Homework # 6

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Parameters

```
clear;
cp_0 = 0.5;
crho = 0.5;
sigma_u = 0.1;
cdelta = 0.95;
```

Question 2

Formulate firm's dynamic optimization problem. Specifically, formulate the Bellman equation, identify state and policy variables, their spaces and transition probabilities. Assume initial stock is between 0 and 100.

$$V(x,p) = max_{x' \in [0,x]} p(x - x') - 0.2x^{1.5} + \delta E\{V(x', p')|p\} \quad (1)$$
$$p_t = p_0 + \rho p_{t-1} + u \quad (2)$$

Question 2

Take a look at tauchen.m in the repository, use it to generate a grid that approximates the process for p_t with 21 grid points

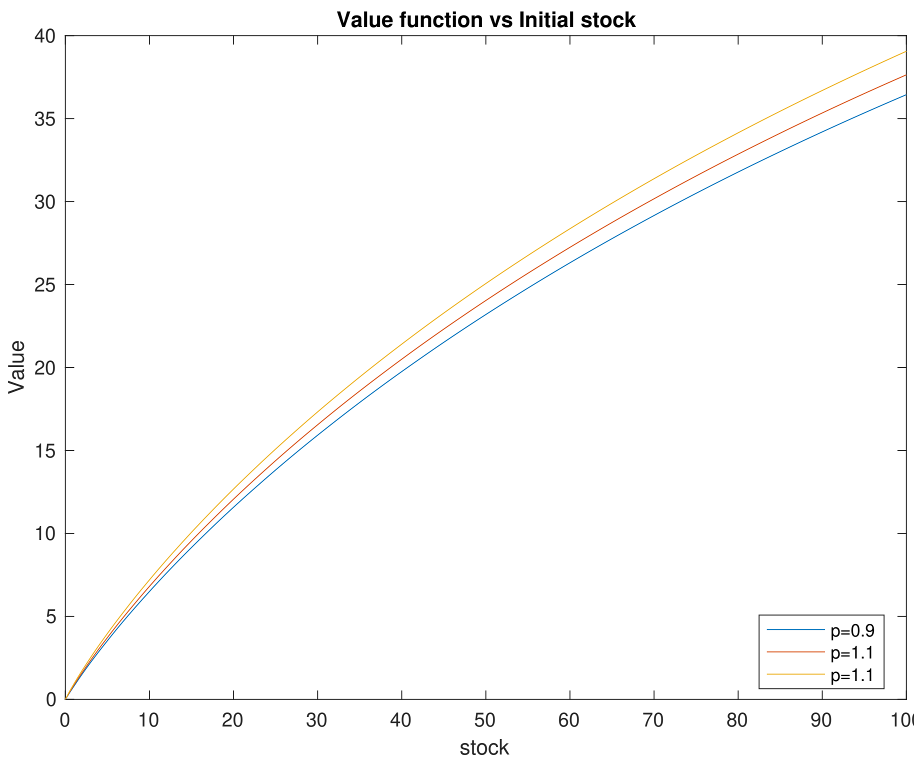
```
% Number of grid points for the price
npgrid = 21;
[prob, pgrid] = tauchen(npgrid, cp_0, crho, sigma_u);

% Number of grid points for the stock of timber
nxgrid = 201;

% Grid for timber stock
xgrid = linspace(0,100, nxgrid)';
```

Question 3

Solve the firm's problem using value function iteration. Plot the value of the firm depending on its initial stock (x-axis) and the current price of timber for $p \in \{0.9, 1, 1.1\}$.

```
% Value function iteration
% Initial Value function matrix
Vold=zeros(nxgrid, npgrid);

% Store updated values here
Vnew=zeros(nxgrid, npgrid);

% Store policy indices here
pol_index=zeros(nxgrid, npgrid);

% Amount Harvested
h=xgrid-xgrid';

% Value function iteration

for j=1:npgrid
 values=period_profit(h,pgrid(j)) + ...
     cdelta*prob(j,:)*Vold';
 Vnew(:,j)=max(values,[],2);
end

diff=norm(Vnew-Vold)/norm(Vnew);

tol=0.00001;

iters=0;
while diff>tol
 Vold=Vnew;
 for j=1:npgrid
  values=period_profit(h,pgrid(j)) + ...
      cdelta*prob(j,:)*Vold';
  [Vnew(:,j), pol_index(:,j)]=max(values,[],2);
 end

 diff=norm(Vnew-Vold)/norm(Vnew);
 iters=iters+1;
end

policy=xgrid(pol_index);

% Plot the value of the firm
v0_9=Vnew(:,8);
v1=Vnew(:,11);
v1_1=Vnew(:,14);
```

```
figure
plot(xgrid, v0_9, xgrid, v1, xgrid, v1_1)
title('Value function vs Initial stock')
xlabel('stock')
ylabel('Value')
legend({'p=0.9', 'p=1.1', 'p=1.1'}, 'Location', ...
    'best')
```

