# Package 'pense'

September 13, 2020

```
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      linear regression. The methods are proposed in
      Cohen Freue, G. V., Kepplinger, D., Salibián-Barrera, M., and Smucler, E.
      (2019) <a href="https://projecteuclid.org/euclid.aoas/1574910036">https://projecteuclid.org/euclid.aoas/1574910036</a>>.
      The package implements the extensions and algorithms described in
      Kepplinger, D. (2020) <doi:10.14288/1.0392915>.
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coef.pense\_cvfit

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coef.pense\_cvfit

Extract Coefficient Estimates

# Description

Extract coefficients from a PENSE (or LS-EN) regularization path with hyper-parameters chosen by cross-validation.

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#### Usage

```
## S3 method for class 'pense_cvfit'
coef(
  object,
  lambda = c("min", "se"),
  se_mult = 1,
  sparse = NULL,
  exact = deprecated(),
  correction = deprecated(),
  ...
)
```

# **Arguments**

object	PENSE with cross-validated hyper-parameters to extract coefficients from.
lambda	either a string specifying which penalty level to use ("min" or "se") or a a single numeric value of the penalty parameter. See details.
se_mult	If lambda = "se", the multiple of standard errors to tolerate.
sparse	should coefficients be returned as sparse or dense vectors? Defaults to the sparse argument supplied to pense_cv(). Can also be set to sparse = 'matrix', in which case a sparse matrix is returned instead of a sparse vector.
exact	deprecated. Always gives a warning if lambda is not part of the fitted sequence and coefficients are interpolated.
correction	defunct.
	currently not used.

## **Details**

If lambda = "se" and object contains fitted estimates for every penalization level in the sequence, extract the coefficients of the most parsimonious model with prediction performance statistically indistinguishable from the best model. This is determined to be the model with prediction performance within se\_mult \* cv\_se from the best model.

#### Value

either a numeric vector or a sparse vector of type dsparse Vector of size p+1, depending on the sparse argument. Note: prior to version 2.0.0 sparse coefficients were returned as sparse matrix of type dgCMatrix. To get a sparse matrix, use sparse = 'matrix'.

# See Also

```
Other functions for extracting components: coef.pense_fit(), predict.pense_cvfit(), predict.pense_fit(), residuals.pense_cvfit(), residuals.pense_fit()
```

# **Examples**

```
# Compute the PENSE regularization path for Freeny's revenue data
# (see ?freeny)
data(freeny)
x <- as.matrix(freeny[ , 2:5])
regpath <- pense(x, freeny$y, alpha = 0.5)</pre>
```

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coef.pense\_fit

Extract Coefficient Estimates

# **Description**

Extract coefficients from a PENSE (or LS-EN) regularization path fitted by pense() or elnet().

## Usage

```
## $3 method for class 'pense_fit'
coef(
  object,
  lambda,
  sparse = NULL,
  exact = deprecated(),
  correction = deprecated(),
  ...
)
```

# **Arguments**

object PENSE regularization path to extract coefficients from.

lambda a single value of the penalty parameter.

sparse should coefficients be returned as sparse or dense vectors? Defaults to the

sparse argument supplied to pense(). Can also be set to sparse = 'matrix',

in which case a sparse matrix is returned instead of a sparse vector.

exact defunct Always gives a warning if lambda is not part of the fitted sequence and

hence coefficients are interpolated.

correction defunct.

... currently not used.

# Value

either a numeric vector or a sparse vector of type dsparse Vector of size p+1, depending on the sparse argument. Note: prior to version 2.0.0 sparse coefficients were returned as sparse matrix of type dgCMatrix. To get a sparse matrix, use sparse = 'matrix'.

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#### See Also

```
coef.pense_cvfit() for extracting coefficients from a PENSE fit with hyper-parameters chosen
by cross-validation
```

Other functions for extracting components: coef.pense\_cvfit(), predict.pense\_cvfit(), predict.pense\_fit(), residuals.pense\_cvfit(), residuals.pense\_fit()

## **Examples**

```
# Compute the PENSE regularization path for Freeny's revenue data
# (see ?freeny)
data(freeny)
x <- as.matrix(freeny[ , 2:5])</pre>
regpath <- pense(x, freeny$y, alpha = 0.5)
plot(regpath)
# Extract the coefficients at a certain penalization level
coef(regpath, lambda = regpath$lambda[40])
# What penalization level leads to good prediction performance?
cv_results <- pense_cv(x, freeny$y, alpha = 0.5, cv_repl = 2,</pre>
                       cv_k = 4
plot(cv_results, se_mult = 1)
# Extract the coefficients at the penalization level with
# smallest prediction error ...
coef(cv_results)
# ... or at the penalization level with prediction error
# statistically indistinguishable from the minimum.
coef(cv_results, lambda = 'se')
```

consistency\_const

Get the Constant for Consistency for the M-Scale

# **Description**

Get the Constant for Consistency for the M-Scale

#### Usage

```
consistency_const(delta, rho)
```

# **Arguments**

delta desired breakdown point (between 0 and 0.5) rho the name of the chosen  $\rho$  function.

#### Value

consistency constant

# See Also

Other miscellaneous functions: rho\_function()

deprecated\_en\_options Deprecated Options for the EN Algorithms

# Description

## **Deprecated**

# Usage

```
en_options_aug_lars(use_gram = c("auto", "yes", "no"), eps = 1e-12)
en_options_dal(
  maxit = 100,
  eps = 1e-08,
  eta_mult = 2,
  eta_start_numerator = 0.01,
  eta_start,
  preconditioner = c("approx", "none", "diagonal"),
  verbosity = 0
)
```

# **Arguments**

```
use_gram
                  ignored. Should the Gram matrix be pre-computed.
                  ignored. Numeric tolerance for convergence.
eps
                  maximum number of iterations allowed.
maxit
                  multiplier to increase eta at each iteration.
eta_mult
eta_start_numerator
                  if eta_start is missing, it is defined by eta_start = eta_start_numerator
                  /lambda.
                  ignored. The start value for eta.
eta_start
preconditioner ignored. Preconditioner for the numerical solver. If none, a standard solver will
                  be used, otherwise the faster preconditioned conjugate gradient is used.
verbosity
                  ignored.
```

#### **Functions**

```
• en_options_aug_lars: Superseded by en_lars_options().
```

```
• en_options_dal: Superseded by en_dal_options()
```

# See Also

```
Other deprecated functions: enpy(), initest_options(), mstep_options(), pense_options(), pensem()
```

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elnet

Compute the Least Squares (Adaptive) Elastic Net Regularization Path

#### **Description**

Compute least squares EN estimates for linear regression with optional observation weights and penalty loadings.

#### Usage

```
elnet(
  Х,
  у,
  alpha,
  nlambda = 100,
  lambda_min_ratio,
  lambda,
  penalty_loadings,
  weights,
  intercept = TRUE,
  en_algorithm_opts,
  sparse = FALSE,
  eps = 1e-06,
  standardize = TRUE,
  correction = deprecated(),
  xtest = deprecated(),
  options = deprecated()
)
```

#### **Arguments**

x n by p matrix of numeric predictors.y vector of response values of length n.

alpha elastic net penalty mixing parameter with  $0 \le \alpha \le 1$ . alpha = 1 is the LASSO

penalty, and alpha = 0 the Ridge penalty.

nlambda number of penalization levels.

lambda\_min\_ratio

Smallest value of the penalization level as a fraction of the largest level (i.e., the smallest value for which all coefficients are zero). The default depends on the sample size relative to the number of variables and alpha. If more observations than variables are available, the default is 1e-3 \* alpha, otherwise 1e-2 \* alpha.

alpha.

lambda optional user-supplied sequence of penalization levels. If given and not NULL,

nlambda and lambda\_min\_ratio are ignored.

penalty\_loadings

a vector of positive penalty loadings (a.k.a. weights) for different penalization

of each coefficient.

weights a vector of positive observation weights.

intercept include an intercept in the model.

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en\_algorithm\_opts

options for the EN algorithm. See en\_algorithm\_options for details.

sparse use sparse coefficient vectors.

eps numerical tolerance.

standardize standardize variables to have unit variance. Coefficients are always returned in

original scale.

correction defunct. Correction for EN estimates is not supported anymore.

xtest deprecated. Instead, extract coefficients with coef.pense\_fit() and compute

predictions manually.

options deprecated. Use en\_algorithm\_opts instead.

#### **Details**

The elastic net estimator for the linear regression model solves the optimization problem

$$argmin_{\mu,\beta}(1/2n)\sum_{i}w_{i}(y_{i}-\mu-x_{i}'\beta)^{2}+\lambda\sum_{j}0.5(1-\alpha)\beta_{j}^{2}+\alpha l_{j}|\beta_{j}|$$

with observation weights  $w_i$  and penalty loadings  $l_i$ .

#### Value

a list-like object with the following items

lambda the sequence of penalization parameters.

estimates a list of estimates. Each estimate contains the following information:

intercept intercept estimate.

beta beta (slope) estimate.

lambda penalization level at which the estimate is computed.

alpha alpha hyper-parameter at which the estimate is computed.

statuscode if > 0 the algorithm experienced issues when computing the estimate.

status optional status message from the algorithm.

call the original call.

predictions if xtest was given, a matrix of predicted values. Each column corresponds to the predictions from the estimate at the lambda value at the same index.

# See Also

```
pense() for an S-estimate of regression with elastic net penalty.
coef.pense_fit() for extracting coefficient estimates.
plot.pense_fit() for plotting the regularization path.
Other functions for computing non-robust estimates: elnet_cv()
```

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#### **Examples**

