ENERGY DATA SCIENCE

Optimal control of energy storage systems

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PREVIOUSLY IN COURSE …

Key takeaways:

- Identification
- Models with exogenous inputs
- Discrete-time models
- Time-domain identification

A BRIEF INTRO

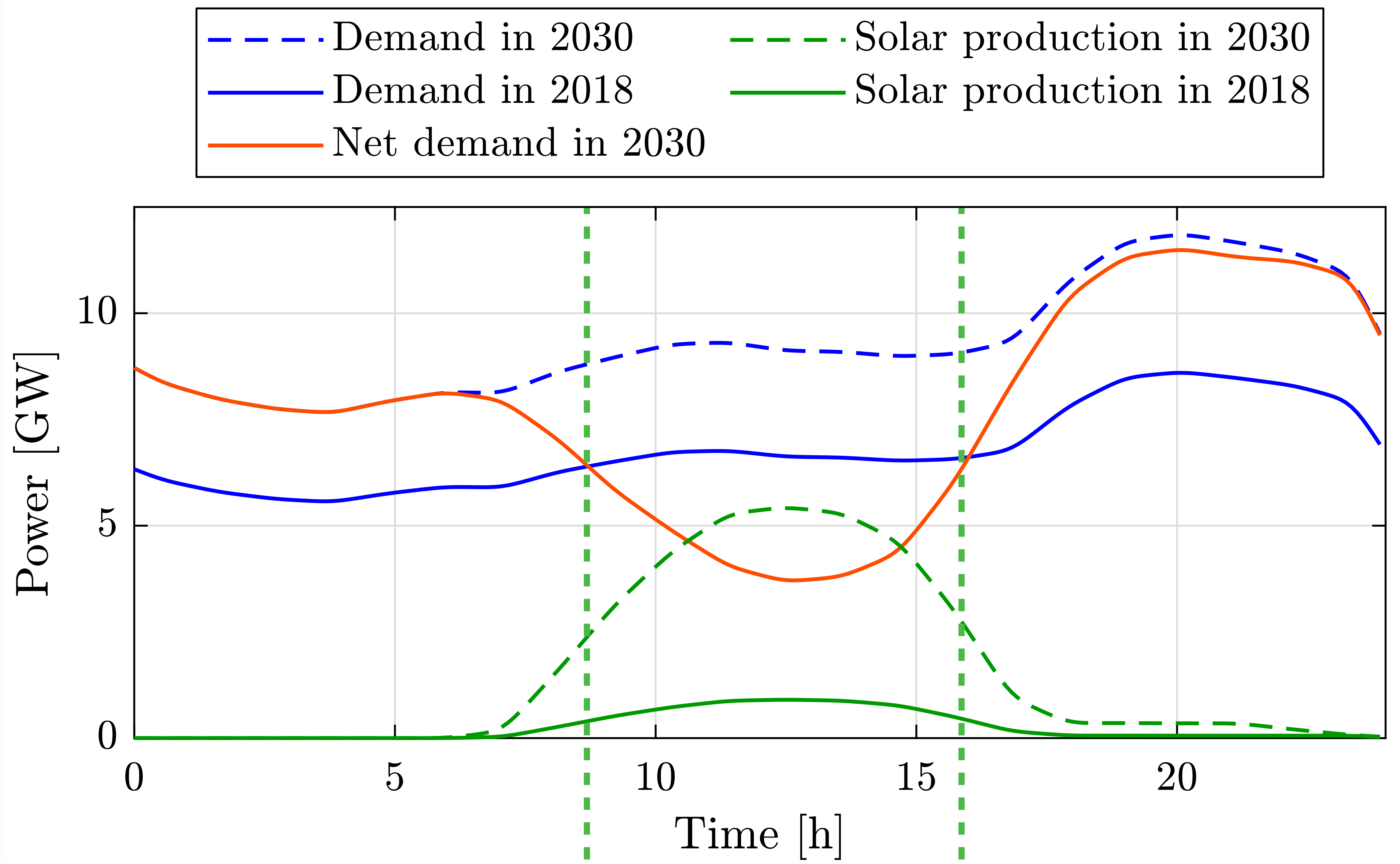
Motivation for shifting towards renewable energy sources (RESs):

- Climate change
- Carbon emissions
- Geopolitical tensions

Challenges:

- Renewable energy is intermittent and require energy storages.
- Recall “Lecture 1” for more detailed overview.

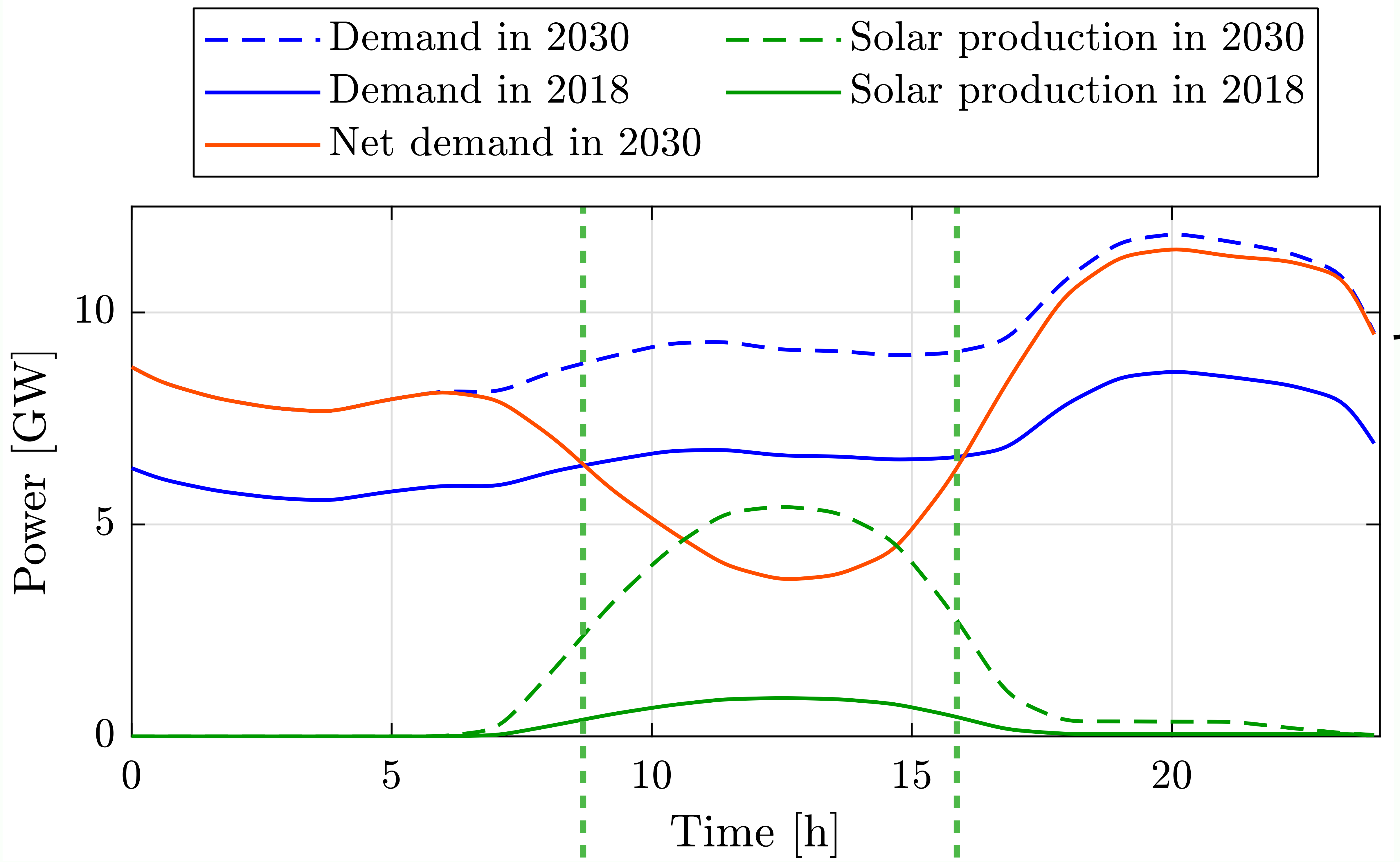
RES & THE DUCK CURVE



Net demand =
demand - solar
production

N. Zargari, R. Ofir, N. R. Chowdhury, J. Belikov, Y. Levron. An optimal control method for storage systems with ramp constraints, based on an on-going trimming process, *IEEE Transactions on Control Systems Technology*, 31(1), pp. 493–496, 2022.

RES & THE DUCK CURVE

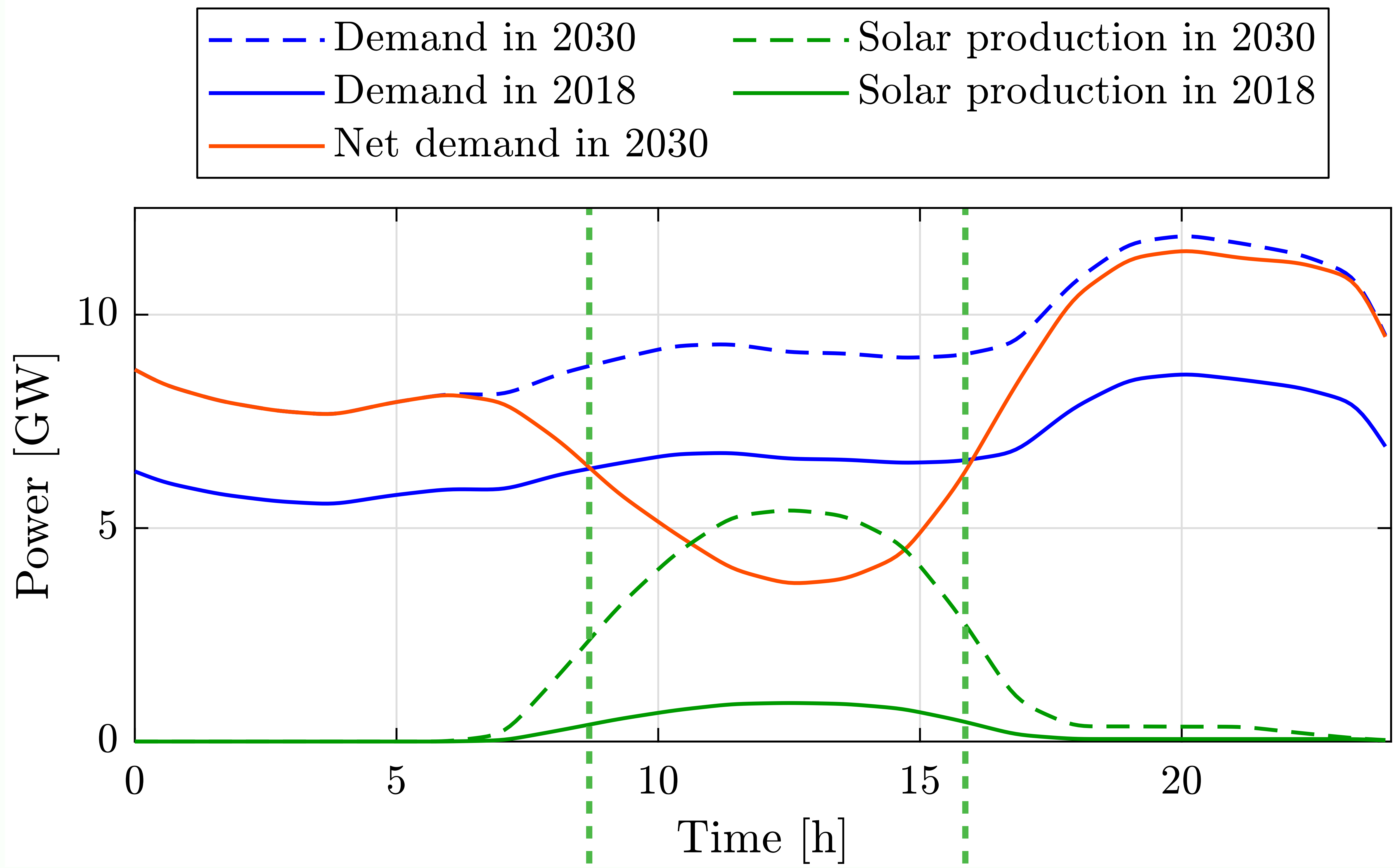


Net demand = demand - solar production

Too much solar energy is produced in times when demand is low → Overgeneration

N. Zargari, R. Ofir, N. R. Chowdhury, J. Belikov, Y. Levron. An optimal control method for storage systems with ramp constraints, based on an on-going trimming process, *IEEE Transactions on Control Systems Technology*, 31(1), pp. 493–496, 2022.

