(High School) Vector

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Vectors

- A vector **v** is a set of numbers

$$\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Column vector

$$\mathbf{v} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

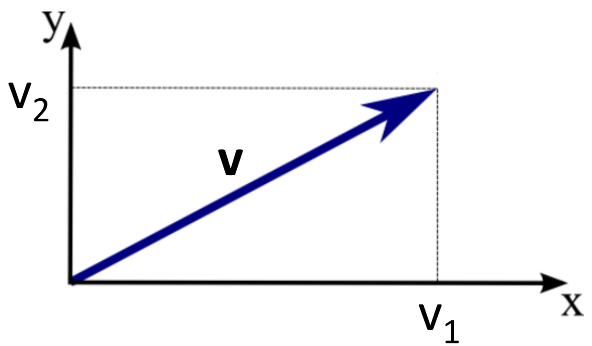
Row vector

In this course, the term **vector** refers to a **column vector** unless being explicitly mentioned otherwise.

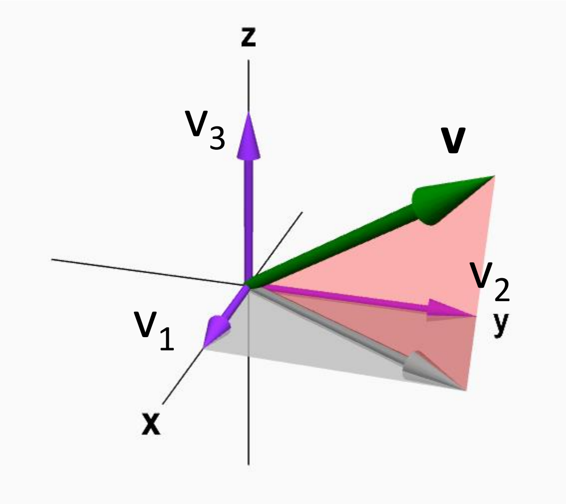
Vectors

$$\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

- components: the entries of a vector.
  - The i-th component of vector **v** refers to v_i
  - $v_1=1$, $v_2=2$, $v_3=3$
- If a vector only has less than four components, you can visualize it.



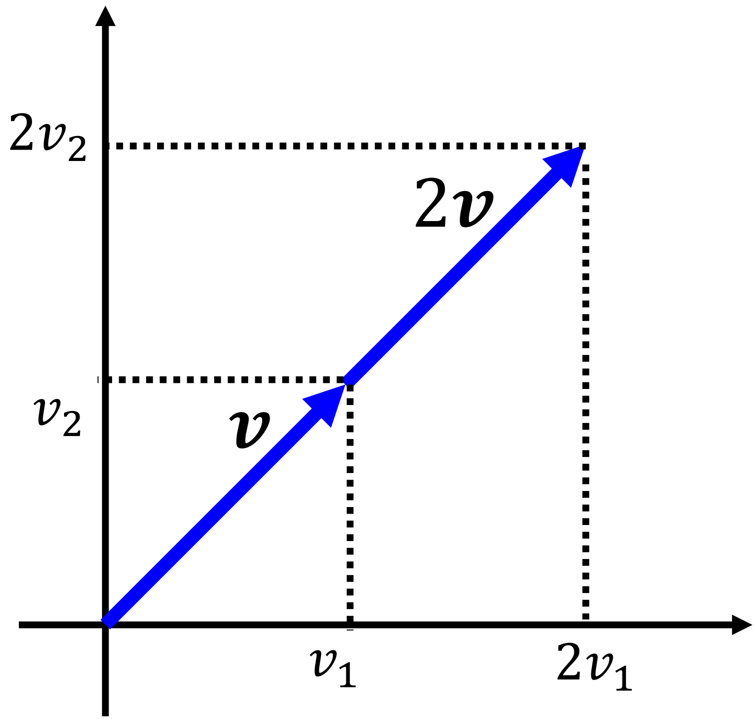
http://mathinsight.org/vectors_cartesian_coordinates_2d_3d#vector3D