```
# Compute the LS-EN regularization path for Freeny's revenue data
# (see ?freeny)
data(freeny)
x <- as.matrix(freeny[ , 2:5])</pre>
regpath <- elnet(x, freeny$y, alpha = 0.75)
plot(regpath)
# Extract the coefficients at a certain penalization level
coef(regpath, lambda = regpath$lambda[5])
# What penalization level leads to good prediction performance?
cv_results <- elnet_cv(x, freeny$y, alpha = 0.75, cv_repl = 10,</pre>
                       cv_k = 4, cv_measure = 'tau')
plot(cv_results, se_mult = 1)
plot(cv_results, se_mult = 1, what = 'coef.path')
# Extract the coefficients at the penalization level with
# smallest prediction error ...
coef(cv_results)
\# ... or at the penalization level with prediction error
\mbox{\tt\#} statistically indistinguishable from the minimum.
coef(cv_results, lambda = 'se')
```

elnet\_cv

Cross-validation for Least-Squares (Adaptive) Elastic Net Estimates

# **Description**

Perform (repeated) K-fold cross-validation for elnet().

#### Usage

```
elnet_cv(
    x,
    y,
    lambda,
    cv_k,
    cv_repl = 1,
    cv_metric = c("rmspe", "tau_size", "mape"),
    fit_all = TRUE,
    cl = NULL,
    ncores = deprecated(),
    ...
```

#### **Arguments**

```
n by p matrix of numeric predictors.
```

y vector of response values of length n.

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lambda optional user-supplied sequence of penalization levels. If given and not NULL, nlambda and lambda\_min\_ratio are ignored. cv\_k number of folds per cross-validation. cv\_repl number of cross-validation replications. cv\_metric either a string specifying the performance metric to use, or a function to evaluate prediction errors in a single CV replication. If a function, it is called with a single numeric vector of prediction errors and must return a scalar number. fit\_all If TRUE, fit the model for all penalization levels. Otherwise, only at penalization level with smallest average CV performance. cl a parallel cluster. Can only be used if ncores = 1, because multi-threading can not be used in parallel R sessions on the same host. deprecated and not used anymore. ncores Arguments passed on to elnet alpha elastic net penalty mixing parameter with  $0 < \alpha < 1$ . alpha = 1 is the LASSO penalty, and alpha = 0 the Ridge penalty. nlambda number of penalization levels. lambda\_min\_ratio Smallest value of the penalization level as a fraction of the largest level (i.e., the smallest value for which all coefficients are zero). The default depends on the sample size relative to the number of variables and

penalty\_loadings a vector of positive penalty loadings (a.k.a. weights) for different penalization of each coefficient.

alpha. If more observations than variables are available, the default is 1e-3

standardize standardize variables to have unit variance. Coefficients are always returned in original scale.

weights a vector of positive observation weights.

intercept include an intercept in the model.

\* alpha, otherwise 1e-2 \* alpha.

sparse use sparse coefficient vectors.

en\_algorithm\_opts options for the EN algorithm. See en\_algorithm\_options for details.

eps numerical tolerance.

xtest deprecated. Instead, extract coefficients with coef.pense\_fit() and compute predictions manually.

options deprecated. Use  $en_algorithm_opts$  instead.

correction defunct. Correction for EN estimates is not supported anymore.

## Details

The built-in CV metrics are

"rmspe" Root mean squared prediction error (default).

"mape" Median absolute prediction error.

"tau\_size"  $\tau$ -size of the prediction error, computed by tau\_size().

#### Value

a list with components:

lambda the sequence of penalization levels.

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cvres data frame of average cross-validated performance.

cv\_replications matrix of cross-validated performance metrics, one column per replication. Rows are in the same order as in cvres.

call the original call.

estimates the estimates fitted on the full data. Same format as returned by elnet().

#### See Also

```
elnet() for computing the LS-EN regularization path without cross-validation.
pense_cv() for cross-validation of S-estimates of regression with elastic net penalty.
coef.pense_cvfit() for extracting coefficient estimates.
plot.pense_cvfit() for plotting the CV performance or the regularization path.
Other functions for computing non-robust estimates: elnet()
```

# **Examples**

```
# Compute the LS-EN regularization path for Freeny's revenue data
# (see ?freeny)
data(freeny)
x <- as.matrix(freeny[ , 2:5])</pre>
regpath <- elnet(x, freeny$y, alpha = 0.75)</pre>
plot(regpath)
# Extract the coefficients at a certain penalization level
coef(regpath, lambda = regpath$lambda[5])
# What penalization level leads to good prediction performance?
cv_results <- elnet_cv(x, freeny$y, alpha = 0.75, cv_repl = 10,</pre>
                       cv_k = 4, cv_measure = 'tau')
plot(cv_results, se_mult = 1)
plot(cv_results, se_mult = 1, what = 'coef.path')
# Extract the coefficients at the penalization level with
# smallest prediction error ...
coef(cv_results)
# ... or at the penalization level with prediction error
# statistically indistinguishable from the minimum.
coef(cv_results, lambda = 'se')
```

enpy

Deprecated: ENPY Initial Estimates for EN S-Estimators

# Description

**Deprecated** Superseded by enpy\_initial\_estimates().

#### Usage

```
enpy(x, y, alpha, lambda, delta, cc, options, en_options)
```

# **Arguments**

X	data matrix with predictors.
У	response vector.
alpha, lambda	EN penalty parameters (NOT adjusted for the number of observations in x).
delta	desired breakdown point of the resulting estimator.
СС	tuning constant for the S-estimator. Default is to chosen based on the breakdown point delta. Should never have to be changed.
options	<b>ignored.</b> Additional options for the initial estimator.
en_options	<b>ignored.</b> Additional options for the EN algorithm.

#### Value

coeff A numeric matrix with one initial coefficient per column

objF A vector of values of the objective function for the respective coefficient

# See Also

```
Other deprecated functions: deprecated_en_options, initest_options(), mstep_options(), pense_options(), pensem()
```

```
enpy_initial_estimates
```

ENPY Initial Estimates for EN S-Estimators

# **Description**

Compute initial estimates for the EN S-estimator using the EN-PY procedure.

# Usage

```
enpy_initial_estimates(
    x,
    y,
    alpha,
    lambda,
    bdp = 0.25,
    cc,
    intercept = TRUE,
    penalty_loadings,
    enpy_opts = enpy_options(),
    mscale_opts = mscale_algorithm_options(),
    eps = 1e-06,
    sparse = FALSE,
    ncores = 1L
)
```

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#### **Arguments**

x n by p matrix of numeric predictors.y vector of response values of length n.

alpha elastic net penalty mixing parameter with  $0 \le \alpha \le 1$ . alpha = 1 is the LASSO

penalty, and alpha = 0 the Ridge penalty.

lambda a vector of positive values of penalization levels.

bdp desired breakdown point of the estimator, between 0 and 0.5.

cc cutoff value for the bisquare rho function. By default, chosen to yield a consis-

tent estimate for the Normal distribution.

intercept include an intercept in the model.

penalty\_loadings

a vector of positive penalty loadings (a.k.a. weights) for different penalization

of each coefficient. Only allowed for alpha > 0.

enpy\_opts options for the EN-PY algorithm, created with the enpy\_options() function.

mscale\_opts options for the M-scale estimation. See mscale\_algorithm\_options() for de-

tails.

eps numerical tolerance.

sparse use sparse coefficient vectors.

ncores number of CPU cores to use in parallel. By default, only one CPU core is used.

May not be supported on your platform, in which case a warning is given.

## **Details**

If these manually computed initial estimates are intended as starting points for pense(), they are by default *shared* for all penalization levels. To restrict the use of the initial estimates to the penalty level they were computed for, use as\_starting\_point(..., specific = TRUE). See as\_starting\_point() for details.

## References

Cohen Freue, G.V.; Kepplinger, D.; Salibián-Barrera, M.; Smucler, E. Robust elastic net estimators for variable selection and identification of proteomic biomarkers. *Ann. Appl. Stat.* **13** (2019), no. 4, 2065–2090 doi: 10.1214/19AOAS1269

#### See Also

Other functions for initial estimates: prinsens(), starting\_point()

enpy\_options Options for the ENPY Algorithm

# **Description**

Additional control options for the elastic net Peña-Yohai procedure.

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#### Usage

```
enpy_options(
  max_it = 10,
  keep_psc_proportion = 0.5,
  en_algorithm_opts,
  keep_residuals_measure = c("threshold", "proportion"),
  keep_residuals_proportion = 0.5,
  keep_residuals_threshold = 2,
  retain_best_factor = 2,
  retain_max = 500
)
```

## Arguments

max\_it maximum number of EN-PY iterations.

keep\_psc\_proportion

how many observations should to keep based on the Principal Sensitivity Components.

en\_algorithm\_opts

options for the LS-EN algorithm. See en\_algorithm\_options for details.

keep\_residuals\_measure

how to determine what observations to keep, based on their residuals. If proportion, a fixed number of observations is kept. If threshold, only observations with residuals below the threshold are kept.

keep\_residuals\_proportion

proportion of observations to kept based on their residuals.

keep\_residuals\_threshold

only observations with (standardized) residuals less than this threshold are kept.

retain\_best\_factor

only keep candidates that are within this factor of the best candidate. If  $\leq 1$ , only keep candidates from the last iteration.

retain\_max ı

maximum number of candidates, i.e., only the best retain\_max candidates are retained.

# Details

The EN-PY procedure for computing initial estimates iteratively cleans the data of observations with possibly outlying residual or high leverage. Least-squares elastic net (LS-EN) estimates are computed on the possibly clean subsets. At each iteration, the Principal Sensitivity Components are computed to remove observations with potentially high leverage. Among all the LS-EN estimates, the estimate with smallest M-scale of the residuals is selected. Observations with largest residual for the selected estimate are removed and the next iteration is started.

## Value

options for the ENPY algorithm.

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en\_admm\_options

Use the ADMM Elastic Net Algorithm

#### **Description**

Use the ADMM Elastic Net Algorithm

## Usage

