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### Project 3 Report

Our Project 3 submission was written in Python and was tested and developed on the student machines. The program consists of two files, `main.py` and `tape.py`, and is invoked using the shell script `tm`.

`Tape.py` defines a `tape` class that is passed to the main function. This `tape` class holds the infinite tape, implemented as a dictionary, which can be traversed in either direction. The class definition also contains several general functions for moving the tape head left or right and writing or reading from a location on the tape.

`Main.py` has all of the data structures to hold the states, alphabets and transitions in its main function. States and alphabets are stored in lists. Transitions are stored in a dictionary. The main function proceeds by first reading from the input file and parsing the lines. The data is then put into the corresponding storage structures based on the first character of the line (Q, A, Z, T, S, F). Checks are done after the structures are filled to make sure the machine is deterministic and that the states / alphabet letters are valid in the machine. The program then waits for the user to input an integer stating how many lines of input will follow. The program then waits for and parses user input lines and proceeds to go through the machine's transitions. If there are more than 1000 transitions, the machine will halt and print `DID NOT HALT`. Otherwise, if the machine reaches an accept state or reject state, the program will halt and print `ACCEPT` or `REJECT` respectively. If the machine has looped through all possible transitions and it

ends on a state that is not an accept state, it is transferred to the reject state and REJECT is printed.

Our arithmetic Turing machine (found in the file subtraction.txt) performs subtraction in order to recognize the language  $L = \{a^i b^j c^k \mid i - j = k \text{ \& } i > 0\}$ . The machine first moves the tape head right until it reaches a 'b', which is then marked with an 'x'. The machine then moves the tape head left until it encounters an 'a', marking that character with an 'x'. Once all the 'b' characters have been replaced, the process repeats for the remaining 'c' characters. Once all characters have been replaced and the head is at the end of the tape, the machine enters its accept state. Inputs that are not part of the language result in the machine becoming trapped in a state without any way of transitioning out. As stated before, once the machine exhausts all possible transitions for a given state it automatically enters its reject state. This machine was tested using the following inputs:

```
a,a,b,b
a,a,a,b,c,c
a,c
a,a,b,c
a,b,c
a,a,b,b,c
a,b,b,c
a,a,a,b,c
```

The first 4 inputs test whether or not the machine properly accepts input from the language. The final 4 test whether or not the machine properly rejects inputs that are not in the language due to  $i - j \neq k$ . The results from these inputs can be found at the end of the report and in the file sub\_output.txt.

Our string operation Turing machine (found in palindrome.txt) was designed to recognize binary palindromes (i.e.  $L = \{w\#w^R \mid w \in \{0,1\}^*\}$ ). The machine first moves

the tape head right until it encounters the '#' character. It then reads the first character that is not an 'x' to the right of the '#' character before replacing it with an 'x'. The machine then moves the tape head to the left until it encounters the first character that is not an 'x' to the left of the '#' character. If the 2 characters that have just been read match, both are marked as an 'x' and the process repeats. If they don't match, the machine enters its reject state. The machine enters its accept state once all characters on either side of the '#' character are marked with an 'x'. This machine was tested using the following inputs:

```

1,0,#,0,1
0,0,#,0,0
1,0,#,1,1
1,0,#,1,0
1,1,0,0,1,1,#,1,1,0,0,1,1
1,0,1,0,0,1,#,1,0,0,1,0,1
1,1,0,1,1,1,#,1,1,0,0,1,1
1,1,0,0,1,1,#,1,1,0,1,1,1

```

Inputs 1, 2, 5, and 6 are used to determine if the machine properly accepts palindromes of varying sizes. Inputs 3, 4, 7, and 8 are used to determine whether or not the machine rejects inputs of varying sizes that are not palindromes. The results from these inputs can be found at the end of this report and in the file palindrome\_output.txt

Our final Turing machine (found in divisionwr.txt) performs division in order to recognize the language  $L = \{a^i b^j c^k d^l \mid i / j = k \text{ with remainder } l \ \& \ i, j, > 0\}$ . The machine first moves the tape head to the right until it encounters a 'b'. It then marks that character with a 'b\*' before moving the tape head left. Once the tape head reaches an 'a', that character is marked with an 'x' and the process repeats itself until all 'b' character's are marked and the first 'c' is encountered. The machine marks the character with an 'x' before moving the tape head right. If another 'c' is encountered, the tape head is moved to the left until an 'a' is encountered, at which point the machine

once again moves the tape head to the left until a 'c' is encountered and marked with an 'x'. If a 'd' is encountered, the character is marked with an 'x' and the machine moves the tape head back and forth to ensure that the remaining amount of 'a' characters matches the amount of 'd' characters. Once all characters have been marked, the machine enters its accept state. Inputs that are not in the language result in the machine becoming trapped in a state with no way to transition out. Once the machine has exhausted all possible transitions from a state, it automatically enters its reject state.

This machine was tested with the following inputs:

```
a,a,b,b,c
a,a,a,b,b,c,d
a,a,a,a,b,b,c,c
a,a,a,a,a,b,b,b,c,d,d
a,b,c,d
a,b,c,c
a,b,b
a,a,b,b,c,d
```

Inputs 1-4 are used to determine whether or not the machine properly accepts inputs from the language while inputs 5-8 are used to determine whether or not the machine properly rejects inputs that are not found in the language. The results from these inputs can be found at the end of the report or in the file `division_output.txt`.

The only major difficulty with the assignment was designing the 3 Turing machines since the simulator program itself utilized a lot of the same code used in the previous 2 projects.

