Package 'dst'

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Description Using the Theory of Belief Functions for evidence calculus. Basic probability assignments, or mass functions, can be defined on the subsets of a set of possible values and combined. A mass function can be extended to a larger frame. Marginalization, i.e. reduction to a smaller frame can also be done. These features can be combined to analyze small belief networks and take into account situations where information cannot be satisfactorily described by probability distributions.
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addTobca

Add some elements of 0 mass to an existing basic chance assignment.

Description

Given a previously defined basic chance assignment (bca), the user may want to add some elements of the set of possible values or some subsets, even if they have zero mass value. This feature is useful, for example, to examine the measure of plausibility of these elements or subsets of zero mass value.

Usage

```
addTobca(x, tt, f)
```

Arguments

x A basic chance assignment (see bca).

A matrix constructed in a boolean style (0,1) or a boolean matrix. The number of columns of the matrix tt must match the number of columns of the tt matrix of x (see bca). Each row of the matrix identify a subset of the set of possible values

f Deprecated. Old name for tt matrix.

Value

x The original basic chance assignment x augmented with the added subsets defined by tt.

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Author(s)

Claude Boivin, Stat.ASSQ

Examples

```
 y \leftarrow bca(tt = matrix(c(1,0,0,1,1,1),nrow=2, \ byrow = TRUE), \\ m = c(0.6, 0.4), \quad cnames = c("a", "b", "c"), \ idvar = 1) \\ addTobca(y, matrix(c(0,1,0,0,0,1, 0,1,1), nrow = 3, \ byrow = TRUE)) \\ x \leftarrow bca(tt = matrix(c(0,1,1,1,1,0,1,1,1),nrow=3, \ byrow = TRUE), m=c(0.2,0.5, 0.3), \\ cnames = c("a", "b", "c"), \ idvar = 1) \\ xy \leftarrow dsrwon(x,y) \\ xy1 \leftarrow addTobca(nzdsr(xy), matrix(c(0,1,0,0,0,1), nrow = 2, \ byrow = TRUE)) \\ xy1 \\ addTobca(x, tt = diag(1, ncol(x$tt))) \# add all singletons
```

ads

The Captain's Problem. ads: Relation between variables Arrival (A), Departure delay (D) and Sailing delay (S)

Description

This dataset is the tt matrix establishing the relation A = D + S, where A = 0.6, D = 0.3 and S = 0.3. The subset made of all the triplets (a,d,s) of $(A \times D \times S)$ where a = d + s is true has a mass value of 1. To construct the tt matrix, we put the variables A, D, S side by side, as in a truth table representation. Each triplet of the subset is described by a row of the matrix as a vector of zeros and ones.

Usage

ads

Format

An integer matrix with 18 rows and 17 columns

```
[1, c(1,2)] value = 0, not used
```

[1, 3:17] Identification numbers of the three variables. Column 3 to 9: variable 1; column 10 to 13: variable 2; column 14 to 17: variable 3.

nospec identification number of the specification

m the value of the specification, a number between 0 and 1

- 6 1 if 6 is part of the specification, 0 otherwise
- 5 1 if 5 is part of the specification, 0 otherwise
- 4 1 if 4 is part of the specification, 0 otherwise
- **3** 1 if 3 is part of the specification, 0 otherwise
- 2 1 if 2 is part of the specification, 0 otherwise
- 1 1 if 1 is part of the specification, 0 otherwise
- **0** 1 if 0 is part of the specification, 0 otherwise

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Author(s)

Claude Boivin, Stat.ASSQ

Source

https://www.researchgate.net/publication/265993533_Fusion_and_Propagation_in_Graphical_ Belief_Models

bca

Basic chance assignment mass function

Description

Function bca is used to define subsets of a finite set Θ of possible values and to assign their corresponding mass value.

The set Θ is called the frame of discernment. Each subset A of Theta with a positive mass value is called a focal element or a proposition. The associated mass value is a number of the (0,1] interval, called "basic chance assignment" (the basic probability assignment of Shafer's book). All other subsets that have not received a positive mass value are assumed to have a mass value of zero.

Usage

```
bca(
   tt,
   m,
   cnames = NULL,
   con = NULL,
   idvar = NULL,
   infovar = NULL,
   varnames = NULL,
   valuenames = NULL,
   inforel = NULL,
   f,
   varnb,
   infovarnames
)
```

Arguments

tt

A (0,1)-matrix or a boolean matrix. The number of columns must match the number of elements (values) of the frame of discernment Θ . Each row is a subset of Θ . The last row is the frame Θ , represented by a vector of 1's.

m

A numeric vector of length equal to the number of rows of the matrix tt. Values of m must lie in the interval (0,1] and must add to one. The mass m(k) represents the chance value allotted to the proposition represented by the row k of the matrix tt.

cnames

A character vector containing the names of the elements of the frame of discernment Θ . The length must be equal to the number of elements of Θ . The names are first searched in the valuenames parameter. If NULL, column names of the matrix tt are taken if present. Otherwise, names are generated.

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con	The measure of conflict can be provided. 0 by default.
idvar	The number given to the variable. A number is necessary to manage relations between variables and make computations on a graph. 0 if omitted.
infovar	A two-column matrix containing variable identification numbers and the number of elements of the variable. Generated if omitted.
varnames	The name of the variable. Generated if omitted.
valuenames	A list of the names of the variables with the name of the elements of their frame of discernment.
inforel	Not used here. Defined within function bcaRe1.
f	Deprecated. Old name for tt matrix.
varnb	Deprecated. Old name for idvar.
infovarnames	Deprecated. Old name for varnames.

Value

y An object of class becaspec called a bea for "basic chance assignment":

- tt The table of focal elements. Rownames of the matrix of focal elements are generated from the column names of the elements of the frame. See nameRows for details.
- spec A two column matrix. First column contains numbers given to the subsets, 1 to nrow(tt). Second column contains the mass values of the subsets.
- con The measure of conflict.
- infovar The number of the variable and the size of the frame of discernment.
- varnames The name of the variable.
- valuenames A list of length 1 consisting of the name of the variable with the names of the elements of the frame of discernment (the column names of the tt matrix).
- inforel Set at 0. used in function bcaRel.