```
en_admm_options(max_it = 1000, step_size, acceleration = 1)
```

#### **Arguments**

max\_it maximum number of iterations.

step\_size step size for the algorithm.

acceleration acceleration factor for linearized ADMM.

# Value

options for the ADMM EN algorithm.

## See Also

```
Other EN algorithms: en_dal_options(), en_lars_options()
```

# Description

The package supports different algorithms to compute the EN estimate for weighted LS loss functions. Each algorithm has certain characteristics that make it useful for some problems. To select a specific algorithm and adjust the options, use any of the en\_\*\*\*\_options functions.

#### **Details**

- en\_lars\_options(): Use the tuning-free LARS algorithm. This computes *exact* (up to numerical errors) solutions to the EN-LS problem. It is not iterative and therefore can not benefit from approximate solutions, but in turn guarantees that a solution will be found.
- en\_admm\_options(): Use an iterative ADMM-type algorithm which needs O(np) operations per iteration and converges sub-linearly.
- en\_dal\_options(): Use the iterative Dual Augmented Lagrangian (DAL) method. DAL needs  $O(n^3p^2)$  operations per iteration, but converges exponentially.

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en\_dal\_options

Use the DAL Elastic Net Algorithm

# Description

Use the DAL Elastic Net Algorithm

# Usage

```
en_dal_options(
  max_it = 100,
  max_inner_it = 100,
  eta_multiplier = 2,
  eta_start_conservative = 0.01,
  eta_start_aggressive = 1,
  lambda_relchange_aggressive = 0.25
)
```

## **Arguments**

```
max_it maximum number of (outer) iterations.

max_inner_it maximum number of (inner) iterations in each outer iteration.

eta_multiplier multiplier for the barrier parameter. In each iteration, the barrier must be more restrictive (i.e., the multiplier must be > 1).

eta_start_conservative conservative initial barrier parameter. This is used if the previous penalty is undefined or too far away.

eta_start_aggressive aggressive initial barrier parameter. This is used if the previous penalty is close.

lambda_relchange_aggressive how close must the lambda parameter from the previous penalty term be to use an aggressive initial barrier parameter (i.e., what constitutes "too far").
```

#### Value

options for the DAL EN algorithm.

# See Also

```
Other EN algorithms: en_admm_options(), en_lars_options()
```

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en\_lars\_options

Use the LARS Elastic Net Algorithm

## **Description**

Use the LARS Elastic Net Algorithm

#### Usage

```
en_lars_options()
```

#### See Also

```
Other EN algorithms: en_admm_options(), en_dal_options()
```

initest\_options

Deprecated Options for Initial Estimates

# **Description**

**Deprecated** Superseded by enpy\_options().

#### Usage

```
initest_options(
  keep_solutions = 5,
  psc_method = c("exact", "rr"),
  maxit = 10,
  maxit_pense_refinement = 5,
  eps = 1e-06,
  psc_keep = 0.5,
  resid_keep_method = c("proportion", "threshold"),
  resid_keep_thresh = 2,
  mscale_eps = 1e-08,
  mscale_maxit = 200
)
```

# **Arguments**

keep\_solutions how many initial estimates should be kept to perform full PENSE iterations?

psc\_method The method to use for computing the principal sensitivity components. See

details for the possible choices.

maxit maximum number of refinement iterations.

maxit\_pense\_refinement

**ignored.** Maximum number of PENSE iterations to refine initial estimator.

eps **ignored.** Numeric tolerance for convergence.

psc\_keep proportion of observations to keep based on the PSC scores.

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resid\_keep\_method

How to clean the data based on large residuals. If "proportion", observations with the smallest resid\_keep\_prop residuals will be retained. If "threshold", all observations with scaled residuals smaller than the threshold resid\_keep\_thresh will be retained.

resid\_keep\_prop, resid\_keep\_thresh

proportion or threshold for observations to keep based on their residual.

mscale\_eps, mscale\_maxit

ignored. Maximum number of iterations and numeric tolerance for the M-scale.

# See Also

Other deprecated functions: deprecated\_en\_options, enpy(), mstep\_options(), pense\_options(), pensem()

mloc

Compute the M-estimate of Location

#### **Description**

Compute the M-estimate of location using an auxiliary estimate of the scale.

# Usage

```
mloc(x, scale, rho, cc, opts = mscale_algorithm_options())
```

# **Arguments**

Х	numeric values. Missing values are verbosely ignored.
scale	scale of the x values. If omitted, uses the mad().
rho	the $\rho$ function to use. See rho_function() for available functions.
СС	value of the tuning constant for the chosen $\rho$ function. By default, chosen to achieve 95% efficiency under the Normal distribution.
opts	a list of options for the M-estimating algorithm, see mscale_algorithm_options() for details

# Value

a single numeric value, the M-estimate of location.

#### See Also

Other functions to compute robust estimates of location and scale: mlocscale(), mscale(), tau\_size()

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mlocscale

Compute the M-estimate of Location and Scale

# **Description**

Simultaneous estimation of the location and scale by means of M-estimates.

# Usage

```
mlocscale(
    x,
    bdp = 0.25,
    scale_cc = consistency_const(bdp, "bisquare"),
    location_rho,
    location_cc,
    opts = mscale_algorithm_options()
)
```

#### **Arguments**

# Value

a vector with 2 elements, the M-estimate of location and the M-scale estimate.

# See Also

Other functions to compute robust estimates of location and scale: mloc(), mscale(), tau\_size()

# Description

Additional options for the MM algorithm to compute EN S- and M-estimates.

20 mscale

## Usage

```
mm_algorithm_options(
  max_it = 500,
  tightening = c("adaptive", "exponential", "none"),
  tightening_steps = 10,
  en_algorithm_opts
)
```

# Arguments

max\_it maximum number of iterations.

tightening how to make inner iterations more precise as the algorithm approaches a local minimum.

tightening\_steps

for *adaptive* tightening strategy, how often to tighten until the desired tolerance is attained.

en\_algorithm\_opts

options for the inner LS-EN algorithm. See en\_algorithm\_options for details.

#### Value

options for the MM algorithm.

mscale

Compute the M-Scale of Centered Values

# **Description**

Compute the M-scale without centering the values.

# Usage

```
mscale(
    x,
    bdp = 0.25,
    cc = consistency_const(bdp, "bisquare"),
    opts = mscale_algorithm_options(),
    delta = deprecated(),
    rho = deprecated(),
    eps = deprecated(),
    maxit = deprecated()
```

## **Arguments**

x numeric values. Missing values are verbosely ignored.

bdp desired breakdown point (between 0 and 0.5).

cc cutoff value for the bisquare rho function. By default, chosen to yield a consis-

tent estimate for the Normal distribution.

opts a list of options for the M-scale estimation algorithm, see mscale\_algorithm\_options()

for details.

delta deprecated. Use bpd instead.

rho, eps, maxit deprecated. Instead set control options for the algorithm with the opts argu-

ments.

#### Value

the M-estimate of scale.

#### See Also

Other functions to compute robust estimates of location and scale: mlocscale(), mloc(), tau\_size()

```
mscale_algorithm_options
```

Options for the M-scale Estimation Algorithm

## **Description**

Options for the M-scale Estimation Algorithm

## Usage

```
mscale_algorithm_options(max_it = 200, eps = 1e-08)
```

# **Arguments**

max\_it maximum number of iterations.

eps numerical tolerance to check for convergence.

#### Value

options for the M-scale estimation algorithm.

mstep\_options

Deprecated Additional Options for the Penalized EN MM-estimator

# **Description**

**Deprecated** Superseded by mm\_algorithm\_options() and options supplied directly to pense().

# Usage

```
mstep_options(
  cc = 3.44,
  maxit = 1000,
  eps = 1e-06,
  adjust_bdp = FALSE,
  verbosity = 0,
  en_correction = TRUE
)
```

## **Arguments**

cc ignored. Tuning constant for the M-estimator.

maxit maximum number of iterations allowed.

eps ignored. Numeric tolerance for convergence.

adjust\_bdp ignored. Should the breakdown point be adjusted based on the effective degrees of freedom?

verbosity ignored. Verbosity of the algorithm.

en\_correction ignored. Should the corrected EN estimator be used to choose the optimal lambda with CV. If TRUE, as by default, the estimator is "bias corrected".

#### See Also

Other deprecated functions: deprecated\_en\_options, enpy(), initest\_options(), pense\_options(), pensem()

pense

Compute (Adaptive) Elastic Net S-Estimates of Regression

# Description

Compute elastic net S-estimates (PENSE estimates) along a grid of penalization levels with optional penalty loadings for adaptive elastic net.

# Usage

```
pense(
  Х,
  у,
  alpha,
  nlambda = 50,
  nlambda_enpy = 10,
  lambda,
  lambda_min_ratio,
  enpy_lambda,
  penalty_loadings,
  intercept = TRUE,
  bdp = 0.25,
  add_zero_based = TRUE,
  enpy_specific = FALSE,
  other_starts,
  eps = 1e-06,
  explore_solutions = 10,
  explore_tol = 0.1,
  max_solutions = 10,
  comparison_tol = sqrt(eps),
  sparse = FALSE,
  ncores = 1,
  standardize = TRUE,
  algorithm_opts = mm_algorithm_options(),
```

