# Advanced array operations in the gRbase package

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#### 1 Tables

This note describes various functions in the gRbase package for operations on tables / arrays in R. Notice that there is a distinction between a table and an array in R. For the purpose of what is described here the concepts can be used interchangably. The important point is that we are working on vectors which have a dim and a dimnames attribute. (Arrays do not need a dimnames attribute, but dimnames are essential in what follows here).

Consider the lizard data in gRbase:

```
R> data(lizard)
R> lizard
, , species = anoli
    height
diam >4.75 <=4.75
    <=4     32     86
    >4     11     35
```

```
, , species = dist
     height
diam >4.75 <=4.75
  <=4
         61
                 73
  >4
                 70
Data is of class table and has dim and dimnames attributes
R> class(lizard)
[1] "table"
R> is.array(lizard)
[1] TRUE
R> dim(lizard)
[1] 2 2 2
R> dimnames(lizard)
$diam
[1] "<=4" ">4"
$height
[1] ">4.75" "<=4.75"
$species
[1] "anoli" "dist"
```

Notice from the output above that the first variable (diam) varies fastest.

# 2 Notation

A formal description of an array is as follows: Let  $\Delta = \{\delta_1, \ldots, \delta_R\}$  be a set of factors where  $\delta_r$  has a finite set  $I_r$  of levels. Let  $|I_r|$  denote the number of levels of  $\delta_r$  and let  $i_r \in I_r$  denote a value of  $\delta_r$ . A configuration of the variables in  $\Delta$  is  $i = i_{\Delta} = (i_1, \ldots, i_R) \in I_1 \times \ldots \times I_R = I$ . The total number of configurations is  $|\Delta| = \prod_r |I_r|$ . An array T is a function which maps I into some domain.

## 3 Algebraic operations on tables

Let U and V be non-empty subsets of  $\Delta$  with configurations  $I_U$  and  $I_V$  and let  $T_U^1$  and  $T_V^2$  be corresponding arrays.

• The product and quotient of  $T_U^1$  and  $T_V^2$  are arrays defined on  $U \cup V$  given by

$$T_{U\cup V}(i_{U\cup V}) := T_U^1(i_U) \times T_V^2(i_V)$$
 and  $T_{U\cup V}(i_{U\cup V}) := T_U^1(i_U)/T_V^2(i_V)$ 

respectively, with the convention that 0/0 = 0.

• If  $W \subset U$  is non-empty<sup>1</sup> then marginalization of  $T_U^1$  onto W is defined as

$$T_W^1(i_W) := \sum_{i_{U \setminus W}} T_U^1(i_{U \setminus W}, i_W)$$

• If  $W \subset U$  is non–empty then a configuration  $i_W^*$  defines a slice of  $T_U^1$  as

$$T^1_{U\setminus V}(i_{U\setminus V}) := T^1_U(i_{U\setminus V}, i_V^*)$$

In a less abstract setting let  $U = \{A, B, C\}$ ,  $V = \{C, D, B\}$  and  $W = \{C, B\}$  where (a, b, c) denotes a specific configuration of  $\{A, B, C\}$  and so on. Then the product and quotient become

$$T_{ABCD}(a, b, c, d) = T_{ABC}^{1}(a, b, c)T_{CDB}^{2}(c, d, b)$$

The marginal becomes

$$T_{CB}^{1} = \sum_{a} T^{1}(a, b, c)$$
 and

Finally the slice defined by  $C = c^*$  and  $B = b^*$  becomes

$$T_A^1(a) = T_{ABC}^1(a, b^*, c^*)$$

To illustrate we find two marginal tables

```
R> T1.U <- tableMargin(lizard, c("species", "height"))
```

```
height
species >4.75 <=4.75
anoli 43 121
dist 102 143
```

R> T1.V <- tableMargin(lizard, c("diam", "species"))

```
species
diam anoli dist
<=4 118 134
>4 46 111
```

Multiplication of these is done with

$$R > T1.UV < -tableOp(T1.U, T1.V, op = "*")$$

<sup>&</sup>lt;sup>1</sup>Marginalization onto an empty set is not implemented.

```
, , height = >4.75
     species
diam anoli
            dist
  <=4 5074 13668
       1978 11322
, , height = <=4.75
     species
diam anoli dist
  <=4 14278 19162
      5566 15873
A slice of a table is obtained with tableSlice:
R> tableSlice(lizard, "species", "anoli")
     height
diam >4.75 <=4.75
         32
  <=4
                86
  >4
                35
         11
A reorganization of the table can be made with tablePerm:
R> tablePerm(T1.UV, c("species", "height", "diam"))
, , diam = <=4
       height
species >4.75 <=4.75
  anoli 5074
              14278
  dist 13668
               19162
, , diam = >4
       height
species >4.75 <=4.75
  anoli 1978
                5566
  dist 11322 15873
```

# 4 Defining tables / arrays

As mentioned above, a table can be represented as an array. In general, arrays do not need dimnames in R, but for the functions described here, the dimnames are essential.

The examples here relate to the chest clinique example of Lauritzen and Spiegelhalter. The following two specifications are equivalent:

```
R> yn <- c('y','n')
R> T.U <- array(c(5,95,1,99), dim=c(2,2), dimnames=list("tub"=yn, "asia"=yn))
R> T.U <- parray(c("tub","asia"), levels=list(yn, yn), values=c(5,95,1,99))</pre>
```

Using parray(), arrays can be normalized in two ways: Normalization can be over the first variable for *each* configuration of all other variables or over all configurations. We illustrate this by defining the probability of tuberculosis given a recent visit to Asia and by defining the marginal probability of a recent visit to Asia:

## 5 Calculations with probability tables

```
The joint distributions is
R> T.all <- tableOp(T.U, T.V, op="*")
    tub
asia
          У
   y 0.0005 0.0095
   n 0.0099 0.9801
The marginal distribution of "tub" is
R> T.W <- tableMargin(T.all, "tub")</pre>
tub
0.0104 0.9896
The conditional distribution of "asia" given "tub" is
R> tableOp(T.all, T.W, op="/")
   asia
tub
  y 0.048076923 0.9519231
  n 0.009599838 0.9904002
```