# The PhyloSim package

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## Contents

1	Get	ting help				
2	Bas	Basic examples				
	2.1 Simulating substitutions under the JC69 model					
		2.1.1	Compact example	2		
		2.1.2	"Unrolled" example	3		
	2.2	Simula	ating substitutions under the HKY model	5		
	2.3	2.3 Simulating among-sites rate variation				
		2.3.1	Simulating under the discrete gamma (GTR+d $\Gamma$ ) model	7		
		2.3.2	Simulating under the invariants and discrete gamma (GTR+I+d $\Gamma$ ) model	8		
	2.4	Simula	ating indels	9		
		2.4.1	Insertion and deletions having the same length distribution	9		
		2.4.2	Insertion and deletions having different length distributions	10		
		2.4.3	Simulating indels under selective constraints	11		
	2.5	Simula	ating partitions	13		
	2.6	Simulating heterotachy				
	2.7	Simula	ating many replications	16		
3	Adv	Advanced examples				
	3.1	Simula	ating "domains" and heterogeneous evolution	17		
	3.2	Evolvi	ng codon sequences	24		
	3.3	Imple	menting a new process	29		
	3.4	Evolvi	ng a genomic region containing a "gene"	31		
4	Det	Details of the fast field deletion model 36				

## 1 Getting help

- PhyloSim is extensively documented; the documentation of the PhyloSim class (as invoked by help("PhyloSim")) is probably a good entry point for beginners and also contains some more basic examples.
- Serious users might consider reading all class documentations which provide more focused examples.
- Additional example scripts can be found at http://github.com/sbotond/phylosim/tree/master/examples
- The 11() method list the methods and virtual field implemented in the immediate class of an object which is useful as "proto-documentation".

## 2 Basic examples

The following examples illustrate how to set up simple simulations.

First load the package:

```
> library(phylosim)
```

Create the tree files used in the basic examples:

```
> cat("((t1:0.3,t2:0.3):0.2,t3:0.1);", file = "3taxa.nwk")
> cat("((t1:0.3,t2:0.3):0.2,(t3:0.1,t4:0.1):0.4);", file = "4taxa.nwk")
```

## 2.1 Simulating substitutions under the JC69 model

We will simulate the evolution of nucleotide sequences of length 50 along a tree with 3 tips under the JC69 model. The following two examples are essentially equivalent.

#### 2.1.1 Compact example

```
> Simulate(PhyloSim(root.seq = sampleStates(NucleotideSequence(len = 50,
     proc = list(list(JC69())))), phylo = read.tree("3taxa.nwk")))$alignment
Simulating edge 1 of 4
Simulating edge 2 of 4
Simulating edge 3 of 4
Simulating edge 4 of 4
                                      8
                                          9
                                              10 11
                                                    12 13 14
                                                                 15
                                                                     16
Root node 4 "C" "T" "T" "T" "C" "C" "G" "G" "G" "G" "A" "C" "G" "A" "A" "A" "A" "C" "C" "T"
           "C" "T" "T" "T" "C" "C" "G" "G" "G" "A" "C" "G" "A" "A" "A" "A" "A" "A" "C"
t.3
           "A" "T" "T" "T" "C" "C" "G" "C" "G" "T" "A" "C" "G" "A"
Node 5
                                                                 "A" "C" "T"
              "C" "C" "T"
                          "C" "C" "G" "C" "G" "T" "A" "A" "A" "A"
                                                                 "C"
                                                                    "C" "T"
t2
           "A" "T" "C" "T" "C" "G" "C" "C" "G" "A" "G" "A" "G" "A"
                                                                 "C" "C" "T"
               21
                  22
                       23
                          24
                              25
                                  26
                                      27
                                          28
                                              29
                                                 30
                                                     31
                                                         32
                                                             33
                                                                 34
                                                                     35
Root node 4 "G" "G" "T" "G"
                          יידיי יידיי
                                  "G"
                                     "A" "G"
                                             "T" "T" "G" "G"
                                                            "C" "A"
                                                                    "A" "C"
           t3
           "G" "G" "T" "G" "T" "T" "G" "A" "G" "T" "T" "G" "G" "G" "C" "A" "T" "C" "G"
Node 5
           "C" "G" "T" "G" "T" "T" "G" "A" "G" "T" "T" "A" "G" "G" "G" "T" "T" "G" "G"
t2
           "G" "G" "G" "G" "G" "T" "G" "A" "G" "T" "A" "G" "G" "C" "A" "T" "C" "G" "T"
t1
```

### 2.1.2 "Unrolled" example

Construct the root sequence object:

```
> root.seq <- NucleotideSequence(length = 50)</pre>
```

Print out the root sequence:

```
> print(root.seq)
```

#### 

Note, that all states are still undefined.

Construct a JC69 substitution process object:

Attach the substitution process to root sequence:

```
> attachProcess(root.seq, p)
```

Sample the site states from the equilibrium distribution of the JC69 substitution process:

```
> sampleStates(root.seq)
```

Print out the root sequence, now with the sampled states:

```
> print(root.seq)
```

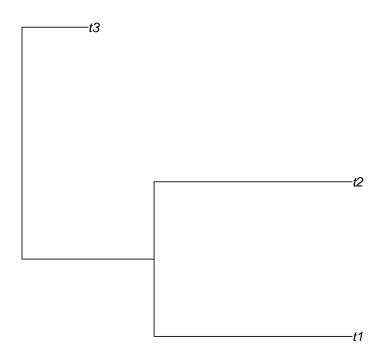
#### $\hbox{\tt [1]} \quad \hbox{\tt "ACATTGCTCTGAGTATGGAGATAGGTCCGCACCGCAAGGTGTACTT"}$

Read in tree from file using APE:

```
> tree <- read.tree("3taxa.nwk")</pre>
```

Plot the tree:

> plot(tree)



 $Construct\ a\ {\tt PhyloSim}\ object:$ 

> sim <- PhyloSim()</pre>

Set the phylo object:

> sim\$phylo <- tree

Set the root sequence:

> sim\$rootSeq <- root.seq</pre>

Run the simulation:

> Simulate(sim)

Simulating edge 1 of 4

Simulating edge 2 of 4

Simulating edge 3 of 4

Simulating edge  $4\ \text{of}\ 4$ 

Display the resulting alignment matrix:

#### > sim\$alignment

```
2 3 4
                          5
                              6 7
                                      8
                                          9
                                              10 11 12 13 14 15 16 17 18
Root node 4 "A" "C" "A" "T" "T" "G" "C" "T" "C" "T" "G" "T" "C" "T" "G" "A" "G" "T" "A"
           "A" "C" "A" "T" "T" "C" "C" "T" "C" "T" "G" "T" "C" "T" "G" "A" "G" "A" "G" "T" "A"
           "A" "C" "A" "T" "T" "G" "C" "T" "C" "T" "G" "T" "C" "T" "G" "T"
Node 5
           "T" "C" "A" "T" "A" "T" "C" "G" "C" "T" "G" "T" "C" "C" "C" "T" "G" "T"
t2
           "A" "C" "A" "C" "T" "T" "C" "T" "C" "T" "G" "T" "C" "A" "G" "A" "G" "T"
               21 22
                       23
                          24
                              25
                                  26
                                      27
                                          28
                                              29
                                                  30
                                                      31
                                                         32
                                                             33
                                                                 34
Root node 4 "T" "G" "G" "A" "G" "A" "T" "A" "G" "G" "T" "C" "C" "G" "C" "A" "C"
           "T" "G" "G" "A" "G" "C" "T" "A" "A" "G" "T" "C" "C" "G" "C" "A" "C" "C" "G"
           "T" "G" "G" "G" "G" "A" "T" "A" "G" "G" "T" "C" "C" "C" "C" "A" "G" "C" "G"
Node 5
           "T" "A" "G" "G" "G" "A" "T" "A" "G" "G" "T" "C" "G" "C" "C" "A" "G" "C" "G"
t2
           "T" "G" "G" "T" "G" "A" "T" "A" "A" "G" "A" "C" "G" "G" "G" "A" "G" "G"
t1
           39 40 41 42 43 44 45 46 47 48 49
Root node 4 "C" "A" "A" "G" "G" "T" "G" "T" "A" "C" "T" "T"
           "C" "A" "G" "G" "T" "G" "T" "A" "C" "T" "T"
t3
           "C" "A" "A" "G" "G" "T" "G" "T" "A" "G" "T" "T"
Node 5
           "C" "T" "A" "G" "G" "T" "G" "A" "A" "G" "T" "T"
t2
           "C" "A" "A" "A" "G" "T" "T" "T" "A" "G" "T" "T"
t1
```