```
mscale_opts = mscale_algorithm_options(),
enpy_opts = enpy_options(),
cv_k = deprecated(),
cv_objective = deprecated(),
...
)
```

#### **Arguments**

x n by p matrix of numeric predictors.

y vector of response values of length n.

alpha elastic net penalty mixing parameter with  $0 \le \alpha \le 1$ . alpha = 1 is the LASSO

penalty, and alpha = 0 the Ridge penalty.

nlambda number of penalization levels.

nlambda\_enpy number of penalization levels where the EN-PY initial estimate is computed.

lambda optional user-supplied sequence of penalization levels. If given and not NULL,

nlambda and lambda\_min\_ratio are ignored.

lambda\_min\_ratio

Smallest value of the penalization level as a fraction of the largest level (i.e., the smallest value for which all coefficients are zero). The default depends on the sample size relative to the number of variables and alpha. If more observations than variables are available, the default is 1e-3 \* alpha, otherwise 1e-2 \* alpha

alpha.

enpy\_lambda optional user-supplied sequence of penalization levels at which EN-PY initial

estimates are computed. If given and not NULL, nlambda\_enpy is ignored.

penalty\_loadings

a vector of positive penalty loadings (a.k.a. weights) for different penalization

of each coefficient. Only allowed for alpha > 0.

intercept include an intercept in the model.

bdp desired breakdown point of the estimator, between 0 and 0.5.

add\_zero\_based also consider the 0-based regularization path. See details for a description.

enpy\_specific use the EN-PY initial estimates only at the penalization level they are computed

for. See details for a description.

other\_starts a list of other staring points, created by starting\_point(). If the output of

enpy\_initial\_estimates() is given, the starting points will be *shared* among all penalization levels. Note that if a the starting point is *specific* to a penalization level, this penalization level is added to the grid of penalization levels (either the manually specified grid in lambda or the automatically generated grid of size

nlambda). If standardize = TRUE, the starting points are also scaled.

eps numerical tolerance.

explore\_solutions

number of solutions to compute up to the desired precision eps.

explore\_tol numerical tolerance for exploring possible solutions. Should be (much) looser

than eps to be useful.

max\_solutions only retain up to max\_solutions unique solutions per penalization level.

comparison\_tol numeric tolerance to determine if two solutions are equal. The comparison is

first done on the absolute difference in the value of the objective function at the solution If this is less than comparison\_tol, two solutions are deemed equal if the squared difference of the intercepts is less than comparison\_tol and the

squared  $L_2$  norm of the difference vector is less than comparison\_tol.

sparse use sparse coefficient vectors.

ncores number of CPU cores to use in parallel. By default, only one CPU core is used.

May not be supported on your platform, in which case a warning is given.

standardize logical flag to standardize the x variables prior to fitting the PENSE estimates.

Coefficients are always returned on the original scale. This can fail for variables with a large proportion of a single value (e.g., zero-inflated data). In this case, either compute with standardize = FALSE or standardize the data manually.

algorithm\_opts options for the MM algorithm to compute the estimates. See mm\_algorithm\_options()

for details.

mscale\_opts options for the M-scale estimation. See mscale\_algorithm\_options() for de-

tails.

enpy\_opts options for the ENPY initial estimates, created with the enpy\_options() func-

tion. See enpy\_initial\_estimates() for details.

cv\_k, cv\_objective

deprecated and ignored. See pense\_cv() for estimating prediction performance

via cross-validation.

... ignored. See the section on deprecated parameters below.

#### Value

a list-like object with the following items

lambda the sequence of penalization levels.

estimates a list of estimates. Each estimate contains the following information:

intercept intercept estimate.

beta beta (slope) estimate.

lambda penalization level at which the estimate is computed.

alpha alpha hyper-parameter at which the estimate is computed.

objf\_value value of the objective function at the solution.

statuscode if > 0 the algorithm experienced issues when computing the estimate.

status optional status message from the algorithm.

call the original call.

#### **Strategies for Using Starting Points**

The function supports several different strategies to compute, and use the provided starting points for optimizing the PENSE objective function.

Starting points are computed internally but can also be supplied via other\_starts. By default, starting points are computed internally by the EN-PY procedure for penalization levels supplied in enpy\_lambda (or the automatically generated grid of length nlambda\_enpy). By default, starting points computed by the EN-PY procedure are *shared* for all penalization levels in lambda (or the automatically generated grid of length nlambda). If the starting points should be *specific* to the penalization level the starting points' penalization level, set the enpy\_specific argument to TRUE.

In addition to EN-PY initial estimates, the algorithm can also use the "0-based" strategy if add\_zero\_based = TRUE (by default). Here, the 0-vector is used to start the optimization at the largest penalization level in lambda. At subsequent penalization levels, the solution at the previous penalization level is also used as starting point.

At every penalization level, all starting points are explored using the loose numerical tolerance explore\_tol. Only the best explore\_solutions are computed to the stringent numerical tolerance eps. Finally, only the best max\_solutions are retained and carried forward as starting points for the subsequent penalization level.

#### **Deprecated Arguments**

Starting with version 2.0.0, cross-validation is performed by separate function pense\_cv(). Arguments related cross-validation cause an error when supplied to pense(). Furthermore, the following arguments are deprecated as of version 2.0.0: initial, warm\_reset, cl, options, init\_options, en\_options. If pense() is called with any of these arguments, warnings detail how to replace them.

#### See Also

```
pense_cv() for selecting hyper-parameters via cross-validation.
coef.pense_fit() for extracting coefficient estimates.
plot.pense_fit() for plotting the regularization path.
Other functions to compute robust estimates: regmest()
```

# **Examples**

```
# Compute the PENSE regularization path for Freeny's revenue data
# (see ?freeny)
data(freeny)
x <- as.matrix(freeny[ , 2:5])</pre>
regpath <- pense(x, freeny$y, alpha = 0.5)
plot(regpath)
# Extract the coefficients at a certain penalization level
coef(regpath, lambda = regpath$lambda[40])
# What penalization level leads to good prediction performance?
cv_results <- pense_cv(x, freeny$y, alpha = 0.5, cv_repl = 2,</pre>
                       cv_k = 4
plot(cv_results, se_mult = 1)
# Extract the coefficients at the penalization level with
# smallest prediction error ...
coef(cv_results)
# ... or at the penalization level with prediction error
# statistically indistinguishable from the minimum.
coef(cv_results, lambda = 'se')
```

pensem

Deprecated Alias of pensem\_cv

## **Description**

```
pensem() is a deprecated alias for pensem_cv().
```

#### Usage

```
pensem(x, ...)
```

# **Arguments**

```
x either a numeric matrix of predictor values, or a cross-validated PENSE fit from pense_cv().
```

ignored. See the section on deprecated parameters below.

#### See Also

```
Other deprecated functions: deprecated_en_options, enpy(), initest_options(), mstep_options(), pense_options()
```

pensem\_cv

Compute Penalized Elastic Net M-Estimates from PENSE

# **Description**

This is a convenience wrapper around pense\_cv() and regmest\_cv(), for the common use-case of computing a highly-robust S-estimate followed by a more efficient M-estimate using the scale of the residuals from the S-estimate.

# Usage

```
pensem_cv(x, ...)
## Default S3 method:
pensem_cv(
    x,
    y,
    alpha = 0.5,
    nlambda = 50,
    lambda_min_ratio,
    lambda_m,
    lambda_s,
    standardize = TRUE,
    penalty_loadings,
    intercept = TRUE,
    bdp = 0.25,
    ncores = 1,
```

```
sparse = FALSE,
  eps = 1e-06,
  cc = 4.7,
  cv_k = 5,
  cv_repl = 1,
  cl = NULL,
  cv_metric = c("tau_size", "mape", "rmspe"),
  add_zero_based = TRUE,
  explore_solutions = 10,
  explore_tol = 0.1,
  max_solutions = 10,
  fit_all = TRUE,
  comparison_tol = sqrt(eps),
  algorithm_opts = mm_algorithm_options(),
  mscale_opts = mscale_algorithm_options(),
  nlambda_enpy = 10,
  enpy_opts = enpy_options(),
)
## S3 method for class 'pense_cvfit'
pensem_cv(
  х,
  scale,
  alpha,
  nlambda = 50,
  lambda_min_ratio,
  lambda_m,
  standardize = TRUE,
  penalty_loadings,
  intercept = TRUE,
  bdp = 0.25,
  ncores = 1,
  sparse = FALSE,
  eps = 1e-06,
  cc = 4.7,
  cv_k = 5,
  cv_repl = 1,
  cl = NULL,
  cv_metric = c("tau_size", "mape", "rmspe"),
  add_zero_based = TRUE,
  explore_solutions = 10,
  explore_tol = 0.1,
  max_solutions = 10,
  fit_all = TRUE,
  comparison_tol = sqrt(eps),
  algorithm_opts = mm_algorithm_options(),
  mscale_opts = mscale_algorithm_options(),
  x_train,
  y_train,
```

## **Arguments**

x either a numeric matrix of predictor values, or a cross-validated PENSE fit from

pense\_cv().

... ignored. See the section on deprecated parameters below.

y vector of response values of length n.

alpha elastic net penalty mixing parameter with  $0 \le \alpha \le 1$ . alpha = 1 is the LASSO

penalty, and alpha = 0 the Ridge penalty.

nlambda number of penalization levels.

lambda\_min\_ratio

Smallest value of the penalization level as a fraction of the largest level (i.e., the smallest value for which all coefficients are zero). The default depends on the sample size relative to the number of variables and alpha. If more observations than variables are available, the default is 1e-3 \* alpha, otherwise 1e-2 \* alpha

alpha.

lambda\_m, lambda\_s

optional user-supplied sequence of penalization levels for the S- and M-estimates. If given and not NULL, nlambda and lambda\_min\_ratio are ignored for the re-

spective estimate (S and/or M).

standardize logical flag to standardize the x variables prior to fitting the PENSE estimates.

Coefficients are always returned on the original scale. This can fail for variables with a large proportion of a single value (e.g., zero-inflated data). In this case, either compute with standardize = FALSE or standardize the data manually.

penalty\_loadings

a vector of positive penalty loadings (a.k.a. weights) for different penalization

of each coefficient. Only allowed for alpha > 0.

intercept include an intercept in the model.

bdp desired breakdown point of the estimator, between 0 and 0.5.

ncores number of CPU cores to use in parallel. By default, only one CPU core is used.

May not be supported on your platform, in which case a warning is given.

sparse use sparse coefficient vectors.

eps numerical tolerance.

cc cutoff constant for Tukey's bisquare  $\rho$  function in the M-estimation objective

function.

cv\_k number of folds per cross-validation.cv\_repl number of cross-validation replications.

cl a parallel cluster. Can only be used if ncores = 1, because multi-threading can

not be used in parallel R sessions on the same host.

cv\_metric either a string specifying the performance metric to use, or a function to evaluate

prediction errors in a single CV replication. If a function, it is called with a single

numeric vector of prediction errors and must return a scalar number.

add\_zero\_based also consider the 0-based regularization path. See details for a description. explore\_solutions

number of solutions to compute up to the desired precision eps.

explore\_tol numerical tolerance for exploring possible solutions. Should be (much) looser

than eps to be useful.

max\_solutions only retain up to max\_solutions unique solutions per penalization level.

fit\_all If TRUE, fit the model for all penalization levels. Otherwise, only at penalization level with smallest average CV performance. comparison\_tol numeric tolerance to determine if two solutions are equal. The comparison is first done on the absolute difference in the value of the objective function at the solution If this is less than comparison\_tol, two solutions are deemed equal if the squared difference of the intercepts is less than comparison\_tol and the squared  $L_2$  norm of the difference vector is less than comparison\_tol. options for the MM algorithm to compute the estimates. See mm\_algorithm\_options() algorithm\_opts for details. mscale\_opts options for the M-scale estimation. See mscale\_algorithm\_options() for denlambda\_enpy number of penalization levels where the EN-PY initial estimate is computed. enpy\_opts options for the ENPY initial estimates, created with the enpy\_options() function. See enpy\_initial\_estimates() for details. initial scale estimate to use in the M-estimation. By default the S-scale from the scale PENSE fit is used. x\_train, y\_train override arguments x and y as provided in the call to pense\_cv(). This is useful if the arguments in the pense\_cv() call are not available in the current environ-

#### Value

an object of cross-validated regularized M-estimates as returned from regmest\_cv().

#### See Also

