

PowerUpR Vignettes

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Problem

Institute of Education Science (IES) reported that evidence on effectiveness of Saxon Math curriculum on secondary school student's algebra subject is lacking and further research is needed (U.S. Department of Education, 2016). To evaluate the effectiveness Saxon Math curriculum, assume a 3-Level cluster randomized trial is considered where schools are randomly assigned to treatment (Saxon Math curriculum) and a control condition. In the grant proposal number of schools, teachers, and students, and the cost associated with data collection from this sample needs to be justified.

Consider the following hypothetical design parameters

- Proportion of variance in algebra posttest at the school level (σ_3^2): .15
- Proportion of variance in algebra posttest at the classroom level (σ_2^2): .05
- Proportion of variance in algebra posttest at the student level (σ_1^2): .65
- Proportion of variance in algebra posttest explained by school mean algebra pretest: (R_3^2): .45
- Proportion of variance in algebra posttest explained by classroom mean algebra pretest: (R_2^2): .50
- Proportion of variance in algebra posttest explained by student algebra pretest: (R_1^2): .55

Then, intraclass correlation coefficients are

- $\rho_2 = \sigma_2^2 / (\sigma_1^2 + \sigma_2^2 + \sigma_3^2) = .05 / (.65 + .15 + .05) = 0.059$
- $\rho_3 = \sigma_3^2 / (\sigma_1^2 + \sigma_2^2 + \sigma_3^2) = .15 / (.65 + .15 + .05) = 0.176$

Further define

- Alpha level (α): 0.05
- Proportion of schools allocated to treatment (P): .40

Scenario 1: Find MDES

Further assumptions

- Number of schools (K): 83
- Average number of classrooms per school (J): 2
- Average number of students per classroom (n): 10
- Statistical power ($1 - \beta$): .80

Given design parameters and further assumptions, what is minimum detectable effect size?

```
library(PowerUpR)
```

```
design1 <- mdes.cra3r3(power=.80, rho2=.06, rho3=.18,
                      g3=1, R12=.55, R22=.50, R32=.45,
                      P=.40, n=10, J=2, K=83)
print(design1$mdes)
```

```
##      mdes 95% LCL 95% UCL
## [1,] 0.23   0.069  0.392
```

Scenario 2: Find Statistical Power

Further assumptions

- Number of schools (K): 60
- Average number of classrooms per school (J): 2
- Average number of students per classroom (n): 10
- Minimum detectable effect size (MDES): .232

Given design parameters and further assumptions, what is statistical power?

```
design2 <- power.cra3r3(mdes=.23, rho2=.06, rho3=.18,
                       g3=1, R12=.55, R22=.50, R32=.45,
                       P=.40, n=10, J=2, K=83)
print(design2$power)
```

```
## [1] 0.8
```

Scenario 3: Find MRSS for Schools (K)

Further assumptions

- Average number of classrooms per school (J): 2
- Average number of students per classroom (n): 10
- Statistical power ($1 - \beta$): .80
- Minimum detectable effect size (MDES): .232

Given design parameters and further assumptions, how many schools should be recruited?

```
design3 <- mrss.cra3r3(power=.80, mdes=.23, rho2=.06, rho3=.18,
                      g3=1, R12=.55, R22=.50, R32=.45,
                      P=.40, n=10, J=2)
print(design3$round.mrss)
```

```
##      n J K mdes power
## [1,] 10 2 83 0.23  0.8
```

Scenario 4: Find MRSS for Classrooms (J)

Further assumptions

- Number of schools (K): 60
- Average number of students per classroom (n): 10
- Statistical power ($1 - \beta$): .80
- Minimum detectable effect size (MDES): .232

Given design parameters and further assumptions, how many classrooms should be recruited per school on average?

```
design4 <- mrss.cra3r3(power=.80, mdes=.23, rho2=.06, rho3=.18,
                      g3=1, R12=.55, R22=.50, R32=.45,
                      P=.40, n=10, K=83)
print(design4$round.mrss)
```

```
##          n J  K mdes power
## [1,] 10 2 83 0.23  0.8
```

Scenario 5: Find MRSS for Students (n)

Further assumptions

- Number of schools (K): 60
- Average number of classrooms per school (J): 2
- Statistical power ($1 - \beta$): .80
- Minimum detectable effect size (MDES): .232

