Array operations in the gRbase package

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1 Introduction

This note describes some operations on arrays in R. These operations have been implemented to facilitate implementation of graphical models and Bayesian networks in R.

2 Arrays/tables in R

The documentation of R states the following about arrays:

An array in R can have one, two or more dimensions. It is simply a vector which is stored with additional attributes giving the dimensions (attribute "dim") and optionally names for those dimensions (attribute "dimnames"). A two-dimensional array is the same thing as a matrix. One-dimensional arrays often look like vectors, but may be handled differently by some functions.

2.1 Cross classified data - contingency tables

Arrays appear for example in connection with cross classified data. The array hec below is an excerpt of the HairEyeColor array in R:

```
hec < c(32, 53, 11, 50, 10, 25, 36, 66, 9, 34, 5, 29)
dim(hec) <- c(2, 3, 2)
dimnames(hec) <- list(Hair = c("Black", "Brown"),</pre>
                     Eye = c("Brown", "Blue", "Hazel"),
                     Sex = c("Male", "Female"))
hec
## , , Sex = Male
##
         Eye
## Hair Brown Blue Hazel
    Black 32 11 10
##
    Brown
             53 50
                        25
##
## , , Sex = Female
##
##
         Eye
## Hair
        Brown Blue Hazel
##
             36
                 9
                         5
    Black
             66
                 34
    Brown
```

Above, hec is an array because it has a dim attribute. Moreover, hec also has a dimnames attribute naming the levels of each dimension. Notice that each dimension is given a name.

An array with named dimensions is in this package called a *named array*; this can be checked with is.named.array()[gRbase]

```
is.named.array( hec )
## [1] TRUE
```

The functionality described below relies heavily on arrays having named dimensions.

Printing arrays takes up a lot of space. A more compact view of data can be achieved with ftable(). Since gRbase imports the pipe operator %>% from the magrittr package we will in this note do:

```
flat <- function(x) {ftable(x, row.vars=1)}
hec %>% flat
```

```
Eye Brown
                           Blue
##
         Sex Male Female Male Female Male Female
## Hair
                                      9
                                                    5
## Black
                 32
                        36
                                            10
                              11
## Brown
                 53
                        66
                             50
                                     34
                                            25
                                                   29
```

2.2 Defining arrays

Arrays can be defined in different ways using standard R code:

where the dimnames part in both cases is optional. Another way is to use ar_new()[gRbase] from gRbase:

```
counts <- c(32, 53, 11, 50, 10, 25, 36, 66, 9, 34, 5, 29)
z3 <- ar_new( ~ Hair:Eye:Sex, levels = dn, value = counts)
z4 <- ar_new(c("Hair", "Eye", "Sex"), levels=dn, values=counts)</pre>
```

Notice that dn when used in $ar_new()[gRbase]$ is allowed to contain superfluous elements. Default dimnames are generated with

```
z5 <- ar_new(~Hair:Eye:Sex, levels=c(2, 3, 2), values = counts)
z5 %>% flat
##
         Eye Eye1
                        Eye2
                                   Eye3
##
         Sex Sex1 Sex2 Sex1 Sex2 Sex1 Sex2
## Hair
                                            5
## Hair1
                32
                     36
                          11
                                 9
                                     10
## Hair2
                53
                     66
                          50
                                34
                                     25
                                           29
```

Using ar_new()[gRbase], arrays can be normalized in two ways: Normalization can be over the first variable for *each* configuration of all other variables or over all configurations. For example:

```
z6 <- ar_new(~Hair:Eye:Sex, levels=c(2, 3, 2), values = counts, normalize="first")
z6 %>% flat
##
         Eye
               Eye1
                              Eye2
                                            Eye3
               Sex1
##
         Sex
                                            Sex1
                      Sex2
                              Sex1
                                     Sex2
                                                    Sex2
## Hair
             0.3765 0.3529 0.1803 0.2093 0.2857 0.1471
## Hair1
             0.6235 0.6471 0.8197 0.7907 0.7143 0.8529
```

3 Operations on arrays

In the following we shall denote the dimnames (or variables) of the array hec by H, E and S and we let (h, e, s) denote a configuration of these variables. The contingency table above shall be denoted by T_{HES} and we shall refer to the (h, e, s)-entry of T_{HES} as $T_{HES}(h, e, s)$.

