Robust Linearly Constrained Filtering and Smoothing: Results and Applications

NAVIgation, Radar and REmote Sensing (NAVIR^{2}ES) Research Group

Institut Supérieur de l'Aéronautique et de l'Espace (ISAE-SUPAERO)

Séminaire SiMul, CRAN, Nancy, 21/10/2021

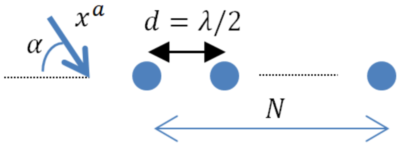

Figure 1: Uniform linear array with a random signal source x^a

Typical model

$$x_k^a = f_{k-1} x_{k-1}^a + w_{k-1}, \quad \boldsymbol{h}_k(d,\alpha) = (1,\ldots,e^{j2\pi \frac{(N-1)d\sin(\alpha)}{\lambda}})^T$$
$$\boldsymbol{y}_k = \boldsymbol{h}_k(\hat{d},\alpha) x_k^a + \boldsymbol{v}_k, \quad \boldsymbol{C}_{\boldsymbol{v_l},\boldsymbol{v_k}} = \boldsymbol{\delta}_k^l \mathrm{I}$$

Potential mismatches

- Miscalibration of the array: $d = \hat{d} + \delta_d$
- A crosstalk with another source x^b, $\boldsymbol{x}_k = \begin{bmatrix} x^a \\ x^b \end{bmatrix}_k = \begin{bmatrix} 1 & d\phi_k \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x^a \\ x^b \end{bmatrix}_{k-1}$, where $d\phi_k$ is unknown. This induces a loss of coherence of x^a.
- An intermittent jammer is located at a broadside angle α_J:

$$\boldsymbol{y}_k = \boldsymbol{h}_k(\hat{d},\alpha) x_k^a + \boldsymbol{h}_k(\hat{d},\alpha_J) x_k^{Jam} + \boldsymbol{v}_k$$

The general misspecified state-space model (SSM) set: nominal SSM $\mathcal{M}_N$, assumed SSM $\mathcal{M}_A$ and true SSM $\mathcal{M}_T$:

$$\mathcal{M}_N : \begin{cases} \boldsymbol{x}_t = \boldsymbol{f}_{t-1}(\boldsymbol{x}_{t-1}, \boldsymbol{u}_{x,t-1}, \boldsymbol{w}_{t-1}, \boldsymbol{\Phi}_{\boldsymbol{f}_{t-1}}), & \boldsymbol{w}_{t-1} \sim p(\boldsymbol{w}_{t-1}|\boldsymbol{\Psi}_{\boldsymbol{w}_{t-1}}). \\ \boldsymbol{y}_t = \boldsymbol{h}_t(\boldsymbol{x}_t, \boldsymbol{u}_{y,t}, \boldsymbol{v}_t, \boldsymbol{\Phi}_{\boldsymbol{h}_t}), & \boldsymbol{v}_t \sim p(\boldsymbol{v}_t|\boldsymbol{\Psi}_{\boldsymbol{v}_t}), \end{cases} \tag{1a}$$

$$\mathcal{M}_A : \begin{cases} \boldsymbol{x}'_t = \widehat{\boldsymbol{f}}_{t-1}(\boldsymbol{x}'_{t-1}, \widehat{\boldsymbol{u}}_{x,t-1}, \boldsymbol{w}'_{t-1}, \widehat{\boldsymbol{\Phi}}_{\boldsymbol{f}_{t-1}}), & \boldsymbol{w}'_{t-1} \sim \widehat{p}(\boldsymbol{w}'_{t-1}|\widehat{\boldsymbol{\Psi}}_{\boldsymbol{w}'_{t-1}}), \\ \boldsymbol{y}_t = \widehat{\boldsymbol{h}}_t(\boldsymbol{x}'_t, \widehat{\boldsymbol{u}}_{y,t}, \boldsymbol{v}'_t, \widehat{\boldsymbol{\Phi}}_{\boldsymbol{h}_t}), & \boldsymbol{v}'_t \sim \widehat{p}(\boldsymbol{v}'_t|\widehat{\boldsymbol{\Psi}}_{\boldsymbol{v}'_t}), \end{cases} \tag{1b}$$

$$\mathcal{M}_T : \begin{cases} \boldsymbol{x}''_t = \boldsymbol{f}_{t-1}(\boldsymbol{x}''_{t-1}, \boldsymbol{u}_{x,t-1}, \boldsymbol{w}_{t-1}, \boldsymbol{\Phi}_{\boldsymbol{f}_{t-1}}) + \boldsymbol{\eta}_{k-1}, \\ \boldsymbol{y}_t = \boldsymbol{h}_t(\boldsymbol{x}''_t, \boldsymbol{u}_{y,t}, \boldsymbol{v}_t, \boldsymbol{\Phi}_{\boldsymbol{h}_t}) + \boldsymbol{j}_k, \end{cases} \tag{1c}$$