Given design parameters and further assumptions, how many students should be recruited per classroom on average?

```
design5 <- mrss.cra3r3(power=.80, mdes=.23, rho2=.06, rho3=.18,
                      g3=1, R12=.55, R22=.50, R32=.45,
                      P=.40, J=2, K=83)
print(design5$round.mrss)
```

```
##          n J  K mdes power
## [1,] 10 2 83 0.23  0.8
```

Scenario 6: Find Optimal Sample (Constrain Power)

Further assumptions

- Statistical power ($1 - \beta$): .80
- Minimum detectable effect size (MDES): .232
- Marginal cost per student (cn): 10
- Marginal cost per classroom (cJ): 200

- Marginal cost per school (cK): 500

Given design parameters and further assumptions, how many schools, classrooms and students should be recruited to achieve 80% power while minimizing the total cost?

```
design6 <- optimal.cra3r3(cn=10, cJ=200, cK=500, constrain="power", power=.80,
                        mdes=.23, rho2=.06, rho3=.18,
                        g3=1, R12=.55, R22=.50, R32=.45,
                        P=.40)
print(design6$round.optim)
```

```
##          n J K cost mdes power
## [1,] 13 1 97 80510 0.231 0.796
```

```
print(design6$integer.optim)
```

```
##          n J K cost mdes power
## [1,] 13 1 98 81340 0.230 0.800
## [2,] 16 1 95 81700 0.230 0.800
## [3,] 14 1 97 81480 0.230 0.801
## [4,] 15 1 96 81600 0.230 0.801
## [5,] 12 1 99 81180 0.230 0.799
## [6,] 14 1 96 80640 0.231 0.797
## [7,] 15 1 95 80750 0.231 0.797
## [8,] 17 1 95 82650 0.229 0.804
## [9,] 13 1 99 82170 0.229 0.804
## [10,] 13 1 97 80510 0.231 0.796
```

Note that MDES values in the output are calculated at specified power values (.80), and power values in the output are calculated at specified MDES values (.23).

Scenario 7: Find Optimal Sample (Constrain Power and Number of Classrooms)

Further assumptions

- Statistical power ($1 - \beta$): .80
- Minimum detectable effect size (MDES): .232
- Average number of classrooms per school (J): 2
- Marginal cost per student (cn): 10
- Marginal cost per classroom (cJ): 200
- Marginal cost per school (cK): 500

Given design parameters and further assumptions, how many schools, and students should be recruited to achieve 80% power while minimizing the total cost?

```
design7 <- optimal.cra3r3(cn=10, cJ=200, cK=500, constrain="power", power=.80, J=2,
                        mdes=.23, rho2=.06, rho3=.18,
                        g3=1, R12=.55, R22=.50, R32=.45,
                        P=.40)
print(design7$round.optim)
```

```
##      n J K cost mdes power
## [1,] 8 2 85 90100 0.231 0.797
```

```
print(design7$integer.optim)
```

```
##      n J K cost mdes power
## [1,] 4 3 87 106140 0.230 0.800
## [2,] 10 2 83 91300 0.230 0.800
## [3,] 2 6 84 152880 0.230 0.800
## [4,] 9 2 84 90720 0.230 0.799
## [5,] 8 2 86 91160 0.230 0.802
## [6,] 3 4 85 120700 0.231 0.798
## [7,] 5 3 83 103750 0.231 0.798
## [8,] 5 3 84 105000 0.229 0.803
## [9,] 3 4 86 122120 0.229 0.803
## [10,] 7 2 87 90480 0.231 0.797
```

Note that MDES values in the output are calculated at specified power values (.80), and power values in the output are calculated at specified MDES values (.23).

