# Package 'fdth'

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<b>Depends</b> R (>= 2.6.0), base, grDevices, graphics	
<b>Description</b> Perform frequency distribution tables, associated histograms and poligons from vector, data.frame and matrix objects.	
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fdth-package

Frequency Distribution Tables, Histograms and Poligons

# **Description**

The fdth package contains a set of functions which easily allows the user to make frequency distribution tables (fdt), its associated histograms and frequency poligons (absolut, relative and cumulative). The fdt can be formatted in many ways which may be suited to publication in many different ways (papers, books, etc). The plot method (S3) is the histogram which can be dealt with the easiness and flexibility of a high level function.

#### **Details**

The frequency of a particular observation is the number of times the observation occurs in the data. The distribution of a variable is the pattern of frequencies of the observation.

Frequency distribution table (fdt) can be used for both ordinal and continuous variables.

The R environment provides a set of functions (generally low level) enabling the user to perfom a fdt and the associated graphical representation, the histogram. A fdt plays an important role to summarize data information and is the basis for the estimation of probability density function used in parametrical inference.

However, for novices or ocasional users of  $\mathbb{R}$ , it can be laborious to find out all necessary funtions and graphical parameters to do a normatized and pretty fdt and the associated histogram ready for publications.

That is the aim of this package, i.e, to allow the user to do (using a few, simple and flexible high level set of S3 functions) with ease and flexibility both: the fdt and histogram. The input data for univariated is generally a vector. For multivariated data can be used both: a data.frame, in this case also allowing grouping all numerical variables according to one categorical, or matrices.

The simplest way to run fdt is done by supplying only the x object, for example: d < - fdt(x). In this case all necessary default values (breaks and right) ("Sturges" and FALSE respectively) will be used.

It can be provided also: a) x and k (number of class intervals); b) x, start (left endpoint of the first class interval) and end (right endpoint of the last class interval); or c) x, start, end and k (class interval width). These options make the fdt very easy and flexible.

The fdt object stores information to be used by methods summary, print and plot. The result of plot is a histogram or poligon (absolut, relative or cummulative). The methods summary, print and plot provide a reasonable set of parameters to format and plot the fdt object in a pretty (and publishable) way.

#### Author(s)

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```

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# See Also

hist provided by graphics; table, cut both provided by base and hist.data.frame provided by Hmisc package.

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```
library (fdth)
#-----
# Vectors: univariated
#-----
set.seed(1)
x \leftarrow rnorm(n=1e3, mean=5, sd=1)
d \leftarrow fdt(x); d
# Histogram
plot(d)
plot(d, main='My title')
plot(d, x.round=3, col='darkgreen')
plot(d, x.las=2)
plot(d, x.round=2, x.las=2, xlab=NULL)
plot(d, x.round=2, x.las=2, xlab=NULL, col=rainbow(11))
# Poligons
plot(d, type='f')
plot(d, type='rf', pch=19, col='violet')
plot(d, type='rfp')
plot(d, type='cf')
plot(d, type='cfp')
# Density
plot(d, type='d')
plot(d, type='cd', pch=19, col='green')
# Summary
summary(d) # the same
print(d) # the same
show(d)
         # the same
                                          # It can not be what you want to publications!
summary(d, format=TRUE)
summary(d, format=TRUE, pattern='%.2f') # Huumm ..., good, but ... Can it be better?
summary(d,
       col=c(1:2, 4, 6),
       format=TRUE, pattern='%.2f')
                                          # Yes, it can!
range(x)
                                          # To know x
summary(fdt(x, start=1, end=9, h=1),
       col=c(1:2, 4, 6),
        format=TRUE, pattern='%d')
                                          # Is it nice now?
# The fdt.object
                                          # Stores the feq. dist. table (fdt)
d[['table']]
                                          # Stores the breaks of fdt
d[['breaks']]
d[['breaks']]['start']
                                          # Stores the left value of the first class
d[['breaks']]['end']
                                         # Stores the right value of the last class
d[['breaks']]['h']
                                         # Stores the class interval
as.logical(d[['breaks']]['right'])
                                         # Stores the right option
# Theoretical curve and fdt
x <- rnorm(1e5, mean=5, sd=1)
```

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```
plot(fdt(x, k=100), type='d', col=heat.colors(100))
curve(dnorm(x, mean=5, sd=1), col='darkgreen', add=TRUE, lwd=2)
# Data.frames: multivariated with categorical
#-----
mdf <- data.frame(X1 = rep(LETTERS[1:4], 25),</pre>
                 X2 = as.factor(rep(1:10, 10)),
                 Y1 = c(NA, NA, rnorm(96, 10, 1), NA, NA),
                 Y2 = rnorm(100, 60, 4),
                 Y3 = rnorm(100, 50, 4),
                 Y4 = rnorm(100, 40, 4))
d <- fdt (mdf); d
# Histograms
plot(d, main=TRUE)
plot(d, col='darkgreen', ylim=c(0, 40), main=TRUE)
plot(d, col=rainbow(8), main=TRUE)
# Poligons
plot(d, type='f')
plot(d, type='rf', pch=19, col='violet')
plot(d, type='rfp')
plot(d, type='cf')
plot(d, type='cfp')
# Density
plot(d, type='d')
plot(d, type='cd')
# Summary
d
summary(d) # the same
print(d) # the same
show(d)
         # the same
summary(d, format=TRUE)
summary(d, format=TRUE, pattern='%05.2f') # regular expression
summary(d, col=c(1:2, 4, 6), format=TRUE, pattern='%05.2f')
print(d, col=c(1:2, 4, 6))
print(d, col=c(1:2, 4, 6), format=TRUE, pattern='%05.2f')
# Using by
levels (mdf$X1)
summary(fdt(mdf, k=5, by='X1'))
plot(fdt(mdf, k=5, by='X1'), col=rainbow(5), main=TRUE)
levels (mdf$X2)
summary(fdt(mdf, breaks='FD', by='X2'), round=3)
plot(fdt(mdf, breaks='FD', by='X2'), main=TRUE)
summary(fdt(iris, k=5), format=TRUE, patter='%04.2f')
plot(fdt(iris, k=5), col=rainbow(5), main=TRUE)
levels(iris$Species)
summary(fdt(iris, k=5, by='Species'), format=TRUE, patter='%04.2f')
```