RES & THE DUCK CURVE



Net demand = demand - solar production

Too much solar energy is produced in times when demand is low → Overgeneration

Might lead to curtailment - producing less renewable energy to avoid damaging the system

N. Zargari, R. Ofir, N. R. Chowdhury, J. Belikov, Y. Levron. An optimal control method for storage systems with ramp constraints, based on an on-going trimming process, *IEEE Transactions on Control Systems Technology*, 31(1), pp. 493–496, 2022.

ENERGY STORAGE SYSTEMS (2)

Typical applications:

- Energy balancing, load levelling, or peak shaving
- Renewable energy integration
- Energy trading

Challenge:

- Maximising efficiency/performance or reliability

Solution: Optimal control

- High numeric complexity: hard to solve
- Not convex: converge to local minima

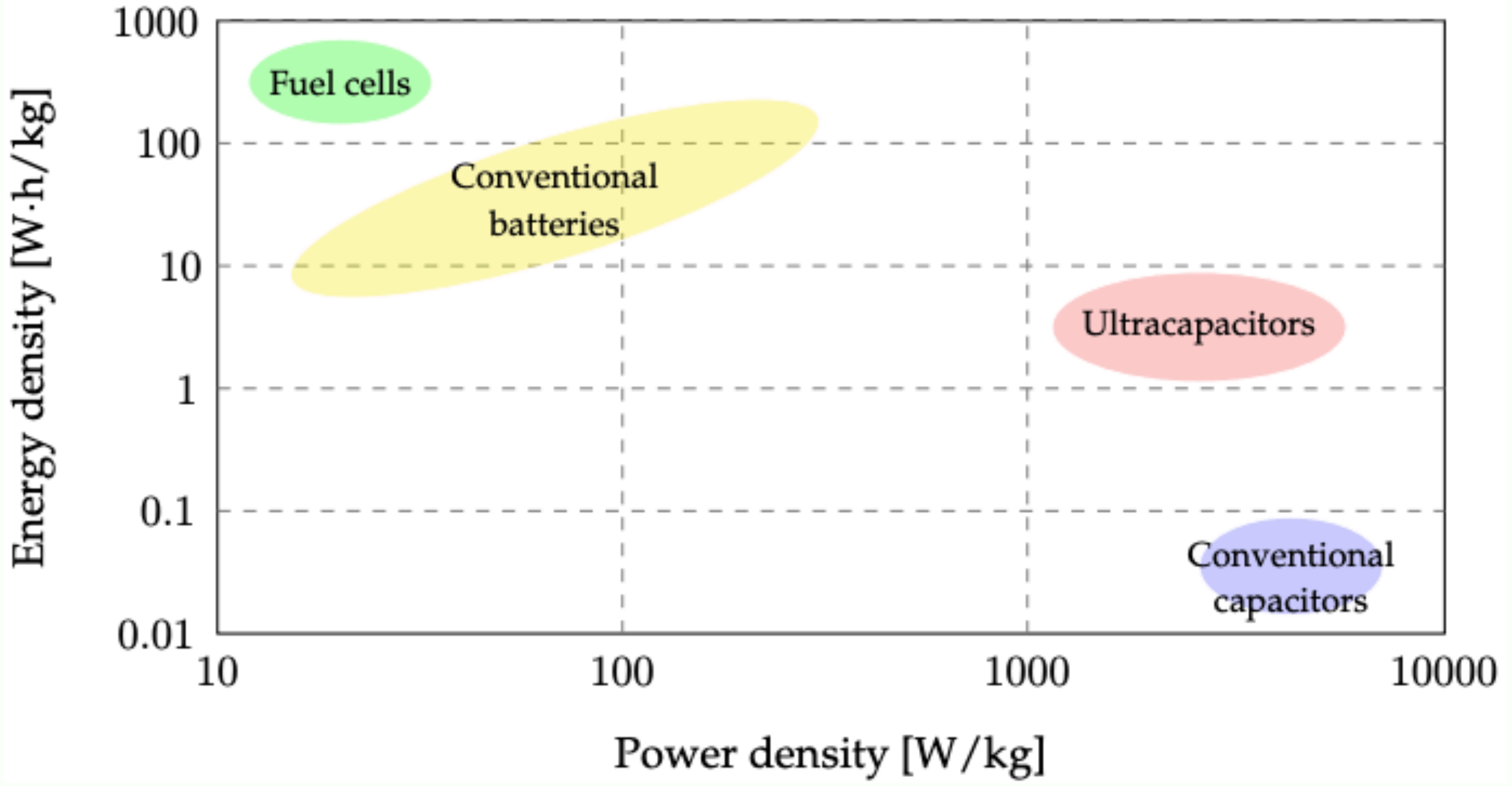
ENERGY STORAGE SYSTEMS

Common energy storage technologies include:

- Mechanical: hydroelectric, flywheels, compressed air, thermal.
- Electrochemical: rechargeable batteries, flow batteries, fuel cells.
- Electrical: capacitors, inductors, superconducting magnetic coils.

Difference

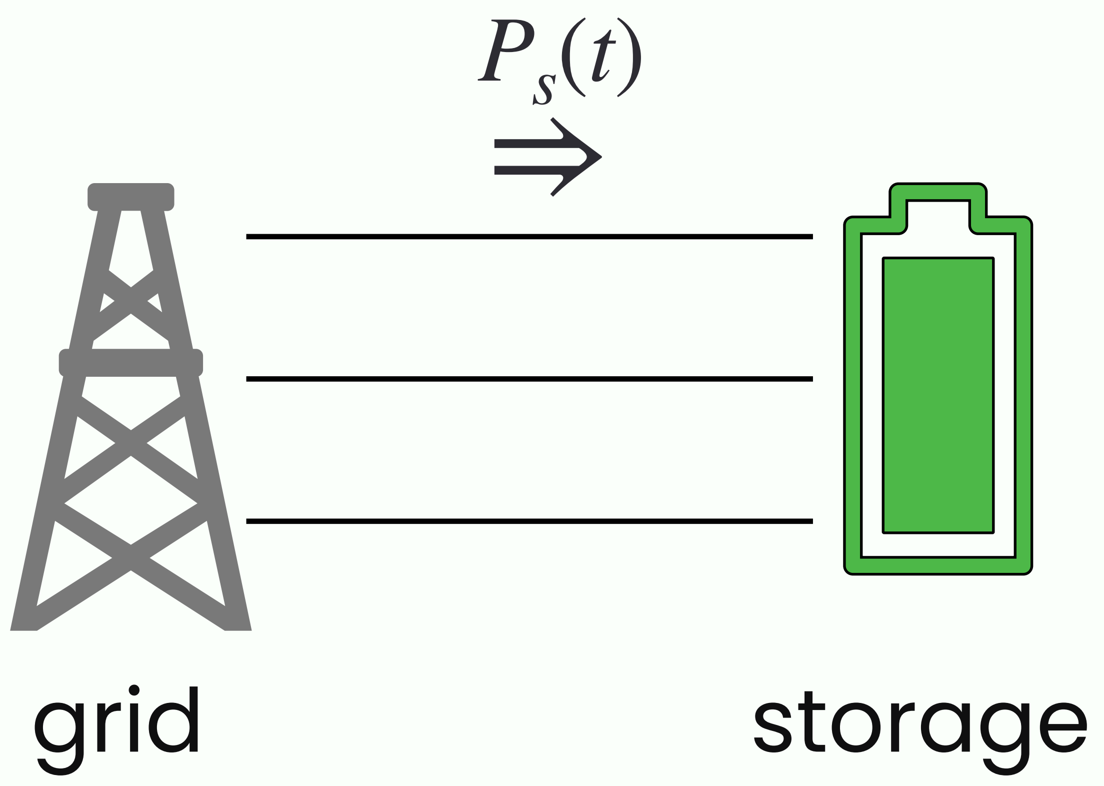
- Parameters: energy density, capacity, power density, efficiency, lifetime, etc.
- Dynamics and constraints
- Different models



Basic dynamic model of a storage device

MODELLING

Consider a grid-connected storage device



A basic dynamic model of the device is:

$$\frac{\mathrm{d}}{\mathrm{d}t}E_s = \begin{cases} \eta_c(E_s, P_s)P_s, & P_s \geq 0, \\ \eta_d^{-1}(E_s, P_s)P_s, & P_s < 0 \end{cases}$$

- ✓ $E_s(t)$ is the stored energy
- ✓ $P_s(t)$ is the total power flowing into the device
- ✓ $\eta_c(t)(E_s, P_s)$ is the charging efficiency
- ✓ $\eta_d(t)(E_s, P_s)$ is the discharging efficiency

Both $\eta_c(t)(E_s, P_s)$ and $\eta_d(t)(E_s, P_s)$ have values in the range $(0,1]$.

MODELLING (2)

The round trip efficiency (η_r) is the efficiency of a charge-discharge cycle.

Consider a device that is charged from E_{low} to E_{high} and then discharged from E_{high} to E_{low}. The round trip efficiency is:

$$\eta_r = \frac{-\int_{P_s(t)<0} P_s(\tau)\mathrm{d}\tau}{\int_{P_s(t)\geq 0} P_s(\tau)\mathrm{d}\tau}.$$

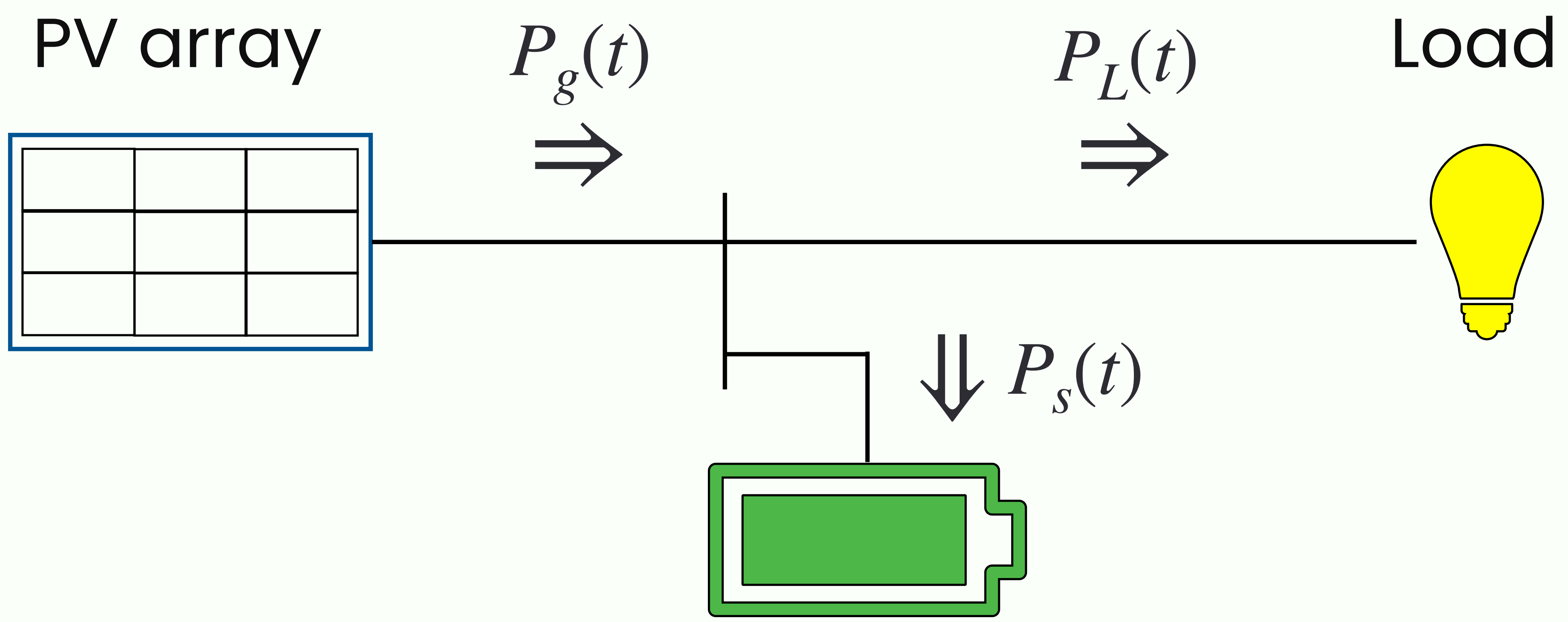
Now, assuming that η_c and η_d are constants and using definition of $\mathrm{d}E_s/\mathrm{d}t$, we get

$$\eta_r = \frac{\int_{E_{low}}^{E_{high}} \eta_d \mathrm{d}E}{\int_{E_{low}}^{E_{high}} \eta_c^{-1} \mathrm{d}E} = \frac{\eta_d(E_{high} - E_{low})}{\eta_c^{-1}(E_{high} - E_{low})} = \eta_c \eta_d.$$

Managing storage & non-controllable power sources

PROBLEM STATEMENT

Consider a power system with a generation unit and a load that are non-controllable.