## TM Simulator Outputs:

subtraction.txt

```

()q0(a,a,b,b)
(a)q1(a,b,b)
(a,a)q1(b,b)
(a)q2(a,x,b)
(a,x)q1(x,b)
(a,x,x)q1(b)
(a,x)q2(x,x)
(a)q2(x,x,x)
()q2(a,x,x,x)
(x)q1(x,x,x)
(x,x)q1(x,x)
(x,x,x)q1(x)
(x,x,x,x)q1()
(x,x,x)q9(x, )
(x,x)q9(x,x, )
(x)q9(x,x,x, )
()q9(x,x,x,x, )
()q9(x,x,x,x, )
()q7(x,x,x,x, ,)
ACCEPT

```

```

()q0(a,a,a,b,c,c)
(a)q1(a,a,b,c,c)
(a,a)q1(a,b,c,c)
(a,a,a)q1(b,c,c)
(a,a)q2(a,x,c,c)
(a,a,x)q1(x,c,c)
(a,a,x,x)q1(c,c)
(a,a,x)q4(x,x,c)
(a,a)q4(x,x,x,c)
(a)q4(a,x,x,x,c)
(a,x)q5(x,x,x,c)
(a,x,x)q5(x,x,c)
(a,x,x,x)q5(x,c)
(a,x,x,x,x)q5©
(a,x,x,x)q4(x,x)
(a,x,x)q4(x,x,x)
(a,x)q4(x,x,x,x)
(a)q4(x,x,x,x,x)
()q4(a,x,x,x,x,x)
(x)q5(x,x,x,x,x)
(x,x)q5(x,x,x,x)
(x,x,x)q5(x,x,x)
(x,x,x,x)q5(x,x)
(x,x,x,x,x)q5(x)
(x,x,x,x,x,x)q5()
(x,x,x,x,x,x)q6(x, )
(x,x,x,x)q6(x, , )
(x,x,x)q6(x, , , )
(x,x)q6(x, , , , )
(x)q6(x, , , , , )

```

()q6(x, , , , , )  
 ()q6( , , , , , )  
 ()q7( , , , , , , )  
 ACCEPT

()q0(a,c)  
 (a)q1(c)  
 ()q4(a,x)  
 (x)q5(x)  
 (x,x)q5()  
 (x)q6(x, )  
 ()q6(x, , )  
 ()q6( , , )  
 ()q7( , , , )  
 ACCEPT

()q0(a,a,b,c)  
 (a)q1(a,b,c)  
 (a,a)q1(b,c)  
 (a)q2(a,x,c)  
 (a,x)q1(x,c)  
 (a,x,x)q1⊙  
 (a,x)q4(x,x)  
 (a)q4(x,x,x)  
 ()q4(a,x,x,x)  
 (x)q5(x,x,x)  
 (x,x)q5(x,x)  
 (x,x,x)q5(x)  
 (x,x,x,x)q5()  
 (x,x,x)q6(x, )  
 (x,x)q6(x, , )  
 (x)q6(x, , , )  
 ()q6(x, , , , )  
 ()q6( , , , , )  
 ()q7( , , , , , )  
 ACCEPT

()q0(a,b,c)  
 (a)q1(b,c)  
 ()q2(a,x,c)  
 (x)q1(x,c)  
 (x,x)q1⊙  
 (x)q4(x,x)  
 ()q4(x,x,x)  
 ()q4(x,x,x)  
 ()q8(x,x,x)  
 REJECT

()q0(a,a,b,b,c)  
 (a)q1(a,b,b,c)  
 (a,a)q1(b,b,c)  
 (a)q2(a,x,b,c)  
 (a,x)q1(x,b,c)  
 (a,x,x)q1(b,c)  
 (a,x)q2(x,x,c)  
 (a)q2(x,x,x,c)

```
( )q2(a,x,x,x,c)
(x)q1(x,x,x,c)
(x,x)q1(x,x,c)
(x,x,x)q1(x,c)
(x,x,x,x)q1@
(x,x,x)q4(x,x)
(x,x)q4(x,x,x)
(x)q4(x,x,x,x)
( )q4(x,x,x,x,x)
( )q4(x,x,x,x,x)
( )q8(x,x,x,x,x)
REJECT
```

```
( )q0(a,b,b,c)
(a)q1(b,b,c)
( )q2(a,x,b,c)
(x)q1(x,b,c)
(x,x)q1(b,c)
(x)q2(x,x,c)
( )q2(x,x,x,c)
( )q2(x,x,x,c)
( )q8(x,x,x,c)
REJECT
```

```
( )q0(a,a,a,b,c)
(a)q1(a,a,b,c)
(a,a)q1(a,b,c)
(a,a,a)q1(b,c)
(a,a)q2(a,x,c)
(a,a,x)q1(x,c)
(a,a,x,x)q1@
(a,a,x)q4(x,x)
(a,a)q4(x,x,x)
(a)q4(a,x,x,x)
(a,x)q5(x,x,x)
(a,x,x)q5(x,x)
(a,x,x,x)q5(x)
(a,x,x,x,x)q5( )
(a,x,x,x,x)q6(x, )
(a,x,x)q6(x, , )
(a,x)q6(x, , , )
(a)q6(x, , , , )
( )q6(a, , , , )
(a)q8( , , , , )
REJECT
```

palindrome.txt

```
( )q0(1,0,#,0,1)
(1)q1(0,#,0,1)
(1,0)q1(#,0,1)
(1,0,#)q2(0,1)
(1,0)q3(#,x,1)
```

```

(1)q4(0,#,x,1)
(1,x)q5(#,x,1)
(1,x,#)q2(x,1)
(1,x,#,x)q2(1)
(1,x,#)q6(x,x)
(1,x)q6(#,x,x)
(1)q7(x,#,x,x)
()q7(1,x,#,x,x)
(x)q8(x,#,x,x)
(x,x)q8(#,x,x)
(x,x,#)q2(x,x)
(x,x,#,x)q2(x)
(x,x,#,x,x)q2()
(x,x,#,x,x, )q10()
ACCEPT

```

```

()q0(0,0,#,0,0)
(0)q1(0,#,0,0)
(0,0)q1(#,0,0)
(0,0,#)q2(0,0)
(0,0)q3(#,x,0)
(0)q4(0,#,x,0)
(0,x)q5(#,x,0)
(0,x,#)q2(x,0)
(0,x,#,x)q2(0)
(0,x,#)q3(x,x)
(0,x)q3(#,x,x)
(0)q4(x,#,x,x)
()q4(0,x,#,x,x)
(x)q5(x,#,x,x)
(x,x)q5(#,x,x)
(x,x,#)q2(x,x)
(x,x,#,x)q2(x)
(x,x,#,x,x)q2()
(x,x,#,x,x, )q10()
ACCEPT

```

```

()q0(1,0,#,1,1)
(1)q1(0,#,1,1)
(1,0)q1(#,1,1)
(1,0,#)q2(1,1)
(1,0)q6(#,x,1)
(1)q7(0,#,x,1)
(1,0)q9(#,x,1)
(1,0,#)q9(x,1)
REJECT

