Overview of the functionalities of the package lavaSearch2

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Load lavaSearch2 in the R session:

```
library(lavaSearch2)
```

1 Inference

1.1 Introductory example

You may have noticed that for simple linear regression, the p-values of the Wald tests from 1m:

```
## simulate data
mSim <- lvm(Y[1:1]~0.3*X1+0.2*X2)
set.seed(10)
df.data <- sim(mSim, 2e1)

## fit linear model
summary(lm(Y~X1+X2, data = df.data))$coef</pre>
```

```
Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.7967775 0.2506767 3.1785069 0.005495832
X1 0.1550938 0.2205080 0.7033477 0.491360483
X2 0.4581556 0.2196785 2.0855736 0.052401103
```

differ from those obtained with the corresponding latent variable model estimated by maximum likelihood:

```
## fit latent variable model
m <- lvm(Y~X1+X2)
e <- estimate(m, data = df.data)

## extract Wald tests
summary(e)$coef</pre>
```

```
Estimate Std. Error Z-value P-value
Y~X1 0.1550938 0.2032984 0.7628877 0.4455303456
Y~X2 0.4581556 0.2025335 2.2621221 0.0236898575
Y~~Y 0.5557910 0.1757566 3.1622777 NA
Y 0.7967775 0.2311125 3.4475747 0.0005656439
```

For instance, the p-value for the effect of X2 is 0.024 in the latent variable model and 0.052 in the linear regression. The discrepancy is due to 2 corrections that 1m applies in order to improve the control of the type 1 error of the Wald tests:

- use of a Student t-distribution instead of a Gaussian distribution (informally using a t-value instead of z-value).
- use of an unbiased estimator of the residuals variance instead of the ML-estimator.

lavaSearch2 attempts to generalize these corrections to models with correlated and heteroschedastic measurements. In the case of a simple linear regression, Wald tests obtained with lavaSearch2 exactly match the results of lm:

```
summary2(e)$coef
```

```
Estimate Std. Error t-value P-value df
Y~X1 0.1550938 0.2205078 0.7033483 0.49136012 17.00
Y~X2 0.4581556 0.2196783 2.0855754 0.05240092 17.00
Y~~Y 0.6538707 0.2242758 2.9154759 NA 4.25
Y 0.7967775 0.2506765 3.1785096 0.00549580 17.00
```

1.2 How it works in a nutshell

When using lava, the p.values that are obtained from the summary (Wald tests) rely on a Gaussian approximation and maximum likelihood estimation. While being asymptotically valid, they usually do not provide a very accurate control of the type 1 error rate in small samples. Simulations have shown that the type 1 error rate tends to be too large, i.e. the p.values are have a downward bias. lavaSearch2 provides two improvements:

- using a Student's t-distribution instead of a Gaussian distribution to account for the uncertainty on the variance of the coefficients. The degrees of freedom are estimated using Satterwaite approximation, i.e. identifying the chi-squared distribution that best fit the observed moments of the variance of the coefficients.
- (partially) correcting for the first order bias in the ML estimates of the variance parameters. This correction also affects the standard error of the estimates.

1.3 Single univariate Wald test

We will illustrate the functionalities using a simulated dataset:

```
## simulate data mSim <- lvm(Y1\simeta,Y2\simeta,Y3\sim0.4+0.4*eta,Y4\sim0.6+0.6*eta,eta\sim0.5*X1+0.7*X2) latent(mSim) <- \simeta set.seed(12) df.data <- sim(mSim, n = 3e1, latent = FALSE) ## display head(df.data)
```

```
Υ1
                    Y2
                                ΥЗ
                                           Y4
                                                      Х1
                                                                 X2
                                   0.2579355
1 -1.7606233 0.1264910 0.66442611
                                               0.2523400 -1.5431527
  3.0459417 2.4631929 0.00283511
                                    2.1714802
                                               0.6423143 -1.3206009
3 -2.1443162 -0.3318033 0.82253070 0.3008415 -0.3469361 -0.6758215
4 -2.5050328 -1.3878987 -0.10474850 -1.7814956 -0.5152632 -0.3670054
5 -2.5307249 0.3012422 1.22046986 -1.0195188 0.3981689 -0.5138722
6 -0.9521366  0.1669496 -0.21422548  1.5954456
                                              0.9535572 -0.9592540
```

We first fit the latent variable model using, as usual, the estimate function:

```
m <- lvm(c(Y1,Y2,Y3,Y4)\simeta, eta\simX1+X2)
e <- estimate(m, data = df.data)
```

We can extract the Wald tests based on the traditional approach using summary:

```
summary(e)$coef[c("Y2","Y3","Y2~eta","Y3~eta","eta~X1","eta~X2"), ]
```

```
Estimate Std. Error Z-value P-value
Y2 0.2335412 0.2448593 0.9537775 0.3401962906
Y3 0.5114275 0.1785886 2.8637186 0.0041869974
Y2~eta 0.9192847 0.2621248 3.5070497 0.0004531045
Y3~eta 0.2626930 0.1558978 1.6850339 0.0919820326
eta~X1 0.5150072 0.2513393 2.0490515 0.0404570768
eta~X2 0.6212222 0.2118930 2.9317729 0.0033703310
```

