



# Classification (Short Version)

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# Classification

- To learn more .....



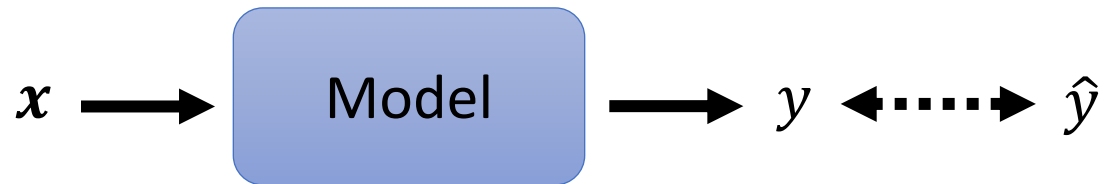
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(in Mandarin)



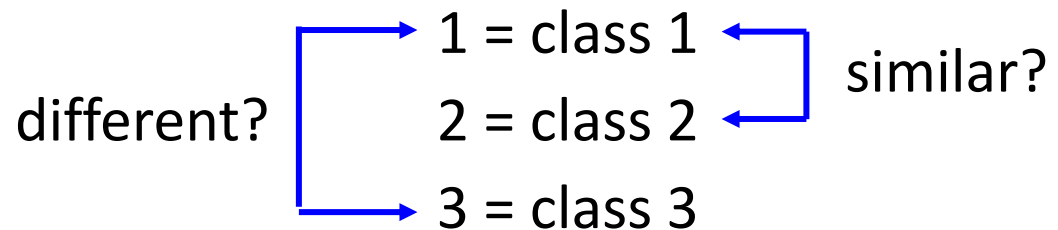
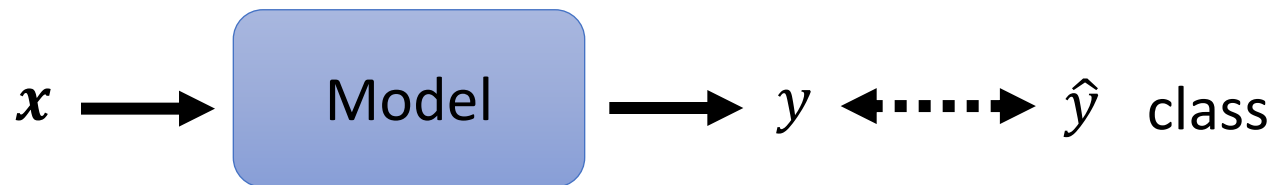
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(in Mandarin)

