### fastJT

Efficient Jonckheere-Terpstra Test Statistics for Robust Machine Learning and Genome-Wide Association Studies

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## Outline

Introduction

Examples

Session Information

### Introduction

- ▶ This document provides an example for using the fastJT package to calculate the Jonckheere-Terpstra test statistics for large data sets (multiple dependent and independent variables) commonly encountered in machine learning or GWAS. The functionality is also included to perform k-fold cross validation or feature sets.
- ➤ The calculation of the standardized test statistic employs the null variance equation as defined by Hollander and Wolfe (1999, eq. 6.19) to account for ties in the data.
- ► The major algorithm in this package is written in C++, which is ported to R by Rcpp, to facilitate fast computation.
- ► Features of this package include:
  - 1  $O(N \times \log(N))$  computational complexity (where N is the number of the samples)
  - 2 OpenMP supported parallelization
  - 3 Customized output of top m significant independent variables for each dependent variable

#### fast.JT

```
res <- fastJT(Y, X, outTopN=15, numThreads=1, standardized=TRUE)</pre>
```

#### Function arguments:

- Y: Matrix of continuous values, representing dependent variables, with sample IDs as row names and variable names as column names.
- X: A matrix of integer values, representing independent variables, with sample IDs as row names and feature IDs as column names.
- outTopN: Number of top statistics to return (i.e., the largest standardized statistics). The default value is 15. If outTopN is set to NA, all results will be returned.
- numThreads: Number of threads to use for parallel computation. The default value is 1 (sequential computation).
- standardized: A boolean to specify whether to return standardized statistics or non-standardized statistics. The default value is TRUE, returning standardized statistics.

Users may wish to consider the dplyr::recode() function for converting non-numeric group indices into ordinal values for argument X.

### Returned Values

#### Function returns:

J: A matrix of standardized/non-standardized Jonckheere-Terpstra test statistics, depending on option standardized, with column names from input Y. If outTopN is not NA, results are sorted within each column.

XIDs: If outTopN is not NA, this is a matrix of column names from X associated with top standardized

Jonckheere-Terpstra test statistics from J. Otherwise this is an unsorted vector of column names from input X.

### fastJT.select

#### Function arguments:

- Y: Matrix of continuous values, representing dependent variables, with sample IDs as row names and variable names as column names.
- X: Matrix of integer values, representing independent variables, with sample IDs as row names and feature IDs as column names.

cvMesh: A user-defined function to specify how to separate the data into training and testing parts. The inputs of this function are a vector of the sample IDs and kFold, an integer representing the number of folds for cross validation. The output of this function is a list of kFold vectors of sample IDs forming the testing subset for each fold. The default value is NULL, and if no function is specified, the data are partitioned sequentially into kFold equal sized subsets. Optional.

### fastJT.select

Function arguments continued:

- kFold: An integer to indicate the number of folds. Optional. The default value is 10.
- selCrit: A user-defined function to specify the criteria for selecting the top features. The inputs of this function include J, the matrix of statistics resulting from fastJT, and P, the matrix of p-values from pvalues(J). The output is a data frame containing the selected feature IDs for each trait of interest. Optional. The default value is NULL, and if no function is specified, the features the largest standardized Jonckheere-Terpstra test statistics are selected.
- outTopN: An integer to indicate the number of top hits to be returned when selCrit=NULL. Unused if selCrit!=NULL. Optional. The default value is 15.
- numThreads: A integer to indicate the number of threads to be used in the computation. Optional. The default value is 1 (sequential computation).

The function withholds one subset while the remaining kFold-1 subsets are used to test the features. The process is repeated kFold times, with each of the subsets withheld exactly once as the validation data.

### Returned Values

Function returns: Three lists of length kFold:

J: A list of matrices of standardized Jonckheere-Terpstra test statistics, one for each cross validation.

Pval: A list of matrices of p-values, one for each cross validation.

XIDs: A list of matrices of the selected feature IDs, one for each cross validation.

### Simulate Data

1 Define the number of markers, samples, and features:

```
num_sample      <- 100
num_marker      <- 4
num_feature      <- 50</pre>
```

- 2 Create two matrices containing marker levels and feature information.
  - a. Data contains the samples' marker levels.
  - b. Feature contains the samples' feature values.

