# The neldermead Package - version 1.0-7

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**neldermead** is a R port of a module originally developed for Scilab version 5.2.1 by Michael Baudin (INRIA - DIGITEO). Information about this software can be found at www.scilab.org. The following documentation as well as the content of the functions .Rd files are adaptations of the documentation provided with the original Scilab neldermead module.

**neldermead** currently does not include any adaptation of the Scilab 'nmplot' function series that is available in the original neldermead module.

# 1 Overview

# 1.1 Description

The goal of this toolbox is to provide several direct search optimization algorithms based on the simplex method. The optimization problem to solve is the minimization of a cost function, with bounds and nonlinear constraints.

```
minf(x)

l_i \le x_i \le h_i, \quad i = 1, n

g_j(x) \ge 0, \quad j = 0, nbineq
```

where f is the cost function, x is the vector of parameter estimates, l and h are vectors of lower and upper bounds for the parameter estimates, n is the number of parameters and nbineq the number of inequality constraints g(x).

The provided algorithms are direct search algorithms, i.e. algorithms which do not use the derivative of the cost function. They are based on the update of a simplex, which is a set of  $k \ge n+1$  vertices, where each vertex is associated with one point and one function value.

The following algorithms are available:

- The fixed shape simplex method of Spendley, Hext and Himsworth: this algorithm solves an unconstrained optimization problem with a fixed shape simplex made of k = n + 1 vertices.
- The variable shape simplex method of Nelder and Mead: this algorithm solves an unconstrained optimization problem with a variable shape simplex made of k = n + 1 vertices [3].
- Box's complex method: this algorithm solves an constrained optimization problem with a variable shape simplex made of an arbitrary k number of vertices (k = 2n is recommended by Box).

# 1.2 Basic object

The basic object used by the **neldermead** package to store the configuration settings and the history of an optimization is a 'neldermead' object, i.e. a list typically created by **neldermead.new** and having a strictly defined structure (see ?neldermead.new for more details).

## 1.3 The cost function

The function element of the neldermead object allows to configure the cost function. The cost function is used to compute the objective function value f. If the nbineqconst element of the neldermead object is configured to a non-zero value, the cost function must also compute the value of the nonlinear, positive, inequality constraints c. The cost function can also take as input/output an additional argument, if the costfargument element is configured. The function should be defined as described in vignette('optimbase', package='optimbase'):

```
costf <- function(x, index, fmsfundata){</pre>
    # Define f and c here #
    return(list(f, g=NULL, c, gc=NULL, index=index,
                  this=list(costfargument = fmsfundata)))
  }
where
 x: is the current point, as a column vector,
 index: (optional), an integer representing the value to compute, and
 fmsfundata: an user-provided input/output argument.
 f: the value of the objective function (a scalar),
 g: typically the gradient of the objective function in the context of the optimbase functions; must
     be set to NULL as the Nelder-Mead is not gradient-based,
 c: the vector of values of non-linear, positive, inequality constraints,
 gc: typically the gradient of the constraints in the context of the optimbase functions; must be set
     to NULL as the Nelder-Mead is not gradient-based,
 this: must be set to list(costfargument = fmsfundata).
   The index input parameter tells the cost function what to return as output arguments (as de-
scribed in vignette ('optimbase', package='optimbase'). It has the following meaning:
 index = 2: compute f,
 index = 5: compute c,
 index = 6: compute f and c
```

The fmsdata argument is both input and output. This feature may be used in the situation where the cost function has to update its environment from call to call. Its simplest use is to count the number of calls to the cost function, but this feature is already available directly. Consider the more practical situation where the optimization requires the execution of an underlying Newton method (a chemical solver for example). This Newton method requires an initial guess  $x_0$ . If the initial guess for this underlying Newton method is kept constant, the Newton method may have problems to converge when the current optimization point get far away from the its initial point. If a costfargument element is defined in the neldermead object, it can be passed to the cost function as the fmsdata argument. In this case, the initial guess for the Newton method can be updated so that it gets the value of the previous call. This way, the Newton method will have less problems to converge and the cost function evaluation may be faster.

We now present how the feature works. Everytime the cost function is called back, the costfargument element is passed to the cost function as an input argument. If the cost function modifies its content in the output argument, the content of the costfargument element is updated accordingly. Once the optimization is performed, the user may call the neldermead.cget function and get back an updated costfargument content.

# 1.4 The output function

The outputcommand element of the neldermead object allows to configure a command which is called back at the start of the optimization, at each iteration and at the end of the optimization. The output function must be defined as follows:

```
outputcmd <- function(state, data, myobj)
where</pre>
```

state: is a string representing the current state of the algorithm. Available values are 'init', 'iter', and 'done',

data: a list containing at least the following entries:

x: the current optimum,

fval: the current function value,

iteration: the current iteration index,

function: the number of function evaluations,

simplex: the current simplex,

step: the previous step in the algorithm. The following values are available: 'init', 'done', 'reflection', 'expansion', 'insidecontraction', 'outsidecontraction', 'reflectionnext', and 'shrink',

myobj: a user-defined parameter. This input parameter is defined with the outputcommandarg element of the neldermead object.

The output function may be used when debugging the specialized optimization algorithm, so that a verbose logging is produced. It may also be used to write one or several report files in a specialized format (ASCII, LATEX, Excel, etc...). The user-defined parameter may be used in that case to store file names or logging options.

The data list argument may contain more fields than the current presented ones. These additionnal fields may contain values which are specific to the specialized algorithm, such as the simplex in a Nelder-Mead method, the gradient of the cost function in a BFGS method, etc...

#### 1.5 Termination

The current package takes into account several generic termination criteria. The following termination criteria are enabled by default:

- maxiter,
- maxfunevals,
- tolxmethod.
- tolsimplexizemethod.

The neldermead.termination function uses a set of rules to compute if the termination occurs and sets optimization status to one of the following: 'continue', 'maxiter', 'maxfunevals', 'tolf', 'tolx', 'tolsize', 'tolsizedeltafv', 'kelleystagnation', 'tolboxf' or 'tolvariance'. The value of the status may also be a user-defined string, in the case where a user-defined termination function has been set.

The following set of rules is examined in this order.

- By default, the status is 'continue' and the terminate flag is FALSE.
- The number of iterations is examined and compared to the maxiter element of the neldermead object: if iterations ≥ maxiter, then the status is set to 'maxiter' and terminate is set to TRUE.
- The number of function evaluations is examined and compared to the maxfunevals elements: if funevals ≥ maxfunevals, then the status is set to 'maxfuneval' and terminate is set to TRUE.
- The tolerance on function value is examined depending on the value of the tolfunmethod.

FALSE: then the criteria is just ignored,

TRUE: if |currentfopt| < tolfunrelative · |previousfopt| + tolfunabsolute, then the status is set to 'tolf' and terminate is set to TRUE.

The relative termination criteria on the function value works well if the function value at optimum is near zero. In that case, the function value at initial guess fx0 may be used as previousfopt. This criteria is sensitive to the tolfunrelative and tolfunabsolute elements. The absolute termination criteria on the function value works if the user has an accurate idea of the optimum function value.

• The tolerance on x is examined depending on the value of the tolxmethod element.

FALSE: then the criteria is just ignored,

TRUE: if norm(currentxopt - previousxopt) < tolxrelative · norm(currentxopt) + tolxabsolute, then the status is set to 'tolx' and terminate is set to TRUE.

This criteria is sensitive to the tolxrelative and tolxabsolute elements. The relative termination criteria on x works well if x at optimum is different from zero. In that case, the condition measures the distance between two iterates. The absolute termination criteria on x works if the user has an accurate idea of the scale of the optimum x. If the optimum x is near 0, the relative tolerance will not work and the absolute tolerance is more appropriate.

• The tolerance on simplex size is examined depending on the value of the tolsimplexizemethod element.

FALSE: then the criteria is just ignored,

TRUE: if ssize < tolsimplexizerelative · simplexsize0 + tolsimplexizeabsolute, where simplexsize0 is the size of the simplex at iteration 0, then the status is set to 'tolsize' and terminate is set to TRUE.

The size of the simplex is computed from the 'sigmaplus' method of the **optimsimplex** package. This criteria is sensitive to the **tolsimplexizeabsolute** and the **tolsimplexizerelative** elements.

• The absolute tolerance on simplex size and absolute difference of function value is examined depending on the value of the tolssizedeltafvmethod element.

FALSE: then the criteria is just ignored,

TRUE: if both the following conditions ssize < tolsimplexizeabsolute and shiftfy < toldeltafv are true where ssize is the current simplex size and shiftfy is the absolute value of the difference of function value between the highest and lowest vertices, then the status is set to 'tolsizedeltafy' and terminate is set to TRUE.

• The stagnation condition based on Kelley sufficient decrease condition is examined depending on the value of the kelleystagnationflag element.

FALSE: then the criteria is just ignored,

TRUE: if newfvmean  $\leq$  oldfvmean - alpha·t(sg)·sg where newfvmean (resp. oldfvmean) is the function value average in the current iteration (resp. in the previous iteration), then the status is set to 'kelleystagnation' and terminate is set to TRUE. Here, alpha is a non-dimensional coefficient and sg is the simplex gradient.

• The termination condition suggested by Box is examined depending on the value of the boxtermination element.

FALSE: then the criteria is just ignored,

TRUE: if both the following conditions shiftfy < boxtolf and boxkount == boxnbmatch are true, where shiftfy is the difference of function value between the best and worst vertices, and boxkount is the number of consecutive iterations where this criteria is met, then the status is set to 'tolboxf' and terminate is set to TRUE. Here, the boxtolf parameter is the value associated with the boxtolf element of the neldermead object and is a user-defined absolute tolerance on the function value. The boxnbmatch parameter is the value associated with the boxnbmatch element and is the user-defined number of consecutive match.

• The termination condition based on the variance of the function values in the simplex is examined depending on the value of the tolvarianceflag element.

FALSE: then the criteria is just ignored,

TRUE: if var < tolrelativevariance · variancesimplex0 + tolabsolutevariance, where var is the variance of the function values in the simplex, then the status is set to 'tolvariance' and terminate is set to TRUE. Here, the tolrelativevariance parameter is the value associated with the tolrelativevariance element of the neldermead

object and is a user-defined relative tolerance on the variance of the function values. The tolabsolutevariance parameter is the value associated with the tolabsolutevariance element and is the user-defined absolute tolerance of the variance of the function values.

• The user-defined termination condition is examined depending on the value of the myterminateflag element.

FALSE: then the criteria is just ignored,

TRUE: if the term boolean output argument returned by the termination function is TRUE, then the status is set to the user-defined status and terminate is set to TRUE.

# 1.6 Kelley's stagnation detection

The stagnation detection criteria suggested by Kelley is based on a sufficient decrease condition, which requires a parameter alpha > 0 to be defined [1]. The kelleynormalizationflag element of the neldermead object allows to configure the method to use to compute this alpha parameter. Two methods are available, where each method corresponds to a different paper by Kelley:

constant: in 'Detection and Remediation of Stagnation in the Nelder-Mead Algorithm Using a Sufficient Decrease Condition', Kelley uses a constant alpha, with the suggested value 1.e-4, which is the typical choice for line search method.

normalized: in 'Iterative Methods for Optimization', Kelley uses a normalized alpha, computed from the following formula: alpha = alpha0 · sigma0 / nsg, where sigma0 is the size of the initial simplex and nsg is the norm of the simplex gradient for the initial guess point.

# 1.7 O'Neill's factorial optimality test

In 'Algorithm AS47 - Function minimization using a simplex procedure', O'Neill presents a fortran 77 implementation of the simplex method [5]. A factorial test is used to check if the computed optimum point is a local minimum. If the restartdetection element of the neldermead object is set to 'oneill', that factorial test is used to see if a restart should be performed.

# 1.8 Implementation notes of the method of Spendley et al.

The original paper may be implemented with several variations, which might lead to different results [6]. This section defines what algorithmic choices have been used in the present package.

The paper states the following rules.