3.1 Normalizing an array

Normalize an array with ar_normalize()[gRbase]

```
ar_normalize(z5, "first") %>% flat
         Eye
##
               Eye1
                              Eye2
                                             Eye3
##
         Sex
               Sex1
                              Sex1
                                      Sex2
                                             Sex1
                       Sex2
                                                     Sex2
## Hair
             0.3765 0.3529 0.1803 0.2093 0.2857 0.1471
## Hair1
## Hair2
             0.6235 0.6471 0.8197 0.7907 0.7143 0.8529
ar_normalize(z5, "all") %>% flat
         Eye
                                                   Eye3
                Eye1
                                  Eye2
##
         Sex
                 Sex1
                         Sex2
                                  Sex1
                                          Sex2
                                                   Sex1
                                                           Sex2
## Hair
## Hair1
             0.08889 0.10000 0.03056 0.02500 0.02778 0.01389
             0.14722 0.18333 0.13889 0.09444 0.06944 0.08056
```

3.2 Subsetting an array – slicing

We can subset arrays (this will also be called "slicing") in different ways. Notice that the result is not necessarily an array. Slicing can be done using standard R code or using ar_slice[gRbase]. The virtue of ar_slice[gRbase] comes from the flexibility when specifying the slice:

The following leads from the original $2 \times 3 \times 2$ array to a $2 \times 2 \times 2$ array by cutting away the Eye=Brown slice of the array:

```
ar_slice(hec, slice=list(Eye=c("Blue", "Hazel")))
                                                     %>% flat
         Eye Blue
                          Hazel
##
         Sex Male Female Male Female
## Hair
## Black
                        9
                             10
                                      5
                11
## Brown
                       34
                             25
                                     29
                50
```

Levels can be written as numerics.¹

```
ar_slice(hec, slice=list(Eye=2:3, Sex="Female"))
```

Suppose we pick the Sex=Female slice of hec. This slice can be regarded as a 2×3 array or as $2 \times 3 \times 1$ array.

```
# 2 x 3 array :
ar_slice(hec, slice=list(Sex="Female")) %>% flat
```

¹Currently names can not be abbreviated, but that might be added later.

```
Eye Brown Blue Hazel
## Hair
## Black
               36
                     9
                           5
## Brown
               66
                    34
                          29
#2x3x1array:
ar_slice(hec, slice=list(Sex="Female"), drop=FALSE) %>% flat
        Eye Brown Blue Hazel
##
        Sex Female Female Female
## Hair
                               5
## Black
                36
                        9
## Brown
                       34
                              29
```

If slicing leads to a one dimensional array, the output will by default not be an array but a vector (without a dim attribute). However, the result can be forced to be a 1-dimensional array:

```
## A vector:
z <- ar_slice(hec, slice=list(Hair=1, Sex="Female")); z
## A 1-dimensional array:
z <- ar_slice(hec, slice=list(Hair=1, Sex="Female"), as.array=TRUE); z</pre>
```

Slicing using standard R code can be done as follows:

```
hec[, 2:3, ] %>% flat ## A 2 x 2 x 2 array
##
        Eye Blue
                       Hazel
##
        Sex Male Female Male Female
## Hair
                    9
                                  5
## Black
              11
                           10
## Brown
              50
                     34
                           25
                                 29
hec[1, , 1]
                     ## A vector
## Brown Blue Hazel
   32
##
           11
                 10
hec[1, , 1, drop=FALSE] ## A 1 x 3 x 1 array
## , , Sex = Male
##
##
         Eye
## Hair
         Brown Blue Hazel
## Black 32 11 10
```