As explain at the begining of this section, lavaSearch2 implements two corrections that can be directly applied by calling the summary2 method:

```
summary2(e)$coef[c("Y2","Y3","Y2~eta","Y3~eta","eta~X1","eta~X2"), ]
```

```
Estimate Std. Error t-value P-value df
Y2 0.2335412 0.2518218 0.9274067 0.371516094 12.328385
Y3 0.5114275 0.1828716 2.7966475 0.009848769 24.707696
Y2~eta 0.9192847 0.2653220 3.4647887 0.031585600 3.515034
Y3~eta 0.2626930 0.1562776 1.6809386 0.143826633 5.993407
eta~X1 0.5150072 0.2642257 1.9491180 0.065414617 20.044312
eta~X2 0.6212222 0.2221293 2.7966698 0.009275494 27.718363
```

To use the Satterthwaite correction alone, set the argument bias.correct to FALSE:

```
summary2(e, bias.correct = FALSE)$coef[c("Y2","Y3","Y2\simeta","Y3\simeta","eta\simX1","eta\simX2"), ]
```

```
Estimate Std. Error t-value P-value df
Y2 0.2335412 0.2448593 0.9537775 0.357711941 12.911877
Y3 0.5114275 0.1785886 2.8637186 0.008210968 25.780552
Y2~eta 0.9192847 0.2621248 3.5070497 0.028396459 3.674640
Y3~eta 0.2626930 0.1558978 1.6850339 0.141185621 6.222912
eta~X1 0.5150072 0.2513393 2.0490515 0.052814794 21.571210
eta~X2 0.6212222 0.2118930 2.9317729 0.006351686 30.370334
```

When using the Satterthwaite correction alone, the standard error are left unchanged compared to the original lava output. The only change is how the p-values are computed, i.e. based on the quantiles of a Student's t-distribution instead of a Gaussian distribution.

To only use the bias correction, set the argument df to FALSE:

```
summary2(e, df = FALSE)$coef[c("Y2","Y3","Y2~eta","Y3~eta","eta~X1","eta~X2"), ]
```

```
Estimate Std. Error t-value P-value df
Y2 0.2335412 0.2518218 0.9274067 0.3537154044 Inf
Y3 0.5114275 0.1828716 2.7966475 0.0051635832 Inf
Y2~eta 0.9192847 0.2653220 3.4647887 0.0005306482 Inf
Y3~eta 0.2626930 0.1562776 1.6809386 0.0927748494 Inf
eta~X1 0.5150072 0.2642257 1.9491180 0.0512813393 Inf
eta~X2 0.6212222 0.2221293 2.7966698 0.0051632271 Inf
```

1.4 Saving computation time with scorrect

For each call to summary2 the small sample size correction(s) will be recalculated. However the calculation of the sample correction(s) can be time consuming.

```
system.time(
   res <- summary2(e, bias.correct = FALSE)
)</pre>
```

```
user system elapsed 0.25 0.00 0.25
```

In such a case one can pre-compute the main terms of the correction (e.g. the derivative of the variance-covariance matrix) once for all using the sCorrect method (sCorrect stands for Satterthwaite correction). When calling sCorrect, the right hand side indicates whether the bias correction should be used (equivalent to bias.correct argument described previously):

```
e2 <- e
sCorrect(e2) <- TRUE
```

sCorrect automatically store the pre-computed terms in the sCorrect slot of the object. It also adds the class lvmfit2 to the object:

```
class(e2)
```

```
[1] "lvmfit2" "lvmfit"
```

Then p-values computed using the small sample correction can be obtained calling the summary method, as usual:

```
summary2(e2)$coef[c("Y2","Y3","Y2~eta","Y3~eta","eta~X1","eta~X2"), ]
```

```
Estimate Std. Error
                              t-value
                                          P-value
                                                          df
Y2
                  0.2518218 0.9274067 0.371516094 12.328385
       0.2335412
Y3
       0.5114275
                  0.1828716 2.7966475 0.009848769 24.707696
Y2~eta 0.9192847
                  0.2653220 3.4647887 0.031585600
Y3~eta 0.2626930
                  0.1562776 1.6809386 0.143826633
                                                    5.993407
eta~X1 0.5150072
                  0.2642257 1.9491180 0.065414617 20.044312
                  0.2221293 2.7966698 0.009275494 27.718363
eta~X2 0.6212222
```

The summary2 methods take approximately the same time as the usual summary method:

```
system.time(
    summary2(e2)
)
```

```
user system elapsed
```

