# Package 'spray'

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Type Package

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<b>Description</b> Sparse arrays interpreted as multivariate polynomials.  Uses 'disordR' discipline (Hankin, 2022, <doi:10.48550 arxiv.2210.03856="">). To cite the package in publications please use Hankin (2022) <doi:10.48550 arxiv.2210.10848="">.</doi:10.48550></doi:10.48550>
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## **Description**

Functionality for sparse arrays, with emphasis on their interpretation as multivariate polynomials.

#### **Details**

Base R has the capability of dealing with arbitrary dimensioned numerical arrays, with the array class.

A *sparse array* is a type of array in which nonzero elements are stored along with an index vector describing their coordinates. This allows for efficient storage and manipulation as base arrays often require the storing of many zero elements which consume computational and memory resources.

In the package, sparse arrays are represented as objects of class spray. They use the C++ standard template library (STL) map class, with keys being (unsigned) integer vectors, and values floats.

One natural application of sparse arrays, for which the package was written, is multivariate polynomials and the package vignette presents an extended discussion. Note that other interpretations exist: the **stokes** and **weyl** packages interpret spray objects as differential forms and elements of a Weyl algebra respectively.

# Author(s)

Robin K. S. Hankin

```
# define a spray using a matrix of indices and a vector of values:
M <- matrix(sample(0:3,21,replace=TRUE),ncol=3)
a <- spray(M,sample(7))

# there are many pre-defined simple sprays:
b <- homog(3,4)

# arithmetic operators work:
a + 2*b
a - a*b^2/4
a+b</pre>
```

arity 3

```
# we can sum over particular dimensions:
asum(a+b,1)

# differentiation is supported:
deriv(a^6,2)

# extraction and replacement work as expected:
b[1,2,1]
b[1,2,1,drop=TRUE]

b[diag(3)] <- 3</pre>
```

arity

The arity of a spray object

# Description

The arity of a spray object: the number of indices needed to retrieve an entry, or the number of columns in the index matrix.

# Usage

arity(S)

# Arguments

S

a spray object

# Value

Returns an integer

# Author(s)

Robin K. S. Hankin

```
(a <- rspray())
arity(a)</pre>
```

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as.array

Coerce spray objects to arrays

#### **Description**

Coerces spray objects to arrays. Includes off-by-one functionality via option offbyone.

#### Usage

```
## S3 method for class 'spray'
as.array(x, offbyone=FALSE, compact=FALSE, ...)
## S3 method for class 'spray'
dim(x)
```

#### **Arguments**

x spray object

offbyone Boolean with default FALSE meaning to interpret the index entries as positions

in their dimension, and TRUE meaning to add one to index values so that zero

entries appear in the first place

compact Boolean with default FALSE meaning to translate the spray as is, and TRUE mean-

ing to add constants to each column of the index matrix so that the resulting array

is as small as possible

... Further arguments, currently ignored

## **Details**

Argument of fbyone defaults to FALSE; but if it is set to TRUE, it effectively adds one from the index matrix, so a zero entry in the index matrix means the first position in that dimension.

After the subtraction, if performed, the function will not operate if any index is less than 1.

## Value

Returns an array of dimension dim(S). The "meat" of the function is

```
out <- array(0, dS)
out[ind] <- coeffs(S)</pre>
```

#### Author(s)

Robin K. S. Hankin

```
(M <- matrix(sample(0:4,28,replace=TRUE),ncol=4))
(S <- spray(M,sample(7),addrepeats=TRUE))
as.array(S,offbyone=TRUE)  # a large object! sprays are terse

S <- spray(matrix(sample(1:4,28,replace=TRUE),ncol=4),sample(7))
A <- as.array(S)  # S has no zero indices [if it did, we would need to use offbyone=TRUE]
stopifnot(all(S[index(S),drop=TRUE] == A[index(S)]))</pre>
```

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as.character

Coerce spray objects to character

## **Description**

Coerces spray objects to a character string or disord character vector.

#### Usage

```
## S3 method for class 'spray'
as.character(x, ..., split=FALSE)
```

# Arguments

x spray object

... Further arguments, currently ignored

split Boolean with default FALSE meaning to return a length-one character vector, and

TRUE meaning to return a disord object with elements being terms, coerced to

character

#### **Details**

The method uses print\_spray\_polyform() and as such is sensitive to option sprayvars, but not polyform.

