1 TSFA Guide

This guide illustrates the steps for estimating a factor model using as an example the data and process which led to results reported in Gilbert and Meijer (2006). The background theory is reported in Gilbert and Meijer (2005).

Plots and some output from the examples below are omitted to save paper. The functions in the tsfa package are made available with

> library("tsfa")

The code from the vignette that generates this guide can be loaded into an editor with edit(vignette("tsfa")). This uses the default editor, which can be changed using options(). In some examples the code runs into the margins and is truncated. (This is a limitation of the current version of RSweave.) If in doubt about the code, please edit the vignette as above or consult its source, which is distributed in the file tsfa/inst/doc/tsfa.Stex. It should also be possible to view the pdf version of the guide for this package with print(vignette("tsfa")).

The data is converted to real per capita data as follows

Investment series goes back only to November 1981 and some of the data ends in November 2004, so the data is truncated to that time window.

```
> z \leftarrow tfwindow(z, start = c(1981, 11), end = c(2004, 11))
> scale \leftarrow tfwindow(1e+08/(popm * cpi), tf = tframe(z))
> MBandCredit \leftarrow sweep(z, 1, scale, "*")
```

Multiplying by 1e8 gives real dollars per person. (Credit aggregates, B and MB numbers are in millions, CPI is in percentage points, popm is in units.) Plots to check the data can be generated with

```
> tfplot(MBandCredit, graphs.per.page = 3)
> tfplot(diff(MBandCredit), graphs.per.page = 3)
```

```
Various sample statistics are checked with
> start(MBandCredit)
[1] 1981
           11
> end(MBandCredit)
[1] 2004
> periods(MBandCredit)
[1] 277
> DX <- diff(MBandCredit, lag = 1)
The number of observations is
> periods(MBandCredit)
[1] 277
The number of series is
> nseries(MBandCredit)
[1] 10
The means of differenced series are
> colMeans(DX)
```

currency	personal cheq.
1.398385	2.880048
NonbankCheq	N-P demand & notice
3.157783	9.156883
N-P term	Investment
4.926683	48.744141
Consumer Credit	Residential Mortgage
12.703423	29.878367
Short Term Business Credit	Other Business Credit
-8.066348	37.283246

The standard deviations of differenced series are

> sqrt(diag(cov(DX)))

currency	personal cheq.
13.46788	42.94902
NonbankCheq	N-P demand & notice
10.74835	46.04252
N-P term	Investment
71.93040	116.98518
Consumer Credit	Residential Mortgage
30.00178	50.32996
Short Term Business Credit	Other Business Credit
71.83893	56.30949

2 Checking for the number of factors

Eigenvalues for the scree plot are

```
> zz <- eigen(cor(diff(MBandCredit, lag = 1)), symmetric = TRUE)[["values"]]
> print(zz)
```

```
[1] 3.2659246 1.6498244 1.0754761 0.8689273 0.8380089 0.6236201 0.5125156 [8] 0.4828785 0.4056941 0.2771303
```

The scree plot (Figure 1) is generated by

```
> par(omi = c(0.1, 0.1, 0.1, 0.1), mar = c(4.1, 4.1, 0.6, 0.1))
> plot(zz, ylab = "Value", xlab = "Eigenvalue Number", pch = 20:20,
    cex = 1, type = "o")
```

 $\mathit{FAfitStats}$ is used to calculate the fit statististics for different numbers of factors (Table 1).

```
> z <- FAfitStats(MBandCredit)
> print(z, digits = 3)
```

