Overview of durmod

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Abstract

This is a walkthrough of an estimation of a generated dataset with the **durmod** package. Also, various tunable parameters and details are provided.

1 A dataset

The **durmod** package fits a mixed proportional hazard model with competing risks to duration data. The model is the one from [Gaure et al., 2007], which was developed in [Heckman and Singer, 1984] based on results in [Lindsay, 1983a, Lindsay, 1983b].

Let's have a look at a generated dataset which simulates an unemployment register with two competing risks. It was generated by durdata <- datagen(5000,400).

```
library(durmod)
data(durdata)
head (durdata, 15)
##
                   x1
                                 x2 alpha
                                                 d
                                                     duration
                                                                    state
##
        1 -0.3317134 -1.34060625
                                        0
                                               job
                                                     0.230954
                                                                    unemp
##
        2 -0.6737615 -0.46594276
                                        0
                                                    13.327312
                                              none
                                                                    unemp
##
    3:
        2 -1.6495569
                       1.54143453
                                        0
                                                    44.964614
                                              none
                                                                    unemp
        2 -2.4295179
##
                       0.37094254
                                        0
                                              none 103.924002
                                                                    unemp
##
        2 -1.6927724
                       1.43136302
                                        0
                                                    28.231373
                                                                    unemp
                                              none
##
    6:
        2 -1.7235513 -0.15882989
                                        0
                                                     9.888351
                                              none
                                                                    unemp
##
    7:
        2 -2.7544434
                       1.51210768
                                        0
                                                    70.451252
                                              none
                                                                    unemp
        2 -4.3062211
##
    8:
                        3.40174960
                                        0
                                              none
                                                    24.068327
                                                                    unemp
    9:
        3
           0.7160858
                                                     5.759026
##
                        0.07710256
                                          program
                                                                    unemp
## 10:
        3
           0.7160858
                       0.07710256
                                        1
                                                     2.085667 onprogram
                                              none
        3 -0.5252669 -0.05042386
## 11:
                                        1
                                              none
                                                     4.710827 onprogram
## 12:
        3 -1.7092856 -0.24886947
                                        1
                                                     2.305123
                                                               onprogram
                                               job
        4 -1.4664934 -1.62605988
                                        0
                                               job
                                                     1.591296
                                                                    unemp
        5 -0.3070180
                                                     0.704823
   14:
                        1.63896593
                                        0
                                              none
                                                                    unemp
        5 -1.5294205
                       0.81822933
                                        0
                                              none
                                                    24.497315
                                                                    unemp
```

There is an id which identifies an individual. The individuals have been through a process. At the outset they are all unemployed, this is recorded by the factor state. As unemployed they face two hazards, i.e. probabilites per time unit. Either they can get a job, or they can enter a labour market programme, like a subsidized wage job or similar.

These transitions are recorded in the d factor. In our simulation, individuals who transition to "job", exit the dataset. If a transition to labour market programme occurs, the state variable changes to "onprogram", and the dummy alpha changes to 1. It is also possible to do a "none" transition, this is typically necessary if a covariate changes, since the model has piecewise constant explanatory covariates. Also, when on a programme, one of the hazards disappear, it is no longer possible to make a transition to a programme, we're already on it.

Each row of the dataset has a duration, this is the time until the transition marked in d occurs.

The rows of an individual are independent, they can be reordered, but the dataset must be sorted on individuals, i.e. the rows for an individual must be together. I suggest using data tables (from the data.table package), then it is easy to sort on individuals: setorder(data,id). If you have factors with hundreds or thousands of levels, it can be time-consuming to compute the Fishermatrix. However, the algorithm which creates the Fisher matrix is more efficient if individuals with almost the same values for these factors (e.g. county, or company) are close to each other in the dataset.

In our dataset, we have an observation period of 400, and the individuals enter the dataset at a random time in this period. When they reach the end of the observation period they are no longer observed. That means that some individuals do not exit the dataset by doing a transition, but with a d=="none".