```

```

()q0(1,0,#,1,0)
(1)q1(0,#,1,0)
(1,0)q1(#,1,0)
(1,0,#)q2(1,0)
(1,0)q6(#,x,0)
(1)q7(0,#,x,0)
(1,0)q9(#,x,0)
(1,0,#)q9(x,0)
REJECT

```



()q0(1,1,0,0,1,1,#,1,1,0,0,1,1)  
 (1)q1(1,0,0,1,1,#,1,1,0,0,1,1)  
 (1,1)q1(0,0,1,1,#,1,1,0,0,1,1)  
 (1,1,0)q1(0,1,1,#,1,1,0,0,1,1)  
 (1,1,0,0)q1(1,1,#,1,1,0,0,1,1)  
 (1,1,0,0,1)q1(1,#,1,1,0,0,1,1)  
 (1,1,0,0,1,1)q1(#,1,1,0,0,1,1)  
 (1,1,0,0,1,1,#)q2(1,1,0,0,1,1)  
 (1,1,0,0,1,1)q6(#,x,1,0,0,1,1)  
 (1,1,0,0,1)q7(1,#,x,1,0,0,1,1)  
 (1,1,0,0,1,x)q8(#,x,1,0,0,1,1)  
 (1,1,0,0,1,x,#)q2(x,1,0,0,1,1)  
 (1,1,0,0,1,x,#,x)q2(1,0,0,1,1)  
 (1,1,0,0,1,x,#)q6(x,x,0,0,1,1)  
 (1,1,0,0,1,x)q6(#,x,x,0,0,1,1)  
 (1,1,0,0,1)q7(x,#,x,x,0,0,1,1)  
 (1,1,0,0)q7(1,x,#,x,x,0,0,1,1)  
 (1,1,0,0,x)q8(x,#,x,x,0,0,1,1)  
 (1,1,0,0,x,x)q8(#,x,x,0,0,1,1)  
 (1,1,0,0,x,x,#)q2(x,x,0,0,1,1)  
 (1,1,0,0,x,x,#,x)q2(x,0,0,1,1)  
 (1,1,0,0,x,x,#,x,x)q2(0,0,1,1)  
 (1,1,0,0,x,x,#,x)q3(x,x,0,1,1)  
 (1,1,0,0,x,x,#)q3(x,x,x,0,1,1)  
 (1,1,0,0,x,x)q3(#,x,x,x,0,1,1)  
 (1,1,0,0,x)q4(x,#,x,x,x,0,1,1)  
 (1,1,0,0)q4(x,x,#,x,x,x,0,1,1)  
 (1,1,0)q4(0,x,x,#,x,x,x,0,1,1)  
 (1,1,0,x)q5(x,x,#,x,x,x,0,1,1)  
 (1,1,0,x,x)q5(x,#,x,x,x,0,1,1)  
 (1,1,0,x,x,x)q5(#,x,x,x,0,1,1)  
 (1,1,0,x,x,x,#)q2(x,x,x,0,1,1)  
 (1,1,0,x,x,x,#,x)q2(x,x,0,1,1)  
 (1,1,0,x,x,x,#,x,x)q2(x,0,1,1)  
 (1,1,0,x,x,x,#,x,x,x)q2(0,1,1)  
 (1,1,0,x,x,x,#,x,x)q3(x,x,1,1)  
 (1,1,0,x,x,x,#,x)q3(x,x,x,1,1)  
 (1,1,0,x,x,x,#)q3(x,x,x,x,1,1)  
 (1,1,0,x,x,x)q3(#,x,x,x,x,1,1)  
 (1,1,0,x,x)q4(x,#,x,x,x,x,1,1)  
 (1,1,0,x)q4(x,x,#,x,x,x,x,1,1)  
 (1,1,0)q4(x,x,x,#,x,x,x,x,1,1)  
 (1,1)q4(0,x,x,x,#,x,x,x,x,1,1)  
 (1,1,x)q5(x,x,x,#,x,x,x,x,1,1)  
 (1,1,x,x)q5(x,x,#,x,x,x,x,1,1)  
 (1,1,x,x,x)q5(x,#,x,x,x,x,1,1)  
 (1,1,x,x,x,x)q5(#,x,x,x,x,1,1)  
 (1,1,x,x,x,x,#)q2(x,x,x,x,1,1)  
 (1,1,x,x,x,x,#,x)q2(x,x,x,1,1)  
 (1,1,x,x,x,x,#,x,x)q2(x,x,1,1)  
 (1,1,x,x,x,x,#,x,x,x)q2(x,1,1)  
 (1,1,x,x,x,x,#,x,x,x,x)q2(1,1)  
 (1,1,x,x,x,x,#,x,x,x)q6(x,x,1)  
 (1,1,x,x,x,x,#,x,x)q6(x,x,x,1)  
 (1,1,x,x,x,x,#,x)q6(x,x,x,x,1)