The system includes:

- ✓PV array with a given output power $P_g(t)$,
- ✓load with a given power $P_L(t)$,
- ✓storage device with power $P_s(t)$.

MANAGING PRINCIPLES

The generated power is given by:

$$P_g(t) = \begin{cases} P_L(t), & \text{if } E_s \geq E_{\max} \text{ and } P_{pv}(t) \geq P_L(t), \\ P_{pv}(t), & \text{otherwise,} \end{cases}$$

where $P_{pv}(t)$ is the maximum power point of the PV array and $E_{\max}$ is the capacity of the storage device.

According to this equation if the storage device is full then the surplus power must be lost. As a result, the power flowing into the storage device is

$$P_s(t) = P_g(t) - P_L(t) = \begin{cases} 0, & \text{if } E_s \geq E_{\max} \text{ and } P_{pv}(t) \geq P_L(t), \\ P_{pv}(t) - P_L(t), & \text{otherwise,} \end{cases}$$

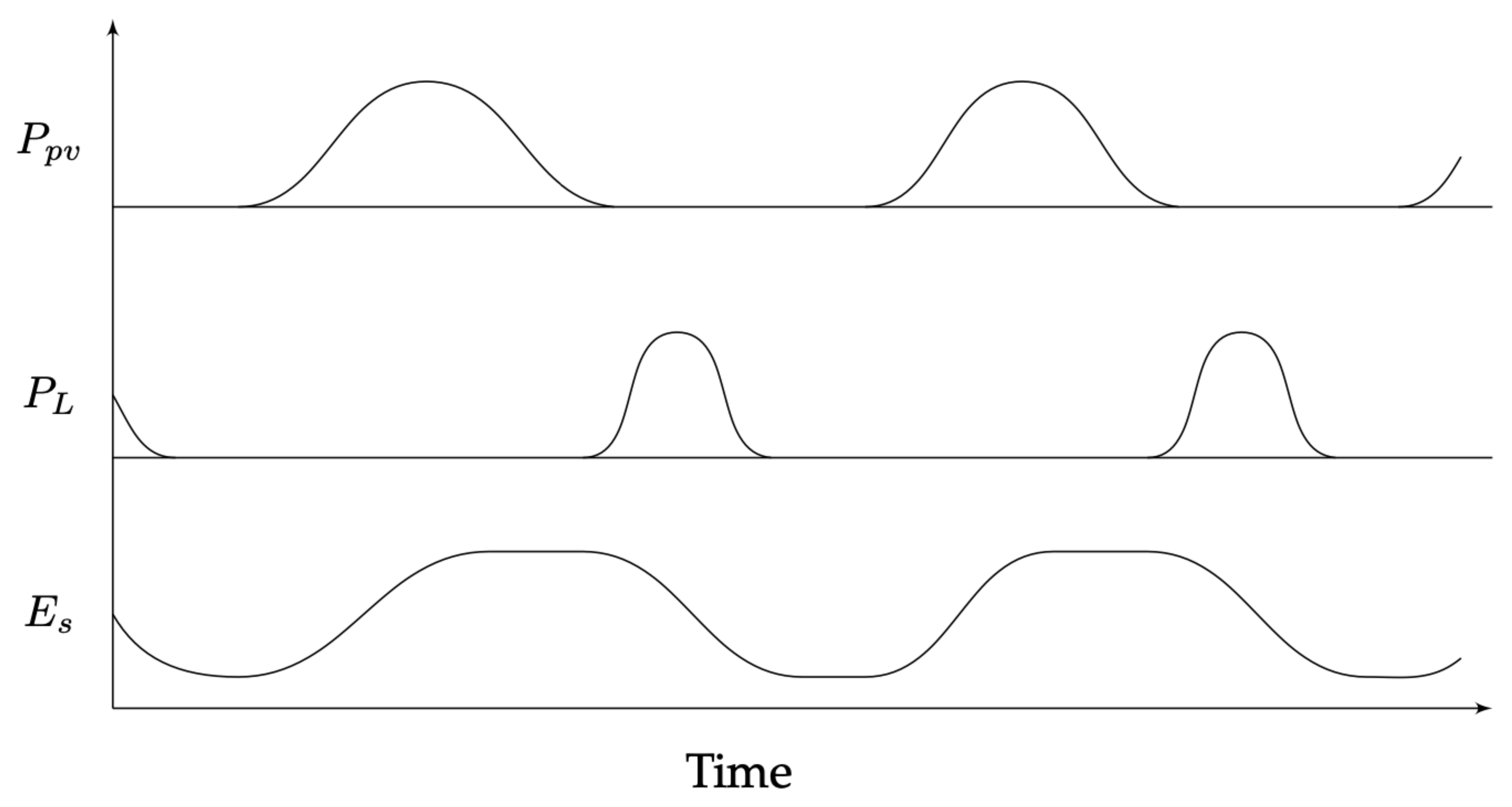
and the stored energy is

$$E_s(t) = \int_0^t P_s(\tau)\mathrm{d}\tau = \int_0^t (P_g(\tau) - P_L(\tau))\mathrm{d}\tau.$$

ILLUSTRATION

Typical waveforms for a system with a storage device and a non-controllable power source.

NB! If the storage device is full then the surplus power must be lost.

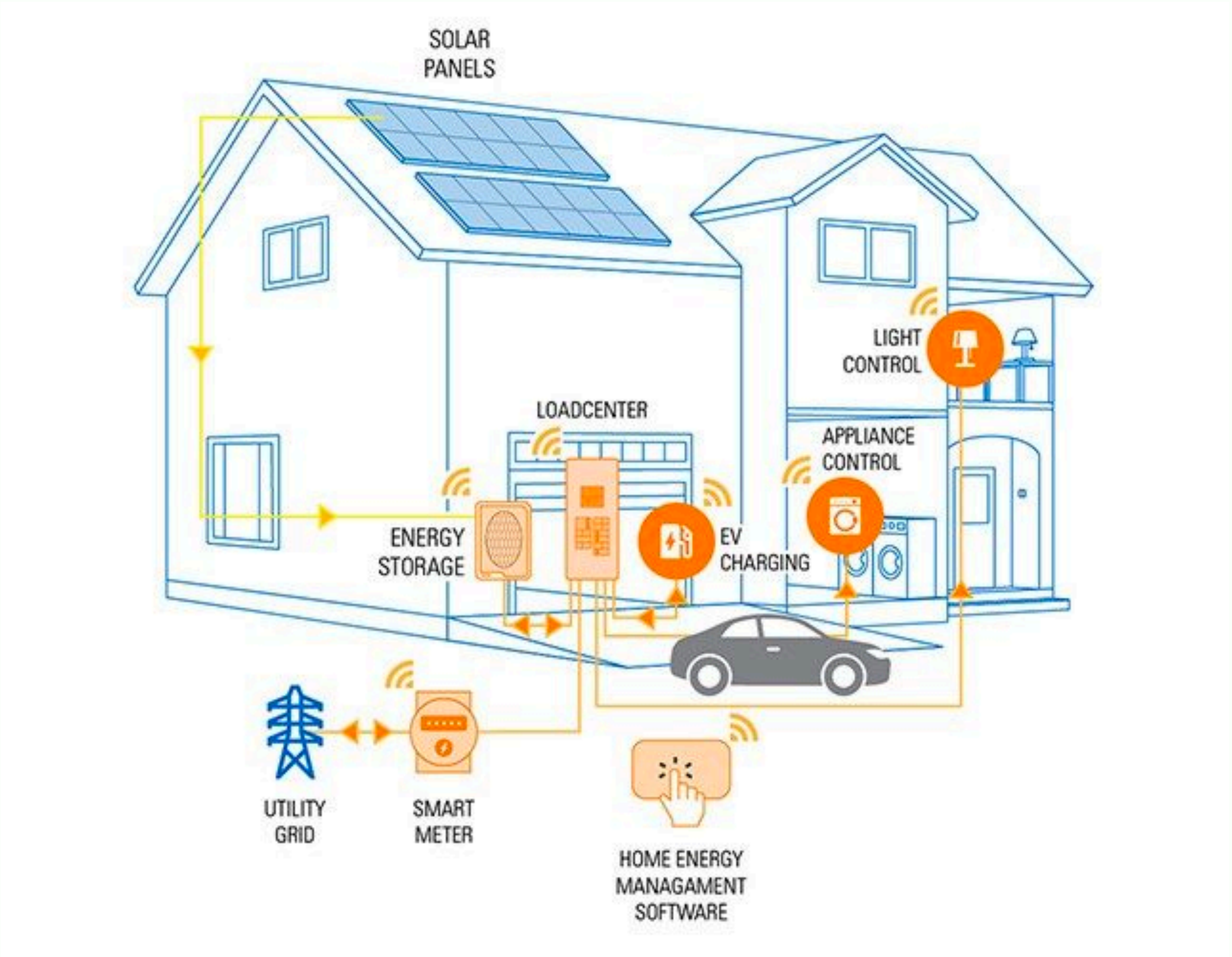


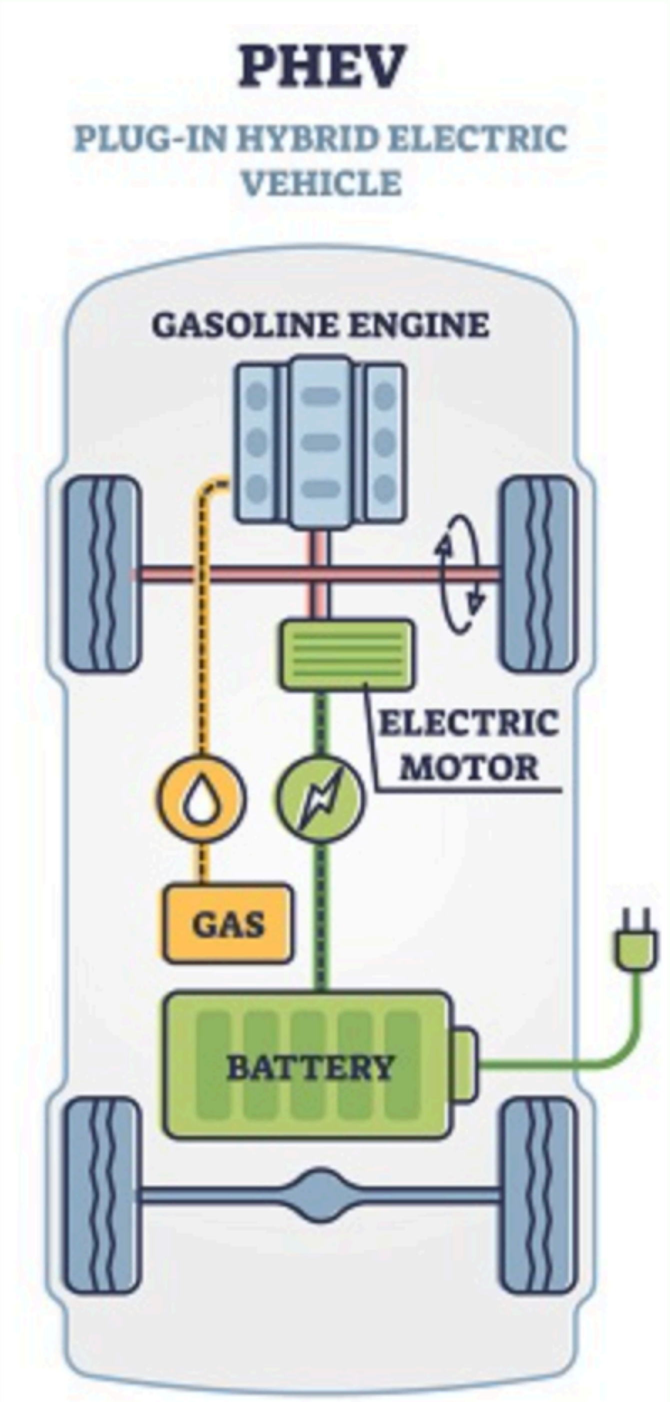
Managing storage & controllable power sources

PROBLEM STATEMENT

Consider a system consisting of generators storage devices and and loads in which the generated power can be controlled.