$\mathbf{2}$ The mixed proportional hazard competing risk model

There are two covariates, x1 and x2. These are assumed to influence the two hazards. We also assume the alpha enters the hazard.

We model the baseline hazard for transition to job as,

$$h^{j}(\mu^{j}) = \exp(x_{1}\beta_{1}^{j} + x_{2}\beta_{2}^{j} + \alpha\beta_{3}^{j} + \mu^{j})$$
(1)

The hazard for transition to programme is,

$$h^{p}(\mu^{p}) = \exp(x_{1}\beta_{1}^{p} + x_{2}\beta_{2}^{p} + \mu^{p})$$
(2)

Here we have included an "intercept", a μ , it could equally well have been written as a multiplicative factor $\exp(\mu)$ instead. This $\exp(\mu)$ -term is the "proportional hazard".

The likelihood for a single observation k consists of two parts. Let $H(\mu) = h^j(\mu^j) + h^p(\mu^p)$ be the sum of the hazards, where μ is the vector (μ^j, μ^p) .

For an observation k there is a survival probability/density up until the transition:

$$s_k(\mu) = \exp(-t_k H(\mu)),\tag{3}$$

where t_k is the duration of the period.

If there is a transition, $s(\mu)$ is multiplied by the transition hazard, $h^d(\mu)$, where d is either p or j. If there is no transition, $h^d(\mu)$ is taken to be 1. Taken together, all the observations for an individual i yields an individual likelihood. We call it $\ell_i(\mu)$.

$$\ell_i(\mu) = \prod_{k \in K_i} h^{d_k}(\mu) s_k(\mu), \tag{4}$$

where K_i is the set of observations for individual i.

However, there is also a mixture part, designed to account for unobserved individual heterogeneity. The μ -vector is stochastic with a discrete distribution. That is, there is an n, a set of probabilites p_j , and vectors μ_j , for j=1..n. Of course, we have $\sum_{j=1}^n p_j=1$. The mixture likelihood for an individual i is $L_i=\sum_j p_j\ell_i(\mu_j)$. The log-likelihood for the dataset is thus, $L=\sum_i \log(L_i)$.

The L must be maximized with respect to the five β s, the n, the probabilities p_i , and the vectors μ_j for j = 1..n.

Estimation 3

The estimation proceeds as follows. We start with n=1, estimate the β s and the two μ s. Then we increase n to 2, let $p_2 = 0$, and find a vector μ_2 such that the derivative of the log-likelihood in the direction of positive p_2 is positive. This is used as starting point for a new likelihood maximization. Then n is increased to 3, and we proceed in this fashion, adding masspoints to the distribution until we are no longer able to increase the likelihood.

In durmod we use the mphcrm function for this purpose. Here is an example. First we create a "riskset", a specification of which hazards are experienced in various states:

```
risksets <- list(unemp=c('job', 'program'), onprogram='job')
```

Note that the names of the list risksets are the same as the levels in the factor state. And that the entries in the list are levels of the factor d, i.e. possible transitions.

Then we create a set of control parameters. Since this vignette is to be created by the busy CRAN repository, we limit ourselves to 4 iterations, i.e. no more than 4 masspoints in the distribution. For the same reason we also limit to 1 cpu, or threads, in the computation. The default is to use all the available cpus/cores.

```
ctrl <- mphcrm.control(iters=4, threads=1)</pre>
```

Then we are ready to estimate. There are a couple of special terms in the formula we use:

The left hand side of the formula, d, is the outcome, the transition that is taken. The C(job, alpha) term is a list of conditional covariates, the alpha should only explain the "job" transition. The ID(id) specifies that the covariate id identifies individuals. The D(duration) specifies that the covariate duration contains the durations of the observations. Finally, the S(state) term specifies that the covariate state is a factor which indexes into the risksets argument. In this application, we could as well have replaced C(job, alpha) with C(job, state) in the formula, since these two covariates are essentially the same.