Author(s)

Claude Boivin, Stat.ASSQ

References

- Shafer, G., (1976). A Mathematical Theory of Evidence. Princeton University Press, Princeton, New Jersey, p. 38: Basic probability assignment.
- Guan, J. W. and Bell, D. A., (1991). Evidence Theory and its Applications. Elsevier Science Publishing company inc., New York, N.Y., p. 29: Mass functions and belief functions

```
tt<- t(matrix(c(1,0,1,1),ncol = 2))
m<- c(.9,.1)
cnames <- c("yes","no")
bca(tt, m)
bca(tt, m, cnames)
bca(tt, m, cnames, idvar = 1)
tt1<- t(matrix(c(1,0,1,1),ncol = 2))
colnames(tt1) <- c("yes", "no")
m <- c(.9, .1)</pre>
```

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bcaPrint

Simple printing of the tt matrix and mass values of a basic chance assignment (bca)

Description

This utility function does a simple printing of a bca

Usage

```
bcaPrint(x)
```

Arguments

Х

A list of class beaspec.

Value

A table of subsets with their associated mass. Subsets are identified by row names.

Author(s)

Claude Boivin, Stat.ASSQ

Examples

```
z \leftarrow bca(tt = matrix(c(0,1,1,1,1,0,1,1,1),nrow = 3, byrow = TRUE), m = c(0.2,0.5, 0.3), cnames = c("a", "b", "c"), varnb = 1) bcaPrint(z)
```

bcaRel

Representation of a mass function in a product space

Description

This function is used to represent a relation between two or more variables in their product space P. The relation can be described by more than one subset of P. Each subset can also include more than one element. Complete disjunctive coding is used to represent one element in the input matrix of the function.

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Usage

```
bcaRel(
   tt,
   spec,
   infovar,
   varnames,
   valuenames,
   relnb = NULL,
   infovarnames,
   infovaluenames
)
```

Arguments

tt The description matrix of the subsets establishing the relation. This matrix is

obtained by putting the variables side by side, as in a truth table representation. For each variable, there are as many columns as possible values. Each row of the matrix is an element of a subset. Each element is described by a sequence of 0 (absence of value of a variable) or 1 (presence of value). This forms a complete

disjunctive coding.

spec A two column matrix. First column: numbers assigned to the sub-assemblies.

Second column: the mass values of the sub-assemblies. If the subset has more than one element, the number of the subset and its associated mass value are

repeated to match the number of elements in the subset.

infovar A two column matrix containing variable identification numbers and the number

of elements of each variable. The identification numbers must be ordered in

increasing number.

varnames The names of the variables.

valuenames A list of the names of the variables with the name of the elements of their frame

of discernment.

relnb A number given to the relation. Set at 0 if omitted.

infovarnames Deprecated. Old name for varnames. infovaluenames Deprecated. Old name for valuenames.

Value

An object of class becaspec called a bea for "basic chance assignment". This is a list containing the following components:

- con The measure of conflict.
- tt The resulting table of subsets. Rownames of the matrix of subsets are generated from the column names of the elements of the product frame. See nameRows for details.
- spec The resulting two-column matrix of specification numbers with associated mass values.
- infovar The two-column matrix of variables number and size given in the input data.
- valuenames A list of the names of the variables with the name of the elements of their frame
 of discernment.
- inforel A two-column matrix containing the relation number and the depth (number of variables) of the relation.

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Author(s)

Claude Boivin, Stat.ASSQ

Examples

```
# A logical implication rule
# A typical relation between two variables is the
\# logical implication a -> b. Let us suppose
# that a stands for Rain: {yes, no} and b stands for
# Roadworks: {yes, no}. From experience,
# I am 75 % sure that there will be RoadWorks if there is no rain.
 # 1. The tt table of the logical implication
 ttrwf \leftarrow matrix(c(0,1,1,0,1,0,1,0,1,0,0,1,1,1,1,1)),
 nrow = 4, byrow = TRUE,
 dimnames = list(NULL, c("rWdy", "rWdn", "Ry", "Rn")) )
 # 2. The mass distribution
 specrw <- matrix(c(1,1,1,2,0.75,0.75,0.75,0.25), ncol = 2,
 dimnames = list(NULL, c("specnb", "mass")))
 # 3. Variables numbers and sizes
 inforw <- matrix(c(4,5,2,2), ncol = 2,
 dimnames = list(NULL, c("varnb", "size")) )
bcaRel(tt = ttrwf, spec = specrw, infovar = inforw,
 varnames = c("RdWorks", "Rain"), relnb = 6)
```

belplau

Calculation of the degrees of Belief and Plausibility of a basic chance assignment (bca).

Description

Degrees of Belief Bel and Plausibility Pl of the focal elements of a bca are computed. The ratio of the plausibility of a focal element against the plausibility of its contrary is also computed. Subsets with zero mass can be excluded from the calculations.

Usage

```
belplau(x, remove = FALSE)
```

Arguments

x A basic chance assignment mass function (see bca).

remove = TRUE: Exclude subsets with zero mass.

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Details

The degree of belief Bel is defined by:

$$bel(A) = Sum((m(B); B \subseteq A))$$

for every subset B of A.

The degree of plausibility pl is defined by:

$$pl(A) = Sum[(m(B); B \cap A \neq \emptyset]$$

for every subset B of the frame of discernment.

The plausibility ratio of a focal element A versus its contrary not A is defined by: Pl(A)/(1 - Bel(A)).

Value

A matrix of M rows by 3 columns is returned, where M is the number of focal elements:

- Column 1: the degree of belief Bel;
- Column 2: the degree of Plausibility P1;
- Column 3: the Plausibility ratio

Author(s)

Claude Boivin, Stat.ASSQ

References

- Shafer, G., (1976). A Mathematical Theory of Evidence. Princeton University Press, Princeton, New Jersey, p. 39-43.
- Williams, P., (1990). An interpretation of Shenoy and Shafer's axioms for local computation. International Journal of Approximate Reasoning 4, pp. 225-232.