1.5 Single multivariate Wald test

0.16

The function compare from the lava package can be use to perform multivariate Wald tests, i.e. to test simultaneously several linear combinations of the coefficients. compare uses a contrast matrix to encode in lines which linear combination of coefficients should be tested. For instance if we want to simultaneously test whether all the mean coefficients are 0, we can create a contrast matrix using createContrast:

```
resC <- createContrast(e2, par = c("Y2=0","Y2~eta=0","eta~X1=0"))
resC
```

\$contrast

user

0.15

system elapsed

0.00

```
Y2 Y3 Y4 eta Y2~eta Y3~eta Y4~eta eta~X1 eta~X2 Y1~~Y1 Y2~~Y2 Y3~~Y3 Y4~~Y4
                                                           0
                                                                   0
                                                                                   0
[Y2] = 0
                   0
                      0
                           0
                                   0
                                           0
                                                   0
                                                                           0
                                                                                           0
                                                                                                   0
                                   1
                                           0
                                                           0
                                                                   0
                                                                           0
                                                                                   0
                                                                                           0
[Y2\text{-eta}] = 0
               0
                           0
                                                   0
                                                                                                   0
[eta~X1] = 0
               0
                  0 0
                                   0
                                           0
                                                   0
                                                           1
                                                                                                   0
              eta~~eta
[Y2] = 0
[Y2~eta] = 0
                      0
[eta~X1] = 0
                      0
```

\$null

```
[Y2] = 0 [Y2~eta] = 0 [eta~X1] = 0
       0
```

\$Q [1] 3

We can then test the linear hypothesis by specifying in compare the left hand side of the hypothesis (argument contrast) and the right hand side (argument null):

```
resTest0 <- lava::compare(e2, contrast = resC$contrast, null = resC$null)
resTest0
```

```
- Wald test -
        Null Hypothesis:
        [Y2] = 0
        [Y2~eta] = 0
        [eta~X1] = 0
data:
chisq = 21.332, df = 3, p-value = 8.981e-05
sample estimates:
                     Std.Err
                                    2.5%
          Estimate
                                             97.5%
[Y2]
         0.2335412 0.2448593 -0.2463741 0.7134566
[Y2~eta] 0.9192847 0.2621248 0.4055295 1.4330399
[eta~X1] 0.5150072 0.2513393 0.0223912 1.0076231
```

compare uses a chi-squared distribution to compute the p-values. Similarly to the Gaussian approximation, while being valid asymptotically this procedure may not provide a very accurate control of the type 1 error rate in small samples. Fortunately, the correction proposed for the univariate Wald statistic can be adapted to the multivariate Wald statistic. This is achieved by compare2:

```
resTest1 <- compare2(e2, contrast = resC$contrast, null = resC$null)
resTest1
```

```
- Wald test -
Null Hypothesis:
[Y2] = 0
[Y2\text{-eta}] = 0
[eta~X1] = 0
```

```
F-statistic = 6.7118, df1 = 3, df2 = 11.1, p-value = 0.007596
sample estimates:
              Estimate
                         Std.Err
                                         df
                                                  2.5%
             0.2335412 0.2518218 12.328385 -0.3135148 0.7805973
```

[Y2~eta] = 0 0.9192847 0.2653220 3.515034 0.1407653 1.6978041

The same result could have been obtained using the par argument to define the linear hypothesis:

```
resTest2 <- compare2(e2, par = c("Y2","Y2~eta","eta~X1"))
identical(resTest1,resTest2)</pre>
```

[1] TRUE

Now a F-distribution is used to compute the p-values. As before on can set the argument bias.correct to FALSE to use the Satterthwaite approximation alone:

```
- Wald test -
        Null Hypothesis:
        [Y2] = 0
        [Y2\text{-eta}] = 0
        [eta~X1] = 0
data:
F-statistic = 7.1107, df1 = 3, df2 = 11.13, p-value = 0.006182
sample estimates:
              Estimate
                         Std.Err
                                        df
                                                    2.5%
                                                             97.5%
             0.2335412 0.2448593 12.91188 -0.295812256 0.7628948
[Y2~eta] = 0 0.9192847 0.2621248 3.67464 0.165378080 1.6731913
[eta~X1] = 0 0.5150072 0.2513393 21.57121 -0.006840023 1.0368543
```

In this case the F-statistic of compare2 is the same as the chi-squared statistic of compare divided by the rank of the contrast matrix:

```
resTestO$statistic/qr(resC$contrast)$rank
```

```
chisq
7.110689
```