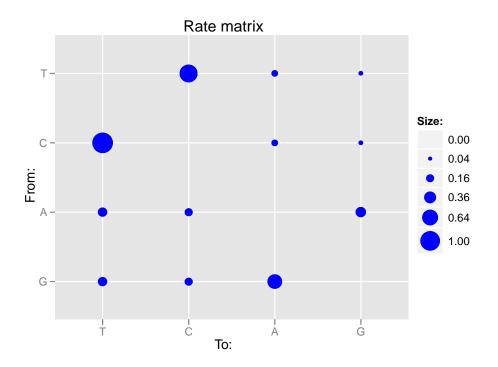
#### 2.2 Simulating substitutions under the HKY model

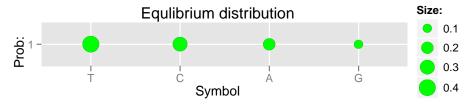
Construct an HKY substitution process object:

```
> p \leftarrow HKY(rate.params = list(Alpha = 10, Beta = 2), base.freqs = c(4, + 3, 2, 1)/10)
```

Get a plot of the instantaneous substitution matrix and equilibrium distribution of the p process (bubble plot):

```
> plot(p, scale = 0.5)
```





Construct the root sequence, attach the substitution process to root sequence via the constructor and sample states:

```
> root.len50.seq <- NucleotideSequence(length = 50, processes = list(list(p)))
> sampleStates(root.len50.seq)
```

Print out root sequence:

> print(root.len50.seq)

### [1] "TCTGTCATTGGAAGAAGTTCCCTCTCCTGACTCTTTTGGCCAATAACCCT"

Construct a PhyloSim object, set the phylo object and the root sequence:

> sim <- PhyloSim(root.seq = root.len50.seq, phylo = read.tree("3taxa.nwk"))</pre>

Run simulation:

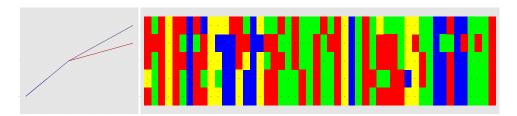
> Simulate(sim)

Simulating edge 1 of 4 Simulating edge 2 of 4 Simulating edge 3 of 4 Simulating edge 4 of 4 Save the resulting alignment (fasta format):

```
> saveAlignment(sim, file = "HKY_sim.fas")
```

Plot the alignment alongside the tree (including sequences at ancestral nodes):

> plot(sim, num.pages = 1)



### 2.3 Simulating among-sites rate variation

#### 2.3.1 Simulating under the discrete gamma ( $GTR+d\Gamma$ ) model

Construct a GTR subsection process object:

```
> p <- GTR(rate.params = list(a = 1, b = 2, c = 3, d = 1, e = 2, f = 3),
+ base.freqs = c(2, 2, 1, 1)/6
```

Summary of object p:

> summary(p)

Name: Anonymous

Id: GTR:Anonymous:59600536

Alphabet:

Type: Nucleotide Symbols: T C A G

Rate parameters: a = 1, b = 2, c = 3, d = 1, e = 2, f = 3

Unscaled rate matrix:

T C A G
T -1.1666667 0.3333333 0.3333333 0.5000000
C 0.3333333 -0.8333333 0.1666667 0.3333333
A 0.6666667 0.3333333 -1.5000000 0.5000000
G 1.0000000 0.6666667 0.5000000 -2.1666667

Equilibrium distribution:

T C A G Prob: 0.3333333 0.3333333 0.1666667 0.1666667

Site specific parameters (1):

Id: rate.multiplier
Name: Rate multiplier

Type: numeric
Default value: 1

Construct the root sequence, attach substitution process:

```
> root.seq <- NucleotideSequence(length = 50, processes = list(list(p)))</pre>
```

Sample rate multipliers from a discrete gamma distribution with 4 categories and shape parameter 0.5:

```
> plusGamma(root.seq, p, 0.5)
```

Get the sampled rate multipliers:

> getRateMultipliers(root.seq, p)

```
[1] 0.82026848 2.89442785 0.25191592 0.82026848 0.03338775 0.25191592 0.82026848 [8] 2.89442785 0.03338775 0.25191592 0.25191592 0.82026848 0.03338775 2.89442785 [15] 0.82026848 0.82026848 0.82026848 0.25191592 0.25191592 0.25191592 0.82026848 [22] 2.89442785 0.82026848 2.89442785 2.89442785 0.03338775 0.25191592 2.89442785 [29] 0.82026848 0.82026848 0.03338775 0.25191592 2.89442785 2.89442785 [36] 0.82026848 0.82026848 2.89442785 0.25191592 0.82026848 0.82026848 0.82026848 0.82026848 0.03338775 0.25191592 0.82026848 0.82026848 0.82026848 0.03338775 0.03338775 0.82026848 0.82026848 0.82026848 0.03338775 0.03338775 0.82026848 0.82026848 [50] 0.03338775
```

Construct the PhyloSim object, sample states and set the phylo object:

```
> sim <- PhyloSim(root.seq = sampleStates(root.seq), phylo = read.tree("3taxa.nwk"))
```

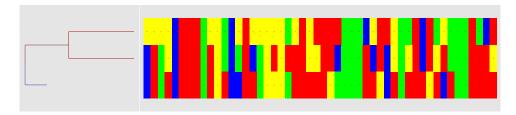
Run the simulation:

#### > Simulate(sim)

```
Simulating edge 1 of 4
Simulating edge 2 of 4
Simulating edge 3 of 4
Simulating edge 4 of 4
```

Plot the alignment alongside the tree, skip sequences at ancestral nodes:

> plot(sim, num.pages = 1, plot.ancestors = FALSE)



Save the resulting alignment, omitting sequences at internal nodes:

```
> saveAlignment(sim, file = "Gamma_sim.fas", skip.internal = TRUE)
```

#### 2.3.2 Simulating under the invariants and discrete gamma $(GTR+I+d\Gamma)$ model

We will reuse the root sequence object and GTR object from the previous example. The process is already attached, but we need to clear the states to have a new root sequence.