10 decode

decode

Find the value in base 10 of a number coded in another base

Description

The aplDecode function of the project APL in R (https://rpubs.com/deleeuw/158476) has been adapted to follow the standard implementation of the APL decode function.

Usage

```
decode(base, ind)
```

Arguments

A scalar or a numeric vector which describes the number system in which the

data is coded.

ind The value to decode represented by a numeric vector in the base system.

Details

If the base value is a number system, e.g. base 2, we need only to enter it as a scalar, which is then processed to match the length of the expression to decode. If length(ind) is less than length(base), zeroes are added to the left of the vector ind to match the length of the two vectors. And vice-versa.

Value

A scalar representing the conversion of the coded number ind to its decimal representation.

Author(s)

Claude Boivin, Stat.ASSQ.

References

- Jan de Leeuw and Masanao Yajima (March 07, 2016) *APL in R (Version 009)*, Source code. https://rpubs.com/deleeuw/158476
- L. Gilman and A. J. Rose.(1974): APL an Interactive Approach, Second Edition, John Wiley, New York.
- APL 68000 Level II language manual. MicroAPL Ltd. 1990.

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d1fm

The Captain's Problem. dlfm: Relation between variables Departure delay (D), Loading delay (L), Forecast of the weather (F), Maintenance delay (M)

Description

This dataset is the tt matrix establishing the relation between the four variables. Each event (loading = true, forecast = foul, Maintenance = true) adds one day of Departure Delay. The elements (d,l,f,m) of $(D \times L \times F \times M)$ satisfying the relation form a subset with a mass value of 1. To construct the tt matrix, we put the variables D,L,F,M side by side, as in a truth table representation. Each 4-tuple of the subset is described by a row of the matrix as a vector of zeros and ones.

Usage

dlfm

Format

An integer matrix with 10 rows and 12 columns.

```
[1,c(1,2)] value = 0, not used
```

[1,3:12] Identification numbers of the four variables. Column 3 to 6: variable 2; columns 7,8: variable 4; columns 9, 10: variable 5: columns 11,12: variable 6.

nospec identification number of the specification

m the value of the specification, a number between 0 and 1

3 1 if d3 is part of the specification, 0 otherwise

2 1 if d2 is part of the specification, 0 otherwise

1 1 if d1 is part of the specification, 0 otherwise

0 1 if d0 is part of the specification, 0 otherwise

true 1 if true is part of the specification, 0 otherwise

false 1 if false is part of the specification, 0 otherwise

foul 1 if foul is part of the specification, 0 otherwise

fair 1 if fair is part of the specification, 0 otherwise

Author(s)

Claude Boivin, Stat.ASSQ

Source

https://www.researchgate.net/publication/265993533_Fusion_and_Propagation_in_Graphical_Belief_Models

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dotprod

Generalized inner product of two matrices

Description

The generalized inner product of two matrices combines two operators in the same manner as the classical inner product defined for the multiplication of two matrices. The number of rows of the second matrix must be equal the number of columns of the first matrix.

Usage

```
dotprod(x, y, g, f)
```

Arguments

```
x A matrix of M rows by K columns.

y A matrix of K rows by N columns.

g Any operator: +, -, *, /, &, l, ==, <=, paste etc.

f Any operator: +, -, *, /, &, l, ==, <=, paste etc.
```

Value

The result of the generalized inner product is returned.

Author(s)

Claude Boivin, Stat.ASSQ

```
print("Standard matrix product")
x \leftarrow y \leftarrow matrix(c(1:6), nrow = 2, byrow = TRUE)
dotprod(x, t(y), g = "+", f = "*") ## same as x %*% t(y)
print("Find some data x2 in the rows of a larger matrix y2")
x2 \leftarrow matrix(c(1,0,0,1,1,1), nrow = 2, byrow = TRUE)
y2 \leftarrow matrix(c(1,0,0,0,1,0,1,1,0,0,1,1,1,1,1))
nrow = 5, byrow = TRUE)
(1:nrow(y2)) * dotprod(x2, t(y2), g = "%", f = "==")
print("Find some names in a long list")
team_names <- matrix(c("Patrick", "Dole", "Amanda",</pre>
 "Dole", "Robert", "Calvin", "Alvina", "Klein",
  "Robert", "Gariepy", "Nellie", "Arcand"),
   ncol = 2, byrow = TRUE)
colnames(team_names) <- c("First_name", "Last_name")</pre>
print("Where in the list are the person with first name Robert and where are the Doles?")
BobandDoles <- matrix(c("Robert", "", "", "Dole"),</pre>
ncol = 2, byrow = TRUE)
dotprod(team_names, t(BobandDoles),g="|",f="==") * (1:nrow(team_names))
```

doubles 13

doubles

Remove duplicate rows in a two-dimensional table

Description

Remove duplicate rows in a two-dimensional table

Usage

```
doubles(x)
```

Arguments

Х

A matrix of numeric, character or logical type.

Value

The submitted matrix with duplicated rows removed from.

Author(s)

Claude Boivin, Stat.ASSQ

Examples

```
td0 <- matrix(c(rep(c(1,0,1),times=3),0,0,1,1,1,1, 1,1,1),ncol = 3,byrow = TRUE)
(doubles(td0))
td1 <- matrix(c(rep(c(1,0,1),times=3),0,0,1,1,1,1),ncol = 3,byrow = TRUE)
(doubles(td1))
td2 <- matrix(c(1:3, 1:3,4:6,1:3),nrow = 4,byrow = TRUE)
(doubles(td2))
td3 <- matrix(c("d","e","f", rep(c("a","b","cc"),times = 3),"g","h","i"),nrow = 5,byrow = TRUE)
(doubles(td3))
td4 <- matrix(as.logical(td1),nrow = 5,byrow = TRUE)
(doubles(td4))</pre>
```

dsrwon

Combination of two mass functions

Description

The unnormalized Dempster's rule is used to combine two mass functions mx and my defined on the same frame of discernment and described by their respective basic chance assignments x and y. Dempster's rule of combination is applied. The normalization is not done, leaving the choice to the user to normalize the results or not (for the normalization operation, see function nzdsr).