The (too general) Problem

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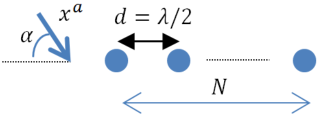
We can distinguish four general scenarios of interest:

1) $\{\mathcal{M}_T = \mathcal{M}_N = \mathcal{M}_A\}$ – no mismatch, no nuisance signals $\rightarrow$ standard filtering solutions
2) $\{\mathcal{M}_T = \mathcal{M}_N\}$ – a possible system mismatch but no nuisance signals
3) $\{\mathcal{M}_A = \mathcal{M}_N\}$ – the mismatch arises only because of nuisance signals
4) $\{\mathcal{M}_T \neq \mathcal{M}_N \neq \mathcal{M}_A\}$ – general misspecified case

Ultimate goal: estimating the nominal state x_t (1a) based on the (true) measurements y_t (1c) and the knowledge of (1b).



$$x_k^a = f_{k-1}x_{k-1}^a + w_{k-1}, \quad \boldsymbol{h}_k(d,\alpha) = (1,\ldots,e^{j2\pi\frac{(N-1)d\sin(\alpha)}{\lambda}})^T$$
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- Miscalibration of the array: $d = \hat{d} + \delta_d$
  $\{\mathcal{M}_T = \mathcal{M}_N \neq \mathcal{M}_A$ (measurement model mismatch)$\}$
- A crosstalk with another source x^b, $\boldsymbol{x}_k = \begin{bmatrix} x^a \\ x^b \end{bmatrix}_k = \begin{bmatrix} 1 & d\phi_k \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x^a \\ x^b \end{bmatrix}_{k-1}$, where $d\phi_k$ is unknown. This induces a loss of coherence of x^a.
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$\{\mathcal{M}_T \neq \mathcal{M}_N = \mathcal{M}_A$ (measurement model mismatch)$\}$

What is the impact of a possible model mismatch ?

$$\mathcal{M}_N : \begin{cases} \mathbf{x}_k = \mathbf{F}_{k-1}\mathbf{x}_{k-1} + \mathbf{m}_{\mathbf{w}_{k-1}} + \mathbf{w}_{k-1} \\ \mathbf{y}_k = \mathbf{H}_k\mathbf{x}_k + \mathbf{m}_{\mathbf{v}_k} + \mathbf{v}_k \end{cases} \; ; \; \mathcal{M}_A : \begin{cases} \mathbf{x}'_k = \widehat{\mathbf{F}}_{k-1}\mathbf{x}'_{k-1} + \widehat{\mathbf{m}}_{\mathbf{w}_{k-1}} + \mathbf{w}_{k-1} \\ \mathbf{y}_k = \widehat{\mathbf{H}}_k\mathbf{x}'_k + \widehat{\mathbf{m}}_{\mathbf{v}_k} + \mathbf{v}_k \end{cases}$$

$$\mathbf{F}_{k-1} = \widehat{\mathbf{F}}_{k-1} + d\mathbf{F}_{k-1} \; ; \; \mathbf{H}_k = \widehat{\mathbf{H}}_k + d\mathbf{H}_k \; ; \; \mathbf{m}_{\mathbf{w}_{k-1}} = \widehat{\mathbf{m}}_{\mathbf{w}_{k-1}} + d\mathbf{m}_{\mathbf{w}_{k-1}} \; ; \; \mathbf{m}_{\mathbf{v}_k} = \widehat{\mathbf{m}}_{\mathbf{v}_k} + d\mathbf{m}_{\mathbf{v}_k}$$