# Load Package

Load fastJT (after installing its dependent packages):

library(fastJT)

## **Example Execution**

```
JTStat <- fastJT(Y=Data, X=Feature, outTopN=10)
summary(JTStat, Y2Print=1:4, X2Print=1:5)</pre>
```

```
##
##
##
                    Johckheere-Terpstra Test for Large Matrices
                      P-values for Top Standardized Statistics
##
                Mrk:1|
                                   Mrk:21
                                                      Mrk:31
                                                                         Mrk:4
        SNPID P-value
                           SNPID P-value!
                                              SNPID P-value
       Ftr:35 1.7e-02|
                        Ftr:35 2.0e-02|
                                             Ftr:20 1.9e-02|
                                                               Ftr:46 1.2e-02
       Ftr:17 1.7e-02|
                         Ftr:7 5.7e-02|
                                             Ftr:49 3.5e-02|
                                                              Ftr:34 1.7e-02
##
       Ftr:27 7.0e-02|
                        Ftr:46 9.1e-02|
                                             Ftr:26 3.8e-02|
                                                               Ftr:47 3.7e-02
       Ftr:28 7.0e-02|
                        Ftr:40 9.4e-02|
                                             Ftr:47 5.6e-02|
                                                               Ftr:23 4.5e-02
       Ftr:14 8.8e-02|
                         Ftr:29 1.5e-01
                                             Ftr:30 9.7e-021
                                                               Ftr:16 5.1e-02
```

## Example Execution: Statistics in the Summary

#### summary(JTStat, Y2Print=1:4, X2Print=1:5, printP=FALSE)

```
##
##
##
                    Johckheere-Terpstra Test for Large Matrices
##
                            Top Standardized Statistics
##
                                                       Mrk:3|
##
                Mrk:1
                                    Mrk:2
                                                                           Mrk:4
        SNPID
                   J*
                            SNPID
                                       J*
                                               SNPID
                                                          .1 * 1
                                                                   SNPID
##
                                                                              .18
##
       Ftr:35 -2.390|
                                               Ftr:20
##
                           Ftr:35 -2.331|
                                                       2.350
                                                                  Ftr:46
                          Ftr:7 1.905
##
       Ftr:17 -2.388|
                                               Ftr:49 -2.106
                                                                  Ftr:34
                                                                           2.396
##
       Ftr:27 -1.813
                           Ftr:46
                                   1.693
                                               Ftr:26 -2.072|
                                                                  Ftr:47 -2.089
       Ftr:28
##
               1.813
                           Ftr:40
                                    1.676
                                               Ftr:47
                                                      1.914
                                                                  Ftr:23
                                                                           2.0001
       Ftr: 14
               1.706
                           Ftr:29
                                   1.4321
                                               Ftr:30 -1.662|
                                                                  Ftr:16
                                                                          1.955
```

# Example Execution: Sorting in the Summary

```
JTAll <- fastJT(Y=Data, X=Feature, outTopN=NA)
summary(JTAll, Y2Print=1:4, outTopN=3)</pre>
```

```
##
##
                  Johckheere-Terpstra Test for Large Matrices
##
##
                    P-values for Top Standardized Statistics
               Mrk:1
                                 Mrk:2
                                                   Mrk:3|
##
                                                                      Mrk:4
       SNPID P-value | SNPID P-value
                                            SNPID P-value | SNPID P-value
      Ftr:35 1.7e-02| Ftr:35 2.0e-02|
                                           Ftr:20 1.9e-02| Ftr:46 1.2e-02|
      Ftr:17 1.7e-021
                                           Ftr:49 3.5e-02| Ftr:34 1.7e-02|
                      Ftr:7 5.7e-02
      Ftr:27 7.0e-021
                      Ftr:46 9.1e-02
                                           Ftr:26 3.8e-021
                                                            Ftr:47 3.7e-02
```

### Example Execution: fastJT.select

## \$J[[5]]

```
## $J
## $J[[1]]
           Mrk:1
                  Mrk:2 Mrk:3
## [1,] -2,449202 -2,401972 2,402878 2,491444
## [2,] -2,356094 1,863353 2,159547 2,333146
## [3,] -1.798716 1.662280 -2.066804 -2.185883
## [4,] 1,796900 1,662069 -1,902853 2,004838
## [5,] 1.754959 1.399926 -1.893419 1.955443
##
## $J[[2]]
           Mrk:1
                  Mrk:2 Mrk:3
                                       Mrk:4
## [1,] -2,449202 -2,401972 2,402878 2,491444
## [2,] -2,356094 1,863353 2,159547 2,333146
## [3,] -1.798716 1.662280 -2.066804 -2.185883
## [4,] 1,796900 1,662069 -1,902853 2,004838
## [5,] 1.754959 1.399926 -1.893419 1.955443
##
## $J[[3]]
           Mrk:1
                    Mrk:2 Mrk:3
## [1,] -2.449202 -2.401972 2.402878 2.491444
## [2,] -2.356094 1.863353 2.159547 2.333146
## [3,] -1.798716 1.662280 -2.066804 -2.185883
## [4,] 1.796900 1.662069 -1.902853 2.004838
## [5,] 1.754959 1.399926 -1.893419 1.955443
##
## $.1[[4]]
                 Mrk:2 Mrk:3
           Mrk:1
## [1,] -2.449202 -2.401972 2.402878 2.491444
## [2,] -2.356094 1.863353 2.159547 2.333146
## [3,] -1.798716 1.662280 -2.066804 -2.185883
## [4,] 1.796900 1.662069 -1.902853 2.004838
## [5,] 1.754959 1.399926 -1.893419 1.955443
##
```