## Author(s)

Robin K. S. Hankin

# **Examples**

```
as.character(rspray())
as.character(rspray(),split=TRUE)
```

as.function.spray

Coerce a spray object to a function

## **Description**

Coerce a spray object to a function

## Usage

```
## S3 method for class 'spray'
as.function(x,...)
```

# Arguments

x spray object, interpreted as a multivariate polynomial

... Further arguments, currently ignored

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#### Value

Returns a function; this function returns a numeric vector.

#### Note

Coercion is possible even if some indices are zero or negative. The function is not vectorized in the arity of its argument.

## Author(s)

Robin K. S. Hankin

# **Examples**

```
(S <- spray(matrix(1:6,3,2),1:3))
(f <- as.function(S))
f(2:3) == 3*2^3*3^6 + 2*2^2*3^5 + 1*2^1*3^4  # should be TRUE

S1 <- spray(matrix(sample(-2:2,replace=TRUE,21),ncol=3),rnorm(7),addrepeats=TRUE)
S2 <- spray(matrix(sample(-2:2,replace=TRUE,15),ncol=3),rnorm(5),addrepeats=TRUE)

f1 <- as.function(S1)
f2 <- as.function(S2)

f3 <- as.function(S1*S2)

x <- 4:6
f1(x)*f2(x)-f3(x)  # should be zero

# coercion is vectorized:
f1(matrix(1:33,ncol=3))</pre>
```

asum

Sum over dimension margins

# Description

Sum over specified dimension margins.

#### Usage

```
## S3 method for class 'spray'
asum(S, dims, drop=TRUE, ...)
asum_inverted(S, dims)
process_dimensions(S,dims)
```

constant 7

# Arguments

S	spray object
dims	Vector of strictly positive integers corresponding to dimensions to be summed over
drop	Boolean, with default TRUE meaning to drop the summed dimensions, and FALSE meaning to retain them.
	Further arguments, currently ignored

#### **Details**

Function asum.spray() is the method for asum(). This takes a spray, and a vector of integers corresponding to dimensions to be summed over.

Function asum\_inverted() is the same, but takes a vector of integers corresponding to dimensions not to sum over. This function is here because there is nice C++ idiom for it.

Function process\_dimensions() ensures that the dims argument is consistent with the spray S and returns a cleaned version thereof.

#### Value

Returns a spray object.

#### Author(s)

Robin K. S. Hankin

# **Examples**

```
S <- spray(matrix(sample(0:2,60,replace=TRUE),ncol=3),addrepeats=TRUE)
S
asum(S,1)
asum(S,1:2)
asum(S,1:2,drop=FALSE)
asum(S,c(1,3)) == asum_inverted(S,2)</pre>
```

constant

Get or set the constant term of a spray object

# Description

The constant term of a spray object is the coefficient corresponding to an index of all zeros. These functions get or set the constant of a spray object.

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#### Usage

```
is.constant(x)
constant(x,drop=FALSE)
constant(x) <- value
drop(x)</pre>
```

## **Arguments**

x Object of class spray

value Numeric value to set the constant coefficient to

drop Boolean, with default FALSE meaning to return a spray object and TRUE meaning

to return a numeric value

## Value

In function constant(), return the coefficient, or a constant multivariate polynomial, depending on the value of drop.

#### Note

The behaviour of the drop argument (sort of) matches that of the spray extractor method. Function drop() returns the elements of the coefficients.

Function constant() ensures that zero spray objects retain the argument's arity.

It might have been better to call is.constant() is.scalar(), for consistency with the stokes and clifford packages. But this is not clear.

# Author(s)

Robin K. S. Hankin

#### See Also

Extract

```
(S <- spray(partitions::blockparts(rep(2,3),3,TRUE)))
constant(S)
constant(S) <- 33
S
drop(constant(S,drop=FALSE))</pre>
```

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deriv

Partial differentiation of spray objects

# Description

Partial differentiation of spray objects interpreted as multivariate polynomials

## Usage

```
## S3 method for class 'spray'
deriv(expr, i , derivative = 1, ...)
aderiv(S,orders)
```

## **Arguments**

expr A spray object, interpreted as a multivariate polynomial

i Dimension to differentiate with respect to

derivative How many times to differentiate
... Further arguments, currently ignored

S spray object

orders The orders of the differentials

#### **Details**

Function deriv. spray() is the method for generic spray(); if S is a spray object, then spray(S,i,n) returns  $\partial^n S/\partial x_i^n = S^{(x_i,\dots,x_i)}$ .