1.6 Robust Wald tests

When one does not want to assume normality distributed residuals, robust standard error can be used instead of the model based standard errors. They can be obtain by setting the argument robust to TRUE when computing univariate Wald tests:

```
summary2(e, robust = TRUE)$coef[c("Y2","Y3","Y2~eta","Y3~eta","eta~X1","eta~X2"),]
```

```
Estimate robust SE
                             t-value
                                          P-value
                                                         df
Y2
       0.2335412 0.2353245 0.9924222 0.3340117610 18.18841
Y3
       0.5114275 0.1897160 2.6957535 0.0099985389 42.79555
Y2~eta 0.9192847 0.1791240 5.1321150 0.0002361186 12.19058
Y3~eta 0.2626930 0.1365520 1.9237585 0.0653095551 26.20919
eta~X1 0.5150072 0.2167580 2.3759546 0.0315112789 14.74859
eta~X2 0.6212222 0.2036501 3.0504389 0.0035239307 54.54181
```

or multivariate Wald test:

- Wald test -

data:

```
compare2(e2, robust = TRUE, par = c("Y2", "Y2~eta", "eta~X1"))
```

```
Null Hypothesis:
         [Y2] = 0
         [Y2\text{-eta}] = 0
         [eta~X1] = 0
F-statistic = 12.526, df1 = 3, df2 = 23.97, p-value = 3.981e-05
```

sample estimates: Estimate robust SE 2.5% df 97.5% 0.2335412 0.2353245 18.18841 -0.26049031 0.7275728 [Y2~eta] = 0 0.9192847 0.1791240 12.190580.52968275 1.3088867

[eta~X1] = 0 0.5150072 0.2167580 14.748590.05231154 0.9777028

Only the standard error is affected by the argument robust, the degrees of freedom are the one of the model-based standard errors. It may be surprising that the (corrected) robust standard errors are (in this example) smaller than the (corrected) model-based one. This is also the case for the uncorrected one:

```
rbind(robust = diag(crossprod(iid(e2))),
     model = diag(vcov(e2)))
```

```
Y2
                          Y3
                                     Y4
                                                        Y2~eta
                                               eta
                                                                   Y3~eta
robust 0.04777252 0.03325435 0.03886706 0.06011727 0.08590732 0.02179453 0.02981895
model 0.05995606 0.03189389 0.04644303 0.06132384 0.06870941 0.02430412 0.03715633
           eta~X1
                      eta~X2
                                Y1~~Y1
                                          Y2~~Y2
                                                     Y3~~Y3
                                                                 Y4~~Y4 eta~~eta
robust 0.05166005 0.05709393 0.2795272 0.1078948 0.03769614 0.06923165 0.3198022
model 0.06317144 0.04489865 0.1754744 0.1600112 0.05112998 0.10152642 0.2320190
```

This may be explained by the fact the robust standard error tends to be liberal in small samples (e.g. see Kauermann 2001, A Note on the Efficiency of Sandwich Covariance Matrix Estimation).

1.7 Assessing the type 1 error of the testing procedure

The function calibrateType1 can be used to assess the type 1 error of a Wald statistic on a specific example. This however assumes that the estimated model is correctly specified. Let's make an example. For this we simulate some data:

```
set.seed(10)
m.generative <- lvm(Y ~ X1 + X2 + Gene)
categorical(m.generative, labels = c("ss","ll")) <- ~Gene
d <- lava::sim(m.generative, n = 50, latent = FALSE)</pre>
```

Let's now imagine that we want to analyze the relationship between Y and Gene using the following dataset:

```
head(d)
```

```
Y X1 X2 Gene
1 -1.14369572 -0.4006375 -0.7618043 ss
2 -0.09943370 -0.3345566 0.4193754 ss
3 -0.04331996 1.3679540 -1.0399434 11
4 2.25017335 2.1377671 0.7115740 ss
5 0.16715138 0.5058193 -0.6332130 ss
6 1.73931135 0.7863424 0.5631747 ss
```

For this we fit define a LVM:

```
myModel <- lvm(Y \sim X1 + X2 + Gene)
```

and estimate the coefficients of the model using estimate:

```
e <- estimate(myModel, data = d)
e
```

```
Estimate Std. Error Z-value P-value
Regressions:
   Y~X1
                     1.02349
                                 0.12017 8.51728
                                                    <1e-12
   Y~X2
                                 0.12380 7.39244
                     0.91519
                                                    <1e-12
                     0.48035
                                 0.23991
                                          2.00224
   Y~Genell
                                                   0.04526
Intercepts:
                    -0.11221
                                 0.15773 -0.71141
                                                    0.4768
Residual Variances:
   Y
                     0.67073
                                 0.13415 5.00000
```

We can now use calibrateType1 to perform a simulation study. We just need to define the null hypotheses (i.e. which coefficients should be set to 0 when generating the data) and the number of simulations:

To save time we only make 50 simulations but much more are necessary to really assess the type 1 error rate. Then we can use the summary method to display the results:

```
summary(mySimulation)
```