## Example User-Defined cvMesh Function

```
Mesh <- function(rownamesData, kFold){</pre>
  numSamples <- length(rownamesData)</pre>
  res <- NULL
  subSampleSize <- floor(numSamples/kFold)</pre>
  for (i in 1:kFold){
    start <- (i-1)*subSampleSize + 1
    if(i < kFold)</pre>
      end <- i*subSampleSize
    else
      end <- numSamples
    if(i == 1)
      res <- list(c(start:end))</pre>
    else
      res[[i]] <- c(start:end)
  res
```

# Example User-Defined selCrit Function

```
whichpart \leftarrow function(x, n=30) {
  nx <- length(x)</pre>
  p < -nx-n
  xp <- sort(x, partial=p)[p]</pre>
  which(x > xp)
selectCrit <- function(J, P){</pre>
  pcut <- 0.95
  hit <- NULL
  for(i in 1:ncol(P)){
    if(i == 1)
      hit <- list(rownames(P)[whichpart(P[,i], 4)])</pre>
    else
      hit[[i]] = rownames(P)[whichpart(P[,i], 4)]
  }
  res <- do.call(cbind, hit)
  colnames(res) <- colnames(P)</pre>
  res
```

### Example Execution with User-Defined Functions

```
## $J
## $J[[1]]
                                     Mrk:3
               Mrk:1
                          Mrk:2
          1.044871948 0.30810327 -0.52243597 -1.41325629
## Ftr:2
          0.064765493 -0.57607202 0.48744556 0.91694304
## Ftr:3
          0.708932518 -0.93211498  0.38072302 -0.99775688
## Ftr:4
          0.177698249 -0.10530267 1.24388774 0.13820975
## Ftr:5 -0.135648047 0.29164330 -1.58029974 1.23439722
          0.626045363 0.20007635 -1.20691219 -0.61313721
## Ftr:7 -1.007934807 1.86335316 0.53712315 1.32623001
## Ftr:8 0.382152784 -1.08715878 -1.30459054 0.17789871
## Ftr:9 -0.627298931 -1.04772268 0.28695589 1.08776304
## Ftr:10 -0.552417489 -1.03827865 0.62562945 -0.54576186
## Ftr:11 0.718808299 0.69218577 0.79867589 -0.06655632
## Ftr:12 0.214931358 -0.73882654 -1.25600512 -1.24928852
## Ftr:13 -0.637403711 -1.34635274 -0.27317302 -0.61138723
## Ftr:14 1.754958762 -0.17582763 -0.27535270 0.53411788
## Ftr:15 -0.987651750 0.20792668 0.01299542 -1.13709905
## Ftr:16 1.487980337 0.09875976 0.72423822 1.95544319
## Ftr:18 0.846622444 1.31915590 0.22970376 0.01312593
## Ftr:19 0.006991013 0.34955065 0.78998447 -1.37023855
## Ftr:20 -0.055126760 -0.45074233 2.40287819 0.49614084
## Ftr:21 -0.201986904 0.05212565 -0.19547120 -0.81446332
## Ftr:23 -0.359172943 0.58773754 0.34611211 2.00483806
## Ftr:24 0.661565280 -0.17050652 -1.35041160 1.05032014
## Ftr:25 0.892047636 0.20079301 1.25413339 -0.41146109
## Ftr:26 0.275352698 -0.93221817 -2.06680399 0.82605809
## Ftr: 27 -1.798716484 -0.26774787 -0.46684245 0.51489976
## Ftr:28 1.796899533 1.09120808 0.31364065 1.17615242
## Ftr:29 0.364631969 1.39992631 0.20184984 -0.71624137
## Ftr:30 0.451127576 0.81993260 -1.89341866 1.28752469
## Ftr:31 -0.868212309 -0.42431429 0.41125846 0.15014198
```

### References

Hollander, M. and Wolfe, D. A., *Nonparametric Statistical Methods*. New York, Wiley, 2nd edition, 1999.

### Session Information

- ► R version 3.3.2 (2016-10-31), x86\_64-pc-linux-gnu
- ▶ Base packages: base, datasets, grDevices, graphics, methods, stats, utils
- ▶ Other packages: fastJT 1.0.1, knitr 1.13
- ▶ Loaded via a namespace (and not attached): Rcpp 0.12.4, evaluate 0.8, formatR 1.2.1, highr 0.5.1, magrittr 1.5, stringi 1.0-1, stringr 1.0.0, tools 3.3.2

```
## [1] "Start Time Mon Feb 6 12:04:55 2017"
## [1] "End Time Mon Feb 6 12:04:55 2017"
```