# Classification as Regression?

- Regression

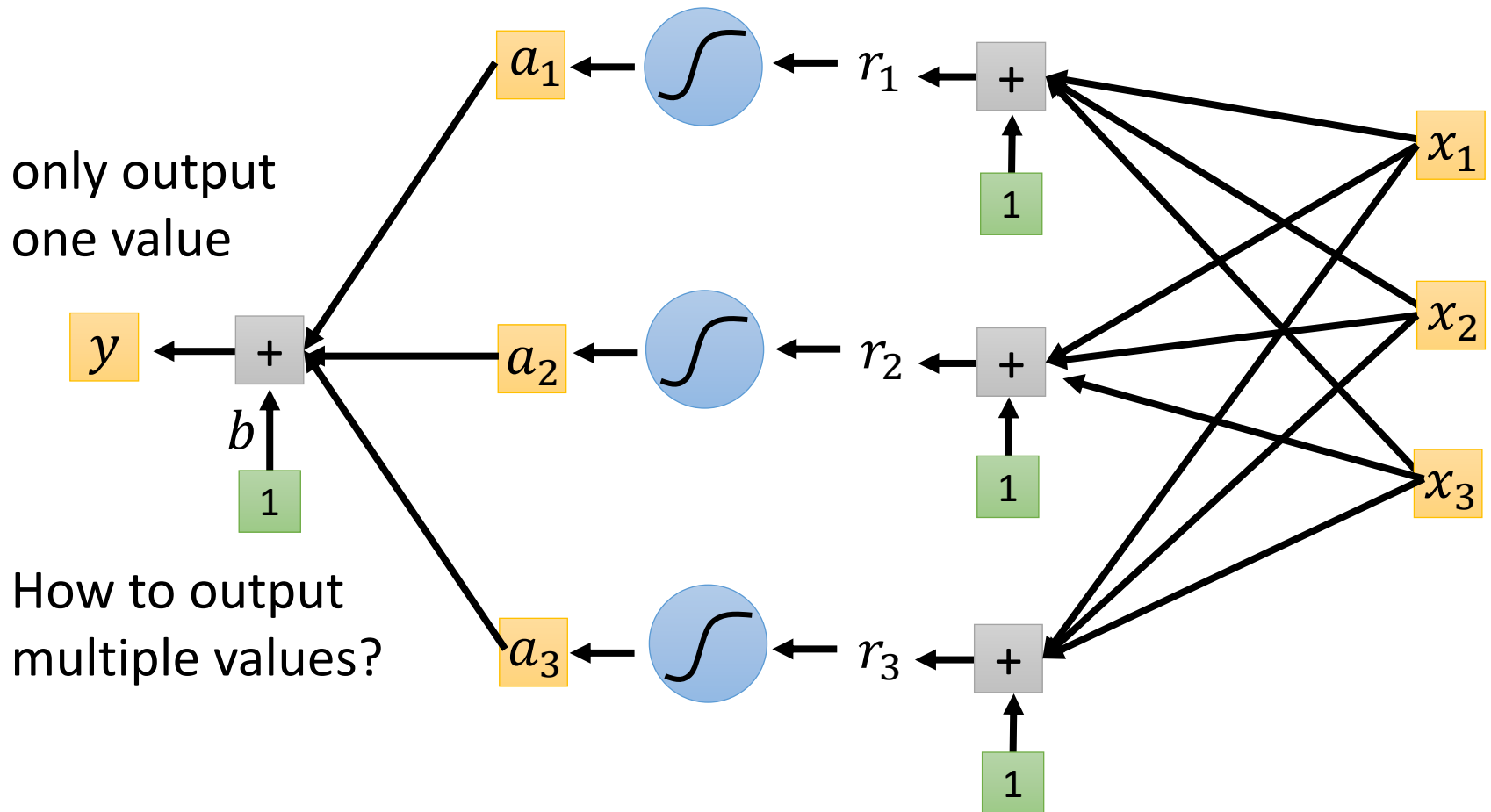


- Classification as regression?



# Class as one-hot vector

$$\hat{y} = \begin{matrix} \text{Class 1} & \text{Class 2} & \text{Class 3} \\ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} & \text{or} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} & \text{or} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{matrix}$$



