CS461 Quiz Four

CS461 Section #:	
Name:	
NetID:	

0. True / False questions.

- In a Bayesian network, a direct edge from A to B indicates a causal and effect relationship between A and B. (True / **False**)
- A joint density has a unique Bayesian network representation. (True / **False**)
- Given a Bayesian network (structure and conditional probability tables), we can quantify any probability of interest within the defined model. (**True** / False)
- When A and B are conditionally independent given C, we can say A and B do not have a direct causal and effect relationship. (**True** / False)
- In the EM algorithm, the parameters updated at each iteration are guaranteed to monotonically increase the log-likelihood of the observed data. (**True** / False)
- K-means is a discriminative algorithm. (True / **False**)

1. Choose all true statements.

1. $T \perp\!\!\!\perp D$ (False)
2. $T \perp\!\!\!\perp D | R$ (True)
3. $T \perp\!\!\!\perp D | S$ (False)
4. $T \perp\!\!\!\perp D | R, S$ (False)
5. $R \perp\!\!\!\perp S$ (False)
6. $R \perp\!\!\!\perp S | T$ (False)
7. $R \perp\!\!\!\perp S | D$ (False)
8. $R \perp\!\!\!\perp S | T, D$ (True)

2. (25) Given the following Bayesian network, compute conditional density $P[X_2|X1 = +, X3 = -]$.

$$\boldsymbol{P[X_0] = 1/3}$$
$$\boldsymbol{P[X_{i+1} = +|x_i = +] = 2/3}$$
$$\boldsymbol{P[X_{i+1} = +|x_i = -] = 1/4}$$

$$\begin{aligned}
P[X_2|X1 = +, X3 = -] &= \alpha P[X_1 = +, X_2, X3 = -] \\
&= \alpha P[X_1 = +]P[X_2|X_1 = +]P[X_3 = -|X_2] \\
&= \alpha \begin{bmatrix} 2/3 \\ 1/3 \end{bmatrix} \begin{bmatrix} 1/3 \\ 3/4 \end{bmatrix} \\
&= \alpha \begin{bmatrix} 2/9 \\ 3/12 \end{bmatrix} \qquad \alpha = 108/51 \\
&= \begin{bmatrix} 24/51 \\ 27/51 \end{bmatrix}
\end{aligned}$$

3. (25) Given the following Bayesian network what is P(W,F)?

$$P(w+, F+) = \sum_{H,S} P(H) \cdot P(S|H) \cdot P(F+|H) \cdot P(\overset{+}{w}|S)$$

$$= \underbrace{\sum_{H} P(H)}_{f_4(H)} \cdot \underbrace{P(F+|H)}_{f_5(H)} \cdot \underbrace{\sum_{S} \cdot \underbrace{P(S|H)}_{f_1(S,H)} \cdot \underbrace{P(w+|S)}_{f_2(S)}}_{f_3(H)}$$

① $f_1(S,H) =$

S \ H	+	−
+	1/10	1/2
−	9/10	1/2

② $f_2(S)$

S	
+	1/2
−	1

③ $$f_3(H) = \frac{1}{2} \cdot \begin{bmatrix} 1/10 \\ 1/2 \end{bmatrix} + 1 \begin{bmatrix} 9/10 \\ 1/2 \end{bmatrix} = \begin{bmatrix} 19/20 \\ 3/4 \end{bmatrix}$$

④ $$f_4(H) = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$$

⑤ $$f_5(H) = \begin{bmatrix} 0 \\ +1/2 \end{bmatrix}$$

⑥ $$P(w+, F+) = \cancel{19/20 \cdot 1/2 \cdot 0} + 3/4 \cdot 1/2 \cdot 1/2$$

$$= 3/16$$