Using the missForest Package

Daniel J. Stekhoven stekhoven@stat.math.ethz.ch

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1 Introduction

1.1 What is this document? (And what it isn't!)

This package vignette is an application focussed user guide for the R package missForest. The functionality is explained using a couple of real data examples. Argument selection with respect to feasibility and accuracy issues are discussed and illustrated using these real data sets. Do not be alarmed by the length of this document which is mainly due to some major R output included for illustrative reasons.

This document is *not* a theoretical primer for the fundamental approach of the missForest algorithm. It also does not contain any simulations or comparative studies with other imputation methods. For this information we point the interested reader to Stekhoven and Bühlmann [2012].

1.2 The missForest algorithm

missForest is a nonparametric imputation method for basically any kind of data. It can cope with mixed-type of variables, nonlinear relations, complex interactions and high dimensionality $(p \gg n)$. It only requires the observation (i.e. the rows of the data frame supplied to the function) to be pairwise independent. The algorithm is based on random forest (Breiman [2001]) and is dependent on its R implementation randomForest by Andy Liaw and Matthew Wiener. Put simple (for those who have skipped the previous paragraph): for each variable missForest fits a random forest on the observed part and then predicts the missing part. The algorithm continues to repeat these two steps until a stopping criterion is met or the user specified maximum of iterations is reached. For further details see Stekhoven and Bühlmann [2012].

To understand the remainder of this user guide it is important to know that missForest is running iteratively, continuously updating the imputed matrix variable-wise, and is assessing its performance between iterations. This assessment is done by considering the difference(s) between the previous imputation result and the new imputation result. As soon as this difference (in case of one type of variable) or differences (in case of mixed-type of variables) increase the algorithm stops.

missForest provides the user with an estimate of the imputation error. This estimate is based on the out-of-bag (OOB) error estimate of random forest. Stekhoven and Bühlmann [2012] showed that this estimate produces an appropriate representation of the true imputation error.

1.3 Installation

The R package missForest is available from the Comprehensive R Archive Network (CRAN, http://cran.r-project.org/) and as such can be installed in the default way using the install.packages function:

> install.packages(missForest, dependencies = TRUE)

Make sure to include the dependencies = TRUE argument to install also the randomForest package unless it is already installed.

2 Missing value imputation with missForest

In this section we describe using the missForest function. We will shed light on all arguments which can or have to be supplied to the algorithm. Also, we will discuss how to make missForest faster or more accurate. Finally, an interpretation of the OOB imputation error estimates is given.

2.1 Description of the data used

Iris data This complete data set contains five variables of which one is categorical with three levels. It is contained in the R base and can be loaded directly by typing data(iris). The data were collected by Anderson [1935].

Oesophageal cancer data This complete data set comes from a case-control study of oesophageal cancer in Ile-et-Vilaine, France. It is contained in the R base and can be loaded directly by typing data(esoph). The data were collected by Breslow and Day [1980].

Musk data This data set describes the shapes of 92 molecules of which 47 are musks and 45 are non-musks. Since a molecule can have many conformations due to rotating bonds, there are n=476 different conformations in the set. The classification into musk and non-musk molecules is removed. For further details see Frank and Asuncion [2010].

2.2 missForest in a nutshell

After you have properly installed missForest you can load the package in your R session:

> library(missForest)

We will load now the famous Iris data set and artificially remove 10% of the entries in the data completely at random using the prodNA function from the missForest package:

```
> data(iris)
> iris.mis <- prodNA(iris, noNA = 0.1)</pre>
> summary(iris.mis)
 Sepal.Length
                   Sepal.Width
                                    Petal.Length
                                                     Petal.Width
        :4.300
                         :2.000
                                          :1.000
                                                           :0.100
 1st Qu.:5.100
                 1st Qu.:2.800
                                   1st Qu.:1.600
                                                    1st Qu.:0.300
Median :5.800
                                   Median :4.400
                 Median :3.000
                                                    Median :1.300
                                                           :1.221
Mean
        :5.862
                 Mean
                         :3.068
                                   Mean
                                          :3.856
                                                    Mean
3rd Qu.:6.400
                 3rd Qu.:3.325
                                   3rd Qu.:5.100
                                                    3rd Qu.:1.800
        :7.900
Max.
                 Max.
                         :4.400
                                   Max.
                                          :6.900
                                                    Max.
                                                           :2.500
NA's
        :17
                  NA's
                         :14
                                   NA's
                                          :17
                                                    NA's
                                                           :14
       Species
setosa
           :46
versicolor:44
virginica:47
NA's
```

