How to generate new distributions in packages "distr", "distrEx"

Peter Ruckdeschel* Matthias Kohl[†]

Institut für Mathematik Fakultät V - Mathematik und Naturwissenschaften Carl von Ossietzky Universität Oldenburg PObox 2503 26111 Oldenburg (Oldb) Germany

e-Mail: peter.ruckdeschel@uni-oldenburg.de

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Abstract

In this vignette, we give short examples how to produce new distributions in packages "distr" and "distrEx". This vignette refers to package version 2.6.2.

Basically there are three ways to produce new distributions in packages "distr" and "distrEx":

- 1. automatic generation of single distribution objects by arithmetics and the like
- 2. using generating functions to produce single distribution objects
- 3. defining new distribution classes / doing it from scratch

We will give short examples of all three of them.

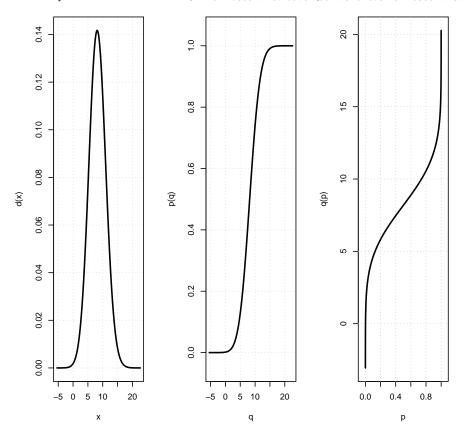
^{*}Universität Oldenburg, Oldenburg

[†]FH Furtwangen

1 Automatic generation by arithmetics and the like

We have made available quite general arithmetical operations to our distribution objects, generating new image distribution objects automatically. As an example, try

```
> require(distr)
> N \leftarrow Norm(mean = 2, sd = 1.3)
> P \leftarrow Pois(lambda = 1.2)
> Z \leftarrow 2*N + 3 + P
> Z
Distribution Object of Class: AbscontDistribution
> plot(Z, panel.first = grid(), lwd=2)
> p(Z)(0.4)
[1] 0.002415387
> q(Z)(0.3)
[1] 6.705068
> Zs \leftarrow \mathbf{r}(Z)(50)
> Zs
 [1]
       7.528781
                  5.122758
                             8.676185
                                        6.753091
                                                   5.054000
                                                              6.905054
 [7]
       7.807627
                  7.907712\ 11.610921
                                        7.265653
                                                   5.392185
                                                               9.991786
 [13]
       9.684809
                  4.540207
                            10.944730
                                        9.727200
                                                   4.036700
                                                              2.835064
 [19]
       6.664548
                  8.802903
                             3.491921 \ 12.545060
                                                  10.673256 \ 13.541608
 25]
      6.819095
                  6.980519
                             7.038506
                                        6.646279
                                                   7.519551
                                                               9.259711
 [31] 13.210811
                  5.999851
                             6.358812
                                        7.282379
                                                   9.621614
                                                               6.026729
 [37] 10.447300
                  9.302980
                             6.353487\ 11.820191
                                                   9.041744
                                                              5.125658
       3.752013 \ 12.955441
                             9.524838
                                       8.654323
                                                   4.150037
                                                              5.601424
[43]
[49]
       8.688506
                 2.267838
```



Comment:

Let N an object of class "Norm" with parameters mean=2, sd=1.3 and let P an object of class "Pois" with parameter lambda=1.2. Assigning to Z the expression 2*N+3+P, a new distribution object is generated —of class "AbscontDistribution" in our case— so that identifying N, P, Z with random variables distributed according to N, P, Z, $\mathcal{L}(Z) = \mathcal{L}(2*N+3+P)$, and writing p(Z)(0.4) we get $P(Z \leq 0.4)$, q(Z)(0.3) the 30%-quantile of Z, and with r(Z)(50) we generate 50 pseudo random numbers distributed according to Z, while the plot command generates the above figure.

There a caveats to take care about; for details refer to the (larger) vignette distr in package "distrDoc".