\$fitStats

+ 00								
	0	1	2	3	4	5	6	saturated
chisq	7.22e+02	2.52e+02	1.26e+02	3.79e+01	17.8435	8.5095	1.217	0.000
df	4.50e+01	3.50e+01	2.60e+01	1.80e+01	11.0000	5.0000	0.000	0.000
pval	8.88e-123	1.38e-34	3.93e-15	4.01e-03	0.0853	0.1303	NaN	NA
${\tt delta}$	6.77e+02	2.17e+02	1.00e+02	1.99e+01	6.8435	3.5095	1.217	0.000
${\tt RMSEA}$	2.33e-01	1.50e-01	1.18e-01	6.33e-02	0.0475	0.0504	Inf	NA
RNI	0.00e+00	6.80e-01	8.52e-01	9.71e-01	0.9899	0.9948	0.998	1.000
CFI	0.00e+00	6.80e-01	8.52e-01	9.71e-01	0.9899	0.9948	0.998	1.000
MCI	2.94e-01	6.75e-01	8.34e-01	9.65e-01	0.9877	0.9937	0.998	1.000
GFI	5.84e-01	8.33e-01	9.13e-01	9.75e-01	0.9881	0.9941	0.999	1.000
AGFI	4.91e-01	7.37e-01	8.16e-01	9.22e-01	0.9406	0.9347	-Inf	1.000
AIC	6.32e+02	1.82e+02	7.43e+01	1.89e+00	-4.1565	-1.4905	1.217	0.000
CAIC	7.88e+02	3.84e+02	3.18e+02	2.83e+02	309.1411	339.5296	365.339	364.122
SIC	7.78e+02	3.64e+02	2.89e+02	2.46e+02	265.1411	289.5296	310.339	309.122
CAK	2.69e+00	1.06e+00	6.68e-01	4.05e-01	0.3835	0.3932	0.403	0.399
CK	2.69e+00	1.06e+00	6.76e-01	4.17e-01	0.3967	0.4082	0.420	0.415

\$seqfitStats

```
0 vs 1 1 vs 2 2 vs 3 3 vs 4 4 vs 5 5 vs 6 6 vs saturated chisq 4.70e+02 1.25e+02 8.84e+01 20.04248 9.334 7.293 1.22 df 1.00e+01 9.00e+00 8.00e+00 7.00000 6.000 5.000 0.00 pval 1.15e-94 1.08e-22 9.91e-16 0.00548 0.156 0.200 NaN
```

> c2withML <- estTSF.ML(MBandCredit, 2)

The sign and order of factors is arbitary. For simulation and estimation comparisons it is useful to put them in the same order when different results are compared. This is done by specifying the *BpermuteTarget* argument in *estTSF.ML*. Other than the sign, this does not affect the estimated values, it only rearranges their order. Here they are arranged so 1=transactions, 2=long term, 3=potential spending, 4=consumer credit, 5=N-P term (which is the order they appear as factors are added). The *BpermuteTarget*, z below was determined by an initial run.

```
> z <- matrix(0, 10, 3)
> z[matrix(c(1, 6, 2, 1:3), 3, 2)] <- c(10, 56, 41)
> c3withML <- estTSF.ML(MBandCredit, 3, BpermuteTarget = z)
> z <- matrix(0, 10, 4)
> z[matrix(c(1, 6, 2, 7, 1:4), 4, 2)] <- c(13, 54, 37, 24)
> c4withML <- estTSF.ML(MBandCredit, 4, BpermuteTarget = z)
> z <- matrix(0, 10, 5)
> z[matrix(c(1, 6, 2, 7, 5, 1:5), 5, 2)] <- c(13, 67, 34, 30, 72)
> c5withML <- estTSF.ML(MBandCredit, 5, BpermuteTarget = z)</pre>
```

The standardized loadings for the four factor model (Table 2) are

> print(DstandardizedLoadings(c4withML))

```
Factor 1
                                Factor 2
                                          Factor 3
                                                   Factor 4
currency
                     1.01983211 0.09441862 -0.09386876 -0.08001695
personal cheq.
                     -0.02898840 0.20319639 0.87632955 -0.07894600
                     0.20214387 -0.13884613 0.36792951 0.11966332
NonbankCheq
N-P demand & notice
                     0.55851463 -0.00326414 0.20169780 0.24090075
N-P term
                     0.02120284 0.10107575 -0.23481508 0.33307726
Investment
                     Consumer Credit
                     0.49350633 0.22622020 0.14854196
Residential Mortgage
                     0.21597515
Short Term Business Credit -0.09041731
                               0.23970572 -0.01376311 0.09703765
Other Business Credit
```

The estimated Φ matrix is

> print(c4withML\$model\$Phi, digits = 3)