We can see that there is an evenly distributed amount of missing values over the variables in the data set. With *completely at random* we mean that the process of deleting entries is not influenced by the data or the data generating process.

The missing data is now imputed by simply handing it over to missForest:

```
> iris.imp <- missForest(iris.mis)</pre>
```

```
missForest iteration 1 in progress...done! missForest iteration 2 in progress...done! missForest iteration 3 in progress...done! missForest iteration 4 in progress...done!
```

Except for the iteration numbering no additional print-out is given. The results are stored in the R object iris.imp which is a list. We can call upon the imputed data matrix by typing iris.imp\$ximp. Note: A common mistake is to use iris.imp instead of iris.imp\$ximp for subsequent analyses.

Additionally, missForest provides an OOB imputation error estimate which can be extracted using the same \$ notation as with the imputed data matrix:

> iris.imp\$00Berror

```
NRMSE PFC
0.14034032 0.04379562
```

As mentioned before the Iris data set contains two types of variables, continuous and categorical. This is why the OOB imputation error supplies two values for the result of the imputation (default setting). The first value is the normalized root mean squared error (NRMSE, see Oba et al. [2003]) for the continuous part of the imputed data set, e.g., Sepal.Length, Sepal.Width, Petal.Length and Petal.Width. The second value is the proportion of falsely classified entries (PFC) in the categorical part of the imputed data set, e.g., Species. In both cases good performance of missForest leads to a value close to 0 and bad performance to a value around 1.

If you are interested in assessing the reliability of the imputation for single variables, e.g., to decide which variables to use in a subsequent data analysis, missForest can return the OOB errors for each variable separately instead of aggregating over the whole data matrix. This can be done using the argument variablewise = TRUE when calling the missForest function.

```
> iris.imp <- missForest(iris.mis, variablewise = TRUE)</pre>
```

```
missForest iteration 1 in progress...done! missForest iteration 2 in progress...done! missForest iteration 3 in progress...done! missForest iteration 4 in progress...done!
```

> iris.imp\$00Berror

```
MSE MSE MSE MSE PFC 0.10837194 0.08829704 0.07126565 0.03286983 0.05109489
```

We can see that the output has the same length as there are variables in the data. For each variable the resulting error and the type of error measure, i.e., mean squared error (MSE) or PFC, is returned. Note that we are not using the NRMSE here.

2.3 Additional output using verbose

In 2.2 the print-out of missForest showed only which iteration is taking place at the moment. Anyhow, if you are imputing a large data set or choose to use ridiculously large mtry and/or ntree arguments (see 2.5) you might be interested in getting additional information on how missForest is performing.

By setting the logical verbose argument to TRUE the print-out is extended threefold:

estimated error(s) The OOB imputation error estimate for the continuous and categorical parts of the imputed data set. Note: If there is only one type of variable there will be only one value with the corresponding error measure.

difference(s) The difference between the previous and the new imputed continuous and categorical parts of the data set. The difference for the set of continuous variables N in the data set is computed by

$$\frac{\sum_{j \in \mathbf{N}} (\mathbf{X}_{new}^{imp} - \mathbf{X}_{old}^{imp})^2}{\sum_{j \in \mathbf{N}} (\mathbf{X}_{new}^{imp})^2},$$

and for the set of categorical variables the difference corresponds to the PFC.

time The runtime of the iteration in seconds.