mphcrm writes diagnostic output, one line per iteration. It contains potentially useful information. There is a time stamp, the iteration number, the number of masspoints, the resulting log likelihood, the 2-norm of the gradient, the smallest probability in the masspoint distribution, the reciprocal condition number of the Fisher matrix, the entropy of the masspoint distribution, and the time used in the iteration.

mphcrm returns a list with one entry for each iteration, it has a special print method which sums up the estimation in reverse order:

```
print(opt)
## iter4: estimate with 4 points, log-likelihood: -22469.0497
##
##
                    job.x2
                             job.alpha program.x1 program.x2
   0.99643337 -0.97520318 -0.04029714 1.01064816 0.34415924
##
##
## Proportional hazard distribution
##
                  prob
                              job
                                     program
## point 1 0.51260711 0.11689179 0.07037034
## point 2 0.29869520 0.03138931 0.03095598
## point 3 0.15736987 0.41290602 0.24906806
## point
         4 0.03132782 0.00926154 0.00557658
## iter3: estimate with 3 points, log-likelihood: -22483.5537
## iter2: estimate with 2 points, log-likelihood: -22568.1711
## iter1: estimate with 1 points, log-likelihood: -23397.2902
## nullmodel: estimate with 1 points, log-likelihood: -28974.1282
```

Unless something has gone wrong, you will normally be interested in the first entry, the one with the largest likelihood. We can look at a summary:

```
best <- opt[[1]]
summary(best)
## $loglik
## [1] -22469.05
##
## $coefs
##
                                                    Pr(>|t|)
                   value
                                 se
             0.99643337 0.01836658 54.252521 0.000000e+00
## job.x1
## job.x2
             -0.97520318 0.02086101 -46.747641 0.000000e+00
## job.alpha -0.04029714 0.04567200 -0.882316 3.776252e-01
## program.x1 1.01064816 0.02322823 43.509478 0.000000e+00
## program.x2 0.34415924 0.02532524 13.589577 1.001345e-41
## $moments
##
                mean
                        variance
          0.13456451 0.016023693 0.12658473
## program 0.08468925 0.005415783 0.07359201
```

It has three entries, "loglik", which is simply the log likelihood, "coefs" which is the values and standard errors of the estimated coefficients. And "moments", which is the first and second moments of the proportional hazard distribution.

We can see how the alpha changes with more points:

```
t(sapply(opt, function(o) summary(o)$coefs["job.alpha",]))

## value se t Pr(>|t|)

## iter4 -0.04029714 0.04567200 -0.8823160 0.377625209

## iter3 -0.04614331 0.04348011 -1.0612510 0.288599125

## iter2 -0.03351990 0.03936906 -0.8514275 0.394550457

## iter1 -0.08024695 0.02459507 -3.2627253 0.001106826

## nullmodel 0.00000000 NA NA NA
```

Here is a pre-made fit:

```
summary(fit[[1]])
## $loglik
## [1] -22444.41
##
## $coefs
##
                                                   Pr(>|t|)
                  value
                                se
             1.0008735 0.01953029 51.247230 0.000000e+00
## job.x1
## job.x2
            -1.0325680 0.02423803 -42.601146 0.000000e+00
                                   3.635683 2.785089e-04
## job.alpha 0.2639588 0.07260225
## program.x1 1.0277278 0.02595800 39.591941 1.256903e-320
## program.x2 0.4615064 0.03425654 13.472069 4.824799e-41
##
## $moments
                        variance
                mean
         0.13617702 0.028997653 0.17028697
## program 0.08489246 0.009307573 0.09647576
```

The full estimation can be rerun with the commands:

```
library(durmod)
data(durdata)
newfit <- eval(attr(fit, 'call'))</pre>
```