NULL

Communalities for the 4 factor model are

> print(1 - c4withML\$estimates\$uniquenesses)

numeric(0)

Communalities for other models are given by the following, but output is omitted here.

```
> print(1 - c2withML$estimates$uniquenesses)
> print(1 - c3withML$estimates$uniquenesses)
> print(1 - c5withML$estimates$uniquenesses)
  Loadings for 4 factor model (Table 3) are
> print(loadings(c4withML))
                                       Factor 2
                                                  Factor 3 Factor 4
                             Factor 1
                           13.7349771 1.2716188 -1.264213 -1.077659
currency
                           -1.2450233 8.7270851 37.637493 -3.390653
personal cheq.
NonbankCheq
                            2.1727138 -1.4923673
                                                    3.954637 1.286184
N-P demand & notice
                           25.7154238 -0.1502893
                                                   9.286676 11.091679
N-P term
                           1.5251290 7.2704190 -16.890343 23.958381
Investment
                           17.5631039 53.6833203 -28.010968 8.028077
Consumer Credit
                           0.5523918 3.1528783
                                                  3.042954 23.867010
Residential Mortgage
                           10.8700198 24.8381514 11.385653 7.476110
Short Term Business Credit -6.4954824 17.2202022 -0.988727 6.971081
Other Business Credit
                            5.4240616 37.3511568 13.511410 2.378538
  Figure 2 is generated by
> tfplot(ytoypc(factors(c4withML)), Title = "Factors from 4 factor model (year-to-year growth)
      lty = c("solid"), col = c("black"), xlab = c(""), ylab = c("factor 1",
          "factor 2", "factor 3", "factor 4"), par = list(omi = c(0.1,
          0.1, 0.1, 0.1), mar = c(3.1, 4.1, 0.6, 0.1)), reset.screen = TRUE)
  Figure 3 is generated by
> tfplot(factors(c4withML), Title = "Factors from 4 factor model",
      lty = c("solid"), col = c("black"), xlab = c(""), ylab = c("factor 1",
          "factor 2", "factor 3", "factor 4"), par = list(omi = c(0.1,
          0.1, 0.1, 0.1), mar = c(3.1, 4.1, 0.6, 0.1)), reset.screen = TRUE)
   Some points are close to zero and cause problems in growth rate graphics.
One solution is to set them to NA, but truncating the graphic works better.
Figure 4 is generated by
> z <- explained(c4withML)
> tfplot(ytoypc(MBandCredit), ytoypc(z), series = 1:5, graphs.per.page = 5,
      lty = c("solid", "dashed"), col = c("black", "red"), ylab = c("currency",
          "personal cheq.", "NonbankCheq", "N-P demand & notice",
          "N-P term"), ylim = list(NULL, NULL, c(-100, 100), NULL,
          NULL), Title = "Explained money indicator 1-5 (year-to-year growth rate) using 4 p
```

Figure 5 is generated by

0.6, 0.1), reset.screen = TRUE)

par = list(omi = c(0.1, 0.1, 0.1, 0.1), mar = c(3.1, 4.1,

```
> tfplot(ytoypc(MBandCredit), ytoypc(explained(c4withML)), series = 6:10,
      graphs.per.page = 5, lty = c("solid", "dashed"), col = c("black",
          "red"), ylab = c("", "", "", "", "Investment", "Consumer Credit",
          "Residential Mortgage", "Short Term Business Credit",
          "Other Business Credit"), Title = "Explained money indicator 6-10 (year-to-year gi
      par = list(omi = c(0.1, 0.1, 0.1, 0.1), mar = c(3.1, 4.1,
          0.6, 0.1), reset.screen = TRUE)
  Figure 6 is generated by
> tfplot(MBandCredit, explained(c4withML), series = 1:5, graphs.per.page = 5,
      lty = c("solid", "dashed"), col = c("black", "red"), Title = "Explained money indicate
      par = list(omi = c(0.1, 0.1, 0.1, 0.1), mar = c(3.1, 4.1,
          0.6, 0.1), reset.screen = TRUE)
  Figure 7 is generated by
> tfplot(MBandCredit, explained(c4withML), series = 6:10, graphs.per.page = 5,
      lty = c("solid", "dashed"), col = c("black", "red"), Title = "Explained money indicate
      par = list(omi = c(0.1, 0.1, 0.1, 0.1), mar = c(3.1, 4.1,
          0.6, 0.1)), reset.screen = TRUE)
The differenced version of the above is not in the paper, but is given by
> tfplot(diff(MBandCredit), diff(explained(c4withML)), graphs.per.page = 3)
Summary information about the model is calculated with
```

