# Array operations in the gRbase package

### Søren Højsgaard

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#### 1 Introduction

This note describes some operations on arrays in R. These operations have been implemented to facilitate implementation of graphical models and Bayesian networks in R.

### 2 Arrays/tables in R

The documentation of R states the following about arrays:

An array in R can have one, two or more dimensions. It is simply a vector which is stored with additional attributes giving the dimensions (attribute "dim") and optionally names for those dimensions (attribute "dimnames"). A two-dimensional array is the same thing as a matrix. One-dimensional arrays often look like vectors, but may be handled differently by some functions.

### 2.1 Cross classified data - contingency tables

Arrays appear for example in connection with cross classified data. The array hec below is an excerpt of the HairEyeColor array in R:

```
hec <- c(32, 53, 11, 50, 10, 25, 36, 66, 9, 34, 5, 29)
dim(hec) <- c(2, 3, 2)
dimnames(hec) <- list(Hair = c("Black", "Brown"),</pre>
                     Eye = c("Brown", "Blue", "Hazel"),
                      Sex = c("Male", "Female"))
hec
## , , Sex = Male
##
##
          Eye
## Hair
          Brown Blue Hazel
     Black
           32 11
                        10
##
     Brown
              53
##
   , , Sex = Female
##
##
##
          Eye
           Brown Blue Hazel
              36 9
     Black
     Brown
                 34
           66
                         29
```

Above, hec is an array because it has a dim attribute. Moreover, hec also has a dimnames attribute naming the levels of each dimension. Notice that each dimension is given a name.

Printing arrays can take up a lot of space. Alternative views on an array can be obtained with ftable() or by converting the array to a dataframe with as.data.frame.table(). We shall do so in the following.

```
##flat <- function(x) {ftable(x, row.vars=1)}
flat <- function(x, n=4) {as.data.frame.table(x) %>% head(n)}
hec %>% flat
## Hair Eye Sex Freq
## 1 Black Brown Male 32
## 2 Brown Brown Male 53
## 3 Black Blue Male 11
## 4 Brown Blue Male 50
```

An array with named dimensions is in this package called a *named array*. The functionality described below relies heavily on arrays having named dimensions. A check for an object being a named array is provided by is.named.array()[gRbase]

```
is.named.array(hec)
## [1] TRUE
```

### 2.2 Defining arrays

Another way is to use tabNew()[gRbase] from gRbase. This function is flexible wrt the input; for example:

```
dn <- list(Hair=c("Black", "Brown"), Eye=~Brown:Blue:Hazel, Sex=~Male:Female)
counts <- c(32, 53, 11, 50, 10, 25, 36, 66, 9, 34, 5, 29)
z3 <- tabNew(~Hair:Eye:Sex, levels=dn, value=counts)
z4 <- tabNew(c("Hair", "Eye", "Sex"), levels=dn, values=counts)</pre>
```

Notice that the levels list (dn above) when used in tabNew()[gRbase] is allowed to contain superfluous elements. Default dimnames are generated with

```
z5 <- tabNew(~Hair:Eye:Sex, levels=c(2, 3, 2), values = counts)
dimnames(z5) %>% str
## List of 3
## $ Hair: chr [1:2] "Hair1" "Hair2"
## $ Eye : chr [1:3] "Eye1" "Eye2" "Eye3"
## $ Sex : chr [1:2] "Sex1" "Sex2"
```

Using tabNew[gRbase], arrays can be normalized to sum to one in two ways: 1) Normalization can be over the first variable for *each* configuration of all other variables and 2) over all configurations. For example:

```
z6 <- tabNew(~Hair:Eye:Sex, levels=c(2, 3, 2), values=counts, normalize="first")
z6 %>% flat
## Hair Eye Sex Freq
## 1 Hair1 Eye1 Sex1 0.3765
## 2 Hair2 Eye1 Sex1 0.6235
## 3 Hair1 Eye2 Sex1 0.1803
## 4 Hair2 Eye2 Sex1 0.8197
```

### 3 Operations on arrays

In the following we shall denote the dimnames (or variables) of the array hec by H, E and S and we let (h, e, s) denote a configuration of these variables. The contingency table above shall be denoted by  $T_{HES}$  and we shall refer to the (h, e, s)-entry of  $T_{HES}$  as  $T_{HES}(h, e, s)$ .