- 'Rule 1. Ascertain the lowest reading y, of yi ... yk+1 Complete a new simplex Sp by excluding the point Vp corresponding to y, and replacing it by V\* defined as above.'
- 'Rule 2. If a result has occurred in (k + 1) successive simplexes, and is not then eliminated by application of Rule 1, do not move in the direction indicated by Rule 1, or at all, but discard the result and replace it by a new observation at the same point.'
- 'Rule 3. If y is the lowest reading in So , and if the next observation made,  $y^*$  , is the lowest reading in the new simplex S , do not apply Rule 1 and return to So from Sp . Move out of S, by rejecting the second lowest reading (which is also the second lowest reading in So).'

We implement the following 'rules' of the Spendley et al. method:

- Rule 1 is strictly applied, but the reflection is done by reflection of the high point, since we minimize a function instead of maximizing it, like Spendley.
- Rule 2 is NOT implemented, as we expect that the function evaluation is not subject to errors.
- Rule 3 is applied, i.e. reflection with respect to next to the high point. The original paper does not mention any shrink step. When the original algorithm cannot improve the function value with reflection steps, the basic algorithm stops. In order to make the current implementation of practical value, a shrink step is included, with shrinkage factor sigma. This perfectly fits into to the spirit of the original paper. Notice that the shrink step makes the rule #3 (reflection with respect to next-to-worst vertex) unnecessary. Indeed, the minimum required steps are the reflection and shrinkage. Nevertheless, the rule #3 has been kept in order to make the algorithm as close as it can be to the original.

# 1.9 Implementation notes on the method of Nelder and Mead

The purpose of this section is to analyse the current implementation of Nelder-Mead's algorithm. The algorithm that we use is described in 'Iterative Methods for Optimization' by Kelley.

The original paper uses a 'greedy' expansion, in which the expansion point is accepted whatever its function value. The current implementation, as most implementations, uses the expansion point only if it improves over the reflection point, that is,

- if fe<fr, then the expansion point is accepted,
- if not, the reflection point is accepted.

The termination criteria suggested by Nelder and Mead is based on an absolute tolerance on the standard deviation of the function values in the simplex. We provide this original termination criteria with the tolvarianceflag element of the neldermead object, which is disabled by default.

## 1.10 Box's complex algorithm implementation notes

In this section, we analyse the current implementation of Box's complex method [4]. The initial simplex can be computed as in Box's paper, but this may not be safe. In his paper, Box suggests that if a vertex of the initial simplex does not satisfy the non linear constraints, then it should be 'moved halfway toward the centroid of those points already selected'. This behaviour is available when the scalingsimplex0 element of the neldermead object is set to 'tocenter'. It may happen, as suggested by Guin [2], that the centroid is not feasible if the constraints are not convex. In this case, the initial simplex cannot be computed. This is why we provide the 'tox0' option, which allows to compute the initial simplex by scaling toward the initial guess, which is always feasible.

In Box's paper, the scaling into the non linear constraints is performed 'toward' the centroid, that is, by using a scaling factor equal to 0.5. This default scaling factor might be sub-optimal in certain situations. This is why we provide the boxineqscaling element, which allows to configure the scaling factor.

In Box's paper, whether we are concerned with the initial simplex or with the simplex at a given iteration, the scaling for the non linear constraints is performed without end. This is because Box's hypothesis is that 'ultimately, a satisfactory point will be found'. As suggested by Guin, if the process fails, the algorithm goes into an infinite loop. In order to avoid this, we perform the scaling until a minimum scaling value is reached, as defined by the guinalphamin element.

We have taken into account the comments by Guin, but it should be emphasized that the current implementation is still as close as possible to Box's algorithm and is not Guin's algorithm. More

precisely, during the iterations, the scaling for the non linear constraints is still performed toward the centroid, be it feasible or not.

# 1.11 User-defined algorithm

The mymethod element of the neldemead object allows to configure a user-defined simplex-based algorithm. The reason for this option is that many simplex-based variants of Nelder-Mead's algorithm have been developed over the years, with specific goals. While it is not possible to provide them all, it is very convenient to use the current structure without being forced to make many developments.

The value of the mymethod element is expected to be a R function with the following structure:

```
> myalgorithm <- function(this) {
+ ...
+ return(this)
+ }</pre>
```

where this is the current neldermead object.

In order to use the user-defined algorithm, the method element must be set to 'mine'. In this case, the component performs the optimization exactly as if the user-defined algorithm was provided by the component.

The user interested in that feature may use the internal scripts provided in the distribution as templates and tune his own algorithm from that point. There is of course no warranty that the user-defined algorithm improves on the standard algorithm, so that users use this feature at their own risks.

### 1.12 User-defined termination

Many termination criteria are found in the literature. Users who aim at reproducing the results exhibited in a particular paper may find that that none of the provided termination criteria match the one which is used in the paper. It may also happen that the provided termination criteria are not suitable for the specific test case. In those situation the myterminate element of the neldermead object allows to configure a user-defined termination function. The value of the myterminate element is expected to be a R function with the following structure:

```
> mystoppingrule <- function(this, simplex) {
+ ...
+ return(list(this = this, terminate = terminate, status = status))
+ }</pre>
```

where this is the current neldermead object and simplex is the current simplex. The terminate output argument is a logical flag which is FALSE if the algorithm must continue and TRUE if the algorithm must stop. The status output argument is a string which is associated with the current termination criteria.

In order to enable the use of the user-defined termination function, the value of the myterminateflag element must be set to TRUE in the neldermead object. At each iteration, if the myterminateflag element has been set to TRUE, the user-defined termination is called. If the terminate output argument is TRUE, then the algorithm is stopped. In that case, the value of the status element of the neldermead.get function output is the value of the status output argument of the user-defined termination function.

# 2 Specialized functions

#### 2.1 fminsearch

The fminsearch function is based on a specialized use of the more general neldermead function bundle and searches for the unconstrained minimum of a given cost function. This function corresponds to the Matlab (or Scilab) fminsearch function. In the context of fminsearch, the function to be minimized is not a cost function as described in Section 1.3 but an objective function (returning a numeric scalar). Additional information and examples are available in ?fminsearch from a R environment.

# 2.2 Direct grid search

Direct grid search, performed by fmin.gridsearch, is a functionality added to the original Scilab neldermead module and constitutes another specialized use of the neldermead package. This function allows to explore the search space of an optimization problem around the initial point  $x_0$ . This optimization problem is defined by an objective function, like for fminsearch, and not a cost function. fmin.gridsearch automatically creates a grid of search points selected around the initial point and evaluates the objective function at each point. The boundaries of the grid are set either by a vector of parameter-specific lower and upper limits, or by a vector of factors  $\alpha$  as follows: $[x_0/\alpha, x_0 \times \alpha]$ . The number npts of points evaluated for each parameter (or dimension of the optimization problem) can also be defined. The total number of points in the grid is therefore  $npts^n$ . At the end of the search, fmin.gridsearch returns a table sorted by value of the objective function. The feasibility of the objective function is also determined at each point, as fmin.gridsearch is a wrapper around optimbase.gridsearch which assesses the feasibility of a cost function in addition to calculating its value at each particular search point. Because fmin.gridsearch does not accept constraints, the objective function should always be feasible. Additional information is available in ?fmin.gridsearch from a R environment.

# 3 Examples

We present in this section basic examples illustrating the use of **neldermead** functions to optimize unconstrained or constrained systems. More complex examples are described in a Scilab-based document written by Michael Baudin and available at http://forge.scilab.org/index.php/p/docneldermead/. Because the R port of the Scilab **neldermead** module is almost literal, the user should be able to reproduce the described examples in R with minimal adaptations.

#### 3.1 Example 1: Basic use

In the following example, we solve a simple quadratic test case. We begin by defining the cost function, which takes 3 input arguments and returns the value of the objective function as the f element of a list. The standard starting point [-1.2 1.0] is used. neldermead.new creates a new neldermead object. Then we use neldermead.configure to configure the parameters of the problem. We use all default settings and perform the search for the optimum. neldermead.get is finally used to retrieve the optimum parameters.

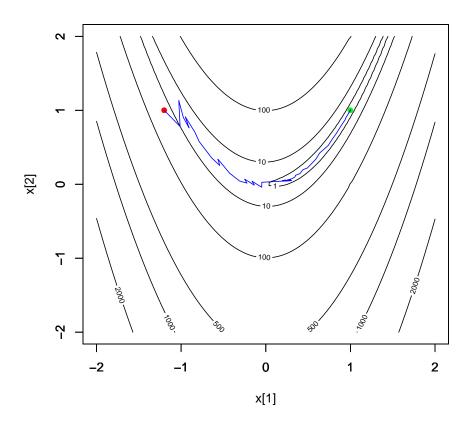
```
> quadratic <- function(x = NULL, index = NULL, fmsfundata = NULL) {
+ return(list(f = x[1]^2 + x[2]^2, g = c(), c = c(), gc = c(),
+ index = index, this = list(costfargument = fmsfundata)))</pre>
```

# 3.2 Example 2: Customized use

In the following example, we solve the Rosenbrock test case. We begin by defining the Rosenbrock function, which takes 3 input arguments and returns the value of the objective function. The standard starting point [-1.2 1.0] is used. neldermead.new creates a new neldermead object. Then we use neldermead.configure to configure the parameters of the problem. The initial simplex is computed from the axes and the single length 1.0 (this is the default, but is explicitly written here as an example). The variable simplex algorithm by Nelder and Mead is used, which corresponds to the -method 'variable' option. neldermead.search performs the search for the minimum. Once the minimum is found, we represent part of the search space using the contour function (this is possible since our problem involves only 2 parameters) and we superimpose the starting point (in red), the optimisation path (in bleu), and the optimum (in green) to the plot. The history of the optimisation can be retrieved (using neldermead.get) because the '-storehistory' option was set to TRUE.

```
> rosenbrock <- function(x = NULL, index = NULL, fmsfundata = NULL) {
      return(list(f = 100 * (x[2] - x[1]^2)^2 + (1 - x[1])^2,
          g = c(), c = c(), gc = c(), index = index, this = list(costfargument = fmsfundata)))
+ }
> x0 \leftarrow transpose(c(-1.2, 1))
> nm <- neldermead.new()</pre>
> nm <- neldermead.configure(nm,
                                 "-numberofvariables", 2)
> nm <- neldermead.configure(nm, "-function", rosenbrock)
> nm <- neldermead.configure(nm, "-x0", x0)
> nm <- neldermead.configure(nm,</pre>
                                 "-maxiter", 200)
> nm <- neldermead.configure(nm, "-maxfunevals", 300)
> nm <- neldermead.configure(nm, "-tolfunrelative", 10 * .Machine$double.eps)
> nm <- neldermead.configure(nm, "-tolxrelative", 10 * .Machine$double.eps)
                                 "-simplex0method", "axes")
> nm <- neldermead.configure(nm,</pre>
> nm <- neldermead.configure(nm, "-simplex0length", 1)
> nm <- neldermead.configure(nm, "-method", "variable")
> nm <- neldermead.configure(nm, "-verbose", 0)
> nm <- neldermead.configure(nm, "-storehistory", TRUE)
> nm <- neldermead.configure(nm, "-verbosetermination", 0)
> nm <- neldermead.search(nm)</pre>
> xmin <- ymin <- -2
> xmax <- ymax <- 2
> nx <- ny <- 100
```

```
> stepy <- stepx <- (xmax - xmin)/nx
> ydata <- xdata <- seq(xmin, xmax, stepx)</pre>
> zdata <- apply(expand.grid(xdata, ydata), 1, function(x) neldermead.function(nm,
      transpose(x)))
> zdata <- matrix(zdata, ncol = length(ydata))</pre>
> optimpath <- matrix(unlist((neldermead.get(nm, "-historyxopt"))),</pre>
      nrow = 2)
> optimpath <- data.frame(x = optimpath[1, ], y = optimpath[2,</pre>
      ])
> contour(xdata, ydata, zdata, levels = c(1, 10, 100, 500,
      1000, 2000))
> par(new = TRUE, ann = TRUE)
> plot(c(x0[1], optimpath$x[158]), c(x0[2], optimpath$y[158]),
      col = c("red", "green"), pch = 16, xlab = "x[1]", ylab = "x[2]",
      xlim = c(xmin, xmax), ylim = c(ymin, ymax))
> par(new = TRUE, ann = FALSE)
> plot(optimpath$x, optimpath$y, col = "blue", type = "l",
      xlim = c(xmin, xmax), ylim = c(ymin, ymax))
```



Setting the '-verbose' element of the neldermead object to 1 allows to get detailed information about the current optimization process. The following is a sample output for an optimization based on the Nelder and Mead variable-shape simplex algorithm. Only the output corresponding to the iteration #156 is displayed. In order to display specific outputs (or to create specific output files and graphics), the '-output command' option should be used.

