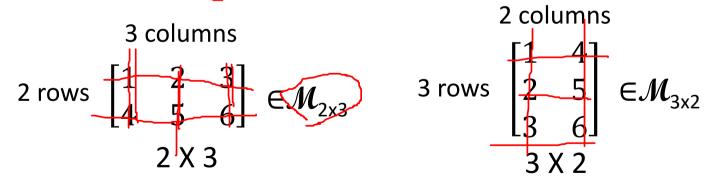
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A matrix is a set of vectors

$$\underline{a_1} = \begin{bmatrix} 2 \\ 3 \\ -2 \end{bmatrix} \qquad \underline{a_2} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} \qquad \underline{a_3} = \begin{bmatrix} 5 \\ -1 \\ 1 \end{bmatrix}$$

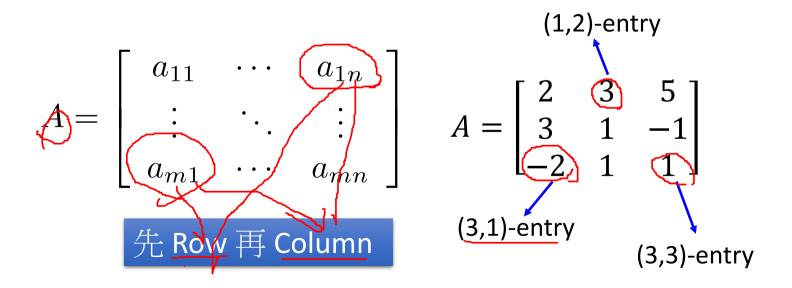
$$A = \begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 5 \\ 3 & 1 & -1 \\ -2 & 1 & 1 \end{bmatrix}$$

- If the matrix has m rows and n columns, we say the size of the matrix is m by n, written m x n
  - The matrix is called square if m=n
  - We use  $M_{mxn}$  to denote the set that contains all matrices whose size is m x n



先 Row 再 Column

Index of component: the scalar in the <u>i-th row</u> and <u>i-th column</u> is called (i,j)-entry of the matrix



- Two matrices with the same size can add or subtract.
- Matrix can multiply by a scalar

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} 6 & 9 \\ 8 & 0 \\ 9 & 2 \end{bmatrix} \qquad 9B$$

$$A + B$$

$$A - B$$

#### Zero Matrix

- zero matrix: matrix with all zero entries, denoted by Q (any size) or  $O_{\underline{m} \times n}$ .

  • For example, a 2-by-3 zero matrix can be denoted

$$O_{2\times 3} = \left[ \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

- Identity matrix: must be square
  - 對角線是 1, 其它都是 0

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Sometimes  $I_n$  is simply written as I (any size).

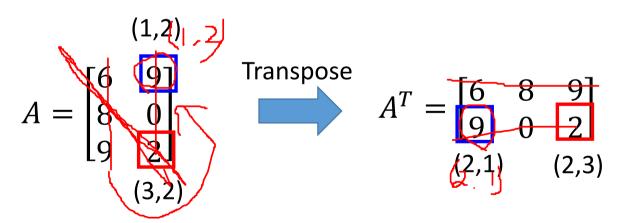
## Properties

- A, B, C are mxn matrices, and s and t are scalars
  - A + B = B + A
  - (A + B) + C = A + (B + C)
  - (st)A = s(tA)
  - s(A + B) = sA + sB
  - (s+t)A = sA + tA



Is "transpose" a linear system?

- If A is an mxn matrix
- $A^T$  (transpose of A) is an <u>nxm</u> matrix whose (i,j)-entry is the (idi)-entry of A



以左上到右下的對角線為軸 進行翻轉

Transpose 
$$A = \begin{bmatrix} 5 & 5 \\ 6 & 6 \end{bmatrix}$$
  $B = \begin{bmatrix} 7 & 7 \\ 8 & 8 \end{bmatrix}$ 

- A and B are mxn matrices, and s is a scalar

$$2A = \begin{bmatrix} 10 & 10 \\ 12 & 12 \end{bmatrix} \quad (2A)^T = \begin{bmatrix} 10 & 12 \\ 10 & 12 \end{bmatrix}$$

•  $(A^{T})^{T} = A$ •  $(3A)^{T} = SA^{T}$ •  $(A + B)^{T} = A^{T} + B^{T}$ 

$$A + B = \begin{bmatrix} 12 & 12 \\ 14 & 14 \end{bmatrix} \qquad (A + B)^T = \begin{bmatrix} 12 & 14 \\ 12 & 14 \end{bmatrix}$$

$$A^{T} = \begin{bmatrix} 5 & 6 \\ 5 & 6 \end{bmatrix} \quad B^{T} = \begin{bmatrix} 7 & 8 \\ 7 & 8 \end{bmatrix} \qquad A^{T} + B^{T} = \begin{bmatrix} 12 & 14 \\ 12 & 14 \end{bmatrix}$$