### 3.1 Normalizing an array

Normalize an array with tabNormalize()[gRbase] Entries of an array can be normalized to sum to one in two ways: 1) Normalization can be over the first variable for *each* configuration of all other variables and 2) over all configurations. For example:

```
tabNormalize(z5, "first") %>% flat
## Hair Eye Sex Freq
## 1 Hair1 Eye1 Sex1 0.3765
## 2 Hair2 Eye1 Sex1 0.6235
## 3 Hair1 Eye2 Sex1 0.1803
## 4 Hair2 Eye2 Sex1 0.8197
```

### 3.2 Subsetting an array – slicing

We can subset arrays (this will also be called "slicing") in different ways. Notice that the result is not necessarily an array. Slicing can be done using standard R code or using tabSlice[gRbase]. The virtue of tabSlice[gRbase] comes from the flexibility when specifying the slice:

The following leads from the original  $2 \times 3 \times 2$  array to a  $2 \times 2$  array by cutting away the Sex=Male and Eye=Brown slice of the array:

```
tabSlice(hec, slice=list(Eye=c("Blue", "Hazel"), Sex="Female"))
## Notice: levels can be written as numerics
## tabSlice(hec, slice=list(Eye=2:3, Sex="Female"))
```

We may also regard the result above as a  $2 \times 2 \times 1$  array:

If slicing leads to a one dimensional array, the output will by default not be an array but a vector (without a dim attribute). However, the result can be forced to be a 1-dimensional array:

```
## A vector:
t1 <- tabSlice(hec, slice=list(Hair=1, Sex="Female")); t1
## A 1-dimensional array:
t2 <- tabSlice(hec, slice=list(Hair=1, Sex="Female"), as.array=TRUE); t2
## A higher dimensional array (in which some dimensions only have one level)
t3 <- tabSlice(hec, slice=list(Hair=1, Sex="Female"), drop=FALSE); t3</pre>
```

The difference between the last two forms can be clarified:

```
t2 %>% flat

## Eye Freq

## 1 Brown 36

## 2 Blue 9

## 3 Hazel 5

t3 %>% flat

## Hair Eye Sex Freq

## 1 Black Brown Female 36

## 2 Black Blue Female 9

## 3 Black Hazel Female 5
```

### 3.3 Collapsing and inflating arrays

Collapsing: The HE-marginal array  $T_{HE}$  of  $T_{HES}$  is the array with values

$$T_{HE}(h,e) = \sum_{s} T_{HES}(h,e,s)$$

Inflating: The "opposite" operation is to extend an array. For example, we can extend  $T_{HE}$  to have a third dimension, e.g. Sex. That is

$$\tilde{T}_{SHE}(s,h,e) = T_{HE}(h,e)$$

so  $\tilde{T}_{SHE}(s, h, e)$  is constant as a function of s.

With gRbase we can collapse arrays with<sup>1</sup>:

```
he <- tabMarg(hec, c("Hair", "Eye"))
he

## Eye
## Hair Brown Blue Hazel
## Black 68 20 15
## Brown 119 84 54
```

```
## Alternatives
tabMarg(hec, ~Hair:Eye)
tabMarg(hec, c(1, 2))
hec %a_% ~Hair:Eye
```

Notice that collapsing is a projection in the sense that applying the operation again does not change anything:

```
he1 <- tabMarg(hec, c("Hair", "Eye"))
he2 <- tabMarg(he1, c("Hair", "Eye"))
tabEqual(he1, he2)
## [1] TRUE</pre>
```