Clear the states of the root sequence object and sample a set of new states:

```
> clearStates(root.seq)
> sampleStates(root.seq)
```

Sample rate multipliers from a  $+I+d\Gamma$  model:

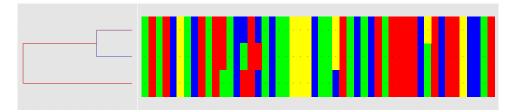
```
> plusInvGamma(root.seq, p, pinv = 0.8, shape = 0.5)
```

Deal with the rest of the simulation (note that for variety we use a random coalescent tree in this example):

> sim <- Simulate(PhyloSim(root.seq = root.seq, phylo = rcoal(3)))

```
Simulating edge 1 of 4
Simulating edge 2 of 4
Simulating edge 3 of 4
Simulating edge 4 of 4
```

> plot(sim, num.pages = 1, plot.ancestors = FALSE)



Save the resulting alignment, skip sequences from internal nodes:

```
> saveAlignment(sim, file = "InvGamma_sim.fas", skip.internal = TRUE)
```

#### 2.4 Simulating indels

#### 2.4.1 Insertion and deletions having the same length distribution

Set up the substitution process and the root sequence:

```
> p <- JC69()
> root.seq <- NucleotideSequence(len = 50, processes = list(list(p)))
> sampleStates(root.seq)
```

Construct a deletion process proposing deletions with rate 0.25 according to a discrete length distribution:

```
> d \leftarrow DiscreteDeletor(rate = 0.25, sizes = c(1, 2), probs = c(1/2, 1/2))
```

Construct an insertion process proposing insertions with rate 0.25 according to a discrete length distribution:

```
> i <- DiscreteInsertor(rate = 0.25, sizes = c(1, 2), probs = c(1/2, 1/2))
```

Set the template sequence for the insertion process:

```
> i$templateSeq <- NucleotideSequence(length = 2, processes = list(list(p)))</pre>
```

The states of the template sequence are undefined. The actual states will be sampled from the equilibrium distribution of the attached substitution process before performing the insertion.

Attaching the indel processes:

```
> attachProcess(root.seq, d)
> attachProcess(root.seq, i)
```

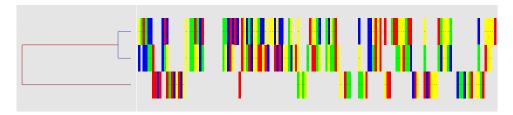
Construct the PhyloSim object, set the phylo object (random coalescent tree for three taxa) and run the simulation:

```
> sim <- Simulate(PhyloSim(root.seq = root.seq, phylo = rcoal(3)))
```

```
Simulating edge 1 of 4
Simulating edge 2 of 4
Simulating edge 3 of 4
Simulating edge 4 of 4
```

Plot the alignment alongside the tree, skip sequences at ancestral nodes:

> plot(sim, num.pages = 1, plot.ancestors = FALSE)



Save the resulting alignment, skip sequences from internal nodes:

```
> saveAlignment(sim, file = "indel1_sim.fas", skip.internal = TRUE)
```

## 2.4.2 Insertion and deletions having different length distributions

The following code reuses the objects constructed in the previous example to simulate deletions and insertions with different length distributions.

Construct a new insertion process proposing insertions of Cs with rate 0.5 according to a discrete length distribution:

```
> i2 <- DiscreteInsertor(rate = 0.25, sizes = c(1, 2, 3, 4), probs = c(1, 2, 3, 4)/10, template.seq = NucleotideSequence(string = "C"))
```

Clear the states of the root sequence object:

```
> clearStates(root.seq)
```

Define a new set of processes (p - substitution, d - deletion, i2 - the new insertion process) for the root sequence using the processes virtual field:

```
> root.seq$processes <- list(list(i2, d, p))</pre>
```

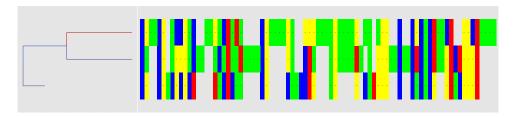
Construct the PhyloSim object, sample states, set the phylo object and run the simulation:

> sim <- Simulate(PhyloSim(root.seq = sampleStates(root.seq), phylo = read.tree("3taxa.nwk")))

```
Simulating edge 1 of 4
Simulating edge 2 of 4
Simulating edge 3 of 4
Simulating edge 4 of 4
```

Plot the alignment alongside the tree, skip sequences at ancestral nodes:

> plot(sim, num.pages = 1, plot.ancestors = FALSE)



Save the resulting alignment, skip sequences from internal nodes:

```
> saveAlignment(sim, file = "indel2_sim.fas", skip.internal = TRUE)
```

#### 2.4.3 Simulating indels under selective constraints

Set up the substitution process and the root sequence:

```
> p <- JC69()
> root.seq <- NucleotideSequence(len = 100, processes = list(list(p)))
> sampleStates(root.seq)
```

Construct a deletion process proposing deletions with rate 1 according to a discrete length distribution:

```
> d \leftarrow Discrete Deletor(rate = 1, sizes = c(1, 2, 3), probs = c(3/6, 2/6, + 1/6))
```

Construct an insertion process proposing insertions with rate 1 according to a discrete length distribution:

```
> i <- DiscreteInsertor(rate = 1, sizes = c(1, 2, 3), probs = c(3/6, 2/6, 1/6))
```

Set the template sequence for the insertion process:

```
> i$templateSeq <- NucleotideSequence(length = 2, processes = list(list(p)))</pre>
```

Attaching the indel processes:

```
> attachProcess(root.seq, d)
> attachProcess(root.seq, i)
```

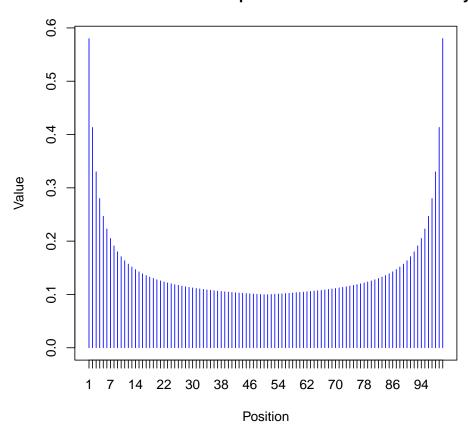
Set deletion tolerance values:

> setDeletionTolerance(root.seq, d, 0.08 + c(1/2:51, 1/51:2))

Plot deletion tolerance values:

> plotParametersAtSites(root.seq, d, "deletion.tolerance")

## arameter deletion.tolerance for process DiscreteDeletor:Anonymo



Set insertion tolerance values:

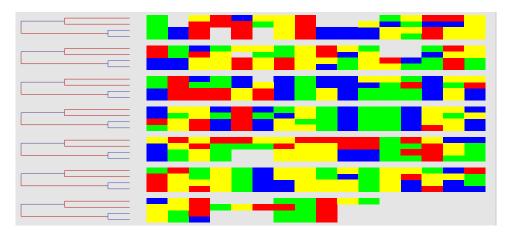
> setInsertionTolerance(root.seq, i, 0.08 + c(1/2:51, 1/51:2))

Construct the PhyloSim object, read phylo object from file:

> sim <- Simulate(PhyloSim(root.seq = root.seq, phylo = read.tree("4taxa.nwk")))</pre>

Simulating edge 1 of 6 Simulating edge 2 of 6 Simulating edge 3 of 6 Simulating edge 4 of 6 Simulating edge 5 of 6 Simulating edge 6 of 6 Plot the alignment alongside the tree, skip sequences at ancestral nodes:

> plot(sim, plot.ancestors = FALSE)



Save the resulting alignment, skip sequences from internal nodes:

> saveAlignment(sim, file = "indel3\_sim.fas", skip.internal = TRUE)

### 2.5 Simulating partitions

The following example demonstrates how to use the processes and site- and process-specific parameters to simulate "partitions" with different properties.