Mismatched estimation error:

$$\widehat{\mathbf{x}}_{k|k}(\mathbf{L}_k) - \mathbf{x}_k = (\mathbf{I} - \mathbf{L}_k\widehat{\mathbf{H}}_k)(\widehat{\mathbf{F}}_{k-1}(\widehat{\mathbf{x}}^b_{k-1|k-1} - \mathbf{x}_{k-1}) - \mathbf{w}_{k-1}) + \mathbf{L}_k\mathbf{v}_k + \boldsymbol{\varepsilon}_k(\mathbf{L}_k),$$

where $\boldsymbol{\varepsilon}_k(\mathbf{L}_k)$ is

$$\boldsymbol{\varepsilon}_k(\mathbf{L}_k) = \mathbf{L}_k d\mathbf{H}_k\mathbf{x}_k + \mathbf{L}_k d\mathbf{m}_{\mathbf{v}_k} - \left(\mathbf{I} - \mathbf{L}_k\widehat{\mathbf{H}}_k\right)\left(d\mathbf{F}_{k-1}\mathbf{x}_{k-1} + d\mathbf{m}_{\mathbf{w}_{k-1}}\right).$$

Can we have $\boldsymbol{\varepsilon}_k(\mathbf{L}_k) = 0$?

$$\begin{aligned} \mathbf{L}_k\left[d\mathbf{H}_k \; d\mathbf{m}_{\mathbf{v}_k}\right] &= \mathbf{L}_k d\mathbb{H}_k = 0 \\ \left(\mathbf{I} - \mathbf{L}_k\widehat{\mathbf{H}}_k\right)\left[d\mathbf{F}_{k-1} \; d\mathbf{m}_{\mathbf{w}_{k-1}}\right] = \left(\mathbf{I} - \mathbf{L}_k\widehat{\mathbf{H}}_k\right) d\mathbb{F}_{k-1} &= 0 \end{aligned}$$

Can we do that in the KF framework, to consistently track the MSE ?

Linear discrete SSM:

$\boldsymbol{x}_k = \boldsymbol{F}_{k-1}\boldsymbol{x}_{k-1} + \boldsymbol{w}_{k-1}$; $\boldsymbol{y}_k = \boldsymbol{H}_k\boldsymbol{x}_k + \boldsymbol{v}_k$ where $\boldsymbol{x}_k \in \mathbb{C}^{n_x}$, $\boldsymbol{y}_k \in \mathbb{C}^{n_y}$ (for $k \geq 1$); $\boldsymbol{F}_{k-1}$ and $\boldsymbol{H}_k$ system matrices; $\boldsymbol{w}_k$, $\boldsymbol{v}_k$, zero mean with covariance $\boldsymbol{C}_{\mathrm{w}_k}$ and $\boldsymbol{C}_{\mathrm{v}_k}$

Idea: the KF is the recursive form of the Wiener filter (under some uncorrelation conditions). Which is the recursive form of the linearly constrained WF?

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Standard Kalman gain

$$\mathbf{K}_k^b = \arg\min_{\mathbf{K}_k} \left\{ \mathbf{P}_{k|k}\left(\mathbf{K}_k\right) \right\}$$

$$\mathbf{P}_{k|k}\left(\mathbf{K}_k\right) = \mathbb{E}\left[\left(\widehat{\mathbf{x}}_{k|k}\left(\mathbf{K}_k\right) - \mathbf{x}_k \right) (\cdot)^H \right]$$

Standard KF

$$\begin{aligned}
\widehat{\mathbf{x}}_{k|k-1}^b &= \mathbf{F}_{k-1}\widehat{\mathbf{x}}_{k-1|k-1}^b \\
\mathbf{P}_{k|k-1}^b &= \mathbf{F}_{k-1}\mathbf{P}_{k-1|k-1}^b\mathbf{F}_{k-1}^H + \mathbf{C}_{\mathbf{w}_{k-1}} \\
\mathbf{S}_{k|k-1}^b &= \mathbf{H}_k\mathbf{P}_{k|k-1}^b\mathbf{H}_k^H + \mathbf{C}_{\mathbf{v}_k} \\
\mathbf{K}_k^b &= \mathbf{P}_{k|k-1}^b\mathbf{H}_k^H\left(\mathbf{S}_{k|k-1}^b\right)^{-1} \\
\widehat{\mathbf{x}}_{k|k}^b &= \widehat{\mathbf{x}}_{k|k-1}^b + \mathbf{K}_k^b\left(\mathbf{y}_k - \mathbf{H}_k\widehat{\mathbf{x}}_{k|k-1}^b\right) \\
\mathbf{P}_{k|k}^b &= \left(\mathbf{I} - \mathbf{K}_k^b\mathbf{H}_k\right)\mathbf{P}_{k|k-1}^b
\end{aligned}$$

