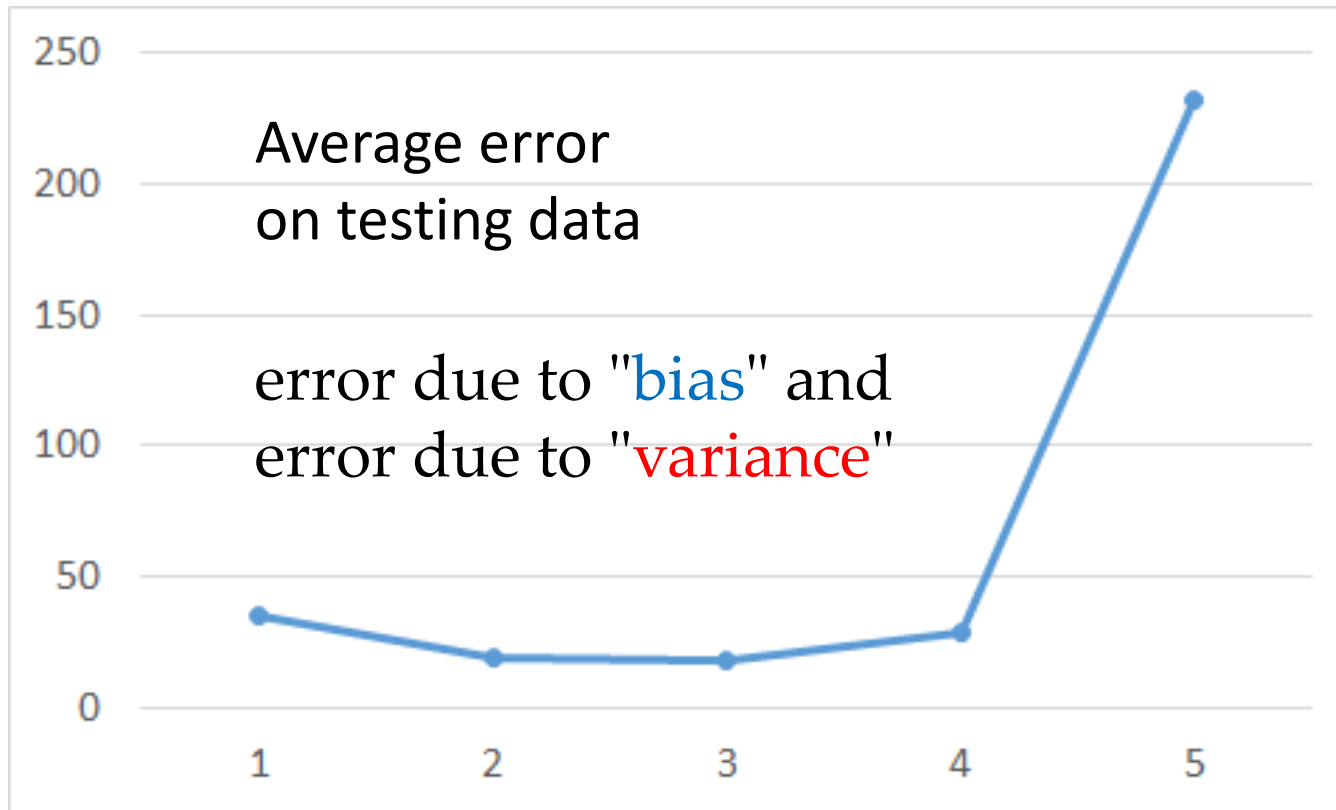


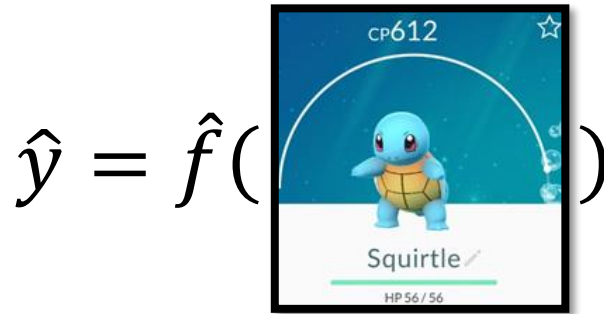
Where does the error
come from?

Review



A more complex model does not always lead to better performance on testing data.

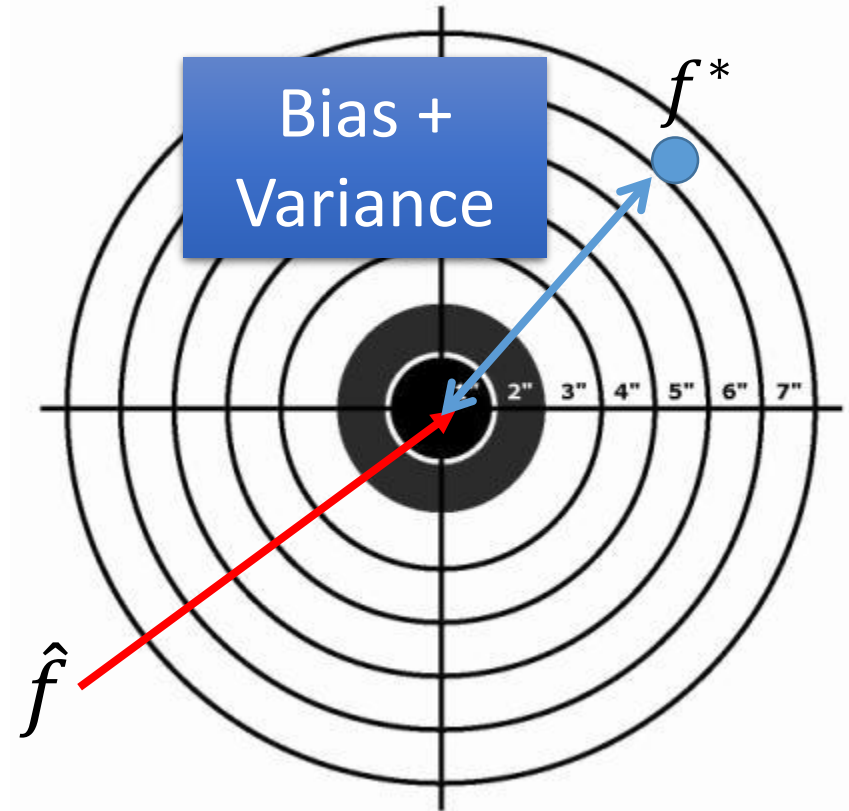
Estimator



Only Niantic knows \hat{f}

From training data,
we find f^*

f^* is an estimator of \hat{f}



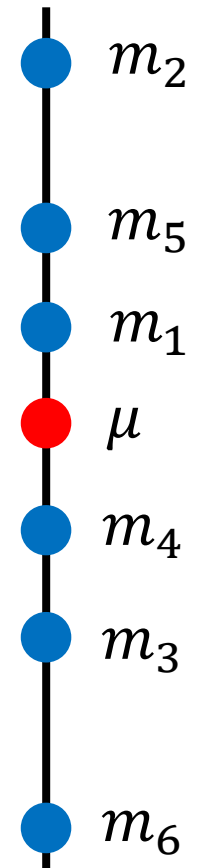
Bias and Variance of Estimator

- Estimate the mean of a variable x
 - assume the mean of x is μ
 - assume the variance of x is σ^2
- Estimator of mean μ
 - Sample N points: $\{x^1, x^2, \dots, x^N\}$

$$m = \frac{1}{N} \sum_n x^n \neq \mu$$

$$E[m] = E\left[\frac{1}{N} \sum_n x^n\right] = \frac{1}{N} \sum_n E[x^n] = \mu$$