Expand an array by adding additional dimensions with tabExpand()[gRbase]:

```
extra.dim <- list(Sex=c("Male", "Female"))</pre>
tabExpand(he, extra.dim)
## , , Sex = Male
##
##
       Hair
## Eye Black Brown
## Brown 68 119
## Blue 20 84
## Hazel 15 54
\#\# , , Sex = Female
##
##
        Hair
## Eye Black Brown
## Brown 68 119
##
    Blue
           20
          15
## Hazel
```

<sup>&</sup>lt;sup>1</sup>FIXME: Should allow for abbreviations in formula and character vector specifications.

```
## Alternatives
he %a^% extra.dim
```

Notice that expanding and collapsing brings us back to where we started:

```
(he %a^% extra.dim) %a_% c("Hair", "Eye")

## Eye

## Hair Brown Blue Hazel

## Black 136 40 30

## Brown 238 168 108
```

### 3.4 Permuting an array

A reorganization of the table can be made with tabPerm[gRbase] (similar to aperm()), but tabPerm[gRbase] allows for a formula and for variable abbreviation:

```
tabPerm(hec, ~Eye:Sex:Hair) %>% flat
## Eye Sex Hair Freq
## 1 Brown Male Black 32
## 2 Blue Male Black 11
## 3 Hazel Male Black 10
## 4 Brown Female Black 36
```

Alternative forms (the first two also works for aperm):

```
tabPerm(hec, c("Eye", "Sex", "Hair"))
tabPerm(hec, c(2, 3, 1))
tabPerm(hec, ~Ey:Se:Ha)
tabPerm(hec, c("Ey", "Se", "Ha"))
```

### 3.5 Equality

Two arrays are defined to be identical 1) if they have the same dimnames and 2) if, possibly after a permutation, all values are identical (up to a small numerical difference):

```
hec2 <- tabPerm(hec, 3:1)
tabEqual(hec, hec2)
## [1] TRUE</pre>
```

```
## Alternative
hec %a==% hec2
```

#### 3.6 Aligning

We can align one array according to the ordering of another:

```
hec2 <- tabPerm(hec, 3:1)
tabAlign(hec2, hec)</pre>
```

```
## Alternative:
tabAlign(hec2, dimnames(hec))
## , , Sex = Male
##
##
       Eye
## Hair Brown Blue Hazel
## Black 32 11 10
##
   Brown
          53 50
##
\#\# , , Sex = Female
##
##
        Eye
## Hair Brown Blue Hazel
## Black 36 9 5
## Brown 66 34
```

### 3.7 Multiplication, addition etc: +, -, \*, /

The product of two arrays  $T_{HE}$  and  $T_{HS}$  is defined to be the array  $\tilde{T}_{HES}$  with entries

$$\tilde{T}_{HES}(h, e, s) = T_{HE}(h, e) + T_{HS}(h, s)$$

The sum, difference and quotient is defined similarly: This is done with tabProd()[gRbase], tabAdd()[gRbase], tabDiff()[gRbase] and tabDiv()[gRbase]:

```
hs <- tabMarg(hec, ~Hair:Eye)
tabMult(he, hs)

## Eye

## Hair Brown Blue Hazel

## Black 4624 400 225

## Brown 14161 7056 2916
```

#### Available operations:

```
tabAdd(he, hs)
tabSubt(he, hs)
tabMult(he, hs)
tabDiv(he, hs)
tabDiv(he, hs) ## Convention 0/0 = 0
```

#### Shortcuts:

```
## Alternative
he %a+% hs
he %a-% hs
he %a*% hs
he %a/% hs
he %a/% hs
```

Multiplication and addition of (a list of) multiple arrays is accomplished with tabProd()[gRbase] and tabSum()[gRbase] (much like prod()[gRbase] and sum()[gRbase]):

```
es <- tabMarg(hec, ~Eye:Sex)
tabSum(he, hs, es)
```

```
## , , Sex = Male
##
      Eye
## Hair Brown Blue Hazel
## Black 221 101 65
   Brown 323 229
##
                   143
##
## , , Sex = Female
##
##
       Eye
## Hair Brown Blue Hazel
## Black 238 83 64
## Brown 340 211 142
## tabSum(list(he, hs, es))
```