Scenario 8: Find Optimal Sample (Constrain MDES and Number of Classrooms)

Further assumptions

- Statistical power ($1 - \beta$): .80
- Minimum detectable effect size (MDES): .20
- Average number of classrooms per school (J): 2
- Marginal cost per student (cn): 10
- Marginal cost per classroom (cJ): 200
- Marginal cost per school (cK): 500

Given design parameters and further assumptions, how many schools, and students should be recruited to achieve an MDES of .20 while minimizing the total cost?

```
design8 <- optimal.cra3r3(cn=10, cJ=200, cK=500, constrain="mdes", mdes=.20, J=2,
                        power=.80, rho2=.06, rho3=.18,
                        g3=1, R12=.55, R22=.50, R32=.45,
                        P=.40)
print(design8$round.optim)
```

```
##      n J K cost mdes power
## [1,] 9 2 107 115560 0.204 0.786
```

```
print(design8$integer.optim)
```

```
##      n J K cost mdes power
## [1,] 13 2 106 122960 0.2 0.800
## [2,] 11 2 108 120960 0.2 0.800
```

```
## [3,] 12 2 107 121980 0.2 0.801
## [4,] 10 2 109 119900 0.2 0.799
## [5,] 14 2 105 123900 0.2 0.799
## [6,] 6 3 107 136960 0.2 0.801
## [7,] 4 4 107 156220 0.2 0.802
## [8,] 15 2 105 126000 0.2 0.802
## [9,] 3 5 106 174900 0.2 0.798
## [10,] 3 5 107 176550 0.2 0.802
```

Note that MDES values in the output are calculated at specified power values (.80), and power values in the output are calculated at specified MDES values (.20).

Scenario 9: Find Optimal Sample (Constrain Cost and Number of Classrooms)

Further assumptions

- Statistical power ($1 - \beta$): .80
- Minimum detectable effect size (MDES): .20
- Average number of classrooms per school (J): 2
- Marginal cost per student (cn): 10
- Marginal cost per classroom (cJ): 200
- Marginal cost per school (cK): 500
- Budget 100,000

Given design parameters and further assumptions, how many schools, and students should be recruited while maximizing the power?

```
design9 <- optimal.cra3r3(cn=10, cJ=200, cK=500, constrain="cost", cost=100000, J=2,
                        power=.80, mdes=.20, rho2=.06, rho3=.18,
                        g3=1, R12=.55, R22=.50, R32=.45,
                        P=.40)
print(design9$round.optim)
```

```
##      n J K   cost mdes power
## [1,] 47 2 55 101200 0.27 0.548
```

```
print(design9$integer.optim)
```

```
##      n J K   cost mdes power
## [1,] 46 2 55 100100 0.270 0.547
## [2,] 44 2 56 99680 0.267 0.554
## [3,] 43 2 57 100320 0.265 0.562
## [4,] 49 2 53 99640 0.275 0.532
## [5,] 48 2 54 100440 0.272 0.540
## [6,] 47 2 54 99360 0.272 0.540
## [7,] 50 2 53 100700 0.275 0.533
## [8,] 45 2 56 100800 0.267 0.555
## [9,] 42 2 57 99180 0.265 0.561
## [10,] 45 2 55 99000 0.270 0.547
```

Note that MDES values in the output are calculated at specified power values (.80), and power values in the output are calculated at specified MDES values (.20).

Scenario 10: Find Optimal Sample (Constrain Cost, Number of Classrooms, Number of Students)

Further assumptions

- Statistical power ($1 - \beta$): .80
- Minimum detectable effect size (MDES): .20
- Average number of students per classroom (J): 20
- Average number of classrooms per school (J): 2
- Marginal cost per student (cn): 10
- Marginal cost per classroom (cJ): 200
- Marginal cost per school (cK): 500
- Budget 100,000

Given design parameters and further assumptions, how many schools should be recruited while maximizing the power?

```
design10 <- optimal.cra3r3(cn=10, cJ=200, cK=500, constrain="cost", cost=100000, n=20, J=2,
                          power=.80, mdes=.20, rho2=.06, rho3=.18,
                          g3=1, R12=.55, R22=.50, R32=.45,
                          P=.40)
print(design10$round.optim)
```

```
##      n J K   cost mdes power
## [1,] 20 2 77 100100 0.231 0.679
```