unbiased



Bias and Variance of Estimator

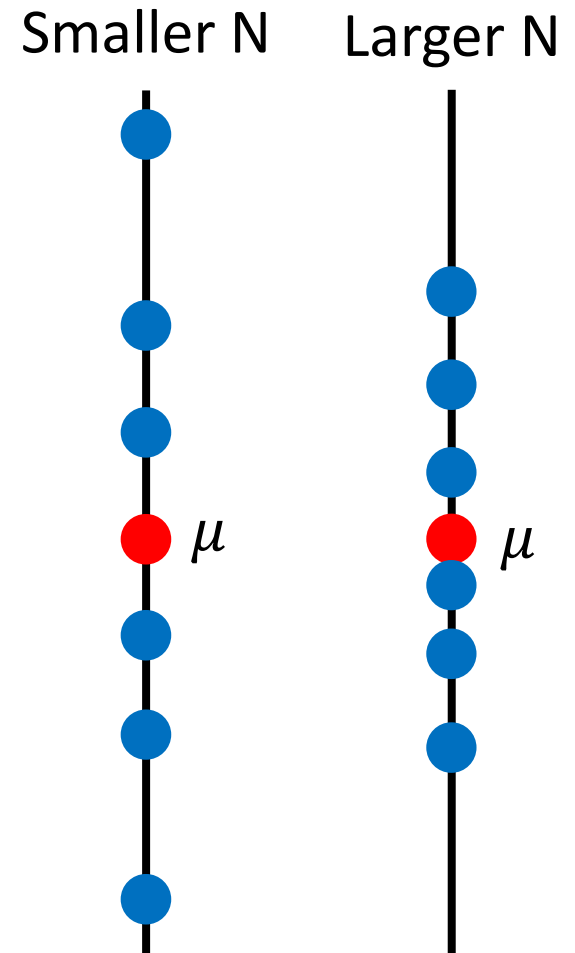
- Estimate the mean of a variable x
 - assume the mean of x is μ
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- Estimator of mean μ
 - Sample N points: $\{x^1, x^2, \dots, x^N\}$

$$m = \frac{1}{N} \sum_n x^n \neq \mu$$

$$\text{Var}[m] = \frac{\sigma^2}{N}$$

Variance depends
on the number of
samples

unbiased



Bias and Variance of Estimator

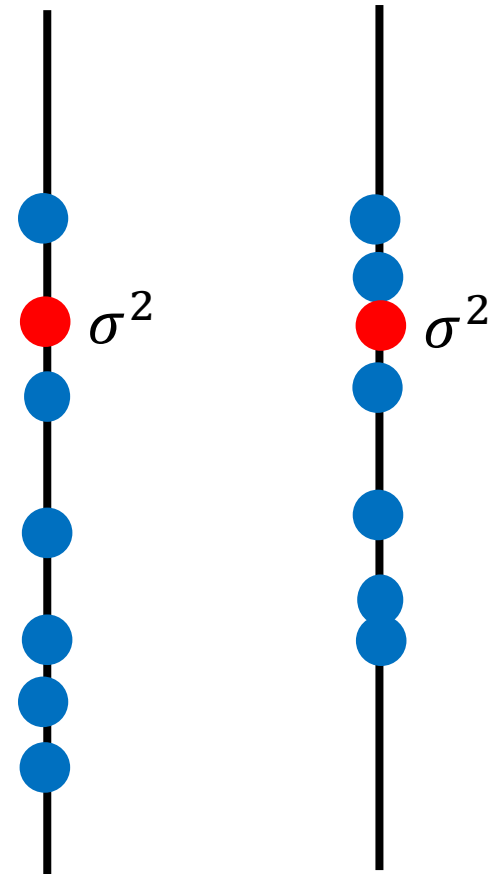
- Estimate the mean of a variable x
 - assume the mean of x is μ
 - assume the variance of x is σ^2
- Estimator of variance σ^2
 - Sample N points: $\{x^1, x^2, \dots, x^N\}$

$$m = \frac{1}{N} \sum_n x^n \quad s^2 = \frac{1}{N} \sum_n (x^n - m)^2$$

Biased estimator

$$E[s^2] = \frac{N-1}{N} \sigma^2 \neq \sigma^2$$

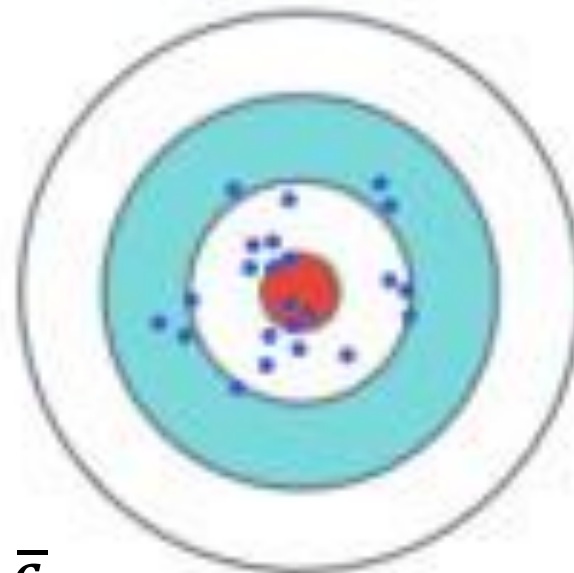
Increase N



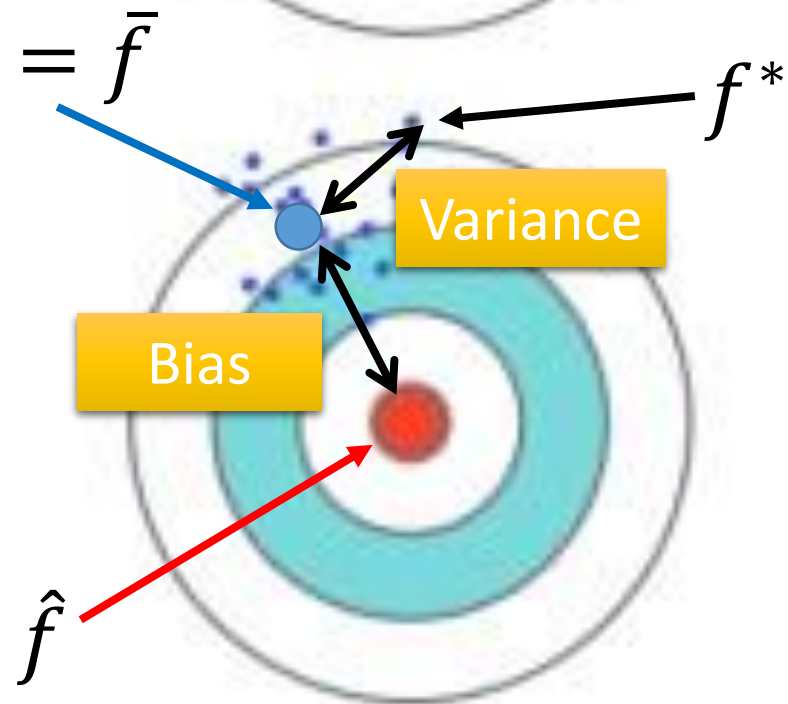
Low Variance

High Variance

Low Bias



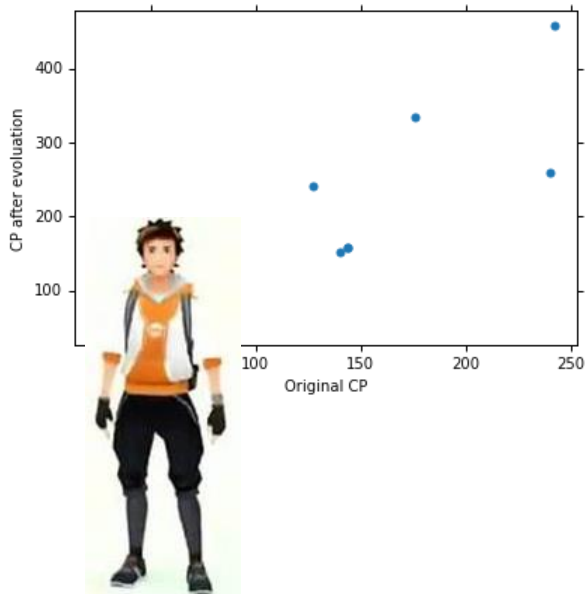
High Bias



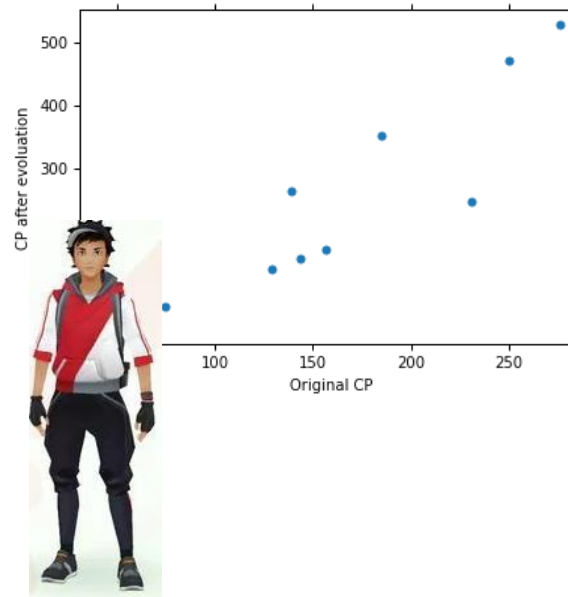
Parallel Universes

- In all the universes, we are collecting (catching) 10 Pokémon as training data to find f^*