We will simulate four partitions:

- Partition 1: sites in range 1:25 evolving by JC+d $\Gamma$  with a shape parameter  $\alpha=1$
- Partition 2: sites in range 26:50 evolving by JC+d $\Gamma$  with a shape parameter  $\alpha=0.5$
- Partition 3: sites in range 51:75 evolving by HKY+d $\Gamma$  with a shape parameter  $\alpha=1$
- Partition 4: sites in range 76:100 evolving by HKY+d $\Gamma$  with a shape parameter  $\alpha=0.5$

First construct two substitution process objects:

```
> jc69 \leftarrow JC69()
> hky \leftarrow HKY(rate.params = list(Alpha = 5, Beta = 2), base.freqs = c(4, + 3, 2, 1)/10)
```

Construct a root sequence object of length 100:

```
> root.seq <- NucleotideSequence(length = 100)</pre>
```

Attach process jc69 to range 1:50:

> attachProcess(root.seq, jc69, 1:50)

Attach process hky to range 51:100:

```
> attachProcess(root.seq, hky, 51:100)
```

Sample rate multipliers in the four partitions:

```
> plusGamma(root.seq, jc69, 1, 1:25)
> plusGamma(root.seq, jc69, 0.5, 26:50)
> plusGamma(root.seq, hky, 1, 51:75)
> plusGamma(root.seq, hky, 0.5, 76:100)
```

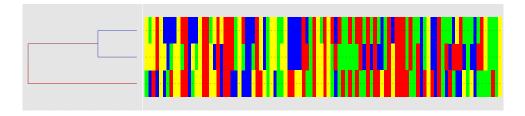
Construct the PhyloSim object, sample states, set root sequence, set the phylo object (random coalescent tree for three taxa) and run the simulation:

```
> sim <- Simulate(PhyloSim(root.seq = sampleStates(root.seq), phylo = rcoal(3)))
```

```
Simulating edge 1 of 4
Simulating edge 2 of 4
Simulating edge 3 of 4
Simulating edge 4 of 4
```

Plot the alignment alongside the tree, skip sequences at ancestral nodes:

> plot(sim, num.pages = 1, plot.ancestors = FALSE)



## 2.6 Simulating heterotachy

Set up the substitution process:

```
> p \leftarrow F84(base.freqs = c(1, 2, 3, 4))
> p$kappa \leftarrow 1
```

Set up the root sequence:

```
> root.seq <- NucleotideSequence(length = 50, processes = list(list(p)))</pre>
```

Sample the rate multipliers from a  $+d\Gamma$  model with shape parameter 1.0:

```
> plusGamma(root.seq, p, 1)
```

Read a tree from file:

```
> tree <- read.tree("4taxa.nwk")</pre>
```

Construct the PhyloSim object, sample states, set root sequence, and set the phylo object:

```
> sim <- PhyloSim(root.seq = sampleStates(root.seq), phylo = tree)
```

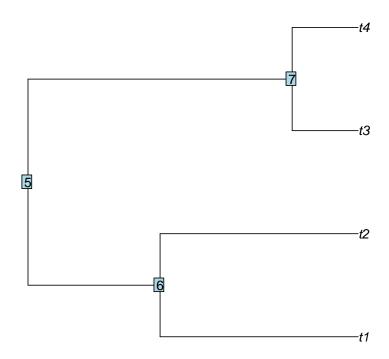
A "node hook" is a function which accepts a Sequence object through the named argument "seq" and returns a Sequence object. After simulating the branch leading to the node, the resulting Sequence object is passed to the node hook and the returned object is used to simulate the downstream branches.

Create a node hook function which will create heterotachy by resampling the site rates from the  $+d\Gamma$  model with shape parameter 1.0:

```
> node.hook <- function(seq) {
+     if (!isAttached(seq$sites[[1]], p)) {
+         return(seq)
+     }
+     cat("Resampling rate multipliers!\n")
+     plusGamma(seq, p, 1)
+     return(seq)
+ }</pre>
```

Plot the PhyloSim object:

```
> plot(sim)
```



Notice that node 6 is the ancestor of taxa t1 and t2.

Attach the hook to node 6:

```
> attachHookToNode(sim, node = 6, fun = node.hook)
```

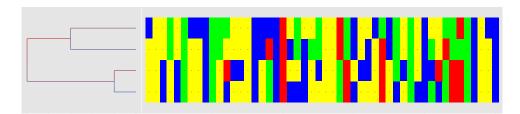
Run the simulation:

```
> Simulate(sim)
```

```
Simulating edge 1 of 6
Resampling rate multipliers!
Simulating edge 2 of 6
Simulating edge 3 of 6
Simulating edge 4 of 6
Simulating edge 5 of 6
Simulating edge 6 of 6
```

Plot the alignment alongside the tree, skip sequences at ancestral nodes:

> plot(sim, num.pages = 1, plot.ancestors = FALSE)



#### 2.7 Simulating many replications

Constructing Sequence objects with a large number of sites is expensive, so it is a good idea to do that outside the cycle when simulating many replications with the same root sequence length.

Reusing the root sequence object is easy, but do not forget to do the modifications needed to get independent simulations (e.g. clearing the states of the root sequence, resampling the rate multipliers).

The following code illustrates how to simulate many replications under the JC69+d $\Gamma$  model.

Construct the root sequence object and attach the substitution process:

```
> p <- JC69()
> root.seq <- NucleotideSequence(length = 10)
> attachProcess(root.seq, p)
```

Read the required phylogeny from file (this will remain fixed in the simulated replications):

```
> tree <- read.tree("3taxa.nwk")</pre>
```

Simulate three replications. Note that the states are cleared and resampled; the rate multipliers are resampled as well. The resulting alignments are stored in files aln\_1.fas, aln\_2.fas, aln\_3.fas.

```
> for (i in 1:3) {
+ cat(paste("\n\nSimulating replication ", i, "\n\n", sep = ""))
+ clearStates(root.seq)
+ plusGamma(root.seq, p, 0.25)
```

```
sampleStates(root.seq)
      sim <- Simulate(PhyloSim(root.seq = root.seq, phylo = tree))</pre>
      saveAlignment(sim, file = paste("aln_", i, ".fas", sep = ""))
+ }
Simulating replication 1
Simulating edge 1 of 4
Simulating edge 2 of 4
Simulating edge 3 of 4
Simulating edge 4 of 4
Simulating replication 2
Simulating edge 1 of 4
Simulating edge 2 of 4
Simulating edge 3\ \text{of}\ 4
Simulating edge 4 of 4
Simulating replication 3
Simulating edge 1 of 4
Simulating edge 2 of 4
Simulating edge 3 of 4
Simulating edge 4 of 4
```

## 3 Advanced examples

Creating the tree file used in the advanced examples:

```
> cat("(((t2:0.1231,t4:0.1231):0.2131,(t3:0.0284,t5:0.0284):0.3078):0.1698,t1:0.5060);", file = "smalldemotree.nwk")
```

## 3.1 Simulating "domains" and heterogeneous evolution

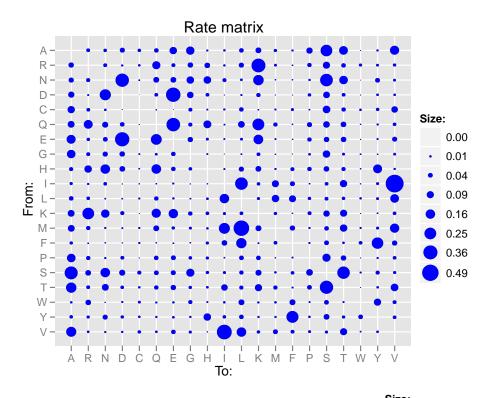
The following code illustrates how to set up a more complicated simulation of amino acid sequences involving "domains" and heterogeneous evolution.