It is necessary to decide at every moment how much energy should be generated, and how much energy should be stored.



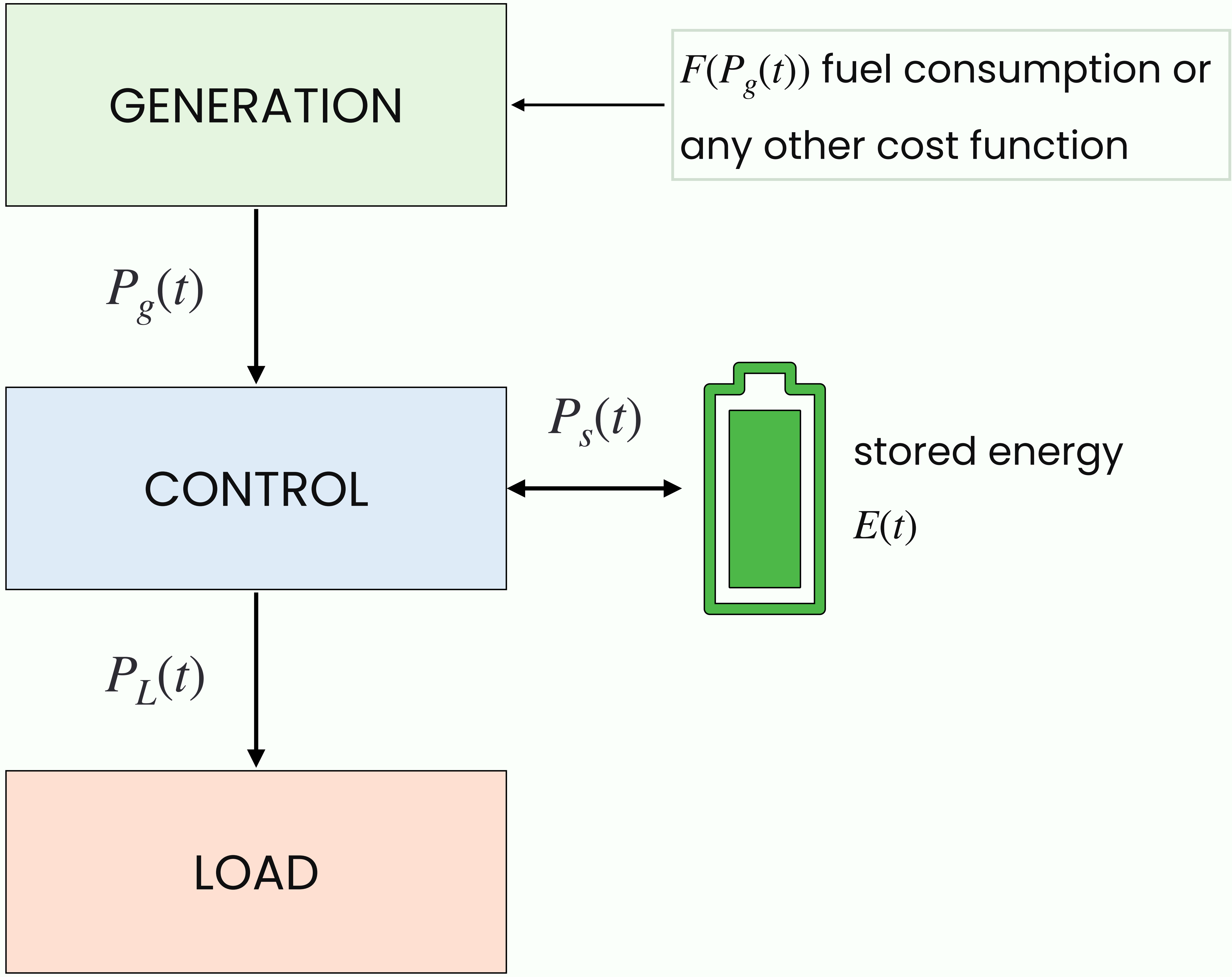


[Accessed June 10, 2025], https://www.transportation.gov/rural/ev/toolkit/ev-basics/vehicle-types

PROBLEM STATEMENT & ASSUMPTIONS

Assumptions:

- ✓Load profile is known or can be estimated over $[0,T]$.
- ✓Ideal storage system $(\eta_c = \eta_d = 1)$.
- ✓No losses.
- ✓$F(P_g)$ is twice differentiable, monotonically increasing and strictly convex.



DEFINITIONS

The generated energy is defined as

$$E_g(t) = \int_0^t P_g(\tau)\mathrm{d}\tau,$$

and the total cost is

$$F_{tot}(t) = \int_0^T F(P_g(t))\mathrm{d}t,$$

We define the load energy as

$$E_L(t) = \int_0^t P_L(\tau)\mathrm{d}\tau.$$

DEFINITIONS (2)

The power flowing into the device is

$$P_s(t) = P_g(t) - P_L(t),$$

and the stored energy is given by

$$E_s(t) = \int_0^t P_s(\tau)\mathrm{d}\tau = \int_0^t (P_g(\tau) - P_L(\tau))\mathrm{d}\tau.$$

Following the above we have

$$E_s(t) = E_g(t) - E_L(t),$$

and the stored energy is limited by the device capacity as

$$0 \leq E_s(t) \leq E_{\max}.$$

OPTIMISATION PROBLEM

The challenge is to determine the generated power $P_g(t)$ that minimises the total cost

$$\begin{aligned} \text{minimize} \quad & F_{tot} = \int_0^T F(P_g(t))\mathrm{d}t, \\ \text{subject to} \quad & E_s(t) = \int_0^t (P_g(\tau) - P_L(\tau))\mathrm{d}\tau, \quad 0 \le E_s(t) \le E_{\max}, \\ & E_s(0) = E_s^i, \quad E_s(T) = E_s^f, \end{aligned}$$

where E_s^i and E_s^f energy stored at the initial and final times, respectively.

OPTIMISATION PROBLEM (2)

Using $E_g(t) = E_s(t) + E_L(t)$ this problem may be reformulated as:

$$
\begin{aligned}
\text{minimize} \qquad & F_{tot} = \int_0^T F(P_g(t))\mathrm{d}t, \\
\text{subject to} \qquad & E_L(t) \leq E_g(t) \leq E_L(t) + E_{\max}, \\
& E_g(0) = E_s^i, \quad E_g(T) = E_s^f + E_L(T), \\
& P_g(t) = \frac{\mathrm{d}}{\mathrm{d}t} E_g(t)\,.
\end{aligned}
$$

In this form we search for a bounded function $E_g(t)$ that minimises the total cost.

PROPERTIES OF THE OPTIMAL SOLUTION

Consider a time interval $[t_1, t_2]$ in which $E_g(t)$ is bounded, i.e.,

$$E_L(t) < E_g(t) < E_L(t) + E_{\max}.$$

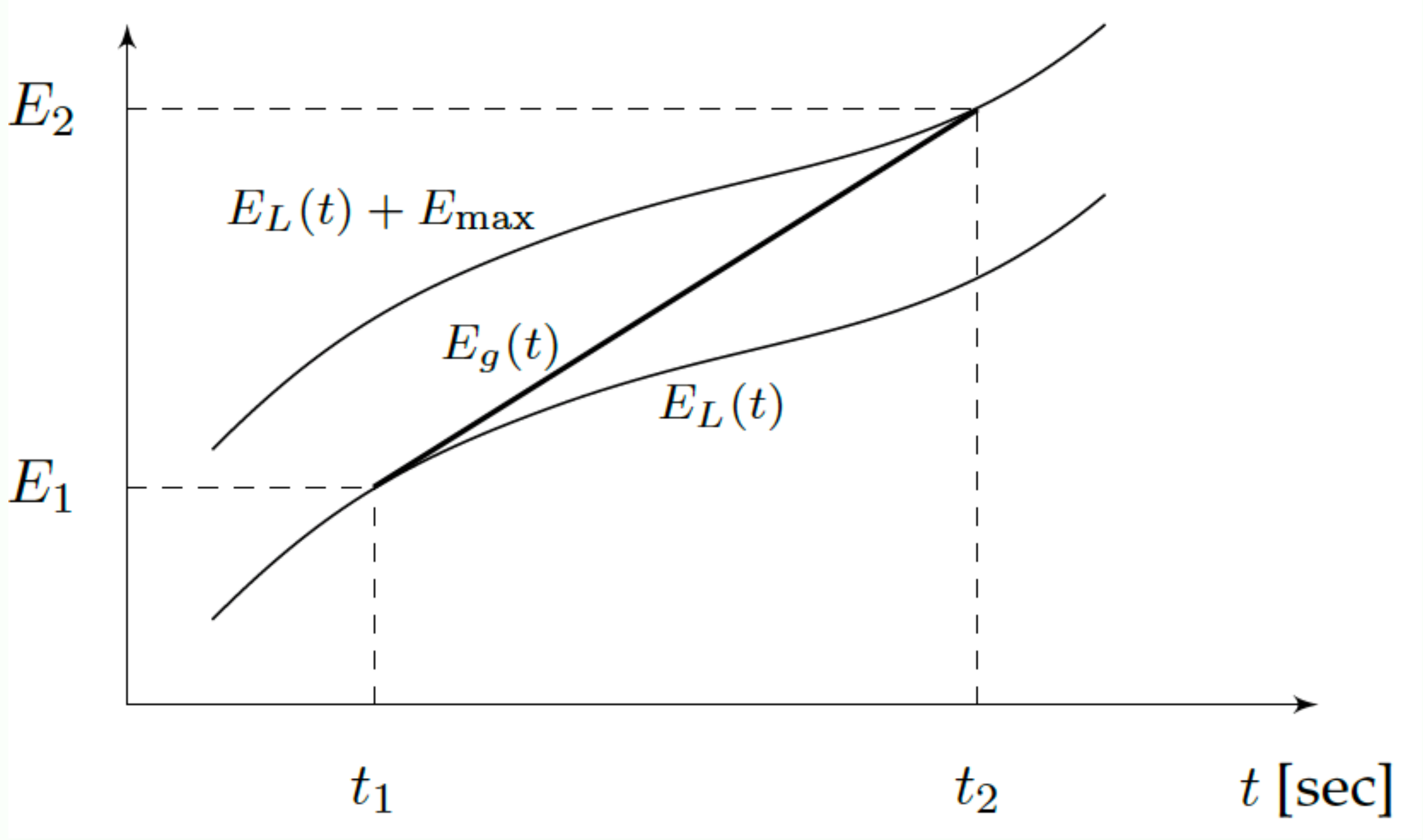
In this interval the optimisation problem may be written as

$$\begin{aligned}
\text{minimize} \qquad & F_{tot} = \int_{t_1}^{t_2} F(P_g(t))\mathrm{d}t, \\
\text{subject to} \qquad & P_g(t) = \frac{\mathrm{d}}{\mathrm{d}t} E_g(t), \\
& E_g(t_1) = E_1, \quad E_g(T) = E_2,
\end{aligned}$$