Function aderiv() is the generalized derivative; if S is a spray of arity 3, then aderiv(S,c(i,j,k)) returns  $\frac{\partial^{i+j+k}S}{\partial x_1^i\partial x_2^j\partial x_3^k}$ .

## Value

Both functions return a spray object.

#### Author(s)

Robin K. S. Hankin

## See Also

asum

```
(S <- spray(matrix(sample(-2:2,15,replace=TRUE),ncol=3),addrepeats=TRUE)) \\ deriv(S,1) \\ deriv(S,2,2) \\ \# \ differentiation \ is \ invariant \ under \ order:
```

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```
aderiv(S,1:3) == deriv(deriv(deriv(S,1,1),2,2),3,3)

# Leibniz's rule:
S1 <- spray(matrix(sample(0:3,replace=TRUE,21),ncol=3),sample(7),addrepeats=TRUE)
S2 <- spray(matrix(sample(0:3,replace=TRUE,15),ncol=3),sample(5),addrepeats=TRUE)
S1*deriv(S2,1) + deriv(S1,1)*S2 == deriv(S1*S2,1)

# Generalized Leibniz:
aderiv(S1*S2,c(1,1,0)) == (
aderiv(S1,c(0,0,0))*aderiv(S2,c(1,1,0)) +
aderiv(S1,c(0,1,0))*aderiv(S2,c(1,0,0)) +
aderiv(S1,c(1,0,0))*aderiv(S2,c(0,1,0)) +
aderiv(S1,c(1,1,0))*aderiv(S2,c(0,0,0))
)</pre>
```

Extract.spray

Extract or Replace Parts of a spray

## **Description**

Extract or replace subsets of sprays.

## Usage

```
## S3 method for class 'spray'
S[..., drop=FALSE]
## S3 replacement method for class 'spray'
S[index, ...] <- value</pre>
```

#### Arguments

S	A spray object
index	elements to extract or replace
value	replacement value
	Further arguments
drop	Boolean, with default FALSE meaning to return a spray object and TRUE meaning to drop the spray structure and return a numeric vector

#### **Details**

These methods should work as expected, although the off-by-one issue might be a gotcha. **disordR** discipline is enforced where appropriate.

In S[index,...], argument drop is FALSE by default, in which case a spray object is returned. If drop is TRUE a numeric vector is returned, with elements corresponding to the rows of index. Compare coeffs(S), which returns a disord object; in S[index,drop=TRUE], the rows of index specify a unique order for the return value.

If a <- spray(diag(3)), for example, then idiom such as a[c(1,2,3)] cannot work, because one would like a[1,2,3] and a[1:3,2,3] to work.

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If p <-1:3, then one might expect idiom such as S[1, p, 1:3] to work but this is problematic and a discussion is given in inst/missing\_accessor.txt.

Functions spray\_extract\_disord() and spray\_replace\_disord() are low-level helper functions which implement idiom such as a [coeffs(a) < 3] and [coeffs(a) < 3] < -99.

## **Examples**

```
(a <- spray(diag(5)))</pre>
a[rbind(rep(1,5))] <- 5
a[3,4,5,3,1] # the NULL polynomial
a[0,1,0,0,0]
a[0,1,0,0,0,drop=TRUE]
a[2,3:5,4,3,3] < -9
options(polyform = TRUE)  # print as a multivariate polynomial
options(polyform = FALSE) # print in sparse array form
(S1 <- spray(diag(5),1:5))
(S2 <- spray(1-diag(5),11:15))
(S3 \leftarrow spray(rbind(c(1,0,0,0,0),c(1,2,1,1,1))))
S1[] <- 3
S1[] <- S2
S1[S3] <- 99
S1
S <- rspray()</pre>
S[coeffs(S) > 4]
S[coeffs(S) < 6] < - 99
```

homog

Various functions to create simple spray objects

## **Description**

Various functions to create simple spray objects such as single-term, homogeneous, and constant multivariate polynomials.

#### Usage

```
product(power)
homog(d,power=1)
```

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```
linear(x,power=1)
lone(n,d=n)
one(d)
as.id(S)
xyz(d)
```

# Arguments

d	An integer; generally, the dimension or arity of the resulting spray object
power	Integer vector of powers
x	Numeric vector of coefficients
S	A spray object
n	In function lone(), the term to raise to power 1

## Value

All functions documented here return a spray object

## Note

The functions here are related to their equivalents in the **multipol** package, but are not exactly the same.