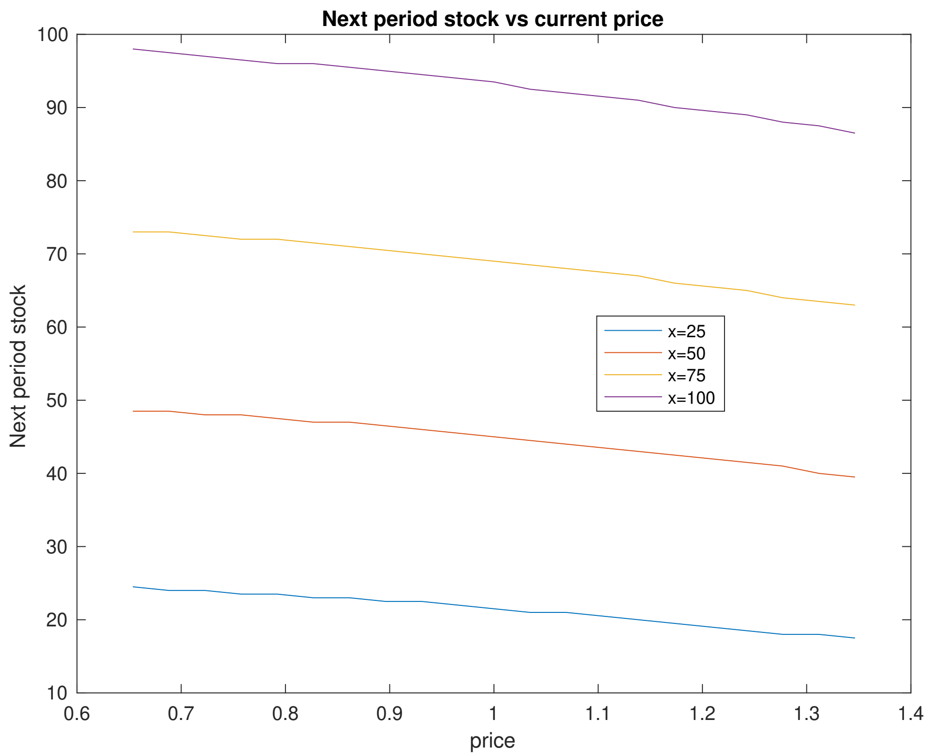


Question 4

Plot next period optimal stock (or harvest amount if you prefer) as a function of today's price for different amounts of lumber left in stock.

```
x25=policy(51,:);
x50=policy(101,:);
x75=policy(151,:);
x100=policy(201,:);

figure
plot(pgrid, x25, pgrid, x50, pgrid, x75, pgrid, x100);
```

```
title('Next period stock vs current price')
xlabel('price')
ylabel('Next period stock')
legend({'x=25', 'x=50', 'x=75', 'x=100'}, ...
    'Location', 'best')
```