2 Adjustment for multiple comparisons

2.1 Univariate Wald test, single model

When performing multiple testing, adjustment for multiple comparisons is necessary in order to control the type 1 error rate, i.e. to provide interpretable p.values. The **multcomp** package enables to do such adjustment when all tests comes from the same lvmfit object:

```
## simulate data
mSim <- lvm(Y ~ 0.25 * X1 + 0.3 * X2 + 0.35 * X3 + 0.4 * X4 + 0.45 * X5 + 0.5 * X6)
set.seed(10)
df.data <- sim(mSim, n = 4e1)

## fit lvm
e.lvm <- estimate(lvm(Y ~ X1 + X2 + X3 + X4 + X5 + X6), data = df.data)
name.coef <- names(coef(e.lvm))
n.coef <- length(name.coef)

## Create contrast matrix
resC <- createContrast(e.lvm, par = pasteO("Y~X",1:6), rowname.rhs = FALSE)
resC$contrast</pre>
```

```
Y Y~X1 Y~X2 Y~X3 Y~X4 Y~X5 Y~X6 Y~~Y
Y~X1 0
                0
                      0
Y~X2 0
           0
                1
                      0
                           0
                                 0
                                       0
                                            0
Y~X3 0
                           0
                                      0
           0
                0
                      1
                                 0
                                            0
Y~X4 0
           0
                0
                      0
                           1
Y~X5 0
           0
                0
                      0
                           0
                                 1
                                      0
                                            0
Y~X6 0
                      0
           0
                0
                           0
                                 0
                                       1
                                            0
```

```
e.glht <- multcomp::glht(e.lvm, linfct = resC$contrast, rhs = resC$null)
summary(e.glht)</pre>
```

Simultaneous Tests for General Linear Hypotheses

```
Fit: estimate.lvm(x = lvm(Y \sim X1 + X2 + X3 + X4 + X5 + X6), data = df.data)
```

Linear Hypotheses:

```
Estimate Std. Error z value Pr(>|z|)
Y \sim X1 == 0
            0.3270
                       0.1589
                                2.058 0.20725
Y \sim X2 == 0
            0.4025
                       0.1596
                                2.523 0.06611 .
Y \sim X3 == 0
            0.5072
                       0.1383 3.669 0.00144 **
Y \sim X4 == 0
            0.3161
                       0.1662 1.902 0.28582
Y \sim X5 == 0
            0.3875
                       0.1498
                                2.586 0.05554 .
Y \sim X6 == 0
            0.3758
                       0.1314
                                2.859 0.02482 *
Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
(Adjusted p values reported -- single-step method)
```

Note that this correction relies on the Gaussian approximation. To use small sample corrections implemented in lavaSearch2, just call glht2 instead of glht:

```
e.glht2 <- glht2(e.lvm, linfct = resC$contrast, rhs = resC$null)
summary(e.glht2)</pre>
```

Simultaneous Tests for General Linear Hypotheses

```
Fit: estimate.lvm(x = lvm(Y \sim X1 + X2 + X3 + X4 + X5 + X6), data = df.data)
Linear Hypotheses:
          Estimate Std. Error t value Pr(>|t|)
Y \sim X1 == 0
          0.3270
                       0.1750 1.869
                                        0.3290
Y \sim X2 == 0
                                2.291
          0.4025
                       0.1757
                                        0.1482
Y \sim X3 == 0
            0.5072
                       0.1522
                                3.333
                                        0.0123 *
Y \sim X4 == 0
          0.3161
                       0.1830 1.727
                                        0.4128
Y \sim X5 == 0
          0.3875
                       0.1650
                                2.349
                                        0.1315
            0.3758
Y \sim X6 == 0
                       0.1447
                                2.597
                                        0.0762 .
Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
```

The single step method is the appropriate correction when one wants to report the most significant p-value relative to a set of hypotheses. If the second most significant p-value is also to be reported then the method "free" is more efficient:

```
summary(e.glht2, test = multcomp::adjusted("free"))
```

Simultaneous Tests for General Linear Hypotheses

(Adjusted p values reported -- single-step method)

```
Fit: estimate.lvm(x = lvm(Y ~ X1 + X2 + X3 + X4 + X5 + X6), data = df.data)
```

Linear Hypotheses:

```
Estimate Std. Error t value Pr(>|t|)
Y \sim X1 == 0
            0.3270
                        0.1750 1.869
                                         0.1291
Y \sim X2 == 0
            0.4025
                        0.1757
                                 2.291
                                         0.0913 .
Y \sim X3 == 0
            0.5072
                        0.1522
                                 3.333
                                         0.0123 *
Y \sim X4 == 0
            0.3161
                        0.1830
                                 1.727
                                         0.1291
Y \sim X5 == 0
            0.3875
                        0.1650
                                 2.349
                                         0.0913 .
Y \sim X6 == 0
            0.3758
                        0.1447
                                 2.597
                                         0.0645 .
___
Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
(Adjusted p values reported -- free method)
```

See the book: "Multiple Comparisons Using R" by Frank Bretz, Torsten Hothorn, and Peter Westfall (2011, CRC Press) for details about the theory underlying the **multcomp** package.