Function zero() is documented at zero.Rd, but is listed below for convenience.

# Author(s)

Robin K. S. Hankin

#### See Also

```
constant, zero
```

```
product(1:3)
                                      x * y^2 * z^3
homog(3)
homog(3,2)
                                      x^2 + xy + xz + y^2 + yz + z^2
linear(1:3)
                                     1*x + 2*y + 3*z
linear(1:3,2)
                                     1*x^2 + 2*y^2 + 3*z^2
lone(3)
lone(2,3)
                                     У
one(3)
                               #
                                     1
                               #
zero(3)
                                     0
                               #
xyz(3)
                                     xyz
```

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knight

Generating function for a chess knight and king

## **Description**

Generating function for a chess knight and king on an arbitrarily-dimensioned chessboard

## Usage

```
knight(d=2)
king(d=2)
```

## **Arguments**

d

Dimensionality of the board, defaulting to 2

#### Value

Returns the generating function of the piece in question.

#### Note

The pieces are forced to move; if they have the option of not moving, add 1 to the returned spray. The vignette contains a short discussion.

# Author(s)

Robin K. S. Hankin

```
knight() # default 2D chess board
king() # ditto

knight()^2 # generating function for two knight's moves

## How many ways can a knight return to its starting square in 6 moves?
constant(knight()^6)

## How many in 6 or fewer?
constant((1+knight())^6)

## Where does a randomly-moving knight end up?
d <- xyz(2)
kt <- (1+knight())*d^2/9
persp(1:25,1:25,as.array(d*kt^6))

## what is the probability that a 4D king is a knight's move from
## (0,0,0,0) after 6 moves?

sum(coeffs(((king(4)/80)^4)[knight(4)]))</pre>
```

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nterms

Number of nonzero terms in a spray object

# Description

Number of nonzero terms in a spray object

## Usage

```
nterms(x)
## S3 method for class 'spray'
length(x)
```

# **Arguments**

Х

Object of class spray

## **Details**

Number of nonzero terms in a spray object. Function length() is defined so that seq\_along() works as expected

## Author(s)

Robin K. S. Hankin

# **Examples**

```
(a <- rspray())
nterms(a)
seq_along(a)</pre>
```

ooom

One-over-one-minus for spray objects

# Description

One-over-one-minus for spray objects; the nearest to 'division' that we can get.

# Usage

```
ooom(S, n)
```

## **Arguments**

S object of class spray

n Order of the approximation

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#### **Details**

Returns the Taylor expansion to order n of 1/(1-S), that is,  $1+S+S^2+S^3+\cdots+S^n$ .

#### Value

Returns a spray object of the same arity as S.

#### Note

Uses Horner's method for efficiency

#### Author(s)

Robin K. S. Hankin

#### **Examples**

```
(x <- spray(matrix(1)))
ooom(x,5) # 1 + x + x^2 + x^3 + x^4 + x^5

(a <- homog(4,2))
d <- (1-a)*ooom(a,3)

constant(d) # should be 1
rowSums(index(d)) # a single 0 and lots of 8s.</pre>
```

Ops.spray

Arithmetic Ops Group Methods for sprays

# **Description**

Allows arithmetic operators to be used for spray calculations, such as addition, multiplication, division, integer powers, etc. Objects of class spray are interpreted as sparse multivariate polynomials.

# Usage

```
## S3 method for class 'spray'
Ops(e1, e2 = NULL)
spray_negative(S)
spray_times_spray(S1,S2)
spray_times_scalar(S,x)
spray_plus_spray(S1,S2)
spray_plus_scalar(S,x)
spray_power_scalar(S,n)
spray_eq_spray(S1,S2)
spray_eq_numeric(S1,x)
```

## **Arguments**

```
e1,e2,S,S1,S2 Objects of class spray, here interpreted as sparse multivariate polynomials x Real valued scalar

Non-negative integer
```

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#### **Details**

The function Ops.spray() passes unary and binary arithmetic operators ("+", "-", "\*", "/", "==", and "^") to the appropriate specialist function.

The most interesting operators are "\*" and "+" which execute multivariate polynomial multiplication and addition respectively.

Testing for equality uses spray\_eq\_spray(). Note that spray\_eq\_spray(S1,S2) is algebraically equivalent to is.zero(S1-S2), but faster (FALSE is returned as soon as a mismatch is found).

#### Value

The functions all return spray objects except "==", which returns a logical.