(1,1,x,x,x,#)q6(x,x,x,x,x,1)  
 (1,1,x,x,x,x)q6(#,x,x,x,x,x,1)  
 (1,1,x,x,x)q7(x,#,x,x,x,x,x,1)  
 (1,1,x,x)q7(x,x,#,x,x,x,x,x,1)  
 (1,1,x)q7(x,x,x,#,x,x,x,x,x,1)  
 (1,1)q7(x,x,x,x,#,x,x,x,x,x,1)  
 (1)q7(1,x,x,x,x,#,x,x,x,x,x,1)  
 (1,x)q8(x,x,x,x,#,x,x,x,x,x,1)  
 (1,x,x)q8(x,x,x,#,x,x,x,x,x,1)  
 (1,x,x,x)q8(x,x,#,x,x,x,x,x,1)  
 (1,x,x,x,x)q8(x,#,x,x,x,x,x,1)  
 (1,x,x,x,x,x)q8(#,x,x,x,x,x,1)  
 (1,x,x,x,x,x,#)q2(x,x,x,x,x,1)  
 (1,x,x,x,x,x,#,x)q2(x,x,x,x,1)  
 (1,x,x,x,x,x,#,x,x)q2(x,x,x,1)  
 (1,x,x,x,x,x,#,x,x,x)q2(x,x,1)  
 (1,x,x,x,x,x,#,x,x,x,x)q2(x,1)  
 (1,x,x,x,x,x,#,x,x,x,x,x)q2(1)  
 (1,x,x,x,x,x,x,#,x,x,x,x,x)q6(x,x)  
 (1,x,x,x,x,x,x,#,x,x,x)q6(x,x,x)  
 (1,x,x,x,x,x,x,#,x,x)q6(x,x,x,x)  
 (1,x,x,x,x,x,x,#,x)q6(x,x,x,x,x)  
 (1,x,x,x,x,x,x,#)q6(x,x,x,x,x,x)  
 (1,x,x,x,x,x,x)q6(#,x,x,x,x,x,x)  
 (1,x,x,x,x,x)q7(x,#,x,x,x,x,x,x)  
 (1,x,x,x)q7(x,x,#,x,x,x,x,x,x)  
 (1,x,x)q7(x,x,x,#,x,x,x,x,x,x)  
 (1,x)q7(x,x,x,x,#,x,x,x,x,x,x)  
 (1)q7(x,x,x,x,x,#,x,x,x,x,x,x)  
 ()q7(1,x,x,x,x,x,#,x,x,x,x,x,x)  
 (x)q8(x,x,x,x,x,#,x,x,x,x,x,x)  
 (x,x)q8(x,x,x,x,#,x,x,x,x,x,x)  
 (x,x,x)q8(x,x,x,#,x,x,x,x,x,x)  
 (x,x,x,x)q8(x,x,#,x,x,x,x,x,x)  
 (x,x,x,x,x)q8(x,#,x,x,x,x,x,x)  
 (x,x,x,x,x,x)q8(#,x,x,x,x,x,x)  
 (x,x,x,x,x,x,#)q2(x,x,x,x,x,x)  
 (x,x,x,x,x,x,#,x)q2(x,x,x,x,x)  
 (x,x,x,x,x,x,x,#,x,x)q2(x,x,x,x)  
 (x,x,x,x,x,x,x,#,x,x,x)q2(x,x,x)  
 (x,x,x,x,x,x,x,x,#,x,x,x,x)q2(x,x)  
 (x,x,x,x,x,x,x,x,x,#,x,x,x,x,x)q2(x)  
 (x,x,x,x,x,x,x,x,x,x,#,x,x,x,x,x,x)q2()  
 (x,x,x,x,x,x,x,x,x,x,x,#,x,x,x,x,x,x,x)q10()  
 ACCEPT

()q0(1,0,1,0,0,1,#,1,0,0,1,0,1)  
 (1)q1(0,1,0,0,1,#,1,0,0,1,0,1)  
 (1,0)q1(1,0,0,1,#,1,0,0,1,0,1)  
 (1,0,1)q1(0,0,1,#,1,0,0,1,0,1)  
 (1,0,1,0)q1(0,1,#,1,0,0,1,0,1)  
 (1,0,1,0,0)q1(1,#,1,0,0,1,0,1)  
 (1,0,1,0,0,1)q1(#,1,0,0,1,0,1)  
 (1,0,1,0,0,1,#)q2(1,0,0,1,0,1)  
 (1,0,1,0,0,1)q6(#,x,0,0,1,0,1)  
 (1,0,1,0,0)q7(1,#,x,0,0,1,0,1)

(1,0,1,0,0,x)q8(#,x,0,0,1,0,1)  
 (1,0,1,0,0,x,#)q2(x,0,0,1,0,1)  
 (1,0,1,0,0,x,#,x)q2(0,0,1,0,1)  
 (1,0,1,0,0,x,#)q3(x,x,0,1,0,1)  
 (1,0,1,0,0,x)q3(#,x,x,0,1,0,1)  
 (1,0,1,0,0)q4(x,#,x,x,0,1,0,1)  
 (1,0,1,0)q4(0,x,#,x,x,0,1,0,1)  
 (1,0,1,0,x)q5(x,#,x,x,0,1,0,1)  
 (1,0,1,0,x,x)q5(#,x,x,0,1,0,1)  
 (1,0,1,0,x,x,#)q2(x,x,0,1,0,1)  
 (1,0,1,0,x,x,#,x)q2(x,0,1,0,1)  
 (1,0,1,0,x,x,#,x,x)q2(0,1,0,1)  
 (1,0,1,0,x,x,#,x,x)q3(x,x,1,0,1)  
 (1,0,1,0,x,x,#)q3(x,x,x,1,0,1)  
 (1,0,1,0,x,x)q3(#,x,x,x,1,0,1)  
 (1,0,1,0,x)q4(x,#,x,x,x,1,0,1)  
 (1,0,1,0)q4(x,x,#,x,x,x,1,0,1)  
 (1,0,1)q4(0,x,x,#,x,x,x,1,0,1)  
 (1,0,1,x)q5(x,x,#,x,x,x,1,0,1)  
 (1,0,1,x,x)q5(x,#,x,x,x,1,0,1)  
 (1,0,1,x,x,x)q5(#,x,x,x,1,0,1)  
 (1,0,1,x,x,x,#)q2(x,x,x,1,0,1)  
 (1,0,1,x,x,x,#,x)q2(x,x,1,0,1)  
 (1,0,1,x,x,x,#,x,x)q2(x,1,0,1)  
 (1,0,1,x,x,x,#,x,x,x)q2(1,0,1)  
 (1,0,1,x,x,x,#,x,x)q6(x,x,0,1)  
 (1,0,1,x,x,x,#,x)q6(x,x,x,0,1)  
 (1,0,1,x,x,x,#)q6(x,x,x,x,0,1)  
 (1,0,1,x,x,x)q6(#,x,x,x,x,0,1)  
 (1,0,1,x,x)q7(x,#,x,x,x,x,0,1)  
 (1,0,1,x)q7(x,x,#,x,x,x,x,0,1)  
 (1,0,1)q7(x,x,x,#,x,x,x,x,0,1)  
 (1,0)q7(1,x,x,x,#,x,x,x,x,0,1)  
 (1,0,x)q8(x,x,x,#,x,x,x,x,0,1)  
 (1,0,x,x)q8(x,x,#,x,x,x,x,0,1)  
 (1,0,x,x,x)q8(x,#,x,x,x,x,0,1)  
 (1,0,x,x,x,x)q8(#,x,x,x,x,0,1)  
 (1,0,x,x,x,x,#)q2(x,x,x,x,0,1)  
 (1,0,x,x,x,x,#,x)q2(x,x,x,0,1)  
 (1,0,x,x,x,x,#,x,x)q2(x,x,0,1)  
 (1,0,x,x,x,x,#,x,x,x)q2(x,0,1)  
 (1,0,x,x,x,x,x)q2(0,1)  
 (1,0,x,x,x,x,x,#,x,x,x)q3(x,x,1)  
 (1,0,x,x,x,x,x,#,x,x)q3(x,x,x,1)  
 (1,0,x,x,x,x,x,#,x)q3(x,x,x,x,1)  
 (1,0,x,x,x,x)q3(#,x,x,x,x,x,1)  
 (1,0,x,x,x)q4(x,#,x,x,x,x,x,1)  
 (1,0,x,x)q4(x,x,#,x,x,x,x,x,1)  
 (1,0,x)q4(x,x,x,#,x,x,x,x,x,1)  
 (1,0)q4(x,x,x,x,#,x,x,x,x,x,1)  
 (1)q4(0,x,x,x,x,#,x,x,x,x,x,1)  
 (1,x)q5(x,x,x,x,#,x,x,x,x,x,1)  
 (1,x,x)q5(x,x,x,#,x,x,x,x,x,1)  
 (1,x,x,x)q5(x,x,#,x,x,x,x,x,1)  
 (1,x,x,x,x)q5(x,#,x,x,x,x,x,1)