Programmatically we can do the above as

```
do.call("[", c(list(hec), list(TRUE, 2:3, TRUE))) %>% flat
do.call("[", c(list(hec), list(1, TRUE, 1)))
do.call("[", c(list(hec), list(1, TRUE, 1), drop=FALSE))
```

gRbase provides two alterntives for each of these three cases above:

```
ar_slice_prim(hec, slice=list(TRUE, 2:3, TRUE)) %>% flat
ar_slice(hec, slice=list(c(2, 3)), margin=2) %>% flat
```

```
ar_slice_prim(hec, slice=list(1, TRUE, 1))
ar_slice(hec, slice=list(1, 1), margin=c(1,3))
ar_slice_prim(hec, slice=list(1, TRUE, 1), drop=FALSE)
ar_slice(hec, slice=list(1, 1), margin=c(1,3), drop=FALSE)
```

3.3 Collapsing and inflating arrays

Collapsing: The HE-marginal array T_{HE} of T_{HES} is the array with values

$$T_{HE}(h,e) = \sum_{s} T_{HES}(h,e,s)$$

Inflating: The "opposite" operation is to extend an array. For example, we can extend T_{HE} to have a third dimension, e.g. Sex. That is

$$\tilde{T}_{SHE}(s, h, e) = T_{HE}(h, e)$$

so $\tilde{T}_{SHE}(s, h, e)$ is constant as a function of s.

With gRbase we can collapse with²:

```
he <- hec %a_% ~Hair:Eye; he %>% flat

## Eye Brown Blue Hazel

## Hair

## Black 68 20 15

## Brown 119 84 54
```

```
## Alternatives
he <- ar_marg(hec, ~Hair:Eye); he
hs <- ar_marg(hec, c("Hair", "Sex"))
es <- ar_marg(hec, c(2, 3))</pre>
```

With gRbase we can inflate with ar_expand()[gRbase]:

```
she <- he %a^% list(Sex=c("Male", "Female"))</pre>
she %>% flat
##
          Eye Brown
                              Blue
          Hair Black Brown Black Brown Black Brown
## Sex
## Male
                   68
                         119
                                20
                                       84
                                              15
                                                    54
                                20
                         119
```

```
## Alternatives
she <- ar_expand(he, list(Sex=c("Male", "Female")))
ar_expand(he, dimnames(hs)) %>% flat
ar_expand(he, hs) %>% flat
```

²FIXME: Should allow for abbreviations in formula and character vector specifications.

3.4 Permuting an array

A reorganization of the table can be made with ar_perm[gRbase] (similar to aperm()), but arperm[gRbase] allows for a formula and for variable abbreviation:

```
ar_perm(hec, ~Eye:Sex:Hair) %>% flat
##
         Sex Male
                           Female
##
         Hair Black Brown Black Brown
## Eye
## Brown
                 32
                        53
                               36
                                      66
                                9
## Blue
                  11
                        50
                                      34
## Hazel
                  10
                        25
                                5
                                      29
```

Alternative forms (the first two also works for aperm):

```
ar_perm(hec, c("Eye", "Sex", "Hair"))
ar_perm(hec, c(2,3,1))
ar_perm(hec, ~Ey:Se:Ha)
ar_perm(hec, c("Ey", "Se", "Ha"))
```

3.5 Equality

Two arrays are defined to be identical 1) if they have the same dimnames and 2) if, possibly after a permutation, all values are identical (up to a small numerical difference):

```
hec2 <- ar_perm(hec, 3:1)
hec %a==% hec2
## [1] TRUE
```

```
## Alternative
ar_equal(hec, hec2)
```

3.6 Aligning

We can align one array according to the ordering of another:³

```
hec2 <- ar_perm(hec, 3:1)
ar_align(hec2, hec)
## ar_align(hec2, dimnames(hec))
## ar_align(hec2, names(dimnames(hec)))</pre>
```

