

Initial State:

A	B		D
			G
		E	
	F	C	H

Goal State:

A	B	C	D
E	F	G	H

If a piece cannot reach a goal state based on the current board, it is said to be infinitely many steps from its goal state.

$S_1$ : 1 item is out of place

$f_1(x) = \# \text{ of moves to get } x_1 \text{ to goal state}$

$$f_1(C) = 3$$

$$f_1(E) = 3$$

$$f_1(G) = 4$$

$S_2$ : 2 items are out of place

$$f_2(x_1, x_2) = \min \{ x_1 \text{ to goal state} + f_1(x_2), x_2 \text{ to goal state} + f_1(x_1) \}$$

$$f_2(C, E) = \min \left\{ \underbrace{C + f_1(E)}_{\infty + 3}, \underbrace{E + f_1(C)}_{3 + 3} \right\} = 6$$

$$f_2(C, G) = \min \left\{ \underbrace{C + f_1(G)}_{3 + 4}, \underbrace{G + f_1(C)}_{\infty + 3} \right\} = 7$$

$$f_2(E, G) = \min \left\{ \underbrace{E + f_1(G)}_{3 + 4}, \underbrace{G + f_1(E)}_{\infty + 3} \right\} = 7$$

$S_3$ : 3 items out of place

$$f_3(x_1, x_2, x_3) = \min \left\{ \begin{array}{l} x_1 \text{ to goal state} + f_2(x_2, x_3) \\ x_2 \text{ to goal state} + f_2(x_1, x_3) \\ x_3 \text{ to goal state} + f_2(x_1, x_2) \end{array} \right\}$$

$$f_3(C, E, G) = \min \left\{ \underbrace{C + f_2(E, G)}_{\infty + 7}, \underbrace{E + f_2(C, G)}_{3 + 7}, \underbrace{G + f_2(C, E)}_{\infty + 6} \right\} = 10$$

Optimal path:

E

C

G

A	B		D
			G
		E	
	F	C	H

A	B		D
			G
E	F	C	H

A	B	C	D
			G
E	F		H

A	B	C	D
E	F	G	H