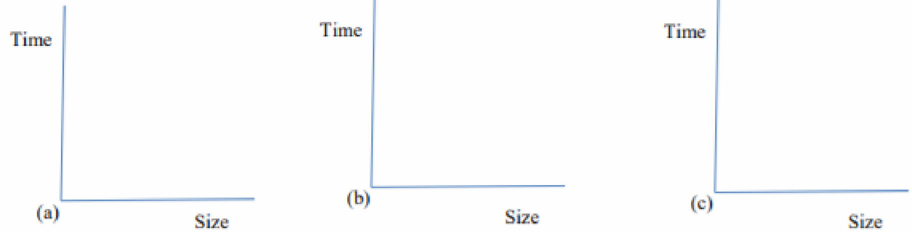
1. (3 pts) Sketch Size vs. Time curves for the two algorithmic complexity classes required in each of the pictures below: for one, write **Impossible** instead: (a) an O(N) algorithm that is **never** faster than an O(N^2) algorithm. (b) an O(N) algorithm that is **always** faster than an O(N^2) algorithm. (c) an O(N) algorithm that is **sometimes** faster than an O(N^2) algorithm.



2. (2 pts) (a) What "better-known"/simpler complexity class is equivalent to **O(N log N²)**; briefly explain why. (b) Explain under what conditions `sorted(set(l))` runs faster than `set(sorted(l))` for a `list l` (they both produce the same answer); state the worst-case complexity class of each.

(a).

(b)

3. (6 pts) In 1972, Intel's **8008** processor could execute 200,000 (200 thousand) instructions per second; at present, an Intel **Core 2** processor can execute 3,000,000,000 (3 billion) instructions per second (15,000 times faster). Let's assume that we program the **8008** to run a fast **O(N Log_2 N)** sorting algorithm, and program the **Core 2** to run a slow **O(N^2)** sorting algorithm. Assume the time to sort **N** values on the **8008** is exactly **10/200,000 N Log_2 N** seconds; assume the time to sort **N** values on the **Core 2** is **2/3,000,000,000 N^2** seconds. Here the constant for fast sorting on the **8008** is 5 times as big as the constant for slow sorting on the **Core 2** (both constants are divided by the speed of the machines the algorithm runs on).

a1) About how long does it take the **8008** to sort 1,000 values?

a2) About how long does it take the **Core 2** to sort 1,000 values?

b1) About how long does it take the **8008** to sort 1,000,000 values?

b2) About how long does it take the **Core 2** to sort 1,000,000 values?

c1) For what problem sizes **N** is it faster to use the **8008** for sorting?

c2) For what problem sizes **N** is it faster to use the **Core 2** for sorting?

In problems c1 and c2 only, compute your answer to the closest integer value (you can ignore decimal places). Use a calculator, spreadsheet, or a program to compute (possibly to guess and refine) your answer.

4. (6 pts) The following two functions each determine the distance between the two closest values in list `l`, with `len(l)` = `N`. (a) Write the complexity class of each statement in the box on its right. (b) Write the **full calculation** that computes the complexity class for the entire function. (c) Simplify what you wrote in (b).

```
def closest(l:[int])->int:
  a = set()
  for i in range(len(l)):
    for j in range(len(l)):
      if i != j:
        a.add(abs(l[i]-l[j]))
  return min(a)
```

(b)

(c)

```
def closest(l:[int])->int:
  a = sorted(l)
  m = -1
  for i in range(len(a)-1):
    if m==-1 or abs(a[i+1]-a[i])<min:
      m = abs(a[i+1]-a[i])
  return m;
```

(b)

(c)

5. (5 pts) Assume that function **f** is in the complexity class **O**($N\ (Log_2N)^3$), and that for **N = 1,000,000** the program runs in **80 seconds**.

(1) Write a formula, **T(N)** that computes the approximate time that it takes to run **f** for any input of size **N**. Show your work/calculations by hand, approximating logarithms, finish/simplify all the arithmetic.

(2) Compute how long it will take to run when **N = 1,000,000,000**. Show your work/calculations by hand, approximating logarithms, finish/simplify all the arithmetic.

6. (3 pts) Fill in the last line (for each complexity class), which shows the size of a problem (**M**) that can be solved in the **same amount of time** on a new machine that **runs twice as fast as the old machine**. Solve by hand when you can, use Excel or a calculator when you must: I used external tools only for **O(N Log_2 N)** and **O(N^2)** solving those to 3 significant figures. Solving a problem in the same amount of time on the new/faster machine is equivalent to solving a problem that takes ten times the amount of time on the old machine. See the **O(N)** for an example. Check that your answers aren't crazy. Hint: write a formula equating problem sizes N and M using the speedup and complexity class. For linear complexity: 2*c*N = c*M; then solve for M.

N = Problem Size	Complexity Class	Time to Solve on Old Machine (secs)	M Solvable in the same Time on a New Machine 2x as Fast
10^6	O(Log_2 N)	1	
10^6	O(N)	1	2 x 10^6
10^6	O(N Log_2 N)	1	
10^6	O(N^2)	1	