Scalar Multiplication

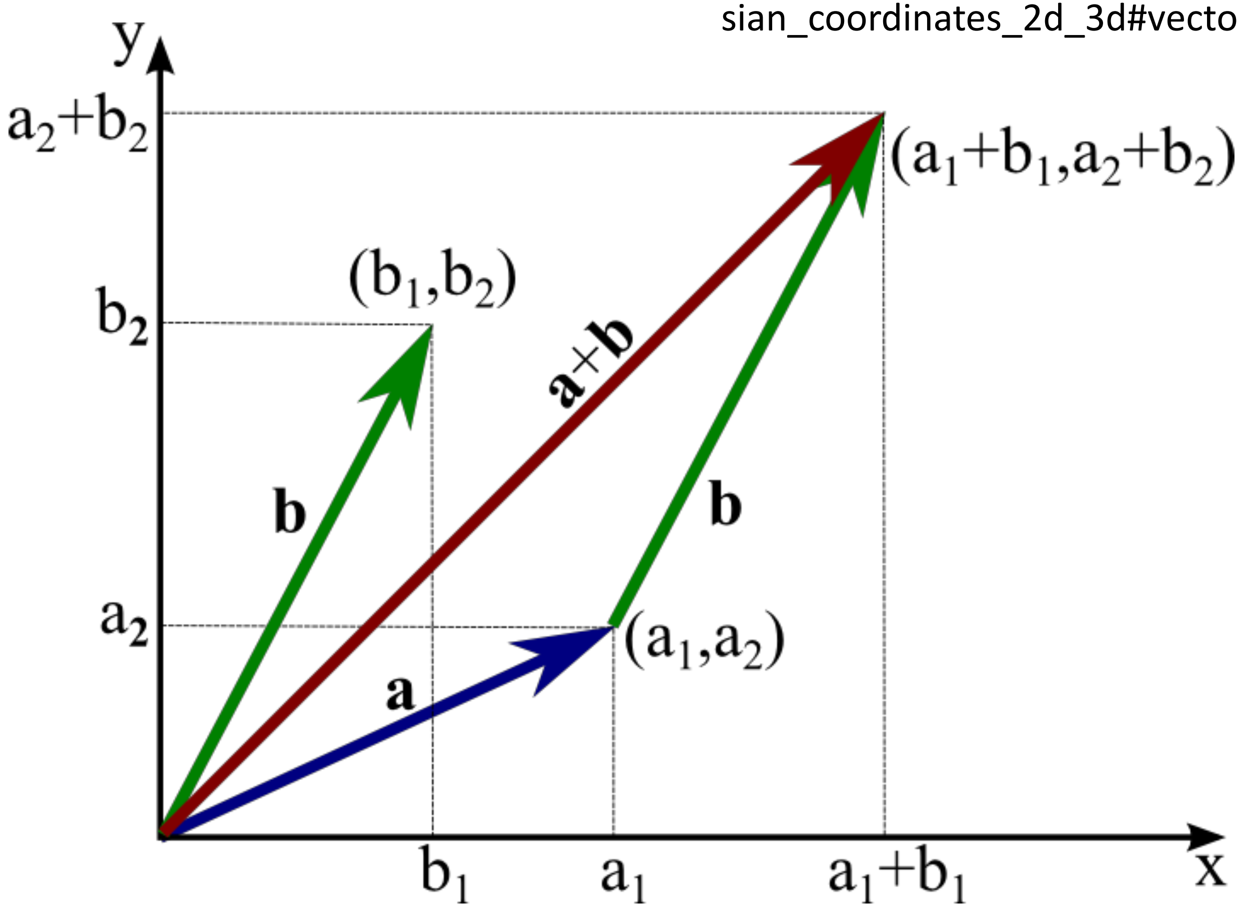
$$\boldsymbol{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$c\boldsymbol{v}$



Vector Addition

http://mathinsight.org/vectors_cartesian_coordinates_2d_3d#vector3D

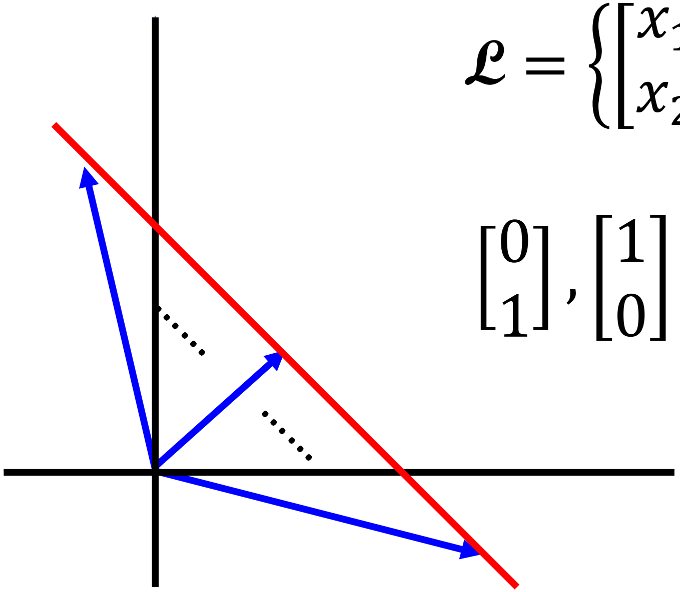


Vector Set

$$\left\{\begin{bmatrix}1\\2\\3\end{bmatrix},\begin{bmatrix}4\\5\\6\end{bmatrix},\begin{bmatrix}6\\8\\9\end{bmatrix},\begin{bmatrix}9\\0\\2\end{bmatrix}\right\}$$