Wiener, Kalman and Linearly Constrained KF

Linear discrete SSM:

$\boldsymbol{x}_k = \boldsymbol{F}_{k-1}\boldsymbol{x}_{k-1} + \boldsymbol{w}_{k-1}$; $\boldsymbol{y}_k = \boldsymbol{H}_k\boldsymbol{x}_k + \boldsymbol{v}_k$ where $\boldsymbol{x}_k \in \mathbb{C}^{n_x}$, $\boldsymbol{y}_k \in \mathbb{C}^{n_y}$ (for $k \geq 1$);
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\boldsymbol{P}_{k|k}^b &= \left(\boldsymbol{I} - \boldsymbol{K}_k^b\boldsymbol{H}_k\right)\boldsymbol{P}_{k|k-1}^b
\end{aligned}$$

Constrained gain

$$\boldsymbol{L}_k^b = \arg\min_{\boldsymbol{L}_k} \left\{ \boldsymbol{P}_{k|k}\left(\boldsymbol{L}_k\right) \right\} \text{ s.t. } \boldsymbol{L}_k\boldsymbol{\Delta}_k = \boldsymbol{T}_k,$$

LCKF

$$\begin{aligned}
\widehat{\boldsymbol{x}}_{k|k-1}^b &= \boldsymbol{F}_{k-1}\widehat{\boldsymbol{x}}_{k-1|k-1}^b \\
\boldsymbol{P}_{k|k-1}^b &= \boldsymbol{F}_{k-1}\boldsymbol{P}_{k-1|k-1}^b\boldsymbol{F}_{k-1}^H + \boldsymbol{C}_{\boldsymbol{w}_{k-1}} \\
\boldsymbol{S}_{k|k-1}^b &= \boldsymbol{H}_k\boldsymbol{P}_{k|k-1}^b\boldsymbol{H}_k^H + \boldsymbol{C}_{\boldsymbol{v}_k} \\
\boldsymbol{K}_k^b &= \boldsymbol{P}_{k|k-1}^b\boldsymbol{H}_k^H\left(\boldsymbol{S}_{k|k-1}^b\right)^{-1} \\
\boldsymbol{\Gamma}_k &= \boldsymbol{T}_k - \boldsymbol{K}_k\boldsymbol{\Delta}_k, \quad \boldsymbol{\Psi}_k = \boldsymbol{\Delta}_k^H(\boldsymbol{S}_{k|k-1}^b)^{-1}\boldsymbol{\Delta}_k \\
\boldsymbol{L}_k^b &= \boldsymbol{K}_k + \boldsymbol{\Gamma}_k\boldsymbol{\Psi}_k^{-1}\boldsymbol{\Delta}_k^H\left(\boldsymbol{S}_{k|k-1}^b\right)^{-1} \\
\widehat{\boldsymbol{x}}_{k|k}^b &= \widehat{\boldsymbol{x}}_{k|k-1}^b + \boldsymbol{L}_k^b\left(\boldsymbol{y}_k - \boldsymbol{H}_k\widehat{\boldsymbol{x}}_{k|k-1}^b\right) \\
\boldsymbol{P}_{k|k}^b &= \left(\boldsymbol{I} - \boldsymbol{K}_k\boldsymbol{H}_k\right)\boldsymbol{P}_{k|k-1}^b + \boldsymbol{\Gamma}_k\boldsymbol{\Psi}_k^{-1}\boldsymbol{\Gamma}_k^H
\end{aligned}$$

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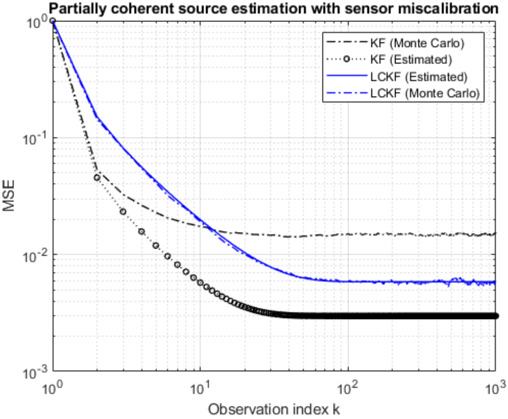


Measurement model mismatch. True inter-sensor distance is $d = 0.98\hat{d}$. LCKF stays consistent.