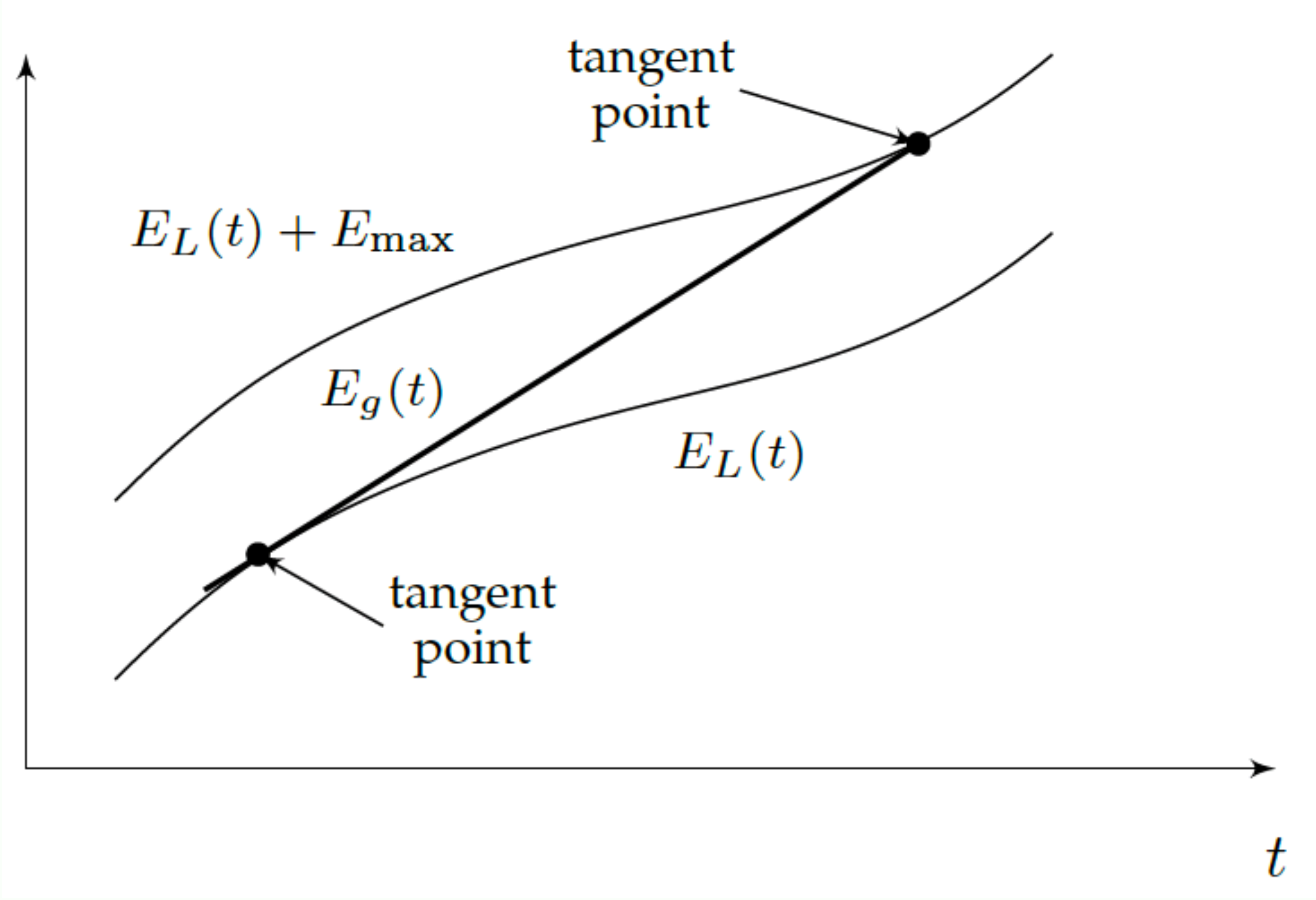
where E_1 and E_2 are given constants.

PROPERTIES OF THE OPTIMAL SOLUTION (2)

Property 1: if solution exists it is characterised by a constant $P_g(t)$ and optimal $E_g(t)$ must be a straight line.



Property 2: $E_g(t)$ must be tangent to the bounds.



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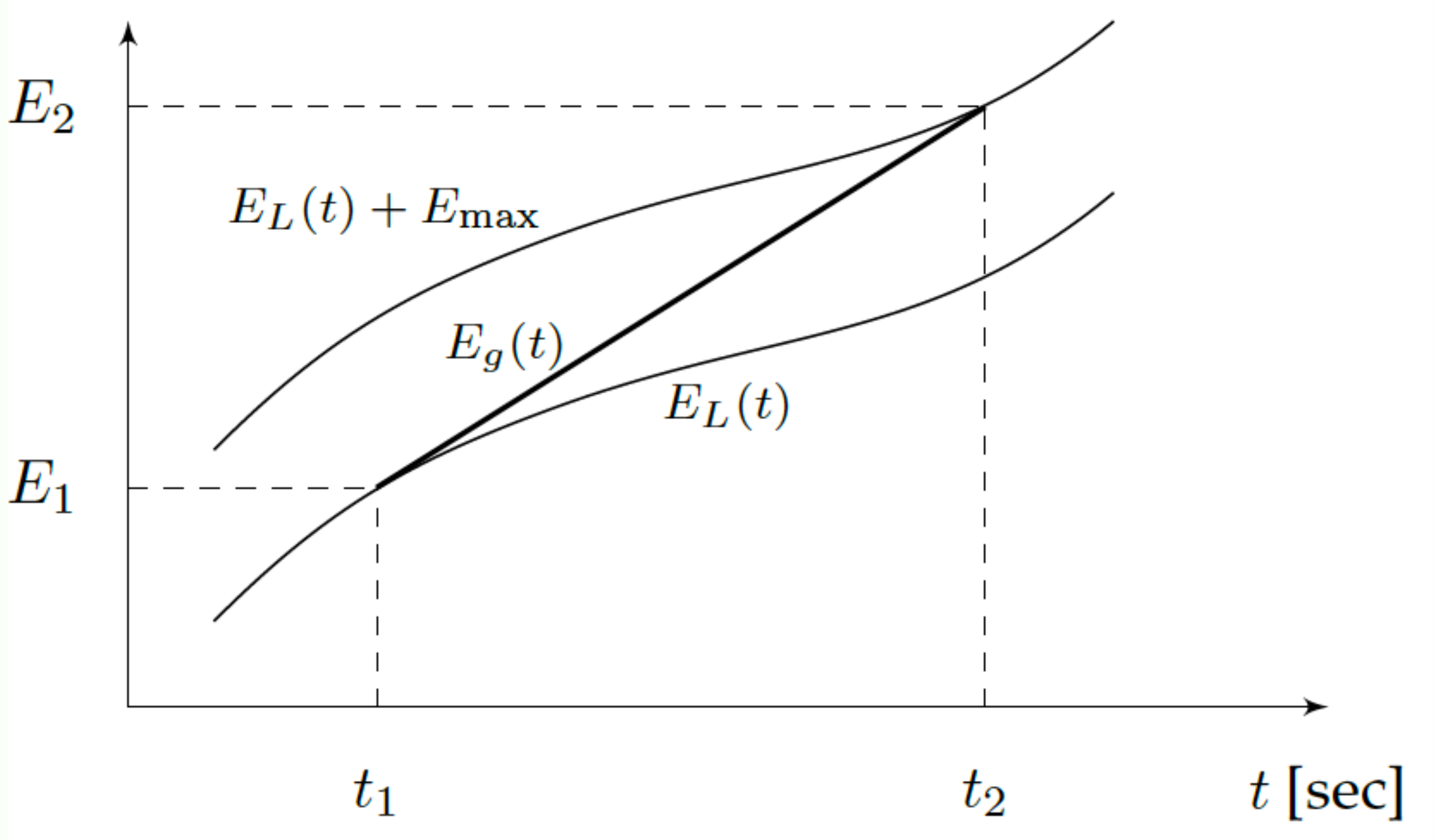

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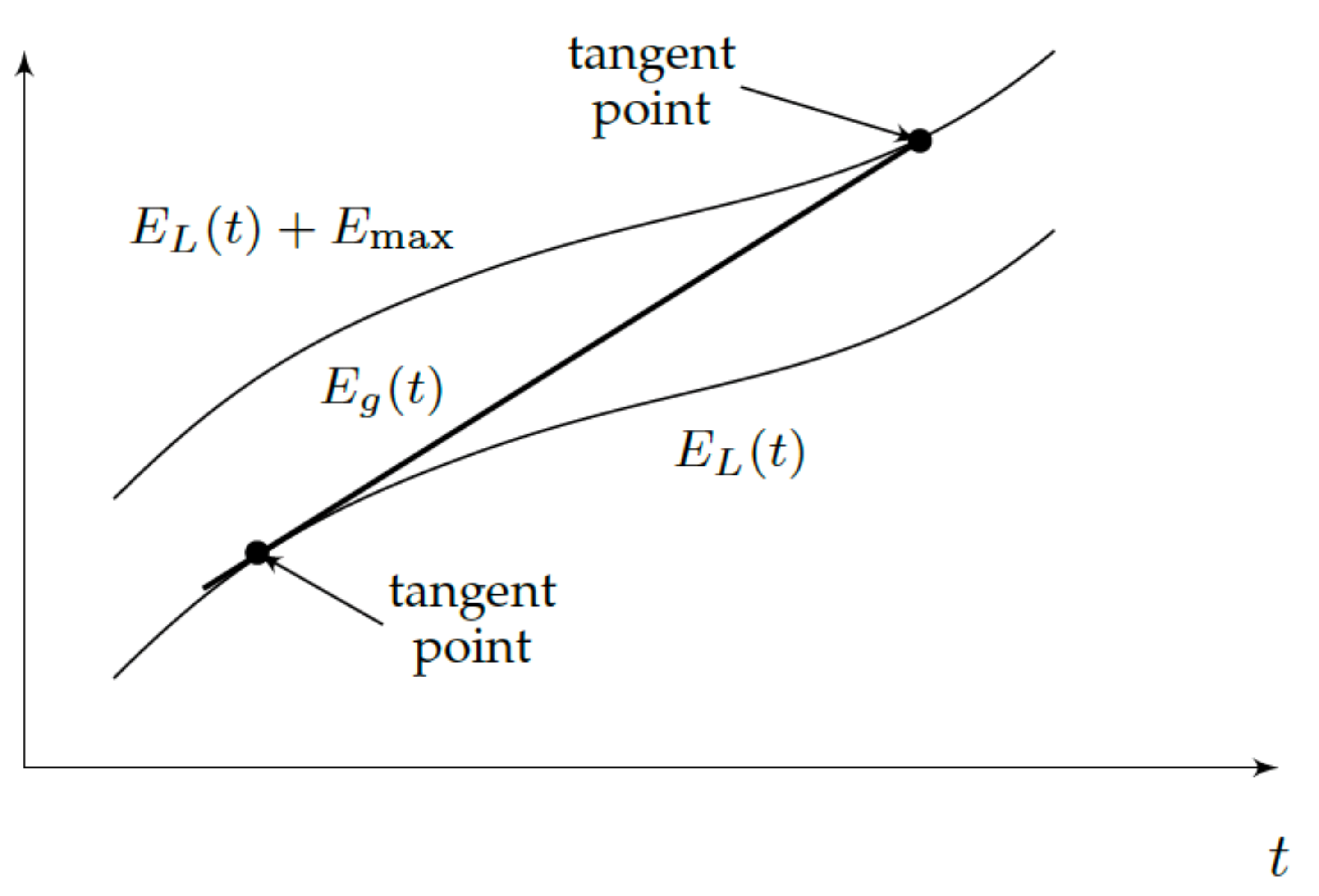

These two properties completely define the optimal solution, and hold regardless of the cost function $F(P_g)$, as long as above assumptions hold.

ILLUSTRATION

Consider the following cost function

$$F(P_g) = \sqrt{1 + P_g^2},$$

and the resulting total cost is

$$F_{tot} = \int_0^T F(P_g(\tau))\mathrm{d}\tau = \int_0^T \sqrt{1 + P_g^2(\tau)}\mathrm{d}\tau = \int_0^T \sqrt{1 + \left(\frac{\mathrm{d}E_g}{\mathrm{d}t} \bigg|_{t=\tau} \right)^2} \mathrm{d}\tau,$$

which is the total length of the curve $E_g(t)$.

ILLUSTRATION

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which is the total length of the curve $E_g(t)$.

> **arc length** Let $y = f(x)$ be the graph of a function f such that f' is *continuous* on $[a, b]$. The length of the arc, or **arc length**, of the curve $y = f(x)$ between $x = a$ and $x = b$ equals
>
> $$\int_a^b \sqrt{1 + (f'(x))^2}\, dx.$$

C. Clapham. Concise Oxford Dictionary of Mathematics, Oxford University Press, 1996

ILLUSTRATION (2)

Based on the arguments above we can conclude that

- ✓ The optimal generated energy $E_g(t)$ does not depend on the cost function $F(\cdot)$.
- ✓ The optimal generated energy minimises the total length of the curve $E_g(t)$.

The optimal generated energy $E_g(t)$ follows the *shortest path* between the bounds $E_L(t)$ and $E_L(t) + E_{\max}$.

GRAPHICAL DESIGN

Sketch of the procedure:

Step 1: Plot the lower bound $E_L(t)$ and the upper bound $E_L(t) + E_{\max}$.