```
pense_cv() to compute the starting S-estimate.
Other functions to compute robust estimates with CV: pense_cv(), regmest_cv()
```

pense\_cv

Cross-validation for (Adaptive) PENSE Estimates

## **Description**

Perform (repeated) K-fold cross-validation for pense(). adapense\_cv() is a convenience wrapper to compute adaptive PENSE estimates.

#### **Usage**

```
pense_cv(
    x,
    y,
    standardize = TRUE,
    lambda,
    cv_k,
    cv_repl = 1,
    cv_metric = c("tau_size", "mape", "rmspe"),
    fit_all = TRUE,
```

ment.

```
cl = NULL,
...
)
adapense_cv(x, y, alpha, alpha_preliminary = 0, exponent = 1, ...)
```

# **Arguments**

x n by p matrix of numeric predictors.

y vector of response values of length n.

standardize whether to standardize the x variables prior to fitting the PENSE estimates. Can

also be set to "cv\_only", in which case the input data is not standardized, but the training data in the CV folds is scaled to match the scaling of the input data. Coefficients are always returned on the original scale. This can fail for variables with a large proportion of a single value (e.g., zero-inflated data). In this case,

either compute with standardize = FALSE or standardize the data manually.

lambda optional user-supplied sequence of penalization levels. If given and not NULL,

nlambda and lambda\_min\_ratio are ignored.

cv\_k number of folds per cross-validation.

cv\_repl number of cross-validation replications.

cv\_metric either a string specifying the performance metric to use, or a function to evaluate

prediction errors in a single CV replication. If a function, it is called with a single

numeric vector of prediction errors and must return a scalar number.

fit\_all If TRUE, fit the model for all penalization levels. Otherwise, only at penalization

level with smallest average CV performance.

cl a parallel cluster. Can only be used if ncores = 1, because multi-threading can

not be used in parallel R sessions on the same host.

... Arguments passed on to pense

nlambda number of penalization levels.

lambda\_min\_ratio Smallest value of the penalization level as a fraction of the largest level (i.e., the smallest value for which all coefficients are zero). The default depends on the sample size relative to the number of variables and alpha. If more observations than variables are available, the default is 1e-3 \* alpha, otherwise 1e-2 \* alpha.

nlambda\_enpy number of penalization levels where the EN-PY initial estimate is computed.

penalty\_loadings a vector of positive penalty loadings (a.k.a. weights) for different penalization of each coefficient. Only allowed for alpha > 0.

enpy\_lambda optional user-supplied sequence of penalization levels at which EN-PY initial estimates are computed. If given and not NULL, nlambda\_enpy is ignored.

other\_starts a list of other staring points, created by starting\_point(). If the output of enpy\_initial\_estimates() is given, the starting points will be *shared* among all penalization levels. Note that if a the starting point is *specific* to a penalization level, this penalization level is added to the grid of penalization levels (either the manually specified grid in lambda or the automatically generated grid of size nlambda). If standardize = TRUE, the starting points are also scaled.

intercept include an intercept in the model.

bdp desired breakdown point of the estimator, between 0 and 0.5.

eps numerical tolerance.

explore\_solutions number of solutions to compute up to the desired precision eps.

explore\_tol numerical tolerance for exploring possible solutions. Should be (much) looser than eps to be useful.

max\_solutions only retain up to max\_solutions unique solutions per penalization level.

comparison\_tol numeric tolerance to determine if two solutions are equal. The comparison is first done on the absolute difference in the value of the objective function at the solution If this is less than comparison\_tol, two solutions are deemed equal if the squared difference of the intercepts is less than comparison\_tol and the squared  $L_2$  norm of the difference vector is less than comparison\_tol.

add\_zero\_based also consider the 0-based regularization path. See details for a description.

enpy\_specific use the EN-PY initial estimates only at the penalization level they are computed for. See details for a description.

sparse use sparse coefficient vectors.

ncores number of CPU cores to use in parallel. By default, only one CPU core is used. May not be supported on your platform, in which case a warning is given.

algorithm\_opts options for the MM algorithm to compute the estimates. See mm\_algorithm\_options() for details.

mscale\_opts options for the M-scale estimation. See mscale\_algorithm\_options() for details.

enpy\_opts options for the ENPY initial estimates, created with the enpy\_options() function. See enpy\_initial\_estimates() for details.

cv\_objective deprecated and ignored. See pense\_cv() for estimating prediction performance via cross-validation.

alpha

elastic net penalty mixing parameter with  $0 \le \alpha \le 1$ . alpha = 1 is the LASSO penalty, and alpha = 0 the Ridge penalty.

alpha\_preliminary

alpha parameter for the preliminary estimate.

exponent

the exponent for computing the penalty loadings based on the preliminary estimate.

## Details

The built-in CV metrics are

"tau\_size"  $\tau$ -size of the prediction error, computed by tau\_size() (default).

"mape" Median absolute prediction error.

"rmspe" Root mean squared prediction error.

adapense\_cv() is a convenience wrapper which performs 3 steps:

- 1. compute preliminary estimates via pense\_cv(...,alpha = alpha\_preliminary),
- 2. computes the penalty loadings from the estimate beta with best prediction performance by adapense\_loadings = 1 / abs(beta)^exponent, and
- 3. compute the adaptive PENSE estimates via pense\_cv(...,penalty\_loadings = adapense\_loadings).

#### Value

```
a list with components:

lambda the sequence of penalization levels.

cvres data frame of average cross-validated performance.

cv_replications matrix of cross-validated performance metrics, one column per replication.

Rows are in the same order as in cvres.

call the original call.

estimates the estimates fitted on the full data. Same format as returned by pense().

the object returned by adapense_cv() has additional components

preliminary the CV results for the preliminary estimate.

penalty_loadings the penalty loadings used for the adaptive PENSE estimate.
```

#### See Also

```
pense() for computing regularized S-estimates without cross-validation.
coef.pense_cvfit() for extracting coefficient estimates.
plot.pense_cvfit() for plotting the CV performance or the regularization path.
Other functions to compute robust estimates with CV: pensem_cv(), regmest_cv()
Other functions to compute robust estimates with CV: pensem_cv(), regmest_cv()
```

#### **Examples**

```
# Compute the adaptive PENSE regularization path for Freeny's
# revenue data (see ?freeny)
data(freeny)
x <- as.matrix(freeny[ , 2:5])</pre>
## Either use the convenience function directly ...
ada_convenience <- adapense_cv(x, freeny$y, alpha = 0.5,
                                cv_repl = 2, cv_k = 4)
## ... or compute the steps manually:
# Step 1: Compute preliminary estimates with CV
preliminary_estimate <- pense_cv(x, freeny$y, alpha = 0,</pre>
                                  cv_repl = 2, cv_k = 4)
plot(preliminary_estimate, se_mult = 1)
# Step 2: Use the coefficients with best prediction performance
# to define the penality loadings:
prelim_coefs <- coef(preliminary_estimate, lambda = 'min')</pre>
pen_loadings <- 1 / abs(prelim_coefs[-1])</pre>
# Step 3: Compute the adaptive PENSE estimates and estimate
# their prediction performance.
ada_manual <- pense_cv(x, freeny$y, alpha = 0.5, cv_repl = 2,
                       cv_k = 4, penalty_loadings = pen_loadings)
# Visualize the prediction performance and coefficient path of
# the adaptive PENSE estimates (manual vs. automatic)
def.par <- par(no.readonly = TRUE)</pre>
```

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```
layout(matrix(1:4, ncol = 2, byrow = TRUE))
plot(ada_convenience$preliminary)
plot(preliminary_estimate)
plot(ada_convenience)
plot(ada_manual)
par(def.par)
```

pense\_options

Deprecated Additional Options for the Penalized EN S-estimator

# **Description**

**Deprecated** Superseded by mm\_algorithm\_options() and options supplied directly to pense().

#### Usage

```
pense_options(
  delta = 0.25,
  maxit = 1000,
  eps = 1e-06,
  mscale_eps = 1e-08,
  mscale_maxit = 200,
  verbosity = 0,
  cc = NULL,
  en_correction = TRUE
)
```

# Arguments

delta desired breakdown point of the resulting estimator.

maxit maximum number of iterations allowed.

eps numeric tolerance for convergence.

mscale\_eps, mscale\_maxit

maximum number of iterations and numeric tolerance for the M-scale.

verbosity **ignored.** Verbosity of the algorithm.

cc **ignored.** Tuning constant for the S-estimator. Default is to chosen based on the

breakdown point delta. Should never have to be changed.

en\_correction ignored. Should the corrected EN estimator be used to choose the optimal

lambda with CV. If TRUE, as by default, the estimator is "bias corrected".