J. Vilà-Valls et al., Recursive Linearly Constrained Wiener Filter for Robust Multi-Channel Signal Processing, Signal Processing, 2020

Timeline

MVDR E. Chaumette et al., "Minimum Variance Distortionless Response Estimators For Linear Discrete State-Space Models", IEEE Trans. Automatic Control, 2017.

MVDR E. Chaumette et al., "On LMVDR Estimators For LDSS Models: Conditions for Existence and Further Applications", IEEE Trans. Automatic Control, 2019.

MVE F. Vincent and E. Chaumette, "Recursive Linearly Constrained Minimum Variance Estimator in Linear Models with Non-Stationary Constraints", Signal Processing, 2018.

LCKF E. Chaumette, F. Vincent, J. Vilà-Valls, "Linearly Constrained Wiener Filter Estimates For Linear Discrete State-Space Models", in Proc. of the IEEE Asilomar, 2018.

LCKF J. Vilà-Valls et al., "Recursive Linearly Constrained Wiener Filter for Robust Multi-Channel Signal Processing", Signal Processing, 2020.

LC-EKF E. Hrustic et al., "Robust Linearly Constrained EKF for Mismatched Nonlinear Systems", International Journal of Robust and Nonlinear Control, 2021.

LC-InEKF P. Chauchat et al., "Robust Linearly Constrained Invariant Filtering for a Class of Mismatched Nonlinear Systems", IEEE Control Systems Letters, 2021.

LC-IF P. Chauchat et al., "Robust Information Filtering under Model Mismatch for Large-Scale Dynamic Systems", IEEE Control Systems Letters, 2021.

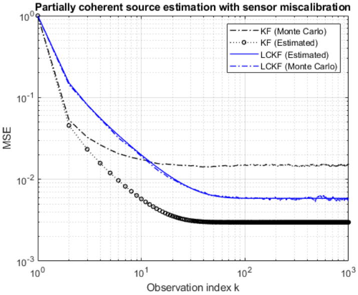
LC-RTS P. Chauchat et al., "Robust Kalman Smoothers with Linear Equality Constraints", IEEE Control Systems Letters, IEEE Control Systems Letters, 2022.

Robust SLAM R. Ben Abdallah et al., "Robust Filter-Based Visual Navigation Solution with Miscalibrated Bi-monocular or Stereo Cameras," IEEE Robotics and Automation Letters, under review.

Robust JPA P. Chauchat et al., "Robust Linearly Constrained Filtering for GNSS Position and Attitude Estimation under Antenna Baseline Mismatch", in the Proc. of the International Conf. on Information Fusion, Nov. 2021.

Asymp. Perf. P. Chauchat et al., "On the Asymptotic Behavior of Linearly Constrained Filters for Robust Multi-Channel Signal Processing," Signal Processing, under review.

LC-SRCKF R. Ben Abdallah et al., "Robust Linearly Constrained Square-Root Cubature Kalman Filter for Mismatched Nonlinear Dynamic Systems," IEEE Control Systems Letters, under review.



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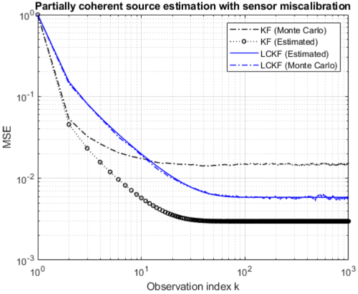

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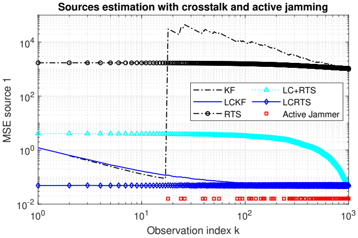
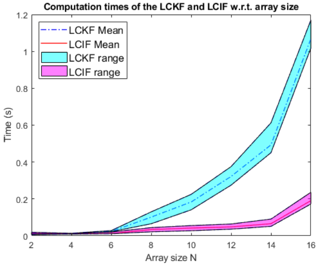


Dual source crosstalk and jamming. $d\phi_k \sim \mathcal{U}(0.8, 1.6) \times 10^{-4}$ and $\widehat{d\phi_k} = 10^{-4}$. $\alpha_J = \alpha - \alpha_{3dB}$, the jammer to noise power is 60 dB with probability of activation of 0.1. Only the LCKF and LCRTS are able to mitigate them.