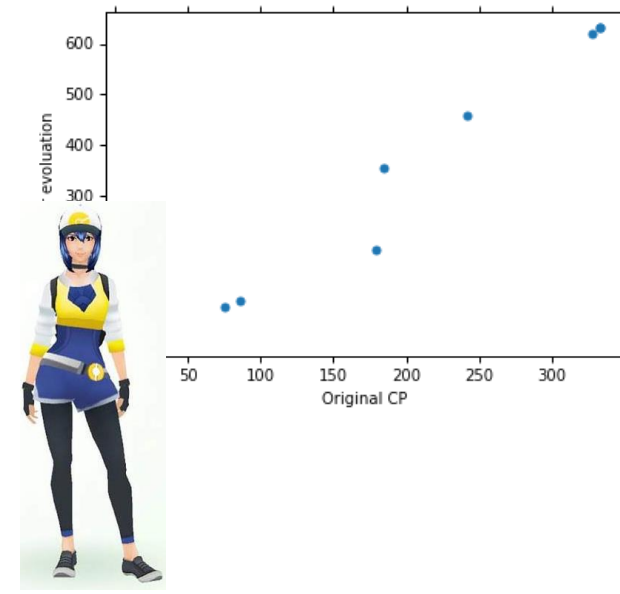
Universe 1



Universe 2



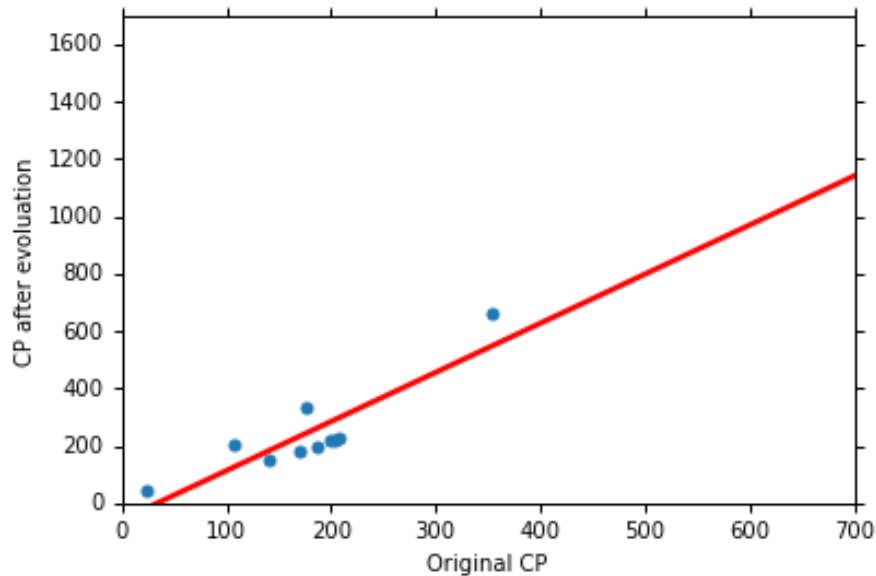
Universe 3



Parallel Universes

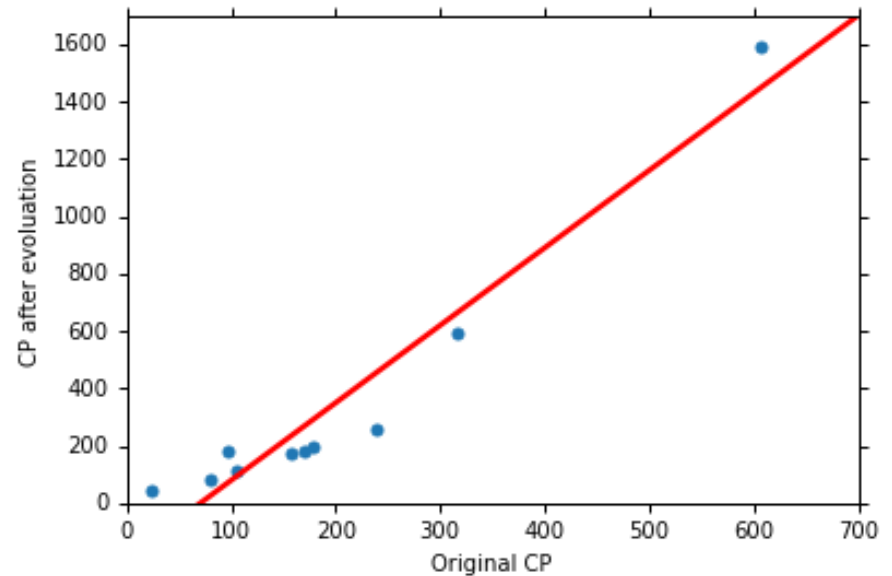
- In different universes, we use the same model, but obtain different f^*

Universe 123



$$y = b + w \cdot x_{cp}$$

Universe 345



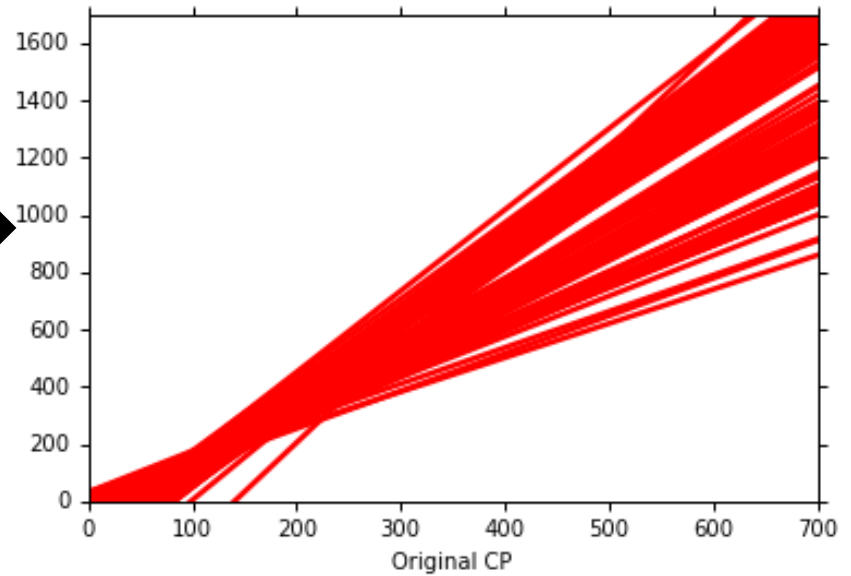
$$y = b + w \cdot x_{cp}$$

f^* in 100 Universes

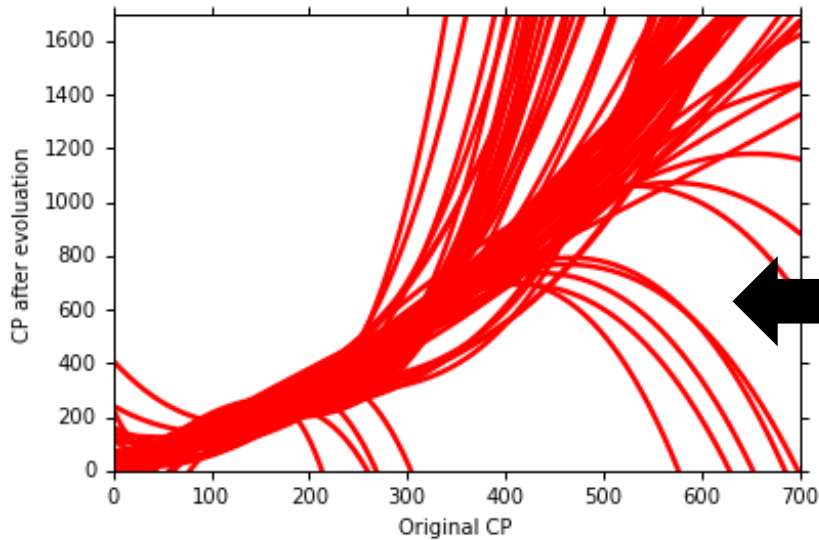
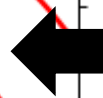
$$y = b + w \cdot x_{cp}$$