2 Using generating functions

If you want to generate a single distribution object (without any particular parameter) generating functions are the method of choice:

Objects of classes LatticeDistribution resp. DiscreteDistribution, AbscontDistribution, may be gen-

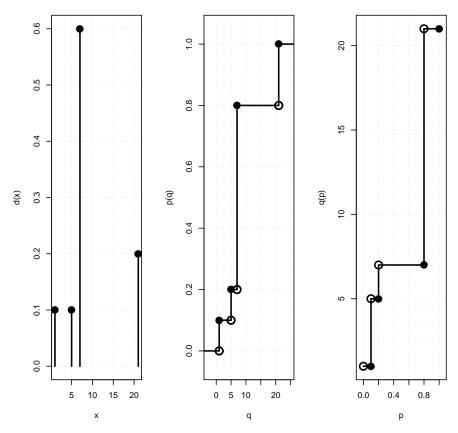
erated using the generating functions LatticeDistribution() resp. DiscreteDistribution() resp. AbscontDistribution(); see also the corresponding help.

E.g., to produce a discrete distribution with support (1, 5, 7, 21) with corresponding probabilities (0.1, 0.1, 0.6, 0.2) we may write

```
> D \leftarrow DiscreteDistribution(supp = c(1,5,7,21), prob = c(0.1,0.1,0.6,0.2)) > D
```

Distribution Object of Class: Discrete Distribution

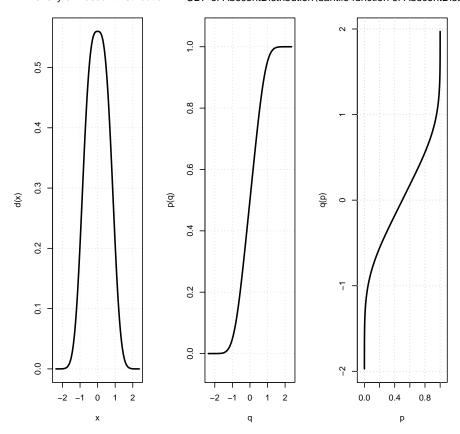
```
> plot(D, panel.first = grid(), lwd = 2)
```



and to generate an absolutely continuos distribution with density proportional to $e^{-|x|^3}$, we write $> AC \leftarrow \mathbf{AbscontDistribution}(\mathbf{d} = \mathbf{function}(\mathbf{x}) \ \mathbf{exp}(-abs(\mathbf{x}) \land 3)$, with Stand = TRUE) > AC

Distribution Object of Class: AbscontDistribution

Density of AbscontDistribution CDF of AbscontDistribution tuantile function of AbscontDistribution



3 Doing it from scratch

If you would like to create new parametric distributions, using already implemented r, d, p, and q functions (e.g. implementing additional distributions realized in another CRAN package), you should probably envisage introducing new distribution S4 (sub-)classes and hence better look at the implementation of some discrete and continuous parametric distribution classes in package "distr". Hint: download the .tar.gz file; extract it to some temp folder; look at subdirectories R and man

The general procedure is as follows

- 1. introduce a new subclass of class Parameter
- 2. introduce a new subclass of LatticeDistribution/DiscreteDistribution (if discrete) or of class AbscontDistribution (if continuous).

- 3. define accessor and replacement functions for the "slots" of the parameter (e.g. "size" and "prob" in the binomial case), possibly with new generics
- 4. (possibly) define a validity function
- 5. define a generating function
- if existing, define particular convolution methods or similar particular methods for this new distribution class
- 7. create .Rd files for the
 - parameter class
 - distribution class
- 8. if analytic expressions are available, define particular E-, var-, skewness-, and kurtosis-methods and if so, also document¹ the corresponding methods in the distribution class .Rd file

Let's go through the steps in the example case of the Binomial implementation in packages "distr" and "distrEx":