#### Author(s)

Robin K. S. Hankin

#### See Also

ooom

## **Examples**

```
M <- matrix(sample(0:3,21,replace=TRUE),ncol=3)
a <- spray(M,sample(7))
b <- homog(3,4)

# arithmetic operators mostly work as expected:
a + 2*b
a - a*b^2/4
a+b

S1 <- spray(partitions::compositions(4,3))
S2 <- spray(diag(3)) # S2 = x+y+z

stopifnot( (S1+S2)^3 == S1^3 + 3*S1^2*S2 + 3*S1*S2^2 + S2^3 )</pre>
```

pmax

Parallel maxima and minima for sprays

## **Description**

Parallel (pairwise) maxima and minima for sprays.

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## Usage

```
maxpair_spray(S1,S2)
minpair_spray(S1,S2)
## S3 method for class 'spray'
pmax(x, ...)
## S3 method for class 'spray'
pmin(x, ...)
```

#### **Arguments**

```
x,S1,S2 Spray objects
... spray objects to be compared
```

#### **Details**

Function maxpair\_spray() finds the pairwise maximum for two sprays. Specifically, if  $S3 < maxpair_spray(S1,S2)$ , then S3[v] = max(S1[v],S2[v]) for every index vector v.

Function pmax.spray() is the method for the generic pmax(), which takes any number of arguments. If  $S3 \leftarrow maxpair_spray(S1,S2,...)$ , then S3[v] = max(S1[v],S2[v],...) for every index vector v.

Function pmax.spray() operates right-associatively:

pmax(S1,S2,S3,S4) == f(S1,f(S2,f(S3,S4))) where f() is short for  $maxpair\_spray()$ . So if performance is important, put the smallest spray (in terms of number of nonzero entries) last.

In these functions, a scalar is interpreted as a sort of global maximum. Thus if S3 <- pmax(S,x) we have S3[v] == max(S[v],x) for every index v. Observe that this operation is not defined if x>0, for then there would be an infinity of v for which S3[v] != 0, an impossibility (or at least counter to the principles of a sparse array). Note also that x cannot have length > 1 as the elements of a spray object are stored in an arbitrary order, following disordR discipline.

Functions minpair\_spray() and pmin.spray() are analogous. Note that minpair\_spray(S1,S2) is algebraically equivalent to -pmax\_spray(-S1,-S2); see the examples.

The value of pmax(S) is problematic. Suppose all(coeffs(S)<0); the current implementation returns pmax(S)==S but there is a case for returning the null polynomial.

## Value

Returns a spray object

## Author(s)

Robin K. S. Hankin

```
S1 <- rspray(100,vals=sample(100)-50)
S2 <- rspray(100,vals=sample(100)-50)
S3 <- rspray(100,vals=sample(100)-50)
# following comparisons should all be TRUE:
jj <- pmax(S1,S2,S3)</pre>
```

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```
jj == maxpair_spray(S1,maxpair_spray(S2,S3))
jj == maxpair_spray(maxpair_spray(S1,S2),S3)

pmax(S1,S2,S3) == -pmin(-S1,-S2,-S3)
pmin(S1,S2,S3) == -pmax(-S1,-S2,-S3)

pmax(S1,-Inf) == S1
pmin(S1, Inf) == S2

pmax(S1,-3)

## Not run:
pmax(S1,3) # not defined

## End(Not run)
```

print.spray

Print methods for spray objects

## **Description**

Print methods for spray objects with options for printing in matrix form or multivariate polynomial form

# Usage

```
## S3 method for class 'spray'
print(x, ...)
print_spray_matrixform(S)
print_spray_polyform(S,give=FALSE)
```

#### **Arguments**

x,S	spray object
give	Boolean, with default FALSE meaning to print the value of S, and TRUE meaning to return a string (without nice formatting); used in as.character.spray()
	Further arguments (currently ignored)

#### **Details**

The print method, print.spray(), dispatches to helper functions print\_spray\_matrixform() and print\_spray\_polyform() depending on the value of option polyform; see the examples section.

Option sprayvars is a character vector with entries corresponding to the variable names for printing. The sprayvars option has no algebraic significance: all it does is affect the print method.

Note that printing a spray object (in either matrix form or polynomial form) generally takes much longer than calculating it.

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#### Value

Returns its argument invisibly.

## Note

There are a couple of hard-wired symbols for multiplication and equality which are defined near the top of the helper functions.

## Author(s)

Robin K. S. Hankin

# **Examples**