CP after evolution



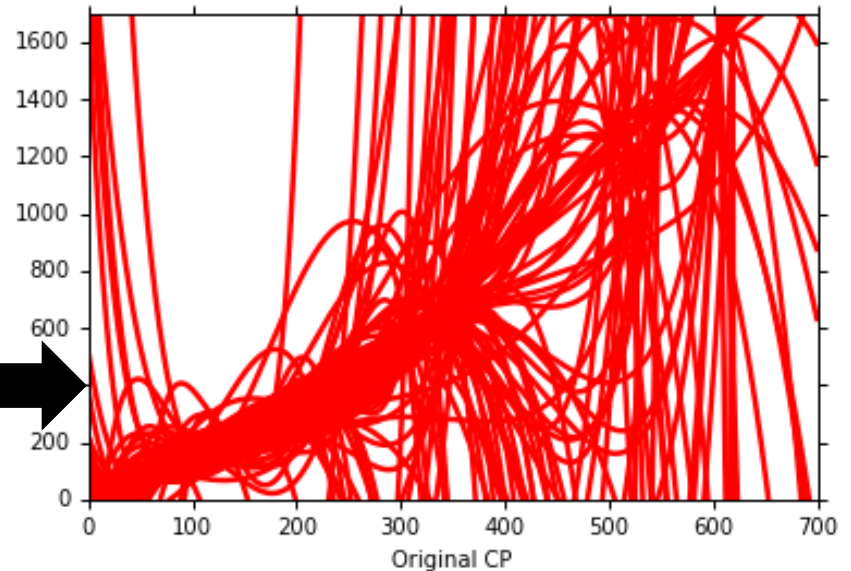
$$y = b + w_1 \cdot x_{cp} + w_2 \cdot (x_{cp})^2 + w_3 \cdot (x_{cp})^3$$



$$y = b + w_1 \cdot x_{cp} + w_2 \cdot (x_{cp})^2 + w_3 \cdot (x_{cp})^3 + w_4 \cdot (x_{cp})^4 + w_5 \cdot (x_{cp})^5$$

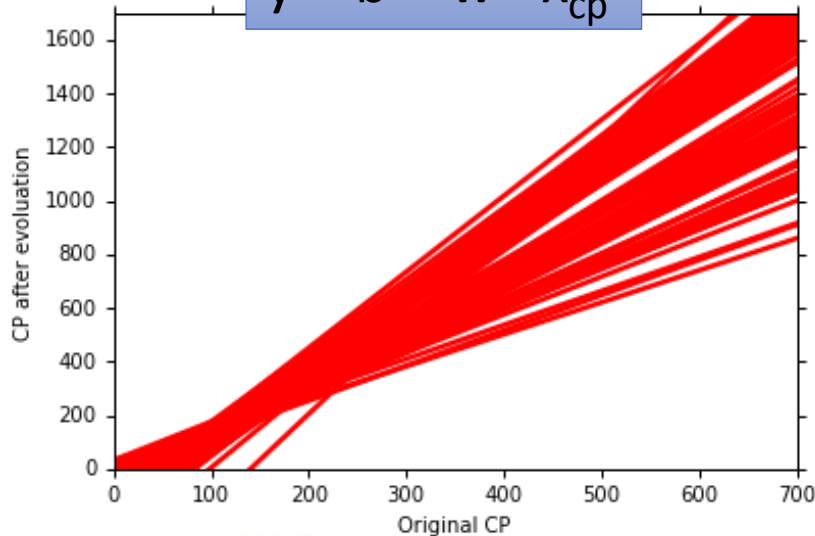


CP after evolution

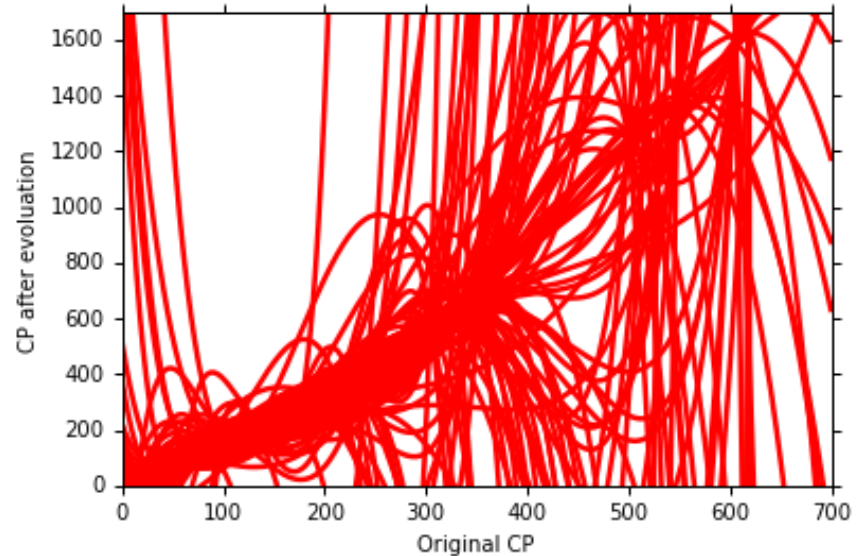


Variance

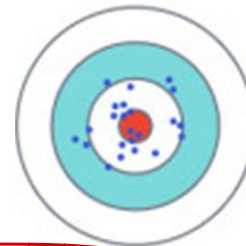
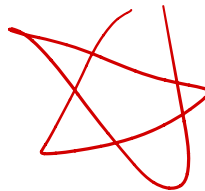
$$y = b + w \cdot x_{cp}$$



$$y = b + w_1 \cdot x_{cp} + w_2 \cdot (x_{cp})^2 + w_3 \cdot (x_{cp})^3 + w_4 \cdot (x_{cp})^4 + w_5 \cdot (x_{cp})^5$$



Small
Variance



Large
Variance

Simpler model is less influenced by the sampled data

Consider the extreme case $f(x) = c$

Bias

$$E[f^*] = \bar{f}$$

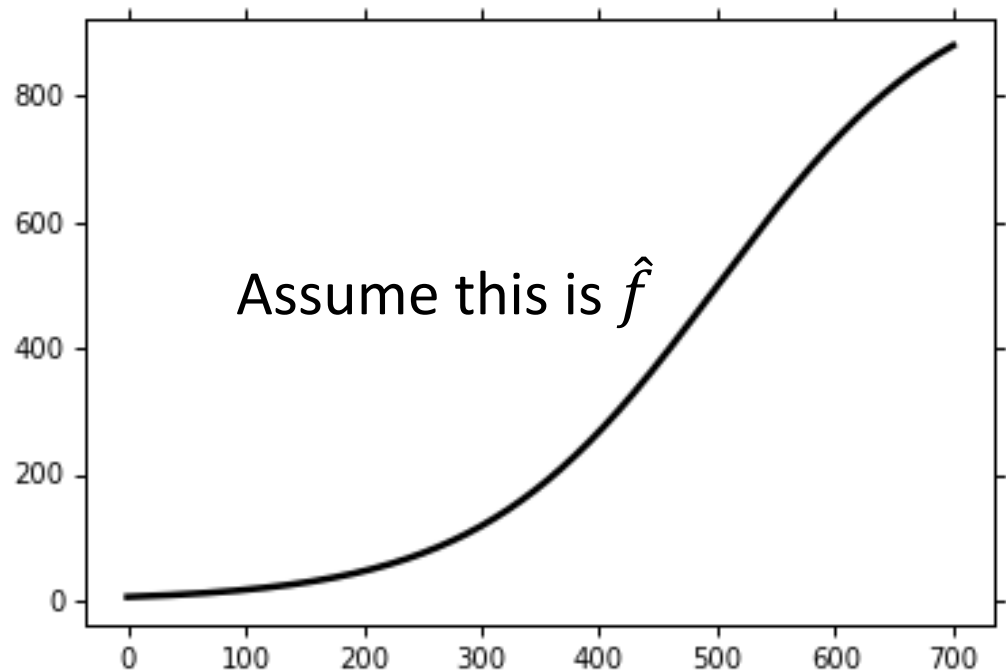
- Bias: If we average all the f^* , is it close to \hat{f}