Step 2: Choose the initial and final values of the generated energy $E_g(t)$. A typical choice is $E_g(0) = E_L(0)$ and $E_g(T) = E_L(T)$, which is equivalent to $E_s(0) = E_s(T) = 0$.

Step 3: Plot the shortest path that connects $E_g(0)$ and $E_g(T)$ and is between bounds. This is the optimal generated energy $E_g(t)$.

Step 4: All the other functions may be computed directly. For instance, the stored energy is given by $E_s(t) = E_g(t) - E_L(t)$.

OBSERVATIONS

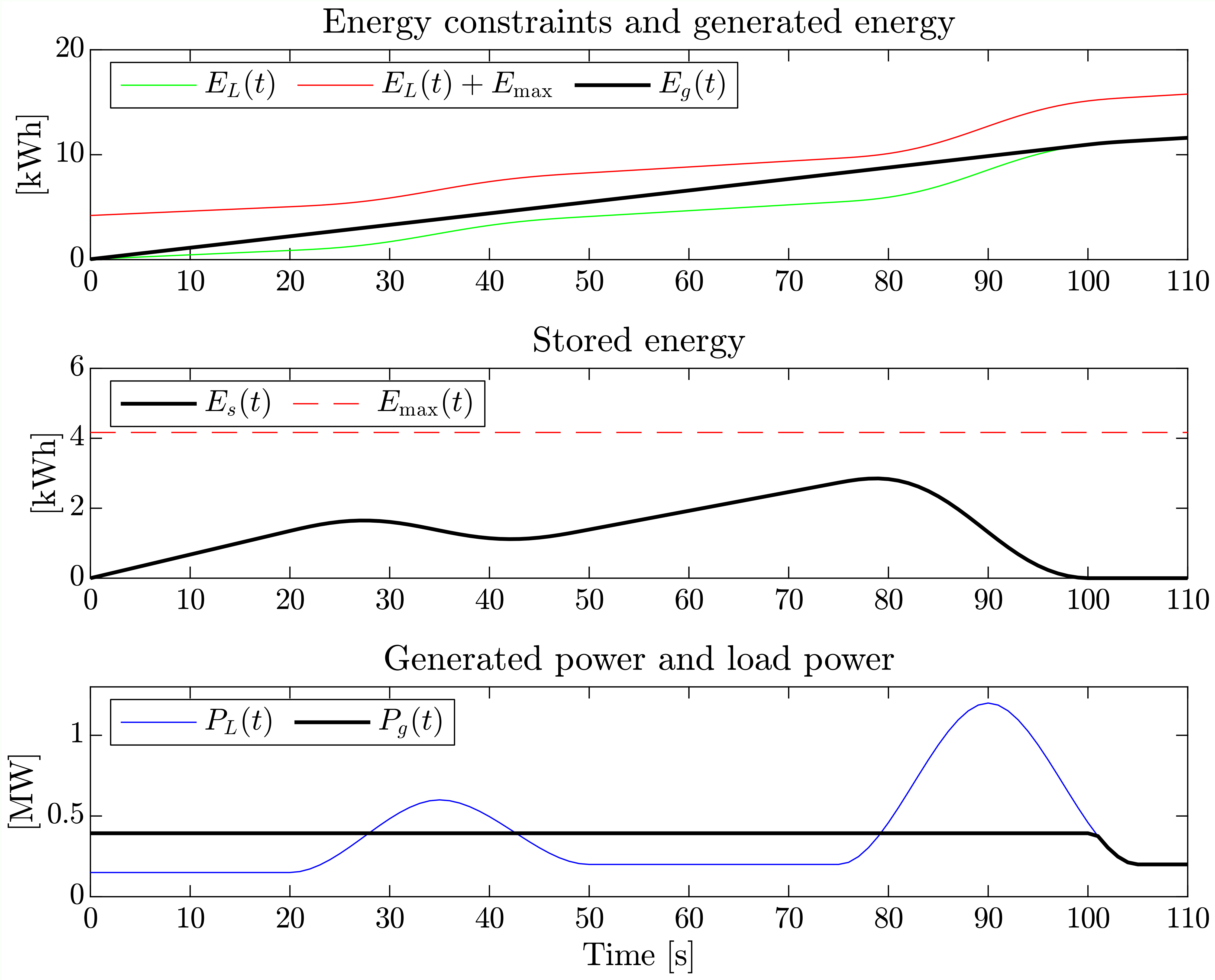
The shortest path method allows simple planning of the optimal generated energy, and enables designers to estimate the size of the storage device for a specific application.

Several properties of the optimal solution are:

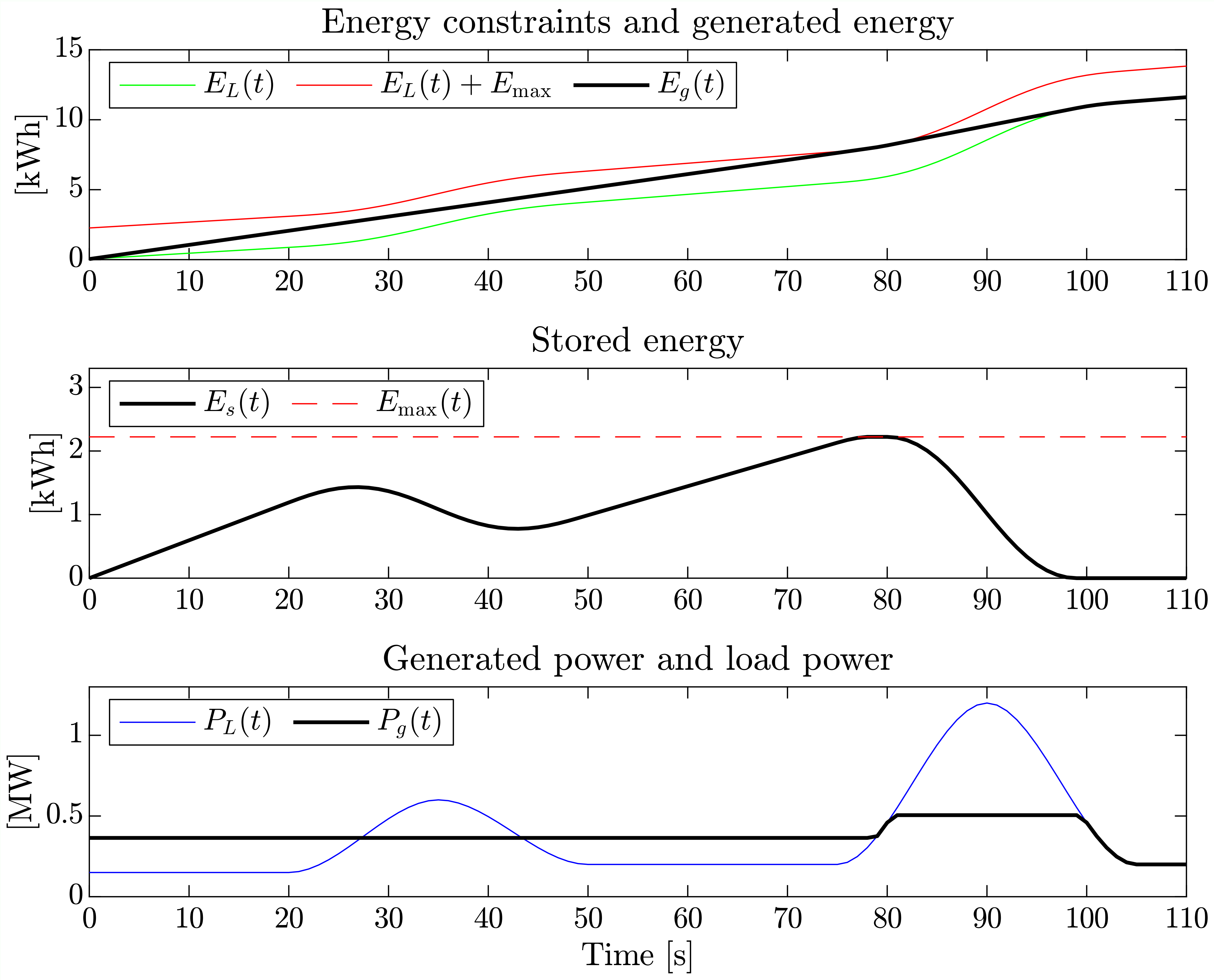
- ✓ If the capacity $E_{\max}(t)$ is very low then the generated power is approximately equal to the load. In this case the storage device has little effect.
- ✓ If the capacity $E_{\max}(t)$ is high enough then the generated energy approximately follows a straight line, and is equal to the average load.

