Singular Value Decomposition

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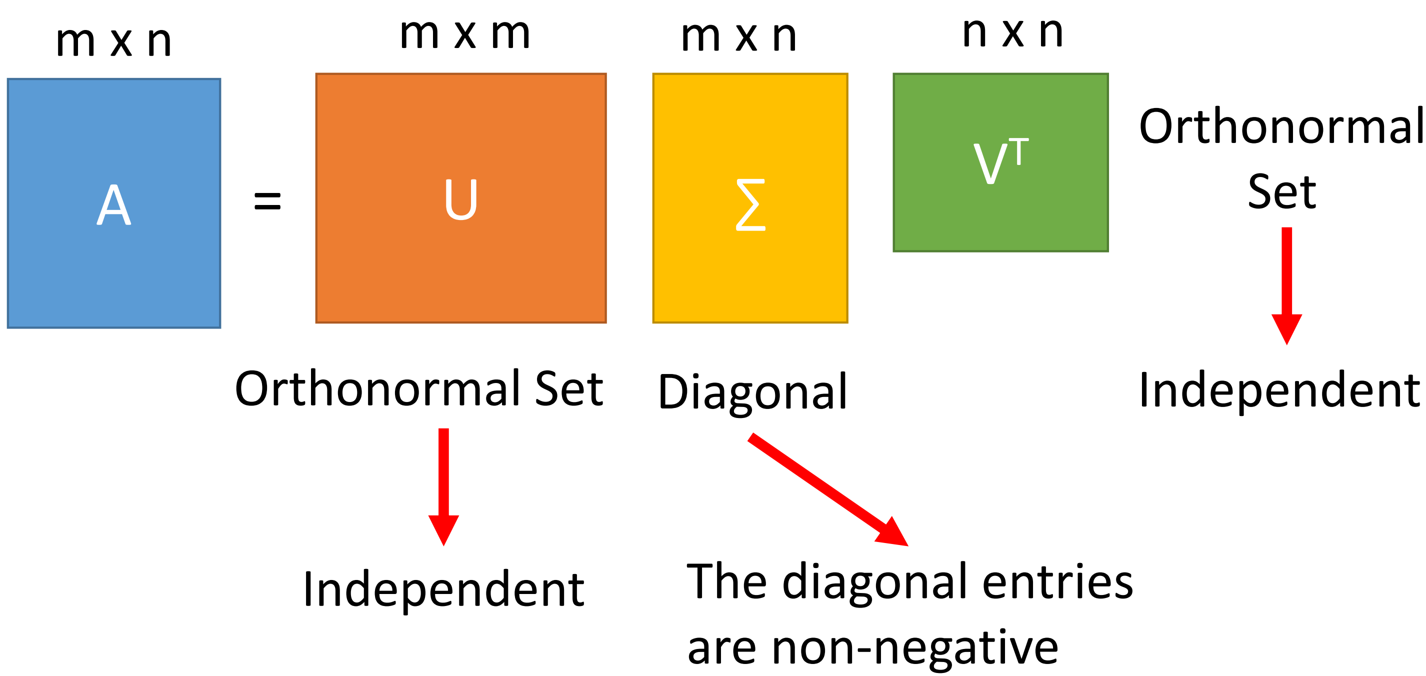
Outline

- Diagonalization can only apply on some square matrices.
- Singular value decomposition (SVD) can apply on any matrix.

- Reference: Chapter 7.7

SVD

- Any m x n matrix A



SVD

(We can exchange some rows and columns to achieve that)

$$\left[\begin{array}{cccc|cccc} \sigma_1 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\ 0 & \sigma_2 & \dots & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & & \sigma_k & 0 & 0 & \dots & 0 \\ \hline 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \end{array}\right]$$

- Any m x n matrix A

m x n		m x m	m x n	n x n	
A	=	U	Σ	V^T	Independent
		Independent	Diagonal		

$= rank(\Sigma)$

What is the rank of A?

If A is a m x n matrix, and B is a n x k matrix.