Large
Bias



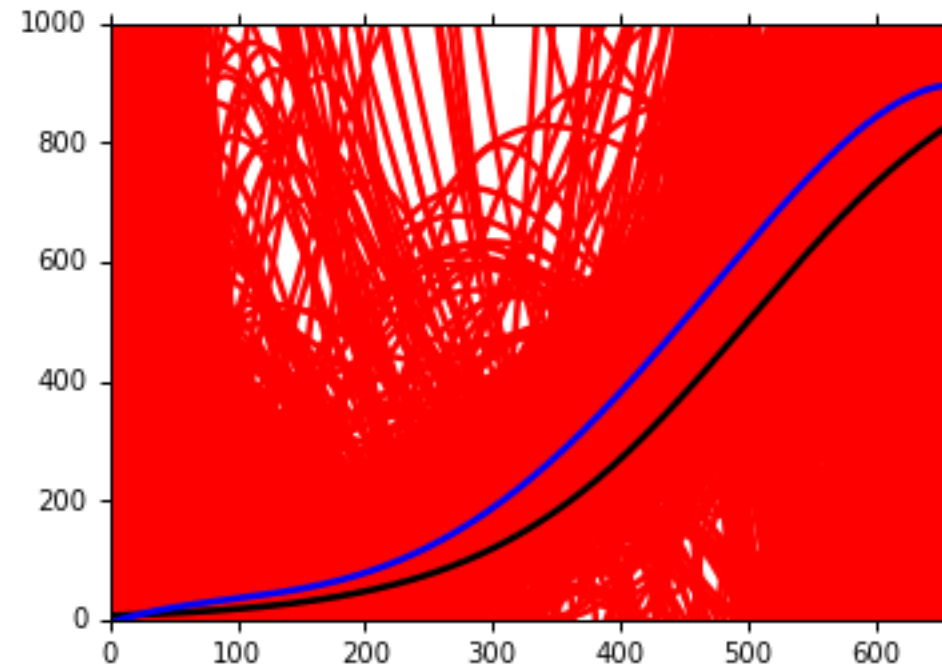
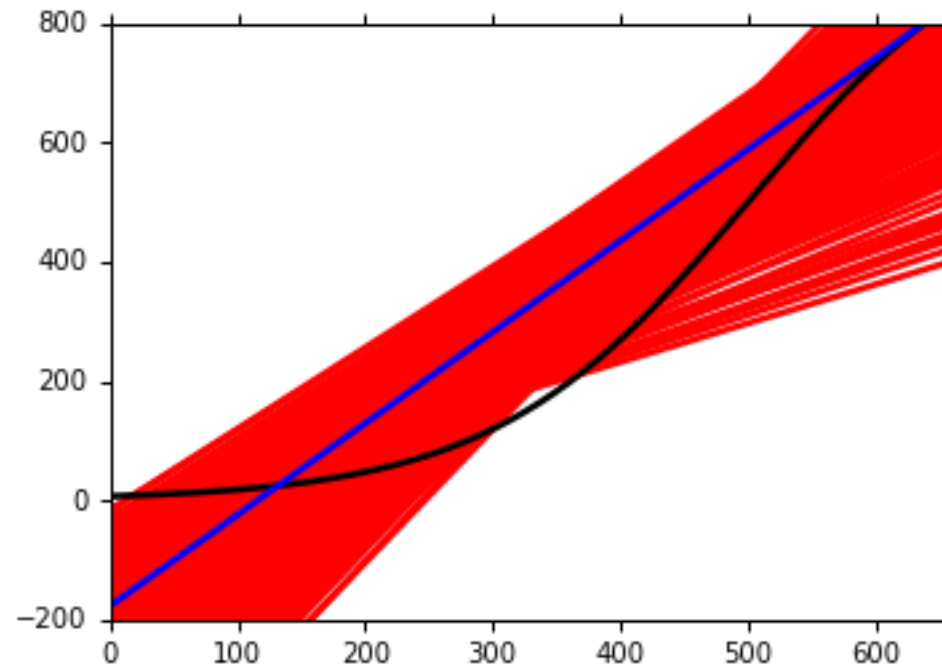
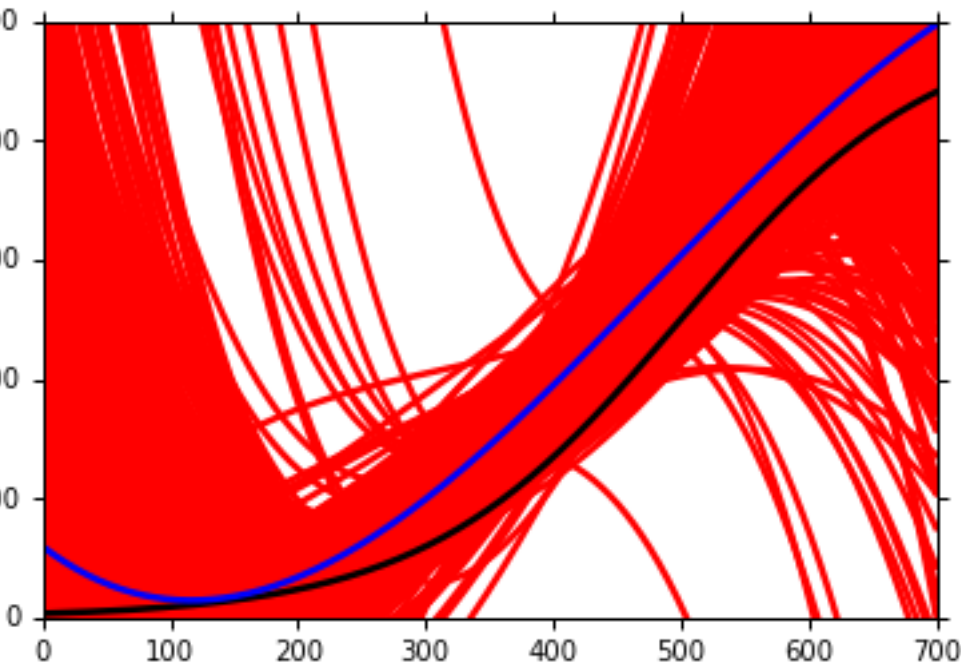
Small
Bias



Black curve: the true function \hat{f}

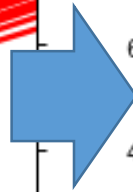
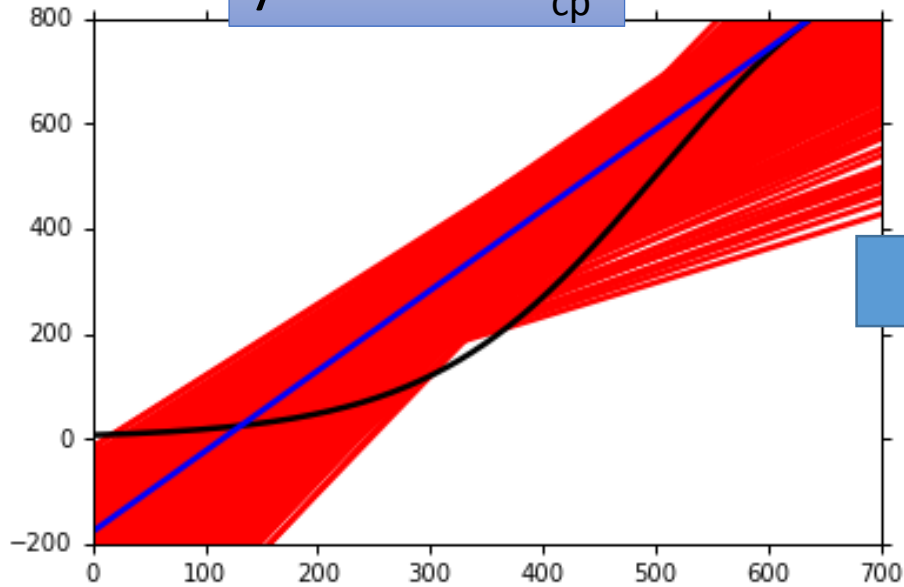
Red curves: 5000 f^*