ILLUSTRATIVE EXAMPLES

$E_{\max} = 15$ kWh

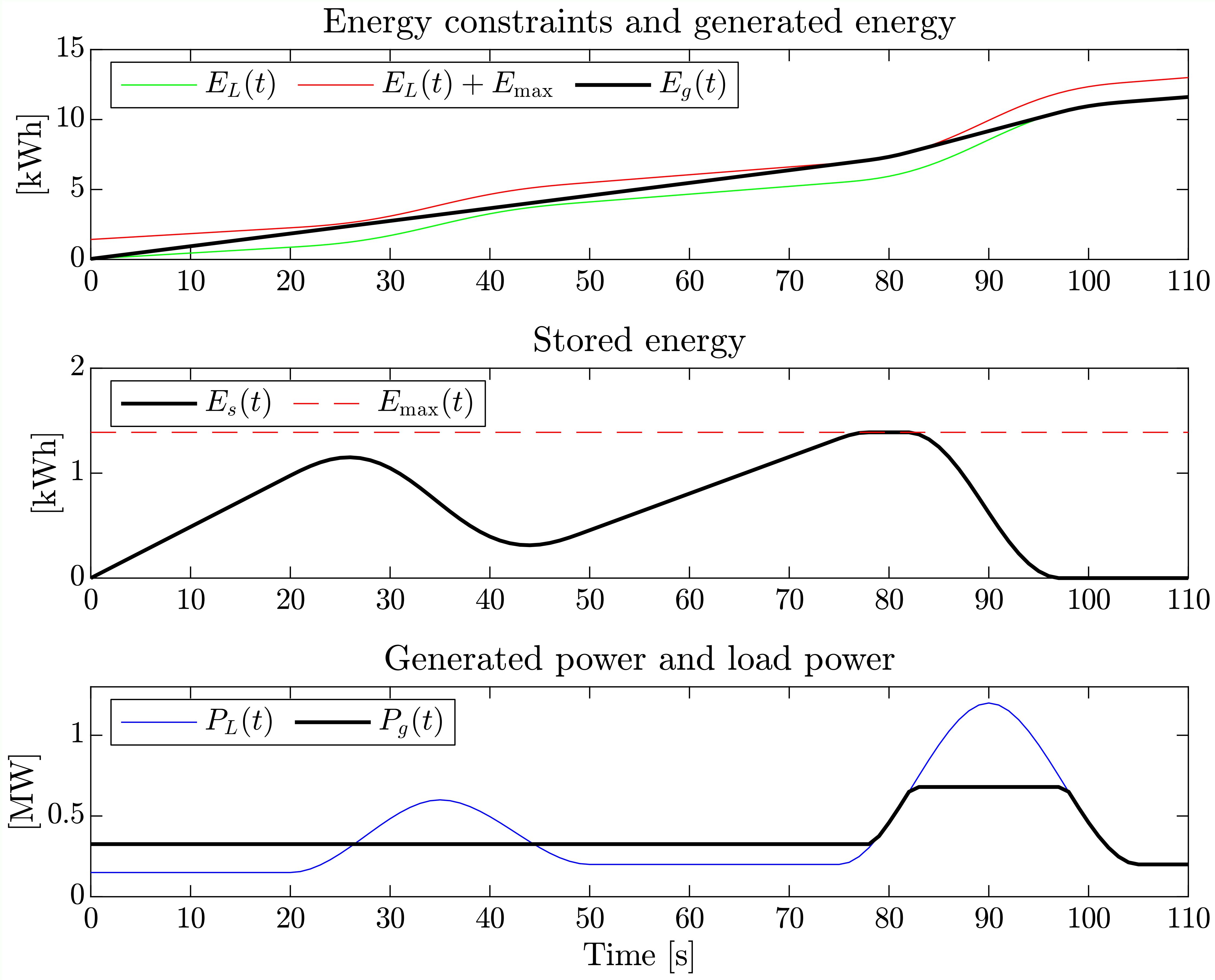


$E_{\max} = 8$ kWh

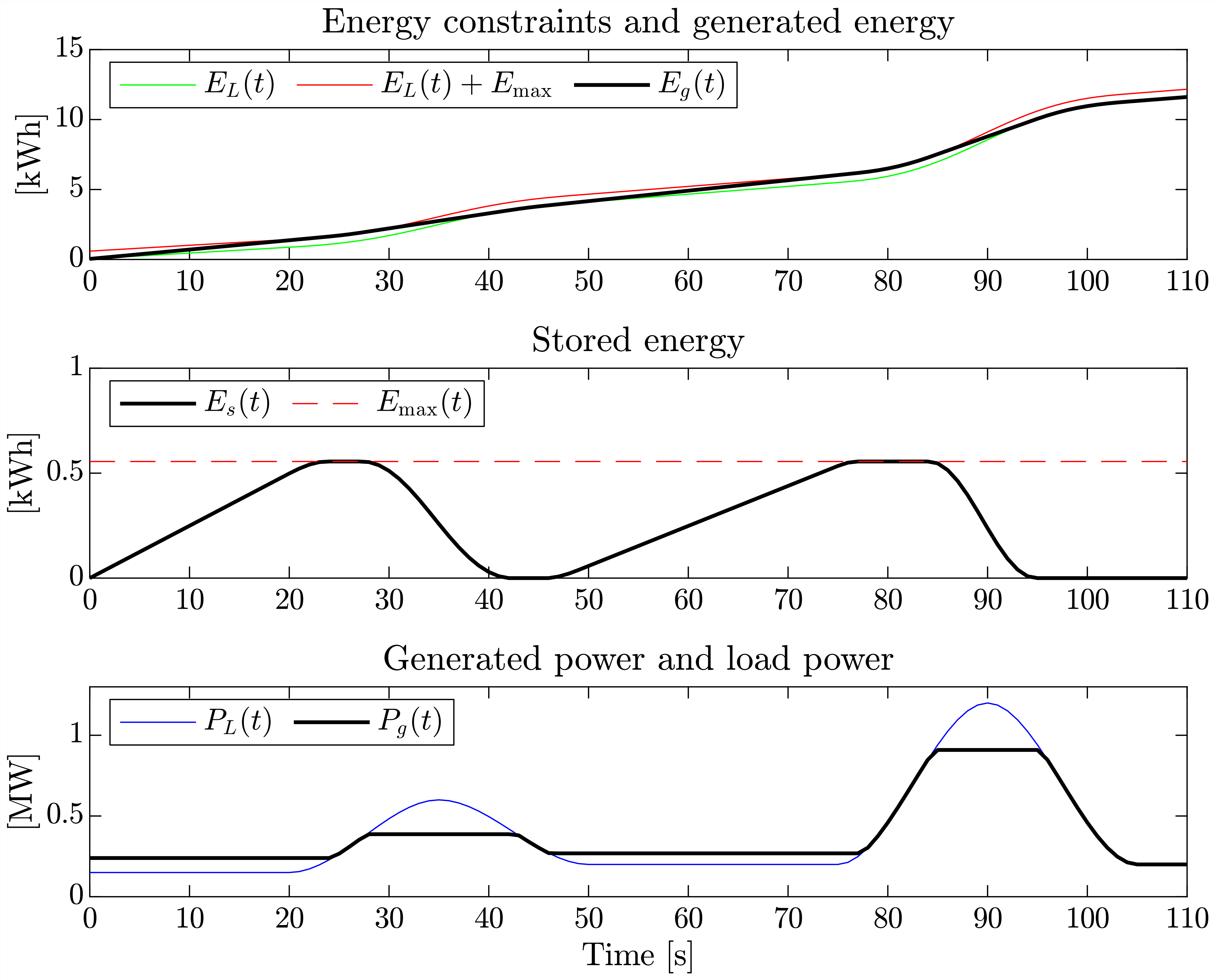


ILLUSTRATIVE EXAMPLES (2)

$E_{\max} = 5$ kWh

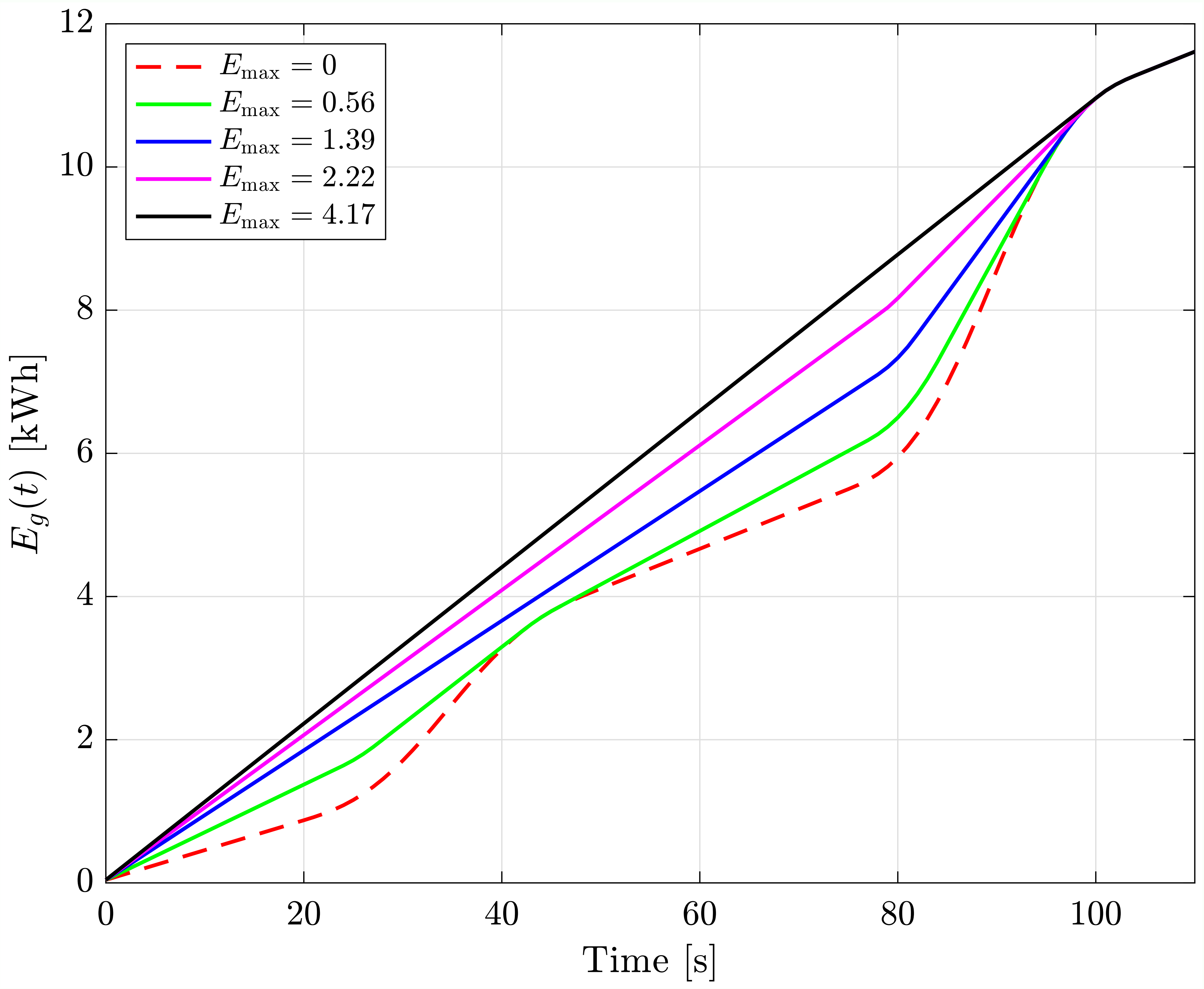


$E_{\max} = 2$ kWh



ILLUSTRATIVE EXAMPLES (3)

The generated energy for various capacities. In each case the optimal generated energy follows the shortest path.



OBSERVATIONS (2)

Property 3: Shortest path method minimises the peak of the generated power $P_g(t)$.

Consider the cost function

$$F(P_g(t)) = |P_g(t)|^m.$$

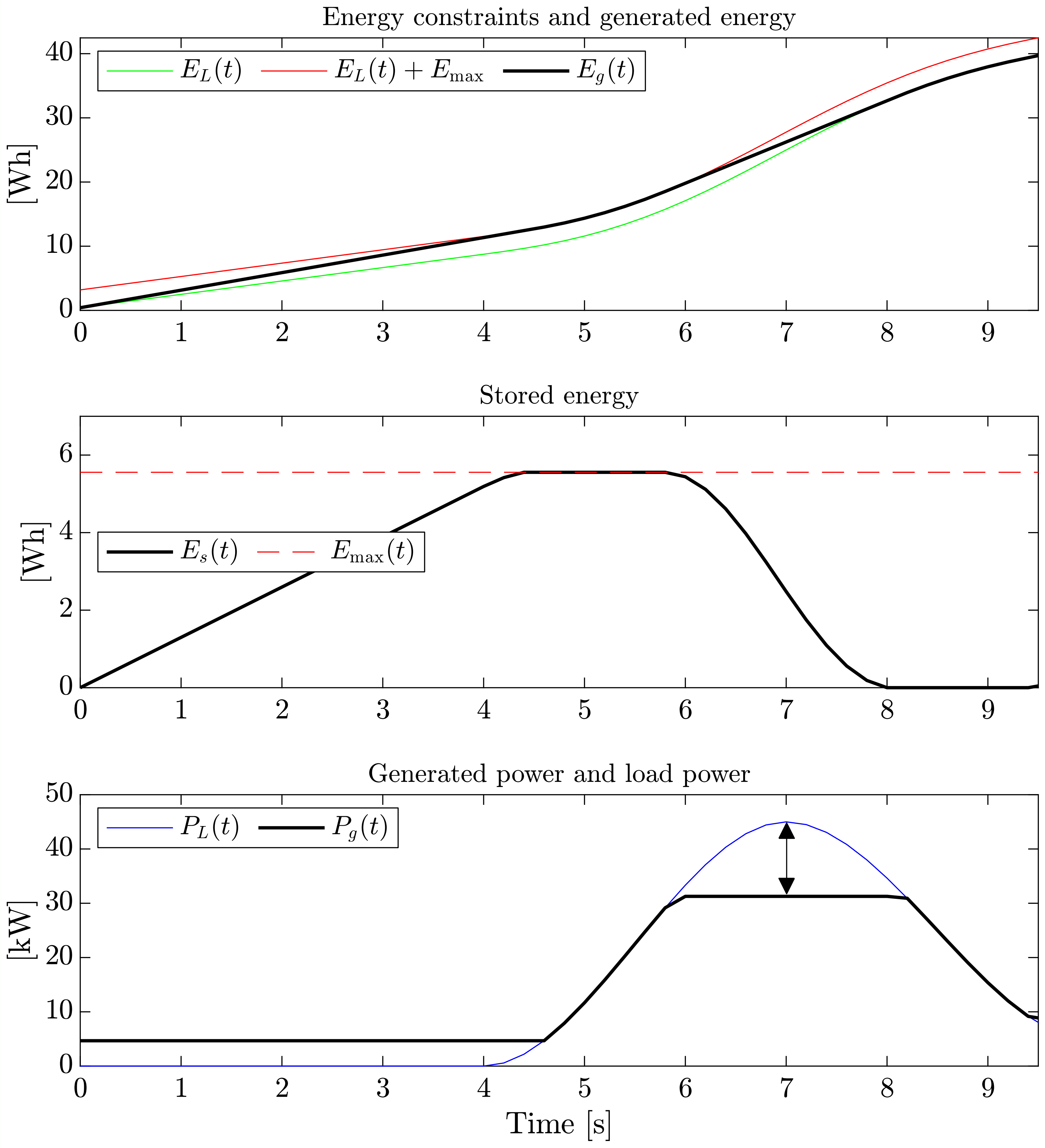
When $m \to \infty$ we have

$$\lim_{m\to\infty} F_{tot}^{\frac{1}{m}} = \lim_{m\to\infty} \left(\int_0^T |P_g(t)|^m \mathrm{d}t \right)^{\frac{1}{m}} = \max_t \{ |P_g(t)| \}.$$