A vector set with 4 elements

- A vector set can contain infinite elements

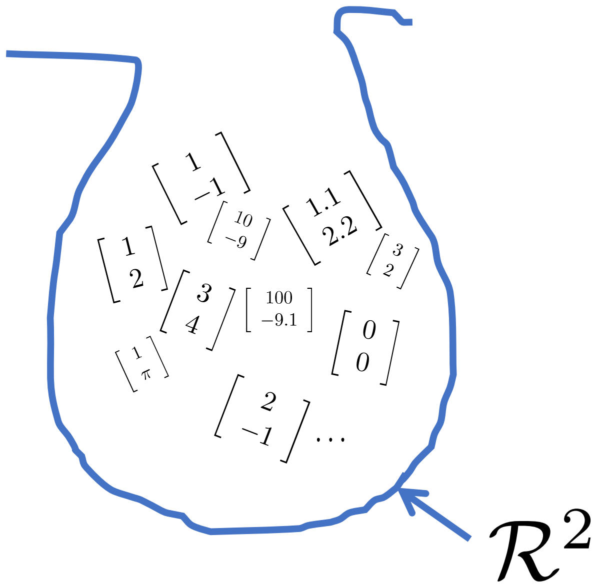


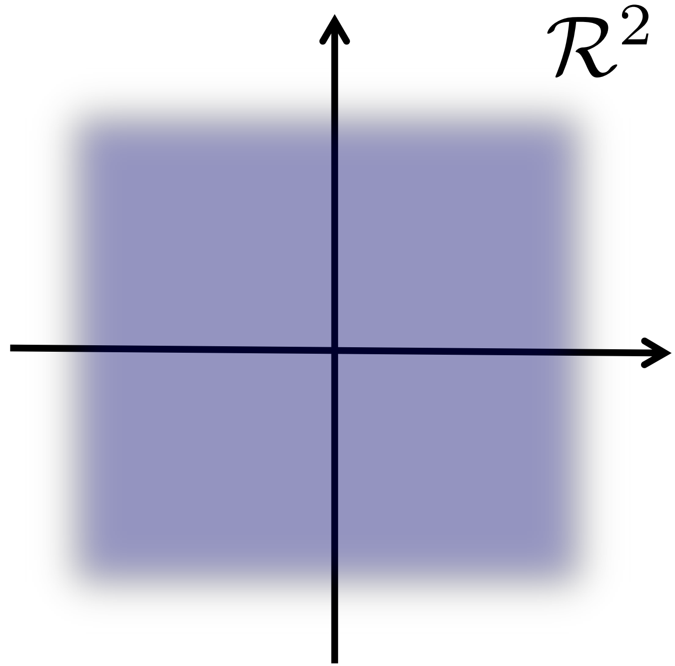
$$\mathcal{L}=\left\{\begin{bmatrix}x_1\\x_2\end{bmatrix}: x_1+x_2=1\right\}$$

$$\begin{bmatrix}0\\1\end{bmatrix},\begin{bmatrix}1\\0\end{bmatrix},\begin{bmatrix}0.3\\0.7\end{bmatrix},\begin{bmatrix}0.7\\0.3\end{bmatrix}\cdots\cdots$$

Vector Set

- $\mathcal{R}^n$:We denote the set of all **vectors** with *n* entries by $\mathcal{R}^n$. 向量空间





Properties of Vector

The objects have the following 8 properties are "vectors".

- For any vectors **u**, **v** and **w** in $\mathcal{R}^n$, and any scalars a and b
  - **u** + **v** = **v** + **u**
  - (**u** + **v**) + **w** = **u** + (**v** + **w**)
  - There is an element $\mathbf{0}$ in $\mathcal{R}^n$ such that $\mathbf{0}$ + **u** = **u**
  - There is an element **u'** in $\mathcal{R}^n$ such that **u'** + **u** = 0
  - 1**u** = **u**
  - (ab)**u** = a(b**u**)
  - a(**u**+**v**) = a**u** + a**v**
  - (a+b)**u** = a**u** + b**u**

$$\mathbf{0} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$$ zero vector

幺元

u' is the additive inverse of **u**

逆元

More Properties of Vector

$\mathbf{0} + \mathbf{u} = \mathbf{u}$

$\mathbf{u}' + \mathbf{u} = \mathbf{0}$

- For any vectors **u**, **v** and **w** in $\mathcal{R}^n$, and any scalar a
  - If $\mathbf{u} + \mathbf{v} = \mathbf{w} + \mathbf{v}$, then $\mathbf{u} = \mathbf{w}$
  - If $\mathbf{u} + \mathbf{v} = \mathbf{u} + \mathbf{w}$, then $\mathbf{v} = \mathbf{w}$
  - The zero vector **0** is unique. It is the only vector in $\mathcal{R}^n$ that satisfies $\mathbf{0} + \mathbf{u} = \mathbf{u}$
  - Each vector in $\mathcal{R}^n$ has exactly one $\mathbf{u}'$
  - $0\mathbf{u} = \mathbf{0}$
  - $a\mathbf{0} = \mathbf{0}$
  - $\mathbf{u}' = -1(\mathbf{u}) = -\mathbf{u}$
  - $(-a)\mathbf{u} = a(-\mathbf{u}) = -(a\mathbf{u})$