Birth Data - Loglinear Models

January 25, 2024

```
library(catdata)

data(birth)
attach(birth)
```

In the following loglinear models are fitted with the binary variables Sex, Membranes, Cesarean and Induced from the "birth" data. As an overview a contingency table is plotted.

```
table1 <- table(Sex, Membranes, Cesarean, Induced)

ftable(table1)
```

Now we start fitting the models. The goal is to find a model with good fit but sparse parametrization.

First the saturated model is fitted, then the model with all 3–factor interactions and the model with all 2–factor interactions, and finally the independence model. To control for model fit we look at the corresponding deviances and degrees of freedom.

```
m4 <- loglin(table1, margin=list(c(1,2,3,4)), fit=TRUE)
cat("deviance(m4)=", m4$lrt, "df(m4)=", m4$df, "\n")

m3 <- loglin(table1, margin=list(c(1,2,3), c(1,2,4), c(1,3,4), c(2,3,4)), fit=TRUE)
cat("deviance(m3)=", m3$lrt, "df(m3)=", m3$df, "\n")

m2 <- loglin(table1, margin=list(c(1,2), c(1,3), c(1,4), c(2,3), c(2,4), c(3,4)),
             fit=TRUE)
cat("deviance(m2)=", m2$lrt, "df(m2)=", m2$df, "\n")

m1 <- loglin(table1, margin=list(c(1), c(2), c(3), c(4)), fit=TRUE)
cat("deviance(m1)=", m1$lrt, "df(m1)=", m1$df, "\n")
```

In order to see if a model or rather the reduction of a model is appropriate we use chi–square tests.

```
(df34 <- m3$df - m4$df)
(dev34 <- m3$lrt - m4$lrt)
1-pchisq(dev34, df34)


(df23 <- m2$df - m3$df)
(dev23 <- m2$lrt - m3$lrt)
1-pchisq(dev23, df23)

(df12 <- m1$df - m2$df)
(dev12 <- m1$lrt - m2$lrt)
1-pchisq(dev12, df12)
```

Since model "m2" fits the data well but model "m1" is definitely rejected we fit submodels of "m2" by leaving out one of the 2–factor interactions.

```
m2.GM <- loglin(table1, margin=list(c(1,3), c(1,4), c(2,3), c(2,4), c(3,4)),
                fit=TRUE)
cat("deviance(m2.GM)=", m2.GM$lrt, "df(m2.GM)=", m2.GM$df, "\n")

m2.MC <- loglin(table1, margin=list(c(1,2), c(1,3), c(1,4), c(2,4), c(3,4)),
                fit=TRUE)
cat("deviance(m2.MC)=", m2.MC$lrt, "df(m2.MC)=", m2.MC$df, "\n")

m2.CI <- loglin(table1, margin=list(c(1,2), c(1,3), c(1,4), c(2,3), c(2,4)),
                fit=TRUE)
cat("deviance(m2.CI)=", m2.CI$lrt, "df(m2.CI)=", m2.CI$df, "\n")

m2.GI <- loglin(table1, margin=list(c(1,2), c(1,3), c(2,3), c(2,4), c(3,4)),
                fit=TRUE)
cat("deviance(m2.GI)=", m2.GI$lrt, "df(m2.GI)=", m2.GI$df, "\n")

m2.GC <- loglin(table1, margin=list(c(1,2), c(1,4), c(2,3), c(2,4), c(3,4)),
                fit=TRUE)
cat("deviance(m2.GC)=", m2.GC$lrt, "df(m2.GC)=", m2.GC$df, "\n")

m2.MI <- loglin(table1, margin=list(c(1,2), c(1,3), c(1,4), c(2,3), c(3,4)),
                fit=TRUE)
cat("deviance(m2.MI)=", m2.MI$lrt, "df(m2.MI)=", m2.MI$df, "\n")
```

These six models all have 6 degrees of freedom so that the difference of degrees of freedom corresponding to model "m2" is 1 in each case.

```
1 - pchisq(m2.GM$lrt - m2$lrt, 1)

1 - pchisq(m2.MC$lrt - m2$lrt, 1)

1 - pchisq(m2.CI$lrt - m2$lrt, 1)
```

```
1 - pchisq(m2.GI$lrt - m2$lrt, 1)

1 - pchisq(m2.GC$lrt - m2$lrt, 1)

1 - pchisq(m2.MI$lrt - m2$lrt, 1)
```

Testing of the 2–factor interactions shows that the interactions "MC", "CI" and "MI" should be kept in the model. In the next step the model that contains these interactions, G—MC—MI—CI, is fitted.

```
m2.GM.GI.GC<- loglin(table1, margin=list(c(1), c(2,3), c(2,4), c(3,4)), fit=TRUE)
cat("deviance(m2.GM.GI.GC)=", m2.GM.GI.GC$lrt, "df(m2.GM.GI.GC)=", m2.GM.GI.GC$df,
    "\n")

1 - pchisq(m2.GM.GI.GC$lrt - m2$lrt, m2.GM.GI.GC$df - m2$df)
```

Comparison with model "m2" shows that reduction is possible. However, reduction to a model in which the main effect "G" is omitted is rejected.

```
m2.G<- loglin(table1, margin=list(c(2,3), c(2,4), c(3,4)), fit=TRUE)
cat("deviance(m2.G)=", m2.G$lrt, "df(m2.G)=", m2.G$df, "\n")

1 - pchisq(m2.G$lrt - m2$lrt, m2.G$df - m2$df)
```