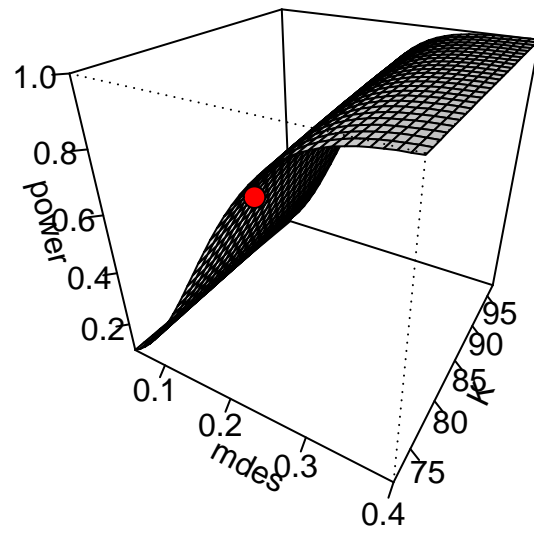
```
print(design10$integer.optim)
```

```
##      n J K   cost mdes power
## [1,] 20 2 77 100100 0.231 0.679
## [2,] 19 2 78  99840 0.230 0.683
## [3,] 21 2 76 100320 0.232 0.674
## [4,] 18 2 79  99540 0.229 0.687
## [5,] 22 2 75 100500 0.234 0.670
## [6,] 21 2 75  99000 0.234 0.668
## [7,] 19 2 79 101120 0.229 0.689
## [8,] 20 2 76  98800 0.233 0.673
## [9,] 20 2 78 101400 0.230 0.684
## [10,] 19 2 77  98560 0.232 0.677
```

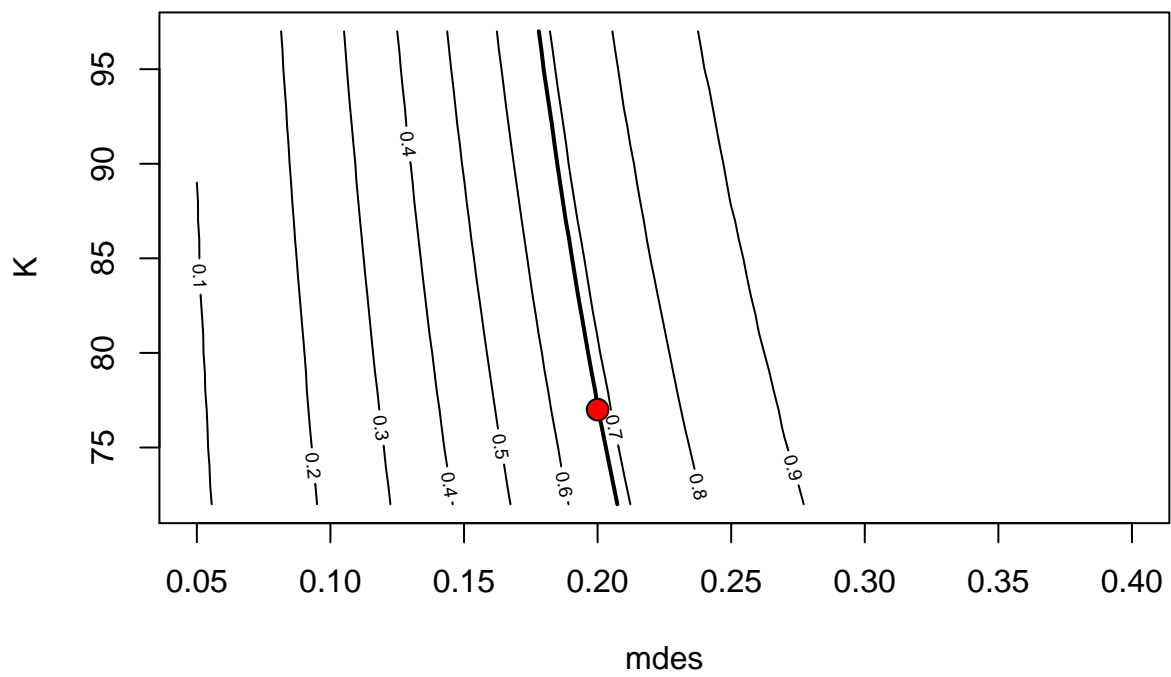
Note that MDES values in the output are calculated at specified power values (.80), and power values in the output are calculated at specified MDES values (.20).

