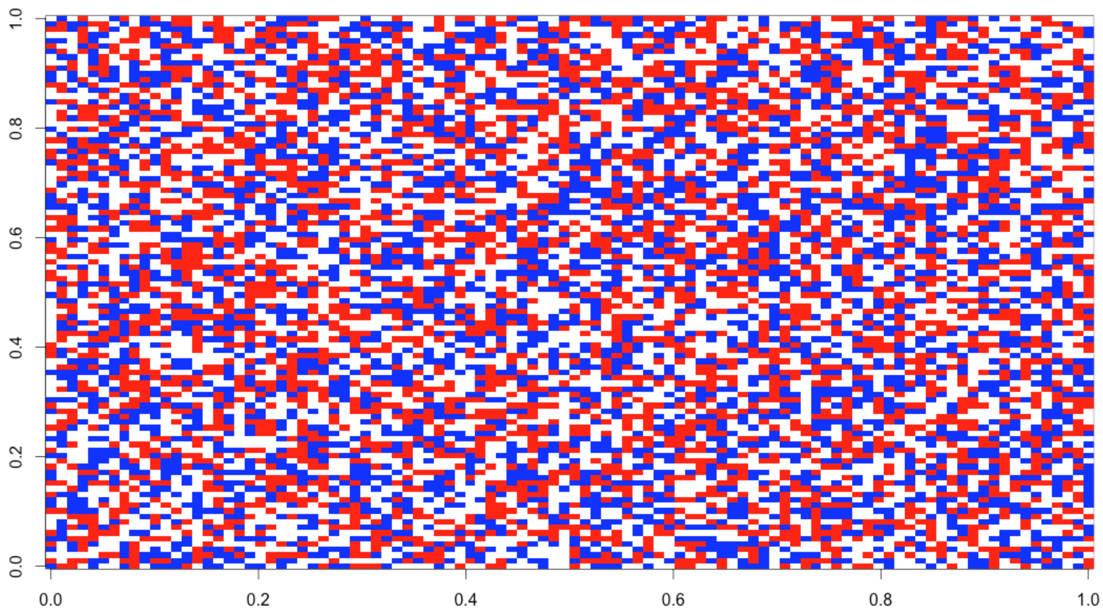


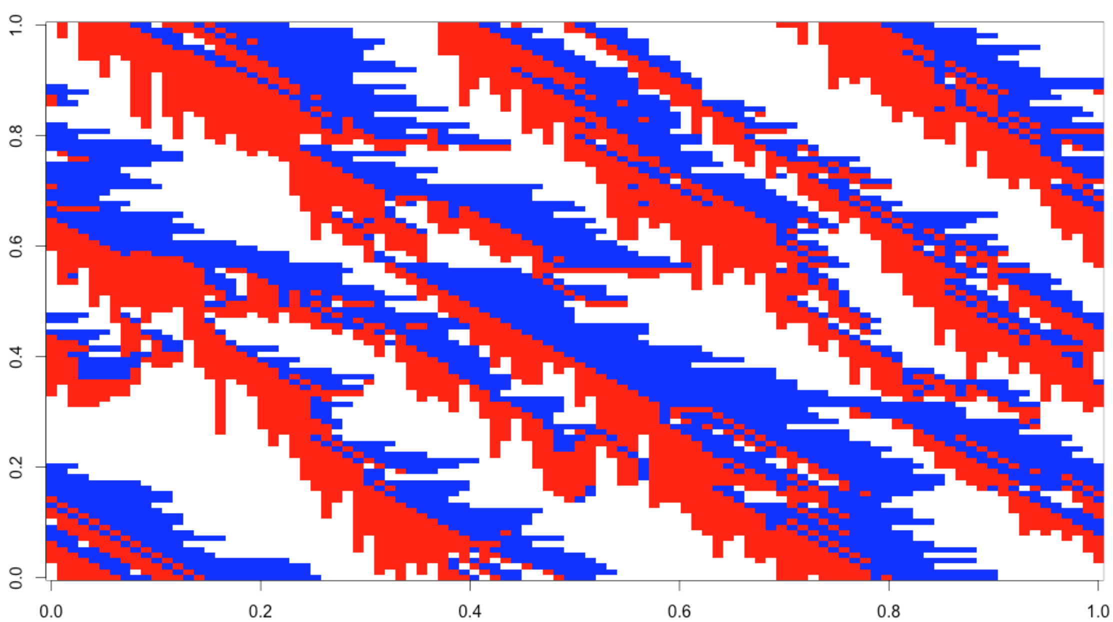
100x100 Lattice with p=.2 :Setup

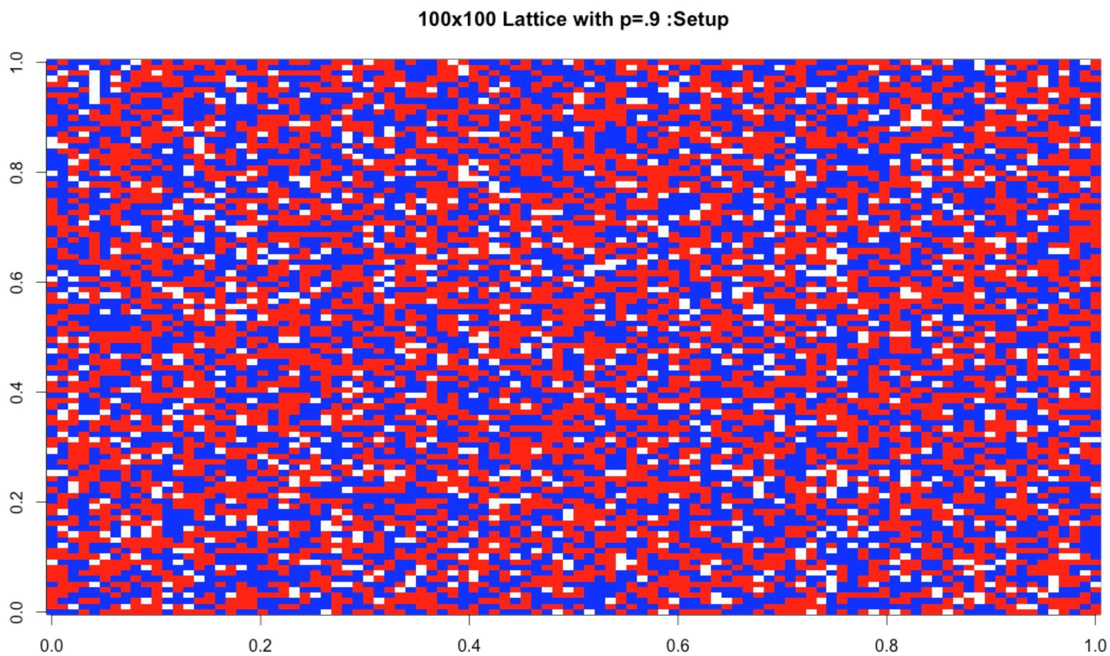


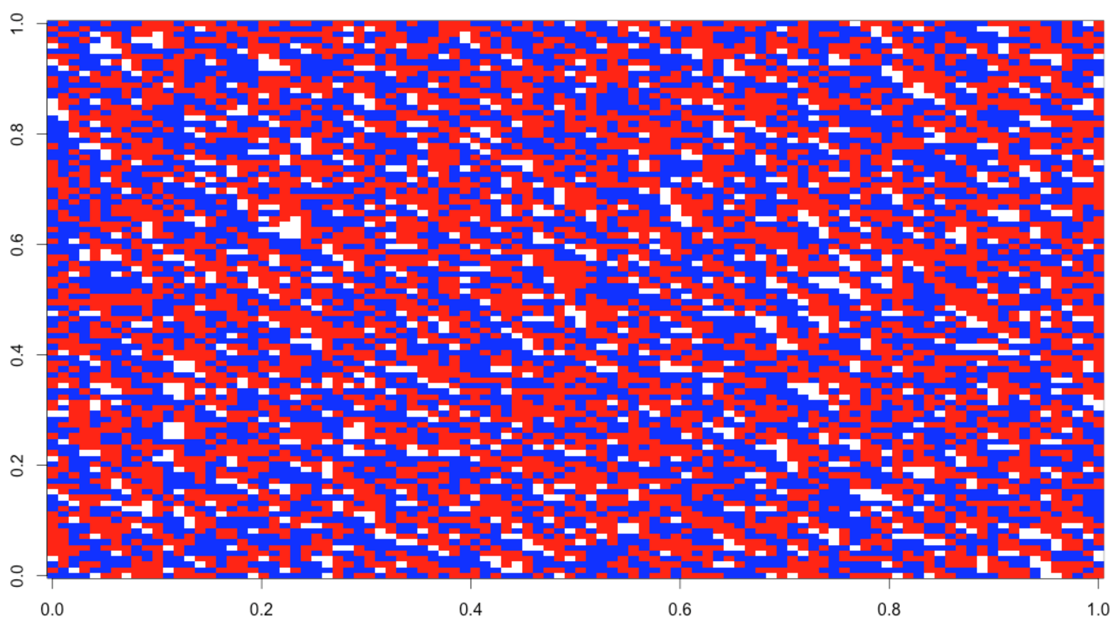


100x100 Lattice with p=.6 :Setup





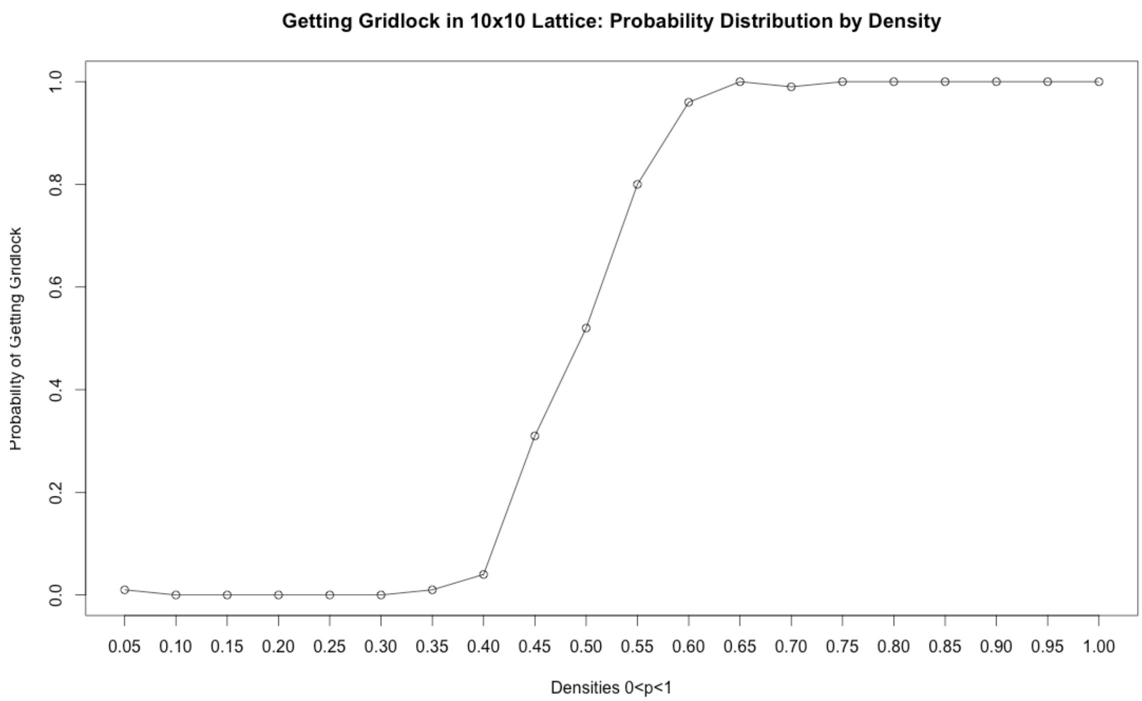


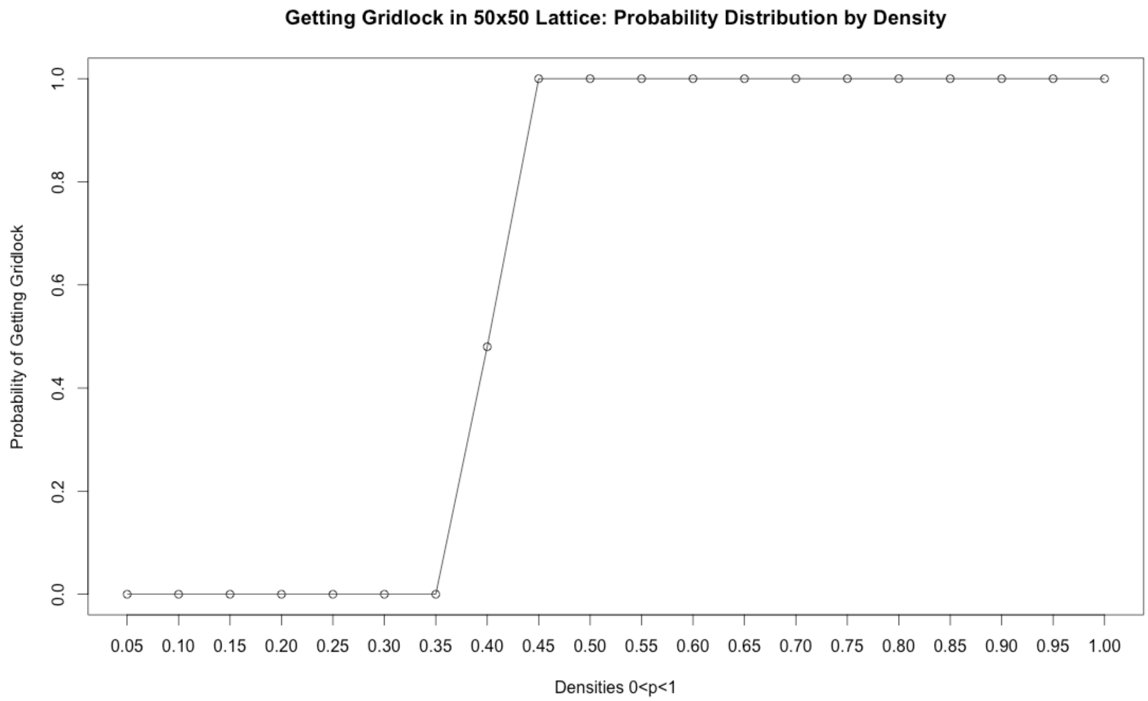


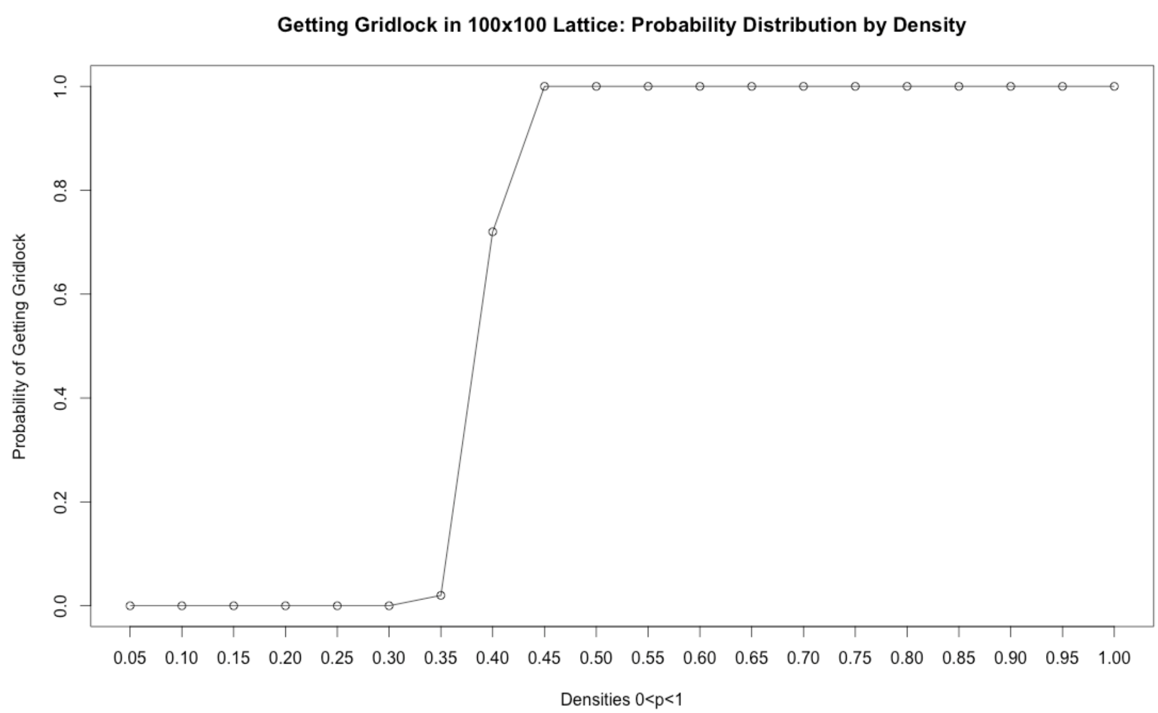
To be able to answer the second part of this question, we will be focusing on analyzing the proportion of trials that hit gridlock in the different lattice sizes as a function of the grid density. With this said, there is expected to be a strong correlationg between the level of the grid desntity and the proportion of cases that