If we rerun the previous imputation of the Iris data ¹ setting verbose = TRUE we get:

```
> set.seed(81)
> iris.imp <- missForest(iris.mis, verbose = TRUE)</pre>
 missForest iteration 1 in progress...done!
    estimated error(s): 0.1471057 0.05109489
    difference(s): 0.008108616 0.06666667
    time: 0.09 seconds
 missForest iteration 2 in progress...done!
    estimated error(s): 0.1379585 0.04379562
    difference(s): 3.88011e-05 0
    time: 0.1 seconds
 missForest iteration 3 in progress...done!
    estimated error(s): 0.1403403 0.04379562
    difference(s): 8.445032e-06 0
    time: 0.11 seconds
 missForest iteration 4 in progress...done!
    estimated error(s): 0.142112 0.05109489
    difference(s): 1.164481e-05 0
    time: 0.11 seconds
```

The above print-out shows that missForest needs four iterations to finish. If we check the final OOB imputation error estimate:

we can see that it used the result from the second last iteration, i.e., the third instead of the last one. This is because the stopping criterion was triggered and the fact that the differences increase indicate that the new imputation is probably a less accurate imputation than the previous one. However, we can also see that the *estimated* error(s) is lower for the last imputation than for the one before. But we will show later on that the true imputation error is lower for iteration 3 (the impatient reader can jump to section 2.9).

2.4 Changing the number of iterations with maxiter

Depending on the composition and structure of the data it is possible that missForest needs more than the typical four to five iterations (see 2.3) until the stopping criterion kicks in. From an optimality point of view we do want missForest to stop due to the stopping criterion and not due to the limit of iterations. However, if the difference between iterations is seriously shrinking towards nought and the estimated error is in a stalemate the only way to keep computation time at a reasonable level is to limit the number of iterations using the argument maxiter.

¹Since random forest – as its name suggests – is using a random number generator (RNG) the result for two trials on the same missing data set will be different. To avoid this from happening in the given illustrative example we use the set.seed function before applying missForest on the iris.mis data set. This causes the RNG to be reset to the same state as before (where we invisibly called set.seed(81) already but did not want to trouble the concerned reader with technical details).

We show this using the esoph data. First, we run missForest on a data set where we removed 5% of the entries at random:

```
> data(esoph)
> esoph.mis <- prodNA(esoph, 0.05)
> set.seed(96)
> esoph.imp <- missForest(esoph.mis, verbose = TRUE)
 missForest iteration 1 in progress...done!
    estimated error(s): 0.4890565 0.7059562
    difference(s): 0.04277602 0.02651515
    time: 0.04 seconds
 missForest iteration 2 in progress...done!
    estimated error(s): 0.3792187 0.7184408
    difference(s): 0.02079341 0
    time: 0.05 seconds
 missForest iteration 3 in progress...done!
    estimated error(s): 0.4038586 0.6702544
    difference(s): 6.112263e-05 0.01136364
    time: 0.05 seconds
 missForest iteration 4 in progress...done!
    estimated error(s): 0.4616839 0.6861481
    difference(s): 0.0003034195 0.01136364
    time: 0.05 seconds
```

We can see that it takes missForest nine iterations to come to a stop. The returned imputation result was reached in iteration 8 having estimated errors of 0.55 and 0.73 and differences of $3 \cdot 10^{-5}$ and 0. In iteration 6 the estimated errors are smaller (i.e. 0.53 and 0.70) and the differences are $1 \cdot 10^{-4}$ and $4 \cdot 10^{-3}$. So why is missForest not simply taking the sixth iteration and calls it a day? Because the difference in the continuous part of the data set is still reduced in each iteration up until iteration 9. This stopping strategy is on average (taking all possible data sets into account) quite good but can have its caveats at specific data sets. In the above case of the esoph data we can get the result of the sixth iteration by doing the following:

```
> set.seed(96)
> esoph.imp <- missForest(esoph.mis, verbose = TRUE, maxiter = 6)
missForest iteration 1 in progress...done!
   estimated error(s): 0.4890565 0.7059562
   difference(s): 0.04277602 0.02651515
   time: 0.04 seconds

missForest iteration 2 in progress...done!
   estimated error(s): 0.3792187 0.7184408
   difference(s): 0.02079341 0
   time: 0.05 seconds

missForest iteration 3 in progress...done!
   estimated error(s): 0.4038586 0.6702544</pre>
```

```
difference(s): 6.112263e-05 0.01136364 time: 0.05 seconds
```

missForest iteration 4 in progress...done! estimated error(s): 0.4616839 0.6861481 difference(s): 0.0003034195 0.01136364

time: 0.03 seconds

The returned result is now given by iteration 6. Quintessentially, there are two uses for the maxiter argument:

- 1. Controlling the run time in case of stagnating performance;
- 2. extract a preferred iteration result not supplied by the stopping criterion.

2.5 Speed and accuracy trade-off manipulating mtry and ntree

missForest grows in each iteration for each variable a random forest to impute the missing values. With a large number of variables p this can lead to computation times beyond today's perception of feasibility. There are two ways to speed up the imputation process of missForest:

- 1. Reducing the number of trees grown in each forest using the argument ntree;
- 2. reducing the number of variables randomly sampled at each split using the argument mtry.

It is imperative to know that reducing either of these numbers will probably result in reduced accuracy. This is why we speak of a speed and accuracy trade-off.

2.5.1 ntree

The effect of reducing ntree on the computation time is linear, e.g., halving ntree will half computation time for a single iteration. The default value in missForest is set to 100 which is fairly large. Smaller values in the tens can give appropriate results already. We show this using the Musk data:

```
> musk.mis <- prodNA(musk, 0.05)
> musk.imp <- missForest(musk.mis, verbose = TRUE, maxiter = 3)

missForest iteration 1 in progress...done!
   estimated error(s): 0.1491825
   difference(s): 0.02383702
   time: 280.739 seconds

missForest iteration 2 in progress...done!
   estimated error(s): 0.1367353
   difference(s): 0.0001208087
   time: 277.011 seconds

missForest iteration 3 in progress...done!
   estimated error(s): 0.137418
   difference(s): 3.836082e-05
   time: 278.287 seconds</pre>
```

The computation time is about 14 minutes and we end up with an estimated NRMSE of 0.14. Note: The response was removed from the Musk data, that is why there is only the estimated NRMSE and also only the difference for the continuous part of the data set.

If we repeat the imputation using the ntree argument and setting it to 20 we get:

```
> musk.imp <- missForest(musk.mis, verbose = TRUE, maxiter = 3, ntree = 20)
missForest iteration 1 in progress...done!
   estimated error(s): 0.1724939
   difference(s): 0.02383371
   time: 56.705 seconds

missForest iteration 2 in progress...done!
   estimated error(s): 0.1576795
   difference(s): 0.0002417658
   time: 55.833 seconds

missForest iteration 3 in progress...done!
   estimated error(s): 0.1591702
   difference(s): 0.0001966117
   time: 56.053 seconds</pre>
```

The computation time is now around 3 minutes which is approximately a fifth of the previous computation time using 100 trees (as a matter of fact, taking the floor values of the iteration times in seconds then the former imputation took exactly five times longer than the latter). The estimated NRMSE has increased to 0.16 – an increase of 14% compared to before. In some application this might seem as an unacceptable increase of imputation error. However, if the number of variables is large enough, e.g., in the thousands like in gene expression data, the amount of computation time saved will surpass the amount of imputation error increased.

2.5.2 mtry

The effect on computation time when changing mtry is not as straight forward as with ntree. It is however more pronounced in settings with high-dimensionality (e.g. $p \gg n$, where n is the number of observations) and complex structures. The default setting in missForest is \sqrt{p} . This choice qualifies for a quite nice trade-off between imputation error and computation time. Anyhow, certain data might demand different choices either putting a focus on better imputation error or better computation time. We leave this delicate choice to the user of these certain data sets.