3.7 Multiplication, addition etc: +, -, *, /

The sum of two arrays T_{HE} and T_{HS} is defined to be the array \tilde{T}_{HES} with entries

$$\tilde{T}_{HES}(h, e, s) = T_{HE}(h, e) + T_{HS}(h, s)$$

³FIXME; see ar_expand()

The difference, product and quotient is defined similarly:

With gRbase this is done with ar_mult()[gRbase]:

```
she \leftarrow he %a+% hs
she %>% flat
        Sex Male
                               Female
##
         Eye Brown Blue Hazel Brown Blue Hazel
## Hair
## Black
               121
                     73
                            68
                                   118
                                         70
                                               65
## Brown
               247 212
                         182
                                  248
                                      213
                                              183
```

Likewise

```
he %a+% hs
he %a-% hs
he %a*% hs
he %a/% hs
he %a/% hs
```

```
ar_add(he, hs) %>% flat
ar_subt(he, hs) %>% flat
ar_mult(he, hs) %>% flat
ar_div(he, hs) %>% flat
ar_div0(he, hs) %>% flat ## Convention 0/0 = 0
```

Multiplication and addition of a list of multiple arrays is accomplished with ar_prod()[gRbase] and ar_sum()[gRbase] (much like prod()[gRbase] and sum()[gRbase]):

```
ar_sum( he, hs, es )
ar_prod( he, hs, es )
```

Lists of arrays are processed with

```
ar_sum_list( list(he, hs, es) )
ar_prod_list( list(he, hs, es) )
```

3.8 An array as a probability density

If an array consists of non–negative numbers then it may be regarded as an (unnormalized) discrete multivariate density. With this view, the following examples should be self explanatory:

```
ar_dist(hec) %>% flat
##
        Eye Brown
                               Blue
                                              Hazel
##
        Sex
               Male Female
                               Male Female
                                             Male
                                                    Female
## Hair
            0.08889 0.10000 0.03056 0.02500 0.02778 0.01389
## Black
## Brown
            0.14722 0.18333 0.13889 0.09444 0.06944 0.08056
ar_dist(hec, marg=~Hair:Eye) %>% flat
```

```
Eve
               Brown
                        Blue
                               Hazel
## Hair
             0.18889 0.05556 0.04167
## Black
             0.33056 0.23333 0.15000
## Brown
ar_dist(hec, cond=~Eye) %>% flat
##
         Sex
                Male
                                       Female
##
         Eye
               Brown
                        Blue
                               Hazel
                                        Brown
                                                 Blue
                                                        Hazel
## Hair
## Black
             0.17112 0.10577 0.14493 0.19251 0.08654 0.07246
             0.28342 0.48077 0.36232 0.35294 0.32692 0.42029
## Brown
ar_dist(hec, marg=~Hair, cond=~Sex) %>% flat
##
         Sex Male Female
## Hair
## Black
             0.2928 0.2793
             0.7072 0.7207
## Brown
```

3.9 Miscellaneous

Multiply values in a slice by some number and all other values by another number:

```
ar_slice_mult(hec, list(Sex="Female"), val=10, comp=0) %>% flat
##
         Eve Brown
                           Blue
                                       Hazel
##
         Sex Male Female Male Female Male Female
## Hair
                 0
                       360
                              0
                                    90
                                            0
                                                  50
## Black
## Brown
                 0
                       660
                              0
                                   340
                                            0
                                                 290
```

4 Examples

4.1 A Bayesian network

A classical example of a Bayesian network is the "sprinkler example", see e.g. http://en.wikipedia.org/wiki/Bayesian_network:

Suppose that there are two events which could cause grass to be wet: either the sprinkler is on or it is raining. Also, suppose that the rain has a direct effect on the use of the sprinkler (namely that when it rains, the sprinkler is usually not turned on). Then the situation can be modeled with a Bayesian network.