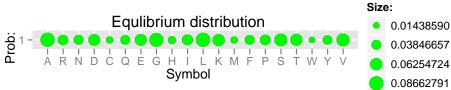
Use the 11() method to list the methods and virtual fields implemented in the Site class:

```
> 11(Site())

Class: Site
Inherits from: PSRoot Object
Fields (0):
Virtual fields (7):
  alphabet
  ancestral
  events
  processes
```

```
sequence
  state
  totalRate
Methods implemented in Site (8):
  as.character
  attachProcess
  checkConsistency
  detachProcess
  flagTotalRate
  is
  isAttached
  summary
Enable the "fast & careless mode":
> PSIM_FAST <- TRUE
Construct substitution process objects:
> wag <- WAG()
> jtt <- JTT()
> lg <- LG()
> pam <- PAM()
Summary of the object wag:
> summary(wag)
Name: Anonymous
Id: WAG:Anonymous:88960056
Alphabet:
  Type: Amino acid
  Symbols: A R N D C Q E G H I L K M F P S T W Y V
PAML data file:: PAMLdat/wag.dat
Unscaled rate matrix: not shown
Equilibrium distribution:
                                               N
                                   R
        Prob: 0.08662791 0.04397200 0.03908940 0.05704511 0.0193078 0.0367281 0.0580589
                                   Н
                                               Ι
                                                                   K
                                                                              М
                                                        L
        Prob: 0.08325181 0.02443130 0.04846600 0.086209 0.0620286 0.0195027 0.0384319 0.04576310
                                  Τ
        Prob: 0.0695179 0.06101271 0.01438590 0.0352742 0.07089561
Site specific parameters (1):
  Id: rate.multiplier
  Name: Rate multiplier
  Type: numeric
  Default value: 1
Get a plot of the instantaneous substitution matrix and equilibrium distribution of the wag process
(bubble plot):
> plot(wag, scale = 0.8)
```





Construct a continuous deletor process:

Construct the template sequence for the cont.ins.lg insertion process:

```
> templ.seq.wag <- AminoAcidSequence(length = 10)</pre>
```

Clone the template sequence for the cont.ins.wag process:

```
> templ.seq.lg <- clone(templ.seq.wag)</pre>
```

Construct continuous insertor process object cont.ins.wag:

Construct continuous insertor process object cont.ins.lg:

Set up the template sequences for the insertion processes:

```
> processes.site.wag <- list(wag, cont.ins.wag, cont.del)
> processes.site.lg <- list(lg, cont.ins.lg, cont.del)
> templ.seq.wag$processes <- list(processes.site.wag)
> templ.seq.lg$processes <- list(processes.site.lg)</pre>
```

Now the cont.ins.lg process samples the states from the equilibrium distribution of the LG model and cont.ins.wag samples the states from the WAG model.

Disabling write protection for the insertion processes:

```
> cont.ins.wag$writeProtected <- FALSE
> cont.ins.lg$writeProtected <- FALSE</pre>
```

Set the template sequence for the insertion processes:

```
> cont.ins.wag$templateSeq <- templ.seq.wag
> cont.ins.lg$templateSeq <- templ.seq.lg</pre>
```

Setting up the insert hook for the insertion processes:

Insert hook functions are called just before inserting the sequence generated by the insertion process. This function allows for arbitrary modifications to be made to the inserted sequence object. In this case the insert hook functions sample the site-process specific rate multipliers of the substitution processes from an invariants plus discrete gamma  $(+I+d\Gamma)$  model:

```
> cont.ins.wag$insertHook <- function(seq, target.seq, event.pos, insert.pos) {
+    plusInvGamma(seq, process = wag, pinv = 0.1, shape = 1)
+    return(seq)
+ }
> cont.ins.lg$insertHook <- function(seq, target.seq, event.pos, insert.pos) {
+    plusInvGamma(seq, process = lg, pinv = 0.1, shape = 1)
+    return(seq)
+ }</pre>
```

Now the processes are in place, so it is time to set up the root sequence.

```
> aa.seq <- AminoAcidSequence(length = 60)</pre>
```

Now we will create a pattern of processes. The "left linker", "core" and "right linker" regions evolve by different sets of processes. The core region has no indel processes attached, so its length will remain constant:

```
> process.pattern <- c(rep(list(list(wag, cont.del, cont.ins.wag)), times = 20),
+ rep(list(list(jtt)), times = 20), rep(list(list(lg, cont.del, cont.ins.lg)),
+ times = 20))</pre>
```

Apply the process pattern to the root sequence:

```
> aa.seq$processes <- process.pattern
```

Set up site specific rates by iterating over sites and sampling rates from a substitution process specific distribution:

Sample the states of the root sequence from the attached substitution processes:

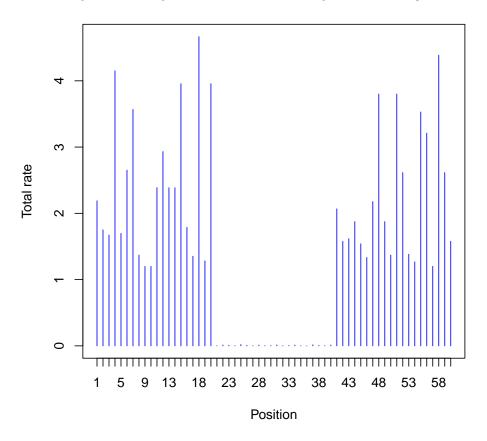
```
> sampleStates(aa.seq)
> print(aa.seq)
```

[1] "DTDEGNHENVGLGGTEANPTDKVGNGGIVWELGQLLSRELCLPIYKKAIQAIVGKEKVIL"

Plot the total rates of the sites:

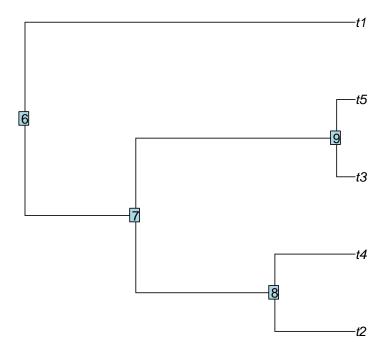
```
> plot(aa.seq)
```

## otal rate plot for sequence AminoAcidSequence: Anonymous: 733



```
Read in a tree using the ape package:
> tree <- read.tree(file = "smalldemotree.nwk")</pre>
Construct the simulation object and get an object summary:
> sim <- PhyloSim(phylo = tree, root.seq = aa.seq)</pre>
> summary(sim)
Name: Anonymous
Id: PhyloSim:Anonymous:84543752
Root Sequence big rate: 93.52729841301
Tree length: 1.4997
Phylo object details:
        Phylogenetic tree with 5 tips and 4 internal nodes.
        Tip labels:
        [1] "t2" "t4" "t3" "t5" "t1"
        Rooted; includes branch lengths.
Alignment: undefined
Log file: PhyloSim_Anonymous_84543752.log
Log level: -1
Plot the simulation object (tree with node labels):
```

> plot(sim)



A "node hook" is a function which accepts a Sequence object through the named argument "seq" and returns a Sequence object. After simulating the branch leading to the node, the resulting Sequence object is passed to the node hook and the returned object is used to simulate the downstream branches.

Create a node hook function:

```
> node.hook <- function(seq) {
+    for (site in seq$sites) {
+        if (isAttached(site, jtt)) {
+           attachProcess(site, pam)
+        }
+        return(seq)
+ }</pre>
```

Attach the hook to node 8:

```
> attachHookToNode(sim, node = 8, fun = node.hook)
```

This node.hook function will attach the pam substitution process to all sites which have the jtt process attached (the "core" region). The affected sites will evolve with a doubled rate by a combination of substitution processes (jtt and pam) in the clade defined by the node - (t4, t2).