(1,x,x,x,x)q5(#,x,x,x,x,1)  
 (1,x,x,x,x,#)q2(x,x,x,x,1)  
 (1,x,x,x,x,#,x)q2(x,x,x,x,1)  
 (1,x,x,x,x,#,x,x)q2(x,x,x,1)  
 (1,x,x,x,x,#,x,x,x)q2(x,x,1)  
 (1,x,x,x,x,#,x,x,x,x)q2(x,1)  
 (1,x,x,x,x,#,x,x,x,x,x)q2(1)  
 (1,x,x,x,x,#,x,x,x,x,x)q6(x,x)  
 (1,x,x,x,x,#,x,x,x,x)q6(x,x,x)  
 (1,x,x,x,x,#,x,x,x,x)q6(x,x,x,x)  
 (1,x,x,x,x,#,x,x,x,x,x)q6(x,x,x,x,x)  
 (1,x,x,x,x,x)q6(#,x,x,x,x,x)  
 (1,x,x,x,x)q7(x,#,x,x,x,x,x)  
 (1,x,x,x)q7(x,x,#,x,x,x,x,x)  
 (1,x,x)q7(x,x,x,#,x,x,x,x,x)  
 (1,x)q7(x,x,x,x,#,x,x,x,x,x)  
 (1)q7(x,x,x,x,x,#,x,x,x,x,x)  
 ()q7(1,x,x,x,x,x,#,x,x,x,x,x)  
 (x)q8(x,x,x,x,x,#,x,x,x,x,x)  
 (x,x)q8(x,x,x,x,#,x,x,x,x,x)  
 (x,x,x)q8(x,x,x,#,x,x,x,x,x)  
 (x,x,x,x)q8(x,x,#,x,x,x,x,x)  
 (x,x,x,x,x)q8(x,#,x,x,x,x,x)  
 (x,x,x,x,x,x)q8(#,x,x,x,x,x)  
 (x,x,x,x,x,x,#)q2(x,x,x,x,x)  
 (x,x,x,x,x,x,#,x)q2(x,x,x,x)  
 (x,x,x,x,x,x,#,x,x)q2(x,x,x)  
 (x,x,x,x,x,x,#,x,x,x)q2(x,x,x)  
 (x,x,x,x,x,x,#,x,x,x,x)q2(x,x)  
 (x,x,x,x,x,x,#,x,x,x,x,x)q2(x)  
 (x,x,x,x,x,x,#,x,x,x,x,x,x)q2()  
 (x,x,x,x,x,x,#,x,x,x,x,x,x, )q10()  
 ACCEPT

()q0(1,1,0,1,1,1,#,1,1,0,0,1,1)  
 (1)q1(1,0,1,1,1,#,1,1,0,0,1,1)  
 (1,1)q1(0,1,1,1,#,1,1,0,0,1,1)  
 (1,1,0)q1(1,1,1,#,1,1,0,0,1,1)  
 (1,1,0,1)q1(1,1,#,1,1,0,0,1,1)  
 (1,1,0,1,1)q1(1,#,1,1,0,0,1,1)  
 (1,1,0,1,1,1)q1(#,1,1,0,0,1,1)  
 (1,1,0,1,1,1,#)q2(1,1,0,0,1,1)  
 (1,1,0,1,1,1)q6(#,x,1,0,0,1,1)  
 (1,1,0,1,1)q7(1,#,x,1,0,0,1,1)  
 (1,1,0,1,1,x)q8(#,x,1,0,0,1,1)  
 (1,1,0,1,1,x,#)q2(x,1,0,0,1,1)  
 (1,1,0,1,1,x,#,x)q2(1,0,0,1,1)  
 (1,1,0,1,1,x,#)q6(x,x,0,0,1,1)  
 (1,1,0,1,1,x)q6(#,x,x,0,0,1,1)  
 (1,1,0,1,1)q7(x,#,x,x,0,0,1,1)  
 (1,1,0,1)q7(1,x,#,x,x,0,0,1,1)  
 (1,1,0,1,x)q8(x,#,x,x,0,0,1,1)  
 (1,1,0,1,x,x)q8(#,x,x,0,0,1,1)  
 (1,1,0,1,x,x,#)q2(x,x,0,0,1,1)  
 (1,1,0,1,x,x,#,x)q2(x,0,0,1,1)

(1,1,0,1,x,x,#,x,x)q2(0,0,1,1)  
 (1,1,0,1,x,x,#,x)q3(x,x,0,1,1)  
 (1,1,0,1,x,x,#)q3(x,x,x,0,1,1)  
 (1,1,0,1,x,x)q3(#,x,x,x,0,1,1)  
 (1,1,0,1,x)q4(x,#,x,x,x,0,1,1)  
 (1,1,0,1)q4(x,x,#,x,x,x,0,1,1)  
 (1,1,0)q4(1,x,x,#,x,x,x,0,1,1)  
 (1,1,0,1)q9(x,x,#,x,x,x,0,1,1)  
 (1,1,0,1,x)q9(x,#,x,x,x,0,1,1)  
 REJECT