We specify conditional probabilities p(r), p(s|r) and p(w|s,r) as follows (notice that the vertical conditioning bar (|) is replaced by the horizontal underscore:

```
yn <- c("y","n")
lev <- list(rain=yn, sprinkler=yn, wet=yn)
r <- ar_new( ~rain, levels = lev, values = c(.2, .8) )
s_r <- ar_new( ~sprinkler:rain, levels = lev, values = c(.01, .99, .4, .6) )
w_sr <- ar_new( ~wet:sprinkler:rain, levels = lev,</pre>
```

```
values = c(.99, .01, .8, .2, .9, .1, 0, 1))
r
## rain
## y n
## 0.2 0.8
s_r %>% flat
## rain y n
## sprinkler
            0.01 0.40
## y
            0.99 0.60
## n
w_sr %>% flat
## sprinkler y
    rain y n y n
## wet
## y
            0.99 0.90 0.80 0.00
           0.01 0.10 0.20 1.00
## n
```

The joint distribution p(r, s, w) = p(r)p(s|r)p(w|s, r) can be obtained with ar_prod()[gRbase]: ways:

What is the probability that it rains given that the grass is wet? We find $p(r, w) = \sum_{s} p(r, s, w)$ and then p(r|w) = p(r, w)/p(w). Can be done in various ways: with $ar_dist()[gRbase]$

```
## Alternative:
rw <- ar_marg(joint, ~rain + wet)
ar_div( rw, ar_marg(rw, ~wet))
## or
rw %a/% (rw %a_% ~wet)</pre>
```

```
ar_dist(x, marg=~rain)
## rain
## y n
## 0.3577 0.6423
```

4.2 Iterative Proportional Scaling (IPS)

We consider the 3-way lizard data from gRbase:

```
data( lizard, package="gRbase" )
lizard %>% flat
## height >4.75 <=4.75
## species anoli dist anoli dist
## diam
## <=4 32 61 86 73
## >4 11 41 35 70
```

Consider the two factor log–linear model for the lizard data. Under the model the expected counts have the form

$$\log m(d, h, s) = a_1(d, h) + a_2(d, s) + a_3(h, s)$$

If we let n(d, h, s) denote the observed counts, the likelihood equations are: Find m(d, h, s) such that

$$m(d,h) = n(d,h), \quad m(d,s) = n(d,s), \quad m(h,s) = n(h,s)$$

where $m(d,h) = \sum_s m(d,h.s)$ etc. The updates are as follows: For the first term we have

$$m(d,h,s) \leftarrow m(d,h,s) \frac{n(d,h)}{m(d,h)}$$

After iterating the updates will not change and we will have equality: $m(d, h, s) = m(d, h, s) \frac{n(d, h)}{m(d, h)}$ and summing over s shows that the equation m(d, h) = n(d, h) is satisfied.

A rudimentary implementation of iterative proportional scaling for log–linear models is straight forward:

```
myips <- function(indata, glist){
    fit <- indata
    fit[] <- 1
    ## List of sufficient marginal tables
    md <- lapply(glist, function(g) ar_marg(indata, g))

for (i in 1:4){
    for (j in seq_along(glist)){
        mf <- ar_marg(fit, glist[[j]])
        # adj <- ar_div( md[[ j ]], mf)
        # fit <- ar_mult( fit, adj )
        ## or
        adj <- md[[ j ]] %a/% mf
        fit <- fit %a*% adj
    }
}
pearson <- sum( (fit - indata)^2 / fit)</pre>
```

```
list(pearson=pearson, fit=fit)
}
glist <- list(c("species","diam"),c("species","height"),c("diam","height"))</pre>
fm1 <- myips( lizard, glist )</pre>
fm1$pearson
## [1] 0.1506
fm1$fit %>% flat
       height >4.75 <=4.75
##
       species anoli dist anoli dist
## diam
## <=4
                32.8 60.2 85.2 73.8
## >4
                10.2 41.8 35.8 69.2
fm2 <- loglin( lizard, glist, fit=T )</pre>
## 4 iterations: deviation 0.009619
fm2$pearson
## [1] 0.1506
fm2$fit %>% flat
                      <=4.75
       height >4.75
##
       species anoli dist anoli dist
## diam
## <=4
                 32.8 60.2 85.2 73.8
                 10.2 41.8 35.8 69.2
## >4
```