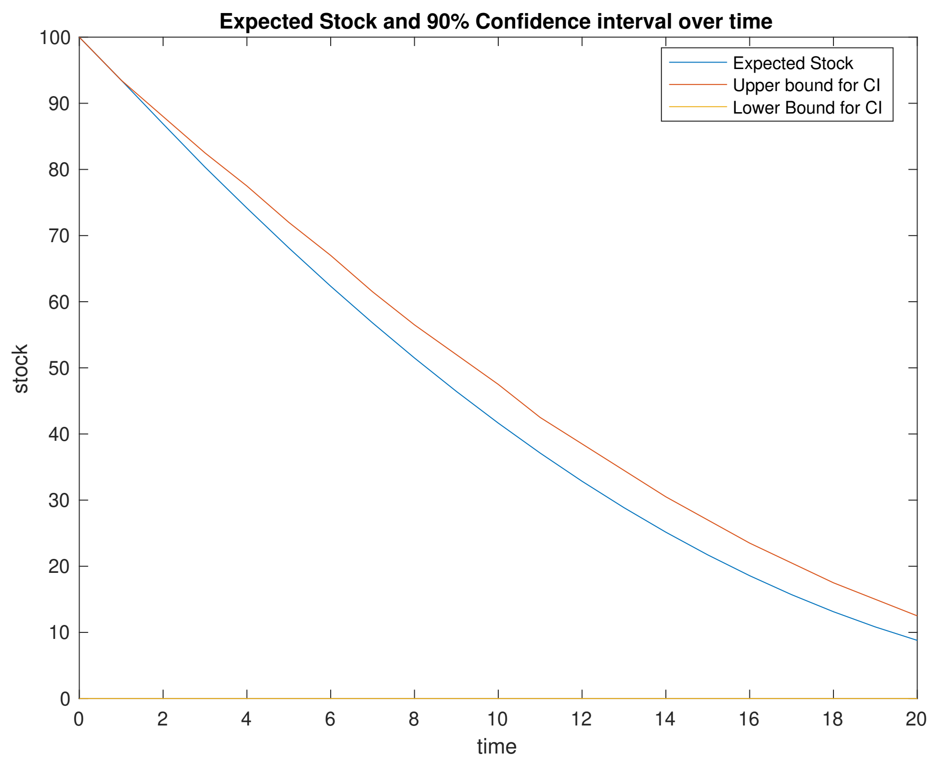
Question 5

Assume firm starts with stock of 100 and todays price is 1. Plot expected stock over time for 20 periods ahead. Include the 90 percent confidence interval

```
P=zeros(nxgrid*npgrid, nxgrid*npgrid);

% Transition matrix for the states
for i=1:npgrid
    for j=1:npgrid
     P((i-1)*nxgrid+1:i*nxgrid, ...
         (j-1)*nxgrid+1:j*nxgrid)=prob(i,j)*(kron(ones(1,nxgrid), ...
         policy(:,i))==kron(ones(nxgrid,1),xgrid'));
    end
```

```
end

% For the following periods ahead
nperiods = 20;

% Store distribution over states here
q = zeros(nperiods + 1,nxgrid*npgrid);

% Start at p=1, x=100
q(1,11*nxgrid) = 1;

for t = 1:nperiods
 q(t + 1,:) = q(t,:)*P;
end

% Expected Values

% Store the marginal probability distributions of x

x_margdist = zeros(nperiods + 1, nxgrid);

for t = 1:nperiods + 1
    mat_q = reshape(q(t,:) ,[nxgrid,npgrid]);
    x_margdist(t,:) =sum(mat_q,2)';
end

% Expected values
mu_x = x_margdist*xgrid;

% Confidence interval
index = 1:nxgrid;

% Cumulative distributions
x_cumsum = cumsum(x_margdist,2);

% Obtain 90th quantiles
indices90 = zeros(nperiods + 1,1);
for t = 1:nperiods + 1
    indices90(t,1) = min(index(x_cumsum(t,:) >= 0.8));
end
x90 = xgrid(indices90);

lb = zeros(nperiods + 1,1);
time = 0:20;

plot(time, mu_x, time, x90, time, lb)
```

```
title('Expected Stock and 90% Confidence interval ...
    over time')
xlabel('time')
ylabel('stock')
legend({'Expected Stock', 'Upper bound for CI', ...
    'Lower Bound for CI'}, 'Location', 'best')
```



Question 6.

Redo 2-4 for the coarse grid of 5 points in Tauchen's representation

Question 6.2. Take a look at tauchen.m in the repository, use it to generate a grid that approximates the process for p_t with 5 grid points

```
% Number of grid points for the price
npgrid = 5;
[prob, pgrid, invdist] = tauchen(npgrid, cp_0, crho, ...
    sigma_u);
```