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fdt

Frequency Distribution Table

#### **Description**

A S3 set of methods to easily perform frequency distribution table (fdt) from vector, data. frame and matrix objects.

# Usage

```
fdt(x, ...)

## Default S3 method:
fdt(x, k, start, end, h, breaks=c("Sturges", "Scott", "FD"),
    right=FALSE, ...)

## S3 method for class 'data.frame':
fdt(x, k, by, breaks=c("Sturges", "Scott", "FD"),
    right=FALSE, ...)

## S3 method for class 'matrix':
fdt(x, k, breaks=c("Sturges", "Scott", "FD"),
    right=FALSE, ...)
```

# **Arguments**

X	A numeric vector, data.frame or matrix object. If $\mathbf{x}$ is data.frame or matrix it must contain at least one numeric column.
k	Number of class intervals.
start	Left endpoint of the first class interval.
end	Right endpoint of the last class interval.
h	Class interval width.
by	Categorical variable used for grouping each numeric variable, useful only on ${\tt data.frames}.$

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breaks	Method used to determine the number of interval classes, c("Sturges", "Scott", "FD").
right	Right endpoints open (default = FALSE).
	Potencial further arguments (required by generic).

#### **Details**

The simplest way to run fdt is done by supplying only the x object, for example: d <- fdt(x). In this case all necessary default values (breaks and right) ("Sturges" and FALSE respectively) will be used.

It can also be provided: a) x and k; b) x, start and end; or c) x, start, end and k. These options make the fdt very easy and flexible to use.

The fdt object stores information to be used by methods summary, print and plot. The result of plot is a histogram. The methods summary, print and plot provide a reasonable set of parameters to format and plot the fdt object in a pretty (and publishable) way.

#### Value

The method fdt.default returns a list of class fdt.default with the slots:

table A data.frame storing the fdt.

breaks A vector of length 4 storing start, end, h and right of the fdt generated

by this method.

data A vector of the data x provided.

The methods fdt.data.frame and fdt.matrix return a list of class fdt.multiple. This list has one slot for each numeric variable of the x provided. Each slot, corresponding to each numeric variable, stores the same slots of the fdt.default described above.

#### Author(s)

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```

#### See Also

hist provided by graphics; table, cut both provided by base and hist.data.frame provided by Hmisc package.