Run the simulation:

> Simulate(sim)

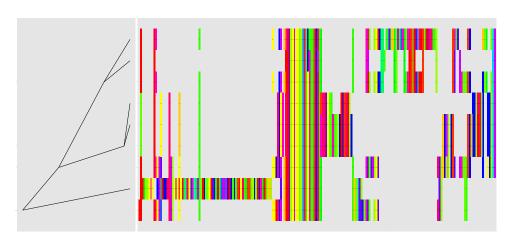
Save the resulting alignment, omitting the internal nodes:

> saveAlignment(sim, file = "example\_V3.1\_aln.fas", skip.internal = TRUE)

Plot the resulting alignment alongside the tree (including sequences at internal nodes):

> plot(sim, num.pages = 1)

Simulating edge 7 of 8 Simulating edge 8 of 8



Disable fast mode:

> rm(PSIM\_FAST)

#### 3.2 Evolving codon sequences

Enable "fast & careless" mode:

> PSIM\_FAST <- TRUE

Construct a GY94 codon substitution model:

> p <- GY94()

Set the transition/transversion rate ratio:

> p\$kappa = 2

Sample codon frequencies from a normal distribution:

```
> codon.freqs <- abs(rnorm(61, mean = 10, sd = 3))</pre>
> codon.freqs <- codon.freqs/sum(codon.freqs)</pre>
> p$equDist <- codon.freqs
```

Get object summary for p:

> summary(p)

Name: Anonymous

Id: GY94:Anonymous:94914304

Kappa: 2

Genetic code table id: 1

Alphabet:

Type: Standard

Symbols: AAA AAC AAG AAT ACA ACC ACG ACT AGA AGC AGG AGT ATA ATC ATG ATT CAA CAC CAG CAT CCA CCC C

Unscaled rate matrix: not shown

Equilibrium distribution:

AAAAAC AAG AAT ACA ACC ACG Prob: 0.009739455 0.02188547 0.01497060 0.0247754 0.008989882 0.01343272 0.006799762 AGC AGG AGT AGA ATA Prob: 0.01837423 0.02483477 0.007672109 0.01951341 0.01020393 0.008812812 0.01583113 ATG ATT CAA CAC CAG CAT Prob: 0.01713388 0.01315921 0.01251611 0.01068059 0.009016164 0.009314366 0.01931166 CCC CCG CCT CGA CGC CGG Prob: 0.01859599 0.02228580 0.01213243 0.01714229 0.01707703 0.01739439 0.008401107 CTC CTG GAC CTA CTT GAA GAG Prob: 0.01602727 0.01213363 0.02475264 0.00969484 0.02132718 0.01168857 0.001373409 GCC GCG GAT GCA GCT GGA Prob: 0.01720035 0.02081518 0.02260029 0.01754163 0.01932578 0.01474071 0.01604953 GGG GGT GTA GTC GTG GTT TAC Prob: 0.02940613 0.01879263 0.02249327 0.02351973 0.01237346 0.02038729 0.01504563 TAT TCA TCC TCG TCT TGC Prob: 0.01951445 0.01832491 0.02417719 0.02058495 0.01807729 0.01798332 0.0195153 TGT TTA TTC TTG

Site specific parameters (2):

Id: rate.multiplier Name: Rate multiplier

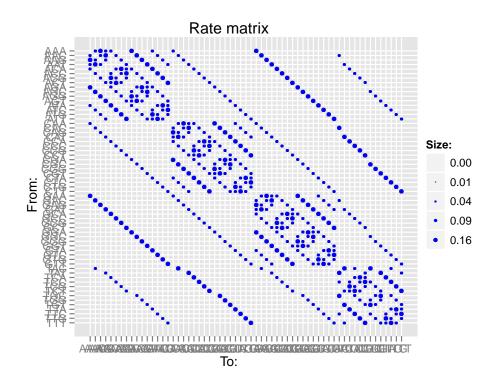
Type: numeric Default value: 1

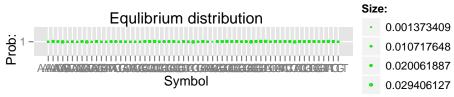
Id: omega Name: Omega Type: numeric Default value: 1

Get a bubble plot of p:

> plot(p, scale = 0.5)

Prob: 0.01514637 0.01739166 0.02220287 0.01858493 0.01121091





Construct a discrete deletor process:

$$>$$
 d <- DiscreteDeletor(rate = 1, sizes = 1:4, probs =  $c(4, 3, 2, 1)/10$ )

Construct a discrete insertor process inserting neutrally evolving sites:

Construct root sequence and attach process p:

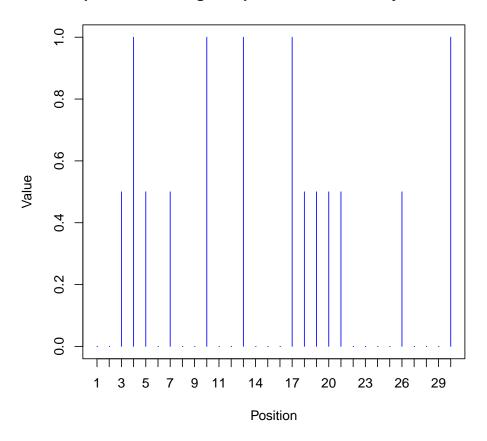
Sample omegas from a discrete model:

$$>$$
 omegaVarM3(s, p, omegas =  $c(0, 0.5, 1)$ , probs =  $c(2/4, 1/4, 1/4)$ )

Plot the omega values across sites:

> plotParametersAtSites(s, p, "omega")

## Plot of parameter omega for process GY94:Anonymous:949143



Sample states:

```
> sampleStates(s)
```

Construct the simulation object:

```
> sim <- PhyloSim(root.seq = s, phylo = read.tree("smalldemotree.nwk"))
```

Create a node hook function and attach to node 8:

```
> node.hook <- function(seq) {
+    setOmegas(seq, p, 1)
+    attachProcess(seq, d)
+    attachProcess(seq, i)
+    return(seq)
+ }
> attachHookToNode(sim, node = 8, fun = node.hook)
```

The node.hook function sets all omegas to 1 and attaches the insertion process i and deletion process d. Hence the sequences will evolve neutrally with indels in the clade defined by node 8 - (t4, t2).

Disable fast mode just before simulation in order to preserve branch statistics:

```
> rm(PSIM_FAST)
```

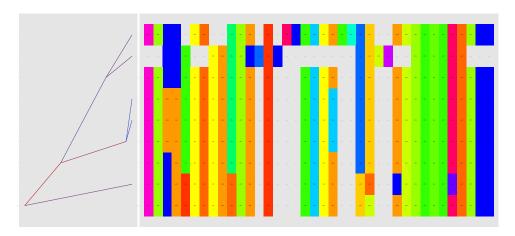
#### Run the simulation:

#### > Simulate(sim)

```
Simulating edge 1 of 8 Simulating edge 2 of 8 Simulating edge 3 of 8 Simulating edge 4 of 8 Simulating edge 5 of 8 Simulating edge 6 of 8 Simulating edge 7 of 8 Simulating edge 8 of 8
```

Plot the resulting alignment alongside the tree:

#### > plot(sim, num.pages = 1)

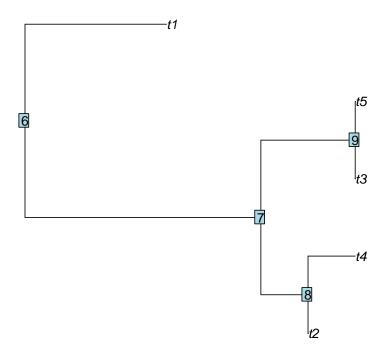


Export the nonsynonymous substitution counts as a phylo object:

> nsyn.subst <- exportStatTree(sim, "nr.nsyn.subst")</pre>

Plot the exported phylo object:

- > plot(nsyn.subst)
- > nodelabels()



Save the resulting alignment:

```
> saveAlignment(sim, file = "example_V3.2_aln.fas", )
```

## 3.3 Implementing a new process

The following code demonstrates how to implement a process which performs inverted duplications. In this example we simply replace the function object stored in the generateBy virtual field of a GeneralInsertor object. Alternatively, we could have defined a new class and set the insert generating function in the constructor method.