There are also some functions for extracting the proportional hazard distribution:

```
round(mphdist(fit[[1]]),6)
               prob
                      job program
## point 1 0.295516 0.181172 0.049529
## point 2 0.175513 0.029290 0.019901
## point 3 0.143208 0.067335 0.103531
## point 4 0.132564 0.065707 0.027457
## point 5 0.073408 0.023677 0.115193
## point 6 0.063346 0.155159 0.336856
## point 7 0.037025 0.515057 0.386386
## point 8 0.027400 0.590028 0.114730
## point 9 0.024802 0.006909 0.036665
## point 10 0.017977 0.010285 0.002475
## point 11 0.009242 1.296720 0.010676
# and the moments,
mphmoments(fit[[1]])
##
                       variance
               mean
          0.13617702 0.028997653 0.17028697
## job
## program 0.08489246 0.009307573 0.09647576
# and covariance matrix
mphcov(fit[[1]])
                 job
                         program
          0.028997653 0.005300171
## job
## program 0.005300171 0.009307573
# and some pseudo R2
pseudoR2(fit)
          mcfadden adjmcfadden coxsnell
## iter10 0.2253580 0.2241846 0.9265992 0.9266078
## iter9 0.2253543 0.2242844 0.9265961 0.9266047
## iter8 0.2253491 0.2243827 0.9265916 0.9266002
## iter7 0.2253142 0.2244514 0.9265620 0.9265706
## iter6 0.2251987 0.2244394 0.9264636 0.9264721
## iter5 0.2247808 0.2241250 0.9261065 0.9261151
## iter4 0.2244687 0.2239165 0.9258388 0.9258474
## iter3 0.2240128 0.2235641 0.9254459 0.9254544
## iter2 0.2210923 0.2207472 0.9228792 0.9228878
## iter1 0.1924765 0.1922349 0.8925506 0.8925589
```

The true values used to generate the dataset was job.x1=1, job.x2=-1, job.alpha=0.2, program.x1=1, and program.x2=0.5. The true proportional hazard moments are for convenience stored as attributes in the dataset

```
attributes(durdata)[c('means','cov')]
## $means
```

```
## job program
## 0.13819404 0.08270112
##
## $cov
## job program
## job 0.030584053 0.005931508
## program 0.005931508 0.010850049
```

In this case the estimated mixture has moments fairly close to the true ones, but beware that the estimation process sometimes finds a couple of points with very low probability and very high hazard. If these very low probabilities are imprecisely estimated, so that they should really have been an order of magnitude smaller (10^{-6} instead of 10^{-5}), the moments of the mixture distribution can be way off. It is good practice to inspect the mixture distribution for such extreme points, and go back to a previous iteration (which has slightly worse likelihood) without such extreme points.

3.1 Overparameterization?

We did note in [Gaure et al., 2007] that picking the number of points which yields the lowest AIC yields satisfactory results. We did not, however, have any theoretical justification for doing this, and still don't. It is easy to look at the estimates with the lowest AIC:

```
summary(fit[[which.min(sapply(fit,AIC))]])
## $loglik
##
   [1] -22445.84
##
##
  $coefs
##
                                                     Pr(>|t|)
                    value
## job.x1
               1.0001819 0.01910274
                                      52.358024 0.000000e+00
##
  job.x2
              -1.0263325 0.02356950 -43.544939 0.000000e+00
##
   job.alpha
               0.2405894 0.06952164
                                        3.460641 5.409340e-04
               1.0261267 0.02570300
                                      39.922443 0.000000e+00
##
   program.x1
##
               0.4512076 0.03273131
                                      13.785200 7.098126e-43
   program.x2
##
##
   $moments
##
                 mean
                          variance
##
           0.13471732 0.023595274 0.15360753
  job
  program 0.08443194 0.008140664 0.09022563
```

AIC is generally used to pick a model which is parsimonious, but still explains the data well, i.e. to avoid overparameterization. AIC has an interpretation as the distance between the model and reality, see e.g. [Burnham and Anderson, 2002]. However, since the points are not found in a canonical order, the AIC is really not well defined in these models. Another estimation of the same data may find the points in a different order, with different log likelihoods along the way, resulting in another set of points having the lowest AIC in the new estimation.