3 Two, three and five factors models

Tables of standardized loadings for differenced models, and the estimated Φ are given by the following (output is omitted).

```
> DstandardizedLoadings(c2withML)
> print(c2withML$model$Phi, digits = 3)
> DstandardizedLoadings(c3withML)
> print(c3withML$model$Phi, digits = 3)
> print(DstandardizedLoadings(c5withML), digits = 3)
> print(c5withML$model$Phi, digits = 3)
> DstandardizedLoadings(c4withML)
> print(c2withML$model$Phi, digits = 3)
```

Figure 8 part 1 is generated by

> summary(MBandCredit)

```
> tfplot(ytoypc(factors(c4withML)), ytoypc(factors(c2withML)),
    ytoypc(factors(c3withML)), ytoypc(factors(c5withML)), series = 1:2,
    xlab = c(""), ylab = c("factor 1", "factor 2"), lty = c("solid",
        "dotdash", "dashed", "dotted"), col = c("black", "green",
        "red", "blue"), Title = paste("Factors transaction and long ",
        "(year-to-year growth rate) using 2, 3, 4 and 5 factor models",
        sep = ""), par = list(omi = c(0.1, 0.1, 0.1, 0.1), mar = c(3.1,
        4.1, 0.6, 0.1)), reset.screen = TRUE)
```

Figure 8 part 2 is generated by

```
> tfplot(ytoypc(factors(c4withML)), ytoypc(factors(c3withML)),
    ytoypc(factors(c5withML)), series = 3, lty = c("solid", "dashed",
        "dotted"), xlab = c(""), ylab = c("", "", "factor 3"),
    col = c("black", "red", "blue"), Title = paste("Factor near ",
        "(year-to-year growth rate) using 3, 4 and 5 factor models",
        sep = ""), par = list(omi = c(0.1, 0.1, 0.1, 0.1), mar = c(3.1,
        4.1, 0.6, 0.1)), reset.screen = TRUE)
```

4 Rotation method sensitivity

BpermuteTarget just helps put the factors in the same order, and with the same signs as c4withML. It does not otherwise affect the estimation or rotation. The oblimin rotation with $\gamma = 0.5$ is given by

```
Factor 1
                                      Factor 2
                                                 Factor 3 Factor 4
                          17.204376 -0.2993325 -2.9221503 -4.161355
currency
personal cheq.
                          -5.888054 11.8489892 40.8218129 -9.350992
                           2.306075 -3.1917702 3.5808888
                                                           2.094587
NonbankCheq
N-P demand & notice
                          28.858827 -9.6930351
                                               4.4360441 12.301306
N-P term
                          -2.519688 1.6858628 -21.7335744 32.446348
Investment
                          14.121979 70.4973501 -30.5725683 -12.535677
Consumer Credit
                          -4.945051 -4.9008176 -0.7956332 33.697057
                          6.854980 29.4305518 10.4874854 -1.255816
Residential Mortgage
Short Term Business Credit -12.201107 22.7939544 -0.5132386 4.095379
Other Business Credit
                          -1.130633 49.8454385 14.9460409 -13.213999
```

> DstandardizedLoadings(c4withMLg0.5)

```
Factor 1 Factor 2 Factor 3 Factor 4 currency 1.27743748 -0.02222566 -0.216971802 -0.30898366 personal cheq. -0.13709404 0.27588499 0.950471411 -0.21772307 NonbankCheq 0.21455145 -0.29695432 0.333156946 0.19487511
```