Enable fast & careless mode:

```
> PSIM_FAST <- TRUE
```

 ${\bf Construct\ a\ Discrete Insertor\ process:}$ 

```
> ivd <- DiscreteInsertor(rate = 0.06, sizes = c(4, 6), probs = c(2/3, + 1/3))
```

Set template sequence just to make the process object happy:

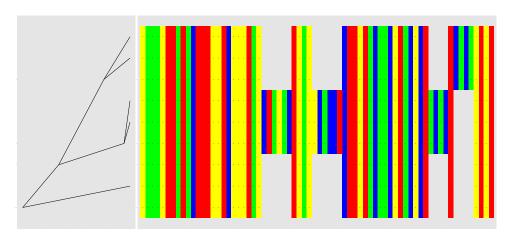
> ivd\$templateSeq <- NucleotideSequence(length = 1)</pre>

Replace the function object stored in the generateBy virtual field. See the documentation of the GeneralInsertor class:

```
> ivd$generateBy <- function(process = NA, length = NA, target.seq = NA,
      event.pos = NA, insert.pos = NA) {
      target.length <- target.seq$length</pre>
     positions <- (insert.pos + 1):(insert.pos + length)</pre>
     positions <- positions[positions > 0 & positions <= target.length]</pre>
      insert <- copySubSequence(target.seq, positions, process)</pre>
      revComp.NucleotideSequence(insert)
      setRateMultipliers(insert, ivd, 0)
      return(insert)
+ }
Now we have a process which performs inverted duplications.
Construct a JC69 process object:
> p <- JC69()
Construct root sequence object:
> s <- NucleotideSequence(length = 50)
Attach processes via virtual field:
> s$processes <- list(list(p, ivd))</pre>
Sample states from the equilibrium distribution of the attached processes:
> sampleStates(s)
Detach the substitution process:
> detachProcess(s, p)
Create among-sites rate variation for the inverted duplication process by sampling rate multipliers from
an I+G model:
> plusInvGamma(s, ivd, pinv = 0.7, shape = 0.5)
Construct simulation object:
> sim <- PhyloSim(root.seq = s, phylo = read.tree("smalldemotree.nwk"))
Run simulation:
> Simulate(sim)
!! WARNING: fast & careless mode is on, most of the error checking is omitted! !!
     Please note that this also disables the saving of branch statistics.
```

Plot tree and alignment:

> plot(sim, num.pages = 1)



Save alignment:

> saveAlignment(sim, file = "example\_V3.3.fas")

Disable fast & careless mode:

> rm(PSIM\_FAST)

#### 3.4 Evolving a genomic region containing a "gene"

The following code demonstrates how to simulate a genomic region containing a gene with introns and noncoding regions.

We will simulate the following features:

- NC1 (length: 1000 nucleotide sites) noncoding region 1, evolving under a GTR substitution model with deletions and insertions (of Cs only).
- E1 (length: 400 codon sites) exon 1, evolving under a nucleotide-scaled GY94 codon model with small indels.
- I1-5 (length: 200 nucleotide sites) introns 1-5, evoling under a F84 substitution model with indels
- E2-5 (length: 200 codon sites) exons 2-5, evolving under a nucleotide-scaled GY94 codon model.

- E6 (length: 400 codon sites) exon 6, evolving under a nucleotide-scaled GY94 codon model with small indels.
- NC2 (length: 1400 nucleotide sites) noncoding region 2, evolving under a K80 substitution model.
- Invariable start codon and splice sites.
- A special substitution process for stop codons.

The total length of the Sequence object is 5000. We will also simulate heterogeneity of nucleotide substitution rates, indel rates and omega ratios.

Enable fast & careless mode:

```
> PSIM_FAST <- TRUE
```

Create a list holding partition information and set partition properties:

Partition E1:

```
> part$e1 <- list(type = "first.exon", len = 400, subst = NA, ins = NA, del = NA, omegas = c(0, 1, 1.1), omega.probs = c(4, 2, 1)/5)
```

> part\$nc1\$del <- DiscreteDeletor(rate = 0.025, sizes = 1:6, probs = 6:1/21)

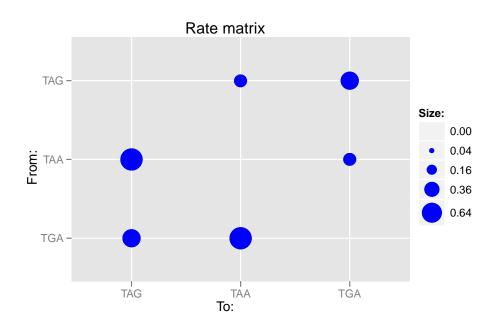
template.seq = NucleotideSequence(string = "CCCCCC"))

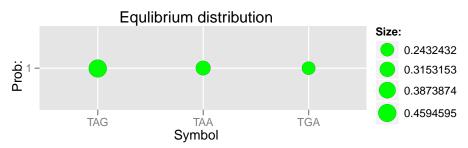
Construct a "nucleotide-scaled" codon model which can be used in mixed sequences:

Partitions E2-5 and I1-4:

```
> del.introns <- DiscreteDeletor(rate = 0.025, sizes = 1:8, probs = 8:1/36)
> ins.introns <- DiscreteInsertor(rate = 0.025, sizes = 1:8, probs = 8:1/36,
+ template.seq = NucleotideSequence(length = 8, processes = list(list(f84))))
> for (i in 1:4) {
+ part[[paste("i", i, sep = "")]] <- list(type = "intron", len = 200,</pre>
```

```
subst = f84, ins = ins.introns, del = del.introns, gamma.shape = abs(rnorm(1,
               mean = 1, sd = 0.2)))
      part[[paste("e", i + 1, sep = "")]] <- list(type = "exon", len = 200,</pre>
          subst = gy94.e2_5, ins = NA, del = NA, omegas = c(0, 0.5), omega.probs = c(3/4, 0.5)
+ }
Partition I5:
> part$i5 <- list(type = "intron", len = 200, subst = f84, ins = ins.introns,
+ del = del.introns, gamma.shape = abs(rnorm(1, mean = 1, sd = 0.1)))
> rm(f84, gy94.e2_5, del.introns, ins.introns)
Partition E6:
> part$e6 <- list(type = "last.exon", len = 400, subst = NA, ins = NA,
     del = NA, omegas = c(0, 1, 1.1), omega.probs = c(4, 2, 1)/5)
> tmp <- clone(part$e1$subst)</pre>
> tmp$kappa <- 0.5
> part$e6$subst <- tmp
> part$e6$del <- DiscreteDeletor(rate = 0.025, sizes = 1:4, probs = c(4,
      3, 2, 1)/10)
> part$e6$ins <- DiscreteInsertor(rate = 0.025, sizes = 1:2, probs = c(1/3,
      2/3), template.seq = CodonSequence(length = 2, processes = list(list(part$e6$subst))))
Partition NC2:
> part$nc2 <- list(type = "noncoding", len = 1400, subst = K80(rate.params = list(Alpha = 2,
      Beta = 1)), ins = NA, del = NA, gamma.shape = 0.5)
Construct root sequence object:
> s <- Sequence(length = 5000)
Construct Alphabet objects:
> nuc <- NucleotideAlphabet()</pre>
> cod <- CodonAlphabet()</pre>
Construct an Alphabet object containing the stop codons:
> stop.alphabet \leftarrow Alphabet(symbols = c("TAG", "TAA", "TGA"))
Construct a substitution process acting on stop codons only:
> stop.subst <- GeneralSubstitution(alphabet = stop.alphabet, rate.list = list(`TAG->TAA` = 1,
      TAG \rightarrow TGA = 2, TAA \rightarrow TAG = 3, TAA \rightarrow TGA = 1, TGA \rightarrow TAG = 2,
      TGA \rightarrow TAA = 3)
Get a bubble plot of stop.subst:
> plot(stop.subst, scale = 0.5)
```