1. in "distr", see source in R/AllClasses.R, lines 187-196

2. in "distr", see source in R/AllClasses.R, lines 972-1000

¹this is new, because so far, all E-, var-, skewness-, and kurtosis-methods for "basic" distributions are documented in the "distrEx" documentation to E, var, ..., but this would not be operational any longer for new derived classes, possibly defined in other, new packages

```
\mathbf{q} = \mathbf{function}(\mathbf{p}, lower.tail = TRUE, log.p = FALSE)
                                       qbinom(p, size = 1, prob = 0.5,
                                               lower.tail = lower.tail, log.p = log.p)
                             img = new("Naturals"),
                             param = new("BinomParameter"),
                             support = 0:1,
                             lattice = new("Lattice",
                                         pivot = 0, width = 1, Length = 2, name =
                                         gettext(
                                            "lattice of a Binomial distribution"
                                         ),
                            .\log Exact = TRUE,
                            .lowerExact = TRUE
                             ),
               contains = "LatticeDistribution"
3. in "distr", see source in R/BinomialDistribution.R, lines 9-16, and 44-54
  ## Access Methods
  setMethod("size", "BinomParameter", function(object) object@size)
  setMethod("prob", "BinomParameter", function(object) object@prob)
  ## Replace Methods
  setReplaceMethod("size", "BinomParameter",
                        function(object , value){ object@size \( \text{value} ; object \) )
  setReplaceMethod("prob", "BinomParameter";
                        function(object , value){ object@prob ← value; object})
  ## wrapped access methods
  \mathbf{setMethod}(\,"\mathbf{prob}\,",\ "Binom"\,,\ \mathbf{function}(\,\mathtt{object}\,)\ \mathbf{prob}(\mathbf{param}(\,\mathtt{object}\,)))
  setMethod("size", "Binom", function(object) size(param(object)))
  ## wrapped replace methods
  \mathbf{setMethod} (\ "\mathbf{prob} \leftarrow "\ , \quad "\mathbf{Binom}"\ ,
                function(object, value) new("Binom", prob = value,
                                                   size = size(object))
  setMethod("size \leftarrow", "Binom",
                function (object, value) new ("Binom", prob = prob (object),
                                                   size = value))
  and R/AllGenerics, lines 159-162
  if(!isGeneric("size"))
      setGeneric("size", function(object) standardGeneric("size"))
  if(!isGeneric("prob"))
      \mathbf{setGeneric}(\ "\mathbf{prob}"\ ,\ \ \mathbf{function}\ (\ \mathrm{object}\ )\ \ \mathbf{standardGeneric}\ (\ "\mathbf{prob}"\ ))
4. in "distr", see source in R/BinomialDistribution.R, lines 19-33
  setValidity("BinomParameter", function(object){
```

```
if(length(prob(object)) != 1)
       stop ("prob has to be a numeric of length 1")
     if (prob (object) < 0)
      stop ("prob has to be in [0,1]")
     if(prob(object) > 1)
      stop("prob has to be in [0,1]")
     if(length(size(object)) != 1)
      stop ("size has to be a numeric of length 1")
     if(size(object) < 1)
      stop("size has to be a natural greater than 0")
     if(!identical(floor(size(object)), size(object)))
      stop ("size has to be a natural greater than 0")
     else return (TRUE)
  })
5. in "distr", see source in R/BinomialDistribution.R, line 42
  Binom 

function(size = 1, prob = 0.5) new("Binom", size = size, prob = prob)
6. in "distr", see source in R/BinomialDistribution.R, lines 55-69
  ## Convolution for two binomial distributions Bin(n1,p1) and Bin(n2,p2)
  ## Distinguish cases
  ## p1 == p2 und p1 != p2
  setMethod("+", c("Binom", "Binom"),
             function (e1, e2)
               newsize \leftarrow size(e1) + size(e2)
               if (isTRUE(all.equal(prob(e1),prob(e2))))
                  return(new("Binom", prob = prob(e1), size = newsize,
                              .withArith = TRUE)
               return(as(e1, "LatticeDistribution") + e2)
             })
7. in "distr", see sources in
     • man/BinomParameter-class.Rd
       \name{BinomParameter-class}
       \docType{class}
       \alias {BinomParameter-class}
       \alias { initialize , BinomParameter-method }
       \title{Class "BinomParameter"}
       description { The parameter of a binomial distribution, used by Binom-class}
       \section{Objects from the Class}{
       Objects can be created by calls of the form
             \code{new("BinomParameter", prob, size)}.
       Usually an object of this class is not needed on its own, it is generated
```

```
automatically when an object of the class Binom
  is instantiated.
  \section{Slots}{
     \describe{
       \item{\code{prob}}}{Object of class \code{"numeric"}:
                the probability of a binomial distribution }
        \item{\code{size}}{Object of class \code{"numeric"}:
                the size of a binomial distribution }
        \item{\code{name}}}{Object of class \code{"character"}:
                a name / comment for the parameters }
   \section { Extends } {
  Class \code{"Parameter"}, directly.
  \section { Methods } {
     \describe{
       \label{linear_code} $$ \left\{ \operatorname{initialize} \right\} \left\{ \operatorname{code} \left\{ \operatorname{signature} \left( \operatorname{.Object} = \operatorname{"BinomParameter"} \right) \right\} : $$
                initialize method }
        \widetilde{prob}_{code}(\widetilde{signature}(\widetilde{object} = "BinomParameter")): returns the slot
       \code\{prob\}\ of the parameter of the distribution \} \idetit{item}\{prob\leftarrow\}\{\code\{signature(object="BinomParameter")\}: modifies the slot}
                \code{prob} of the parameter of the distribution }
        \item{size}{\code{signature(object = "BinomParameter")}: returns the slot
       \code{size} of the parameter of the distribution }
\item{size \code{signature(object = "BinomParameter")}: modifies the slot
                \code{size} of the parameter of the distribution}
  }
  \setminus author {
     Thomas Stabla \email{statho3@web.de},\cr
     Florian Camphausen \email{fcampi@gmx.de},\cr
     Peter Ruckdeschel \email{peter.ruckdeschel@uni-oldenburg.de},\cr
     Matthias Kohl \email{Matthias.Kohl@stamats.de}
  \seealso{
  \code{\link{Binom-class}}
  \code{\left\{\operatorname{link}\left\{\operatorname{Parameter-class}\right\}\right\}}
  \examples{
      W \( \text{new("BinomParameter", prob=0.5, size=1)} \)