()q0(1,1,0,0,1,1,#,1,1,0,1,1,1)  
 (1)q1(1,0,0,1,1,#,1,1,0,1,1,1)  
 (1,1)q1(0,0,1,1,#,1,1,0,1,1,1)  
 (1,1,0)q1(0,1,1,#,1,1,0,1,1,1)  
 (1,1,0,0)q1(1,1,#,1,1,0,1,1,1)  
 (1,1,0,0,1)q1(1,#,1,1,0,1,1,1)  
 (1,1,0,0,1,1)q1(#,1,1,0,1,1,1)  
 (1,1,0,0,1,1,#)q2(1,1,0,1,1,1)  
 (1,1,0,0,1,1)q6(#,x,1,0,1,1,1)  
 (1,1,0,0,1)q7(1,#,x,1,0,1,1,1)  
 (1,1,0,0,1,x)q8(#,x,1,0,1,1,1)  
 (1,1,0,0,1,x,#)q2(x,1,0,1,1,1)  
 (1,1,0,0,1,x,#,x)q2(1,0,1,1,1)  
 (1,1,0,0,1,x,#)q6(x,x,0,1,1,1)  
 (1,1,0,0,1,x)q6(#,x,x,0,1,1,1)  
 (1,1,0,0,1)q7(x,#,x,x,0,1,1,1)  
 (1,1,0,0)q7(1,x,#,x,x,0,1,1,1)  
 (1,1,0,0,x)q8(x,#,x,x,0,1,1,1)  
 (1,1,0,0,x,x)q8(#,x,x,0,1,1,1)  
 (1,1,0,0,x,x,#)q2(x,x,0,1,1,1)  
 (1,1,0,0,x,x,#,x)q2(x,0,1,1,1)  
 (1,1,0,0,x,x,#,x,x)q2(0,1,1,1)  
 (1,1,0,0,x,x,#,x)q3(x,x,1,1,1)  
 (1,1,0,0,x,x,#)q3(x,x,x,1,1,1)  
 (1,1,0,0,x,x)q3(#,x,x,x,1,1,1)  
 (1,1,0,0,x)q4(x,#,x,x,x,1,1,1)  
 (1,1,0,0)q4(x,x,#,x,x,x,1,1,1)  
 (1,1,0)q4(0,x,x,#,x,x,x,1,1,1)  
 (1,1,0,x)q5(x,x,#,x,x,x,1,1,1)  
 (1,1,0,x,x)q5(x,#,x,x,x,1,1,1)  
 (1,1,0,x,x,x)q5(#,x,x,x,1,1,1)  
 (1,1,0,x,x,x,#)q2(x,x,x,1,1,1)  
 (1,1,0,x,x,x,#,x)q2(x,x,1,1,1)  
 (1,1,0,x,x,x,#,x,x)q2(x,1,1,1)  
 (1,1,0,x,x,x,#,x,x,x)q2(1,1,1)  
 (1,1,0,x,x,x,#,x,x)q6(x,x,1,1)  
 (1,1,0,x,x,x,#,x)q6(x,x,x,1,1)  
 (1,1,0,x,x,x,#)q6(x,x,x,x,1,1)  
 (1,1,0,x,x,x)q6(#,x,x,x,x,1,1)  
 (1,1,0,x,x)q7(x,#,x,x,x,x,1,1)  
 (1,1,0,x)q7(x,x,#,x,x,x,x,1,1)  
 (1,1,0)q7(x,x,x,#,x,x,x,x,1,1)  
 (1,1)q7(0,x,x,x,#,x,x,x,x,1,1)  
 (1,1,0)q9(x,x,x,#,x,x,x,x,1,1)  
 (1,1,0,x)q9(x,x,#,x,x,x,x,1,1)

REJECT

divisionwr.txt

()q0(a,a,b,b,c)  
 (a)q1(a,b,b,c)  
 (a,a)q1(b,b,c)  
 (a)q2(a,b\*,b,c)  
 (a,x)q1(b\*,b,c)  
 (a,x,b\*)q1(b,c)  
 (a,x)q2(b\*,b\*,c)  
 (a)q2(x,b\*,b\*,c)  
 ()q2(a,x,b\*,b\*,c)  
 (x)q1(x,b\*,b\*,c)  
 (x,x)q1(b\*,b\*,c)  
 (x,x,b\*)q1(b\*,c)  
 (x,x,b\*,b\*)q1(c)  
 (x,x,b\*,b\*,x)q3()  
 (x,x,b\*,b\*)q7(x, )  
 (x,x,b\*)q7(b\*,x, )  
 (x,x)q7(b\*,b\*,x, )  
 (x)q7(x,b\*,b\*,x, )  
 ()q7(x,x,b\*,b\*,x, )  
 ()q7(x,x,b\*,b\*,x, )  
 ()q8(x,x,b\*,b\*,x, , )  
 ACCEPT

()q0(a,a,a,b,b,c,d)  
 (a)q1(a,a,b,b,c,d)  
 (a,a)q1(a,b,b,c,d)  
 (a,a,a)q1(b,b,c,d)  
 (a,a)q2(a,b\*,b,c,d)  
 (a,a,x)q1(b\*,b,c,d)  
 (a,a,x,b\*)q1(b,c,d)  
 (a,a,x)q2(b\*,b\*,c,d)  
 (a,a)q2(x,b\*,b\*,c,d)  
 (a)q2(a,x,b\*,b\*,c,d)  
 (a,x)q1(x,b\*,b\*,c,d)  
 (a,x,x)q1(b\*,b\*,c,d)  
 (a,x,x,b\*)q1(b\*,c,d)  
 (a,x,x,b\*,b\*)q1(c,d)  
 (a,x,x,b\*,b\*,x)q3(d)  
 (a,x,x,b\*,b\*)q4(x,x)  
 (a,x,x,b\*)q4(b\*,x,x)  
 (a,x,x)q4(b\*,b\*,x,x)  
 (a,x)q4(x,b\*,b\*,x,x)  
 (a)q4(x,x,b\*,b\*,x,x)  
 ()q4(a,x,x,b\*,b\*,x,x)  
 (x)q5(x,x,b\*,b\*,x,x)  
 (x,x)q5(x,b\*,b\*,x,x)  
 (x,x,x)q5(b\*,b\*,x,x)  
 (x,x,x,b\*)q5(b\*,x,x)  
 (x,x,x,b\*,b\*)q5(x,x)  
 (x,x,x,b\*,b\*,x)q5(x)