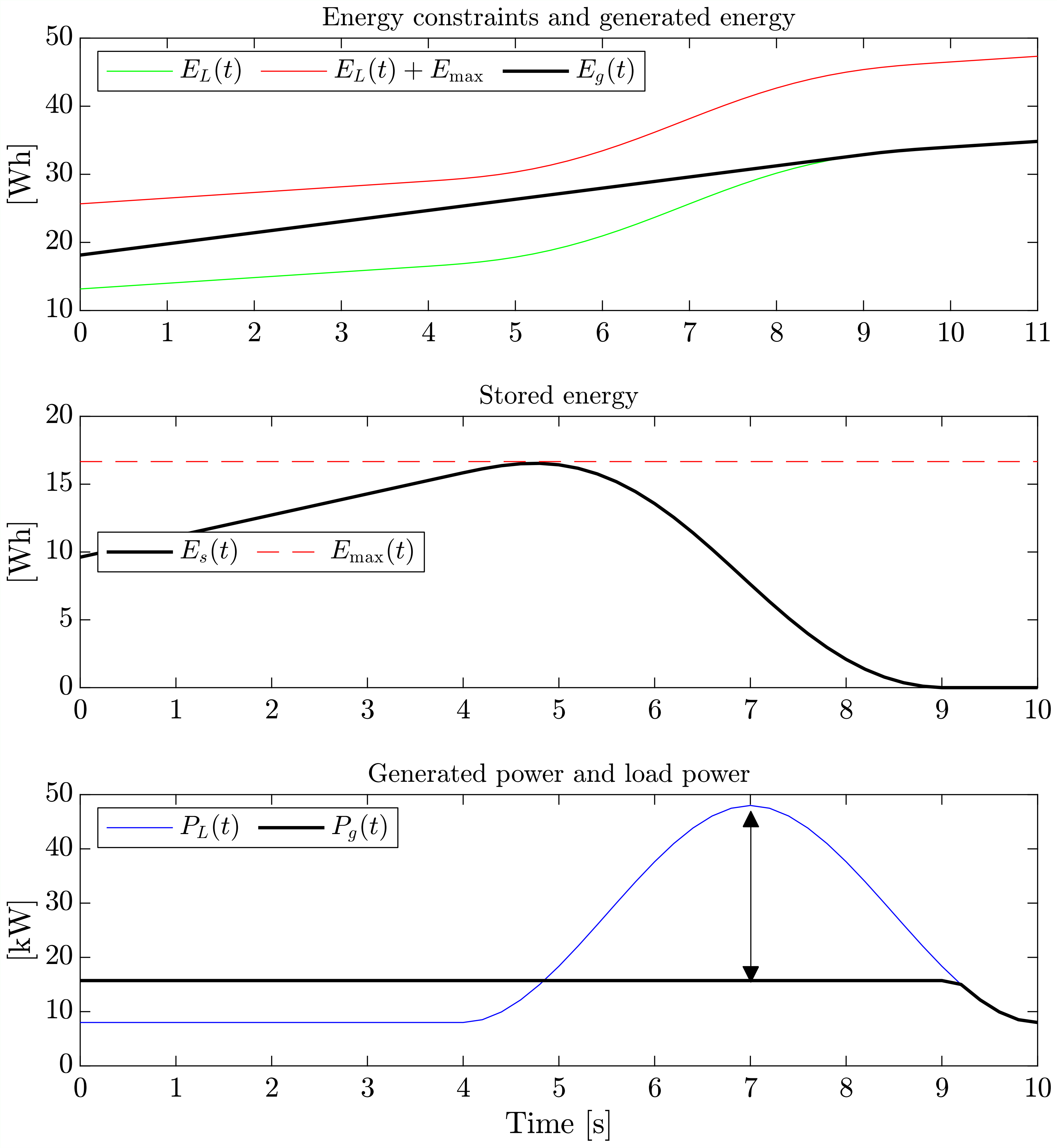
Since typically the optimal solution is the same for every value of m, the shortest path method minimises the peak power $\max\{ |P_g(t)| \}$.

ILLUSTRATIVE EXAMPLES 2

Low capacity, high peak power



High capacity, low peak power



Energy trading

PROBLEM STATEMENT

Consider the following energy trading optimisation problem:

$$
\begin{aligned}
\text{maximize} \quad & -\int_0^T C(t)P_s(t)\mathrm{d}t, \\
\text{subject to} \quad & 0 \le E_s(t) \le E_{\max}, \\
& \frac{\mathrm{d}E_s}{\mathrm{d}t} = \begin{cases} \eta_c(E_s, P_s)P_s, & P_s \ge 0, \\ \eta_d^{-1}(E_s, P_s)P_s, & P_s < 0, \end{cases}
\end{aligned}
$$

✓$C(t)$ is the cost of energy

✓$E_s(t)$ is the stored energy

✓$P_s(t)$ is the power flowing into the storage device

✓$\eta_c(E_s, P_s)$ is the charging efficiency

✓$\eta_d(E_s, P_s)$ is the discharging efficiency

PROBLEM STATEMENT

Consider the following energy trading optimisation problem:

$$
\begin{aligned}
&\text{maximize} && \boxed{-\int_0^T C(t)P_s(t)\mathrm{d}t,} \quad \longleftarrow \text{Profit} \\
&\text{subject to} && 0 \leq E_s(t) \leq E_{\max}, \\
& && \frac{\mathrm{d}E_s}{\mathrm{d}t} = \begin{cases} \eta_c(E_s, P_s)P_s, & P_s \geq 0, \\ \eta_d^{-1}(E_s, P_s)P_s, & P_s < 0, \end{cases}
\end{aligned}
$$

✓ $C(t)$ is the cost of energy

✓ $E_s(t)$ is the stored energy

✓ $P_s(t)$ is the power flowing into the storage device

✓ $\eta_c(E_s, P_s)$ is the charging efficiency

✓ $\eta_d(E_s, P_s)$ is the discharging efficiency

OPTIMAL SOLUTION

For simplicity assume that

$$C(t) = \begin{cases} C_1, & 0 \le t < \alpha T, \\ C_2, & \alpha T \le t < T, \end{cases}$$

$$\eta_c(E_s, P_s) = \eta_c = \text{constant},$$

$$\eta_d(E_s, P_s) = \eta_d = \text{constant},$$

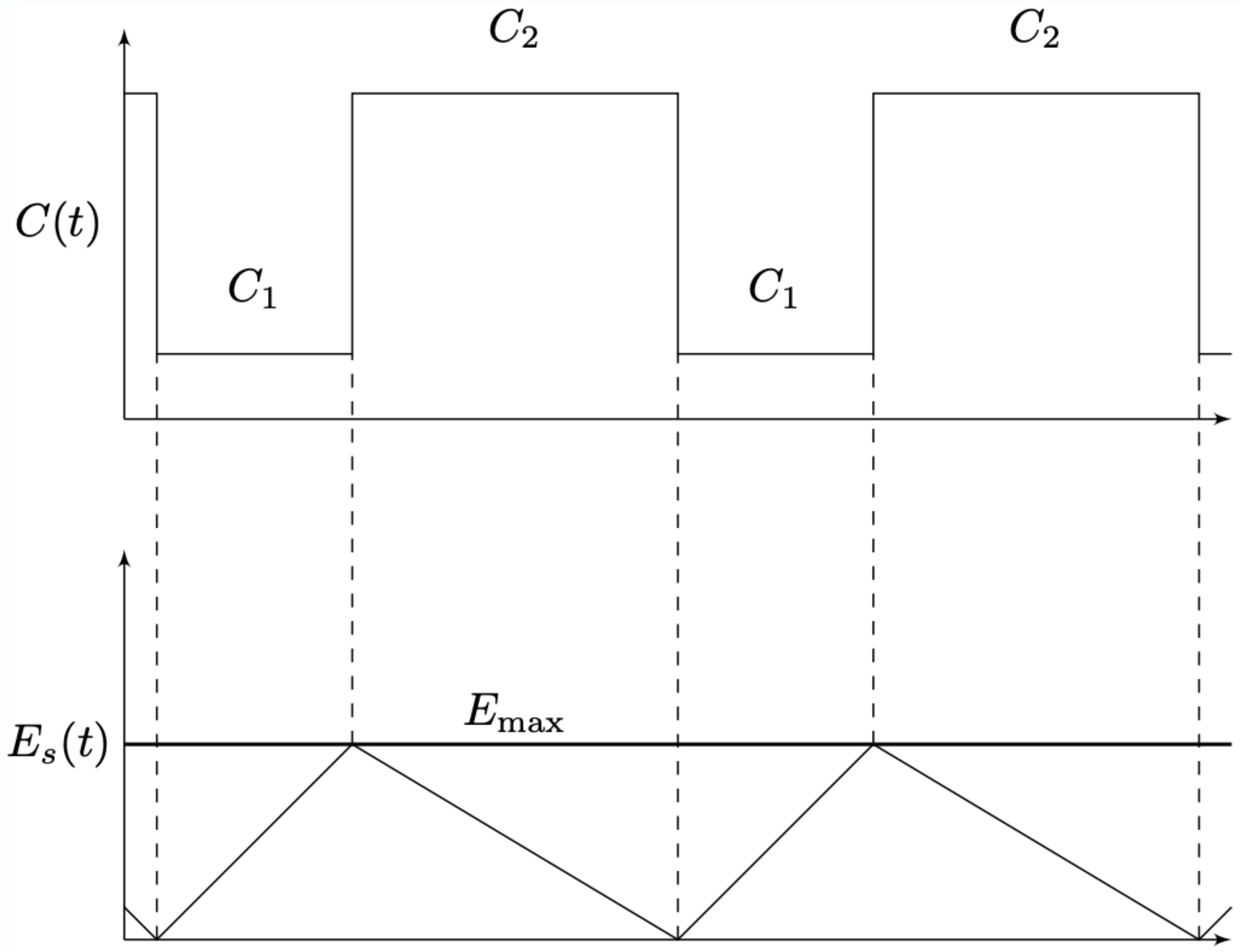
with $0 < C_1 < C_2$ and $0 < \alpha < 1$.

Optimal strategy: Energy is bought when the price is low, and sold when the price is high.

OPTIMAL SOLUTION (2)

One optimal solution is

$$P_s(t) = \begin{cases} \dfrac{E_{\max}}{\eta_c \alpha T}, & 0 \le t < \alpha T, \\ -\dfrac{\eta_d E_{\max}}{(1-\alpha)T}, & \alpha T \le t < T\,. \end{cases}$$



OPTIMAL SOLUTION (3)

The profit over one cycle is

$$-\int_0^T C(t)P_s(t)\mathrm{d}t = -C_1\frac{E_{\max}}{\eta_c} + C_2\eta_d E_{\max} = \left(C_2\eta_d - \frac{C_1}{\eta_c}\right)E_{\max},$$

and it is positive if

$$C_2\eta_d - \frac{C_1}{\eta_c} > 0 \quad \Rightarrow \quad \eta_c\eta_d > \frac{C_1}{C_2}.$$

OPTIMAL SOLUTION (4)

To make a profit the round-trip efficiency must be higher that the price ratio.

For instance:

- ✓ If the storage device is lossless $(\eta_c = \eta_d = 1)$ then the profit over a cycle is $(C_2 - C_1)E_{\text{max}}$.
  In this case the profit is proportional to the price difference, and to the device capacity.
- ✓ If there is no price difference $(C_2 = C_1)$ then the profit is zero or negative.
- ✓ If $C_1 = 0$ a positive profit is guaranteed, regardless of the efficiency.

OUTRO

In practice, most energy storage control problems are more complex, and are generally solved by numeric techniques such as linear programming, dynamic programming, or Pontryagin's minimum principle.

Useful to read:

- R. Machlev, N. Zargari, N. R. Chowdhury, J. Belikov, and Y. Levron. A review of optimal control methods for energy storage systems - energy trading, energy balancing and electric vehicles, Journal of Energy Storage, vol. 32, p. 101787, 2020.
- Y. Levron and D. Shmilovitz. Optimal power management in fueled systems with finite storage capacity, IEEE Transactions on Circuits and Systems I: Regular Papers, 57(8), pp. 2221–2231, 2010.
- Y. Levron and D. Shmilovitz. Power systems' optimal peak-shaving applying secondary storage, Electric Power Systems Research, 89, pp. 80–84, 2012.
- Y. Levron, J. M. Guerrero, and Y. Beck. Optimal power flow in microgrids with energy storage, IEEE Transactions on Power Systems, 28(3), pp. 3226–3234, 2013.

Thank you!

Questions?