#### See Also

Other deprecated functions: deprecated\_en\_options, enpy(), initest\_options(), mstep\_options(), pensem()

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plot.pense\_cvfit

Plot Method for Penalized Estimates With Cross-Validation

# **Description**

Plot the cross-validation performance or the coefficient path for fitted penalized elastic net S- or LS-estimates of regression.

## Usage

```
## S3 method for class 'pense_cvfit'
plot(x, what = c("cv", "coef.path"), se_mult = 1, ...)
```

#### **Arguments**

x fitted estimates with cross-validation information.
 what plot either the CV performance or the coefficient path.
 se\_mult if plotting CV performance, multiplier of the estimated SE.
 currently ignored.

#### See Also

Other functions for plotting and printing: plot.pense\_fit(), prediction\_performance(), summary.pense\_cvfit()

# **Examples**

```
# Compute the PENSE regularization path for Freeny's revenue data
# (see ?freeny)
data(freeny)
x <- as.matrix(freeny[ , 2:5])</pre>
regpath <- pense(x, freeny$y, alpha = 0.5)</pre>
plot(regpath)
# Extract the coefficients at a certain penalization level
coef(regpath, lambda = regpath$lambda[40])
# What penalization level leads to good prediction performance?
cv_results <- pense_cv(x, freeny$y, alpha = 0.5, cv_repl = 2,</pre>
                        cv_k = 4
plot(cv_results, se_mult = 1)
# Extract the coefficients at the penalization level with
# smallest prediction error ...
coef(cv_results)
# ... or at the penalization level with prediction error
\mbox{\tt\#} statistically indistinguishable from the minimum.
coef(cv_results, lambda = 'se')
```

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plot.pense\_fit

Plot Method for Penalized Estimates

# **Description**

Plot the coefficient path for fitted penalized elastic net S- or LS-estimates of regression.

#### Usage

```
## S3 method for class 'pense_fit'
plot(x, ...)
```

# **Arguments**

x fitted estimates.

... currently ignored.

#### See Also

Other functions for plotting and printing: plot.pense\_cvfit(), prediction\_performance(), summary.pense\_cvfit()

#### **Examples**

```
# Compute the PENSE regularization path for Freeny's revenue data
# (see ?freeny)
{\sf data(freeny)}
x <- as.matrix(freeny[ , 2:5])</pre>
regpath <- pense(x, freeny$y, alpha = 0.5)
plot(regpath)
# Extract the coefficients at a certain penalization level
coef(regpath, lambda = regpath$lambda[40])
# What penalization level leads to good prediction performance?
cv_results <- pense_cv(x, freeny$y, alpha = 0.5, cv_repl = 2,</pre>
                       cv_k = 4
plot(cv_results, se_mult = 1)
# Extract the coefficients at the penalization level with
# smallest prediction error ...
coef(cv_results)
\# ... or at the penalization level with prediction error
# statistically indistinguishable from the minimum.
coef(cv_results, lambda = 'se')
```

36 predict.pense\_cvfit

# Description

Predict response values using a PENSE (or LS-EN) regularization path with hyper-parameters chosen by cross-validation.

# Usage

```
## S3 method for class 'pense_cvfit'
predict(
  object,
  newdata,
  lambda = c("min", "se"),
  se_mult = 1,
  exact = deprecated(),
  correction = deprecated(),
  ...
)
```

# **Arguments**

ماء شاء م	DENICE with an analidated house an arrangement of the state from
object	PENSE with cross-validated hyper-parameters to extract coefficients from.
newdata	an optional matrix of new predictor values. If missing, the fitted values are computed.
lambda	either a string specifying which penalty level to use or a a single numeric value of the penalty parameter. See details.
se_mult	If lambda = "se", the multiple of standard errors to tolerate.
exact	deprecated. Always gives a warning if lambda is not part of the fitted sequence and coefficients are interpolated.
correction	defunct.
	currently not used.

#### **Details**

If lambda = "se" and object contains fitted estimates for every penalization level in the sequence, extract the residuals of the most parsimonious model with prediction performance statistically indistinguishable from the best model. This is determined to be the model with prediction performance within se\_mult \* cv\_se from the best model.

#### Value

a numeric vector of residuals for the given penalization level.

# See Also

```
Other functions for extracting components: coef.pense_cvfit(), coef.pense_fit(), predict.pense_fit(), residuals.pense_cvfit(), residuals.pense_fit()
```

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#### **Examples**

```
# Compute the LS-EN regularization path for Freeny's revenue data
# (see ?freeny)
data(freeny)
x <- as.matrix(freeny[ , 2:5])</pre>
regpath <- elnet(x, freeny$y, alpha = 0.75)
# Predict the response using a specific penalization level
predict(regpath, newdata = freeny[1:5, 2:5],
        lambda = regpath$lambda[10])
# Extract the residuals at a certain penalization level
residuals(regpath, lambda = regpath$lambda[5])
# Select penalization level via cross-validation
cv_results <- elnet_cv(x, freeny$y, alpha = 0.5, cv_repl = 10,</pre>
                       cv_k = 4
# Predict the response using the "best" penalization level
predict(cv_results, newdata = freeny[1:5, 2:5])
# Extract the residuals at the "best" penalization level
residuals(cv_results)^2
# Extract the residuals at a more parsimonious penalization level
residuals(cv_results, lambda = 'se')
```

predict.pense\_fit

Predict Method for PENSE Fits

# **Description**

Predict response values using a PENSE (or LS-EN) regularization path fitted by pense() or elnet().

# Usage

```
## S3 method for class 'pense_fit'
predict(
   object,
   newdata,
   lambda,
   exact = deprecated(),
   correction = deprecated(),
   ...
)
```

# **Arguments**

object PENSE regularization path to extract residuals from.

newdata an optional matrix of new predictor values. If missing, the fitted values are

computed.

lambda a single value of the penalty parameter.

exact defunct Always gives a warning if lambda is not part of the fitted sequence and coefficients need to be interpolated.

correction defunct.

currently not used.

#### Value

a numeric vector of residuals for the given penalization level.

#### See Also

```
Other functions for extracting components: coef.pense_cvfit(), coef.pense_fit(), predict.pense_cvfit(), residuals.pense_cvfit(), residuals.pense_fit()
```

# **Examples**

```
# Compute the LS-EN regularization path for Freeny's revenue data
# (see ?freeny)
data(freeny)
x <- as.matrix(freeny[ , 2:5])</pre>
regpath <- elnet(x, freeny$y, alpha = 0.75)
# Predict the response using a specific penalization level
predict(regpath, newdata = freeny[1:5, 2:5],
        lambda = regpath$lambda[10])
# Extract the residuals at a certain penalization level
residuals(regpath, lambda = regpath$lambda[5])
# Select penalization level via cross-validation
cv_results <- elnet_cv(x, freeny$y, alpha = 0.5, cv_repl = 10,</pre>
                       cv_k = 4
# Predict the response using the "best" penalization level
predict(cv_results, newdata = freeny[1:5, 2:5])
# Extract the residuals at the "best" penalization level
residuals(cv_results)^2
# Extract the residuals at a more parsimonious penalization level
residuals(cv_results, lambda = 'se')
```

prediction\_performance

Prediction Performance of Adaptive PENSE Fits

# **Description**

Extract the prediction performance of one or more (adaptive) PENSE fits.

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#### Usage

```
prediction_performance(..., lambda = c("min", "se"), se_mult = 1)
## S3 method for class 'pense_pred_perf'
print(x, ...)
```

#### **Arguments**

one or more (adaptive) PENSE fits with cross-validation information.
 lambda a string specifying which penalty level to use ("min" or "se"). See details.
 se\_mult If lambda = "se", the multiple of standard errors to tolerate.
 x an object with information on prediction performance created with prediction\_performance().

#### **Details**

If lambda = "se" and the cross-validation was performed with multiple replications, use the penalty level whit prediction performance within se\_mult of the best prediction performance.

#### Value

a data frame with details about the prediction performance of the given PENSE fits. The data frame has a custom print method summarizing the prediction performances.