-----

```
Iteration \fint 156 (total = 156)
Function Eval \#298
Xopt: 0.999999999999 0.9999999999816
Fopt: 8.997809e-27
DeltaFv: 4.492261e-26
Center: 1.0000000000003 1.0000000000007
Size: 4.814034e-13
Vertex \frac{42}{3}: fv=2.649074e-26, x=1.000000e+00 1.000000e+00
Vertex \#3/3 : fv=5.392042e-26, x=1.000000e+00 1.000000e+00
Reflect
xbar=1.0000000000001 1.0000000000003
Function Evaluation \#299 at [0.99999999999 ]
Function Evaluation \#299 at [0.9999999999907 ]
> Perform reflection
Sort
```

# 3.3 Example 3: Optimization with bound constraints

In the following example, we solve a simple quadratic test case used in Example 1 but in the case where bounds are set for parameter estimates. We begin by defining the cost function, which takes 3 input arguments and returns the value of the objective function as the f element of a list. The starting point [1.2 1.9] is used. neldermead.new creates a new neldermead object. Then we use neldermead.configure to configure the parameters of the problem including the lower -boundsmin and upper -boundsmax bounds. The initial simplex is computed from boxnbpoints random points within the bounds. The variable simplex algorithm by Box is used, which corresponds to the -method 'box' option. neldermead.search finally performs the search for the minimum.

# 3.4 Example 4: Optimization with nonlinear inequality constraints

In the following example, we solve Michalewicz's  $G_6$  test problem using Box's methods [7] <sup>1</sup>. This problem consists in minimizing:  $G_6(x) = (x_1 - 10)^3 + (x_2 - 20)^3$ , given the nonlinear constraints:

```
c1: (x_1 - 5)^2 + (x_2 - 5)^2 - 100 \ge 0

c2: -(x_1 - 6)^2 - (x_2 - 5)^2 + 82.81 \ge 0

and bounds: 13 \le x_1 \le 100, 0 \le x_2 \le 100.
```

We begin by defining the michalewicz function, which takes 3 input arguments and return the value of the objective function and the constraint evaluations as the f and c elements of a list. neldermead.new creates a new neldermead object. Then we use neldermead.configure to configure the parameters of the problem, including the lower -boundsmin and upper -boundsmax bounds. The initial simplex is computed from boxnbpoints random points within the bounds. The variable simplex algorithm by Box is used, which corresponds to the -method 'box' option. neldermead.search finally performs the search for the minimum. The starting point ([15 4.99]) like all the vertices of the optimization simplex must be feasible, i.e. they must satisfy all constraints and bounds. Constraints are enforced by ensuring that all arguments of c in the cost function output are positive or null. Note that the boundaries were set to stricter ranges to limit the sensitivity of the solution to the initial guesses.

```
> michalewicz <- function(x = NULL, index = NULL, fmsfundata = NULL) {
      f <- c()
      c <- c()
      if (index == 2 | index == 6)
          f \leftarrow (x[1] - 10)^3 + (x[2] - 20)^3
      if (index == 5 \mid index == 6)
          c \leftarrow c((x[1] - 5)^2 + (x[2] - 5)^2 - 100, 82.81 -
               ((x[1] - 6)^2 + (x[2] - 5)^2)
      varargout \leftarrow list(f = f, g = c(), c = c, gc = c(), index = index,
          this = list(costfargument = fmsfundata))
      return(varargout)
+ }
> set.seed(0)
> x0 \leftarrow transpose(c(15, 4.99))
> nm <- neldermead.new()</pre>
> nm <- neldermead.configure(nm, "-numberofvariables", 2)
> nm <- neldermead.configure(nm, "-nbineqconst", 2)
> nm <- neldermead.configure(nm, "-function", michalewicz)
> nm <- neldermead.configure(nm, "-x0", x0)
> nm <- neldermead.configure(nm, "-maxiter", 300)
> nm <- neldermead.configure(nm, "-maxfunevals", 1000)
```

<sup>&</sup>lt;sup>1</sup>Example suggested by Pascal Grandeau

# 3.5 Example 5: Passing data to the cost function

In the following example, we use a simple example to illustrate how to pass user-defined arguments to a user-defined cost function. We try to find the mean and standard deviation of some normally distributed data using maximum likelihood (actually a modified negative log-likelihood approach)<sup>2</sup>.

We begin by defining the negLL function, which takes 3 input arguments and return the value of the objective function. The random dataset is then generated and stored in the list fmsdundata. neldermead.new creates a new neldermead object. Then we use neldermead.configure to configure the parameters of the problem, including costfargument, set to fmsdundata, and the lower -boundsmin and upper -boundsmax bounds (the standard deviations has to be positive). The variable simplex algorithm by Box is used. neldermead.search finally performs the search for the minimum.

```
> negLL <- function(x = NULL, index = NULL, fmsfundata = NULL) {
      mn < -x[1]
      sdv \leftarrow x[2]
      out <- -sum(dnorm(fmsfundata$data, mean = mn, sd = sdv,
          log = TRUE)
      return(list(f = out, index = index, this = list(costfargument = fmsfundata)))
+ }
> set.seed(12345)
> fmsfundata <- list(data = rnorm(500, mean = 50, sd = 2))</pre>
> attr(fmsfundata, "type") <- "T_FARGS"
> x0 \leftarrow transpose(c(45, 3))
> nm <- neldermead.new()
> nm <- neldermead.configure(nm, "-numberofvariables", 2)
> nm <- neldermead.configure(nm,</pre>
                                  "-function", negLL)
> nm <- neldermead.configure(nm, "-x0", x0)</pre>
> nm <- neldermead.configure(nm, "-costfargument", fmsfundata)
> nm <- neldermead.configure(nm, "-maxiter", 500)</pre>
> nm <- neldermead.configure(nm, "-maxfunevals", 1500)
> nm <- neldermead.configure(nm, "-method", "box")</pre>
> nm <- neldermead.configure(nm, "-storehistory", TRUE)
```

 $<sup>^2 \</sup>rm Example$  suggested by Mark Taper

# 3.6 Example 6: Direct grid search

In the following example, we use the Rosenbrock test case introduced as Example 2 to illustrate the direct grid search capacity of neldermead. We begin by defining the Rosenbrock function, which takes only one input argument and returns the value of the objective function. We request 6 points per dimension of the problem and set the range of search around the standard starting point [-1.2 1.0] by providing limits. fmin.gridsearch performs the search and return a table sorted by value of the cost function.

```
> rosenbrock <- function(x = NULL) {</pre>
      f \leftarrow 100 * (x[2] - x[1]^2)^2 + (1 - x[1])^2
+ }
> x0 <- c(-1.2, 1)
> npts <- 6
> xmin <- c(-2, -2)
> xmax <- c(2, 2)
> fmin.gridsearch(fun = rosenbrock, x0 = x0, xmin = xmin, xmax = xmax,
      npts = npts, alpha = alpha)
The grid contains 30 unique combinations.
  Evaluating combination number: 1/30
 Evaluating combination number: 2/30
 Evaluating combination number: 3/30
 Evaluating combination number: 4/30
  Evaluating combination number: 5/30
 Evaluating combination number: 6/30
  Evaluating combination number: 7/30
  Evaluating combination number: 8/30
  Evaluating combination number: 9/30
  Evaluating combination number: 10/30
  Evaluating combination number: 11/30
  Evaluating combination number: 12/30
  Evaluating combination number: 13/30
  Evaluating combination number: 14/30
  Evaluating combination number: 15/30
  Evaluating combination number: 16/30
  Evaluating combination number: 17/30
  Evaluating combination number: 18/30
```

```
Evaluating combination number: 19/30
 Evaluating combination number: 20/30
 Evaluating combination number: 21/30
 Evaluating combination number: 22/30
 Evaluating combination number: 23/30
 Evaluating combination number: 24/30
 Evaluating combination number: 25/30
 Evaluating combination number: 26/30
 Evaluating combination number: 27/30
 Evaluating combination number: 28/30
 Evaluating combination number: 29/30
 Evaluating combination number: 30/30
               f feasible
    x1 x2
22 1.0 1
             0.0
15 0.0 0
             1.0
                        1
20 -1.0 1
             4.0
                        1
24 -1.2 1
            24.2
                        1
30 -1.2 2
            36.2
                        1
16 1.0 0 100.0
                        1
28
  1.0 2 100.0
                        1
   0.0 -1 101.0
21 0.0 1 101.0
                        1
14 -1.0 0 104.0
                        1
26 -1.0 2 104.0
                       1
18 -1.2 0 212.2
10 1.0 -1 400.0
                       1
   0.0 -2 401.0
3
                        1
27 0.0 2 401.0
                        1
29 2.0 2 401.0
8 -1.0 -1 404.0
                       1
25 -2.0 2 409.0
                       1
12 -1.2 -1 600.2
                        1
4 1.0 -2 900.0
                       1
23 2.0 1 901.0
                        1
2 -1.0 -2 904.0
                        1
19 -2.0 1 909.0
                        1
6 -1.2 -2 1188.2
                        1
17 2.0 0 1601.0
                        1
13 -2.0 0 1609.0
                        1
11 2.0 -1 2501.0
7 -2.0 -1 2509.0
                        1
5 2.0 -2 3601.0
                        1
1 -2.0 -2 3609.0
```

# 4 References

- C.T. Kelley. Iterative Methods for Optimization. SIAM Frontiers in Applied Mathematics, Philadelphia, PA, 1999.
- [2] J.A. Guin. Discussion and correspondence: modification of the complex method of constrained optimization. *The Computer Journal*, 10(4):416–417, 1968.
- [3] J.A. Nelder and R. Mead. A Simplex Method for Function Minimization. *The Computer Journal*, 7(4):308–313, 1965.
- [4] M.J. Box. A New Method of Constrained Optimization and a Comparison With Other Methods. *The Computer Journal*, 1(8):42–52, 1965.
- [5] R. O'Neill. Algorithm AS47 Function minimization using a simplex procedure. *Applied Statistics*, 20:338–345, 1971.
- [6] W. Spendley and G.R. Hext and F.R. Himsworth. Sequential Application of Simplex Designs in Optimisation and Evolutionary Operation. *Technometrics*, 4:441–461, 1962.
- [7] Z. Michalewicz and D.B. Fogel. How to solve it: modern heuristics, chapter Constraint-handling techniques, pages 231–270. Springer, 2004.

# 5 Dependencies of fminsearch

We illustrate in the figures below the network of functions of the **neldermead**, **optimbase**, and **optimsimplex** packages that are called from the **fminsearch** functions. This large network is broken down in 6 plots, which are shown in the order functions are called. Green boxes represent functions that are not expanded on a given plot but on a previous or later one.