      size (W) # size of this distribution is 1. size (W) \leftarrow 2 # size of this distribution is now 2.
  \keyword{ distribution }
  \concept { parameter }
  \concept{Binomial distribution}
  \concept{S4 parameter class}
• man/Binom-class.Rd
  \mathbf{name}\{\mathrm{Binom-class}\}
  \docType{class}
```

```
\alias {Binom-class}
\alias (Binom)
\alias { initialize , Binom-method }
\title{Class "Binom" }
\description{The binomial distribution with <math>\description{ size } \end{size} \
      \{eqn\{=1\}, and
      \label{eq:code} $$ \left\{ \begin{array}{l} \mathbf{prob} \right\} \ \left\{ \mathbf{pn} \right\} , \ \mathbf{by} \ \mathbf{default} \ \left\{ \mathbf{qn} \right\} = 0.5 \right\}, \ \mathrm{has} \ \mathbf{density} \\ \left\{ \mathbf{deqn} \left\{ \mathbf{p}(\mathbf{x}) = \left\{ \mathbf{n} \ \left\{ \mathbf{choose} \ \mathbf{x} \right\} \ \left\{ \mathbf{p} \right\} \wedge \left\{ \mathbf{x} \right\} \ \left\{ (1-\mathbf{p}) \right\} \wedge \left\{ \mathbf{n} - \mathbf{x} \right\} \right\} \right\} $$
           p(x) = choose(n,x) p \land x (1-p) \land (n-x)
      for \{x = 0, \{x \in \mathbb{N}\}.
     C. f.\code{\link[stats:Binomial]{rbinom}}
\section{Objects from the Class}{
Objects can be created by calls of the form \colon beta = by calls of the form <math>\colon beta = by calls of the form by calls of
This object is a binomial distribution.
\section{Slots}{
      \describe{
             \item{\code{img}}{Object of class \code{"Naturals"}: The space of the
               image of this distribution has got dimension 1 and the
               name "Natural Space". }
             \item{\code{param}}{Object of class \code{"BinomParameter"}: the parameter
                                 of this distribution (\code{prob}, \code{size}), declared at its
                                 instantiation }
             \item{\code{r}}{Object of class \code{"function"}: generates random
                                 numbers (calls function \code{rbinom}) }
             \item{\code{d}}{Object of class \code{"function"}: density function (calls
                                 function \code{dbinom}) }
             \item{\code{p}}}{Object of class \code{"function"}: cumulative function
                                  (calls function \code{pbinom})
             \item{\code{q}}}{Object of class \code{"function"}: inverse of the
                                    cumulative function (calls function \code{qbinom}).
            The quantile is defined as the smallest value x such that F(x) \geq p, where
                                      F is the cumulative function. }
             \item{\code{support}}{Object of class \code{"numeric"}: a (sorted)
                                       vector containing the support of the discrete density function}
             \item{\code{. with Arith}}{logical: used internally to issue warnings as to
                                       interpretation of arithmetics}
             \item{\code{.withSim}}{logical: used internally to issue warnings as to
                                 accuracy }
             \item{\code{.logExact}}{logical: used internally to flag the case where
             there are explicit formulae for the log version of density, cdf, and
             quantile function}
             \item{\code \{ .lowerExact \}\} \{ logical: used internally to flag the case where
             there are explicit formulae for the lower tail version of cdf and quantile
             function }
             \item{\code{Symmetry}}{object of class \code{"DistributionSymmetry"};
                used internally to avoid unnecessary calculations.}
      }
\section { Extends } {
Class \code{"DiscreteDistribution"}, directly.\cr
\textbf{Class } \\ \textbf{\code} \\ \text{\code} \\ \text{\code}
Class \code{"Distribution"}, by class \code{"DiscreteDistribution"}.
\section { Methods } {
```

```
\describe{
     \item{+}{\code{signature(e1 = "Binom", e2 = "Binom")}: For two binomial
                  distributions with equal probabilities the exact convolution
                 formula is implemented thereby improving the general numerical
     \item{initialize}{\code{signature(.Object = "Binom")}: initialize method}
     \item{prob}{\code{signature(object = "Binom")}: returns the slot \code{prob}
     of the parameter of the distribution } \item{prob} {\code{signature(object = "Binom")}: modifies the slot
                 \code{prob} of the parameter of the distribution }
     \item{size}{\code{signature(object = "Binom")}: returns the slot \code{size}
     of the parameter of the distribution } \item{size \in }{\code{signature(object = "Binom")}: modifies the slot
                 \code{size} of the parameter of the distribution }
}
\author{
  Thomas Stabla \email{statho3@web.de},\cr
  Florian Camphausen \email{fcampi@gmx.de},\cr
  Peter Ruckdeschel \email{peter.ruckdeschel@uni-oldenburg.de},\cr
  Matthias Kohl \email{Matthias.Kohl@stamats.de}
  }
\seealso{
\code{\link{BinomParameter-class}}
\code{\link{DiscreteDistribution-class}}
\code{\link{Naturals-class}}
\code {\link [stats: Binomial] {rbinom}}
\examples{
    B \leftarrow \mathbf{Binom}(\mathbf{prob}{=}0.5\ , \mathbf{size}{=}1) \ \textit{\# B is a binomial distribution with prob}{=}0.5\ \textit{and size}{=}1.
    \mathbf{r}\left(B\right)\left(1\right) # # one random number generated from this distribution, e.g. 1
    \mathbf{d}(B)(1) # Density of this distribution is 0.5 for x=1.
    \mathbf{p}(\mathrm{B})(0.4) # Probability that x<0.4 is 0.5.
    \mathbf{q}(B)(.1) # x=0 is the smallest value x such that p(B)(x) \geq 0.1. \mathbf{size}(B) # size of this distribution is 1.
    \mathbf{size}\left(B\right) \;\leftarrow\; 2 \;\; \textit{\# size of this distribution is now 2}.
    \mathbf{C} \leftarrow \mathbf{Binom}(\mathbf{prob} = 0.5 \;,\;\; \mathbf{size} = 1) \;\; \textit{\# C is a binomial distribution with prob=0.5 and size=1.}
    \mathbf{D} \leftarrow \mathbf{Binom}(\mathbf{prob} = 0.6, size = 1) # D is a binomial distribution with \mathbf{prob} = 0.6 and size = 1.
    \rm E \leftarrow \rm B + C # E is a binomial distribution with prob=0.5 and size=3.
   \mathbf{F} \leftarrow \mathbf{B} + \mathbf{D} # F is an object of class LatticeDistribution. \mathbf{G} \leftarrow \mathbf{B} + \mathbf{as}(\mathbf{D}, \mathbf{DiscreteDistribution}) ## DiscreteDistribution
\keyword{ distribution }
\concept{discrete distribution}
\concept { lattice distribution }
\concept{Binomial family}
\concept{Binomial distribution}
\concept{S4 distribution class}
\concept{generating function}
```