J. Vilà-Valls et al., Recursive Linearly Constrained Wiener Filter for Robust Multi-Channel Signal Processing, Signal Processing, 2020
P. Chauchat et al., "Robust Kalman Smoothers with Linear Equality Constraints", IEEE Control Systems Letters, under review.

$\boldsymbol{y}_k = \boldsymbol{h}_k(d_x, d_y, \boldsymbol{a})x + \boldsymbol{v}_k$ and array miscalibration $d_x = 0.99\hat{d}_x, d_y = 1.01\hat{d}_y$.



Computation time comparison between the LC-KF and the LC-IF with respect to the number of sensors.

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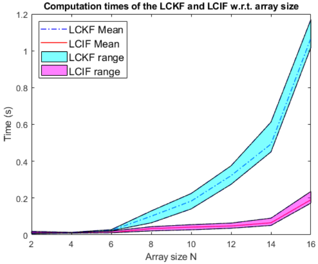

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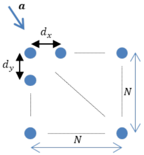
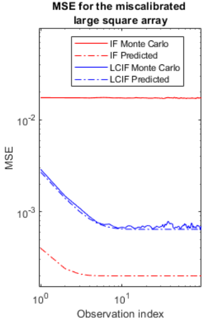




Left: illustration, each dot is a sensor. Right: MSE of IF and LC-IF for $N = 50$.

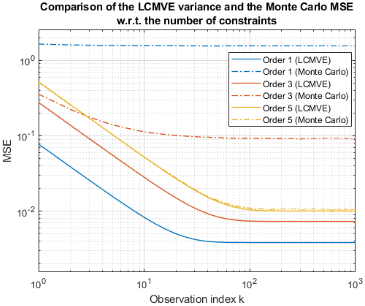
P. Chauchat et al., "Robust Information Filtering under Model Mismatch for Large-Scale Dynamic Systems", IEEE Control Systems Letters, 2021.

Increasing the consistency for nonlinear measurement mismatches

Consider a mismatched observation model depending on a scalar parameter $\boldsymbol{H}_k = \boldsymbol{H}_k(\theta_k)$. Let $\widehat{\theta}_k$ be the assumed value of this parameter,

$$\boldsymbol{H}_k(\theta_k) = \boldsymbol{H}_k(\widehat{\theta}_k) + \sum_m \frac{1}{m!} \left. \frac{\partial^m \boldsymbol{H}_k}{\partial \theta^m} \right|_{\widehat{\theta}_k} (\theta_k - \widehat{\theta}_k)^m. \quad (2)$$

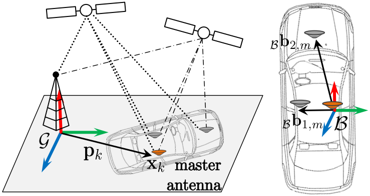
$$\boldsymbol{L}_k \left[\left. \frac{\partial^m \boldsymbol{H}_k}{\partial \theta^m} \right|_{\widehat{\theta}_k} \right]_{m \geq 0} = 0$$



LCKF consistency w.r.t. Taylor expansion, for a 20% mismatch.

Objective: estimate orientation and position (and ambiguities):

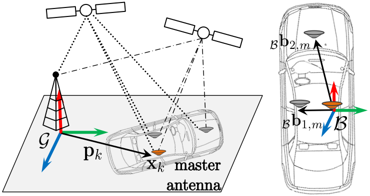
$$\boldsymbol{x}_k = (\boldsymbol{q}_k, \boldsymbol{a}_k, \boldsymbol{p}_k) \in \mathcal{S}^3 \times \mathbb{Z}^M \times \mathbb{R}^3$$



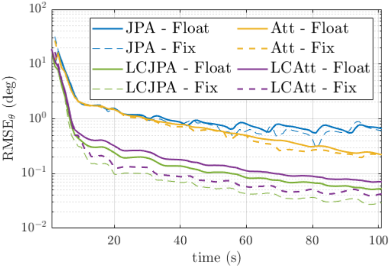
GNSS-based JPA problem: the base station, the rover with multiple antennas. On the right, the configuration of the sensors on the body frame. Master antenna in orange.