2.2 Univariate Wald test, multiple models

Pipper et al. in "A Versatile Method for Confirmatory Evaluation of the Effects of a Covariate in Multiple Models" (2012, Journal of the Royal Statistical Society, Series C) developed a method to assess the effect of an exposure on several outcomes when a different model is fitted for each outcome. This method has been implemented in the mmm function from the multcomp package for glm and Cox models. lavaSearch2 extends it to lvm.

Let's consider an example where we wish to assess the treatment effect on three outcomes X, Y, and Z. We have at hand three measurements relative to outcome Z for each individual:

```
Z2
                                                                               Z3
         X
                Age Treatment
                                       Y Gender
                                                        Z1
1 39.12289 39.10415
                      placebo 0.6088958 female
                                                1.8714112
                                                            2.2960633 -0.09326935
2 39.56766 39.25191
                         SSRI
                               1.0001325 female 0.9709943 0.6296226 1.31035910
3 41.68751 43.05884
                               2.1551047 female -1.1634011 -0.3332927 -1.30769267
                      placebo
                               0.3852728 female -1.0305476 0.6678775 0.99780139
4 44.68102 44.78019
                         SSRI
5 41.42559 41.13105
                      placebo -0.8666783
                                           male -1.6342816 -0.8285492 1.20450488
                         SSRI -1.0710170 female -1.2198019 -1.9602130 -1.85472132
6 42.64811 41.75832
   treatment
1 1.1639675
2 -1.5233846
3 -2.5183351
4 -0.7075292
5 -0.2874329
6 -0.4353083
```

We fit a model specific to each outcome:

```
\label{eq:lmx} $$\lim X <- \lim(X \sim \text{Age + Treatment, data} = \text{df.data})$$ $\lim Y <- \text{estimate}(\lim Y \sim \text{Gender + Treatment}), \ data = \text{df.data})$$ $\lim X <- \text{estimate}(\lim (C(Z1,Z2,Z3) \sim 1*\text{eta, eta} \sim -1 + \text{Treatment}), \ data = \text{df.data})$$
```

and combine them into a list of lymfit objects:

```
mmm.lvm <- multcomp::mmm(X = lmX, Y = lvmY, Z = lvmZ)
```

We can then generate a contrast matrix to test each coefficient related to the treatment:

```
resC <- createContrast(mmm.lvm, var.test = "Treatment", add.variance = TRUE)
resC$contrast</pre>
```

```
X: (Intercept) X: Age X: TreatmentSSRI X: sigma2 Y: Y
X: TreatmentSSRI
                                    0
                                           0
                                                              1
Y: Y~TreatmentSSRI
                                    0
                                           0
                                                              0
                                                                        0
                                                                              0
Z: eta~TreatmentSSRI
                                    0
                                           0
                                                              0
                      Y: Y~Genderfemale Y: Y~TreatmentSSRI Y: Y~~Y Z: Z1 Z: Z2 Z: Z3
X: TreatmentSSRI
                                       0
                                                            0
                                                                    0
                                                                          0
Y: Y~TreatmentSSRI
                                       0
                                                            1
                                                                    0
                                                                          0
                                                                                 0
                                                                                       0
Z: eta~TreatmentSSRI
                                       0
                                                            0
                                                                          0
                      Z: eta~TreatmentSSRI Z: Z1~~Z1 Z: Z2~~Z2 Z: Z3~~Z3 Z: eta~~eta
X: TreatmentSSRI
                                          0
                                                     0
                                                                0
                                                                          0
Y: Y~TreatmentSSRI
                                                     0
                                                                0
                                          0
                                                                          0
                                                                                       0
Z: eta~TreatmentSSRI
                                          1
                                                     0
                                                                0
                                                                          0
                                                                                       0
```

```
lvm.glht2 <- glht2(mmm.lvm, linfct = resC$contrast, rhs = resC$null)
summary(lvm.glht2)</pre>
```

Simultaneous Tests for General Linear Hypotheses

Linear Hypotheses:

```
Estimate Std. Error t value Pr(>|t|) X: TreatmentSSRI == 0 0.4661 0.2533 1.840 0.187 Y: Y~TreatmentSSRI == 0 -0.5421 0.2613 -2.074 0.117 Z: eta~TreatmentSSRI == 0 -0.6198 0.4404 -1.407 0.393 (Adjusted p values reported -- single-step method)
```

This can be compared to the unadjusted p.values:

```
summary(lvm.glht2, test = multcomp::univariate())
```

Simultaneous Tests for General Linear Hypotheses

Linear Hypotheses:

```
Estimate Std. Error t value Pr(>|t|)

X: TreatmentSSRI == 0     0.4661     0.2533     1.840     0.0720 .

Y: Y~TreatmentSSRI == 0     -0.5421     0.2613     -2.074     0.0435 *

Z: eta~TreatmentSSRI == 0     -0.6198     0.4404     -1.407     0.1659

---

Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1

(Univariate p values reported)
```

3 Model diagnostic

3.1 Detection of local dependencies

The modelsearch function of lava is a diagnostic tool for latent variable models. It enables to search for local dependencies (i.e. model misspecification) and add them to the model. Obviously it is a data-driven procedure and its usefulness can be discussed, especially in small samples:

- the procedure is instable, i.e. is likely to lead to two different models when applied on two different dataset sampled from the same generative model.
- it is hard to define a meaningful significance threshold since p-values should be adjusted for multiple comparisons and sequential testing. However traditional methods like Bonferroni-Holm tend to over corrected and therefore reduce the power of the procedure since they assume that the test are independent.