- you could have: man/Binom.Rd for the generating function; in the Binomial case, documentation is in Binom-class.Rd; but in case of the Gumbel distribution, in package "distrEx", there is such an extra .Rd file
- 8. in "distrEx", see sources in

```
• Expectation.R, lines 446-467
  setMethod("E", signature(object = "Binom",
                                  fun = "missing",
                                  cond = "missing"),
       function (object, low = NULL, upp = NULL, ...){
       if(!is.null(low)) if(low \le \min(support(object))) low \le NULL
       if(!is.null(upp)) if(upp \ge max(support(object))) upp \leftarrow NULL
       if(is.null(low) && is.null(upp))
            return(size(object)*prob(object))
       else{
            if(is.null(low)) low \leftarrow -Inf
            if(is.null(upp)) upp \leftarrow Inf
            if(low = -Inf)
                if (upp == Inf) return(size(object)*prob(object))
                \mathbf{else} \ \mathbf{return} (\mathbf{m1df} ( \, \mathbf{object} \, \, , \, \, \, \mathbf{upper} \, = \, \mathrm{upp} \, , \, \, \, \dots \, ) \, )
            }else{
                E1 \leftarrow m1df(object, upper = low, ...)
                E2 \leftarrow if(upp = Inf)
                            size(object)*prob(object) else m1df(object, upper = upp, ...)
                return (E2-E1)
      })
• Functionals.R, lines 219-230
  setMethod("var", signature(x = "Binom"),
       function (x, ...)
       dots \leftarrow match.call(call = sys.call(sys.parent(1)),
                               expand.dots = FALSE) $ "..."
       fun ← NULL; cond ← NULL; low ← NULL; upp ← NULL
       if(hasArg(low)) low \leftarrow dots$low
       if(hasArg(upp)) upp \leftarrow dots$upp
       if(hasArg(fun)||hasArg(cond)||!is.null(low)||!is.null(upp))
            return(var(as(x, "DiscreteDistribution"),...))
       else
            return(size(x)*prob(x)*(1-prob(x)))
       })
• skewness.R, lines 69-80
  setMethod("skewness", signature(x = "Binom"),
       function (x, \dots)
       dots \leftarrow match.call(call = sys.call(sys.parent(1)),
                               expand.dots = FALSE)$"..."
       \operatorname{fun} \leftarrow \operatorname{NULL}; \operatorname{cond} \leftarrow \operatorname{NULL}; \operatorname{low} \leftarrow \operatorname{NULL}; \operatorname{upp} \leftarrow \operatorname{NULL}
       if(hasArg(low)) low \leftarrow dots$low
       if(hasArg(upp)) upp \leftarrow dots\$upp
       if (hasArg(fun) | | hasArg(cond) | | !is.null(low) | | !is.null(upp))
           return(skewness(as(x, "DiscreteDistribution"),...))
```

