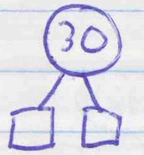
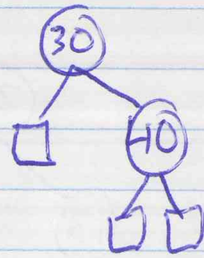
CSC 226 - ASSIGNMENT 1 - SOLUTIONS

1.

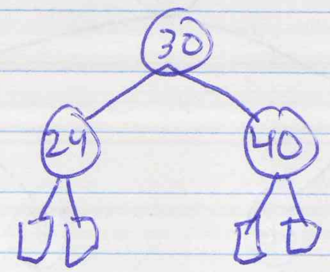
(1.) insert(30)



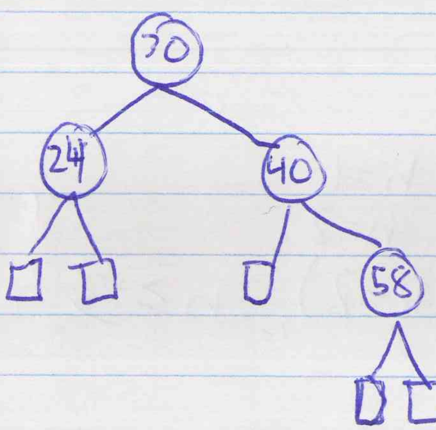
(2.) insert(40)



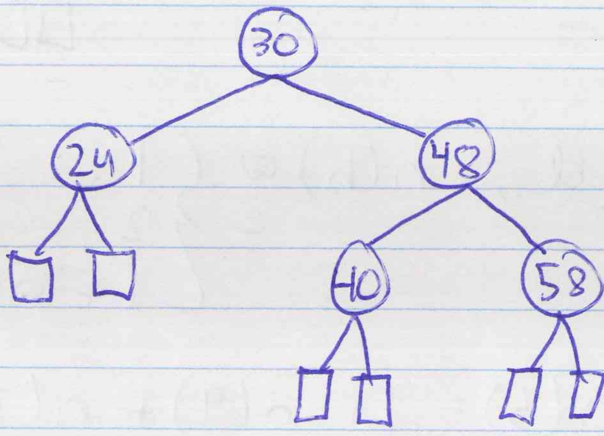
(3.) insert(24)



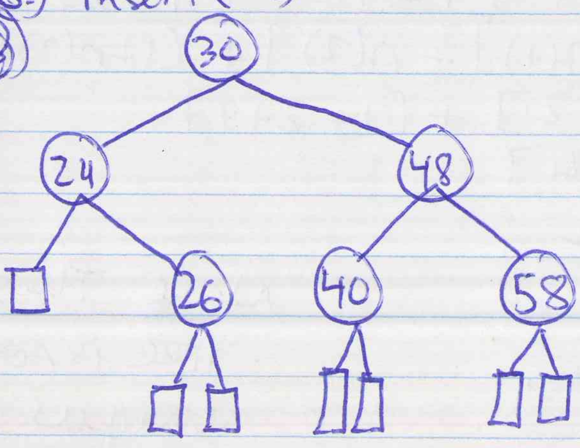
(4.) insert(58)



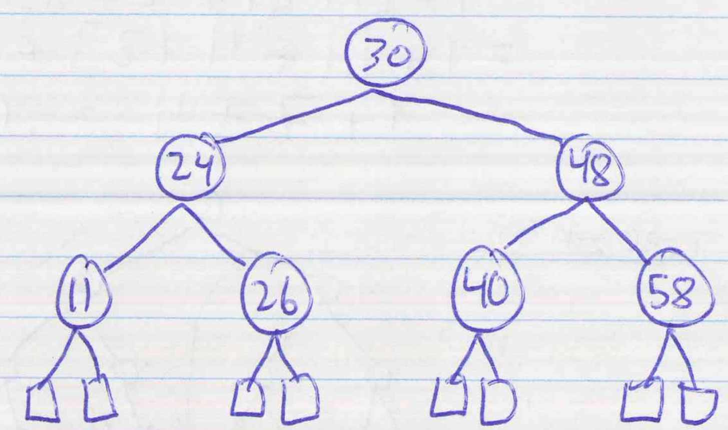
(5.) insert(48)