reach gridlock, and hence for us to visualize a convergence to 1 as said desnity increases.
First we analyze the cases with Square Matrices (10x10,50x50,100x100).
After computing a total of 100 trials for every single density in the range from 0.05 to 1.00 with an increase of .05 between each density, it was possible to estimate the proportion of cases where the matrix reached gridlock before the arbitrary number of iterations chosen since the beginning (10,000). With this proportion of gridlock cases to the total number of trials, we can better understand how the different densities affect each different matrix size.

Square Matrices:

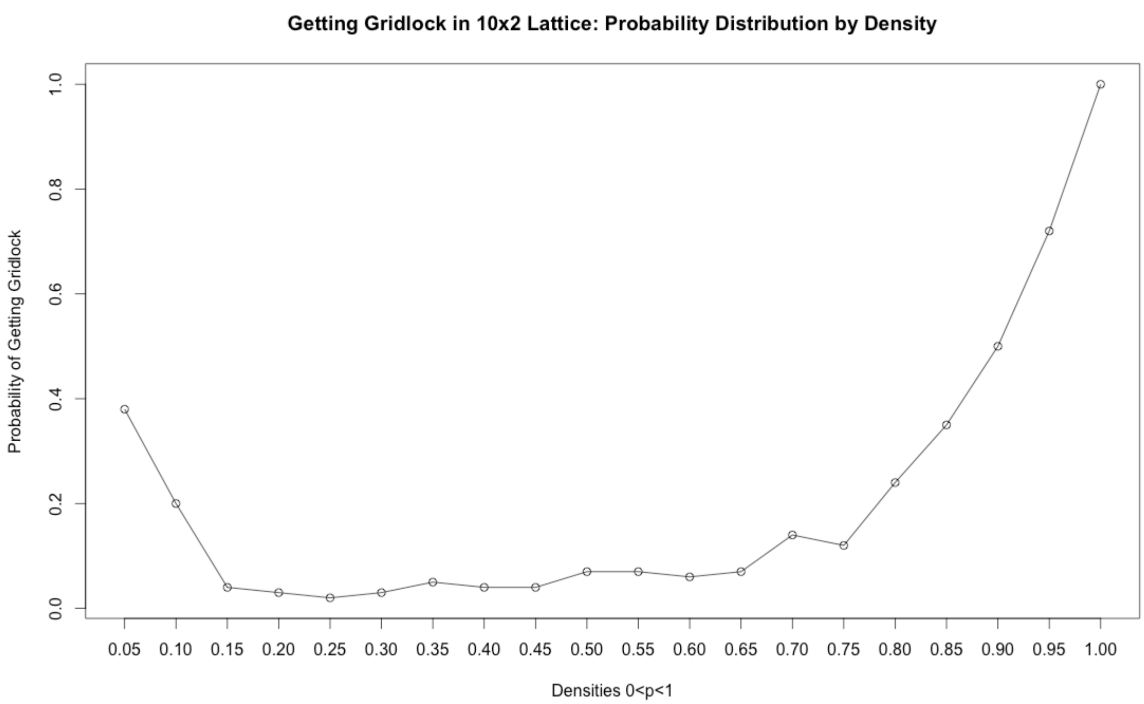



Getting Gridlock in 50x50 Lattice: Probability Distribution by Density
Probability of Getting Gridlock
0.0
0.2
0.4
0.6
0.8
1.0
0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85 0.90 0.95 1.00
Densities 0<p<1

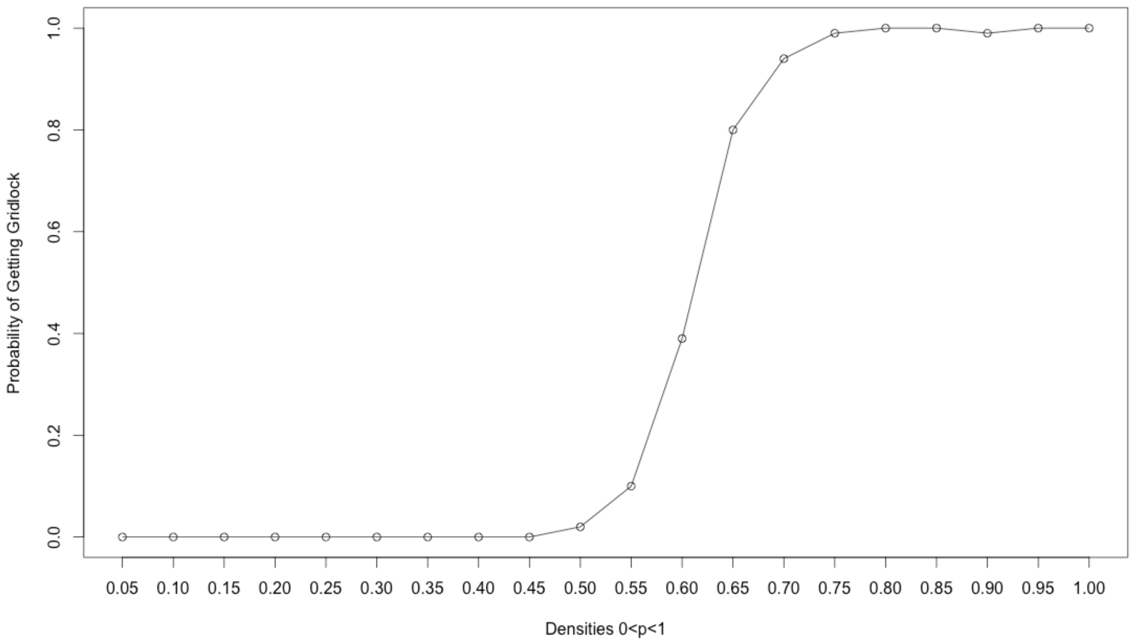

Getting Gridlock in 100x100 Lattice: Probability Distribution by Density
Probability of Getting Gridlock
0.0
0.2
0.4
0.6
0.8
1.0
0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85 0.90 0.95 1.00
Densities 0<p<1