5 Some low level functions

For e.g. a $2 \times 3 \times 2$ array, the entries are such that the first variable varies fastest so the ordering of the cells are (1,1,1), (2,1,1), (1,2,1), (2,2,1), (1,3,1) and so on. To find the value of such a cell, say, (j,k,l) in the array (which is really just a vector), the cell is mapped into an entry of a vector

For example, cell (2,3,1) (Hair=Brown, Eye=Hazel, Sex=Male) must be mapped to entry 4 in

```
hec
## , , Sex = Male
##
##
       Eye
## Hair Brown Blue Hazel
  Black 32 11 10
##
   Brown 53 50
                   25
##
## , , Sex = Female
##
##
        Eye
## Hair Brown Blue Hazel
## Black 36 9 5
```

```
## Brown 66 34 29

c(hec)

## [1] 32 53 11 50 10 25 36 66 9 34 5 29
```

For illustration we do:

```
cell2name <- function(cell, dimnames){
   unlist(lapply(1:length(cell), function(m) dimnames[[m]][cell[m]]))
}
cell2name(c(2,3,1), dimnames(hec))
## [1] "Brown" "Hazel" "Male"</pre>
```

5.1 cell2entry(), entry2cell() and next_cell()

The map from a cell to the corresponding entry is provided by cell2entry()[gRbase]. The reverse operation, going from an entry to a cell (which is much less needed) is provided by entry2cell()[gRbase].

```
cell2entry(c(2,3,1), dim=c(2,3,2))
## [1] 6
entry2cell(6, dim=c(2,3,2))
## [1] 2 3 1
```

Given a cell, say i = (2, 3, 1) in a $2 \times 3 \times 2$ array we often want to find the next cell in the table following the convention that the first factor varies fastest, that is (1, 1, 2). This is provided by $next_cell()[gRbase]$.

```
next_cell(c(2,3,1), dim=c(2,3,2))
## [1] 1 1 2
```

5.2 next_cell_slice() and slice2entry()

Given that we look at cells for which for which the index in dimension 2 is at level 3 (that is Eye=Hazel), i.e. cells of the form (j,3,l). Given such a cell, what is then the next cell that also satisfies this constraint. This is provided by next_cell_slice()[gRbase].⁴

```
next_cell_slice(c(1,3,1), slice_marg=2, dim=c(2,3,2))
## [1] 2 3 1

next_cell_slice(c(2,3,1), slice_marg=2, dim=c(2,3,2))
## [1] 1 3 2
```

Given that in dimension 2 we look at level 3. We want to find entries for the cells of the form (j,3,l).⁵

⁴FIXME: sliceset should be called margin.

⁵FIXME:slicecell and sliceset should be renamed

```
slice2entry(slice_cell=3, slice_marg=2, dim=c( 2, 3, 2 ))
## [1] 5 6 11 12
```

To verify that we indeed get the right cells:

5.3 fact_grid() - Factorial grid

Using the operations above we can obtain the combinations of the factors as a matrix:

```
head( fact_grid( c(2, 3, 2) ), 6 )
       [,1] [,2] [,3]
## [1,]
        1 1 1
## [2,]
          2
             1 1
## [3,]
          1
               2
                   1
## [4,]
          2
               2
                   1
               3
                    1
## [5,]
          1
               3
                    1
## [6,]
          2
```

A similar dataframe can also be obtained with the standard R function expand.grid (but factGrid is faster)

```
head( expand.grid(list(1:2, 1:3, 1:2)), 6)
##
    Var1 Var2 Var3
## 1
       1
           1
                1
## 2
       2
            1
                 1
            2
## 3
       1
                 1
            2
## 4
       2
                 1
## 5
       1
            3
                 1
## 6
       2
```