Blue curve: the average of 5000 f^*
 $= \bar{f}$

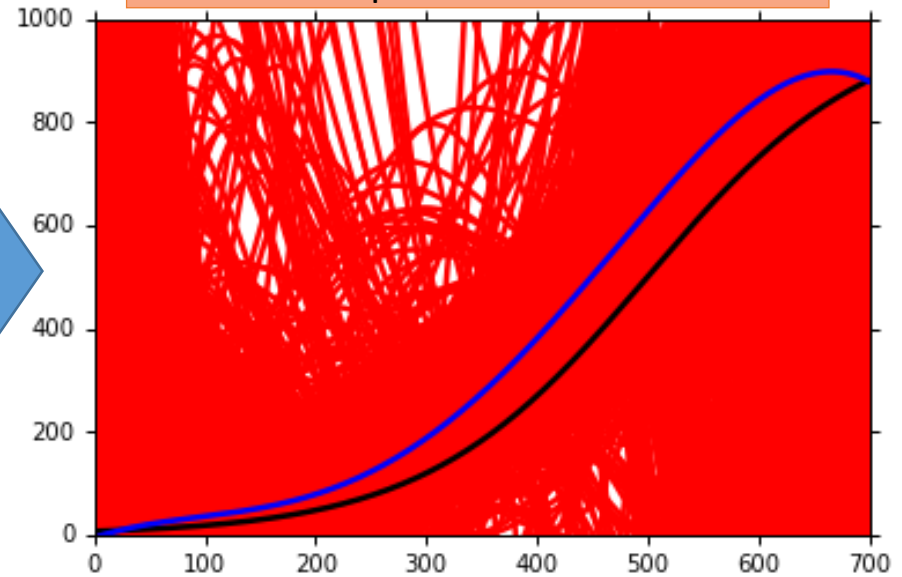


Bias

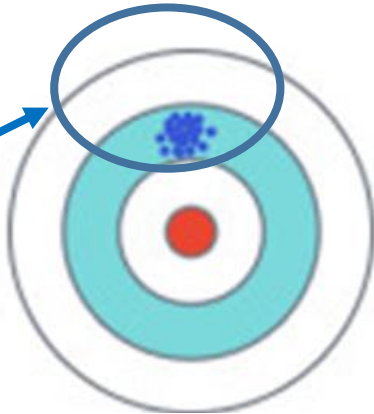
$$y = b + w \cdot x_{cp}$$



$$y = b + w_1 \cdot x_{cp} + w_2 \cdot (x_{cp})^2 + w_3 \cdot (x_{cp})^3 + w_4 \cdot (x_{cp})^4 + w_5 \cdot (x_{cp})^5$$

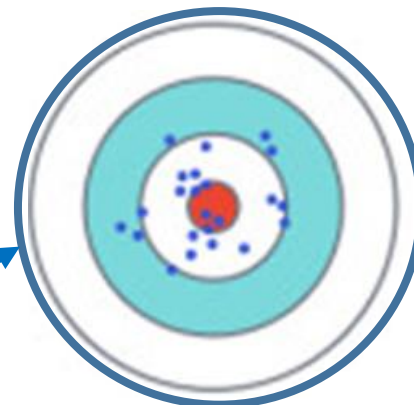


model



Large
Bias

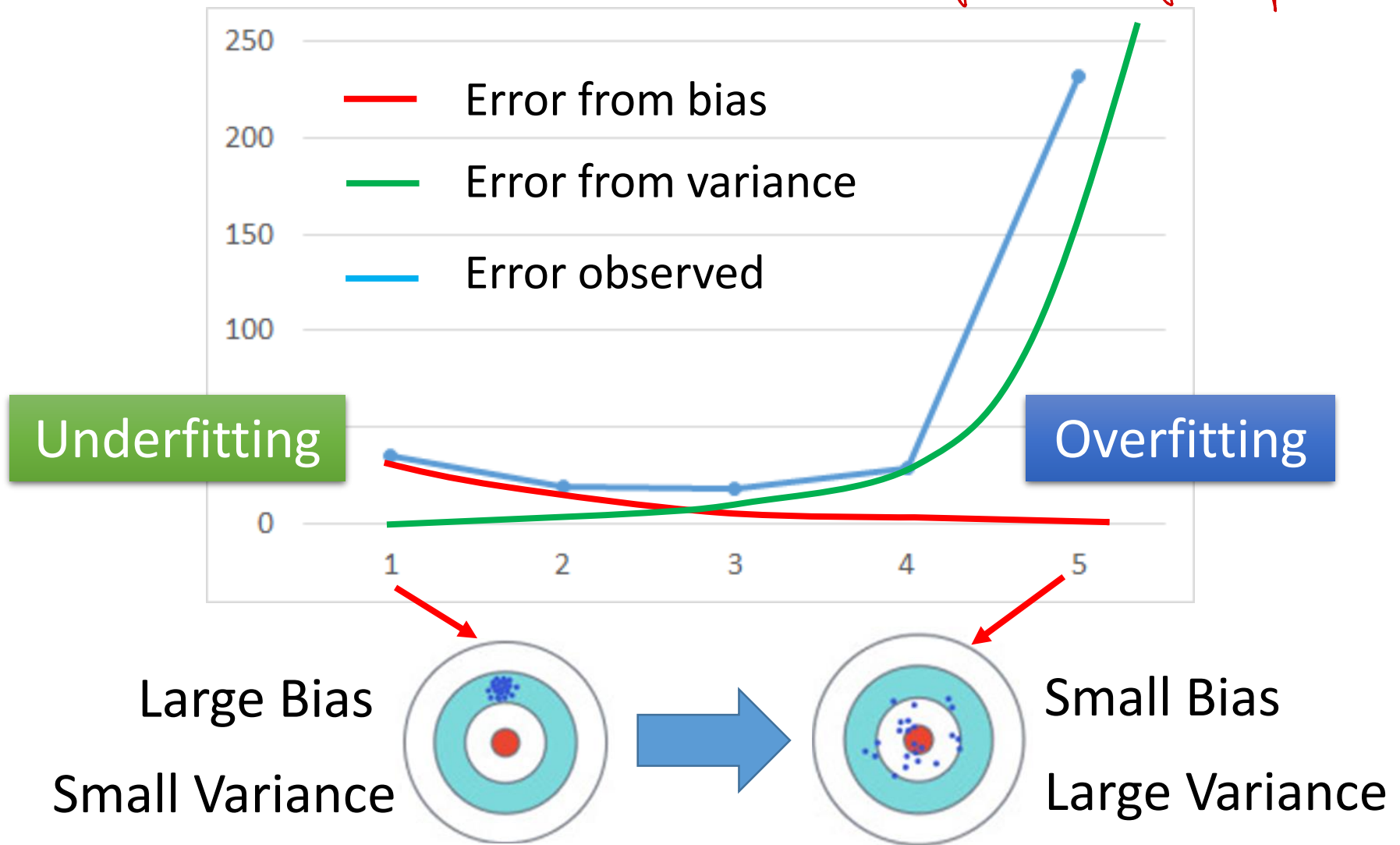
model



Small
Bias

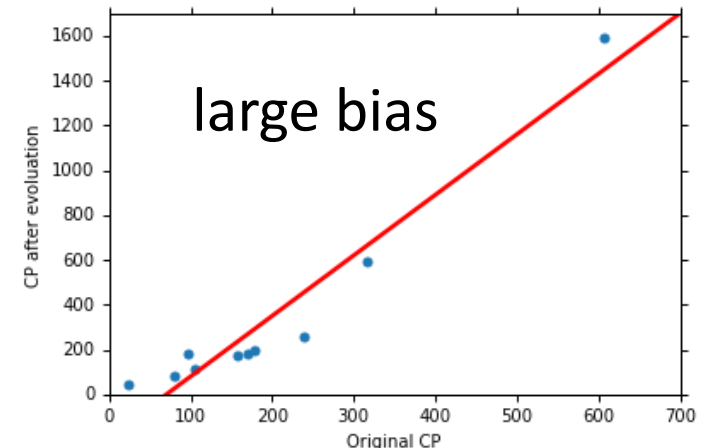
model 是一个 function set, 简单 model 这个 set 很小, 未必包含 f^* , 所以 f^* 始终与 f 有很大 bias, 而复杂 model 的 set 很大, 可能包含 f^* , 所以 f^* 与 f bias 小

Bias v.s. Variance



What to do with large bias?

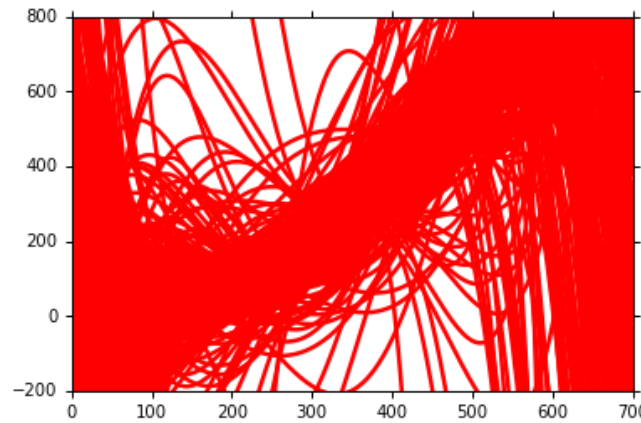
- Diagnosis:
 - If your model cannot even fit the training examples, then you have large bias **Underfitting**
 - If you can fit the training data, but large error on testing data, then you probably have large variance **Overfitting**
- For bias, redesign your model:
 - Add more features as input
 - A more complex model



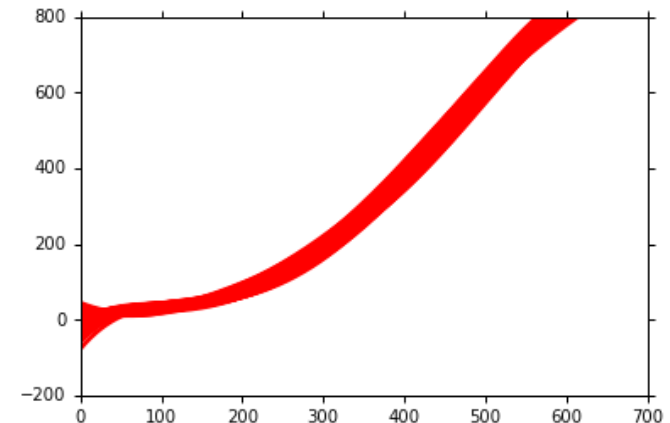
What to do with large variance?