#### See Also

```
summary.pense_cvfit() for a summary of the fitted model.
Other functions for plotting and printing: plot.pense_cvfit(), plot.pense_fit(), summary.pense_cvfit()
```

prinsens

Principal Sensitivity Components

# Description

Compute Principal Sensitivity Components for Elastic Net Regression

# Usage

```
prinsens(
    x,
    y,
    alpha,
    lambda,
    intercept = TRUE,
    penalty_loadings,
    en_algorithm_opts,
    eps = 1e-06,
    sparse = FALSE,
    ncores = 1L,
    method = deprecated()
)
```

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#### **Arguments**

x n by p matrix of numeric predictors.y vector of response values of length n.

alpha elastic net penalty mixing parameter with  $0 \le \alpha \le 1$ . alpha = 1 is the LASSO

penalty, and alpha = 0 the Ridge penalty.

lambda optional user-supplied sequence of penalization levels. If given and not NULL,

nlambda and lambda\_min\_ratio are ignored.

intercept include an intercept in the model.

penalty\_loadings

a vector of positive penalty loadings (a.k.a. weights) for different penalization

of each coefficient. Only allowed for alpha > 0.

en\_algorithm\_opts

options for the LS-EN algorithm. See en\_algorithm\_options for details.

eps numerical tolerance.

sparse use sparse coefficient vectors.

ncores number of CPU cores to use in parallel. By default, only one CPU core is used.

May not be supported on your platform, in which case a warning is given.

method defunct. PSCs are always computed for EN estimates. For the PY procedure for

unpenalized estimation use package pyinit.

#### Value

a list of principal sensitivity components, one per element in lambda. Each PSC is itself a list with items lambda, alpha, and pscs.

#### References

Cohen Freue, G.V.; Kepplinger, D.; Salibián-Barrera, M.; Smucler, E. Robust elastic net estimators for variable selection and identification of proteomic biomarkers. *Ann. Appl. Stat.* **13** (2019), no. 4, 2065–2090 doi: 10.1214/19AOAS1269

Pena, D., and Yohai, V.J. A Fast Procedure for Outlier Diagnostics in Large Regression Problems. J. Amer. Statist. Assoc. **94** (1999). no. 446, 434–445. doi: 10.2307/2670164

# See Also

Other functions for initial estimates: enpy\_initial\_estimates(), starting\_point()

regmest Compute (Adaptive) Elastic Net M-Estimates of Regression

# **Description**

Compute elastic net M-estimates along a grid of penalization levels with optional penalty loadings for adaptive elastic net.

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#### Usage

```
regmest(
  х,
 у,
  alpha,
 nlambda = 50,
  lambda,
  lambda_min_ratio,
  scale,
  starting_points,
  penalty_loadings,
  intercept = TRUE,
  cc = 4.7,
  eps = 1e-06,
  explore_solutions = 10,
  explore_tol = 0.1,
 max_solutions = 10,
  comparison_tol = sqrt(eps),
  sparse = FALSE,
 ncores = 1,
  standardize = TRUE,
  algorithm_opts = mm_algorithm_options(),
  add_zero_based = TRUE,
 mscale\_bdp = 0.25,
 mscale_opts = mscale_algorithm_options()
)
```

# **Arguments**

x n by p matrix of numeric predictors.

y vector of response values of length n.

alpha elastic net penalty mixing parameter with  $0 \le \alpha \le 1$ . alpha = 1 is the LASSO

penalty, and alpha = 0 the Ridge penalty.

nlambda number of penalization levels.

lambda optional user-supplied sequence of penalization levels. If given and not NULL,

 $\verb|nlambda| and \verb|lambda| min\_ratio| are ignored.$ 

lambda\_min\_ratio

Smallest value of the penalization level as a fraction of the largest level (i.e., the smallest value for which all coefficients are zero). The default depends on the sample size relative to the number of variables and alpha. If more observations than variables are available, the default is 1e-3 \* alpha, otherwise 1e-2 \* alpha.

alpha.

scale fixed scale of the residuals.

starting\_points

a list of staring points, created by starting\_point(). The starting points are shared among all penalization levels.

penalty\_loadings

a vector of positive penalty loadings (a.k.a. weights) for different penalization of each coefficient. Only allowed for alpha > 0.

intercept include an intercept in the model.

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cc cutoff constant for Tukey's bisquare  $\rho$  function.

eps numerical tolerance.

explore\_solutions

number of solutions to compute up to the desired precision eps.

explore\_tol numerical tolerance for exploring possible solutions. Should be (much) looser

than eps to be useful.

max\_solutions only retain up to max\_solutions unique solutions per penalization level.

comparison\_tol numeric tolerance to determine if two solutions are equal. The comparison is

first done on the absolute difference in the value of the objective function at the solution If this is less than comparison\_tol, two solutions are deemed equal if the squared difference of the intercepts is less than comparison\_tol and the

squared  $L_2$  norm of the difference vector is less than comparison\_tol.

sparse use sparse coefficient vectors.

ncores number of CPU cores to use in parallel. By default, only one CPU core is used.

May not be supported on your platform, in which case a warning is given.

standardize logical flag to standardize the x variables prior to fitting the M-estimates. Coef-

ficients are always returned on the original scale. This can fail for variables with a large proportion of a single value (e.g., zero-inflated data). In this case, either

compute with standardize = FALSE or standardize the data manually.

 $algorithm\_opts \quad options \ for \ the \ MM \ algorithm \ to \ compute \ estimates. \ See \ mm\_algorithm\_options()$ 

for details.

add\_zero\_based also consider the 0-based regularization path in addition to the given starting

points.

mscale\_bdp, mscale\_opts

options for the M-scale estimate used to standardize the predictors (if standardize

= TRUE).

#### Value

a list-like object with the following items

lambda the sequence of penalization levels.

scale the used scale of the residuals.

estimates a list of estimates. Each estimate contains the following information:

intercept intercept estimate.

beta beta (slope) estimate.

lambda penalization level at which the estimate is computed.

alpha alpha hyper-parameter at which the estimate is computed.

objf\_value value of the objective function at the solution.

statuscode if > 0 the algorithm experienced issues when computing the estimate.

status optional status message from the algorithm.

call the original call.

## See Also

```
regmest_cv() for selecting hyper-parameters via cross-validation.
coef.pense_fit() for extracting coefficient estimates.
plot.pense_fit() for plotting the regularization path.
Other functions to compute robust estimates: pense()
```

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regmest\_cv

Cross-validation for (Adaptive) Elastic Net M-Estimates

# Description

Perform (repeated) K-fold cross-validation for regmest(). adamest\_cv() is a convenience wrapper to compute adaptive elastic-net M-estimates.

# Usage

```
regmest_cv(
    x,
    y,
    standardize = TRUE,
    lambda,
    cv_k,
    cv_repl = 1,
    cv_metric = c("tau_size", "mape", "rmspe"),
    fit_all = TRUE,
    cl = NULL,
    ...
)
adamest_cv(x, y, alpha, alpha_preliminary = 0, exponent = 1, ...)
```

# Arguments

X	n by p matrix of numeric predictors.
у	vector of response values of length n.
standardize	whether to standardize the x variables prior to fitting the PENSE estimates. Can also be set to "cv_only", in which case the input data is not standardized, but the training data in the CV folds is scaled to match the scaling of the input data. Coefficients are always returned on the original scale. This can fail for variables with a large proportion of a single value (e.g., zero-inflated data). In this case, either compute with standardize = FALSE or standardize the data manually.
lambda	optional user-supplied sequence of penalization levels. If given and not NULL, nlambda and lambda_min_ratio are ignored.
cv_k	number of folds per cross-validation.
cv_repl	number of cross-validation replications.
cv_metric	either a string specifying the performance metric to use, or a function to evaluate prediction errors in a single CV replication. If a function, it is called with a single numeric vector of prediction errors and must return a scalar number.
fit_all	If TRUE, fit the model for all penalization levels. Otherwise, only at penalization level with smallest average CV performance.
cl	a parallel cluster. Can only be used if ncores = 1, because multi-threading can not be used in parallel R sessions on the same host.
	Arguments passed on to regmest

scale fixed scale of the residuals.

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nlambda number of penalization levels.

lambda\_min\_ratio Smallest value of the penalization level as a fraction of the largest level (i.e., the smallest value for which all coefficients are zero). The default depends on the sample size relative to the number of variables and alpha. If more observations than variables are available, the default is 1e-3 \* alpha, otherwise 1e-2 \* alpha.

penalty\_loadings a vector of positive penalty loadings (a.k.a. weights) for different penalization of each coefficient. Only allowed for alpha > 0.

starting\_points a list of staring points, created by starting\_point(). The starting points are shared among all penalization levels.

intercept include an intercept in the model.

add\_zero\_based also consider the 0-based regularization path in addition to the given starting points.

cc cutoff constant for Tukey's bisquare  $\rho$  function.

eps numerical tolerance.

explore\_solutions number of solutions to compute up to the desired precision eps.

explore\_tol numerical tolerance for exploring possible solutions. Should be (much) looser than eps to be useful.

max\_solutions only retain up to max\_solutions unique solutions per penalization level.

comparison\_tol numeric tolerance to determine if two solutions are equal. The comparison is first done on the absolute difference in the value of the objective function at the solution If this is less than comparison\_tol, two solutions are deemed equal if the squared difference of the intercepts is less than comparison\_tol and the squared  $L_2$  norm of the difference vector is less than comparison\_tol.

sparse use sparse coefficient vectors.

ncores number of CPU cores to use in parallel. By default, only one CPU core is used. May not be supported on your platform, in which case a warning is given.

algorithm\_opts options for the MM algorithm to compute estimates. See mm\_algorithm\_options() for details.

mscale\_bdp options for the M-scale estimate used to standardize the predictors (if standardize = TRUE).

 $mscale\_opts$  options for the M-scale estimate used to standardize the predictors (if standardize = TRUE).

alpha

elastic net penalty mixing parameter with  $0 \le \alpha \le 1$ . alpha = 1 is the LASSO penalty, and alpha = 0 the Ridge penalty.

alpha\_preliminary

alpha parameter for the preliminary estimate.

exponent

the exponent for computing the penalty loadings based on the preliminary estimate.

#### **Details**

The built-in CV metrics are

"tau\_size"  $\tau$ -size of the prediction error, computed by tau\_size() (default).