> DstandardizedLoadings(c4withMLg0.5) - DstandardizedLoadings(c4withML)

```
Factor 1
                                  Factor 2
                                             Factor 3
                       0.25760538 -0.11664428 -0.123103042 -0.22896671
currency
                     -0.10810564 0.07268860 0.074141866 -0.13877707
personal cheq.
NonbankCheq
                     0.01240759 -0.15810820 -0.034772560 0.07521179
                     0.06827175 -0.20725940 -0.105351130 0.02627195
N-P demand & notice
N-P term
                     -0.05623237 -0.07763833 -0.067332182 0.11800249
Investment
                     Consumer Credit
                     -0.18323724 -0.26844063 -0.127945338 0.32764886
Residential Mortgage
                     Short Term Business Credit -0.07942246 0.07758680 0.006618814 -0.04002985
Other Business Credit
                  -0.11640480 0.22188590 0.025477600 -0.27690780
```

Summary information is produced with (output omitted)

> summary(c4withMLg0.5)

Other rotation results are produced by the following, but outputs are omitted.

- > c4withMLgneg1.0 <- estTSF.ML(MBandCredit, 4, BpermuteTarget = loadings(c4withML), rotation = "oblimin", rotationArgs = list(gam = -1))
- > loadings(c4withMLgneg1.0)
- > DstandardizedLoadings(c4withMLgneg1.0)
- > DstandardizedLoadings(c4withMLgneg1.0) DstandardizedLoadings(c4withML)
- > summary(c4withMLgneg1.0)
- > loadings(c4withMLbQ)
- > DstandardizedLoadings(c4withMLbQ)
- > DstandardizedLoadings(c4withMLbQ) DstandardizedLoadings(c4withML)
- > summary(c4withMLbQ)

```
Figure 9 is generated by
```

Geomin factors 2 and 3 each have one modestly different loading. Factor 2 has personal chequing mixed in with investment and credit. Factor 3 explains on currency, personal chequing, and investment, so the separation is not so interesting. Output is omitted from these.

The difference between the estimates can be checked with (output omitted)

 $\verb| > DstandardizedLoadings(c4withMLgm)| - DstandardizedLoadings(c4withML)|$

The summary of the 4 factor geomin estimate is given by

> summary(c4withMLgm)

Figure 10 is generated by

There is only a qualitative statement about the next in the paper (outputs omitted).

- > DstandardizedLoadings(c4withML)
- > DstandardizedLoadings(c4withMLnotNorm)
- > DstandardizedLoadings(c4withML) DstandardizedLoadings(c4withMLnotNorm)

5 Sensitivity to sample period

> z <- matrix(0, 10, 4)

BpermuteTarget=loadings(c4withML) is not good enough in some cases. There are difficulties interpreting factors 2 and 3 in here.