Usage

```
dsrwon(x, y, varnames = NULL, relnb = NULL, infovarnames)
```

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Arguments

X	A basic chance assignment (see bca).
У	A A basic chance assignment (see bca).
varnames	A character string to name the resulting variable. named "z" if omitted.
relnb	Identification number of the relation. Can be omitted.
infovarnames	Deprecated. Old name for varnames.

Details

The two bca's x and y must be defined on the same frame of discernment for the combination to take place. The relation number of the x input is given to the output result.

Value

A basic chance assignment with these two components added:

- I12 Intersection table of subsets.
- Sort_order Sort order of subsets.

Author(s)

Claude Boivin, Stat.ASSQ

References

Shafer, G., (1976). A Mathematical Theory of Evidence. Princeton University Press, Princeton, New Jersey, pp. 57-61: Dempster's rule of combination.

Examples

```
x1 <- bca(tt = matrix(c(0,1,1,1,1,0,1,1,1),nrow = 3,
byrow = TRUE), m = c(0.2,0.5, 0.3),
cnames = c("a", "b", "c"),
varnames = "x", varnb=1)
x2 <- bca(tt = matrix(c(1,0,0,1,1,1),nrow = 2,
byrow = TRUE), m = c(0.6, 0.4),
cnames = c("a", "b", "c"),
varnames = "x", varnb = 1)
dsrwon(x1,x2)
frame <- bca(matrix(c(1,1,1), nrow = 1), m = 1, cnames = c("a","b","c"))
dsrwon(frame, frame)</pre>
```

dst

Manipulation and combination of belief functions

Description

Package dst allows you to make basic probability assignments on subsets of a set of possibilities (events) and combine these events with Dempster's rule of combination.

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Details

The main operations that can be done are:

- definition of a basic chance assignment (bca) distribution on a variable,
- combination of two bca's defined on the same variable,
- definition of a bca which establish a relation between two or more variables,
- extension of a bca,
- · marginalization of a bca.

Author(s)

Claude Boivin, Stat.ASSQ <webapp.cb@gmail.com>

elim

Reduction of a relation

Description

This function works on a relation defined on a product of two variables or more. Having fixed a variable to eliminate from the relation, the reduced product space is determined and the corresponding reduced bca is computed. This operation is also called "marginalization".

Usage

```
elim(rel, xnb)
```

Arguments

rel The relation to reduce, an object of class beaspec.

xnb Identification number of the variable to eliminate.

Value

r The reduced relation

Author(s)

Claude Boivin, Stat.ASSQ

```
# We construct a relation between two variables to show marginalization. wr_tt <- matrix(c(1,rep(0,3),rep(c(1,0),3),0,1,1,1,0,0, 1,0,rep(1,5),0,1,1,0,rep(1,5)), ncol = 4, byrow = TRUE) colnames(wr_tt) <- c("Wy Ry", "Wy Rn", "Wn Ry", "Wn Rn") rownames(wr_tt) <- nameRows(wr_tt) wr_spec = matrix(c(1:8, 0.017344, 0.046656, 0.004336, 0.199456,0.011664,0.536544,0.049864, 0.134136), ncol = 2, dimnames = list(NULL, c("specnb", "mass"))) wr_infovar = matrix(c(4,5,2,2), ncol = 2,
```

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```
dimnames = list(NULL, c("varnb", "size")) )
wr_rel <- list(tt = wr_tt, con = 0.16, spec=wr_spec,
   infovar = wr_infovar, varnames = c("Roadworks","Rain"),
   valuenames = list( RdWorks = c("Wy", "Wn"), Rain=c("Ry", "Rn") ))
class(wr_rel) <- "bcaspec"
bcaPrint(elim(wr_rel, xnb = 5))
bcaPrint(elim(wr_rel, xnb = 4))</pre>
```

encode

Convert a value to its representation in another chosen base

Description

The aplEncode function of the project APL in R (https://rpubs.com/deleeuw/158476) has been adapted to follow the standard implementation of the APL encode function.

Usage

```
encode(base, ind)
```

Arguments

A numeric vector which describes the number system in which we want to re-

code the data.

ind The value to convert represented by a number or a numeric vector.

Value

A vector or a matrix of the data converted.

Author(s)

Claude Boivin, Stat.ASSQ.

References

- Jan de Leeuw and Masanao Yajima (March 07, 2016) *APL in R (Version 009)*, Source code. https://rpubs.com/deleeuw/158476
- L. Gilman and A. J. Rose.(1974): *APL an Interactive Approach*, Second Edition, John Wiley, New York.
- APL 68000 Level II language manual. MicroAPL Ltd. 1990.

```
encode(c(2,2,2,2), 11) # find the base 2 representation of number 11 encode(c(365,24,60), 2997) # convert 2997 minutes to days-hrs-min.
```

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extmin

Extension of a relation

Description

This function works on a basic chance assignment (bca) x defined on a single variable or more. A relation of reference is given, and an extension of the space of x is made to the larger product space of the relation of reference. The basic chance assignment to extend and the relation of reference must have at least one common variable for the extension to occur.

Usage

```
extmin(rel1, relRef)
```

Arguments

rel1 An object of class beaspec, i.e. a basic chance assignment defined on one vari-

able or a relation.

relRef The relation of reference. It can be an existing relation, or it can be constructed

as a vacuous function.

Details

The relRef parameter is used to extract all the information on the variables, namely their identification numbers and the number of elements of each variable, variables names and columns names of the tt matrix. The relation of reference relRef may be a relation already existing or simply the the vacuous relation defined on the product set of variables of interest.

Value

the resulting extended bca.

Author(s)

Claude Boivin, Stat.ASSQ

References

G. Shafer and P. P. Shenoy. Local Computations in Hypertrees. School of Business, University of Kansas, Lawrence, KS, 1991. See p. 78, vacuous extension of a belief function.