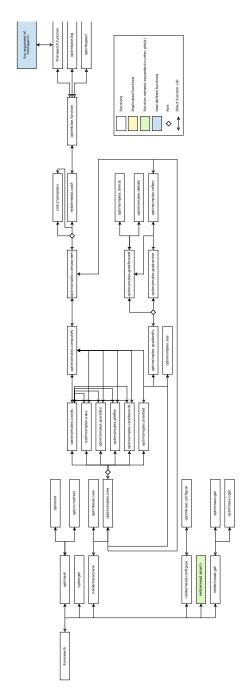


Figure 1: fminsearch function network (1/6)

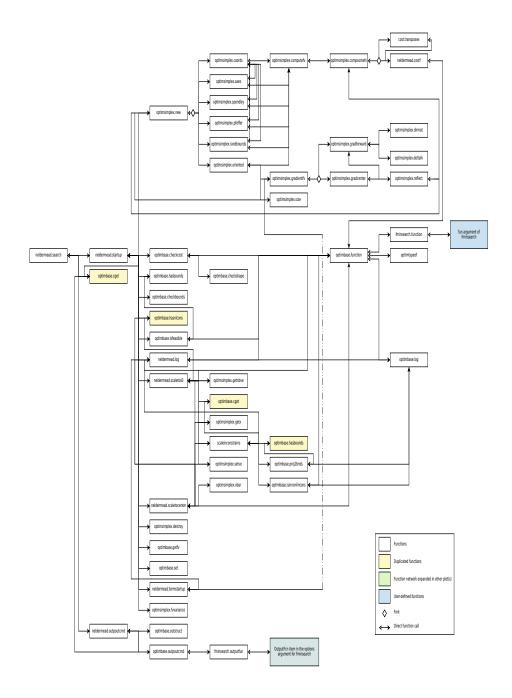


Figure 2: fminsearch function network (2/6)

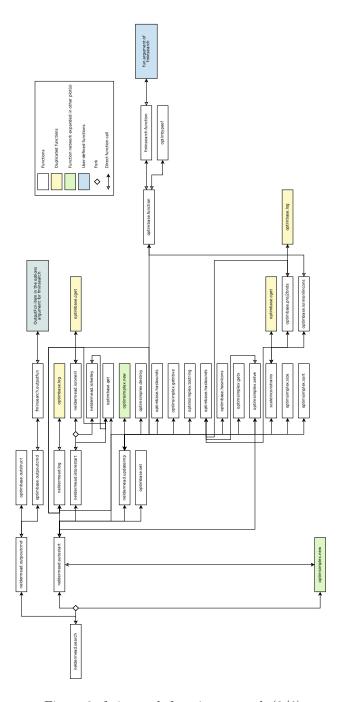


Figure 3: fminsearch function network (3/6)

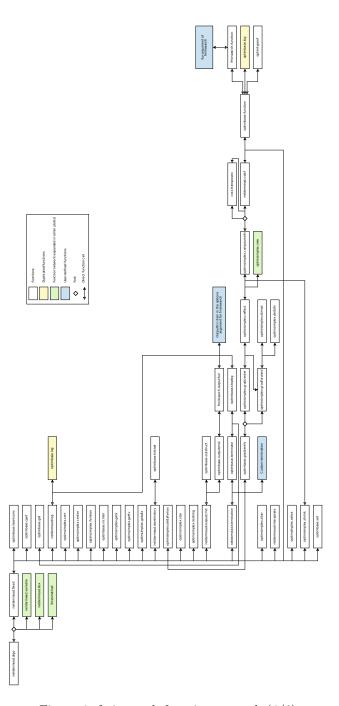


Figure 4: fminsearch function network (4/6)

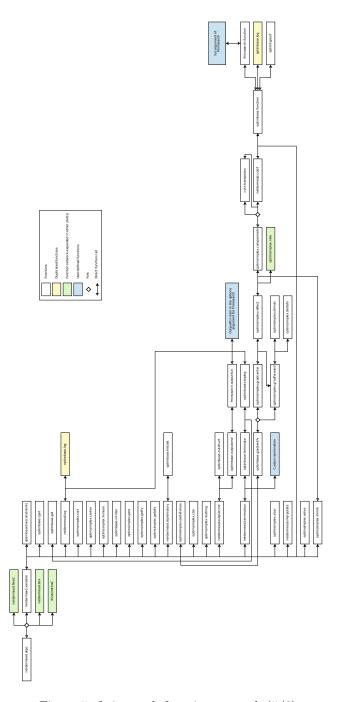


Figure 5: fminsearch function network (5/6)

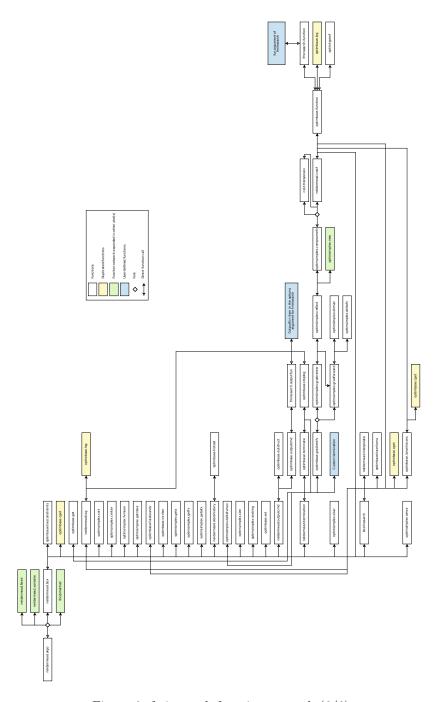


Figure 6: fminsearch function network (6/6)

# 6 Help on neldermead functions

neldermead-package R port of the Scilab neldermead module

## Description

The goal of this package is to provide a Nelder-Mead direct search optimization method. That Nelder-Mead algorithm may be used in the following optimization context:

- there is no need to provide the derivatives of the objective function,
- the number of parameters is small (up to 10-20),
- there are bounds and/or non linear constraints.

#### Design

This package provides the following components:

- neldermead provides various Nelder-Mead variants and manages for Nelder-Mead specific settings, such as the method to compute the initial simplex, the specific termination criteria,
- fminsearch provides a simplified Nelder-Mead algorithm. Specific termination criteria, initial simplex and auxiliary settings are automatically configured.
- optimset, optimget provide commands to emulate their Scilab counterparts.
- optimplotfunccount, optimplotx and optimplotfval provide plotting features for the fminsearch function (Not implemented yet).
- nmplot provides a high-level component which provides directly output pictures for Nelder-Mead algorithm. (Not implemented yet).

The current component is based on the following packages

- optimbase: provides an abstract class for a general optimization component, including the number of variables, the minimum and maximum bounds, the number of non linear inequality constraints, the loggin system, various termination criteria, the cost function, etc...
- optimsimplex: provides a class to manage a simplex made of an arbitrary number of vertices, including the computation of a simplex by various methods (axes, regular, Pfeffer's, randomized bounds), the computation of the size by various methods (diameter, sigma+, sigma-, etc...),

#### **Features**

The following is a list of features the Nelder-Mead prototype algorithm currently provides:

- Provides 3 algorithms, including
  - the fixed shape algorithm of Spendley et al.,
  - the variable shape algorithm of Nelder and Mead,
  - Box's 'complex' algorithm managing bounds and nonlinear inequality constraints based on arbitrary number of vertices in the simplex.

- Manage various simplex initializations:
  - initial simplex given by user,
  - initial simplex computed with a length and along the coordinate axes,
  - initial regular simplex computed with formula of Spendley et al.,
  - initial simplex computed by a small perturbation around the initial guess point.
- Manage cost function:
  - optionnal additionnal argument,
  - direct communication of the task to perform: cost function or inequality constraints.
- Manage various termination criteria, including maximum number of iterations, tolerance on function value (relative or absolute):
  - tolerance on x (relative or absolute),
  - tolerance on standard deviation of function value (original termination criteria in Box 1965),
  - maximum number of evaluations of cost function,
  - absolute or relative simplex size.
- Manage the history of the convergence, including:
  - history of function values,
  - history of optimum point,
  - history of simplices,
  - history of termination criterias.
- Provide a plot command which allows to graphically see the history of the simplices toward the optimum (Not yet implemented).
- Provide query features for the status of the optimization process: number of iterations, number of function evaluations, status of execution, function value at initial point, function value at optimal point, etc...
- Kelley restart based on simplex gradient.
- O'Neill restart based on factorial search around optimum.

#### **Details**

Package: neldermead Type: Package Version: 1.0-7Date: 2012-01-03 License: CeCILL-2

LazyLoad: yes

See vignette('neldermead', package='neldermead') for more information.

# Author(s)

Author of Scilab neldermead module: Michael Baudin (INRIA - Digiteo) Author of R adaptation: Sebastien Bihorel (<sb.pmlab@gmail.com>)

#### References

'Sequential Application of Simplex Designs in Optimisation and Evolutionary Operation', Spendley, W. and Hext, G. R. and Himsworth, F. R., American Statistical Association and American Society for Quality, 1962

'A Simplex Method for Function Minimization', Nelder, J. A. and Mead, R., The Computer Journal, 1965

'A New Method of Constrained Optimization and a Comparison With Other Methods', M. J. Box, The Computer Journal 1965 8(1):42-52, 1965 by British Computer Society

'Discussion and correspondence: modification of the complex method of constrained optimization', J. A. Guin, The Computer Journal, 1968

'Detection and Remediation of Stagnation in the Nelder-Mead Algorithm Using a Sufficient Decrease Condition', Kelley C. T., SIAM J. on Optimization, 1999

'Iterative Methods for Optimization', C. T. Kelley, SIAM Frontiers in Applied Mathematics, 1999

'Algorithm AS47 - Function minimization using a simplex procedure', O'Neill, R., Applied Statistics, 1971

#### See Also

optimbase optimsimplex

costf.transposex

Cost Function Call

#### Description

Call the cost function after transposition of the value of the point estimate x, so that the input row vector, given by optimsimplex, is transposed into a column vector as required by the cost function.

## Usage

```
costf.transposex(x = NULL, this = NULL)
```

#### Arguments

**x** The point estimate provide as a row matrix.

this A neldermead object.

### Value

Return the value of the cost function (called by neldermead.costf.)

## Author(s)

```
Author of Scilab neldermead module: Michael Baudin (INRIA - Digiteo)
Author of R adaptation: Sebastien Bihorel (<sb.pmlab@gmail.com>)
```

#### See Also

neldermead.costf

fmin.gridsearch Grid evaluation of an unconstrained cost function
---

# Description

Evaluate an unconstrained cost function on a grid of points around a given initial point estimate.

#### Usage

### Arguments

fun	An unconstrained cost function returning a numeric scalar, similar to those used in the fminsearch function.
x0	The initial point estimate, provided as a numeric vector.
xmin	Optional: a vector of lower bounds.
xmax	Optional: a vector of upper bounds.
npts	An integer scalar greater than 2, indicating the number of evaluation points will be used on each dimension to build the search grid.
alpha	A vector of numbers greater than 1, which give the factor(s) used to calculate the evaluation range of each dimension of the search grid (see Details). If alpha length is lower than that of x0, elements of alpha are recycled. If its length is higher than that of x0, alpha is truncated.

## **Details**

fmin.gridsearch evaluates the cost function at each point of a grid of npts^length(x0) points. If lower (xmin) and upper (xmax) bounds are provided, the range of evaluation points is limited by those bounds and alpha is not used. Otherwise, the range of evaluation points is defined as [x0/alpha,x0\*alpha].

The actual evaluation of the cost function is delegated to optimbase.gridsearch.

## Value

Return a data frame with the coordinates of the evaluation point, the value of the cost function and its feasibility. Because the cost function is unconstrained, it is always feasible. The data frame is ordered by feasibility and increasing value of the cost function.

#### Author(s)

Sebastien Bihorel (<sb.pmlab@gmail.com>)

#### See Also

fminsearch, optimbase.gridsearch

fminsearch.function fminsearch Cost Function Call

#### Description

This function calls the cost function and makes it match neldermead requirements. It is used in the fminsearch function as the function element of the neldermead object (see ?neldermead.new and ?neldermead.configure).

#### Usage

fminsearch.function(x = NULL, index = NULL, fmsfundata = NULL)

#### Arguments

x A single column vector of parameter estimates.

index An integer variable set to 2, indicating that only the cost function is to be

computed by the algorithm.

fmsfundata A list with a type attribute set to 'T\_FARGS' and with (at least) a fun element,

which contain the user-defined cost function.

#### Value

Returns a list with the following elements:

**f** The value of the cost function at the current point estimate.

index The same index variable.

this A list with a single element costargument which contains fmsfundata.

### Author(s)

Author of Scilab neldermead module: Michael Baudin (INRIA - Digiteo) Author of R adaptation: Sebastien Bihorel (<sb.pmlab@gmail.com>)

#### See Also

fminsearch, neldermead.new, neldermead.configure,

## Description

This function calls the output function and make it match neldermead requirements. It is used in the fminsearch function as the outputcommand element of the neldermead object (see ?neldermead.new and ?neldermead.configure).

# Usage

```
fminsearch.outputfun(state = NULL, data = NULL, fmsdata = NULL)
```

#### **Arguments**

state The current state of the algorithm either 'init', 'iter' or 'done'.

data The data at the current state. This is a list with a 'type' attribute set to

'T\_NMDATA' and with the following elements:

 $\mathbf{x}$  The current parameter estimates.