为什么说要用 one-hot 形式？

一是前页提到的类距离问题

二是 one-hot 本身就是真实概率分布，可用于交叉熵损失

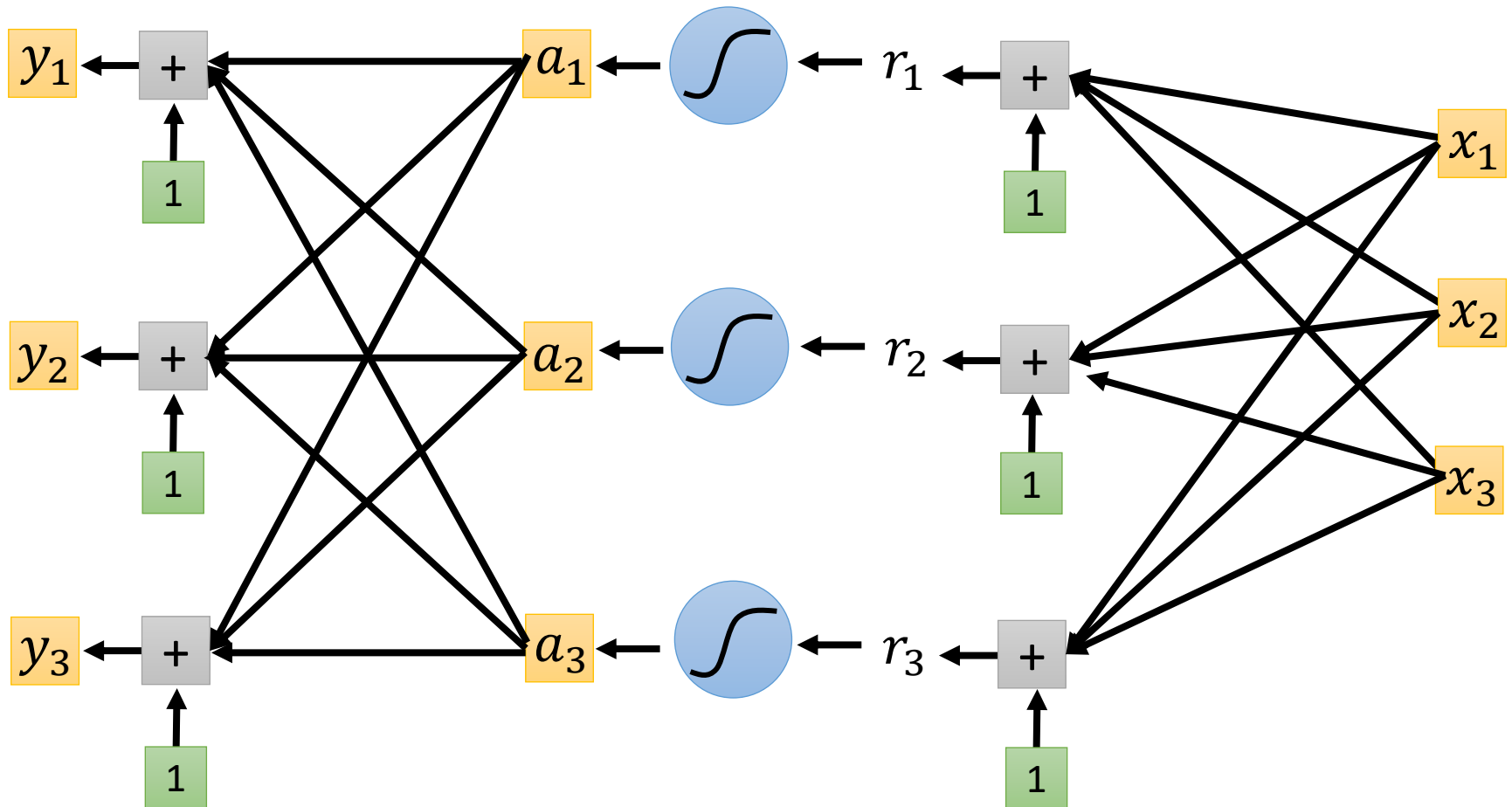
# Class as one-hot vector

Class 1

Class 2

Class 3

$$\hat{y} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$



## Regression

label

$$\hat{y} \longleftrightarrow y = b + c^T \sigma \left( b + W x \right)$$

feature

## Classification

feature

$$y = b' + W' \sigma \left( b + W x \right)$$

label

$$\hat{y} \longleftrightarrow y' = \text{softmax}(y)$$