However, there is something crucial that needs to be noted: While there is a recognizable pattern in the square matrices (10x10,50x50, and 100x100) in terms of the increase in the proportion of gridlock cases when the density of the grid increases, there is no recognizable pattern when the grid is non-square (something that can be seen specially in the case where we consider a lattice of dimensions 10x2, where the proportion of gridlocks begins by decreasing as the density increases, and then reaches a point where it starts increasing once again). This can be seen in the following plots:

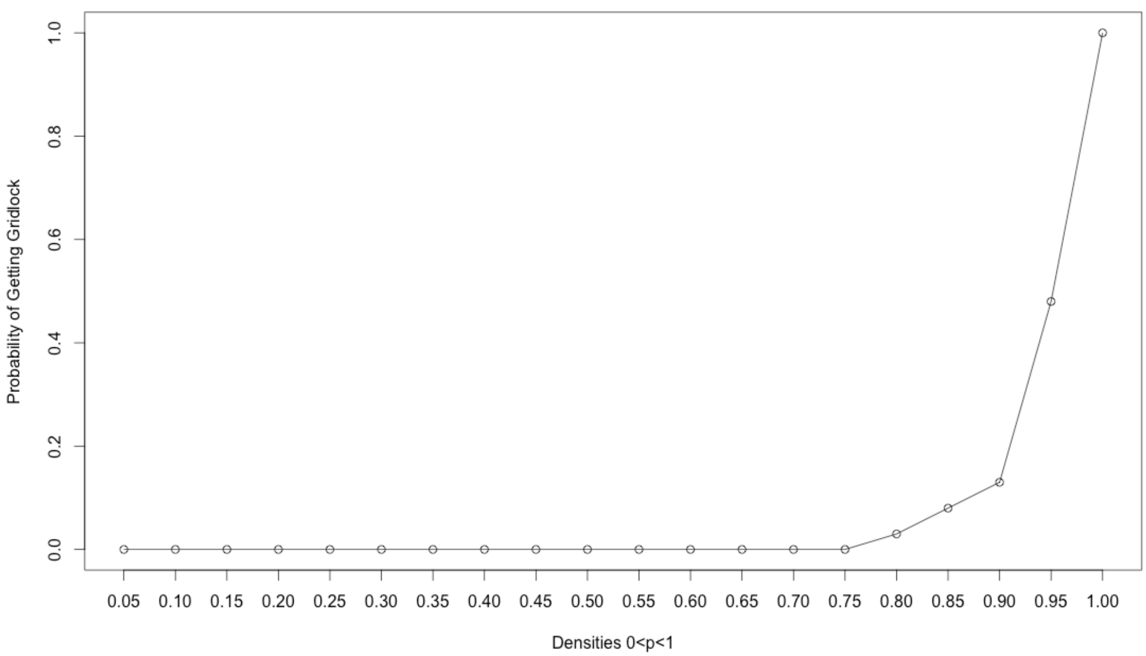
Non-Squares Mattrices:



Getting Gridlock in 30x8 Lattice: Probability Distribution by Density



Getting Gridlock in 50x3 Lattice: Probability Distribution by Density



As a final not to this sections, it is important to conclude that the only similarity seen between the behavior of the square and non-square matrices in terms of the pattern followed by their proportion of gridlocks as a function of the grid density is the fact that in all cases when the grid density is bigger than .80, we observe a proportion close or equal to 1. Intuitively, this can be thought of as the proportion of gridlock occurrences s a function of the "free space and possible moves" in the lattice.

At the same time, the latter plots help us understand when we fins a mixture of free flows and jams. In every plot, we can identify as constant free flow in every single trial given a specific density when the proportion of gridlocks is 0. On the other hand, the other extreme case tells us that we can find cases as "inevitable gridlocks" whenever we have a gridlock proportion of 1, meaning that every single simulation ran with that specific grid density returned a lattice with a gridlock. Everything else which doesn't correspond to one of those two cases mentioned and that can be found in the middle part of this graphs corresponds to those said mixtures of free flows and jams, which make by definition the proportion of gridlocks less than 1, but bigger than 0 necessarily.

2. How many simulation steps did you need to run before observing this behavior?

As it can be seen in the code with which the simulation was run, an arbitrary number of simulation steps was first chosen based on my own judgement. I certainly believed that 10,000 simulation steps was sufficient in order to prove whether any given matrix (of a reasonable size) would reach a gridlock state.

In a more analytical way, we can study the number of steps need for gridlock for every given density from 0.05 to 1.00. With this, we can see when the variability of reaching gridlock converges to 0, and therefore conclude that no more steps would have been needed in order to demonstrate how that certain lattice could end up in said gridlock.

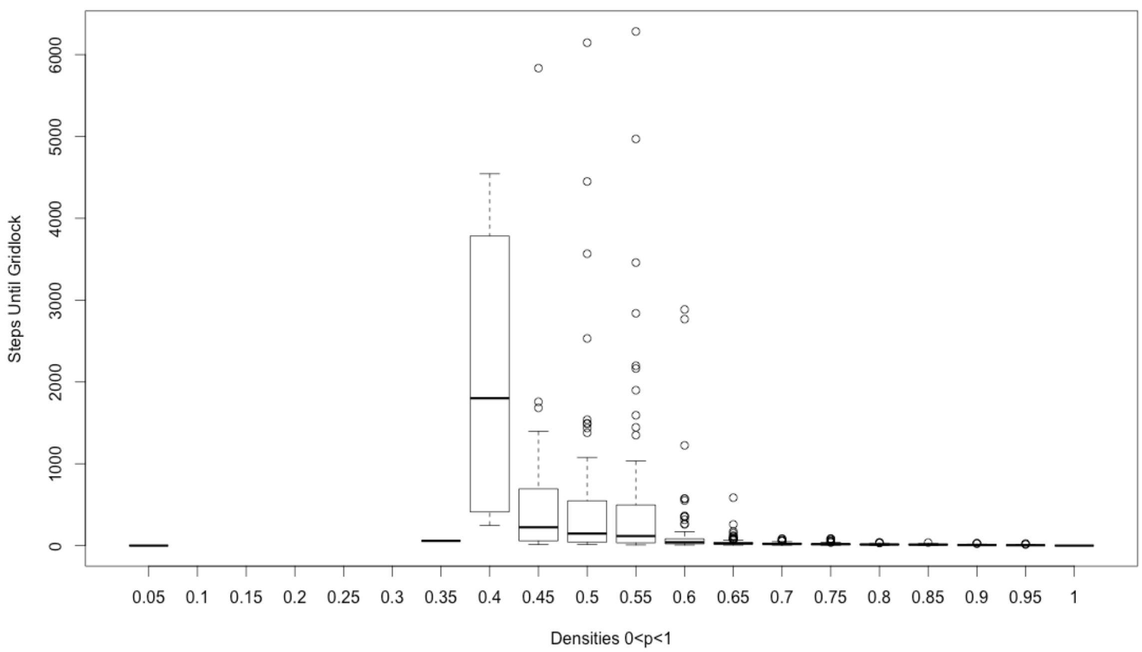
With the following boxplots, we can study not only the variability described above, but also the behavior of the average given a certain density.

For square matrices, we find a consistent and coherent pattern (just like we found in their probability distributions in Question #1), with said convergence starting at around .4 in grid density. However, when analyzing non-square matrices, we observe that there is not such pattern of convergence both for the variability of gridlocks, and for the average, especially in the 50x3 and 10x2 Lattices.