Also, keep in mind that the method of using a discrete distribution in this way is *not* an "approximation". When the likelihood can't be improved by adding more points, we have actually reached the likelihood which would result if we were to integrate with the true distribution, be it discrete or continuous. This is the content of [Lindsay, 1983a, Theorems 3.1 and 4.1]. Thus, there is really no theoretical risk of adding "too many" points in the distribution, save for those which may result from numerical problems.

The drawback with the method is that when the likelihood can't be improved by adding more points, the purely technical reason is that something degenerates, i.e. something breaks down, at least the Fisher matrix. That is, we will necessarily run into numerical problems at the end of the estimation, and this may result in irrelevant points with very low probabilities being added, because they seem to increase the likelihood by some small amount due to numerical inaccuracies.

This may be the case if the algorithm has run several iterations at the end which barely moved the likelihood.

4 More options

4.1 Interval timing

The example above had exactly recorded time. For some applications we do have that, while in other applications we only have a time interval when the transition is known to have taken place. The data above is actually a prime example, perhaps we only have labour data on a monthly basis. When a transition takes place, it is only registered at the end of the month, and there is no record of the day. In this case, the duration would be 1 for every observation, and one should use the timing="interval" argument in mphcrm. The observation likelihood is replaced by,

$$h^{d_k}(\mu) \exp(-t_k H(\mu)) \frac{1 - \exp(-t_k H(\mu))}{H(\mu)}.$$
 (5)

It is the fractional part which distinguishes it from the exact model.

If the hazards are small and we use unit intervals, the difference between the interval and exact model is quite small, so one may opt for using the exact model instead.

4.2 No timing

In some applications there isn't any time. A transition occurs, or not. In this case the specification timing="none" can be used. It will use a logit model for the transition probabilities.

4.3 Factors

mphcrm treats factors specially. There is, I hope, nothing special to see, but internally mphcrm does not create a large model matrix filled with dummy variables. This means that factors with many levels are quite fast to estimate, there is no wasteful multiplication by zero.

mphcrm removes factor levels as explanatory for a transition if they are not observed in any state with a risk for this transition. There can, however, be problems with what the contrasts/references should be if you interact a factor in one transition, but keeps it uninteracted in another transition. For the purpose of automatic removal of reference levels in factors, it is assumed that all terms in all transitions are present everywhere. If this problem occurs, you can create a copy with a different name, so it looks like different factors in different transitions. This may be fixed in a later version, if I find a sensible way to do it.

5 Control parameters

There are many control parameters. Rather than scattering more or less arbitrary constants around the program, I have collected them here with their defaults. Some of them you may want to tinker with.

• threads=getOption("durmod.threads"). An integer. The number of parallel threads used by mphcrm. The default is taken from getOption("durmod.threads"), which is initialized from the environment variable DURMOD_THREADS, OMP_NUM_THREADS, OMP_THREAD_LIMIT, NUMBER_OF_PROCESSORS, or else from parallel::detectCores().

It is not always true that the estimation runs twice as fast on twice as many cpus. This depends on the cpu- and memory architecture of your computer, as well as on the implementation of OpenMP in the compiler used to compile the C++ parts of **durmod**. Besides, not all parts of **durmod** run in parallel, so by Amdahl's law you may not expect linear speedup when the number of cpus tends to infinity.

Also, if you intend to use your computer for something else while mphcrm runs, you should not give it all your cpus, but save one or two for your other work. If one of the 16 threads

in mphcrm shares a cpu with your mail program trying to sort your inbox, the speed may be halved.

For creation of the Fisher matrix, mphcrm calls into the BLAS from a single thread. For large datasets with many coefficients to estimate, there can be some benefit from linking R with a highly optimized and parallel BLAS, like mkl from Intel. See the R documentation for how to do this.

In case you run on a cluster (see the cluster control parameter), the threads parameter is the number of threads on each node in the cluster. In this case, threads can be a vector of integers, if you have different number of cpus on each node.