```
# Making a vacuous reference relation and extending a bca to its space.
init_tt = matrix(rep(1,10),nrow = 1,
dimnames = list(NULL, c("3", "2", "1", "0",
   "true", "false", "foul", "fair", "true", "false")) )
init_spec <- matrix(c(1,1), ncol = 2,
dimnames = list(NULL, c("specnb", "mass")))
init_info <- matrix(c(3,4,7,8,4,2,2,2), ncol = 2,
   dimnames = list(NULL, c("varnb", "size")) )
relRef <- bcaRel(tt = init_tt, spec = init_spec,</pre>
```

18 fw

```
infovar = init_info,
  varnames = c("Sail", "Loading", "Weather", "Repairs"),
  relnb = 0)
# a bcaspec defined on one variable
l_rel <- bca(tt = matrix(c(1,0,1,0,1,1), ncol = 2),
  m = c(0.3,0.5,0.2), cnames = c("true", "false"),
  infovar = matrix(c(4,2), ncol = 2,
  dimnames = list(NULL, c("varnb", "size"))),
  varnames = c("Loading"),
  inforel = matrix(c(7,1), ncol = 2,
  dimnames = list(NULL, c("relnb", "depth"))))
  z <- extmin(l_rel, relRef)
  prmatrix(t(z$tt), collab = rep("", nrow(z$tt)))</pre>
```

fw

The Captain's Problem. fw: Relation between variables Forecast of the weather (F) and Weather at sea (W)

Description

This dataset is the tt matrix establishing the relation between the two variables. An accurate forecast is described by this subset of two events: (Forecast = foul, Weather = foul) and (Forecast = fair, Weather = fair). We assign a mass value of 0.8 to this subset. The remaining mass of 0.2 is allotted to the frame. To construct the tt matrix, we put the variables F and W side by side, as in a truth table representation. Each pair of the subset is described by a row of the matrix as a vector of zeros and ones.

Usage

fw

Format

An integer matrix with 4 rows and 6 columns.

```
[1,c(1,2)] value = 0, not used
```

[1,3:6] Identification numbers of the two variables. Column 3,6: variable 5; columns 5,6: variable 7.

nospec identification number of the specification

m the value of the specification, a number between 0 and 1

foul 1 if foul is part of the specification, 0 otherwise

fair 1 if fair is part of the specification, 0 otherwise

Author(s)

Claude Boivin, Stat.ASSQ

Source

https://www.researchgate.net/publication/265993533_Fusion_and_Propagation_in_Graphical_ Belief_Models inters 19

inters

Intersection of two tables of propositions

Description

Function inters returns a table of the intersection between two (0,1) or boolean matrices or two vectors. The two matrices must have the same number of columns. The two vectors must be of the same length. This function generalizes the intersection of two subsets represented by boolean vectors to the intersection of two matrices of subsets.

Usage

```
inters(x, y)
```

Arguments

x A (0,1)-matrix or a boolean matrix of M rows by K columns, or a vector of length K.

y A (0,1)-matrix or a boolean matrix of N rows by K columns or a vector of length K.

Value

The result is a (0,1)-table of dimensions $(M \times K) \times N$. In the case of vectors, the result is a (0,1)-table of dimensions $(1 \times K) \times 1$

Author(s)

Claude Boivin, Stat.ASSQ

```
mx <- matrix(c(0,1,0,0,1,1,1,1,1),nrow = 3, byrow = TRUE, dimnames = list(NULL, c("a", "b", "c")))
  rownames(mx) <- nameRows(mx)

my<-matrix(c(0,0,1,1,1,1),nrow = 2, byrow = TRUE, dimnames = list(NULL, c("a", "b", "c")))
  rownames(my) <- nameRows(my)

inters(mx,my)

b1 <- c(FALSE, TRUE, TRUE)

b2 <- c(TRUE, TRUE, FALSE)

names(b1) <- names(b2) <- c("c1","c2","c3")

inters(b1,b2)

x3<-matrix(c(1,1,0,1), ncol = 2, dimnames = list(NULL, c("a","b")))

y3<-matrix(c(0,1,1,1), ncol = 2, dimnames = list(NULL, c("a","b")))

inters(x3,y3)

x4 <-matrix(c(1,0,1,1,1,1,1,1),nrow = 2, byrow = TRUE, dimnames = list(NULL, c("a", "b", "c","d")))

y4 <-matrix(c(1,0,0,1,1,1,1,1,1),nrow = 2, byrow = TRUE, dimnames = list(NULL, c("a", "b", "c","d")))

inters(x4,y4)</pre>
```

20 matrixToMarray

marrayToMatrix	Transformation of an array data to its matrix representation
iliai i ay i oria ti 1x	Transformation of an array acid to its matrix representation

Description

The array representation or product space representation is converted to the matrix representation of the corresponding relation.

Usage

```
marrayToMatrix(mtt)
```

Arguments

mtt

The matrix tt of the relation in array format

Value

The matrix representation of the data.

Author(s)

Claude Boivin, Stat.ASSQ

Examples

matrixToMarray

Transformation of the tt matrix of a relation

Description

The matrix representation of a relation is converted to the array representation or product space representation.

Usage

```
matrixToMarray(tt, valuenames)
```

Arguments

tt A(0,1)-matrix or a boolean matrix establishing the relation between two or more

variables. The matrix is constructed by placing the variables side by side, as in

a truth table representation.

valuenames A list of the names of the variables with the name of each value of their frame

of discernment.

mrf 21

Value

mtt The array (product space) representation of the tt matrix.

Author(s)

Claude Boivin, Stat.ASSO

Examples

```
# Define wr_tt, a matrix describing the relation between two variables wr_tt <- matrix(c(1,rep(0,3),rep(c(1,0),3),0,1,1,1,0,0,1,0,rep(1,5),0,1,1,0,rep(1,5)), ncol=4, byrow = TRUE) colnames(wr_tt) <- c("Wy Ry", "Wy Rn", "Wn Ry", "Wn Rn") rownames(wr_tt) <- nameRows(wr_tt) vars = list( RdWorks = c("Wy", "Wn") , Rain = c("Ry", "Rn")) print(zmToa <- matrixToMarray(tt = wr_tt, valuenames = vars ) )
```

mrf

The Captain's Problem. mrf: Relation between variables No Maintenance (M = false) and Repairs at sea (R)

Description

This dataset is the tt matrix establishing a set of two relations between the two variables. First, Repairs = true if Maintenance = false in $(M \times R)$. We are 20% sure that there will be Repairs if no maintenance. Second, Repairs = false if Maintenance = false in $(M \times R)$. We are 20% sure that there will be no repairs if no maintenance.

Usage

mrf

Format

A (0,1) matrix with 4 rows and 6 columns.

