Weekly Homework #1

Dan McArdle
CIS 275 – Discrete Math

Oct 14, 2013

Problem 5

Claim: Let A, B, C be sets. Then, $(A \backslash B) \backslash C \subseteq (A \backslash C) \backslash (B \backslash C)$.

Proof: (direct) It is sufficient to show the following two subclaims.

1. $(A \backslash B) \backslash C \subseteq (A \backslash C) \backslash (B \backslash C)$
2. $(A \backslash B) \backslash C \supseteq (A \backslash C) \backslash (B \backslash C)$

Subclaim 1: $(A \backslash B) \backslash C \subseteq (A \backslash C) \backslash (B \backslash C)$
Let x be an arbitrary element in $(A \backslash B) \backslash C$.
It follows that $x \in A \backslash B$ and $x \notin C$.
Since $x \in A \backslash B$, it follows that $x \in A$ and $x \notin B$.

Since we know $x \in A$ and $x \notin C$, it follows that $x \in A \backslash C$.
Since we know that $x \notin B$, it follows that $x \notin B \backslash C$.

Because $x \in A \backslash C$ and $x \notin B \backslash C$, we have shown $x \in (A \backslash C) \backslash (B \backslash C)$.
Therefore, since x was an arbitrary element in $(A \backslash B) \backslash C)$, we have shown that $(A \backslash B) \backslash C \subseteq (A \backslash C) \backslash (B \backslash C)$. Subclaim 1 is true.

Subclaim 2: $(A \backslash B) \backslash C \supseteq (A \backslash C) \backslash (B \backslash C)$ [This is where we would prove that subclaim 2 is true. I leave it as an exercise for the reader!]

Because subclaims 1 and 2 are true, our original claim is true.