**fval** The current value of the cost function.

simplex The current simplex object.

**iteration** The number of iterations performed. **function** The number of function evaluations.

**step** The type of step in the previous iteration.

fmsdata This is a list with a 'type' attribute set to 'T\_FARGS' which contains specific

data of the fminsearch algorithm:

**Display** what to display

OutputFcn the array of output functions
PlotFcns the array of plot functions

#### Value

This function does not return any data, but execute the output function(s).

#### Author(s)

Author of Scilab neldermead module: Michael Baudin (INRIA - Digiteo) Author of R adaptation: Sebastien Bihorel (<sb.pmlab@gmail.com>)

### See Also

fminsearch, neldermead.new, neldermead.configure,

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Computation of the unconstrained minimimum of given function with the Nelder-Mead algorithm.

## Description

This function searches for the unconstrained minimum of a given cost function. The provided algorithm is a direct search algorithm, i.e. an algorithm which does not use the derivative of the cost function. It is based on the update of a simplex, which is a set of k>=n+1 vertices, where each vertex is associated with one point and one function value. This algorithm is the Nelder-Mead algorithm. This function is based on a specialized use of the more general neldermead function bundle. Users who want to have a more flexible solution based on direct search algorithms should consider using the neldermead functions instead of the fminsearch function.

#### Usage

```
fminsearch(fun = NULL, x0 = NULL, options = NULL)
```

#### Arguments

fun A cost function return a numeric scalar.

x0 A numerical vector of initial guesses (length n).

options A list of optimization options, which drives the behaviour of fminsearch.

These options must be set with the optimset function (see ?optimset) which

returns a list with the following elements:

MaxIter The maximum number of iterations. The default is 200 \* n.

MaxFunEvals The maximum number of evaluations of the cost function.

The default is 200 \* n.

**TolFun** The absolute tolerance on function value. The default value is 1.e-4.

**TolX** The absolute tolerance on simplex size. The default value is 1.e-4.

**Display** The verbose level.

OutputFcn The output function, or a list of output functions called at the end of each iteration. The default value is NULL.

**PlotFcns** The plot function, or a list of plotput functions called at the end of each iteration. The default value is empty.

#### **Details**

#### Termination criteria

In this section, we describe the termination criteria used by fminsearch. The criteria is based on the following variables:

ssize the current simplex size,

**shiftfv** the absolute value of the difference of function value between the highest and lowest vertices.

If both ssize < options\$TolX and shiftfv < options\$TolFun conditions are true, then the iterations stop. The size of the simplex is computed using the 'sigmaplus' method of the optim-simplex package. The 'sigmamplus' size is the maximum length of the vector from each vertex to the first vertex. It requires one loop over the vertices of the simplex.

#### The initial simplex

The fminsearch algorithm uses a special initial simplex, which is an heuristic depending on the initial guess. The strategy chosen by fminsearch corresponds to the content of simplex0method element of the neldermead pbject (set to 'pfeffer'). It is applied using the content of the simplex0deltausual (0.05) and simplex0deltazero (0.0075) elements. Pfeffer's method is an heuristic which is presented in 'Global Optimization Of Lennard-Jones Atomic Clusters' by Ellen Fan. It is due to L. Pfeffer at Stanford. See in the help of optimismplex for more details.

#### The number of iterations

In this section, we present the default values for the number of iterations in fminsearch.

The options input argument is an optionnal list which can contain the MaxIter field, which stores the maximum number of iterations. The default value is 200n, where n is the number of variables. The factor 200 has not been chosen by chance, but is the result of experiments performed against quadratic functions with increasing space dimension. This result is presented in 'Effect of dimensionality on the nelder-mead simplex method' by Lixing Han and Michael Neumann. This paper is based on Lixing Han's PhD, 'Algorithms in Unconstrained Optimization'. The study is based on numerical experiments with a quadratic function where the number of terms depends on the dimension of the space (i.e. the number of variables). Their study showed that the number of iterations required to reach the tolerance criteria is roughly 100n. Most iterations are based on inside contractions. Since each step of the Nelder-Mead algorithm only require one or two function evaluations, the number of required function evaluations in this experiment is also roughly 100n.

# Output and plot functions

The optimset function can be used to configure one or more output and plot functions. The output or plot function is expected to have the following definition:

```
myfun <- function(x , optimValues , state)</pre>
```

The input arguments x, optimValues and state are described in detail in the optimset help page. The optimValues\$procedure field represents the type of step preformed at the current iteration and can be equal to one of the following strings:

- " (the empty string),
- 'initial simplex',
- 'expand',
- 'reflect',
- 'contract inside',
- 'contract outside'.

#### Value

Return a list with the following fields:

**x** The vector of n numeric values, minimizing the cost function.

**fval** The minimum value of the cost function.

exitflag The flag associated with exist status of the algorithm. The following values are available:

- -1 The maximum number of iterations has been reached.
- **0** The maximum number of function evaluations has been reached.
- 1 The tolerance on the simplex size and function value delta has been reached. This signifies that the algorithm has converged, probably to a solution of the problem.

**output** A list which stores detailed information about the exit of the algorithm. This list contains the following fields:

**algorithm** A string containing the definition of the algorithm used, i.e. 'Nelder-Mead simplex direct search'.

funcCount The number of function evaluations.

iterations The number of iterations.

message A string containing a termination message.

## Author(s)

Author of Scilab neldermead module: Michael Baudin (INRIA - Digiteo) Author of R adaptation: Sebastien Bihorel (<sb.pmlab@gmail.com>)

#### References

'Sequential Application of Simplex Designs in Optimisation and Evolutionary Operation', Spendley, W. and Hext, G. R. and Himsworth, F. R., American Statistical Association and American Society for Quality, 1962

'A Simplex Method for Function Minimization', Nelder, J. A. and Mead, R., The Computer Journal, 1965

'Iterative Methods for Optimization', C. T. Kelley, SIAM Frontiers in Applied Mathematics, 1999

'Algorithm AS47 - Function minimization using a simplex procedure', O'Neill, R., Applied Statistics, 1971

'Effect of dimensionality on the nelder-mead simplex method', Lixing Han and Michael Neumann, Optimization Methods and Software, 21, 1, 1–16, 2006.

'Algorithms in Unconstrained Optimization', Lixing Han, Ph.D., The University of Connecticut, 2000.

'Global Optimization Of Lennard-Jones Atomic Clusters' Ellen Fan, Thesis, February 26, 2002, McMaster University

#### See Also

optimset

#### Examples

#In the following example, we use the fminsearch function to compute the minimum #of the Rosenbrock function. We first define the function 'banana', and then use #the fminsearch function to search the minimum, starting with the initial guess #(-1.2, 1.0). In this particular case, 85 iterations are performed with 159 #function evaluations

```
banana <- function(x){
   y <- 100*(x[2]-x[1]^2)^2 + (1-x[1])^2
}
sol <- fminsearch(banana, c(-1.2,1))
sol</pre>
```

#In the following example, we configure the absolute tolerance on the size of #the simplex to a larger value, so that the algorithm performs less iterations. #Since the default value of 'TolX' for the fminsearch function is 1.e-4, we #decide to use 1.e-2. The optimset function is used to create an optimization #option list and the field 'TolX' is set to 1.e-2. The options list is then #passed to the fminsearch function as the third input argument. In this #particular case, the number of iterations is 70 with 130 function evaluations.

```
opt <- optimset(TolX=1.e-2)
sol <- fminsearch(banana, c(-1.2,1), opt)
sol</pre>
```

#In the following example, we want to produce intermediate outputs of the #algorithm. We define the outfun function, which takes the current point x as #input argument. The function plots the current point into the current graphic #window with the plot function. We use the 'OutputFcn' feature of the optimset #function and set it to the output function. Then the option list is passed #to the fminsearch function. At each iteration, the output function is called #back, which creates and update a plot. While this example creates a 2D plot, #the user may customized the output function so that it writes a message in #the console, write some data into a data file, etc... The user can distinguish #between the output function (associated with the 'OutputFcn' option) and the #plot function (associated with the 'PlotFcns' option). See the optimset for #more details on this feature.

```
outfun <- function(x, optimValues, state){
  plot(x[1],x[2],xlim=c(-1.5,1.5),ylim=c(-1.5,1.5))
  par(new=TRUE)
}
opt <- optimset(OutputFcn=outfun)
sol <- fminsearch(banana, c(-1.2,1), opt)
sol</pre>
```

#The 'Display' option allows to get some input about the intermediate steps of #the algorithm as well as to be warned in case of a convergence problem. #In the following example, we present what happens in case of a convergence #problem. We set the number of iterations to 10, instead of the default 400 #iterations. We know that 85 iterations are required to reach the convergence #criteria. Therefore, the convergence criteria is not met and the maximum number #of iterations is reached.

```
opt <- optimset(MaxIter=10)
sol <- fminsearch(banana, c(-1.2,1), opt)

#Since the default value of the 'Display' option is 'notify', a message is
#generated, which warns the user about a possible convergence problem. The
#previous script produces the following output.
# Exiting: Maximum number of iterations has been exceeded
# - increase MaxIter option.
# Current function value: 4.1355598

#In the following example, we present how to display intermediate steps used by
#the algorithm. We simply set the 'Display' option to the 'iter' value. This
#option allows to see the number of function evaluations, the minimum function
#value and which type of simplex step is used for the iteration.
opt <- optimset(Display='iter')
sol <- fminsearch(banana, c(-1.2,1), opt)
sol</pre>
```

neldermead.algo

Nelder-Mead Algorithm

#### Description

neldermead.algo performs an optimization without restart using the method associated with the method element of the neldermead object; neldermead.fixed, neldermead.variable, neldermead.box, boxlinesearch, neldermead.storehistory, neldermead.termination, and neldermead.interpolate are utility functions for neldermead.algo.

#### Usage

## Arguments

this A neldermead object.n Number of variables.

xbar The centroid.xhigh The high point.

fhigh The value of the cost function at xhigh.

rho The reflection factor.

The current value of the function at the current optimum point estimate.

xopt The current optimum point estimate.

**xcoords** Matrix of size  $n \times n+1$ , coordinates of the n+1 vertices

fvinitial The initial cost function value.

oldfvmean The old cost function value average on the simplex.

The new cost function value average on the simplex.

previousxopt The previous point estimate.
currentxopt The current point estimate.

simplex The simplex. The best point estimate in the simplex is expected to be stored

at 1, while the worst point estimate in the simplex is expected to be stored at

n+1.

x1 The first reference point estimate to perform the interpolation.

x2 The second reference point estimate to perform the interpolation.

fac A factor to perform the interpolation.

#### Details

neldermead.fixed The simplex algorithm with fixed size simplex. We implement the following 'rules' of the method of Spendley et al.

- Rule 1 is strictly applied, but the reflection is done by reflection of the high point, since we minimize a function instead of maximizing it, like Spendley.
- Rule 2 is NOT implemented, as we expect that the function evaluation is not subject to errors
- Rule 3 is applied, i.e. reflection with respect to next to high point. A shrink step is included, with shrinkage factor sigma.

Rule 1. Ascertain the lowest reading y, of yi ... Yk+1 Complete a new simplex Sp by excluding the point Vp corresponding to y, and replacing it by V\* defined as above.

Rule 2. If a result has occurred in (k + 1) successive simplexes, and is not then eliminated by application of Rule 1, do not move in the direction indicated by Rule 1, or at all, but discard the result and replace it by a new observation at the same point.