```
[1,c(1,2)] value = 0, not used
```

[1,3:6] Identification numbers of the two variables. Column 3,4: variable 6; columns 5,6: variable 8

nospec identification number of the specification

m the value of the specification, a number between 0 and 1

true 1 if true is part of the specification, 0 otherwise

false 1 if false is part of the specification, 0 otherwise

Details

These two relations are implication rules. The remaining mass of 0.6 is allotted to the frame. To construct the tt matrix, we put the variables M and R side by side, as in a truth table representation. Each pair of the subset is described by a row of the matrix as a vector of zeros and ones.

22 mrt

Author(s)

Claude Boivin, Stat.ASSQ

Source

https://www.researchgate.net/publication/265993533_Fusion_and_Propagation_in_Graphical_Belief_Models

mrt

The Captain's Problem. mrt: Relation between variables Maintenance done (M = true) and Repairs at sea (R)

Description

This dataset is the tt matrix establishing a set of two relations between the two variables. First, Repairs = true if Maintenance = true in $(M \times R)$. We are 10% sure that there will be Repairs if maintenance is done. Second, Repairs = false if Maintenance = true in $(M \times R)$. We are 70% sure that there will be no repairs if maintenance is done.

Usage

mrt

Format

A (0,1) matrix with 4 rows and 6 columns.

[1,c(1,2)] value = 0, not used

[1,3:6] Identification numbers of the two variables. Column 3,4: variable 6; columns 5,6: variable

nospec identification number of the specification

m the value of the specification, a number between 0 and 1

true 1 if true is part of the specification, 0 otherwise

false 1 if false is part of the specification, 0 otherwise

Details

These two relations are implication rules. The remaining mass of 0.2 is allotted to the frame. To construct the tt matrix, we put the variables M and R side by side, as in a truth table representation. Each pair of the subset is described by a row of the matrix as a vector of zeros and ones.

Author(s)

Claude Boivin, Stat.ASSQ

Source

https://www.researchgate.net/publication/265993533_Fusion_and_Propagation_in_Graphical_Belief_Models

nameCols 23

Description

This utility function makes use of the valuenames and size parameters of a set of variables to assign values names to the columns of a tt matrix.

Usage

```
nameCols(valuenames, size)
```

Arguments

valuenames A list of the names of the variables with the name of the elements of their frame

of discernment.

size A vector of the size of the variables.

Value

A character vector of length equal to the sum of the sizes of the variables.

Author(s)

Claude Boivin, Stat.ASSQ

Examples

```
infoval <- list(A = c("a1", "a2"), B = c("b1", "b2", "b3")) sizes <- c(2,3) print(nameCols(valuenames = infoval, size = sizes))
```

nameCols_prod

Naming the columns of the tt matrix of a product space

Description

This utility function makes use of the valuenames and size parameters of a set of variables to assign values names to the columns of the tt matrix of their product space.

Usage

```
nameCols_prod(valuenames, size)
```

Arguments

valuenames A list of the names of the variables with the name of the elements of their frame

of discernment.

size A vector of the size of the variables.

24 nameRows

Value

A character vector of length equal to the product of the sizes of the variables.

Author(s)

Claude Boivin, Stat.ASSQ

Examples

```
infoval <- list(A = c("a1", "a2"), B = c("b1", "b2", "b3")) sizes <- c(2,3) print(nameCols_prod(valuenames = infoval, size = sizes))
```

nameRows

Combining the column names of a matrix to construct names for the rows

Description

This function determines the name of a row from all the columns of the tt that show 1 for that row.

Usage

```
nameRows(tt, f)
```

Arguments

tt A(0,1)-matrix or a boolean matrix. f Deprecated. Old name for tt matrix.

Details

The row containing only 1s is called "frame", to avoid too long a label. The empty set is identified by its code "u00f8". The "+" sign is used to represent the logical "or" operator. The space " " is used to represent the logical "and" operator. Note that in the case of a product space defined on many variables, row labels can become very long.

Value

A character vector of labels obtained for the rows of the tt matrix. The length of the result is nrow(tt).

Author(s)

Claude Boivin, Stat.ASSQ

nzdsr 25

nzdsr

Normalization of a basic chance assignment

Description

It may occur that the result of the combination of two basic chance assignments with Dempster's Rule of combination contains a non-zero mass allocated to the empty set. The function nzdsr normalizes the result of function dsrwon by dividing the mass value of the non-empty subsets by 1 minus the mass of the empty set.

Usage

```
nzdsr(x)
```

Arguments

Х

A basic chance assignment, i.e. a object of class beaspec.

Value

z The normalized basic chance assignment.

Author(s)

Claude Boivin, Stat.ASSQ

References

Shafer, G., (1976). A Mathematical Theory of Evidence. Princeton University Press, Princeton, New Jersey, pp. 57-61: Dempster's rule of combination.

```
x1 <- bca(tt= matrix(c(1,0,1,1),nrow = 2, byrow = TRUE), m = c(0.9,0.1), cnames = c("yes", "no"), varnames = "x", varnb = 1)  
x2 <- bca(tt = matrix(c(0,1,1,1),nrow = 2, byrow = TRUE), m = c(0.5,0.5), cnames = c("yes", "no"), varnames = "x", varnb = 1)  
print("combination of x1 and x2")  
x1x2 <- dsrwon(x1,x2, varname = "x")  
nzdsr(x1x2)  
print("normalization of a bca definition.")  
y2 <- bca(f = matrix(c(0,0,0,1,0,0,1,1,1),nrow = 3, byrow = TRUE), m = c(0.2,0.5,0.3), cnames = c("a", "b", "c"), varnb = 1)  
nzdsr(y2)
```

26 peeling

peeling	The peeling algorithm	

Description

An implementation of the peeling algorithm based on its description in terms of hypergraphs by R. Almond [1989].

Usage