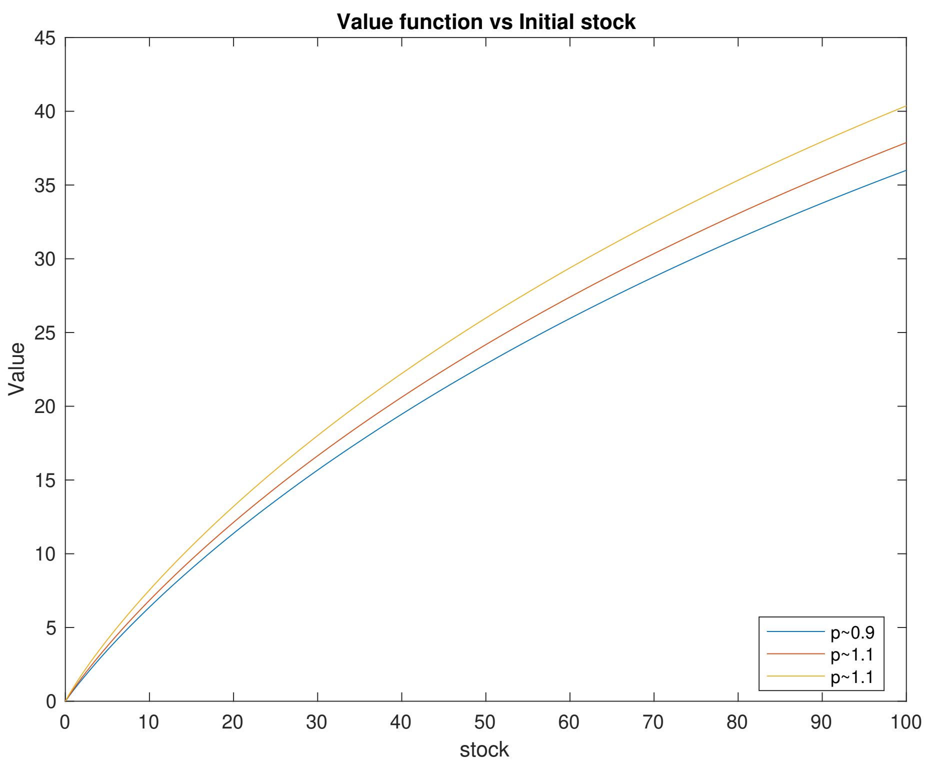
```
% Number of grid points for the stock of timber
nxgrid = 201;

% Grid for timber stock
xgrid = linspace(0,100, nxgrid)';
```

Question 6.3

Solve the firm's problem using value function iteration. Plot the value of the firm depending on its initial stock (x-axis) and the current price of timber for $p \in \{0.9, 1, 1.1\}$.

```
% Value function iteration
% Initial Value function matrix
Vold = zeros(nxgrid, npgrid);

% Store updated values here
Vnew = zeros(nxgrid, npgrid);

% Store policy indices here
pol_index = zeros(nxgrid, npgrid);

% Amount Harvested
h = xgrid-xgrid';

% Value function iteration

for j = 1:npgrid
 values = period_profit(h,pgrid(j)) + ...
     cdelta*prob(j,:)*Vold';
 Vnew(:,j) = max(values,[],2);
end

diff = norm(Vnew-Vold)/norm(Vnew);

tol = 0.00001;

iters = 0;
while diff > tol
 Vold = Vnew;
 for j = 1:npgrid
  values = period_profit(h,pgrid(j)) + ...
      cdelta*prob(j,:)*Vold';
```

```
    [Vnew(:,j), pol_index(:,j)]=max(values,[],2);
  end

  diff=norm(Vnew-Vold)/norm(Vnew);
  iters=iters+1;
end

policy=xgrid(pol_index);

% Plot the value of the firm
v0_9=Vnew(:,2);
v1=Vnew(:,3);
v1_1=Vnew(:,4);

figure
plot(xgrid, v0_9, xgrid, v1, xgrid, v1_1)
title('Value function vs Initial stock')
xlabel('stock')
ylabel('Value')
legend({'p~0.9', 'p~1.1', 'p~1.1'}, 'Location', 'best')
```



Question 6.4

Plot next period optimal stock (or harvest amount if you prefer) as a function of today's price for different amounts of lumber left in stock.

```
x25 = policy(51,:);
x50 = policy(101,:);
x75 = policy(151,:);
x100 = policy(201,:);

figure
plot(pgrid, x25, pgrid, x50, pgrid, x75, pgrid, x100);
title('Next period stock vs current price')
xlabel('price')
ylabel('Next period stock')
legend({'x=25', 'x=50', 'x=75', 'x=100'}, ...
    'Location', 'best')
```