```
(a <- spray(diag(3)))

options(polyform = FALSE)
a^3

options(polyform = TRUE)
a^3

options(sprayvars=letters)
a <- diag(26)
spray(a)

## Following example from mpoly:
a[1 + cbind(0:25, 1:26) %% 26] <- 2
spray(a)</pre>
```

rspray

Random spray objects

# Description

Creates random spray objects as quick-and-dirty examples of multivariate polynomials

## Usage

```
rspray(n=9, vals = seq_len(n), arity = 3, powers = 0:2)
```

# Arguments

n	Number of distinct rows (maximum); repeated rows are merged (argument addrepeats is TRUE)
vals	Values to use for coefficients
arity	Arity of the spray; the number of columns in the index matrix
powers	Set from which to sample the entries of the index matrix

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#### Value

Returns a spray object

## Note

If the index matrix contains repeated rows, the returned spray object will contain fewer than n entries

## Author(s)

Robin K. S. Hankin

#### See Also

```
spray
```

# **Examples**

```
rspray()
rspray(4)*rspray(3,rnorm(3))
rspray(3,arity=7,powers=-2:2)^3
rspray(1000,vals=rnorm(1000))
```

spray

Sparse arrays: spray objects

# Description

Create, coerce, and test for sparse array objects

# Usage

```
spray(M, x, addrepeats=FALSE)
spraymaker(L, addrepeats=FALSE, arity=ncol(L[[1]]))
is.spray(S)
as.spray(arg1, arg2, addrepeats=FALSE, offbyone=FALSE)
index(S)
coeffs(S)
coeffs(S) <- value
is_valid_spray(L)</pre>
```

## **Arguments**

М	Integer matrix with rows corresponding to index positions
x	Numeric value with elements corresponding to spray entries
S	Object to be tested for being a spray

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L	A list, nominally of two elements (index matrix and value) which is to be tested for acceptability to be coerce to class spray
arg1,arg2	Various arguments to be coerced to a spray
addrepeats	Boolean, with default FALSE meaning to check for repeated index rows and, if any are found, return an error
value	In the assignment operator coeffs<-(), a disord object (or a length-one numeric vector), so that coeffs(S) <- x works as expected
offbyone	In function as.spray(), when converting from an array. Argument offbyone is Boolean with default FALSE meaning to insert array elements in positions corresponding to index elements, and TRUE meaning to add one
arity	In function spraymaker(), integer specifying the arity (number of columns of the index matrix $L[[1]]$ ); ignored if L is non-empty. See details

#### **Details**

Spray objects are sparse arrays interpreted as multivariate polynomials. They can be added and subtracted; "\*" is interpreted as polynomial multiplication.

To create a spray object the user should use spray(), if a matrix of indices and vector of values is available, or as.spray() which tries hard to do the Right Thing (tm).

Function spraymaker() is the formal creator function, and it is written to take the output of the C++ routines and return a spray object. The reason this needs an arity argument is that C++ sometimes returns NULL (in lieu of a zero-row matrix, which it cannot deal with). In this case, we need some way to tell R the arity of the corresponding spray object.

Functions index() and coeffs() are accessor methods.

There is an extensive vignette available; type vignette("spray") at the command line.

#### Note

```
Function coeffs() was formerly known as value(). Technically, index() breaks disordR discipline.
```

# Author(s)

Robin K. S. Hankin

#### See Also

Ops, spray-package

```
S <- spray(diag(5))  # missing second argument interpreted as '1'.
as.array(S,offbyone=TRUE) # zero indices interpreted as ones.

M <- matrix(1:5,6,5) # note first row matches the sixth row

## Not run: spray(M,1:6) # will not work because addrepeats is not TRUE

spray(M,1:6,addrepeats=TRUE) # 7=1:6</pre>
```

spray\_cpp

```
S <- spray(matrix(1:7,5,7))
a <- as.array(S)  # will not work if any(M<1)
S1 <- as.spray(a)
stopifnot(S==S1)

a <- rspray(20)
coeffs(a)[coeffs(a) %% 2 == 1] <- 99  # every odd coefficient -> 99
```

spray-class

Class "spray"

# Description

The formal S4 class for sprays.

# **Objects from the Class**

Objects *can* be created by calls of the form new("spray", ...) but this is not encouraged. Use functions spray() or as.spray() instead.

#### **Slots**

