CSC 226 - Assignment 4 - Solutions

1. I first did this iteratively, using x + y directly. So, I check if x[i] = y[j] at any [i, j] in the array to determine if llcs[i,j] came from llcs[i-1, j-1] or not. (equiv. to a "↖" in the path [i,j] position).
If x[i] = y[j] then I store x[i] (or y[j]) in s[k], the substring to be printed. Otherwise, we need to determine if llcs[i,j] came from llcs[i-1, j] or llcs[i, j-1] (the bigger of the two). If they are equal it comes from llcs[i, j-1] by virtue of the LCS algorithm on slide 98 of Lecture 17.

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Algorithm printLCS(Array llcs[][], String x, String y)
   Input: Array of lengths of LCS of strings x + y.
   Ouput: String s, the lcs of x + y.

   n ← x.length()
   m ← y.length()
   new string s of length llcs[m][n]
   k ← s.length() - 1
   i ← n
   j ← m
   while (i>0 && j>0) do
       if x[i] = y[j] then
           s[k] = x[i]
           i ← i-1; j ← j-1; k ← k-1
       else if llcs[i-1,j] > llcs[i,j-1] then
           i ← i-1;
       else
           j ← j-1;
                                    → Print(s);
```

→ Note: 1. It can also be done recursively, in which case you can print the strings out directly as you see it.

2. It seems you can also do it without having to look at the strings x + y by just looking at the lengths.

↳ as you scan left in a row, as soon as the value decreases by 1 you have found a character in the substring.

2. Let N be a flow network and let f be a flow for N. Furthermore, let X be an st-cut of N. Let $s \in S$ + $t \in T$, and let

$$F = \sum_{v \in S} \left(\sum_{e \in E^+(v)} f(e) - \sum_{e \in E^-(v)} f(e) \right)$$

the sum of the total net flow through all vertices in S, some of which include edges across the cut.

First, note that $\forall\, v \neq s \in S$, $\sum_{e \in E^+(v)} f(e) - \sum_{e \in E^-(v)} f(e) = 0$

by the conservation rule. Thus,

$$F = \sum_{e \in E^+(s)} f(e) - \sum_{e \in E^-(s)} f(e) = |f|.$$

Furthermore, note that for every $e \in E$ that is not a crossing edge one of two cases apply:

(1) Both $f(e)$ & $-f(e)$ are part of sum F for a combined value of 0, or

(2) Both $f(e)$ + $-f(e)$ are not part of sum F, also for net value of 0.

This leaves only crossing edges contributing to F, f(e) for those going from S to T and $-f(e)$ for those going from T to S. Thus, it is also true that

$$F = f(x)$$

and therefore

$$|f| = F = f(x).$$

→ Notes: There are ~~prop~~ probably many ways to prove this. I suspect many will do induction or contradiction proofs.

3.

V	S	A	B	C	D	E	F
D[V]	0	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$
pass 1	0	2	9	3	$+\infty$	$+\infty$	$+\infty$
pass 2	0	2	9	3	6	7	5
pass 3	0	2	6	3	6	7	5
pass 4	0	2	6	3	6	7	5

→ No more iterations are necessary as there were no changes from iteration ~~to~~ 3 to iteration 4.