0 or 1      Make all values between 0 and 1      Can have any value

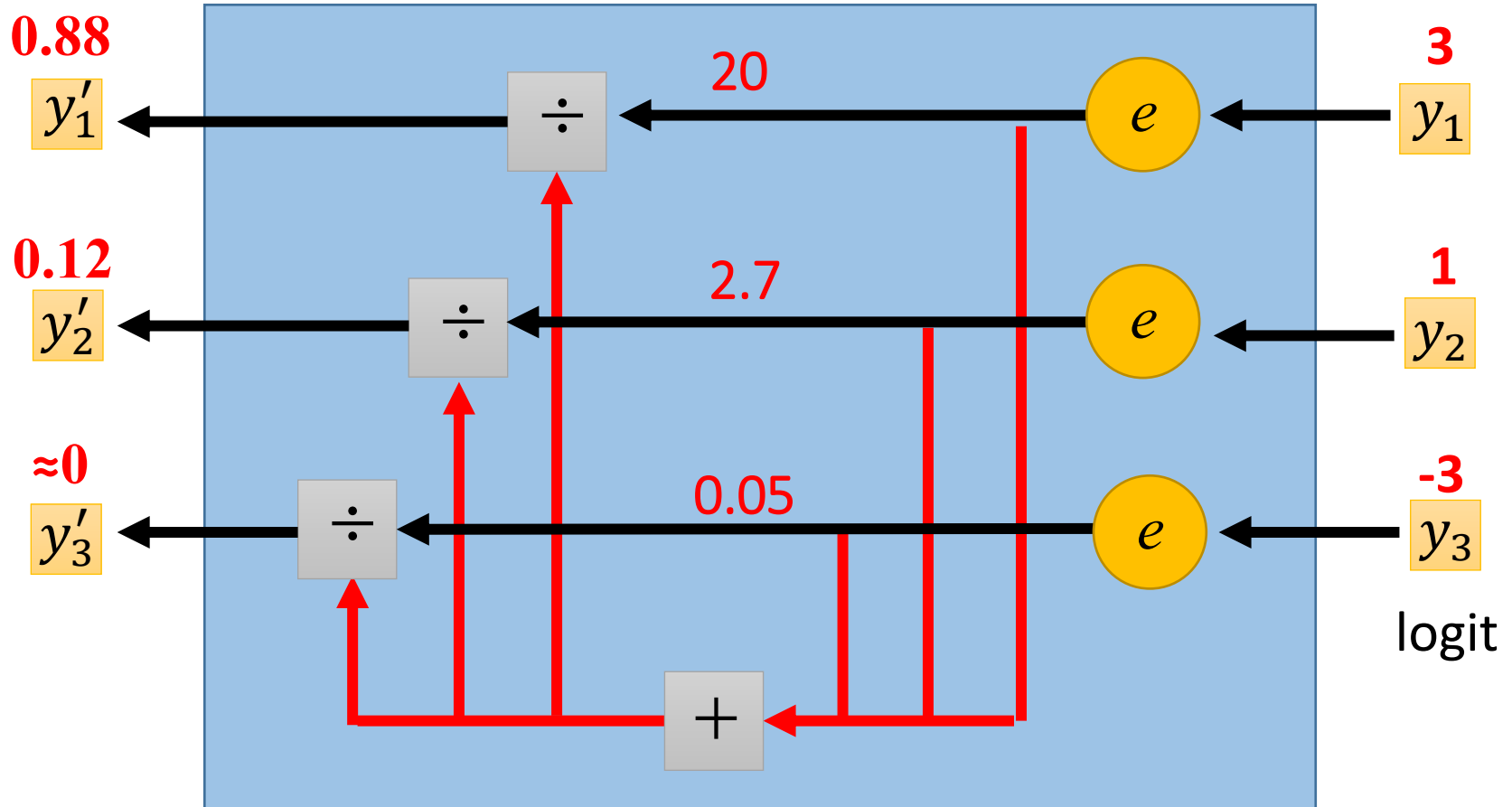
# Soft-max

$$y'_i = \frac{\exp(y_i)}{\sum_j \exp(y_j)}$$

- $1 > y'_i > 0$
- $\sum_i y'_i = 1$

Softmax

How about **binary classification**? ☺





为什么两个类别的 softmax 和 直接用 sigmoid 等价?

$$\begin{aligned}\text{二分类时 } S(x) &= \frac{1}{e^{z_1} + e^{z_2}} \cdot \begin{bmatrix} e^{z_1} \\ e^{z_2} \end{bmatrix} \\ &= \frac{1}{1 + e^{z_2 - z_1}} \cdot \begin{bmatrix} 1 \\ e^{z_2 - z_1} \end{bmatrix}\end{aligned}$$