(x,x,x,b\*,b\*,x,x)q5()  
 (x,x,x,b\*,b\*,x,x, )q6()  
 (x,x,x,b\*,b\*,x,x, )q8()  
 ACCEPT

()q0(a,a,a,a,b,b,c,c)  
 (a)q1(a,a,a,b,b,c,c)  
 (a,a)q1(a,a,b,b,c,c)  
 (a,a,a)q1(a,b,b,c,c)  
 (a,a,a,a)q1(b,b,c,c)  
 (a,a,a)q2(a,b\*,b,c,c)  
 (a,a,a,x)q1(b\*,b,c,c)  
 (a,a,a,x,b\*)q1(b,c,c)  
 (a,a,a,x)q2(b\*,b\*,c,c)  
 (a,a,a)q2(x,b\*,b\*,c,c)  
 (a,a)q2(a,x,b\*,b\*,c,c)  
 (a,a,x)q1(x,b\*,b\*,c,c)  
 (a,a,x,x)q1(b\*,b\*,c,c)  
 (a,a,x,x,b\*)q1(b\*,c,c)  
 (a,a,x,x,b\*,b\*)q1(c,c)  
 (a,a,x,x,b\*,b\*,x)q3(c)  
 (a,a,x,x,b\*,b\*)q10(x,c)  
 (a,a,x,x,b\*)q10(b\*,x,c)  
 (a,a,x,x)q10(b\*,b,x,c)  
 (a,a,x)q10(x,b,b,x,c)  
 (a,a)q10(x,x,b,b,x,c)  
 (a)q10(a,x,x,b,b,x,c)  
 (a,a)q1(x,x,b,b,x,c)  
 (a,a,x)q1(x,b,b,x,c)  
 (a,a,x,x)q1(b,b,x,c)  
 (a,a,x)q2(x,b\*,b,x,c)  
 (a,a)q2(x,x,b\*,b,x,c)  
 (a)q2(a,x,x,b\*,b,x,c)  
 (a,x)q1(x,x,b\*,b,x,c)  
 (a,x,x)q1(x,b\*,b,x,c)  
 (a,x,x,x)q1(b\*,b,x,c)  
 (a,x,x,x,b\*)q1(b,x,c)  
 (a,x,x,x)q2(b\*,b\*,x,c)  
 (a,x,x)q2(x,b\*,b\*,x,c)  
 (a,x)q2(x,x,b\*,b\*,x,c)  
 (a)q2(x,x,x,b\*,b\*,x,c)  
 ()q2(a,x,x,x,b\*,b\*,x,c)  
 (x)q1(x,x,x,b\*,b\*,x,c)  
 (x,x)q1(x,x,b\*,b\*,x,c)  
 (x,x,x)q1(x,b\*,b\*,x,c)  
 (x,x,x,x)q1(b\*,b\*,x,c)  
 (x,x,x,x,b\*)q1(b\*,x,c)  
 (x,x,x,x,b\*,b\*)q1(x,c)  
 (x,x,x,x,b\*,b\*,x)q1(c)  
 (x,x,x,x,b\*,b\*,x,x)q3()  
 (x,x,x,x,b\*,b\*,x)q7(x, )  
 (x,x,x,x,b\*,b\*)q7(x,x, )  
 (x,x,x,x,b\*)q7(b\*,x,x, )  
 (x,x,x,x)q7(b\*,b\*,x,x, )  
 (x,x,x)q7(x,b\*,b\*,x,x, )  
 (x,x)q7(x,x,b\*,b\*,x,x, )

(x)q7(x,x,x,b\*,b\*,x,x, )  
 ()q7(x,x,x,b\*,b\*,x,x, )  
 ()q7(x,x,x,b\*,b\*,x,x, )  
 ()q8(x,x,x,b\*,b\*,x,x, ,)  
 ACCEPT