2.6 Use subsampling instead of bootstrapping by setting replace to FALSE

Like in the original paper by Breiman [2001] missForest uses bootstrap samples to grow its trees on. Another possibility would be to use subsamples instead. Randomly selected observations are replaced in the data when bootstrapping is performed, i.e., a single observation can be selected several times. In subsampling these observations are not replaced and thus a single observation can only be selected once. If replace=FALSE then sampsize (controlling the size of the sample drawn from the data to grow a tree) is reduced from n to 0.632n. This is because otherwise there would be no more OOB observations and an error prediction would be impossible. The number 0.632 is the expected proportion of observations selected when using bootstrapping, i.e., selecting with replacements n observations.

```
> set.seed(81)
> iris.imp.sub <- missForest(iris.mis, verbose = TRUE, replace = FALSE)
  missForest iteration 1 in progress...done!
    estimated error(s): 0.1469176 0.04379562
    difference(s): 0.008129208 0.06666667
    time: 0.06 seconds
  missForest iteration 2 in progress...done!
    estimated error(s): 0.1398355 0.05109489
    difference(s): 3.863383e-05 0
    time: 0.1 seconds
  missForest iteration 3 in progress...done!
    estimated error(s): 0.1397525 0.04379562
    difference(s): 1.272535e-05 0
    time: 0.06 seconds
  missForest iteration 4 in progress...done!
    estimated error(s): 0.1382179 0.05839416
    difference(s): 1.127379e-05 0
    time: 0.09 seconds
  missForest iteration 5 in progress...done!
    estimated error(s): 0.1413583 0.05109489
    difference(s): 7.488807e-06 0
    time: 0.08 seconds
  missForest iteration 6 in progress...done!
    estimated error(s): 0.1391338 0.05839416
    difference(s): 6.838686e-06 0
    time: 0.08 seconds
  missForest iteration 7 in progress...done!
    estimated error(s): 0.1377166 0.05109489
    difference(s): 9.768528e-06 0
    time: 0.08 seconds
> iris.imp.sub$00Berror
     NRMSE
                  PFC.
0.13913382 0.05839416
```

We can see that there is no substantial improvement compared to bootstrapping in the previous example in section 2.3 for the iris data. However, in some cases subsampling can be superior to bootstrapping. Therefore, if time allows explore both strategies and settle for the better performing one.

2.7 Imbalanced data, stratified sampling and focussed selection (classwt, cutoff, strata, sampsize)

From version 1.3 on missForest offers the possibility to pass more arguments to the randomForest function at its core. These include:

classwt adding priors to the classes in categorical variables; cutoff setting cutoffs for each class in categorical variables; strata perform stratified sampling for categorical variables; sampsize define size of samples drawn from a variable.

For each of these arguments the user has to generate a list containing the appropriate object for each variable at the corresponding list entry, i.e., the third entry of the list corresponds to the third variable in the data, etc. The first three arguments in the above list do only make sense when used with categorical variables. However, the generated list has to have an entry for each variable - include for continuous variables NULL (for cutoff use 1). The sampsize argument can be used for both types of data. In case of continuous variables a single integer and in case of categorical variables a vector of the same length as there are classes in the variable.

```
> iris.sampsize <- list(12, 12, 12, 12, c(10, 15, 10))
> iris.imp.sampsize <- missForest(iris.mis, sampsize = iris.sampsize)</pre>
```

Note how we set the list entry for sampsize in case of the fifth variable Species to a vector with three entries. An example for the use of cutoff could be:

```
> iris.cutoff <- list(1, 1, 1, 1, c(0.3, 0.6, 0.1))
> iris.imp.cutoff <- missForest(iris.mis, cutoff = iris.cutoff)</pre>
```

we set the cutoff for setosa to 0.3, for versicolor to 0.6 and for virginica to 0.1 (not that this would make any sense - it is simply to show how the arguments have to be generated). Equivalently, using a NULL instead of 1 for the continuous variables the input for classwt looks as:

```
> iris.classwt <- list(NULL, NULL, NULL, NULL, c(10, 30, 20))
> iris.imp.classwt <- missForest(iris.mis, classwt = iris.classwt)</pre>
```

2.8 Controlling terminal nodes w.r.t. nodesize and maxnodes

We can control the structural tree growing process two fold:

- by setting the maximum number of terminal nodes in the tree;
- by defining the minimum number of observations in a terminal node.