### 3.8 An array as a probability density

If an array consists of non–negative numbers then it may be regarded as an (unnormalized) discrete multivariate density. With this view, the following examples should be self explanatory:

```
tabDist(hec, marg=~Hair:Eye)
##
        Eye
## Hair Brown Blue Hazel
## Black 0.1889 0.05556 0.04167
## Brown 0.3306 0.23333 0.15000
tabDist(hec, cond=~Sex)
## , , Sex = Male
##
##
        Eye
## Hair Brown
                Blue Hazel
## Black 0.1768 0.06077 0.05525
##
   Brown 0.2928 0.27624 0.13812
##
## , , Sex = Female
##
##
       Eye
## Hair Brown Blue Hazel
## Black 0.2011 0.05028 0.02793
## Brown 0.3687 0.18994 0.16201
tabDist(hec, marg=~Hair, cond=~Sex)
## Sex
## Hair Male Female
## Black 0.2928 0.2793
## Brown 0.7072 0.7207
```

#### 3.9 Miscellaneous

Multiply values in a slice by some number and all other values by another number:

```
tabSliceMult(es, list(Sex="Female"), val=10, comp=0)
## Sex
## Eye Male Female
## Brown 0 1020
## Blue 0 430
## Hazel 0 340
```

### 4 Examples

### 4.1 A Bayesian network

A classical example of a Bayesian network is the "sprinkler example", see e.g. http://en.wikipedia.org/wiki/Bayesian\_network:

Suppose that there are two events which could cause grass to be wet: either the sprinkler is on or it is raining. Also, suppose that the rain has a direct effect on the use of the sprinkler (namely that when it rains, the sprinkler is usually not turned on). Then the situation can be modeled with a Bayesian network.

We specify conditional probabilities p(r), p(s|r) and p(w|s,r) as follows (notice that the vertical conditioning bar (|) is replaced by the horizontal underscore:

```
yn <- c("y", "n")</pre>
lev <- list(rain=yn, sprinkler=yn, wet=yn)</pre>
r <- tabNew(~rain, levels=lev, values=c(.2, .8))
s_r <- tabNew(~sprinkler:rain, levels = lev, values = c(.01, .99, .4, .6))
w_sr <- tabNew( ~wet:sprinkler:rain, levels=lev,</pre>
           values=c(.99, .01, .8, .2, .9, .1, 0, 1))
r
## rain
## y n
## 0.2 0.8
s_r %>% flat
   sprinkler rain Freq
## 1
     у у 0.01
## 2
          n y 0.99
## 3
          y n 0.40
## 4
          n n 0.60
w_sr %>% flat
## wet sprinkler rain Freq
## 1 y y 0.99
## 2 n
              у у 0.01
## 3 y
             n y 0.80
             n y 0.20
```

The joint distribution p(r, s, w) = p(r)p(s|r)p(w|s, r) can be obtained with tabProd()[gRbase]: ways:

```
joint <- tabProd(r, s_r, w_sr); joint %>% flat
## wet sprinkler rain Freq
## 1 y y y 0.00198
```

```
## 2 n y 0.00002
## 3 y n y 0.15840
## 4 n n y 0.03960
```

What is the probability that it rains given that the grass is wet? We find  $p(r, w) = \sum_{s} p(r, s, w)$  and then p(r|w) = p(r, w)/p(w). Can be done in various ways: with tabDist()[gRbase]

```
## Alternative:
rw <- tabMarg(joint, ~rain + wet)
tabDiv(rw, tabMarg(rw, ~wet))
## or
rw %a/% (rw %a_% ~wet)</pre>
```

### 4.2 Iterative Proportional Scaling (IPS)

We consider the 3-way lizard data from gRbase:

```
data(lizard, package="gRbase")
lizard %>% flat
## diam height species Freq
## 1 <=4 >4.75 anoli 32
## 2 >4 >4.75 anoli 11
## 3 <=4 <=4.75 anoli 86
## 4 >4 <=4.75 anoli 35</pre>
```