```
peeling(vars_def, hgm, hg_rel_names, elim_order, verbose = FALSE)
```

Arguments

vars_def	A list of variables and their possible values. Concatenate the valuenames pa-
	rameter of all the variables of the hypergraph to obtain this list.

hgm The incidence matrix of the hypergraph (bipartite graph), which is the descrip-

tion of the relations between the variables. The variables are the nodes of the hypergraph, and the relations are the edges. Each column describes a relation between the variables by a (0,1) vector. A "1" indicates that a variable belongs to the relation and a "0" not. This matrix must have row and column names. These names are used to show the graph eventually. They need not be the same as variables and relations names of the set of bca's to be analyzed. Use short

names to obtain a clear graph.

hg_rel_names The names of the relations, which are objects of class "bcaspec".

elim_order The order of elimination of the variables. A vector of length nrow(hgm). Vari-

ables are identified by numbers. The first number gives the first variable to

eliminate. The variable of interest comes last.

verbose = TRUE: print steps on the console. Default = FALSE.

Details

The peeling algorithm works on an undirected graph. Nodes of the graph (variables) are removed one by one until only the variable of interest remains. An order of elimination (peeling) of the variables must be chosen by the user. No algorithm is provided for that matter. At each step, a procedure of extension is applied to the bca's to merge, and marginalization is applied to eliminate a variable. The marginalization has the effect to integrate in the remaining nodes the information of the eliminated variable.

Value

A bca class object.

Author(s)

Claude Boivin, Stat.ASSQ

peeling 27

References

• Almond, R. G. (1989) Fusion and Propagation of Graphical Belief Models: An Implementation and an Example. Ph. D. Thesis, the Department of Statistics, Harvard University. 288 pages (for the description of the peeling algorithm, see pages 52-53).

```
# Zadeh's Example
# 1. Defining variables and relations
# (for details, see vignette: Zadeh_Example)
e1 <- bca(tt = matrix(c(1,0,0,1,1,1)), ncol = 2, byrow = TRUE),
m = c(0.99, 0.01, 0), cnames = c("M", "T"),
varnames = "D1", idvar = 1)
e2 \leftarrow bca(tt = matrix(c(1,0,0,1,1,1), ncol = 2, byrow = TRUE),
m = c(0.99, 0.01, 0), cnames = c("C", "T"),
varnames = "D2", idvar = 2)
p_{diag} \leftarrow bca(tt = matrix(c(1,1,1), ncol = 3, byrow = TRUE),
m = 1, cnames = c("M", "T", "C"),
varnames = "D", idvar = 3)
# Defining the relation between the variables
# tt matrix
tt_r1 \leftarrow matrix(c(1,0,1,0,1,0,0,1,0,1,0,0,0,1,
1,0,0,1,1,0,0,1,0,0,1,0,1,0,0,1,1,0,0,1,0,
ncol = 7,byrow = TRUE
colnames(tt_r1) = c("M", "T", "C", "T", "M", "T", "C")
# The mass function
spec_r1 \leftarrow matrix(c(rep(1,7),2, rep(1,7), 0), ncol = 2, dimnames = list(NULL, c("specnb", "mass")))
# Variables numbers and dimension of their frame
info_r1 \leftarrow matrix(c(1:3, 2,2,3), ncol = 2, dimnames = list(NULL, c("varnb", "size")))
# The relation between e1, e2 and a patient p
r1 <- bcaRel(tt = tt_r1, spec = spec_r1, infovar = info_r1,
 varnames = c("D1", "D2", "D"), relnb = 1)
# 2. Setting the incidence matrix of the grapph
rel1 <- 1*1:3 %in% r1$infovar[,1]
ev1 <- 1*1:3 %in% e1$infovar[,1]
ev2 <- 1*1:3 %in% e2$infovar[,1]
meddiag_hgm <- matrix(c(ev1,ev2, rel1), ncol = 3,</pre>
dimnames = list(c("D1", "D2", "D"), c("e1", "e2", "r1")))
# 3. Setting the names of the variables and their frame of discernment
meddiag_vars1 <- c(e1$valuenames, e2$valuenames, p_diag$valuenames)</pre>
# 4. Names of bca specifications (evidence and relations)
meddiag_rel_names <- c("e1", "e2", "r1")</pre>
# 5. Order of elimination of variables
elim_order <- c(1,2,3)
tabresul(peeling(vars_def = meddiag_vars1, hgm = meddiag_hgm,
hg_rel_names = meddiag_rel_names, elim_order = c(1, 2, 3)) )
```

28 plautrans

plautrans

Plausibility transformation of the singletons of a frame

Description

Given a mass function defined on some subsets of a frame Θ , the application of the plausibility transformation to the singletons of Θ yields the probability distribution associated with this mass function.

Usage

```
plautrans(x)
```

Arguments

Х

A bca mass function.

Details

We compute the plausibility measure of all the singletons of the frame of discernment. The probability distribution of the singletons is obtained from their plausibility measures.

Value

The matrix of singletons with their plausibility transformation added in the last column.

Author(s)

Claude Boivin, Stat.ASSQ

References

Cobb, B. R. and Shenoy, P.P. (2006). On the plausibility transformation method for translating belief function models to probability models. Journal of Approximate Reasoning, 41(3), April 2006, 314–330.

```
x \leftarrow bca(tt = matrix(c(0,1,1,1,1,0,1,1,1),nrow = 3, byrow = TRUE), m = c(0.2,0.5, 0.3), cnames = c("a", "b", "c"), varnames = "x", varnb = 1) plautrans(x)
```

productSpace 29

produce space representation of a retation	productSpace	Product space representation of a relation
--	--------------	--

Description

This utility function takes the input matrix of a relation between two or more variables and yields its product space representation.

Usage

```
productSpace(tt, specnb, infovar)
```

Arguments

tt	A (0,1) or boolean matrix, where the variables are set side by side, as in a truth table. Each variable has a number of columns equal to the number of possible values.
specnb	A vector of integers ranging from 1 to k, where k is the number of subsets of the tt matrix. Values must start at one and can be increased by 1 or not. They determine the partitioning of the rows of the tt matrix between the k subsets.
infovar	A two-column matrix containing identification numbers of the variables and the number of elements of each variable (size of the frame).

Value

The matrix of the product space representation of the relation.

Author(s)

Claude Boivin, Stat.ASSQ