$$Rank(AB) \leq min\big(Rank(A), Rank(B)\big)$$

If B is a matrix of rank n, then $Rank(AB) = Rank(A)$

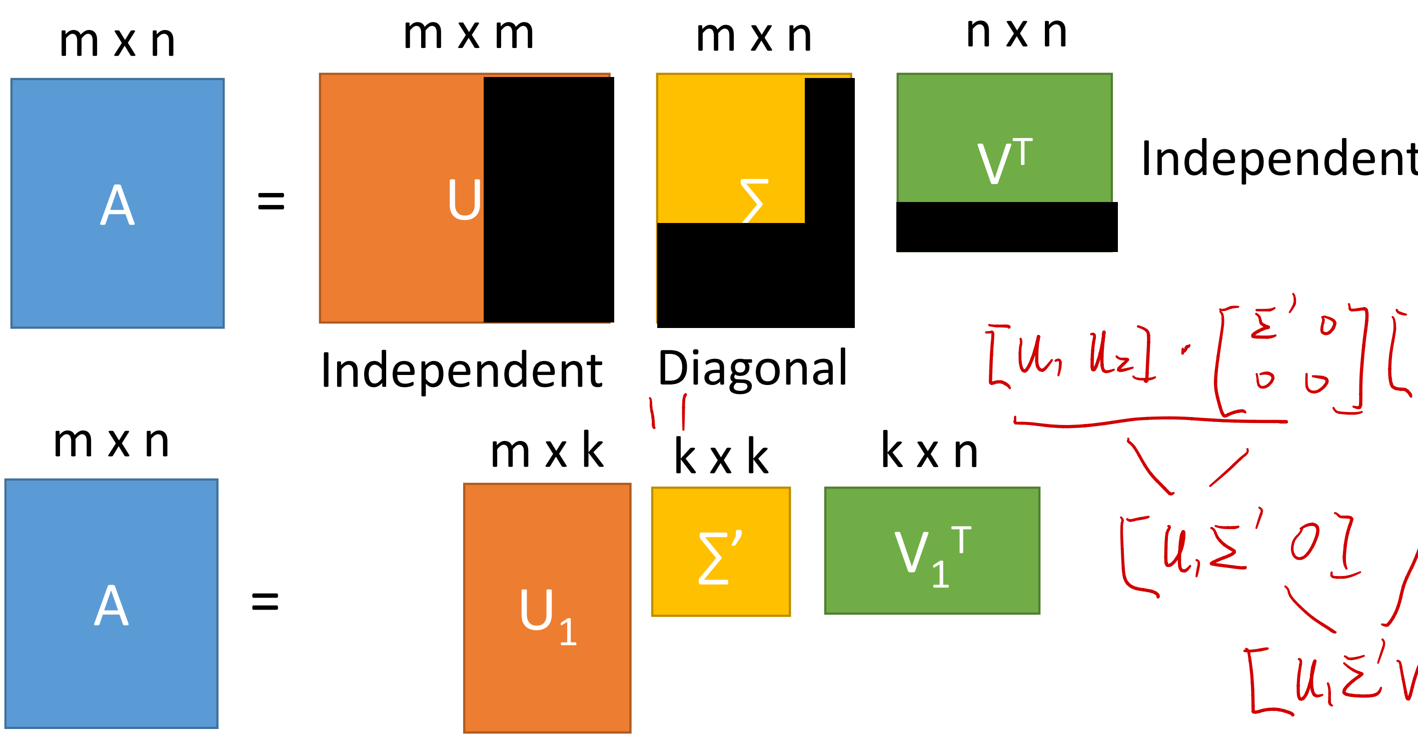
If A is a matrix of rank n, then $Rank(AB) = Rank(B)$

SVD

$$\begin{bmatrix} \sigma_1 & 0 & \dots & 0 \\ 0 & \sigma_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & & \sigma_k \end{bmatrix}$$

[redacted]

- Any m x n matrix A



$[u_1\ u_2] \cdot \begin{bmatrix} \Sigma' & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_1^T \\ v_2^T \end{bmatrix}$

$[u_1 \Sigma'\ 0]$

$[u_1 \Sigma' v_1^T]$

SVD

$$\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_k > 0$$

σ_k is deleted

σ_1 0 …
0 σ_2 …
⋮ ⋮ σ_{k-1}
[redacted]

- Any m x n matrix A

m x n		m x m	m x n	n x n	
A	=	U [redacted]	Σ [redacted]	V^T [redacted]	Independent
		Independent	Diagonal		

What is the rank of A'?

k-1

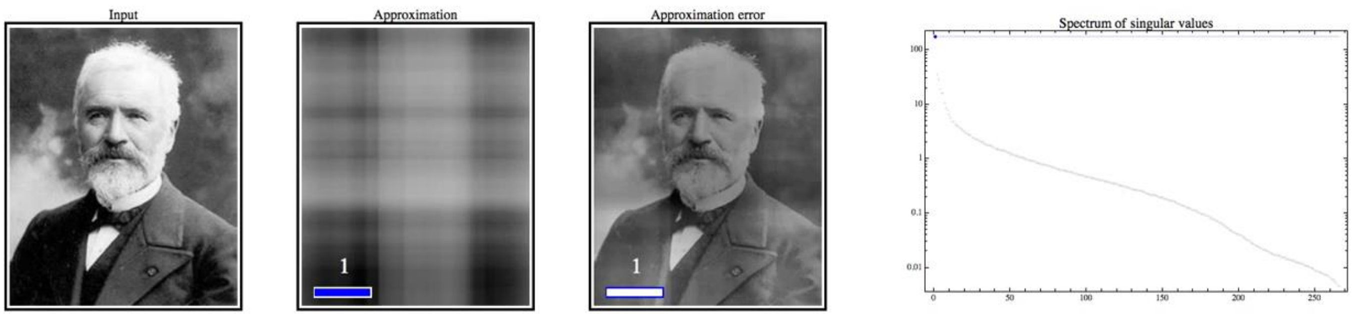
m x n		m x (k-1)	(k-1) x (k-1)	(k-1) x n
A'	=	U_1	Σ'	$V_1{}^T$