Plots

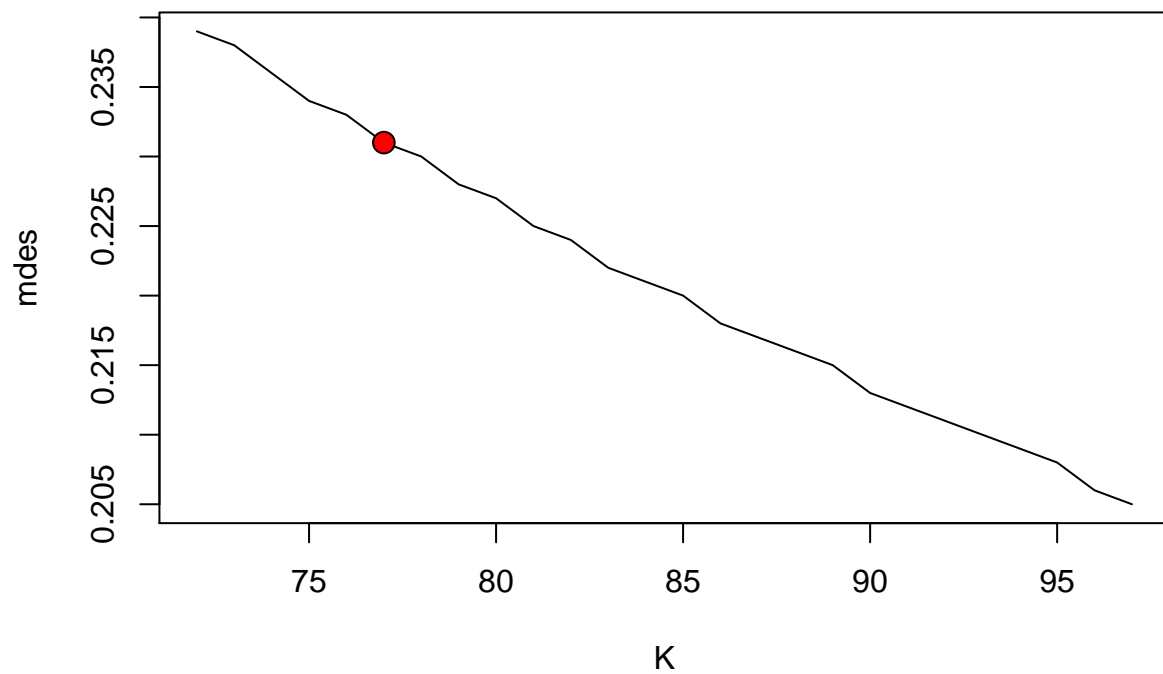
```
# Red points indicates the design location
# Perspective plot
plot(design10, type="p")
```



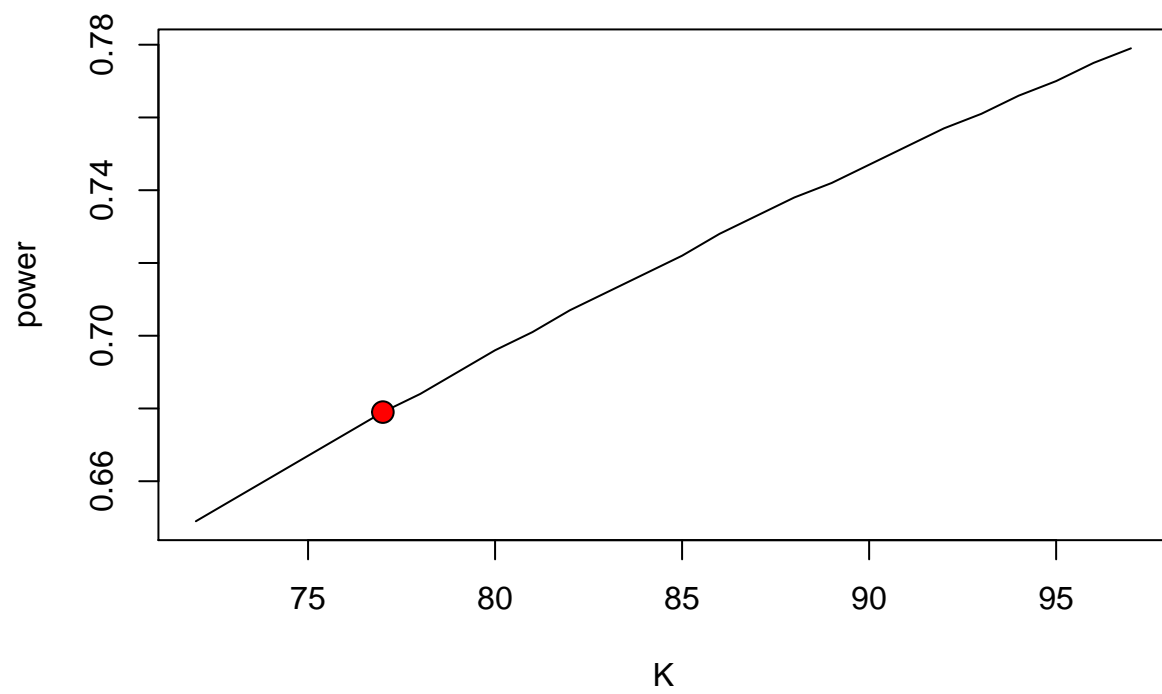
```
# Contour plot  
plot(design10, type="c")
```

```
# MDES - MRSS(K)
plot(design10, pars=c("mdes", "K"))
```