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```
d <- fdt(x, breaks='Scott'); d</pre>
# x, k
d \leftarrow fdt(x, k=20); d
# x, star, end
range(x)
d \leftarrow fdt(x, start=1.5, end=9); d
# x, start, end, h
d \leftarrow fdt(x, start=1, end=9, h=1); d
# Effect of right
x < -rep(1:3, 3); sort(x)
d \leftarrow fdt(x, start=1, end=4, h=1); d
d <- fdt(x, start=0, end=3, h=1, right=TRUE); d</pre>
#-----
# Data.frames: multivariated with categorical
#-----
mdf <- data.frame(X1 = rep(LETTERS[1:4], 25),</pre>
                  X2 = as.factor(rep(1:10, 10)),
                  Y1 = c(NA, NA, rnorm(96, 10, 1), NA, NA),
                  Y2 = rnorm(100, 60, 4),
                  Y3 = rnorm(100, 50, 4),
                  Y4 = rnorm(100, 40, 4))
d <- fdt(mdf); d
levels(mdf$X1)
d \leftarrow fdt (mdf, k=5, by='X1'); d
d <- fdt(mdf, breaks='FD', by='X1')</pre>
str(d)
levels (mdf$X2)
d <- fdt(mdf, breaks='FD', by='X2'); d</pre>
d \leftarrow fdt (mdf, k=5, by='X2'); d
d \leftarrow fdt(iris, k=5); d
d \leftarrow fdt(iris, k=10); d
levels(iris$Species)
d \leftarrow fdt(iris, k=5, by='Species'); d
require (MASS)
levels (Cars93$Origin)
d <- fdt(Cars93, k=5, by='Origin'); d
d <- fdt(Cars93, breaks='FD', by='Origin'); d</pre>
#-----
# Matrices: multivariated
```

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```
make.fdt.format.classes
```

This function formats the presentation's appearance of the interval classes

# **Description**

This function uses part of the fdt results which contains the interval classes description and format it according to a pattern wich is a regular expression, the same as used by sprintf. It is mainly for internal use of the fdth package.

#### Usage

```
make.fdt.format.classes(x, right, pattern)
```

#### **Arguments**

x A fdt object.

right Intervals right open (default = FALSE).

pattern Same as fmt in sprintf.

# **Details**

This function uses the object[["table"]] of the fdt results (object) which contains the interval classes description and format it according to a pattern wich is a regular expression, the same as used by sprintf. It is called by the generic function summary and is mainly for internal use of the fdth package.

#### Value

The function make.fdt.format.classes returns a data.frame which contains the formatted interval class values.

#### Note

This function is mainly for internal use in the fdt package, and may not remain available (unless we see a good reason).

# Author(s)

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```

# See Also

```
sprintf and gsub provided by base package.
```

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```
make.fdt.multiple A function which makes the table using the number of interval classes, a method used for breaks, and choose the endpoint
```

# **Description**

A function which makes the table by using parameters defining the number of interval classes, the method used for breaks, "Sturges", "Scott", "FD", and the closure of the interval class endpoints being left or right.

# Usage

```
make.fdt.multiple(x, k, breaks = c("Sturges", "Scott", "FD"), right)
```

#### **Arguments**

X	A data.frame or matrix object containing at last one numeric column.
k	Number of class intervals.

breaks Method to determine number of classes, c("Sturges", "Scott", "FD").

right Intervals right open (default = FALSE).

# **Details**

A function which makes the table by using parameters defining the number of interval classes, the method used for breaks, "Sturges", "Scott", "FD", and the closure of the interval class endpoints being left or right.

#### Value

The function make.fdt.multiple returns a list with the slots:

table A data.frame storing the fdt.

breaks A vector of length 4 storing start, end, h and right of the fdt generated

by this method.

data A vector of the data x provided.

#### Note

This function is mainly for internal use in the fdt package, and may not remain available (unless we see a good reason).

#### Author(s)

```
Jose Claudio Faria (<joseclaudio.faria@gmail.com>)
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```

# See Also

```
table and cut provided by base package.
```

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make.fdt.simple	A function which makes the table with absolute and relative frequen-
-	cies, cumulative frequencies in numbers and percentages.

# **Description**

This function is called by fdt.default and make.fdt.multiple. It makes a table from a vector of numbers. The table consists of absolute and relative frequencies, cumulative frequencies in numbers and percentages.

# Usage

```
make.fdt.simple(x, start, end, h, right)
```

#### **Arguments**

X	A numeric vector object.
start	The left value of the interval of the first class.
end	The last value of the interval of the last class.
h	The class interval.
right	Intervals right open (default = FALSE).

#### **Details**

This function is called by fdt.default and makes a table from a vector of numbers. The table consists of absolute and relative frequencies, cumulative frequencies in numbers and percentages. The result is used by the generic function summary to format it in a table suited for publication.