"mape" Median absolute prediction error.

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```
"rmspe" Root mean squared prediction error.
```

adamest\_cv() is a convenience wrapper which performs 3 steps:

- 1. compute preliminary estimates via regmest\_cv(...,alpha = alpha\_preliminary),
- 2. computes the penalty loadings from the estimate beta with best prediction performance by adamest\_loadings = 1 / abs(beta)^exponent, and
- 3. compute the adaptive PENSE estimates via regmest\_cv(...,penalty\_loadings = adamest\_loadings).

#### Value

```
a list with components:
```

lambda the sequence of penalization levels.

scale the used scale of the residuals.

cvres data frame of average cross-validated performance.

cv\_replications matrix of cross-validated performance metrics, one column per replication. Rows are in the same order as in cvres.

call the original call.

estimates the estimates fitted on the full data. Same format as returned by regmest().

the object returned by adamest\_cv() has additional components

preliminary the CV results for the preliminary estimate.

penalty\_loadings the penalty loadings used for the adaptive elastic net M-estimate.

#### See Also

```
regmest() for computing regularized S-estimates without cross-validation.
coef.pense_cvfit() for extracting coefficient estimates.
plot.pense_cvfit() for plotting the CV performance or the regularization path.
Other functions to compute robust estimates with CV: pense_cv(), pensem_cv()
Other functions to compute robust estimates with CV: pense_cv(), pensem_cv()
```

#### **Examples**

46 residuals.pense\_cvfit

residuals.pense\_cvfit Extract Residuals

# **Description**

Extract residuals from a PENSE (or LS-EN) regularization path with hyper-parameters chosen by cross-validation.

# Usage

```
## $3 method for class 'pense_cvfit'
residuals(
  object,
  lambda = c("min", "se"),
  se_mult = 1,
  exact = deprecated(),
  correction = deprecated(),
  ...
)
```

# **Arguments**

object	PENSE with cross-validated hyper-parameters to extract coefficients from.
lambda	either a string specifying which penalty level to use or a a single numeric value of the penalty parameter. See details.
se_mult	If lambda = "se", the multiple of standard errors to tolerate.
exact	deprecated. Always gives a warning if lambda is not part of the fitted sequence and coefficients are interpolated.
correction	defunct.
	currently not used.

residuals.pense\_fit 47

#### **Details**

If lambda = "se" and object contains fitted estimates for every penalization level in the sequence, extract the residuals of the most parsimonious model with prediction performance statistically indistinguishable from the best model. This is determined to be the model with prediction performance within se\_mult \* cv\_se from the best model.

#### Value

a numeric vector of residuals for the given penalization level.

#### See Also

```
Other functions for extracting components: coef.pense_cvfit(), coef.pense_fit(), predict.pense_cvfit(), predict.pense_fit(), residuals.pense_fit()
```

#### **Examples**

```
# Compute the LS-EN regularization path for Freeny's revenue data
# (see ?freeny)
data(freeny)
x <- as.matrix(freeny[ , 2:5])</pre>
regpath <- elnet(x, freeny$y, alpha = 0.75)
# Predict the response using a specific penalization level
predict(regpath, newdata = freeny[1:5, 2:5],
        lambda = regpath$lambda[10])
# Extract the residuals at a certain penalization level
residuals(regpath, lambda = regpath$lambda[5])
# Select penalization level via cross-validation
cv_results <- elnet_cv(x, freeny$y, alpha = 0.5, cv_repl = 10,</pre>
                       cv_k = 4
# Predict the response using the "best" penalization level
predict(cv_results, newdata = freeny[1:5, 2:5])
# Extract the residuals at the "best" penalization level
residuals(cv_results)^2
# Extract the residuals at a more parsimonious penalization level
residuals(cv_results, lambda = 'se')
```

## **Description**

Extract residuals from a PENSE (or LS-EN) regularization path fitted by pense() or elnet().

# Usage

```
## S3 method for class 'pense_fit'
residuals(object, lambda, exact = deprecated(), correction = deprecated(), ...)
```

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#### **Arguments**

object PENSE regularization path to extract residuals from.

lambda a single value of the penalty parameter.

exact defunct Always gives a warning if lambda is not part of the fitted sequence and

coefficients need to be interpolated.

correction defunct.

... currently not used.

#### Value

a numeric vector of residuals for the given penalization level.

#### See Also

```
Other functions for extracting components: coef.pense_cvfit(), coef.pense_fit(), predict.pense_cvfit(), predict.pense_fit(), residuals.pense_cvfit()
```

# **Examples**

```
# Compute the LS-EN regularization path for Freeny's revenue data
# (see ?freeny)
data(freeny)
x <- as.matrix(freeny[ , 2:5])</pre>
regpath <- elnet(x, freeny$y, alpha = 0.75)
# Predict the response using a specific penalization level
predict(regpath, newdata = freeny[1:5, 2:5],
        lambda = regpath$lambda[10])
# Extract the residuals at a certain penalization level
residuals(regpath, lambda = regpath$lambda[5])
# Select penalization level via cross-validation
cv_results <- elnet_cv(x, freeny$y, alpha = 0.5, cv_repl = 10,</pre>
                       cv_k = 4
# Predict the response using the "best" penalization level
predict(cv_results, newdata = freeny[1:5, 2:5])
# Extract the residuals at the "best" penalization level
residuals(cv_results)^2
# Extract the residuals at a more parsimonious penalization level
residuals(cv_results, lambda = 'se')
```

rho\_function

List Available Rho Functions

#### **Description**

List Available Rho Functions

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## Usage

```
rho_function(rho)
```

#### **Arguments**

rho

the name of the  $\rho$  function to check for existence.

#### Value

if rho is missing returns a vector of supported  $\rho$  function names, otherwise the internal integer representation of the  $\rho$  function.

#### See Also

Other miscellaneous functions: consistency\_const()

starting\_point

Create Starting Points for the PENSE Algorithm

#### **Description**

Create a starting point for starting the PENSE algorithm in pense(). Multiple starting points can be created by combining starting points via c(starting\_point\_1, starting\_point\_2,...).

# Usage

```
starting_point(beta, intercept, lambda)
as_starting_point(object, specific = FALSE, ...)
## S3 method for class 'enpy_starting_points'
as_starting_point(object, specific = FALSE, ...)
## S3 method for class 'pense_fit'
as_starting_point(object, specific = FALSE, lambda, ...)
## S3 method for class 'pense_cvfit'
as_starting_point(
   object,
   specific = FALSE,
   lambda = c("min", "se"),
   se_mult = 1,
   ...
)
```

## **Arguments**

beta

beta coefficients at the starting point. Can be a numeric vector, a sparse vector of class dsparse Vector, or a sparse matrix of class dgCMatrix with a single column.

intercept

intercept coefficient at the starting point.

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lambda	optionally either a string specifying which penalty level to use ("min" or "se") or a numeric vector of the penalty levels to extract from object. Penalization levels not present in object are ignored with a warning. If NULL, all estimates in object are extracted.
object	an object with estimates to use as starting points.
specific	whether the estimates should be used as starting points only at the penalization level they are computed for. Defaults to using the estimates as starting points for all penalization levels.
	further arguments passed to or from other methods.
se mult	If lambda = "se", the multiple of standard errors to tolerate.

#### **Details**

A starting points can either be *shared*, i.e., used for every penalization level PENSE estimates are computed for, or *specific* to one penalization level. To create a specific starting point, provide the penalization level as lambda. If lambda is missing, a shared starting point is created. Shared and specific starting points can all be combined into a single list of starting points, with pense() handling them correctly. Note that specific starting points will lead to the lambda value being added to the grid of penalization levels. See pense() for details.

Starting points computed via enpy\_initial\_estimates() are by default *shared* starting points but can be transformed to *specific* starting points via enpy\_starting\_point(...,specific = TRUE).

When creating starting points from cross-validated fits, it is possible to extract only the estimate with best CV performance (lambda = "min"), or the estimate with CV performance statistically indistinguishable from the best performance (lambda = "se"). This is determined to be the estimate with prediction performance within se\_mult \* cv\_se from the best model.

#### Value

an object of type starting\_points to be used as starting point for pense().

## See Also

Other functions for initial estimates: enpy\_initial\_estimates(), prinsens()

```
summary.pense_cvfit Summarize Cross-Validated PENSE Fit
```

#### **Description**

If lambda = "se" and object contains fitted estimates for every penalization level in the sequence, extract the coefficients of the most parsimonious model with prediction performance statistically indistinguishable from the best model. This is determined to be the model with prediction performance within se\_mult \* cv\_se from the best model.

# Usage

```
## S3 method for class 'pense_cvfit'
summary(object, lambda = c("min", "se"), se_mult = 1, ...)
## S3 method for class 'pense_cvfit'
print(x, lambda = c("min", "se"), se_mult = 1, ...)
```

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# **Arguments**

object, x an (adaptive) PENSE fit with cross-validation information. lambda either a string specifying which penalty level to use ("min" or "se") or a a single numeric value of the penalty parameter. See details. se\_mult If lambda = "se", the multiple of standard errors to tolerate. ignored. . . .

#### See Also

prediction\_performance() for information about the estimated prediction performance. coef.pense\_cvfit() for extracting only the estimated coefficients.

Other functions for plotting and printing: plot.pense\_cvfit(), plot.pense\_fit(), prediction\_performance()

tau\_size

Compute the Tau-Scale of Centered Values

# **Description**

Compute the  $\tau$ -scale without centering the values.

#### Usage

```
tau_size(x)
```

# **Arguments**

numeric values. Missing values are verbosely ignored. Х

# Value

the  $\tau$  estimate of scale of centered values.

# See Also

Other functions to compute robust estimates of location and scale: mlocscale(), mloc(), mscale()

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