Consider the two factor log-linear model for the lizard data. Under the model the expected counts have the form

$$\log m(d, h, s) = a_1(d, h) + a_2(d, s) + a_3(h, s)$$

If we let n(d, h, s) denote the observed counts, the likelihood equations are: Find m(d, h, s) such that

$$m(d,h) = n(d,h), \quad m(d,s) = n(d,s), \quad m(h,s) = n(h,s)$$

where  $m(d,h) = \sum_{s} m(d,h.s)$  etc. The updates are as follows: For the first term we have

$$m(d,h,s) \leftarrow m(d,h,s) \frac{n(d,h)}{m(d,h)}$$

After iterating the updates will not change and we will have equality:  $m(d, h, s) = m(d, h, s) \frac{n(d, h)}{m(d, h)}$  and summing over s shows that the equation m(d, h) = n(d, h) is satisfied.

A rudimentary implementation of iterative proportional scaling for log–linear models is straight forward:

```
myips <- function(indata, glist){</pre>
    fit <- indata
    fit[] <- 1
    ## List of sufficient marginal tables
    md <- lapply(glist, function(g) tabMarg(indata, g))</pre>
    for (i in 1:4){
        for (j in seq_along(glist)){
            mf <- tabMarg(fit, glist[[j]])</pre>
            \# adj \leftarrow tabDiv(md[[j]], mf)
            # fit <- tabMult( fit, adj )</pre>
            ## or
            adj <- md[[ j ]] %a/% mf
            fit <- fit %a*% adj
        }
    }
    pearson <- sum((fit - indata)^2 / fit)</pre>
    list(pearson=pearson, fit=fit)
}
glist <- list(c("species", "diam"),c("species", "height"),c("diam", "height"))</pre>
fm1 <- myips(lizard, glist)</pre>
fm1$pearson
## [1] 664.6
fm1$fit %>% flat
## species diam height Freq
## 1 anoli <=4 >4.75 32.8
## 2 dist <=4 >4.75 60.2
## 3 anoli >4 >4.75 10.2
## 4 dist >4 >4.75 41.8
fm2 <- loglin(lizard, glist, fit=T)</pre>
## 4 iterations: deviation 0.009619
fm2$pearson
## [1] 0.1506
fm2$fit %>% flat
## diam height species Freq
## 1 <=4 >4.75 anoli 32.8
## 2 >4 >4.75 anoli 10.2
## 3 <=4 <=4.75 anoli 85.2
## 4 >4 <=4.75 anoli 35.8
```

### 5 Some low level functions

For e.g. a  $2 \times 3 \times 2$  array, the entries are such that the first variable varies fastest so the ordering of the cells are (1,1,1), (2,1,1), (1,2,1), (2,2,1), (1,3,1) and so on. To find the value of such a cell, say, (j,k,l) in the array (which is really just a vector), the cell is mapped into an entry of a vector.

For example, cell (2,3,1) (Hair=Brown, Eye=Hazel, Sex=Male) must be mapped to entry 4 in

```
hec
## , , Sex = Male
##
##
         Eye
## Hair
        Brown Blue Hazel
    Black 32 11
##
                       10
##
    Brown
             53
                 50
                        25
##
  , , Sex = Female
##
##
         Eye
## Hair
        Brown Blue Hazel
##
    Black 36
                 9
                         5
##
    Brown
             66
                  34
                        29
c(hec)
## [1] 32 53 11 50 10 25 36 66 9 34 5 29
```

For illustration we do:

```
cell2name <- function(cell, dimnames){
    unlist(lapply(1:length(cell), function(m) dimnames[[m]][cell[m]]))
}
cell2name(c(2,3,1), dimnames(hec))
## [1] "Brown" "Hazel" "Male"</pre>
```