The function modelsearch2 in lavaSearch2 partially solves the second issue by adjusting the p-values for multiple testing. Let's see an example:

```
## simulate data  mSim \leftarrow lvm(c(y1,y2,y3)\sim u, u\sim x1+x2)   latent(mSim) \leftarrow \sim u   covariance(mSim) \leftarrow y2\sim y3   transform(mSim, Id\sim u) \leftarrow function(x)\{1:NROW(x)\}   set.seed(10)   df.data \leftarrow lava::sim(mSim, n = 125, latent = FALSE)   head(df.data)
```

```
y1 y2 y3 x1 x2 Id
1 5.5071523 4.883752014 6.2928016 0.8694750 2.3991549 1
2 -0.6398644 0.025832617 0.5088030 -0.6800096 -0.0898721 2
3 -2.5835495 -2.616715027 -2.8982645 0.1732145 -0.8216484 3
4 -2.5312637 -2.518185427 -2.9015033 -0.1594380 -0.2869618 4
5 1.6346220 -0.001877577 0.3705181 0.7934994 0.1312789 5
6 0.4939972 1.759884014 1.5010499 1.6943505 -1.0620840 6
```

```
## fit model m \leftarrow 1 vm(c(y1,y2,y3) \sim u, u \sim x1) latent(m) \leftarrow u addvar(m) \leftarrow x2 e.lvm \leftarrow estimate(m, data = df.data)
```

modelsearch2 can be used to sequentially apply the modelsearch function with a given correction for the p.values:

```
resScore <- modelsearch2(e.lvm, alpha = 0.1, trace = FALSE)
displayScore <- summary(resScore)</pre>
```

Sequential search for local dependence using the score statistic The variable selection procedure retained 2 variables:

```
p.value adjusted.p.value dp.Info selected nTests
    link statistic
                                    5.008615e-08
    u~x2 6.036264 1.577228e-09
                                                       1
                                                             TRUE
2 y2~~y3 2.629176 8.559198e-03
                                    6.055947e-02
                                                       1
                                                             TRUE
                                                                       9
3 y3~x1 1.770997 7.656118e-02
                                    2.814424e-01
                                                       1
                                                            FALSE
                                                                       8
Confidence level: 0.9 (two sided, adjustement: fastmax)
```

This indeed matches the highest score statistic found by modelsearch:

```
resScore0 <- modelsearch(e.lvm, silent = TRUE)
c(statistic = sqrt(max(resScore0$test[,"Test Statistic"])),
  p.value = min(resScore0$test[,"P-value"]))</pre>
```

```
statistic p.value
6.036264e+00 1.577228e-09
```

We can compare the adjustment using the max distribution to bonferroni:

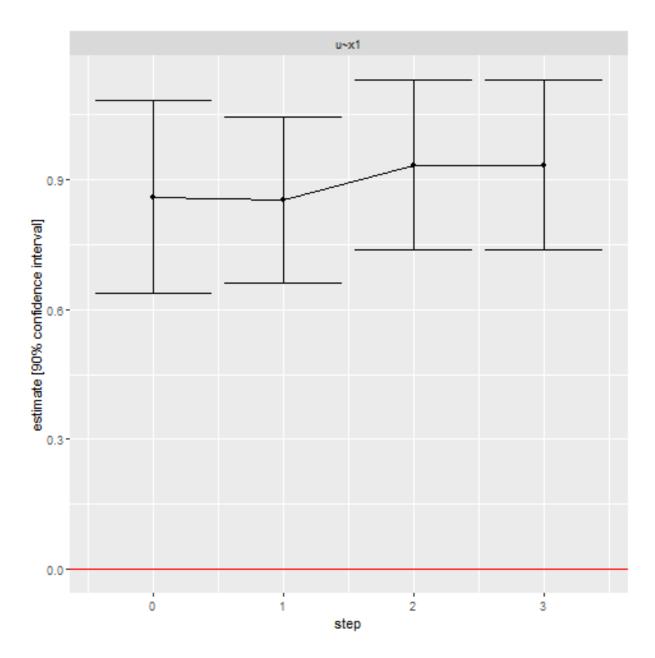
```
link none bonferroni max

1 u~x2 1.577228e-09 1.577228e-08 5.008615e-08

2 y2~~y3 8.559198e-03 8.559198e-02 6.055947e-02

3 y3~x1 7.656118e-02 7.656118e-01 2.814424e-01
```