> z[matrix(c(1, 6, 2, 7, 1:4), 4, 2)] <- c(11, 104, 20, 13)

```
> c4withMLbefore90 <- estTSF.ML(tfwindow(MBandCredit, end = c(1989, c4))
      12)), 4, BpermuteTarget = z)
> c4withMLafter95 <- estTSF.ML(tfwindow(MBandCredit, start = c(1995,
      1)), 4, BpermuteTarget = loadings(c4withML))
> z \leftarrow matrix(0, 10, 4)
> z[matrix(c(1, 6, 2, 7, 1:4), 4, 2)] <- c(11, 104, 20, 13)
> c4withMLbefore95 <- estTSF.ML(tfwindow(MBandCredit, end = c(1994,
      12)), 4, BpermuteTarget = z)
> c4withMLafter00 <- estTSF.ML(tfwindow(MBandCredit, start = c(2000,
      1)), 4, BpermuteTarget = loadings(c4withML))
> c4withML90to00 <- estTSF.ML(tfwindow(MBandCredit, start = c(1990,
      1), end = c(2000, 1), 4, BpermuteTarget = loadings(c4withML))
  Figure 11 is generated by
> tfplot(ytoypc(factors(c4withML)), ytoypc(factors(c4withMLbefore90)),
      ytoypc(factors(c4withMLbefore95)), ytoypc(factors(c4withMLafter95)),
      ytoypc(factors(c4withMLafter00)), ytoypc(factors(c4withML90to00)),
      xlab = c(""), ylab = c("factor 1", "factor 2", "factor 3",
          "factor 4"), ylim = list(NULL, c(-20, 20), c(-25, 40),
          NULL), graphs.per.page = 4, lty = c("dashed", "dotted",
          "dotdash", "longdash", "dotted", "twodash"), col = c("red",
          "blue", "green", "pink", "violet", "brown"), Title = paste("Factors (year to year
          "ML estimation with quartimin rotation objective", sep = ""),
      par = list(omi = c(0.1, 0.1, 0.1, 0.1), mar = c(3.1, 4.1,
          0.6, 0.1), reset.screen = TRUE)
   Figure 12 is generated by
> tfplot(ytoypc(MBandCredit), ytoypc(explained(c4withML)), ytoypc(explained(c4withMLbefore90
      ytoypc(explained(c4withMLbefore95)), ytoypc(explained(c4withMLafter95)),
      ytoypc(explained(c4withMLafter00)), ytoypc(explained(c4withML90to00)),
      series = 1:5, graphs.per.page = 5, ylab = c("currency", "personal cheq.",
          "NonbankCheq", "N-P demand & notice", "N-P term"), ylim = list(NULL,
          NULL, c(-70, 70), NULL, c(-70, 70)), lty = c("solid",
          "dashed", "dotted", "dottash", "longdash", "dotted",
          "twodash"), col = c("black", "red", "blue", "green",
          "pink", "violet", "brown"), Title = paste("Explained money indicators 1-5 (year to
          "using 4 factors, full sample and sub-samples", sep = ""),
      par = list(omi = c(0.1, 0.1, 0.1, 0.1), mar = c(3.1, 4.1,
          0.6, 0.1), reset.screen = TRUE)
```

```
Figure 13 is generated by
```

6 Comparison with Aggregates

```
Compute aggregates M1+ and M2++
> M1 < -tfwindow(M1total, start = c(1981, 11), end = c(2004, 11)) *
> seriesNames(M1) <- "Real per Capita M1"</pre>
> z < - tframed(MB2001 + MB486 + MB487p + MB452 + MB452adj + MB472 +
      NonbankCheq)
> M1p < -tfwindow(z, start = c(1981, 11), end = c(2004, 11)) *
      scale
> seriesNames(M1p) <- "Real per Capita M1+"
> M2pp <- tfwindow(M1total + MB472 + MB473 + MB452 + MB453 + MB454 +
      NonbankCheq + NonbankNonCheq + NonbankTerm + +MB2046 + MB2047 +
      MB2048 + MB2057 + MB2058, start = c(1981, 11), end = c(2004, 11)
      11)) * scale
> seriesNames(M2pp) <- "Real per Capita M2++"
and put factors on the same scale.
> f <- tframed(factors(c4withML)[, 1:2], tf = tframe(factors(c4withML)))</pre>
> mnF <- colMeans(f)
> mnM <- colMeans(cbind(M1p, M2pp))</pre>
> f <- sweep(f, 2, mnM/mnF, "*")
   Now compare the transaction factor with M1+ and the long term factor with
M2++. Figure 14 is generated by
> tfplot(ytoypc(f), ytoypc(cbind(M1, M2pp)), graphs.per.page = 2,
      lty = c("dashed", "solid"), col = c("red", "black"), Title = paste("(year to year grown))
          "computed using ordinary ML parameters (dashed)", sep = ""),
```

ylab = c("M1+ vs. factor 1", "M2++ vs. factor 2"), par = list(omi = c(0.1, 0.1, 0.1, 0.1), mar = c(3.1, 4.1, 0.6, 0.1)), reset.screen = TRUE)

7 References

Paul D. Gilbert and Erik Meijer, (2006) "Money and Credit Factors", Bank of Canada Working Paper 2006-3, Available at http://www.bank-banque-canada.ca/en/res/wp/wp(y)_2006.html.

Paul D. Gilbert and Erik Meijer, (2005) "Time Series Factor Analysis with an Application to Measuring Money", Research Report 05F10, University of Groningen, SOM Research School. Available at

<http://som.eldoc.ub.rug.nl/reports/themeF/2005/05F10/>.