```
ttfw <- matrix(c(1,0,1,0,0,1,0,1,1,1,1,1),nrow = 3,
  byrow = TRUE,
  dimnames = list(NULL, c("foul", "fair", "foul", "fair")))
specfw <- c(1,1,2)
infovarfw <- matrix(c(5,7,2,2), ncol = 2,
  dimnames = list(NULL, c("varnb", "size")))
rownames(ttfw) <- nameRows(ttfw)
ttfw
productSpace(tt = ttfw, specnb = specfw, infovar = infovarfw)</pre>
```

30 reduction

reduction

Summary of a vector for any operator.

Description

This utility function is used to obtain a summary of a vector of data for many operators. The function is taken from the project APL in R (https://rpubs.com/deleeuw/158476).

Usage

```
reduction(x, f)
```

Arguments

x A vector of numbers or a character string.

f The operator. Must be compatible with the type of input vector (numeric or character)

Value

The result of applying the chosen operator to all the elements of the vector is an object of length 1.

Author(s)

Claude Boivin, Stat.ASSQ.

References

- Jan de Leeuw and Masanao Yajima (March 07, 2016) *APL in R (Version 009)*, Source code. https://rpubs.com/deleeuw/158476
- G. Helzer. (1989): An Encyclopedia of APL, second edition, I-APL LTD, St. Albans, G.B.
- L. Gilman and A. J. Rose.(1974): *APL an Interactive Approach*, Second Edition, John Wiley, New-York.

```
reduction(c(1,2,3,4), f = "-")
reduction(c(1,0,1,1,0), f = "|")
reduction(c("a", "b", "c"), f = "paste")
```

shape 31

shape	Obtain dimensions of an array or length of a vector with a single command
	munu

Description

shape returns the dimension of given array or returns the length of a given vector. The function is taken from the project APL in R (https://rpubs.com/deleeuw/158476).

Usage

```
shape(a)
```

Arguments

а

An array or a vector.

Value

The dimension of the array a or the length of the vector a.

Author(s)

Claude Boivin, Stat.ASSQ.

References

- Jan de Leeuw and Masanao Yajima (March 07, 2016) APL in R (Version 009), Source code. https://rpubs.com/deleeuw/158476
- G. Helzer. (1989): An Encyclopedia of APL, second edition, I-APL LTD, St. Albans, G.B.
- L. Gilman and A. J. Rose.(1974): *APL an Interactive Approach*, Second Edition, John Wiley, New-York.

Examples

```
shape(array(c(1:6), c(2,3)))
shape(c("a", "b"))
```

swr

The Captain's Problem. swr: Relation between variables Sailing delay (S), Weather at sea (W), and Repairs at sea (R)

Description

This dataset is the tt matrix establishing a relation between S, W and R, where S = 0.3, W = foul, fair and R = true, false. The goal of this relation is to account for other causes of sailing delay. All the elements (s,w,r) of $(S \times W \times R)$ where W or R is true add one day of sailing delay. We put a mass value of 0.9 to this subset. The remaining mass of 0.1 is allotted to the frame.

32 tabresul

Usage

swr

Format

An integer matrix with 6 rows and 10 columns.

[1,c(1,2)] value = 0, not used

[1,3:10] Identification numbers of the three variables. Column 3 to 6: variable 3; columns 7,8: variable 7, columns 9,10: variable 8

nospec identification number of the specification

m the value of the specification, a number between 0 and 1

3 1 if 3 is part of the specification, 0 otherwise

2 1 if 2 is part of the specification, 0 otherwise

1 1 if 1 is part of the specification, 0 otherwise

0 1 if 0 is part of the specification, 0 otherwise

foul 1 if foul is part of the specification, 0 otherwise

fair 1 if fair is part of the specification, 0 otherwise

true 1 if true is part of the specification, 0 otherwise

false 1 if false is part of the specification, 0 otherwise

Details

To construct the tt matrix, we put the variables S, W, R side by side, as in a truth table representation. Each triplet of the subset is described by a row of the matrix as a vector of zeros and ones.

Author(s)

Claude Boivin, Stat.ASSQ

Source

https://www.researchgate.net/publication/265993533_Fusion_and_Propagation_in_Graphical_ Belief_Models

tabresul

Prepare a table of results

Description

This utility function is a more detailed version of the belplau function. Different tables of measures of belief, plausibility and of the plausibility ratio can be obtained, namely by removing subsets with zero mass if present, or by asking for singletons only.

Usage

```
tabresul(x, singletonsOnly = FALSE, removeZeroes = FALSE)
```

tabresul 33

Arguments

```
x A basic chance assignment (bca)
singletonsOnly = TRUE reduces the table of results to elementary events (singletons).
removeZeroes = TRUE removes subsets with 0 mass.
```

Value

A list of two elements:

- mbp The table of focal elements with the addition of their associated mass, degree of belief, plausibility and the plausibility ratio.
- con The measure of conflict between subsets.

Author(s)

Claude Boivin, Stat.ASSQ

```
x \leftarrow bca(tt = matrix(c(0,1,1,1,1,0,1,1,1),nrow = 3,
byrow = TRUE), m = c(0.2, 0.5, 0.3),
cnames = c("a", "b", "c"),
varnames = "x", idvar = 1)
y \leftarrow bca(tt = matrix(c(1,0,0,1,1,1),nrow = 2,
byrow = TRUE), m = c(0.6, 0.4),
cnames = c("a", "b", "c"), varnames = "y", idvar = 1)
xy <- dsrwon(x,y)</pre>
xyNorm <- nzdsr(xy)</pre>
tabresul(xyNorm)
## print("Show all elementary events")
xy1 <- addTobca(nzdsr(dsrwon(x,y)),</pre>
matrix(c(0,1,0,0,0,1),
nrow = 2, byrow = TRUE))
tabresul(xy1)
## print("Remove focal elements with 0 mass")
tabresul(xy1, removeZeroes = TRUE)
print("Retain singletons only")
tabresul(xy1, singletonsOnly = TRUE)
```

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