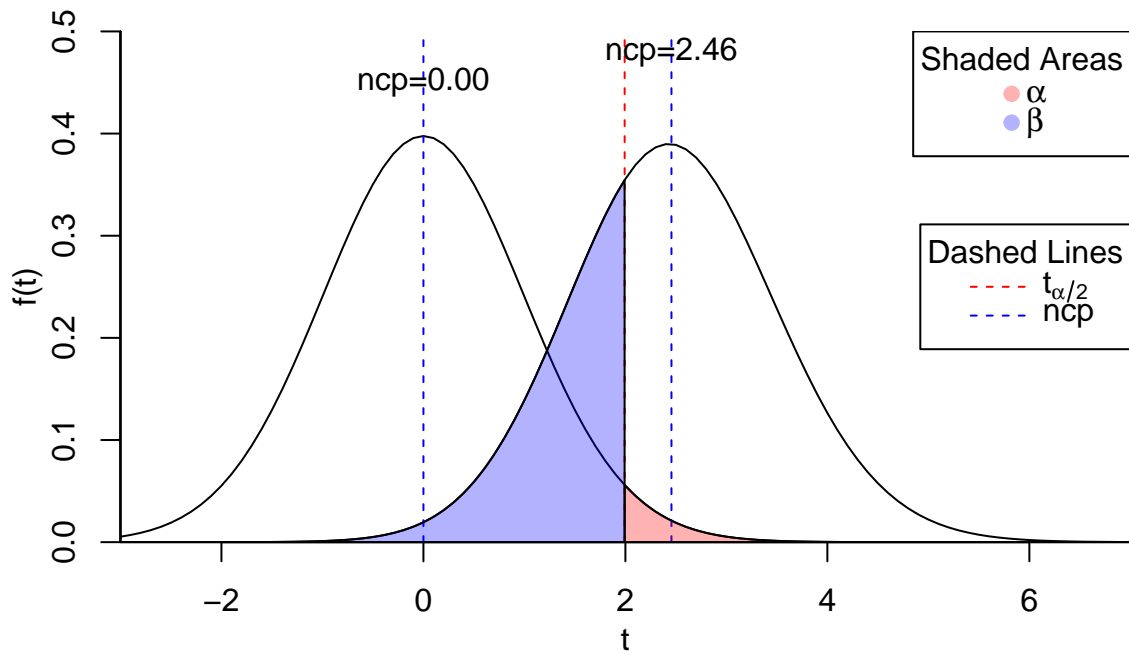


```
# Power - MRSS(K)  
plot(design10, pars=c("power", "K"))
```



```
# Type I and Type II Error Rates  
tlt2.error(design10)
```

Type I (α) and Type II (β) Error Rates



Type I = 0.05 , Type II = 0.321 , Power = 0.679

Reference

U.S. Department of Education, Institute of Education Sciences, What Works Clearinghouse. (2016, May). Secondary Mathematics intervention report: Saxon Math. Retrieved from https://ies.ed.gov/ncee/wwc/Docs/InterventionReports/wwc_saxon_052416.pdf