### 5.1 cell2entry(), entry2cell() and next\_cell()

The map from a cell to the corresponding entry is provided by cell2entry()[gRbase]. The reverse operation, going from an entry to a cell (which is much less needed) is provided by entry2cell()[gRbase].

```
cell2entry(c(2,3,1), dim=c(2, 3, 2))
## [1] 6

entry2cell(6, dim=c(2, 3, 2))
## [1] 2 3 1
```

Given a cell, say i = (2, 3, 1) in a  $2 \times 3 \times 2$  array we often want to find the next cell in the table following the convention that the first factor varies fastest, that is (1, 1, 2). This is provided by  $next_cell()[gRbase]$ .

```
next_cell(c(2,3,1), dim=c(2, 3, 2))
## [1] 1 1 2
```

### 5.2 next\_cell\_slice() and slice2entry()

Given that we look at cells for which for which the index in dimension 2 is at level 3 (that is Eye=Hazel), i.e. cells of the form (j,3,l). Given such a cell, what is then the next cell that also satisfies this constraint. This is provided by next\_cell\_slice()[gRbase].<sup>2</sup>

```
next_cell_slice(c(1,3,1), slice_marg=2, dim=c( 2, 3, 2 ))
## [1] 2 3 1

next_cell_slice(c(2,3,1), slice_marg=2, dim=c( 2, 3, 2 ))
## [1] 1 3 2
```

Given that in dimension 2 we look at level 3. We want to find entries for the cells of the form (j,3,l).

```
slice2entry(slice_cell=3, slice_marg=2, dim=c( 2, 3, 2 ))
## [1] 5 6 11 12
```

To verify that we indeed get the right cells:

### 5.3 fact\_grid() - Factorial grid

Using the operations above we can obtain the combinations of the factors as a matrix:

```
head( fact_grid( c(2, 3, 2) ), 6 )
##
       [,1] [,2] [,3]
## [1,]
         1 1 1
## [2,]
          2
               1
                    1
## [3,]
          1
               2
                    1
## [4,]
          2
               2
                    1
## [5,]
          1
               3
                    1
## [6,]
               3
```

A similar dataframe can also be obtained with the standard R function expand.grid (but factGrid is faster)

<sup>&</sup>lt;sup>2</sup>FIXME: sliceset should be called margin.

<sup>&</sup>lt;sup>3</sup>FIXME:slicecell and sliceset should be renamed

```
head( expand.grid(list(1:2, 1:3, 1:2)), 6 )
## Var1 Var2 Var3
## 1 1 1 1
    2
        1 1
## 2
        2
            1
    1
## 3
## 4
     2
             1
## 5
    1
         3
             1
## 6 2 3
```

### A More about slicing

Slicing using standard R code can be done as follows:

```
hec[, 2:3, ] %>% flat ## A 2 x 2 x 2 array
## Hair Eye Sex Freq
## 1 Black Blue Male 11
## 2 Brown Blue Male
## 3 Black Hazel Male
## 4 Brown Hazel Male
hec[1, , 1]
                     ## A vector
## Brown Blue Hazel
## 32 11 10
hec[1, , 1, drop=FALSE] ## A 1 x 3 x 1 array
\#\# , , Sex = Male
##
##
       Eye
## Hair Brown Blue Hazel
## Black 32 11 10
```

Programmatically we can do the above as

```
do.call("[", c(list(hec), list(TRUE, 2:3, TRUE))) %>% flat
do.call("[", c(list(hec), list(1, TRUE, 1)))
do.call("[", c(list(hec), list(1, TRUE, 1), drop=FALSE))
```

gRbase provides two alterntives for each of these three cases above:

```
tabSlicePrim(hec, slice=list(TRUE, 2:3, TRUE)) %>% flat
tabSlice(hec, slice=list(c(2, 3)), margin=2) %>% flat

tabSlicePrim(hec, slice=list(1, TRUE, 1))
tabSlice(hec, slice=list(1, 1), margin=c(1, 3))

tabSlicePrim(hec, slice=list(1, TRUE, 1), drop=FALSE)
tabSlice(hec, slice=list(1, 1), margin=c(1, 3), drop=FALSE)
```