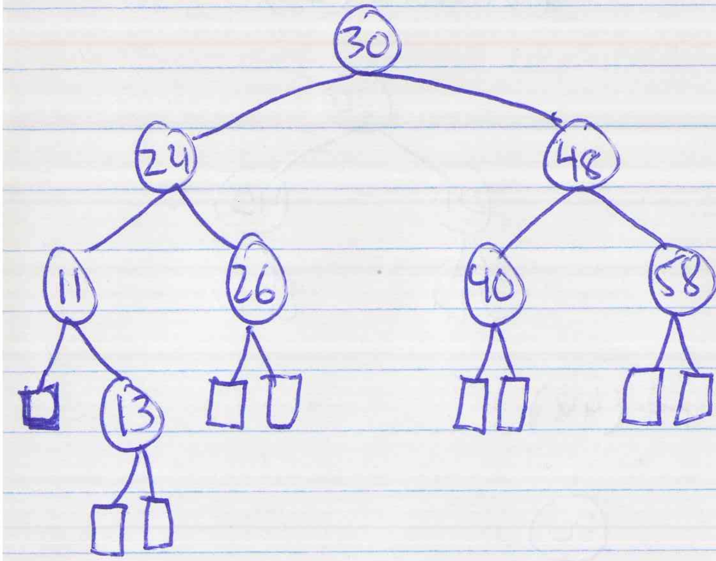
(6.) insert(26)



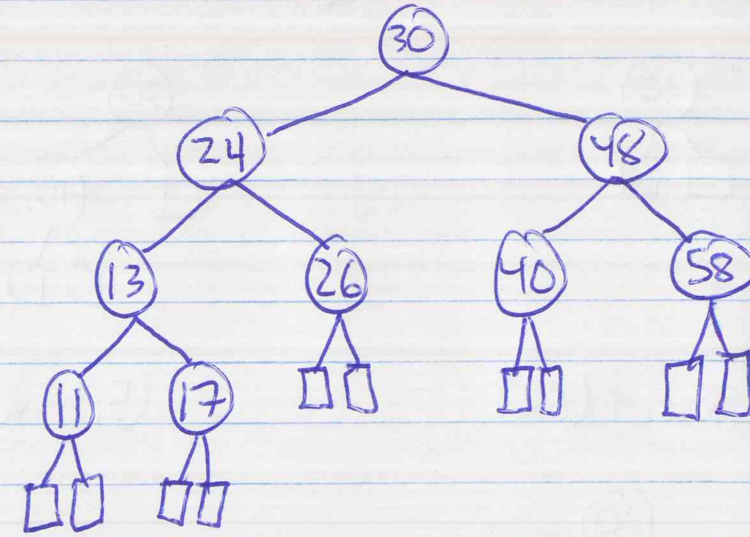
(7.) incert(11)



(8.) insert(13)



(9.) insert(17)

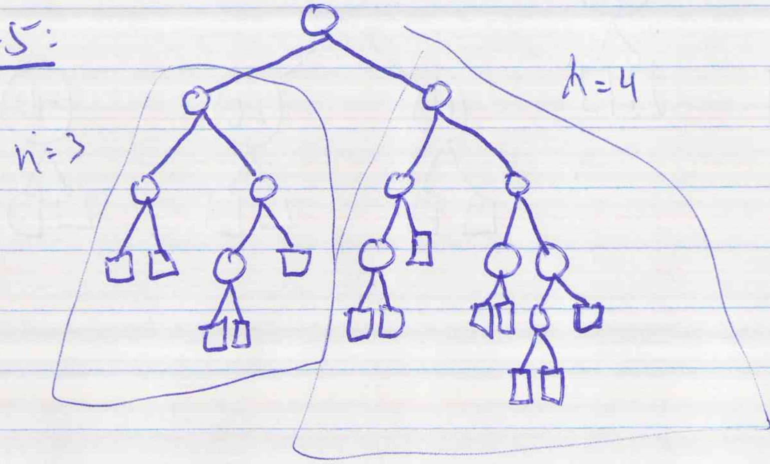


2. Recall,

$$n(h) = \begin{cases} 1, & h=1 \\ 2, & h=2 \\ 1 + n(h-1) + n(h-2), & h \geq 3 \end{cases}$$

$$\begin{aligned} \therefore\ n(5) &= 1 + n(4) + n(3) \\ &= 1 + [1 + n(3) + n(2)] + [1 + n(2) + n(1)] \\ &= 1 + [1 + [1 + n(2) + n(1)] + n(2)] + (1 + n(2) + n(1) \\ &= 1 + [1 + [1 + 2 + 1] + 2] + [1 + 2 + 1] \\ &= 1 + [1 + 4 + 2] + [4] \\ &= 1 + 7 + 4 = 12 \end{aligned}$$

Tree - h=5:



Note: This tree is not unique. As long as it's AVL + has 12 intern nodes

3. $\Rightarrow$ Let $f(n) \in \Theta(g(n))$, then there exists real numbers $c_1, c_2 > 0$ such that

$$c_1 g(n) \le f(n) \le c_2 g(n)$$

for all $n \ge n_0 > 0$.
Since $f(n) \le c_2 g(n)$ for all $n \ge n_0$ then $f(n) \in O(g(n))$.
Furthermore, since $f(n) \ge c_1 g(n)$ for all $n \ge n_0$ then

$$f(n) \in \Omega(g(n)).$$

$\Leftarrow$ Let $f(n) \in O(g(n))$ and $f(n) \in \Omega(g(n))$, then there exists $c_1, c_2 > 0$ such that

$$f(n) \ge c_1 g(n) \quad \text{for all } n \ge n_1 > 0$$

and

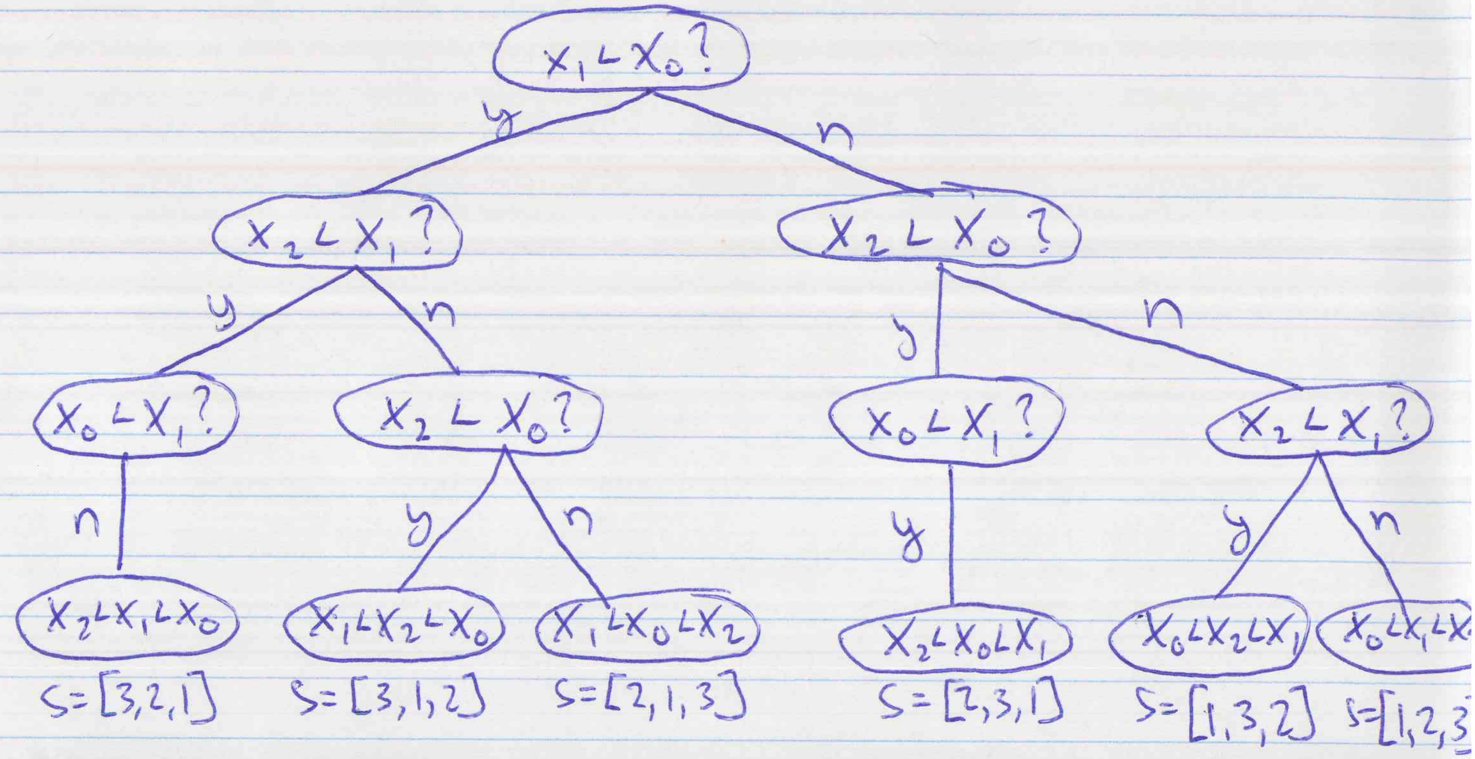
$$f(n) \le c_2 g(n) \quad \text{for all } n \ge n_2 > 0$$

for some n_1 and n_2, respectively. Thus,

$$c_1 g(n) \le f(n) \le c_2 g(n) \quad \text{for all } n \ge n_0 > 0$$

where $n_0 = \max(n_1, n_2)$.

4.



5. Basically, if we do randomized quicksort we know that the expected height of the sort tree is $2 \log_{4/3} n$ when there is n input elements.
In the course of partitioning the music files into these less than + those greater than the pivot we need to do n comparisons (i.e. at each level of the sort tree.).
We expect it will take 2 comparisons each time to get a result since there is a 50-50 chance of fault error. That means expected $2n$ comparisons at each level of the sort tree of ~~height~~ expected height $2 \log_{4/3} n$, for a total of

$$2n \cdot 2\log_{4/3} n = 4 n \log_{4/3} n.$$