Rule 3. If y is the lowest reading in So , and if the next observation made,  $y^*$  , is the lowest reading in the new simplex S , do not apply Rule 1 and return to So from Sp . Move out of S, by rejecting the second lowest reading (which is also the second lowest reading in So).

neldermead.variable The original Nelder-Mead algorithm, with variable-size simplex.

neldermead.box The Nelder-Mead algorithm, with variable-size simplex and modifications by Box for bounds and inequality constraints.

boxlinesearch Called by neldermead.box, i.e. Box's method. Perform a line search from xbar, on the line (xhigh,xbar). The reflected point estimate satisfies the following constraints:

- fr < fhigh
- xr satisfies the bounds constraints
- xr satisfies the nonlinear positive inequality constraints
- xr satisfies the linear positive inequality constraints

The method is based on projection and scaling toward the centroid.

neldermead.storehistory Store the optimization history into the neldermead object.

neldermead.termination Determine if the algorithm must continue or terminate. The function uses the cost function average in the simplex instead of the best cost function value. This is because the function average changes at each iteration. Instead, the best function value has a step-by-step evolution and may not change between two successive iterations, leading to a stop of the algorithm.

neldermead.interpolate Compute the point estimate xi as an interpolation between x1 and x2, as follows: xi = (1+fac)x1 - fac\*x2

#### Value

neldermead.fixed, neldermead.variable, and neldermead.box Return the updated neldermead object, containing the optimum point estimate.

boxlinesearch Return a list with the following elements:

this The updated neldermead object.

status TRUE if the search is successful, FALSE otherwise.

**xr** The reflected point estimate.

fr The value of the cost function at xr.

neldermead.storehistory Return the updated neldermead object.

neldermead.termination Return a list with the following elements:

this The updated neldermead object

terminate TRUE if the algorithm terminates, FALSE if the algorithm must continue.

status The termination status: 'continue', 'maxiter', 'maxfuneval', 'tolf', 'tolx', 'tolsize', 'tolsizedeltafv', 'kelleystagnation', 'tolboxf', 'tolvariance' or the user-defined termination status.

neldermead.interpolate Return a new point estimate, i.e. a column vector.

### Author(s)

Author of Scilab neldermead module: Michael Baudin (INRIA - Digiteo)
Author of R adaptation: Sebastien Bihorel (<sb.pmlab@gmail.com>)

## Description

Configure the current neldermead object with the given value for the given key.

## Usage

```
neldermead.configure(this = NULL, key = NULL, value = NULL)
```

### Arguments

this The current neldermead object.

key The key to configure. See details for the list of possible keys.

value The value to assign to the key.

### Details

neldermead.configure sets the content of the key element of the neldermead object this to value. If key is a sub-element of this soptbase, value is assigned by optimbase.configure.

The main available keys are the following:

- '-verbose' Set to 1 to enable verbose logging.
- '-verbosetermination' Set to 1 to enable verbose termination logging.
- '-x0' The initial guess, as a n x 1 column vector, where n is the number of variables.
- '-maxfunevals' The maximum number of function evaluations. If this criteria is triggered during optimization, the status of the optimization is set to 'maxfuneval'.
- '-maxiter' The maximum number of iterations. If this criteria is triggered during optimization, the status of the optimization is set to 'maxiter'.option
- '-tolfunabsolute' The absolute tolerance for the function value.
- '-tolfunrelative' The relative tolerance for the function value.
- '-tolfunmethod' The method used for the tolerance on function value in the termination criteria. The following values are available: TRUE, FALSE. If this criteria is triggered, the status of the optimization is set to 'tolf'.
- '-tolxabsolute' The absolute tolerance on x.
- '-tolxrelative' The relative tolerance on x.
- '-tolxmethod' The method used for the tolerance on x in the termination criteria. The following values are available: TRUE, FALSE. If this criteria is triggered during optimization, the status of the optimization is set to 'tolx'.
- '-function' The objective function, which computes the value of the cost and the non linear constraints, if any. See vignette('neldermead', package='neldermead') for the details of the communication between the optimization system and the cost function.

- '-costfargument' An additionnal argument, passed to the cost function.
- '-outputcommand' A command which is called back for output. See vignette('neldermead', package='neld for the details of the communication between the optimization system and the output command function.
- '-outputcommandarg' An additionnal argument, passed to the output command option
- '-number of variables' The number of variables to optimize.
- '-storehistory' Set to TRUE to enable the history storing.
- '-boundsmin' The minimum bounds for the parameters.
- '-boundsmax' The maximum bounds for the parameters.
- '-nbineqconst' The number of inequality constraints.
- '-method' The name of the algorithm to use. The following methods are available:
  - 'fixed' the fixed simplex shape algorithm of Spendley et al. This algorithm is for unconstrained problems (i.e. bounds and non linear constraints are not taken into account)
  - 'variable' the variable simplex shape algorithm of Nelder and Mead. This algorithm is for unconstrained problems (i.e. bounds and non linear constraints are not taken into account)
  - 'box' Box's complex algorithm. This algorithm takes into account bounds and nonlinear inequality constraints.
  - 'mine' the user-defined algorithm, associated with the mymethod element. See vignette ('neldermead', pa for details.
- '-simplex0method' The method to use to compute the initial simplex. The first vertex in the simplex is always the initial guess associated with the x0 element. The following methods are available:
  - 'given' The coordinates associated with the coords0 element are used to compute the initial simplex, with arbitrary number of vertices. This allows the user to setup the initial simplex by a specific method which is not provided by the current package (for example with a simplex computed from a design of experiments). This allows also to configure the initial simplex so that a specific behaviour of the algorithm is to be reproduced (for example the Mac Kinnon test case). The given matrix is expected to have nove rows and n columns, where n is the dimension of the problem and nove is the number of vertices.
  - 'axes' The simplex is computed from the coordinate axes and the length associated with the simplexOlength element.
  - 'spendley' The simplex is computed so that it is regular with the length associated with the simplexOlength element (i.e. all the edges have the same length).
  - 'pfeffer' The simplex is computed from an heuristic, in the neighborhood of the initial guess. This initial simplex depends on the -simplex0deltausual and -simplex0deltazero.
  - 'randbounds' The simplex is computed from the bounds and a random number. This option is available only if bounds are available: if bounds are not available, an error is generated. This method is usually associated with Box's algorithm. The number of vertices in the simplex is taken from the boxnbpoints element.
- '-coords0' The coordinates of the vertices of the initial simplex. If the simplex0method element is set to 'given', these coordinates are used to compute the initial simplex. This matrix is expected to have shape nove x n, where nove is the number of vertices and n is the number of variables.

- '-simplex0length' The length to use when the initial simplex is computed with the 'axes' or 'spendley' methods. If the initial simplex is computed from 'spendley' method, the length is expected to be a scalar value. If the initial simplex is computed from 'axes' method, it may be either a scalar value or a vector of values, of length n, where n is the number of variables.
- '-simplex0deltausual' The relative delta for non-zero parameters in 'pfeffer' method.
- '-simplex0deltazero' The absolute delta for non-zero parameters in 'pfeffer' method.
- '-rho' The reflection coefficient. This parameter is used when the method element is set to 'fixed' or 'variable'.
- '-chi' The expansion coefficient. This parameter is used when the method element is set to 'variable'.
- '-gamma' The contraction coefficient. This parameter is used when the method element is set to 'variable'.
- '-sigma' The shrinkage coefficient. This parameter is used when the method element is set to 'fixed' or 'variable'.
- '-tolsimplexizemethod' Set to FALSE to disable the tolerance on the simplex size. If this criteria is triggered, the status of the optimization is set to 'tolsize'. When this criteria is enabled, the values of the tolsimplexizeabsolute and tolsimplexizerelative elements are used in the termination criteria. The method to compute the size is the 'sigmaplus' method.
- '-tolsimplexizeabsolute' The absolute tolerance on the simplex size.
- '-tolsimplexizerelative' The relative tolerance on the simplex size.
- '-tolssizedeltafvmethod' Set to TRUE to enable the termination criteria based on the size of the simplex and the difference of function value in the simplex. If this criteria is triggered, the status of the optimization is set to 'tolsizedeltafv'. This termination criteria uses the values of the tolsimplexizeabsolute and toldeltafv elements.option
- '-toldeltafv' The absolute tolerance on the difference between the highest and the lowest function values.
- '-tolvarianceflag' Set to TRUE to enable the termination criteria based on the variance of the function value. If this criteria is triggered, the status of the optimization is set to 'tolvariance'. This criteria is suggested by Nelder and Mead.
- '-tolabsolutevariance' The absolute tolerance on the variance of the function values of the simplex.
- '-tolrelativevariance' The relative tolerance on the variance of the function values of the simplex.
- '-kelleystagnationflag' Set to TRUE to enable the termination criteria using Kelley's stagnation detection, based on sufficient decrease condition. If this criteria is triggered, the status of the optimization is set to 'kelleystagnation'.
- '-kelleynormalizationflag' Set to FALSE to disable the normalization of the alpha coefficient in Kelley's stagnation detection, i.e. use the value of the kelleystagnationalpha0 element as is. Default value is TRUE, i.e. the simplex gradient of the initial simplex is takeoptionn into account in the stagnation detection.
- '-kelleystagnationalpha0' The parameter used in Kelley's stagnation detection.

- '-restartflag' Set to TRUE to enable the automatic restart of the algorithm.
- '-restartdetection' The method to detect if the automatic restart must be performed. The following methods are available:
  - 'oneill' The factorial local optimality test by O'Neill is used. If the test finds a local point which is better than the computed optimum, a restart is performed.
  - 'kelley' The sufficient decrease condition by O'Neill is used. If the test finds that the status of the optimization is 'kelleystagnation', a restart is performed. This status may be generated if the -kelleystagnationflag option is set to TRUE.
- '-restartmax' The maximum number of restarts, when automatic restart is enabled via the -restartflag option.
- '-restarteps' The absolute epsilon value used to check for optimality in the factorial O'Neill restart detection.
- '-restartstep' The absolute step length used to check for optimality in the factorial O'Neill restart detection.
- '-restartsimplexmethod' The method to compute the initial simplex after a restart. The following methods are available.
  - 'given' The coordinates associated with the coords0 element are used to compute the initial simplex, with arbitrary number of vertices. This allow the user to setup the initial simplex by a specific method which is not provided by the current package (for example with a simplex computed from a design of experiments). This allows also to configure the initial simplex so that a specific behaviour of the algorithm is to be reproduced (for example the Mc Kinnon test case). The given matrix is expected to have now rows and n columns, where n is the dimension of the problem and now is the number of vertices.
  - 'axes' The simplex is computed from the coordinate axes and the length associated with the -simplex0length option.
  - 'spendley' The simplex is computed so that it is regular with the length associated with the -simplex0length option (i.e. all the edges have the same length).
  - 'pfeffer' The simplex is computed from an heuristic, in the neighborhood of the initial guess. This initial simplex depends on the -simplex0deltausual and -simplex0deltazero.
  - 'randbounds' The simplex is computed from the bounds and a random number. This option is available only if bounds are available: if bounds are not available, an error is generated. This method is usually associated with Box's algorithm. The number of vertices in the simplex is taken from the -boxnbpoints option.
  - 'oriented' The simplex is computed so that it is oriented, as suggested by Kelley.
- '-scalingsimplex0' The algorithm used to scale the initial simplex into the nonlinear constraints. The following two algorithms are provided:
  - 'tox0' scales the vertices toward the initial guess.
  - 'tocentroid' scales the vertices toward the centroid, as recommended by Box.
  - If the centroid happens to be unfeasible, because the constraints are not convex, the scaling of the initial simplex toward the centroid may fail. Since the initial guess is always feasible, scaling toward the initial guess cannot fail.
- '-boxnbpoints' The number of points in the initial simplex, when the -simplex0method is set to 'randbounds'. The value of this option is also use to update the simplex when a restart is

performed and the -restartsimplexmethod option is set to 'randbounds'. The default value is so that the number of points is twice the number of variables of the problem.

- '-boxineqscaling' The scaling coefficient used to scale the trial point for function improvement or into the constraints of Box's algorithm.
- '-guinalphamin' The minimum value of alpha when scaling the vertices of the simplex into nonlinear constraints in Box's algorithm.
- '-boxreflect' The reflection factor in Box's algorithm.
- '-boxtermination' Set to TRUE to enable Box's termination criteria.
- '-boxtolf' The absolute tolerance on difference of function values in the simplex, suggested by Box. This tolerance is used if the -boxtermination element is set to TRUE.
- '-boxnbmatch' tThe number of consecutive match of Box's termination criteria.
- '-boxboundsalpha' The parameter used to project the vertices into the bounds in Box's algorithm.
- '-mymethod' A user-derined simplex algorithm. See vignette('neldermead', package='neldermead') for details.
- '-myterminate' A user-defined terminate function. See vignette('neldermead', package='neldermead') for details.
- '-myterminateflag' Set to TRUE to enable the user-defined terminate function.