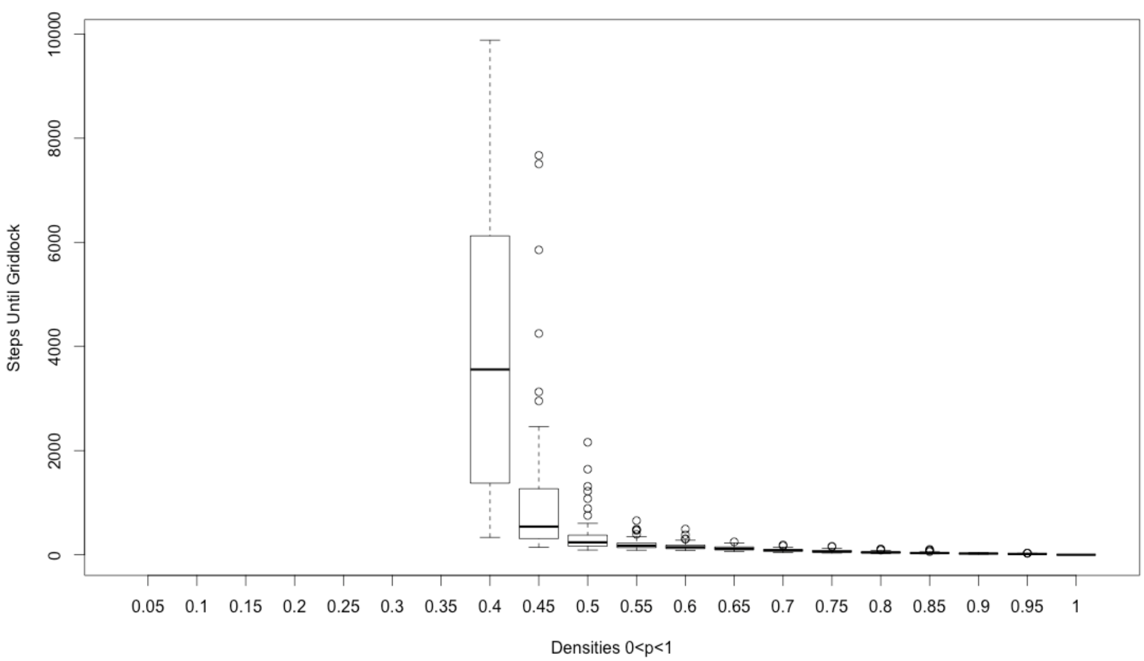
Lastly, it is important to note that no information was included for the cases where the lattice didn't reach gridlock in less than 10,000 iterations, This is the main reason for the lack of information in the boxplots for really low grid densities.

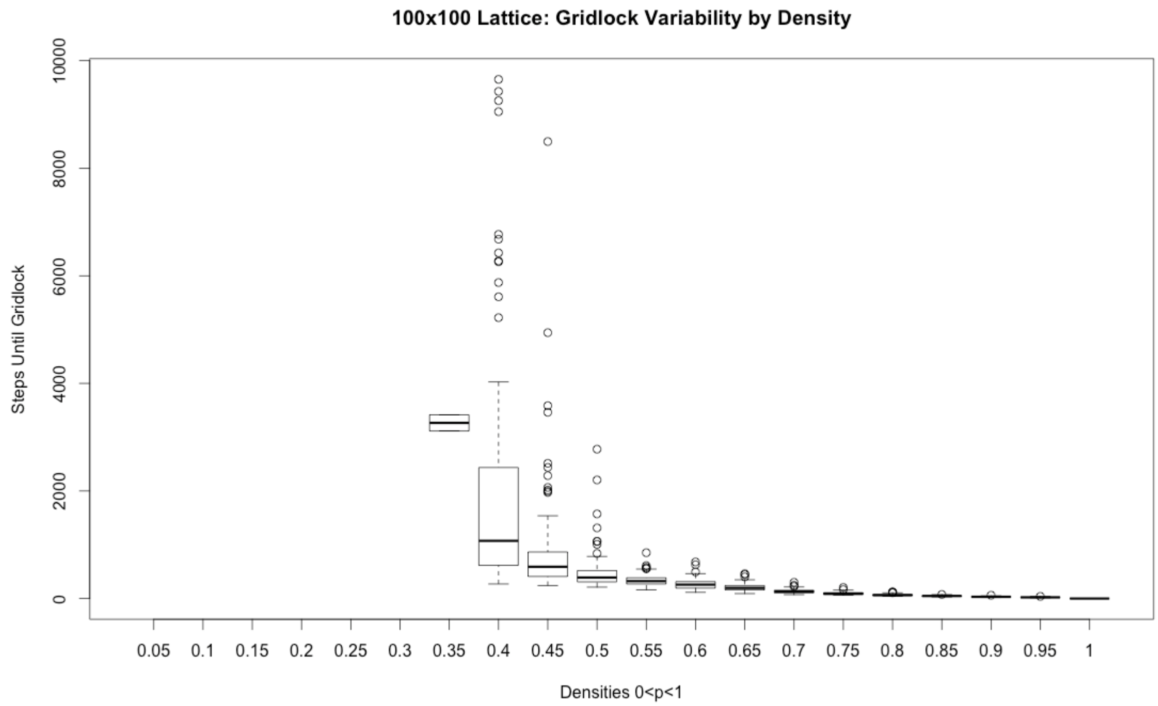
Squares:

10x10 Lattice: Gridlock Variability by Density

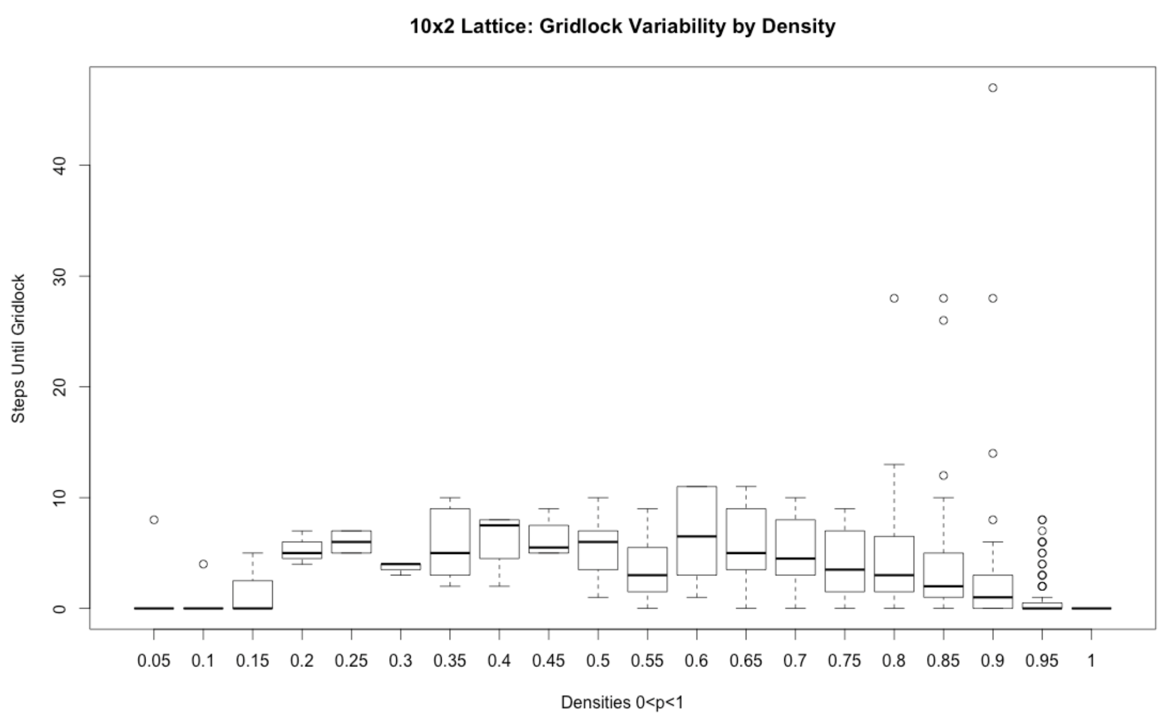


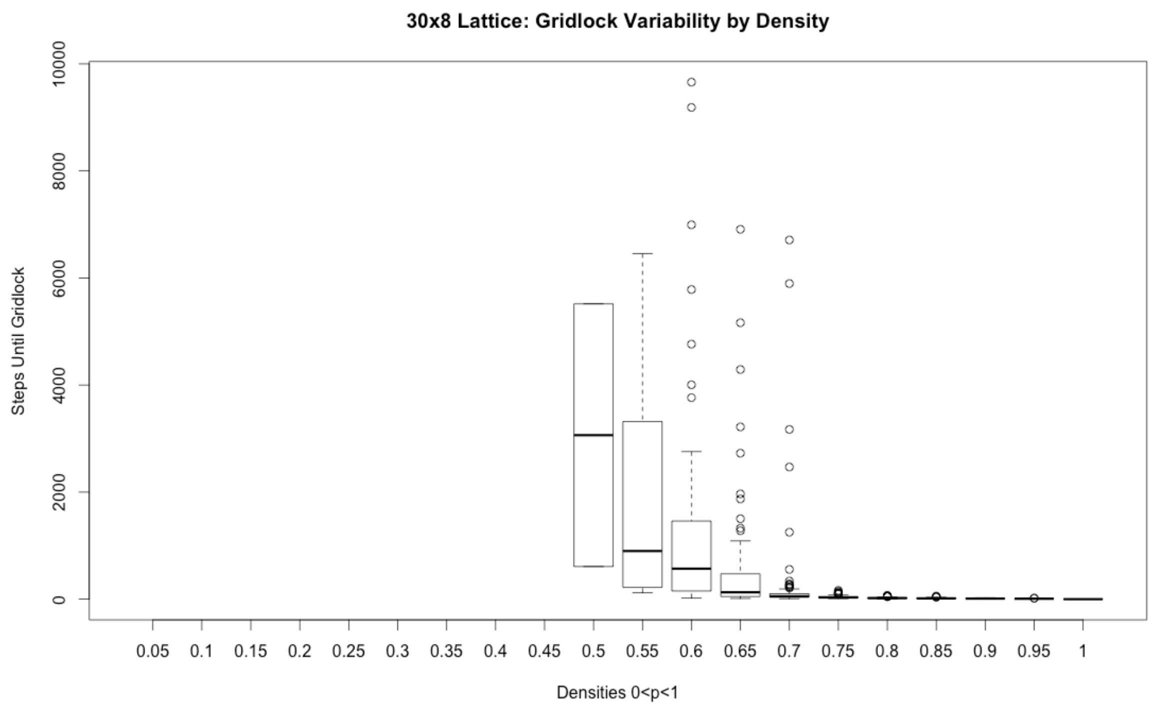
50x50 Lattice: Gridlock Variability by Density

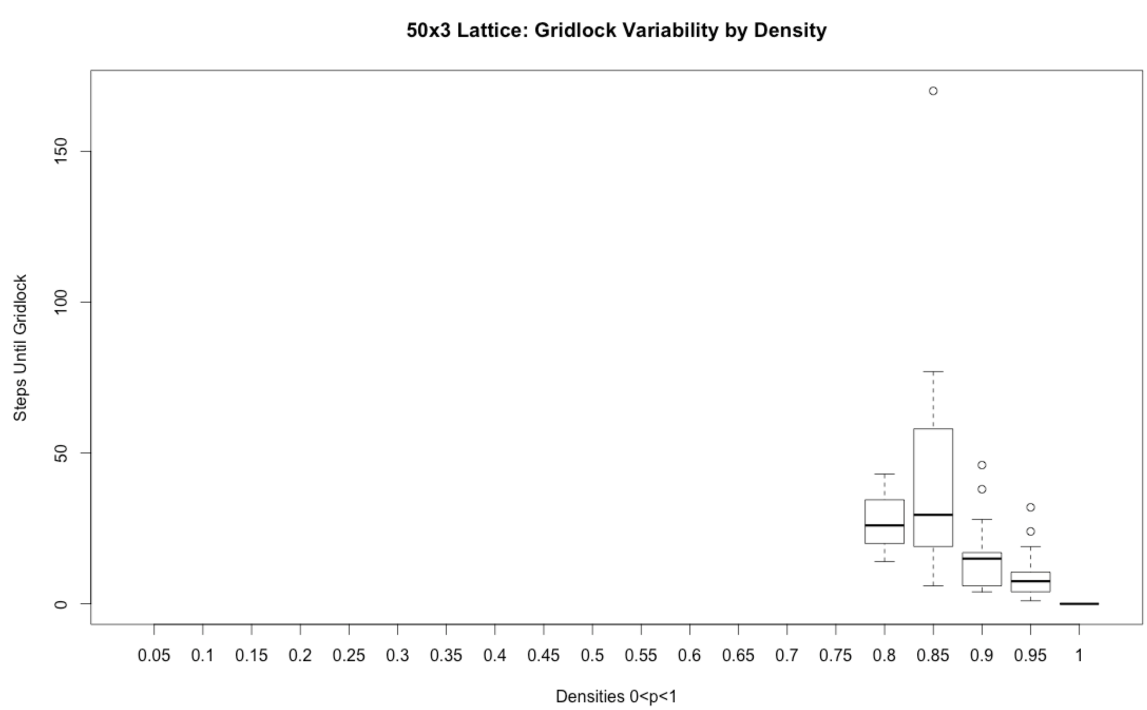




Non-square Matrices:




30x8 Lattice: Gridlock Variability by Density
Steps Until Gridlock
0
2000
4000
6000
8000
10000
0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45 0.5 0.55 0.6 0.65 0.7 0.75 0.8 0.85 0.9 0.95 1
Densities 0<p<1

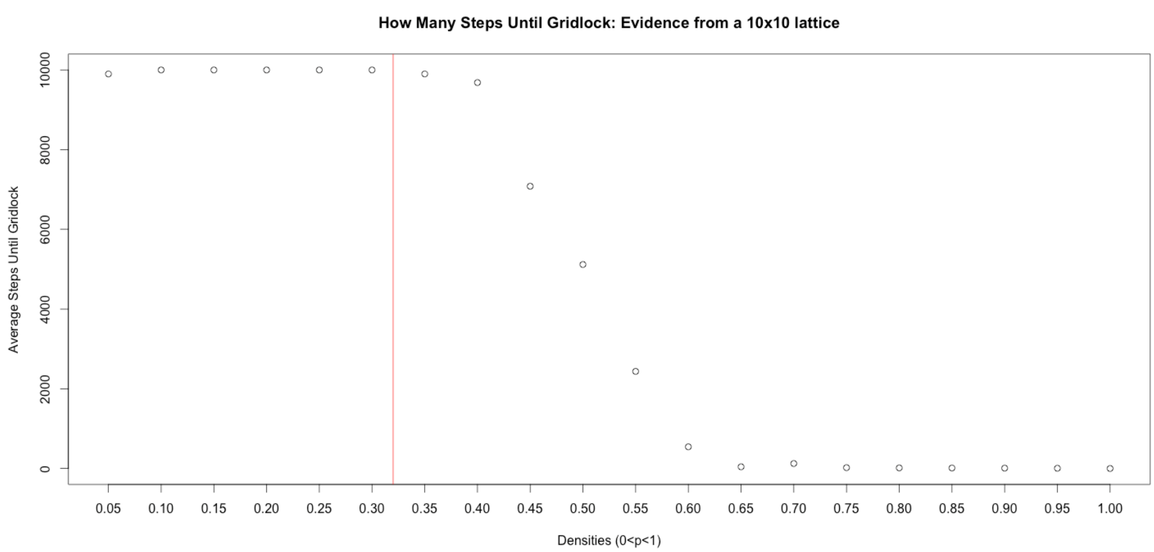

50x3 Lattice: Gridlock Variability by Density
Steps Until Gridlock
0
50
100
150
0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45 0.5 0.55 0.6 0.65 0.7 0.75 0.8 0.85 0.9 0.95 1
Densities 0<p<1