The default setting for the maximum number of nodes is given by the maximum possible in the tree growing process, subject to the limits of nodesize, which in turn has the default setting of 1 for continuous and 5 for categorical variables. The maxnodes argument is simply specified by an integer. For nodesize the user needs to supply a vector of length 2 where the first entry corresponds to continuous and the second entry to categorical variables:

```
> iris.imp.term \leftarrow missForest(iris.mis, nodesize = c(3, 7))
```

In the above call to missForest we set the number of observations in terminal nodes to 3 for continuous variables and 7 for categorical variables. Especially, the maxnodes argument can have a strong effect on computation time.

2.9 Testing the appropriateness by supplying xtrue

Whenever imputing data with real missing values the question arises how good the imputation was. In missForest the estimated OOB imputation error gives a nice indication at what you have to expect. A wary user might want to make an additional assessment (or back the OOB estimate up) by performing cross-validation or – in the optimal case – testing missForest previously on complete data. For both cases missForest offers the xtrue argument which simply takes in the same data matrix as xmis but with no missing values present. The strategy for testing the performance is the same as shown in the previous examples using prodNA:

- 1. Generate a data matrix with missing values;
- 2. impute this artificially generated data matrix;
- 3. compare the complete and imputed data matrices.

The functions to use for this strategy are prodNA, missForest and mixError. Using again the Iris data this would look like:

```
> iris.mis <- prodNA(iris, noNA = 0.1)
> iris.imp <- missForest(iris.mis)

> iris.err <- mixError(iris.imp$ximp, iris.mis, iris)
> print(iris.err)

NRMSE     PFC
0.1277563 0.0000000
```

Note: We want to point out once more that the user has to extract the imputed matrix from the missForest output using the \$ list notation. Not doing so will generate the following error:

```
> iris.err <- mixError(iris.imp, iris.mis, iris)
Error in mixError(iris.imp, iris.mis, iris) :
   Wrong input for 'xmis' - you probably forgot to point at the
   list element $ximp from the missForest output object.</pre>
```

We can simplify the above strategy by using xtrue. If combined with verbose = TRUE the user even gets additional information on the performance of missForest between iterations:

```
> iris.imp <- missForest(iris.mis, xtrue = iris, verbose = TRUE)

missForest iteration 1 in progress...done!
  error(s): 0.1253896 0
  estimated error(s): 0.1471057 0.05109489
  difference(s): 0.008108616 0.06666667
  time: 0.11 seconds

missForest iteration 2 in progress...done!
  error(s): 0.1275875 0
  estimated error(s): 0.1379585 0.04379562
  difference(s): 3.88011e-05 0
  time: 0.11 seconds</pre>
```

```
missForest iteration 3 in progress...done!
  error(s): 0.1277563 0
  estimated error(s): 0.1403403 0.04379562
  difference(s): 8.445032e-06 0
  time: 0.11 seconds

missForest iteration 4 in progress...done!
  error(s): 0.1237153 0
  estimated error(s): 0.142112 0.05109489
  difference(s): 1.164481e-05 0
  time: 0.11 seconds
```

Supplying xtrue adds the line error(s) to the missForest output. We can observe that the true imputation error really is lower for the second last iteration as mentioned in section 2.2. Additionally, the output object (in the above example iris.imp) contains now a list element error which can be called directly:

3 Concluding remarks

Imputation using missForest can be done very easily. The OOB imputation error estimate facilitates the interpretation of such imputation results. However, it should always be kept in mind that imputing data with missing values does not increase the information contained within this data. It is only a way to have completeness for further data analysis. Many methods of data analysis require complete observations. In such complete case analyses observations missing only a single entry will be completely removed from the data and therefore the information content is reduced. Imputing the data beforehand prevents this reduction. For further details on the effect of imputation on the subsequent data analysis we suggest the books of Schafer [1997] and Little and Rubin [1987].

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