A'

A' is the rank k-1 matrix minimizing $||A - \cancel{B}||$

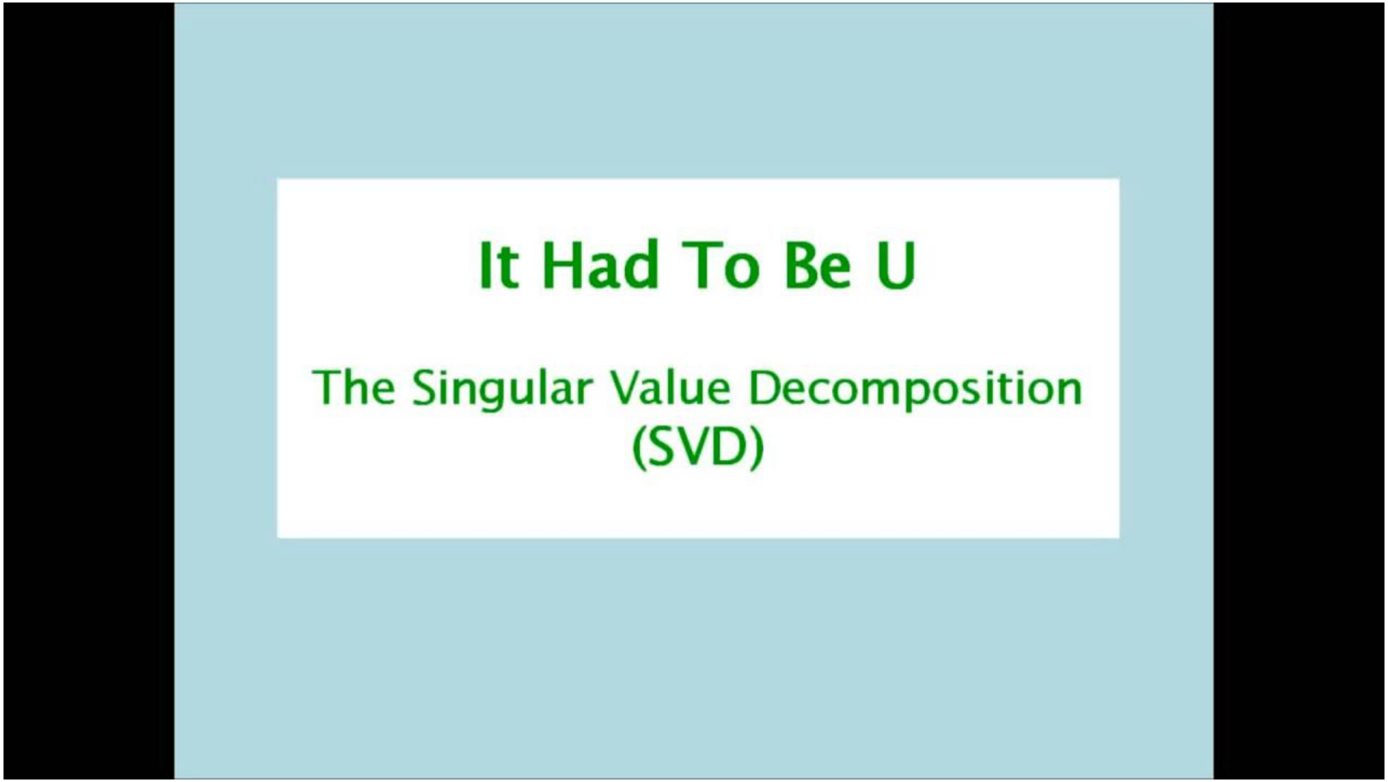
A'是所有rank为k的矩阵中最接近A的

Low rank approximation using the singular value decomposition



k=1

https://www.youtube.com/watch?v=pAiVb7gWUrM https://www.youtube.com/watch?v=fKVRSbFKnEw



Thank You for Your Attention

https://www.youtube.com/watch?v=R9UoFyqJca8