```
index: Index matrix
```

value: Numeric vector holding coefficients

# Author(s)

Robin K. S. Hankin

# See Also

spray

spray\_cpp

Low-level functions that call C++ source code

# Description

Low-level functions that call C++ source code, as detailed in the automatically generated RcppExports.R file.

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#### Usage

```
spray_maker(M, d)
spray_add(M1, d1, M2, d2)
spray_mult(M1, d1, M2, d2)
spray_overwrite(M1, d1, M2, d2)
spray_accessor(M, d, Mindex)
spray_setter(M1, d1, M2, d2)
spray_equality(M1, d1, M2, d2)
spray_asum_include(M,d,n)
spray_asum_exclude(M,d,n)
spray_deriv(M,d,n)
spray_pmax(M1,d1,M2,d2)
spray_pmin(M1,d1,M2,d2)
spray_power(M,d,pow)
spray_spray_accessor()
spray_spray_add()
spray_spray_asum_exclude()
spray_spray_asum_include()
spray_spray_deriv()
spray_spray_equality()
spray_spray_maker()
spray_spray_mult()
spray_spray_overwrite()
spray_spray_pmax()
spray_spray_pmin()
spray_spray_setter()
spray_spray_power()
```

## **Arguments**

M,M1,M2,Mindex	Integer valued matrices with rows corresponding to array indices
d,d1,d2	Vector of values corresponding to nonzero array entries
n	Integer vector corresponding to dimensions to sum over for the sum functions
pow	Nonnegative integer for spray_power()

## Value

These functions return a two-element list which is coerced to an object of class spray by function spraymaker().

#### Note

These functions aren't really designed for the end-user.

Function spray\_equality() cannot simply check for equality of \$value because the order of the index rows is not specified in a spray object. Function spray\_crush() has been removed as it is redundant.

## Author(s)

Robin K. S. Hankin

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#### See Also

```
spraymaker, spray
```

```
spray_missing_accessor
```

Discussion document

#### **Description**

Discussion about the difficulties of implementing idiom like S[1,,5,,] in the package

#### Usage

```
spray_missing_accessor(S, dots)
```

#### **Arguments**

S Object of class spray dots further arguments

#### **Details**

File inst/missing\_accessor.txt presents an extended discussion of the difficulties of implementing idiom like S[1,,5,,] in the package.

## Author(s)

Robin K. S. Hankin

spraycross

Cross product for spray objects

# Description

Provides a natural cross product for spray objects, useful for tensors and k-forms

## Usage

```
spraycross(S, ...)
spraycross2(S1,S2)
```

## **Arguments**

```
S, S1, S2, ... spray objects
```

#### **Details**

Tensor products for sprays. This is *not* an algebraic product of sprays interpreted as multivariate polynomials. The function is used in the **stokes** package.

Function spraycross2() is a helper function that takes exactly two arguments. Function spraycross() is a more general function that takes any number of arguments.

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#### Value

Returns a spray object

# Author(s)

Robin K. S. Hankin

## **Examples**

```
a <- spray(matrix(1:4,2,2),c(2,5))
b <- spray(matrix(c(10,11,12,13),2,2),c(7,11))
a
b
spraycross2(a,b)
spraycross2(b,a)
spraycross(a,b,b)</pre>
```

subs

Substitute values into a spray object

# Description

Substitute values into a spray object, interpreted as a multivariate polynomial

# Usage

```
subs(S, dims, x, drop=TRUE)
```

## **Arguments**

S	spray object
dims	Integer or logical vector with entries corresponding to the dimensions to be substituted
x	Numeric vector of values to be substituted
drop	Boolean, with default TRUE meaning to return the drop() of the result, and FALSE meaning to return a spray object consistently

## Note

It is much easier if argument dims is sorted into increasing order. If not, caveat emptor!

# Author(s)

Robin K. S. Hankin

## See Also

```
process_dimensions
```

26 summary.spray

#### **Examples**

```
(S <- spray(matrix(sample(0:3,60,replace=TRUE),nrow=12)))
subs(S,c(2,5),1:2)
P <- homog(3,3)
subs(P,1,2)</pre>
```

summary.spray

Summaries of spray objects

#### **Description**

A summary method for spray objects, and a print method for summaries.

#### Usage

```
## S3 method for class 'spray'
summary(object, ...)
## S3 method for class 'summary.spray'
print(x, ...)
```

## **Arguments**