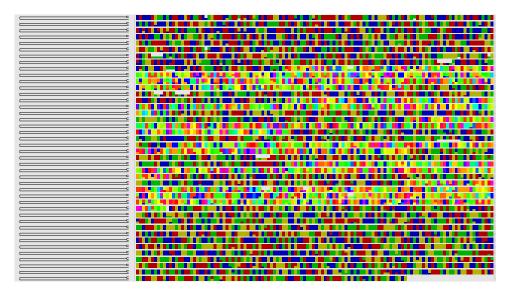
Iterate over partitions, set up processes and rate multipliers, fix start codon and splice sites, attach the stop.subst process to stop codon:

```
> pos <- 0
> for (i in part) {
      beg <- pos + 1
      end <- pos + i$len
      range <- beg:end
      if ((i$type == "noncoding") | (i$type == "intron")) {
          setAlphabets(s, list(nuc), range)
      }
      else {
          setAlphabets(s, list(cod), range)
      if (!is.na(i$del)) {
          attachProcess(s, i$del, range)
      }
      if (!is.na(i$ins)) {
          attachProcess(s, i$ins, range)
      if ((i$type == "noncoding") | (i$type == "intron")) {
          attachProcess(s, i$subst, range)
          plusInvGamma(this = s, process = i$subst, pinv = 0.6, shape = i$gamma.shape,
              range)
          if (i$type == "intron") {
```

```
setStates(s, c("G", "T", "A", "G"), c(beg, beg + 1, end - G)
                 1, end))
             setRateMultipliers(s, i$subst, 0, c(beg, beg + 1, end -
                 1, end))
             setInsertionTolerance(s, i$ins, 0, c(beg, beg + 1, end -
                 1, end))
             setDeletionTolerance(s, i$del, 0, c(beg, beg + 1, end -
                 1, end))
         }
     }
     else if (i$type == "exon") {
         attachProcess(s, i$subst, range)
         omegaVarM3.CodonSequence(s, i$subst, i$omegas, i$omega.probs,
             range)
     if (i$type == "first.exon") {
         attachProcess(s, i$subst, range)
         setStates(s, "ATG", beg)
         setRateMultipliers(s, i$subst, 0, beg)
         setInsertionTolerance(s, i$ins, 0, beg)
         setDeletionTolerance(s, i$del, 0, beg)
     }
     if (i$type == "last.exon") {
         attachProcess(s, i$subst, range)
         detachProcess(s, i$subst, end)
         setAlphabets(s, list(stop.alphabet), end)
         attachProcess(s, stop.subst, end)
         sampleStates(s, end)
         setInsertionTolerance(s, i$ins, 0, end)
         setDeletionTolerance(s, i$del, 0, end)
     }
     pos <- end
Sample remaining site states:
> sampleStates(s)
Generate a random coalescent tree with 3 leaves and construct simulation object:
> sim <- PhyloSim(root.seq = s, phylo = rcoal(3))</pre>
Scale tree length to 0.15:
> scaleTree(sim, 0.15/sim$treeLength)
Run simulation:
> Simulate(sim)
!! WARNING: fast & careless mode is on, most of the error checking is omitted! !!
     Please note that this also disables the saving of branch statistics.
```

Plot tree and alignment, omitting ancestral sequences:

> plot(sim, num.pages = "auto", plot.ancestors = FALSE)



Save alignment:

```
> saveAlignment(sim, file = "example_V3.4.fas")
```

Disable fast & careless mode:

> rm(PSIM\_FAST)

#### 4 Details of the fast field deletion model

A natural way to incorporate deletions into the Gillespie framework is to assign an individual rate to every possible deletion event. Modelling in this manner is extremely general but requires a lot of specification: not only individual sites' tolerance to deletion but also of how they interact with neighbouring sites. Instead we propose a more restricted "field model" of deletion that generalises previous work to allow the rate at which deletions occur to vary across the sequence but only requires one parameter per site – its deletion tolerance – to be specified. Under this model, deletions are proposed in same manner as other events, specifying a rate of occurrence and a distribution of lengths and then assuming that the location and orientation of the deletion is chosen uniformly, but proposed deletions may then be rejected based on sites they propose to remove.

Firstly consider only single-site deletions and let each site, i, in the sequence have an associated deletion tolerance parameter,  $d_i \in [0,1]$ , representing the probability that it is actually deleted given that a deletion is proposed. A proposal / acceptance step is equivalent to just proposing at a slower rate, so this is just a special case of the most general model but one that can be implemented efficiently as a single

Gillespie event (a deletion occurred somewhere) rather than a large number of slower events (treating every possible deletion separately). Sites where  $d_i = 1$  are deleted at the background rate, sites with  $d_i < 1$  are deleted more slowly, and sites with  $d_i = 0$  are never deleted. For proposed deletions that span multiple sites,  $\mathcal{I}$ , each site is considered independently and the proposed deletion is accepted if and only if every site accepts it: the total probability of acceptance is therefore  $\prod_{i \in \mathcal{I}} d_i$ .

It is natural to think of the background rate of deletion as a neutral rate but this is not necessary and can lead to the Gillespie algorithm becoming inefficient: for example, an extremely deletion intolerant sequence will reject almost all deletions proposed and so waste many steps. Instead we can rescale the process ("fast field deletion model") so that deletions are proposed at a rate equal to what would occur if the entire sequence had a deletion tolerance equal to its most tolerant site (deletion tolerance d) and then accept a deletion spanning sites  $\mathcal{I}$  with probability  $d_i/d$ . Table 1 gives the rate scaling factor and distribution of deletion lengths after scaling for a variety of distributions that could be used to model deletion length. In most cases, the rescaled distribution of deletion lengths is a member of the same family as the "neutral" process.

Distribution	Density	Scale factor	Rescaled distribution
Geometric, $Geom(\lambda)$	$\lambda^{k-1}(1-\lambda)$	$d\frac{1-\lambda}{1-d\lambda}$	$Geom(d\lambda)$
	$a^{-\lambda} \lambda k - 1$	. \(1 J)	- (1)
Poisson + 1, $Po_{+1}(\lambda)$	$\frac{e^{-\lambda}\lambda^{k-1}}{\Gamma(k)}$	$de^{-\lambda(1-d)}$	$Po_{+1}(d\lambda)$
Conway-Maxwell Poisson + 1, $CMP_{+1}(\lambda, \nu)$	$rac{\lambda^{k-1}}{\Gamma(k)^{ u}\mathcal{Z}(\lambda, u)}$	$d\frac{\mathcal{Z}(\lambda,\nu)}{\mathcal{Z}(d\lambda,\nu)}$	$CMP_{+1}(d\lambda, \nu)$
Negative Binomial + 1, $NB_{+1}(\lambda, r)$	$\frac{\Gamma(r+k-1)}{\Gamma(k)\Gamma(r)}(1-\lambda)^r\lambda^{k-1}$	$d\frac{(1-\lambda)^r}{(1-d\lambda)^r}$	$NB_{+1}(d\lambda, r)$

Table 1: The rate scaling factor and distribution of deletion lengths after scaling for a variety of distributions that could be used to model deletion length.