- More data

Very effective,
but not always
practical

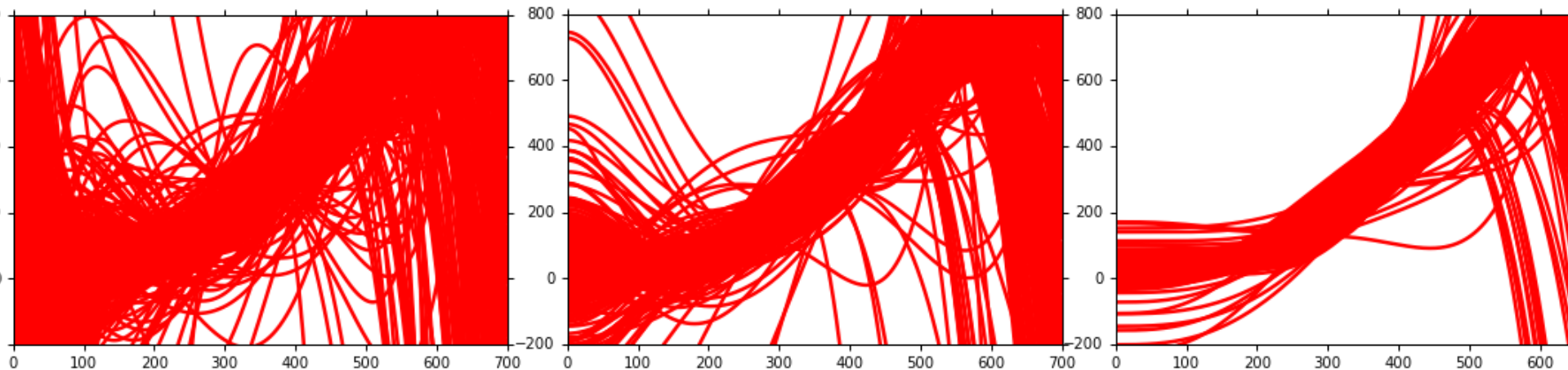


10 examples



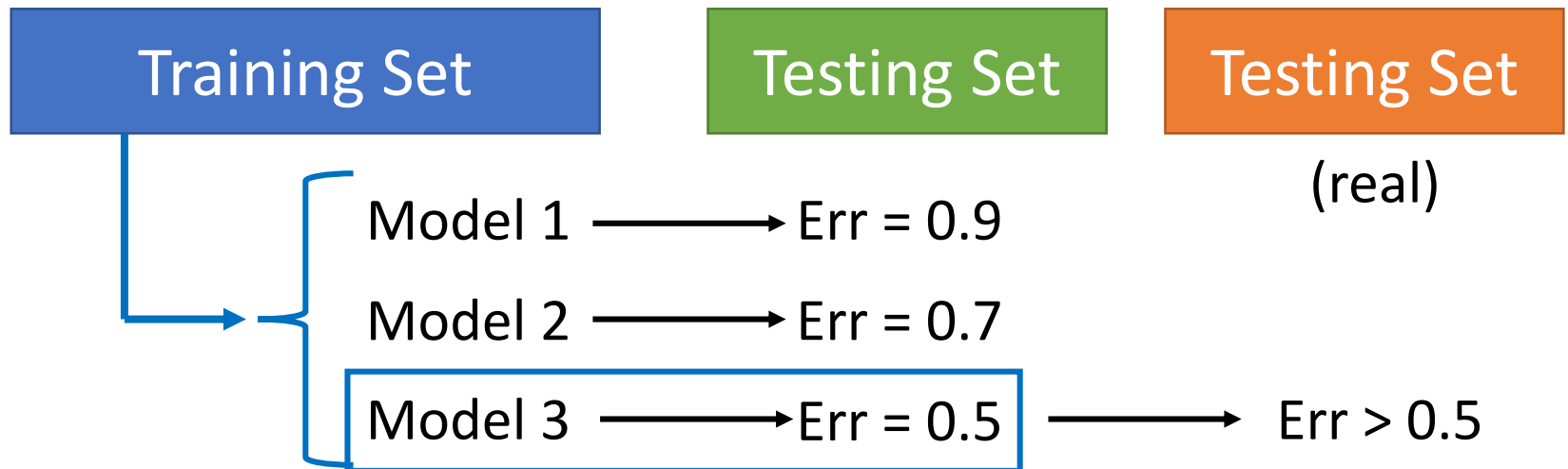
100 examples

- Regularization



Model Selection

- There is usually a trade-off between bias and variance.
- Select a model that balances two kinds of error to minimize total error
- What you should NOT do:



Homework

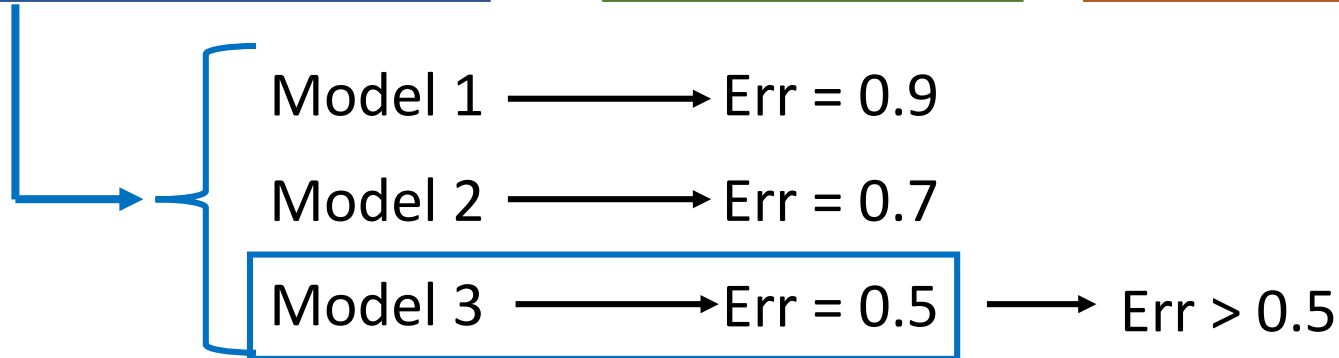
public

private

Training Set

Testing Set

Testing Set

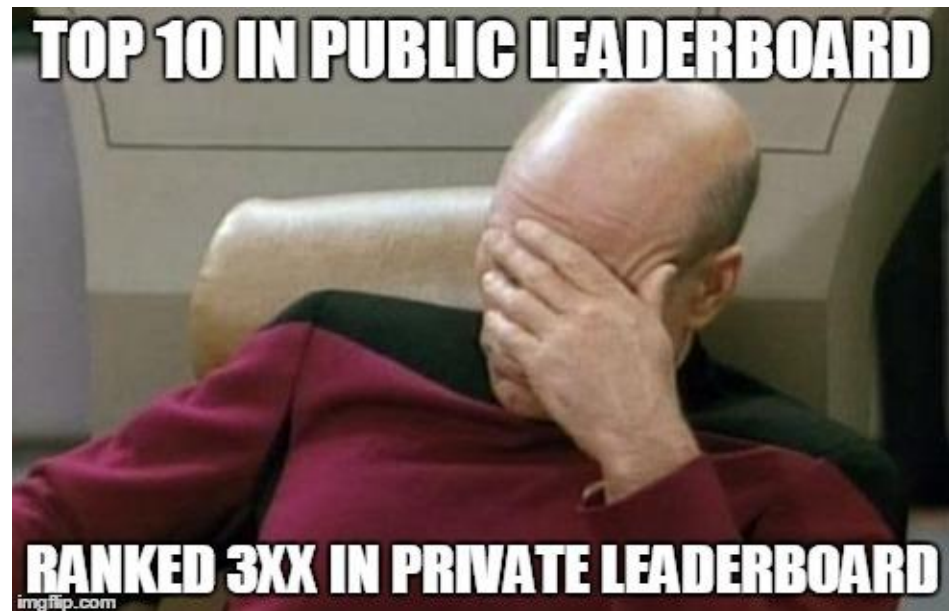


I beat baseline!