else

```
return((1-2*prob(x))/sqrt(size(x)*prob(x)*(1-prob(x))))
          })
• kurtosis.R, lines 90-102
   setMethod("kurtosis", signature(x = "Binom"),
          function(x, \dots)
          dots \leftarrow match.call(call = sys.call(sys.parent(1)),
                                           expand.dots = FALSE) $"..."
          \operatorname{fun} \leftarrow \operatorname{NULL}; \operatorname{cond} \leftarrow \operatorname{NULL}; \operatorname{low} \leftarrow \operatorname{NULL}; \operatorname{upp} \leftarrow \operatorname{NULL}
          if(hasArg(low)) low \leftarrow dots\$low
          if(hasArg(upp)) upp \leftarrow dots$upp
          \mathbf{if}\left(\mathbf{hasArg}(\mathit{fun}\,)\,|\,|\,\mathbf{hasArg}(\mathbf{cond}\,)\,|\,|\,!\,\mathbf{is.null}\left(\mathit{low}\,)\,|\,|\,!\,\mathbf{is.null}\left(\mathit{upp}\,\right)\right)
               return(kurtosis(as(x, "DiscreteDistribution"), ...))
          else
                \mathbf{p} \leftarrow \mathbf{prob}(\mathbf{x})
                 return((1-6*p*(1-p))/(size(x)*p*(1-p)))
          })
```

The procedure will be similar for any new class of distributions.

Comment In the classes in package "distr" (historically the "oldest" in the development of this project), we still use initialize methods; this is no longer needed, if you provide generating functions; for this "more recent" approach, confer the realization of class Gumbel in package "distrEx".

4 Help needed / collaboration welcome

You are — as announced on http://distr.r-forge.r-project.org — very welcome to collaborate in this project! See in particular http://distr.r-forge.r-project.org/HOWTO-collaborate.txt With this you should be able to start working.

References

- [1] Ruckdeschel P. and Kohl, M. (2014): General Purpose Convolution Algorithm for Distributions in S4-Classes by means of FFT. J. Statist. Software, **59**(4): 1–25.
- [2] Ruckdeschel P., Kohl M., Stabla T., and Camphausen F. (2006): S4 Classes for Distributions. R-News, 6(2): 10–13. https://CRAN.R-project.org/doc/Rnews/Rnews_2006-2.pdf