()q0(a,a,a,a,b,b,b,c,d,d)  
 (a)q1(a,a,a,b,b,b,c,d,d)  
 (a,a)q1(a,a,a,b,b,b,c,d,d)  
 (a,a,a)q1(a,a,b,b,b,c,d,d)  
 (a,a,a,a)q1(a,b,b,b,c,d,d)  
 (a,a,a,a,a)q1(b,b,b,c,d,d)  
 (a,a,a,a)q2(a,b\*,b,b,c,d,d)  
 (a,a,a,a,x)q1(b\*,b,b,c,d,d)  
 (a,a,a,a,x,b\*)q1(b,b,c,d,d)  
 (a,a,a,a,x)q2(b\*,b\*,b,c,d,d)  
 (a,a,a,a)q2(x,b\*,b\*,b,c,d,d)  
 (a,a,a)q2(a,x,b\*,b\*,b,c,d,d)  
 (a,a,a,x)q1(x,b\*,b\*,b,c,d,d)  
 (a,a,a,x,x)q1(b\*,b\*,b,c,d,d)  
 (a,a,a,x,x,b\*)q1(b\*,b,c,d,d)  
 (a,a,a,x,x,b\*,b\*)q1(b,c,d,d)  
 (a,a,a,x,x,b\*)q2(b\*,b\*,c,d,d)  
 (a,a,a,x,x)q2(b\*,b\*,b\*,c,d,d)  
 (a,a,a,x)q2(x,b\*,b\*,b\*,c,d,d)  
 (a,a,a)q2(x,x,b\*,b\*,b\*,c,d,d)  
 (a,a)q2(a,x,x,b\*,b\*,b\*,c,d,d)  
 (a,a,x)q1(x,x,b\*,b\*,b\*,c,d,d)  
 (a,a,x,x)q1(x,b\*,b\*,b\*,c,d,d)  
 (a,a,x,x,x)q1(b\*,b\*,b\*,c,d,d)  
 (a,a,x,x,x,b\*)q1(b\*,b\*,c,d,d)  
 (a,a,x,x,x,b\*,b\*)q1(b\*,c,d,d)  
 (a,a,x,x,x,b\*,b\*,b\*)q1(c,d,d)  
 (a,a,x,x,x,b\*,b\*,b\*,x)q3(d,d)  
 (a,a,x,x,x,b\*,b\*,b\*)q4(x,x,d)  
 (a,a,x,x,x,b\*,b\*)q4(b\*,x,x,d)  
 (a,a,x,x,x,b\*)q4(b\*,b\*,x,x,d)  
 (a,a,x,x,x)q4(b\*,b\*,b\*,x,x,d)  
 (a,a,x,x)q4(x,b\*,b\*,b\*,x,x,d)  
 (a,a,x)q4(x,x,b\*,b\*,b\*,x,x,d)  
 (a,a)q4(x,x,x,b\*,b\*,b\*,x,x,d)  
 (a)q4(a,x,x,x,b\*,b\*,b\*,x,x,d)  
 (a,x)q5(x,x,x,b\*,b\*,b\*,x,x,d)  
 (a,x,x)q5(x,x,b\*,b\*,b\*,x,x,d)  
 (a,x,x,x)q5(x,b\*,b\*,b\*,x,x,d)  
 (a,x,x,x,x)q5(b\*,b\*,b\*,x,x,d)  
 (a,x,x,x,x,b\*)q5(b\*,b\*,x,x,d)  
 (a,x,x,x,x,b\*,b\*)q5(b\*,x,x,d)  
 (a,x,x,x,x,b\*,b\*,b\*)q5(x,x,d)  
 (a,x,x,x,x,b\*,b\*,b\*,x)q5(x,d)  
 (a,x,x,x,x,b\*,b\*,b\*,x,x)q5(d)  
 (a,x,x,x,x,b\*,b\*,b\*,x)q4(x,x)  
 (a,x,x,x,x,b\*,b\*,b\*)q4(x,x,x)  
 (a,x,x,x,x,b\*,b\*)q4(b\*,x,x,x)  
 (a,x,x,x,x,b\*)q4(b\*,b\*,x,x,x)  
 (a,x,x,x,x)q4(b\*,b\*,b\*,x,x,x)



(a,x,x,x)q4(x,b\*,b\*,b\*,x,x,x)  
 (a,x,x)q4(x,x,b\*,b\*,b\*,x,x,x)  
 (a,x)q4(x,x,x,b\*,b\*,b\*,x,x,x)  
 (a)q4(x,x,x,x,b\*,b\*,b\*,x,x,x)  
 ()q4(a,x,x,x,x,b\*,b\*,b\*,x,x,x)  
 (x)q5(x,x,x,x,b\*,b\*,b\*,x,x,x)  
 (x,x)q5(x,x,x,b\*,b\*,b\*,x,x,x)  
 (x,x,x)q5(x,x,b\*,b\*,b\*,x,x,x)  
 (x,x,x,x)q5(x,b\*,b\*,b\*,x,x,x)  
 (x,x,x,x,x)q5(b\*,b\*,b\*,x,x,x)  
 (x,x,x,x,x,b\*)q5(b\*,b\*,x,x,x)  
 (x,x,x,x,x,b\*,b\*)q5(b\*,x,x,x)  
 (x,x,x,x,x,b\*,b\*,b\*)q5(x,x,x)  
 (x,x,x,x,x,b\*,b\*,b\*,x)q5(x,x)  
 (x,x,x,x,x,b\*,b\*,b\*,x,x)q5(x)  
 (x,x,x,x,x,b\*,b\*,b\*,x,x,x)q5()  
 (x,x,x,x,x,b\*,b\*,b\*,x,x,x, )q6()  
 (x,x,x,x,x,b\*,b\*,b\*,x,x,x, , )q8()  
 ACCEPT

()q0(a,b,c,d)  
 (a)q1(b,c,d)  
 ()q2(a,b\*,c,d)  
 (x)q1(b\*,c,d)  
 (x,b\*)q1(c,d)  
 (x,b\*,x)q3(d)  
 (x,b\*)q4(x,x)  
 (x)q4(b\*,x,x)  
 ()q4(x,b\*,x,x)  
 ()q4(x,b\*,x,x)  
 ()q9(x,b\*,x,x)  
 REJECT

()q0(a,b,c,c)  
 (a)q1(b,c,c)  
 ()q2(a,b\*,c,c)  
 (x)q1(b\*,c,c)  
 (x,b\*)q1(c,c)  
 (x,b\*,x)q3(c)  
 (x,b\*)q10(x,c)  
 (x)q10(b\*,x,c)  
 ()q10(x,b,x,c)  
 ()q10(x,b,x,c)  
 ()q9(x,b,x,c)  
 REJECT

()q0(a,b,b)  
 (a)q1(b,b)  
 ()q2(a,b\*,b)  
 (x)q1(b\*,b)  
 (x,b\*)q1(b)  
 (x)q2(b\*,b\*)  
 ()q2(x,b\*,b\*)  
 ()q2(x,b\*,b\*)

()q9(x,b\*,b\*)  
REJECT

()q0(a,a,b,b,c,d)  
(a)q1(a,b,b,c,d)  
(a,a)q1(b,b,c,d)  
(a)q2(a,b\*,b,c,d)  
(a,x)q1(b\*,b,c,d)  
(a,x,b\*)q1(b,c,d)  
(a,x)q2(b\*,b\*,c,d)  
(a)q2(x,b\*,b\*,c,d)  
()q2(a,x,b\*,b\*,c,d)  
(x)q1(x,b\*,b\*,c,d)  
(x,x)q1(b\*,b\*,c,d)  
(x,x,b\*)q1(b\*,c,d)  
(x,x,b\*,b\*)q1(c,d)  
(x,x,b\*,b\*,x)q3(d)  
(x,x,b\*,b\*)q4(x,x)  
(x,x,b\*)q4(b\*,x,x)  
(x,x)q4(b\*,b\*,x,x)  
(x)q4(x,b\*,b\*,x,x)  
()q4(x,x,b\*,b\*,x,x)  
()q4(x,x,b\*,b\*,x,x)  
()q9(x,x,b\*,b\*,x,x)  
REJECT