4.4 Ice Cream Parlor – Raiders of the Lost Cone (Switches)
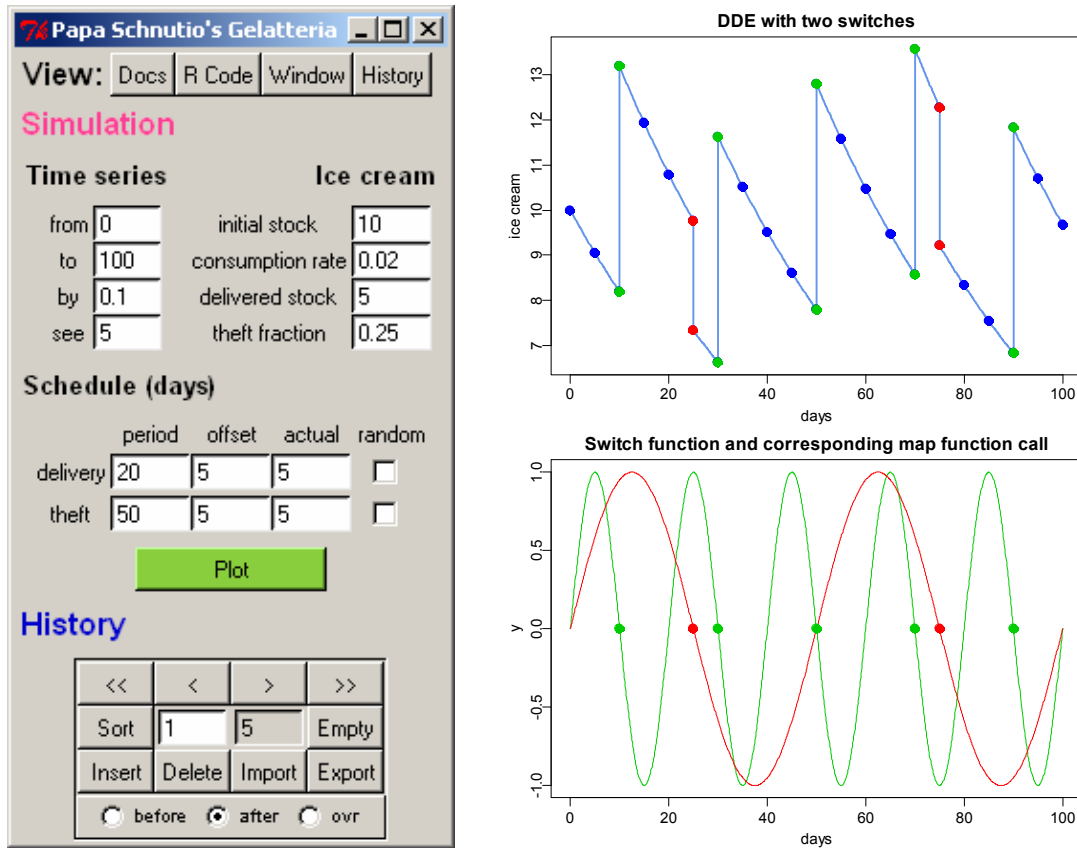

Figure 4. The ice cream parlor receives deliveries (switch 1, green) and experiences theft (switch 2, red).

To illustrate switches, Alex (author ACB) suggested an ice cream parlor that gets restocked periodically. But Jon (author JTS) wanted at least two switches, so he suggested that thieves might occasionally raid the parlor and steal some of the stock. Comments in Alex's code soon suggested a snappy title: *Raiders of the Lost Cone*. Rowan (author RH) and Alex quickly agreed that the parlor should have a flashing sign with the logo "Papa Schnutio's" that puts an Italian twist on Jon's Germanic last name. Jon hesitated, but he once lived in Italy for a year and couldn't resist the idea of Italian ice cream (*gelato*). So he agreed to an establishment named

<p style="text-align:center">*Papa Schnutio's Gelatteria*</p>

Our model assumes an exponential depletion of the ice cream stock $y(t)$ with rate r:

$$\frac{dy}{dt} = -ry,$$

perhaps because a lower stock would offer fewer choices and thus discourage consumption. (Sometimes a *gelatteria* just doesn't have that perfect flavour you came for. It was there yesterday, but not today – a great disappointment!) We need two switch functions that we chose to be the sinusoids

$$s_i = \sin\left[2\pi\left(a_i + \frac{t}{p_i}\right)\right]$$

with offset parameters a_i and periods p_i for $i = 1, 2$. Switch 1 triggers restocking

$$y(t_+) = y(t) + Y$$

and switch 2 triggers theft events

$$y(t_+) = (1 - f)y(t),$$

where $y(t_+)$ denotes the value of y after the switch, Y is the fixed amount of ice cream added to the stock, and f is the fraction of the stock removed by thieves.

In this example, key features of the code involve the sine wave used for switching

```
sinWave <- function(t,aa,pp) { sin( 2*pi*(aa + (t/pp)) ) }
```

the switch function

```
mySwitch <- function(t,y) {
  c( sinWave(t,a[1],p[1]), sinWave(t,a[2],p[2]) ) }
```

the map function

```
myMap <- function(t,y,swID) {
  if (swID==1) y <- y + Y else y <- (1-f)*y }
```

the gradient function

```
myGrad <- function(t,y) { -r*y }
```

and the call to the main routine

```
yout <- dde(y=y0, times=tt, func=myGrad,
  switchfunc=mySwitch, mapfunc=myMap)
```

where the initial stock `y0`, the desired output times `tt`, the consumption rate `r`, the amount `Y` brought by the supplier, the theft fraction `f`, the offset vector `a` (of length 2), and the period vector `p` (of length 2) correspond to values prescribed by the GUI.