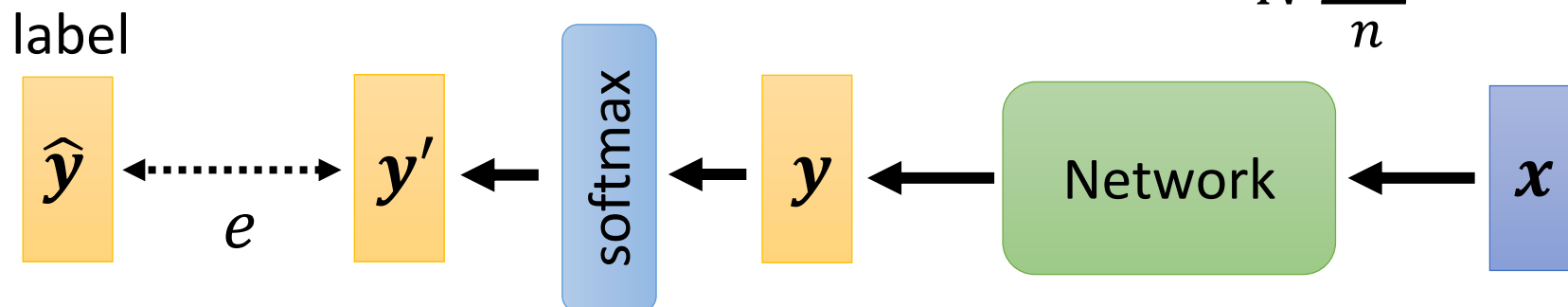
$$\text{令 } z = z_1 - z_2$$

$$\text{则 } S(x) = \frac{1}{1 + e^{-z}} \begin{bmatrix} 1 \\ e^{-z} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{1 + e^{-z}} \\ 1 - \frac{1}{1 + e^{-z}} \end{bmatrix} = \text{logistic } \sigma(z)$$

# Loss of Classification

$$L = \frac{1}{N} \sum_n e_n$$



Mean Square Error (MSE)