### Value

An updated neldermead object.

## Author(s)

Author of Scilab neldermead module: Michael Baudin (INRIA - Digiteo)
Author of R adaptation: Sebastien Bihorel (<sb.pmlab@gmail.com>)

## See Also

neldermead.new

neldermead.destroy

Erase a neldermead object.

## Description

 ${\tt neldermead.destroy}\ \ {\tt calls}\ \ {\tt optimbase.destroy}\ \ {\tt and}\ \ {\tt optimsimplex.destroy}\ \ {\tt to}\ \ {\tt erase}\ \ {\tt the}\ \ {\tt content}\ \ {\tt of}\ \ {\tt this\$optbase}\ \ {\tt and}\ \ {\tt this\$simplex0}.$ 

## Usage

neldermead.destroy(this = NULL)

## Arguments

this

A neldermead object.

### Value

Return an updated neldermead object.

## Author(s)

```
Author of Scilab neldermead module: Michael Baudin (INRIA - Digiteo)
Author of R adaptation: Sebastien Bihorel (<sb.pmlab@gmail.com>)
```

### See Also

```
optimbase.destroy, optimsimplex.destroy
```

Get functions

Get the value for the given key

## Description

Get the value for the given key in a neldermead object.

### Usage

```
neldermead.get(this = NULL, key = NULL)
neldermead.cget(this = NULL, key = NULL)
```

## Arguments

this

A neldermead object.

key

The name of the key to quiery. The list of available keys for query with neldermead.get is: '-historysimplex', '-simplexopt', '-simplex0', and '-restartnb'. If key is different, the query is delegated to optimbase.get.

The list of available keys for query with neldermead.cget is: '-method', '-

coords0', '-simplex0method', '-simplex0length', '-simplex0deltausual', '-simplex0deltazero', '-rho', '-chi', '-gamma', '-sigma', '-tolsimplexizeabsolute', '-tolsimplexizerelative', '-tolsimplexizemethod', '-toldeltafv', '-tolssizedeltafvmethod', '-restartmax', '-restarteps', '-restartstep', '-kelleystagnationflag', '-kelleynormalizationflag', '-kelleystagnationalpha0', '-restartflag', '-restartdetection', '-restartsimplexmethod', '-checkcostfunction', '-boxnbpoints', '-boxnbpoints', '-scalingsimplex0', '-guinalphamin',

'-boxtermination', '-boxtolf', '-boxnbmatch', '-boxreflect', '-mymethod', '-myterminate', '-myterminateflag', '-tolvarianceflag', '-tolabsolutevariance', '-tolrelativevariance', and '-greedy'. If key is different, the query is delegated to optimbase.cget.

### Value

Return the value of the list element key, or an error message if key does not exist.

## Author(s)

Author of Scilab neldermead module: Michael Baudin (INRIA - Digiteo)
Author of R adaptation: Sebastien Bihorel (<sb.pmlab@gmail.com>)

### See Also

neldermead.configure, optimbase.cget, optimbase.get

neldermead.new

Initialize Neldermead Object

## Description

Creates a new neldermead object.

# Usage

neldermead.new()

## Value

Return a new neldermead object, i.e. a list with a 'type' attribute set to 'T\_NELDERMEAD' and containing the following elements:

optbase An optimization object, created by optimbase.new(), i.e. list of 'type' attribute set to 'T\_OPTIMIZATION' and containing the following elements:

**verbose** The verbose option, controlling the amount of messages. Set to 0.

**x0** The initial guess. Set to NULL.

fx0 The value of the function for the initial guess. Set to NULL.

**xopt** The optimum parameter. Set to 0.

**fopt** The optimum function value. Set to 0.

tolfunabsolute The absolute tolerance on function value. Default is 0.

tolfunrelative The relative tolerance on function value. Default is .Machine\$double.eps.

**tolfunmethod** Logical flag for the tolerance on function value in the termination criteria. This criteria is suitable for functions which minimum is associated with a function value equal to 0. Set to FALSE.

tolxabsolute The absolute tolerance on x. Set to 0.

tolxrelative The relative tolerance on x. Set to .Machine\$double.eps.

tolxmethod Possible values: FALSE, TRUE. Set to TRUE.

funevals The number of function evaluations. Set to 0.

maxfunevals The maximum number of function evaluations. Set to 100.

maxiter The maximum number of iterations. Set to 100.

iterations The number of iterations. Set to 0.

fun The cost function. Set to ".

status The status of the optimization. Set to ".

historyfopt The vector to store the history for fopt. The values of the cost function will be stored at each iteration in a new element, so the length of historyfopt at the end of the optimization should be the number of iterations. Set to NULL.

historyxopt The list to store the history for xopt. The vectors of estimates will be stored on separated levels of the list, so the length of historyfopt at the end of the optimization should be the number of iterations. Set to NULL.

verbosetermination The verbose option for termination criteria. Set to 0.

outputcommand The command called back for output. Set to ".

**outputcommandarg** The outputcommand argument is initialized as a string. If the user configure this element, it is expected that a matrix of values or a list is passed so that the argument is appended to the name of the function. Set to ".

number of variables to optimize. Set to 0.

storehistory The flag which enables/disables the storing of the history. Set to FALSE.

**costfargument** The costf argument is initialized as a string. If the user configure this element, it is expected that a matrix of values or a list is passed so that the argument is appended to the name of the function. Set to ".

boundsmin Minimum bounds for the parameters. Set to NULL.

boundsmax Maximum bounds for the parameters. Set to NULL.

**nbineqconst** The number of nonlinear inequality constraints. Default is 0.

logfile The name of the log file. Set to ".

logfilehandle The handle for the log file. Set to 0.

logstartup Set to TRUE when the logging is started up. Set to FALSE.

withderivatives Set to TRUE when the method uses derivatives. Set to FALSE.

method The name of the algorithm to use. Set to 'variable'.

simplex0 A simplex object created by optimsimplex.new()\$newobj, i.e. list of type attribute
set to 'T\_SIMPLEX' and containing the following elements:

verbose The verbose option, controlling the amount of messages. Set to 0.

**x** The coordinates of the vertices, with size nove x n. Set to NULL.

**n** The dimension of the space. Set to 0.

fv The function values, with size nove x 1. Set to NULL.

**nbve** The number of vertices. Set to 0.

simplex0method The method to use to compute the initial simplex. Set to 'axes'.

**simplex0length** The length to use when the initial simplex is computed with the 'axes' or 'spendley' methods. Set to 1.

**rho** The reflection coefficient. This parameter is used when the method element is set to 'fixed' or 'variable'. Set to 1.

**chi** The expansion coefficient. This parameter is used when the method element is set to 'variable'. Set to 2.

**gamma** The contraction coefficient. This parameter is used when the method element is set to 'variable'. Set to 0.5.

sigma The shrinkage coefficient. This parameter is used when the method element is set to 'fixed' or 'variable'. Set to 0.5.

tolfstdeviation The tolerance for the standard deviation. Set to 0.

tolfstdeviationmethod Set to FALSE.

- tolsimplexizeabsolute The absolute tolerance on the simplex size. Set to 0.
- tolsimplexizerelative The relative tolerance on the simplex size. Set to .Machine\$double.eps.
- tolsimplexizemethod Logical flag to enable/disable the tolerance on the simplex size. When this criteria is enabled, the values of the tolsimplexizeabsolute and tolsimplexizerelative elements are used in the termination criteria. The method to compute the size is the 'sigmaplus' method. Set to FALSE.
- simplexsize Initial size of the simplex, for the tolerance on the simplex size. Set to 0.
- **toldeltafv** The absolute tolerance on the difference between the highest and the lowest function values. Set to .Machine\$double.eps.
- tolssizedeltafvmethod Logical flag to enable/disable the termination criteria based on the size of the simplex and the difference of function value in the simplex. If this criteria is triggered, the status of the optimization is set to 'tolsizedeltafv'. This termination criteria uses the values of the tolsimplexizeabsolute and toldeltafv elements. This criteria is identical to Scilab's fminsearch. Set to FALSE.
- historysimplex The list to store the history for simplex. The simplex will be stored on a new level of the list at each iteration, so the length of historyfopt at the end of the optimization should be the number of iterations. Set to NULL.
- ${f simplex0deltausual}$  The relative delta for non-zero parameters in 'pfeffer' method. Set to 0.05
- $\mathbf{simplex0deltazero}$  The absolute delta for non-zero parameters in 'pfeffer' method. Set to 0.0075
- **simplexopt** The optimum simplex, after one optimization process. Set to NULL.
- restartsimplexmethod The method to compute the initial simplex after a restart. Set to 'oriented'.
- restartmax The maximum number of restarts, when automatic restart is enabled via the restartflag element. Set to 3.
- restarteps The absolute epsilon value used to check for optimality in the factorial O'Neill restart detection. Set to .Machine\$double.eps.
- **restartstep** The absolute step length used to check for optimality in the factorial O'Neill restart detection. Set to 1.
- **kelleystagnationflag** Logical flag to enable/disable the termination criteria using Kelley's stagnation detection, based on sufficient decrease condition. If this criteria is triggered, the status of the optimization is set to 'kelleystagnation'. Set to FALSE.,
- kelleynormalizationflag Logical flag to enable/disable the normalization of the alpha coefficient in Kelley's stagnation detection, i.e. use the value of the kelleystagnationalpha0 element as is. Set to TRUE, i.e. the simplex gradient of the initial simplex is taken into account in the stagnation detection.

kelleystagnationalpha0 The parameter used in Kelley's stagnation detection. Set to 1.e-4.

**kelleyalpha** The current value of Kelley's alpha, after normalization, if required. Set to 1.e-4. **restartnb** Number of restarts performed. Set to 0.

restartflag Logical flag to enable/disable the automatic restart of the algorithm. Set to FALSE.

**restart detection** The method to detect if the automatic restart must be performed. Set to 'oneill'.

startupflag Set to TRUE when the startup has been performed. Set to FALSE.

boxnbpoints The number of points in the initial simplex, when the simplexOmethod is set to 'randbounds'. The value of this element is also use to update the simplex when a restart is performed and the restartsimplexmethod element is set to 'randbounds'. The default value is so that the number of points is twice the number of variables of the problem. Set to '2n'.

**boxnbpointseff** The effective number of points required in the simplex for Box's algorithm. Set to 0.

**boxineqscaling** The scaling coefficient used to scale the trial point for function improvement or into the constraints of Box's algorithm. Set to 0.

**checkcostfunction** Logical flag to enable/disable the checking of the connection of the cost function. Set to TRUE.

scalingsimplex0 The algorithm used to scale the initial simplex into the nonlinear constraints.

The following two algorithms are provided:

'tox0' scales the vertices toward the initial guess.

'tocentroid' scales the vertices toward the centroid, as recommended by Box.

If the centroid happens to be unfeasible, because the constraints are not convex, the scaling of the initial simplex toward the centroid may fail. Since the initial guess is always feasible, scaling toward the initial guess cannot fail. Set to 'tox0'.

**guinalphamin** The minimum value of alpha when scaling the vertices of the simplex into nonlinear constraints in Box's algorithm. Set to 1.e-5.

**boxboundsalpha** The parameter used to project the vertices into the bounds in Box's algorithm. Set to 1.e-6.

boxtermination Logical flag to enable/disable Box's termination criteria. Set to FALSE.

**boxtolf** The absolute tolerance on difference of function values in the simplex, suggested by Box. This tolerance is used if the **boxtermination** element is set to TRUE. Set to 1.e-5.

boxnbmatch The number of consecutive match of Box's termination criteria. Set to 5.

boxkount Current number of consecutive match. Set to 0.

**boxreflect** The reflection factor in Box's algorithm. Set to 1.3.

tolvarianceflag Logical flag to enable/disable the termination criteria based on the variance of the function value. If this criteria is triggered, the status of the optimization is set to 'tolvariance'. This criteria is suggested by Nelder and Mead. Set to FALSE.