```
object,x Object of class spray
... Further arguments, passed to head()
```

# **Details**

A summary. spray object is summary of a spray object x: a list with first element being a summary() of the coefficients (which is a disord object), and the second being a spray object comprising a few selected index-coefficient pairs. The selection is done by head().

# Note

The "representative selection" is impermentation-specific, as it uses disordR::elements() to extract rows of the index matrix and coefficients.

## Author(s)

Robin K. S. Hankin

```
a <- rspray()^2
a
summary(a)
summary(a,2)

options(polyform=TRUE)
summary(a^4,3)
options(polyform=FALSE) # restore default</pre>
```

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zap

Zap small values in a spray object

# Description

```
Generic version of zapsmall()
```

# Usage

```
zap(x, digits = getOption("digits"))
## S4 method for signature 'spray'
zapsmall(x, digits = getOption("digits"))
```

# **Arguments**

```
x spray objectdigits number of digits to retain
```

## **Details**

Given a spray object, coefficients close to zero are 'zapped', i.e., replaced by '0', using base::zapsmall(). Function zap() is an easily-typed alias; zapsmall() is the S4 generic.

Note, zap() actually changes the numeric value, it is not just a print method.

## Author(s)

Robin K. S. Hankin

# **Examples**

```
(S <- spray(matrix(sample(1:50),ncol=2),10^-(1:25)))
zap(S)

S-zap(S)  # print method will probably print zeros...
coeffs(S-zap(S)) # ...but they are nevertheless nonzero</pre>
```

zero

The zero polynomial

# Description

Test for the zero, or empty, polynomial

# Usage

```
zero(d)
is.zero(x)
is.empty(L)
```

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## **Arguments**

L,x A two-element list of indices and values, possibly a spray object or numeric vector

d Integer specifying dimensionality of the spray (the arity)

#### **Details**

Functions is.empty() and is.zero() are synonyms. If spray objects are interpreted as multivariate polynomials, "is.zero()" is more intuitive, if sprays are interpreted as sparse arrays, "is.empty()" is better (for me).

Passing a zero-row index matrix can have unexpected effects:

```
> dput(spray(matrix(0,0,5),9))
structure(list(structure(numeric(0), .Dim = c(0L, 5L)), numeric(0)), class = "spray")
```

Above, the index matrix has zero rows (and no elements) but the fact that it has five columns is retained. Arguably spray() should return an error here, as the number of rows of the index matrix should match the length of the coefficient vector and they do not: the index has zero rows and the coefficient vector has length 1 (although they match in the returned value). The returned spray object has no coefficients [specifically, numeric(0)]; this is consistent with the index matrix having zero rows.

Zero coefficients are discarded by the back end:

```
> spray(matrix(1,1,5),0)
empty sparse array with 5 columns
> dput(spray(matrix(1,1,5),0))
structure(list(structure(numeric(0), dim = c(0L, 5L)), numeric(0)), class = "spray")
```

Above, the index matrix given to spray() has one row but the coefficient is a length-one vector with element zero. The resulting spray object has a NULL index matrix [because rows with zero coefficients are removed] and a NULL coefficient. It is also permissible to pass a a zero-row matrix:

```
spray(matrix(0,0,5),0)
empty sparse array with 5 columns

dput(spray(matrix(0,0,5),0))
structure(list(structure(numeric(0), dim = c(0L, 5L)), numeric(0)), class = "spray")
```

In previous versions of the package, the index matrix in the returned spray object could be NULL under some circumstances. If so, the arity of the spray object is lost. It is probably worth noting that spray(), given a zero-row index matrix, loses a length one coefficients vector, but complains about a length-two coefficient vector:

```
> dput(spray(matrix(0,0,5),0))
structure(list(structure(numeric(0), dim = c(0L, 5L)), numeric(0)), class = "spray")
> dput(spray(matrix(0,0,5),3))
structure(list(structure(numeric(0), dim = c(0L, 5L)), numeric(0)), class = "spray")
> dput(spray(matrix(0,0,5),1:2))
Error in is_valid_spray(L) : nrow(L[[1]]) == length(L[[2]]) is not TRUE
>
> identical(spray(matrix(0,0,5),0),spray(matrix(0,0,5),3))
[1] TRUE
```

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```
(a <- lone(1,3))
is.zero(a-a) # should be TRUE
is.zero(zero(6))

x <- spray(t(0:1))
y <- spray(t(1:0))
is.zero((x+y)*(x-y)-(x^2-y^2)) # TRUE</pre>
```

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