$$e = \sum_i (\hat{y}_i - y'_i)^2$$

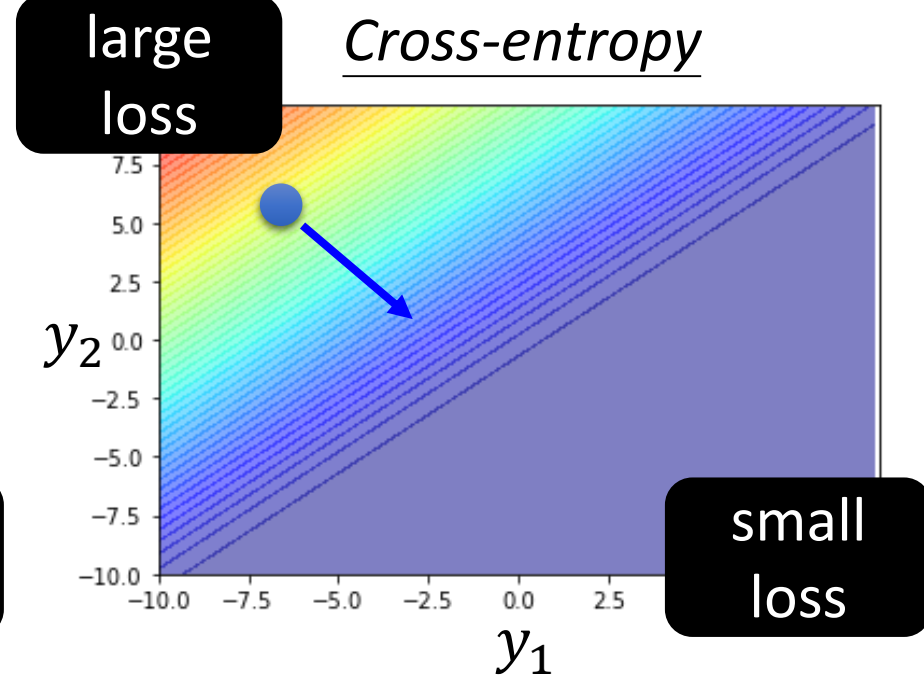
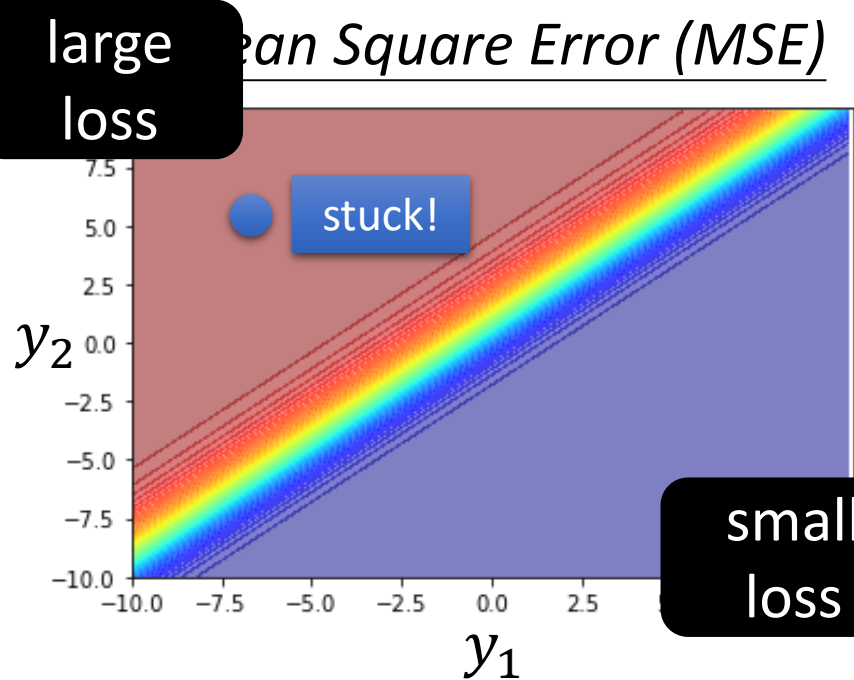
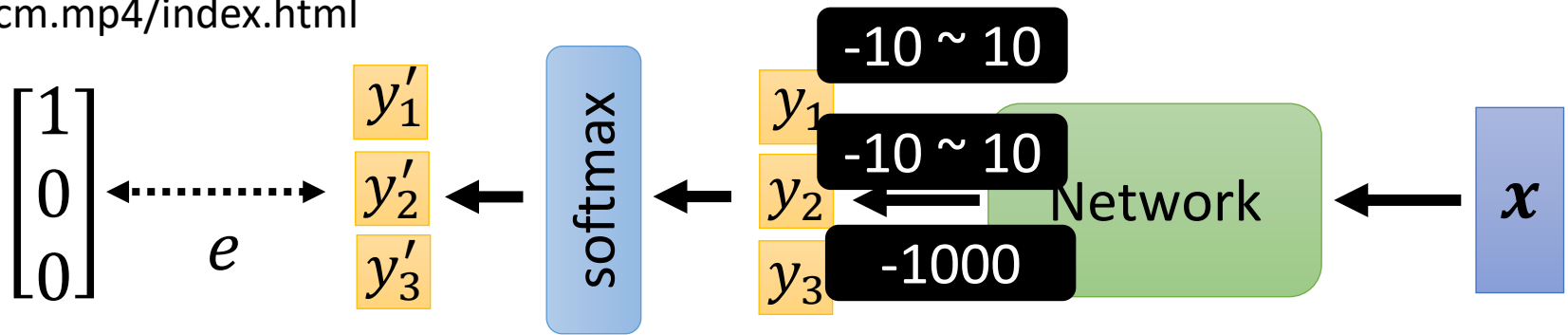
Cross-entropy



$$e = - \sum_i \hat{y}_i \ln y'_i$$

**Minimizing cross-entropy** is equivalent to **maximizing likelihood**.

pytorch 中 选用 cross-entropy 默认使用 softmax



**Changing the loss function can change the difficulty of optimization.**