- iters=50. An integer. The number of iterations to perform. The estimation may stop earlier, if neither the log likelihood *nor* the entropy improves.
- 11.improve=0.001. A numeric. The amount the log-likelihood must increase with to be considered an improvement.
- e.improve=0.001. A numeric. The amount the entropy of the hazard distribution must change with to be considered an improvement.
- gdiff=TRUE. A logical. When searching for a new point, should we try one with zero probability, and positive derivative in the direction of positive probability? This usually works, but in case of problems with finding new points, it can be set to FALSE, and a search for a location with a small probability is done instead. See newprob.
- startprob=10⁻⁴. A numeric. When a location point has been found with zero probability, we can not use a zero probability for further optimization due to our parametrization of probabilities which do not allow it. We must set it to some small value startprob. Setting it too small will make it difficult to move away from it.
- overshoot=0.001. A numeric. When searching for a positive directional derivative, this is the threshold we must be above. It is positive to avoid marginally positive derivatives due to numerical inaccuracies.
- newprob=10⁻⁴. A numeric. When searching for a new masspoint with gdiff=FALSE, a new support point is added to the distribution with a small probability, then a search is done for a location for this probability which increases the likelihood. newprob is this small probability. If the search fails, the new probability is set to zero, and a search for a positive directional derivative of the likelihood (in the direction of positive probability) is done.

If mphcrm after optimization seems to find a series of points with probability equal to newprob, it has perhaps got stuck for numerical reasons. Try to increase it to find higher probability points first.

- minprob=10⁻²⁰. A numeric. Masspoints with probability below this are removed.
- eqtol=10⁻⁴. A numeric. It sometimes happens that two location points in the mixed proportional hazard distribution turns out to be equal. One of them can then be removed. eqtol is the threshold below which the ℓ^{∞} distance between two location points is so small that the two points are thought to be equal. Should be lowered if the hazards generally are very small.
- newpoint.maxtime=120. A numeric. When searching for a new location point, a global search algorithm from package nloptr is used. newpoint.maxtime is its time limit in seconds. Should be increased if mphcrm repeatedly complains about not being able to find a new point. However, when there are no new points to be found at the end of the estimation, this will necessarily happen. See also lowint.
- addmultiple=∞. A positive numeric. The algorithm tries to add more points to the distribution without optimizing the structural parameters as long as the improvement from the last point is larger than addmultiple. If the improvement is less, all parameters are optimized before more points are attempted. Setting addmultiple=1 or smaller may save some

time if there are many structural parameters, but setting it too small risks ending up with too many points which later may be discarded, and possible convergence problems.

- callback=mphcrm.callback. A function. If the one-line diagnostic from mphcrm is insufficient, it is possible to write your own. It will replace the default callback (which you can call from your function). In this way you can e.g. get diagnostics on particular coefficients, save intermediate results to file, or other partakings. See mphcrm.callback.
- jobname="mphcrm". A character string. The initial portion of the one-line diagnostic. Useful if you e.g. use parallel::mclapply to run several estimations in parallel. They can have individual names so you can see the progress.
- tspec="%T". A character string. The format of the time stamp in the one-line diagnostic, as described in help(format.POSIXct). Use "%c" to get both date and time.
- trap.interrupt=interactive(). A logical. If you decide to interrupt an interactive estimation before it has terminated, either because you don't want to wait, or because it seems to have run astray, the default behaviour for mphcrm is to catch the interrupt, and return gracefully with the result of the estimation so far. This behaviour can be switched off with trap.interrupt=FALSE.
- tol=10⁻⁴. A numeric. The (absolute) tolerance for the log-likelihood maximization.
- itfac=20. An integer. The maximum number of iterations in the BFGS-method is itfac*K, where K is the number of parameters to estimate.
- fishblock=128. An integer. The Fisher matrix is created by calling the BLAS dsyrk with blocks of individual gradients. This is the size of the blocks. The optimal size depends on the BLAS version and details of your computing contraption. If memory use is an issue with a large number of coefficients to estimate, it can be lowered, typically to some other power of two.
- lowint=4. A numeric, possibly a vector of length the number of transitions. When searching for the location of a new point, an interval centered at the mean of the old points is used. lowint is how far from the mean (in log units) the interval goes to the left. mphcrm makes no effort at analyzing where the location points may lie, as described in [Lindsay, 1983b], but does a brute force search in a (hyper)rectangle. Setting this parameter too high, increases the risk of finding numerically unfavourable points, in particular large ones (if highint is too large), but if mphrcm repeatedly complains about not being able to find new points, it can be increased. See also newpoint.maxtime.
- highint=2. A numeric. Like lowint, but to the right.
- method="BFGS". A character string. The default is the most robust method. In case of convergence problems, or for fun, one may try one of the local gradient based NLOPT-methods:

 "TNEWTON_PRECOND", "TNEWTON", "SLSQP", "MMA", "TNEWTON_RESTART", "TNEWTON_PRECOND_RESTART",

 "VAR1", "VAR2". The Newton-methods can achieve a smaller gradient than BFGS. For these,
 it could be a good idea to increase itfac.
- cluster=NULL. Cluster specification from package parallel or snow. In addition to utilizing all the cores/cpus on a computer, mphcrm may also spread across several computers. It supports running on a cluster from package parallel or snow. The dataset will be split among the cluster nodes, with approximately equally many observations on each. The nodes will then do their share of the likelihood computations. If using a cluster, the threads parameter will be the number of cpus used on each cluster node. It can also be a vector, different number of threads on each node, in which case the dataset will be split unevenly to approximately match the number of threads.

If running on a professional cluster with a resource manager such as SLURM, TORQUE or similar, it is usually possible to pick up information about the cluster nodes from the resource manager, typically from environment variables. With SLURM you can e.g. obtain

the number of allocated cpus on each node of your job with something like threads <-clusterEvalQ(cluster, as.integer(Sys.getenv("SLURM_CPUS_ON_NODE"))). In general, you will have to check local documentation for how this can be done, and how to best set up a cluster with package parallel. (There can be some catches, in particular with MPI which spin-waits to lower latency, and thus takes up a CPU for the "dormant" master process.)

In general, when using parallelization, one should make sure that the cpus are not overbooked and that the nodes you are running on are approximately equally fast. There is some overhead when distributing the computation among many nodes, depending on how the nodes are interconnected, and due to somewhat sub-optimal implementations in package **parallel** which e.g. does not utilize collective MPI-calls. **mphcrm** will usually run faster if using **threads** on a single node, or few nodes, both because the thread algorithm uses shared memory and because the division of data between the threads are dynamic.

• nodeshares=NULL. A numeric vector. When running on a cluster, the default is to share the data equally among the nodes, or in the case threads is a vector of node-specific threads, equally among the threads on each node. If this is unsatisfactory, e.g. because the nodes have different computing speeds, you can specify a vector of node specific shares of the data set.

Say you have 4 nodes in the cluster, but the last two are half as fast as the first two, just specify nodeshares=c(2,2,1,1), and the last two nodes will get half as much data as the first two. If in addition, the first two have 8 cpus, and the last two have four, specify: threads=c(8, 8, 4, 4), and nodeshares=c(2*8, 2*8, 4, 4). The entries in the nodeshares vector will be divided by their sum, so it is not necessary that they sum to 1, it is their relative sizes which matter.

- fisher=TRUE. A logical. Should the Fisher matrix be computed? You will normally want this, it is used to compute the standard errors. However, it may take some time if you have a large number of covariates. If you do not need standard errors, e.g. if you do a bootstrap or Monte Carlo simulation or similar, you may switch off the computation of the Fisher matrix and save some time.
- gradient=TRUE. A logical. As fisher, but for the gradient. I doubt much time can be saved by switching this off.

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