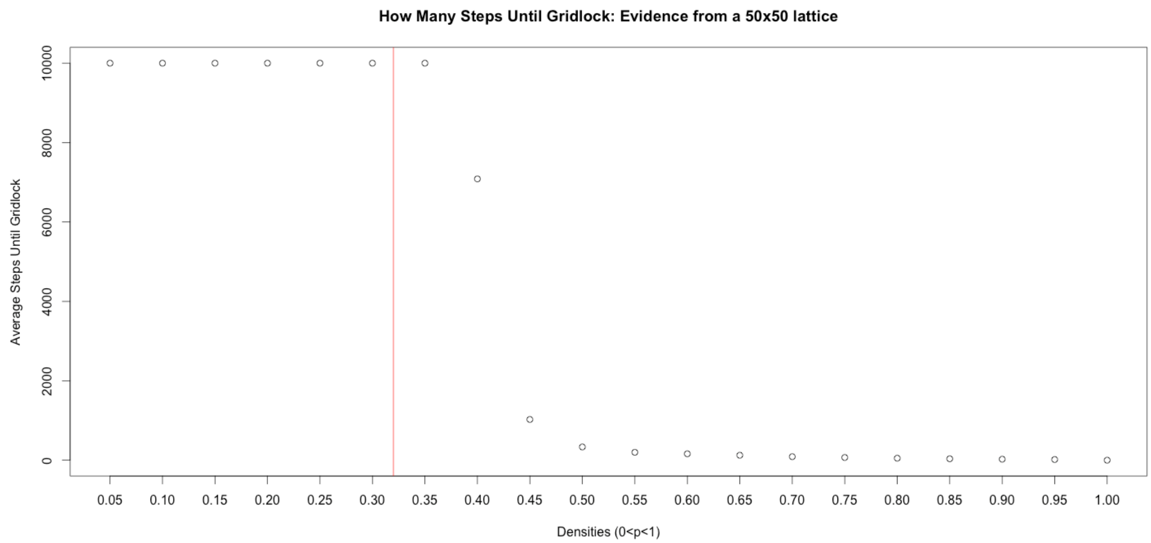
3. Does the transition depend on the size or shape of the grid?

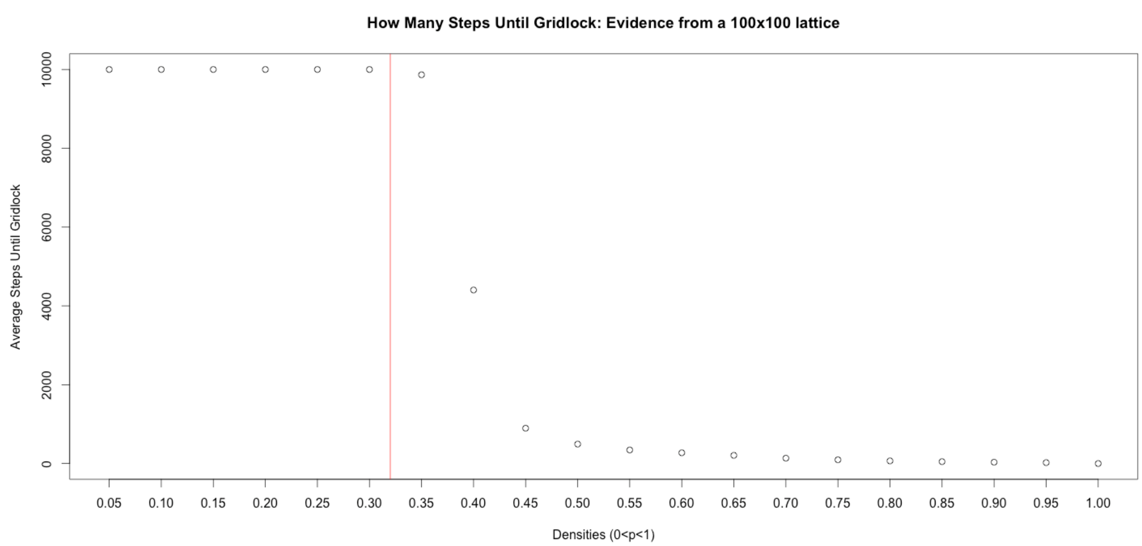
For this last part of the analysis, we are interested in the transition from free-flow to gridlock sate on the different sizes of both the square and non-square matrices. We found that the most insightful way of doing this was to compare the increase of the density with the average number of steps until the lattice hits the gridlock state. After reviewing the first two sections of this analysis, it is evident that we would expect a pattern of convergence to 0 on the average number of steps as the grid density increases, which does happen for the studies Square Lattices. Moreover, we can see that the present simulation significantly resembles the original BML Simulation Study, where the first density where the lattices began reaching gridlock was around .32. We reference this point in the plots, just as a matter of reference.

This being said, it is also important to note how there is no distinguishable pattern of convergence when talking about the non-square lattices, just like what was expected after analyzing their behavior in the previous sections.

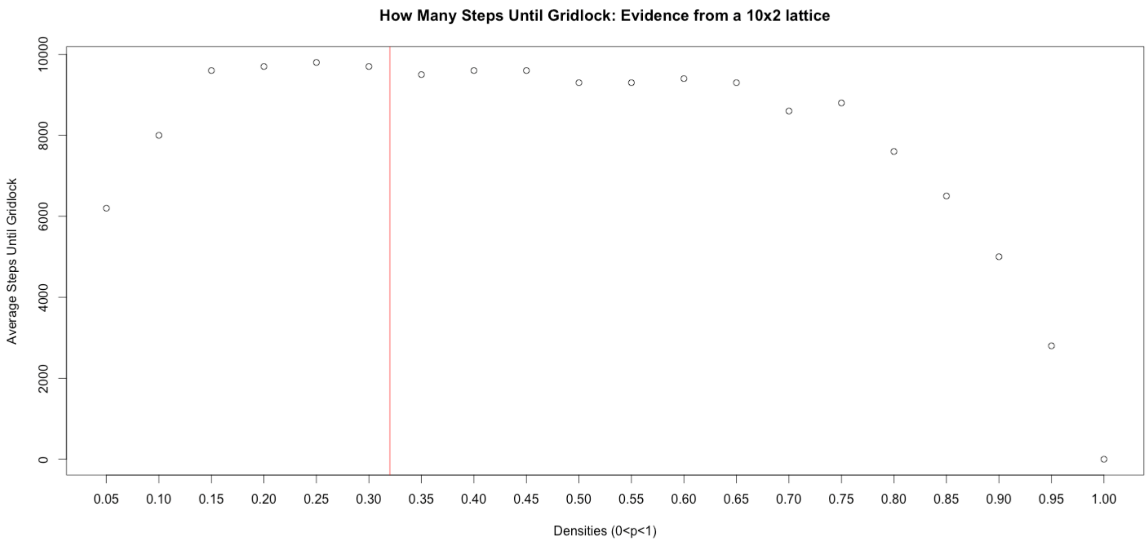
Squares:



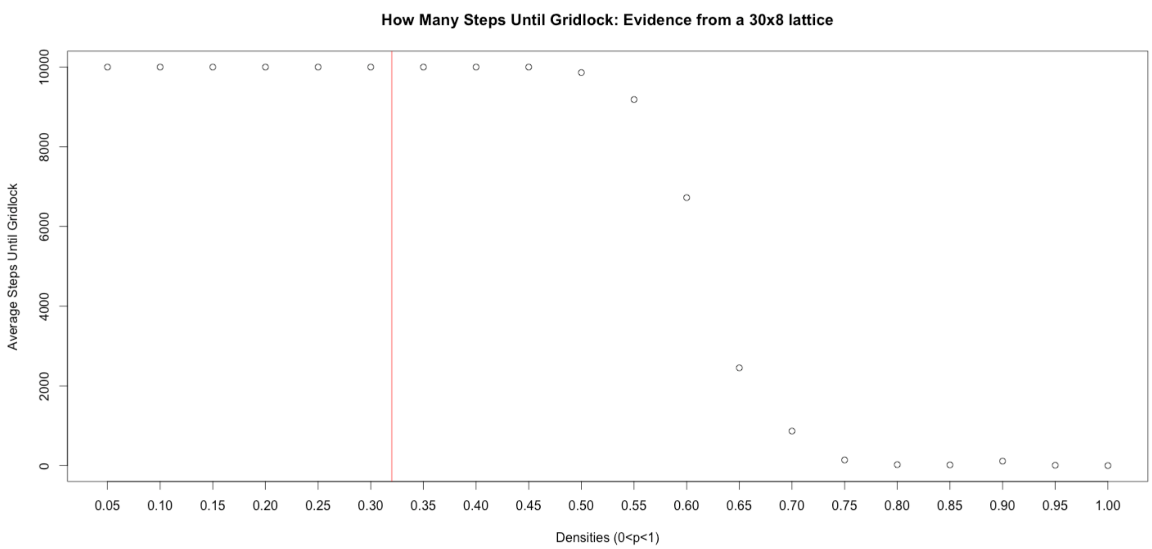

How Many Steps Until Gridlock: Evidence from a 50x50 lattice
Average Steps Until Gridlock
0
2000
4000
6000
8000
10000
0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85 0.90 0.95 1.00
Densities (0<p<1)


How Many Steps Until Gridlock: Evidence from a 100x100 lattice
Average Steps Until Gridlock
0
2000
4000
6000
8000
10000
0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85 0.90 0.95 1.00
Densities (0<p<1)

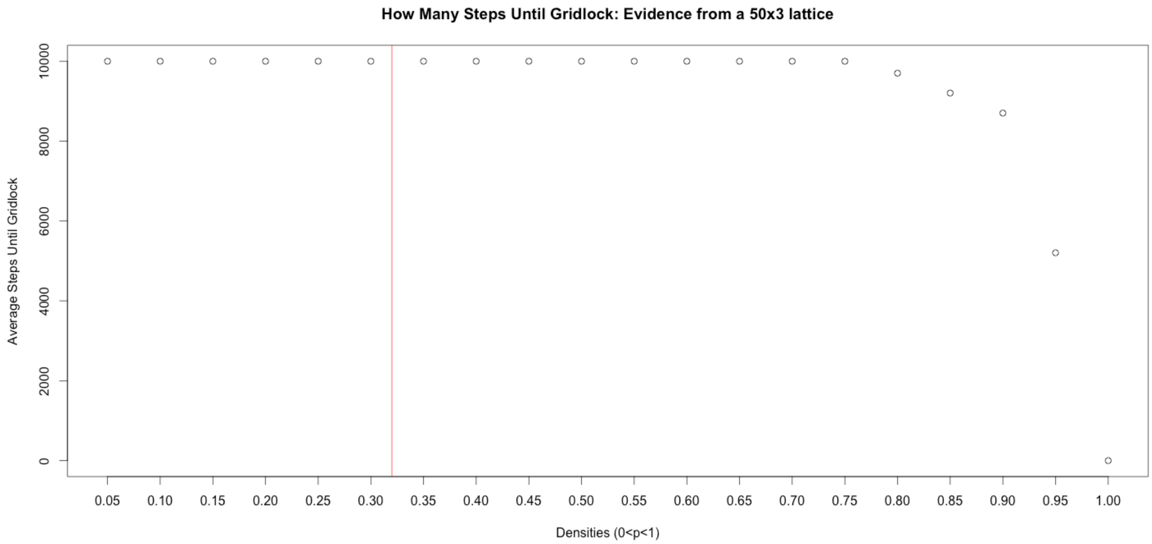
Non-Squares:



How Many Steps Until Gridlock: Evidence from a 10x2 lattice
Average Steps Until Gridlock
0
2000
4000
6000
8000
10000
0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85 0.90 0.95 1.00
Densities (0<p<1)



How Many Steps Until Gridlock: Evidence from a 30x8 lattice
Average Steps Until Gridlock
0
2000
4000
6000
8000
10000
0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85 0.90 0.95 1.00
Densities (0<p<1)



After reviewing the scatter plots above, we go into more detail by quantitatively presenting the average steps when gridlock was reached for both the square and non-square lattices. This gives us a far more accurate picture of the densities that begin to have a significant impact in the movement of the cars through the lattice. We can confirm what he had previously explained: there is a somewhat consistent density for which the gridlock being reaching gridlock, and can be found around .35 to 4, whereas there is no pattern or specific density for the non-square lattices (10x2 always reaches gridlock, the 30x8 begins at .5, and the 50x3 at .8).

	Densities	10x10	50x50	100x100
1	0.05	9900.00	10000.00	10000.00
2	0.10	10000.00	10000.00	10000.00
3	0.15	10000.00	10000.00	10000.00
4	0.20	10000.00	10000.00	10000.00
5	0.25	10000.00	10000.00	10000.00
6	0.30	10000.00	10000.00	10000.00
7	0.35	9900.58	10000.00	9865.31
8	0.40	9683.93	7085.31	4403.71
9	0.45	7081.81	1026.05	895.70
10	0.50	5118.63	333.23	492.85
11	0.55	2436.15	197.90	343.84
12	0.60	543.66	161.19	272.25
13	0.65	42.13	124.21	207.39
14	0.70	124.81	88.36	135.21
15	0.75	21.01	67.32	94.79
16	0.80	14.82	49.56	66.64
17	0.85	12.18	35.62	47.58
18	0.90	8.70	24.90	32.90
19	0.95	5.72	16.92	22.82
20	1.00	0.00	0.00	0.00

	Densities	10x2	30x8	50x3
1	0.05	6200.08	10000.00	10000.00
2	0.10	8000.04	10000.00	10000.00
3	0.15	9600.05	10000.00	10000.00
4	0.20	9700.16	10000.00	10000.00
5	0.25	9800.12	10000.00	10000.00
6	0.30	9700.11	10000.00	10000.00
7	0.35	9500.29	10000.00	10000.00
8	0.40	9600.25	10000.00	10000.00
9	0.45	9600.25	10000.00	10000.00
10	0.50	9300.38	9861.27	10000.00
11	0.55	9300.26	9184.82	10000.00
12	0.60	9400.39	6724.54	10000.00
13	0.65	9300.41	2452.62	10000.00
14	0.70	8600.72	865.58	10000.00
15	0.75	8800.49	141.28	10000.00
16	0.80	7601.17	23.47	9700.83
17	0.85	6501.53	17.10	9203.89
18	0.90	5001.60	112.15	8702.22
19	0.95	2800.76	8.24	5204.02
20	1.00	0.00	0.00	0.00