In theory, the correction based on the max statistic should give a p value that is smaller or equal than the p value adjusted using Bonferroni. However for for very small p-values, the max-correction can be numerically inaccurate and result in p-values that are slightly larger. The evolution of the estimation of a given coefficient across the sequential search can be displayed using autoplot:



In many cases, all links are not plausible so the user should indicates which links should be investigated by modelsearch2. This can be done via the argument link:

```
resRed <- modelsearch2(e.lvm, link = c("y1\simv2","y1\simv3","y2\simv3"), trace = FALSE) print(resRed)
```

```
Sequential search for local dependence using the score statistic

The variable selection procedure did not retain any variable
link statistic p.value adjusted.p.value dp.Info selected nTests

1 y1~~y3 1.754102 0.07941299 0.1818963 1 FALSE 3

Confidence level: 0.95 (two sided, adjustement: fastmax)
```

The function findNewLink can help the user to identify the set of relevant links:

```
findNewLink(e.lvm$model, type = "covariance")$link
```

```
[1] "y1~~y2" "y1~~y3" "y2~~y3"
```

3.2 Checking that the names of the variables in the model match those of the data

When estimating latent variable models using **lava**, it sometimes happens that the model does not converge:

```
## simulate data
set.seed(10)
df.data <- sim(lvm(Y~X1+X2), 1e2)

## fit model
mWrong <- lvm(Y ~ X + X2)
eWrong <- estimate(mWrong, data = df.data)</pre>
```

Warning messages:

```
1: In estimate.lvm(mWrong, data = df.data) :
   Lack of convergence. Increase number of iteration or change starting values.
2: In sqrt(diag(asVar)) : NaNs produced
```

This can have several reasons:

- the model is not identifiable.
- the optimization routine did not managed to find a local optimum. This may happen for complex latent variable model where the objective function is not convex or locally convex.
- the user has made a mistake when defining the model or has not given the appropriate dataset.

The checkData function enables to check the last point. It compares the observed variables defined in the model and the one given by the dataset. In case of mismatch it returns a message:

```
checkData(mWrong, df.data)
```

Missing variable in data: X

In presence of latent variables, the user needs to explicitly define them in the model, otherwise checkData will identify them as an issue:

```
## simulate data
set.seed(10)
mSim <- lvm(c(Y1,Y2,Y3)~eta)
latent(mSim) <- ~eta
df.data <- sim(mSim, n = 1e2, latent = FALSE)

## fit model
m <- lvm(c(Y1,Y2,Y3)~eta)
checkData(m, data = df.data)</pre>
```

Missing variable in data: eta

```
latent(m) <- ~eta
checkData(m, data = df.data)</pre>
```

No issue detected

4 Information about the R session used for this document

```
sessionInfo()
R version 3.5.1 (2018-07-02)
Platform: x86_64-w64-mingw32/x64 (64-bit)
Running under: Windows 7 x64 (build 7601) Service Pack 1
Matrix products: default
locale:
[1] LC_COLLATE=Danish_Denmark.1252 LC_CTYPE=Danish_Denmark.1252
[3] LC_MONETARY=Danish_Denmark.1252 LC_NUMERIC=C
[5] LC_TIME=Danish_Denmark.1252
attached base packages:
[1] stats
              graphics grDevices utils
                                            datasets methods
                                                                 base
other attached packages:
[1] lavaSearch2_1.5.1 lava_1.6.4
                                        ggplot2_3.1.0
loaded via a namespace (and not attached):
 [1] Rcpp_1.0.0
                       pillar_1.3.1
                                         compiler_3.5.1
                                                           plyr_1.8.4
 [5] bindr_0.1.1
                       tools_3.5.1
                                         tibble_2.0.1
                                                           gtable_0.2.0
 [9] lattice_0.20-35
                       pkgconfig_2.0.2
                                         rlang_0.3.1
                                                           Matrix_1.2-14
[13] parallel_3.5.1
                       mvtnorm_1.0-8
                                         bindrcpp_0.2.2
                                                           withr_2.1.2
[17] dplyr_0.7.8
                       stringr_1.3.1
                                         grid_3.5.1
                                                           tidyselect_0.2.5
[21] glue_1.3.0
                       R6_2.3.0
                                         survival_2.42-6
                                                           multcomp_1.4-8
[25] TH.data_1.0-9
                       purrr_0.3.0
                                         reshape2_1.4.3
                                                           magrittr_1.5
[29] scales_1.0.0
                       codetools_0.2-15 MASS_7.3-50
                                                           splines_3.5.1
[33] assertthat_0.2.0
                                         numDeriv_2016.8-1 labeling_0.3
                       colorspace_1.3-2
[37] sandwich_2.5-0
                                                           munsell_0.5.0
                       stringi_1.2.4
                                         lazyeval_0.2.1
```

zoo_1.8-4

[41] crayon_1.3.4