Objective: estimate orientation and position (and ambiguities):

$$\boldsymbol{x}_k = (\boldsymbol{q}_k, \boldsymbol{a}_k, \boldsymbol{p}_k) \in \mathcal{S}^3 \times \mathbb{Z}^M \times \mathbb{R}^3$$



GNSS-based JPA problem: the base station, the rover with multiple antennas. On the right, the configuration of the sensors on the body frame. Master antenna in orange.



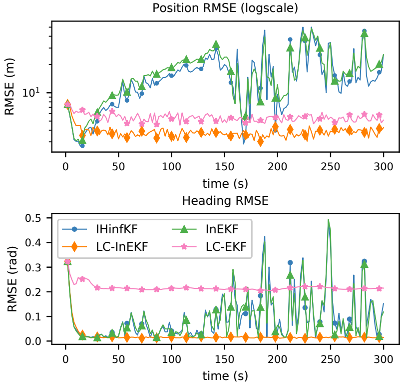
Attitude RMSE for mismatched master-slave baseline lengths (5%).

P. Chauchat et al., "Robust Linearly Constrained Filtering for GNSS Position and Attitude Estimation under Antenna Baseline Mismatch", in the Proc. of the International Conf. on Information Fusion, Nov. 2021.

Filter on the Lie group SE_2: $\chi \equiv \begin{bmatrix} \boldsymbol{R} & \boldsymbol{x} \\ 0 & \boldsymbol{I} \end{bmatrix}$

2D navigation problem: orientation θ and position $\boldsymbol{x}$ of a mobile system are to be estimated.
Sensors give access to its odometry and position measurements.

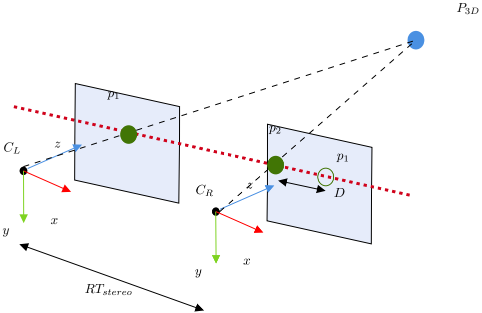
Mismatch: scale and frame mismatch for the outputs, and scale mismatch for the inputs (sleepy wheel).



P. Chauchat et al., "Robust Linearly Constrained Invariant Filtering for a Class of Mismatched Nonlinear Systems", IEEE Control Systems Letters, 2021.

The state $\boldsymbol{x}_k$ used for a standard EKF SLAM/odometry approach contain the left camera pose $\boldsymbol{p}$ and orientation $\boldsymbol{R}$, the velocity $\boldsymbol{v}$ and the list of landmarks (3D points clouds), $\boldsymbol{x}_k = \left(\boldsymbol{p}, \boldsymbol{R}, \boldsymbol{v}, \boldsymbol{p}_{3D_1}, \ldots, \boldsymbol{p}_{3D_N}\right)$.

Problem: you need the extrinsic **calibration** between left and right cameras.

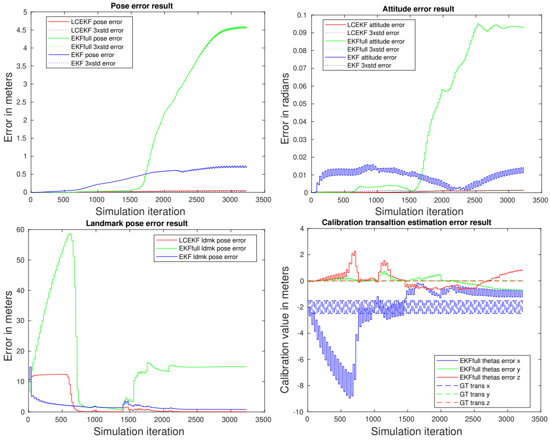


Stereo vision observation model. A 3D point is reprojected in both cameras.

The LCEKF at least equal to the EKF with online calibration. If the vibrations are fast and strong enough, the LCEKF is better.

R. Ben Abdallah et al., "Robust Filter-Based Visual Navigation Solution with Miscalibrated Bi-monocular or Stereo Cameras," IEEE Robotics and Automation Letters, under review.

SLAM with high amplitude periodic noise



R. Ben Abdallah et al., "Robust Filter-Based Visual Navigation Solution with Miscalibrated Bi-monocular or Stereo Cameras," IEEE Robotics and Automation Letters, under review.

- Asymptotical behavior
    - We have results for measurement mismatch only
- Other applications ?

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