**tolabsolutevariance** The absolute tolerance on the variance of the function values of the simplex. Set to 0.

**tolrelativevariance** The relative tolerance on the variance of the function values of the simplex. Set to .Machine\$double.eps.

variancesimplex0 Relative tolerance on variance. Set to .Machine\$double.eps.

mymethod A user-derined simplex algorithm. Set to NULL.

myterminate A user-defined terminate function. Set to NULL.

myterminateflag Logical flag to enable/disable the user-defined terminate function. Set to FALSE.

greedy Logical flag to enable/disable greedy Nelder-Mead. Set to FALSE.

## Author(s)

```
Author of Scilab neldermead module: Michael Baudin (INRIA - Digiteo)
Author of R adaptation: Sebastien Bihorel (<sb.pmlab@gmail.com>)
```

## See Also

```
optimbase.new, optimsimplex.new
```

neldermead.restart

Restart neldermead search.

## Description

Update the simplex with neldermead.updatesimp and restart the search with neldermead.search.

## Usage

```
neldermead.restart(this = NULL)
```

## Arguments

this

A neldermead object.

### Value

Returns an updated neldermead object.

# Author(s)

```
Author of Scilab neldermead module: Michael Baudin (INRIA - Digiteo)
Author of R adaptation: Sebastien Bihorel (<sb.pmlab@gmail.com>)
```

### See Also

neldermead.updatesimp, neldermead.search,

neldermead.search

 $Starts\ the\ optimization$ 

## Description

Performs the optimization associated with the method associated with the method element of the neldermead object and find the optimum. If the restartflag element is enabled, automatic restarts are performed, based on the restartdetection element.

## Usage

```
neldermead.search(this = NULL)
```

## Arguments

this

A neldermead object.

## Value

Return an updated neldermead object.

# Author(s)

```
Author of Scilab neldermead module: Michael Baudin (INRIA - Digiteo)
Author of R adaptation: Sebastien Bihorel (<sb.pmlab@gmail.com>)
```

## See Also

fminsearch, neldermead.new, neldermead.configure,

```
Secondary search functions
```

Secondary functions for neldermead.search

# Description

Utility functions for neldermead.serch and dependent functions.

## Usage

```
neldermead.startup(this = NULL)
neldermead.log(this = NULL, msg = NULL)
neldermead.scaletox0(this = NULL, simplex0 = NULL)
neldermead.scaletocenter(this = NULL, simplex0 = NULL, x0 = NULL)
neldermead.termstartup(this = NULL)
neldermead.outputcmd(this = NULL, state = NULL, simplex = NULL, step = NULL)
neldermead.autorestart(this = NULL)
neldermead.istorestart(this = NULL)
neldermead.isroneill(this = NULL)
neldermead.isrkelley(this = this)
neldermead.updatesimp(this = NULL)
scaleinconstraints(this = NULL, x = NULL, xref = NULL)
neldermead.costf(x = NULL, this = NULL)
```

## Arguments

this	A neldermead object.
msg	A character string.
simplex0	The initial simplex object.
x0	A column matrix of initial parameters.
state	The state of the algorithm, either 'init', 'done' or 'iter'.
simplex	The current simplex object.
step	The type of step performed during the iteration: 'init', 'done', 'reflection', 'expansion', 'insidecontraction', 'outsidecontraction', 'reflectionnext' or 'shrink'.
x	The point estimate to scale.
xref	The reference point estimate.

### **Details**

neldermead.startup Startup the algorithm. Compute the initial simplex, depending on the content of the simplexOmethod element of the neldermead object ('given', 'axes', 'spendley', 'pfeffer' or 'randbounds').

neldermead.log Print a message to the log file using optimbase.log.

neldermead.scaletox0 Scale the simplex into the nonlinear inequality constraints, if any. Scale toward x0, which is feasible.

neldermead.scaletocenter Scale the simplex into the nonlinear inequality constraints, if any. Scale to the centroid of the points which satisfy the constraints. This is Box's method for scaling. It is unsure, since the centroid of the points which satisfy the constraints may not be feasible.

neldermead.termstartup Initialize Kelley's stagnation detection system when normalization is required, by computing kelleyalpha. If the simplex gradient is zero, then use alpha0 as alpha.

neldermead.outputcmd Call the array of user-defined output functions

- neldermead.autorestart Perform an optimization with automatic restart. The loop processes for i = 1 to restartmax + 1. This is because a RE-start is performed after one simulation has been performed, hence the 'RE'.
- neldermead.istorestart Determine if the optimization is to restart using neldermead.isroneill or neldermead.isrkelley depending on the content of the restartdetection element.
- neldermead.isroneill Determine if the optimization is to restart. Use O'Neill method as a criteria for restart. It is an axis-by-axis search for optimality.
- neldermead.isrkelley Determine if the optimization is to restart. Use kelleystagnation as a criteria for restart.
- neldermead.updatesimp Update the initial simplex simplex0 for a restart.
- scaleinconstraints Given a point reference to scale and a reference point which satisfies the constraints, scale the point towards the reference point estimate until it satisfies all the constraints.
- neldermead.costf Call the cost function and return the value. This function is given to the simplex function class as a callback. Input/Output arguments are swapped w.r.t. optimbase.function, so that it matches the requirements of simplex methods.

#### Value

neldermead.startup Return an updated neldermead object this.

neldermead.log Return the neldermead object this.

neldermead.scaletox0 Return an updated simplex.

neldermead.scaletocenter Return an updated simplex.

neldermead.termstartup Return an updated neldermead object this.

neldermead.outputcmd Do not return any data, but execute the output function(s).

neldermead.autorestart Return an updated neldermead object this.

neldermead.istorestart Return a list with the following elements:

this The input neldermead object.

istorestart Set to TRUE if the optimization is to restart, to FALSE otherwise.

neldermead.isroneill Return a list with the following elements:

this The input neldermead object.

**istorestart** Set to TRUE if the optimization is to restart, to FALSE otherwise.

neldermead.isrkelley Return a list with the following elements:

this The input neldermead object.

**istorestart** Set to TRUE if the optimization is to restart, to FALSE otherwise.

neldermead.updatesimp Return an updated neldermead object this.

scaleinconstraints Return a list with the following elements:

this The updated neldermead object.

isscaled TRUE if the procedure has succeeded before boxnbnlloops, FALSE if it has failed.

**p** The scaled parameters.

neldermead.costf Return a list with the following elements:

**f** The value of the cost function.

this The updated neldermead object.

## Author(s)

Author of Scilab neldermead module: Michael Baudin (INRIA - Digiteo) Author of R adaptation: Sebastien Bihorel (<sb.pmlab@gmail.com>)

optimget

Queries an optimization option list

## Description

This function allows to make queries on an existing optimization option list. This list must have been created and updated by the optimset function. The optimget allows to retrieve the value associated with a given key.

# Usage

```
optimget(options = NULL, key = NULL, value = NULL)
```

## Arguments

options A list created or modifies by optimset.

key A single character string, which should be the name of the field in options to

query (case insensitive).

value A default value.

### **Details**

key is matched against the field names of options using grep and a case-insensitive regular expression. If key is not found in options, the function returns NULL. If several matches are found, optimget is stopped.

### Value

Return options\$key if key is found in options. Return value, otherwise.

## Author(s)

```
Author of Scilab neldermead module: Michael Baudin (INRIA - Digiteo)
Author of R adaptation: Sebastien Bihorel (<sb.pmlab@gmail.com>)
```

### See Also

optimset

## Examples

```
opt <- optimset(method='fminsearch')
optimget(opt,'Display')
optimget(opt,'abc','!@')</pre>
```

Default set of optimization options

optimset.method

## Description

This function returns a default set of optimization options for defined 'methods'; optimset.method is called by optimset when a method was provided as input. Currently, the only valid method is 'fminsearch'.

## Usage

```
optimset.method(method = NULL)
```

## Arguments

method

A character string.

## Value

Returns a list with the following fields: Display, FunValCheck, MaxFunEvals, MaxIter, Output-Fcn, PlotFcns, TolFun, and TolX.

## Author(s)

```
Author of Scilab neldermead module: Michael Baudin (INRIA - Digiteo)
Author of R adaptation: Sebastien Bihorel (<sb.pmlab@gmail.com>)
```

## See Also

optimset

### Examples

```
optimset.method('fminsearch')
## Not run: optimset.method('abc')
```

optimset

Configures and returns an optimization data structure.

## Description

This function creates or updates a list which can be used to modify the behaviour of optimization methods. The goal of this function is to manage the options list with a set of fields (for example, 'MaxFunEvals', 'MaxIter', etc...). The user can create a new list with empty fields or create a new structure with default fields which correspond to a particular algorithm. The user can also configure each field and set it to a particular value. Finally, the user passes the list to an optimization function so that the algorithm uses the options configured by the user.

# Usage

optimset(method = NULL, Display = NULL, FunValCheck = NULL, MaxFunEvals = NULL, MaxIter = NULL, OutputFcn = NULL, PlotFcns = NULL, TolFun = NULL, TolX = NULL)

## Arguments

method If provided, the method argument overrides all the others and optimset.method

is called. If the content of method is recognized, a defuault set of options are returned. The only current recognized character string is 'fminsearch'.

Display The verbose level. The default value is 'notify'. The following is a list of

available verbose levels.

'off' The algorithm displays no message at all.

'notify' The algorithm displays message if the termination criteria is not reached at the end of the optimization. This may happen if the maximum number or iterations of the maximum number of function evaluations is reached and warns the user of a convergence problem.

'final' The algorithm displays a message at the end of the optimization, showing the number of iterations, the number of function evaluations and the status of the optimization. This option includes the messages generated by the 'notify' option i.e. warns in case of a convergence problem.

'iter' The algorithm displays a one-line message at each iteration. This option includes the messages generated by the 'notify' option i.e. warns in case of a convergence problem. It also includes the message generated by the 'final' option.

FunValCheck A logical flag to enable the checking of function values.

MaxFunEvals The maximum number of evaluations of the cost function.

MaxIter The maximum number of iterations.

A function which is called at each iteration to print out intermediate state of OutputFcn

the optimization algorithm (for example into a log file).

PlotFcns A function which is called at each iteration to plot the intermediate state of

the optimization algorithm (for example into a 2D graphic).

TolFun The absolute tolerance on function value.

TolX The absolute tolerance on the variable x.

## **Details**

Most optimization algorithms require many algorithmic parameters such as the number of iterations or the number of function evaluations. If these parameters are given to the optimization function as input parameters, this forces both the user and the developper to manage many input parameters. The goal of the optimset function is to simplify the management of input arguments, by gathering all the parameters into a single list.

While the current implementation of the optimset function only supports the fminsearch function, it is designed to be extended to as many optimization function as required. Because all optimization algorithms do not require the same parameters, the data structure aims at remaining flexible. But, most of the time, most parameters are the same from algorithm to algorithm, for example, the tolerance parameters which drive the termination criteria are often the same, even if the termination criteria itself is not the same.

Output and plot functions The 'OutputFcn' and 'PlotFcns' options accept as argument a function (or a list of functions). In the client optimization algorithm, this output or plot function is called back once per iteration. It can be used by the user to display a message in the console, write into a file, etc... The output or plot function is expected to have the following definition:

```
myfun <- function(x, optimValues, state)
```

where the input parameters are:

**x** The current point estimate.

optimValues A list which contains the following fields:

function the number of function evaluations.

**fval** The best function value.

iteration The current iteration number.

**procedure** The type of step performed. This string depends on the specific algorithm (see fminsearch for details).

state the state of the algorithm. The following states are available:

'init' when the algorithm is initializing,

'iter' when the algorithm is performing iterations,

'done' when the algorithm is terminated.

#### Value

Return a list with the following fields: Display, FunValCheck, MaxFunEvals, MaxIter, Output-Fcn, PlotFcns, TolFun, and TolX.

## Author(s)

```
Author of Scilab neldermead module: Michael Baudin (INRIA - Digiteo)
Author of R adaptation: Sebastien Bihorel (<sb.pmlab@gmail.com>)
```

### See Also

```
optimset.method,fminsearch
```

#### Examples

```
optimset()
optimset(Display='iter')
```

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- 13.2 Failing an amicable solution within two (2) months as from their occurrence, and unless emergency proceedings are necessary, the disagreements or disputes shall be referred to the Paris Courts having jurisdiction, by the more diligent Party.

Version 2.0 dated 2006-09-05.