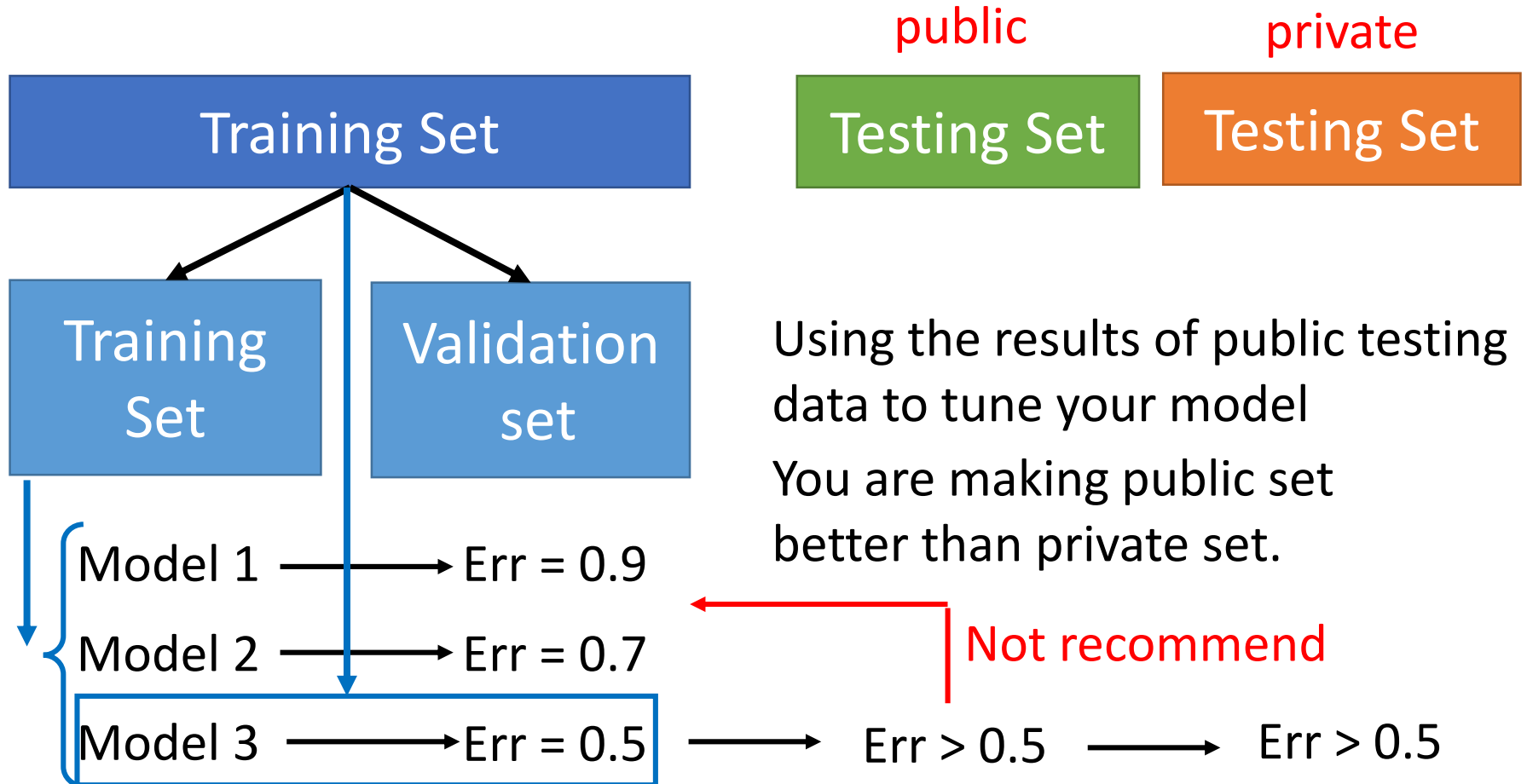
No, you don't

What will happen
next Friday?

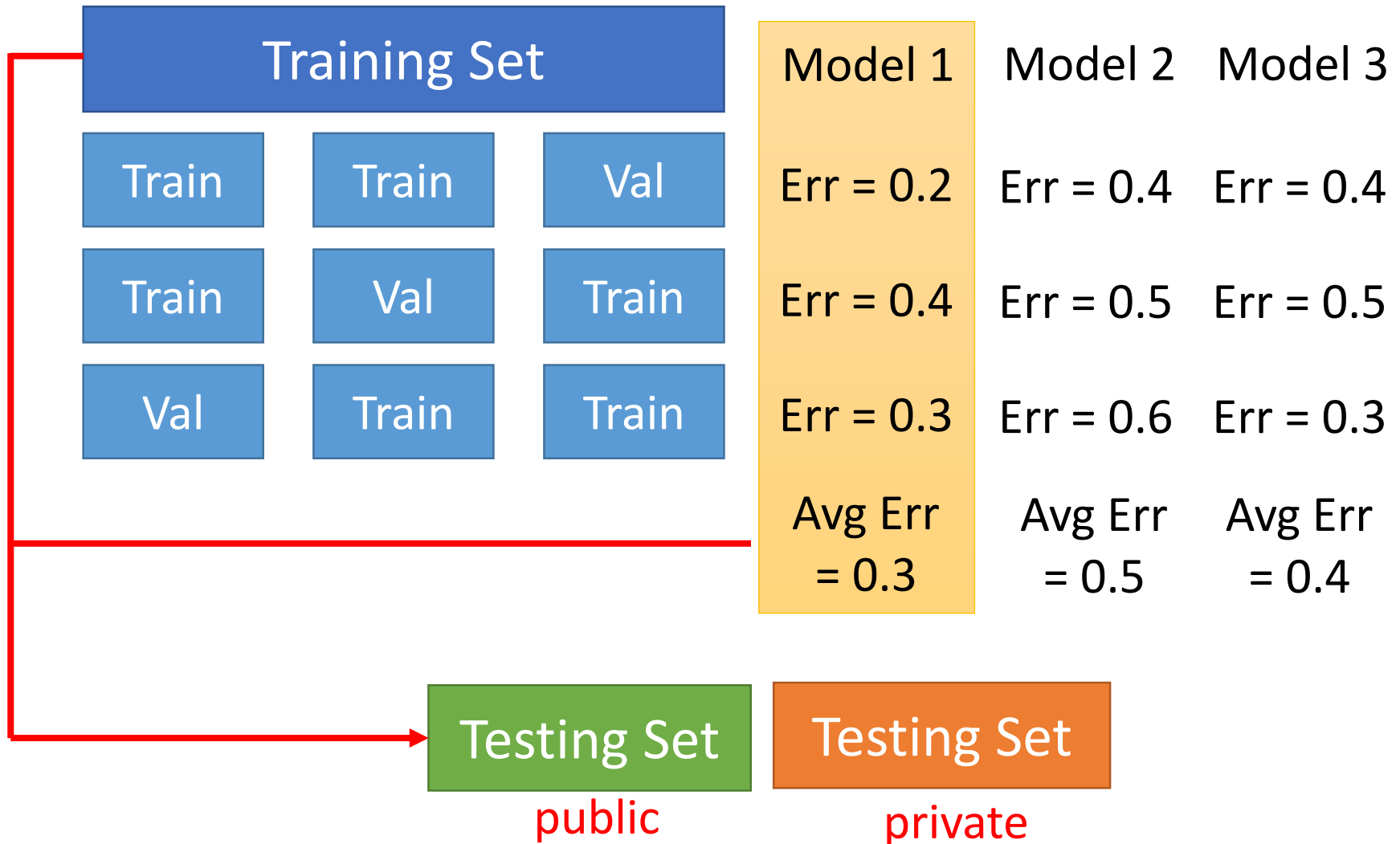
<http://www.chioka.in/how-to-select-your-final-models-in-a-kaggle-competitio/>



Cross Validation



N-fold Cross Validation



Reference

- Bishop: Chapter 3.2