# Value

The function make.fdt.simple returns a data.frame which contains the table with interval classes and frequencies. The following are the columns:

Class	limits	Interval classes, character
f		Absolute frequency, numeric
rf		Relative frequency, numeric
rf(%)		Relative frequency in percentages, numeric
cf		Cumulative frequency; numeric
cf(%)		Cumulative frequency in percentages, numeric

#### Note

This function is mainly for internal use in the fdt package, and may not remain available (unless we see a good reason).

# Author(s)

```
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```

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#### See Also

table and cut provided by base package.

plot.fdt

Plot fdt.default and fdt.multiple objects

# **Description**

S3 methods for fdt.default and fdt.multiple objects. It is possible to plot histograms and poligons (absolute, relative and cumulative).

# Usage

```
## S3 method for class 'fdt.default':
plot(x, type=c('h', 'f', 'rf', 'rfp', 'd', 'cd', 'cf', 'cfp'),
    xlab="Class limits", ylab=NULL, col="gray", xlim=NULL, ylim=NULL,
    main=NULL, x.round=2, x.las=1, ...)
## S3 method for class 'fdt.multiple':
plot(x, type=c('h', 'f', 'rf', 'rfp', 'd', 'cd', 'cf', 'cfp'),
    xlab="Class limits", ylab=NULL, col="gray", xlim=NULL, ylim=NULL,
    main=NULL, x.round=2, x.las=1, ...)
```

# **Arguments**

X	A fdt object.
type	The type of the plot. h histogram, f frequency poligon, rf relative frequency poligon, rfp relative frequency poligon (%), d density, cd cumulative density, cf cumulative frequency poligon, cfp cumulative frequency poligon (%).
xlab	A label for the x axis.
ylab	A label for the y axis.
col	A vector of colors.
xlim	The x limits of the plot.
ylim	The x limits of the plot.
main	A title for the plot.
x.round	A numeric value to round the x ticks.
x.las	An integer which controls the orientation of the x axis labels (0: parallel to the axes, 1: horizontal, 2: perpendicular to the axes, 3: vertical)
	Optional plotting parameters.

# **Details**

The result is a single histogram or poligon (absolute, relative or cummulative) for fdt.default or a set of histograms or poligon (absolute, relative or cummulative) for fdt.multiple objects. Both default and multiple try to compute the maximum number of histograms that will fit on one page, then it draws a matrix of histograms. More than one graphical device may be opened to show all histograms.

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#### Author(s)

```
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```

#### See Also

hist.data.frame provided by Hmisc package.

```
library(fdth)
#=========
# Vectors: univariated
#-----
set.seed(1)
x \leftarrow rnorm(n=1e3, mean=5, sd=1)
d \leftarrow fdt(x); d
# Histogram
plot(d)
plot(d, main='My title')
plot(d, x.round=3, col='darkgreen')
plot(d, x.las=1)
plot(d, x.round=2, x.las=2, xlab=NULL)
# Poligons
plot(d, type='f')
plot(d, type='rf', pch=19, col='violet')
plot(d, type='rfp')
plot(d, type='cf')
plot(d, type='cfp')
# Density
plot(d, type='d')
plot(d, type='cd')
# Theoretical curve and fdt
x \leftarrow rnorm(1e5, mean=5, sd=1)
plot(fdt(x, k=100), type='d', col=heat.colors(100))
curve(dnorm(x, mean=5, sd=1), col='darkgreen', add=TRUE, lwd=2)
#----
# Data.frames: multivariated with categorical
#----
mdf <- data.frame(X1 = rep(LETTERS[1:4], 25),</pre>
                X2 = as.factor(rep(1:10, 10)),
                 Y1 = c(NA, NA, rnorm(96, 10, 1), NA, NA),
                 Y2 = rnorm(100, 60, 4),
                Y3 = rnorm(100, 50, 4),
                 Y4 = rnorm(100, 40, 4))
# Histograms
d <- fdt(mdf); d
plot(d, main=TRUE)
```

```
plot(d, col='darkgreen', ylim=c(0, 40), main=TRUE)
plot(d, col=rainbow(8), ylim=c(0, 40), main=TRUE)
# Poligons
plot(d, type='f')
plot(d, type='rf')
plot(d, type='rfp')
plot(d, type='cf')
plot(d, type='cfp')
# Density
plot(d, type='d')
plot(d, type='cd')
levels (mdf$X1)
plot(fdt(mdf, k=5, by='X1'), ylim=c(0, 12), main=TRUE)
levels (mdf$X2)
plot(fdt(mdf, breaks='FD', by='X2'), main=TRUE)
plot(fdt(mdf, k=5, by='X2'), main=TRUE)
                                                     # It is dificult to compare
plot(fdt(mdf, k=5, by='X2'), ylim=c(0, 8), main=TRUE) # Easy
plot(fdt(iris, k=5), main=TRUE)
plot(fdt(iris, k=5), main=TRUE, col=rainbow(5))
d \leftarrow fdt(iris, k=10)
plot(d, main=TRUE)
plot(d, type='d', main=TRUE)
require (MASS)
levels (Cars93$Origin)
plot(fdt(Cars93, k=5, by='Origin'), col=heat.colors(5), main=TRUE)
plot(fdt(Cars93, breaks='FD', by='Origin'), main=TRUE)
#----
# Matrices: multivariated
#-----
plot(fdt(state.x77), main=TRUE)
plot(fdt(volcano), main=TRUE)
```

summary.fdt

Summary and Print Methods for fdt Objects

# **Description**

S3 methods to return (and print) a data.frame (the frequency distribution table - fdt) for fdt.default and fdt.multiple objects.

#### Usage

```
## S3 method for class 'fdt.default':
```

```
summary(object, columns=1:6, round=2,
    format.classes=FALSE, pattern="%09.3e", ...)
## S3 method for class 'fdt.multiple':
summary(object, columns=1:6, round=2,
    format.classes=FALSE, pattern="%09.3e", ...)
## S3 method for class 'fdt.default':
print(x, ...)
## S3 method for class 'fdt.multiple':
print(x, ...)
```

# **Arguments**

object A fdt object. x A fdt object.

columns A vector of integers to select colums of the data.frame table.

round Rounds the fractionary columns of fdt to the specified number of decimal places

(default 2).

format.classes

Logical, if TRUE the first column of the data.frame table will be formated using

regular expression. The default is "%09.3e".

pattern Same as fmt in sprintf.

... Potential further arguments (require by generic).

#### **Details**

The methods print and show are wrappers for the method summary.

It is possible to select what columns of the table (a data.frame) will be shown, as well as the pattern of the first column. The columns are: 1 = Class limits, 2 = f (absolut frequency), 3 = rf (relative frequency), 4 = rf (%) (relative frequency, %), 5 = cf (cumulative frequency), 6 = cf (%) (cumulative frequency, %).

The available parameters offer an easy and powerful way to format the fdt for publications and other purposes.

# Value

A single data.frame for fdt.default or multiple data.frames for fdt.multiple.

#### Author(s)

```
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```

```
d \leftarrow fdt(x)
str(d)
summary(d) # the same
print(d) # the same
show(d)
         # the same
summary(d, format=TRUE)
                                        # It can not be what you want to publications!
summary(d, format=TRUE, pattern='%.2f')
                                        # Huumm ..., good, but ... Can it be better?
summary(d,
       col=c(1:2, 4, 6),
       format=TRUE, pattern='%.2f')
                                        # Yes, it can!
range(x)
                                         # To know x
summary(fdt(x, start=1, end=9, h=1),
       col=c(1:2, 4, 6),
       format=TRUE, pattern='%d')
                                        # Is it nice now?
d[['table']]
                                         # Stores the feq. dist. table (fdt)
d[['breaks']]
                                         # Stores the breaks of fdt
d[['breaks']]['start']
                                         # Stores the left value of the first class
d[['breaks']]['end']
                                        # Stores the right value of the last class
d[['breaks']]['h']
                                        # Stores the class interval
as.logical(d[['breaks']]['right'])
                                        # Stores the right option
#----
# Data.frames: multivariated with categorical
#-----
mdf <- data.frame(X1 = rep(LETTERS[1:4], 25),</pre>
                 X2 = as.factor(rep(1:10, 10)),
                 Y1 = c(NA, NA, rnorm(96, 10, 1), NA, NA),
                 Y2 = rnorm(100, 60, 4),
                 Y3 = rnorm(100, 50, 4),
                 Y4 = rnorm(100, 40, 4))
d <- fdt (mdf)
str(d)
summary(d) # the same
print(d) # the same
          # the same
show(d)
summary(d, format=TRUE)
summary(d, format=TRUE, pattern='%05.2f') # regular expression
summary(d, col=c(1:2, 4, 6), format=TRUE, pattern='%05.2f')
print(d, col=c(1:2, 4, 6))
print(d, col=c(1:2, 4, 6), format=TRUE, pattern='%05.2f')
levels(mdf$X1)
summary (fdt (mdf, k=5, by='X1'))
levels (mdf$X2)
summary(fdt(mdf, breaks='FD', by='X2'), round=3)
summary(fdt(mdf, k=5, by='X2'), format=TRUE, round=3)
```

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