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When Mathematical Modeling Meets Cycling Race — Stamina Allocation and Power Output Model

Summary

In cycling race, athletes' physiques and abilities vary from person to person. Hence it has been a challenge to develop individual strategies and physical distribution to win the final race since ancient times. Nevertheless, with the development of information technology, this problem has been solved by mathematical modeling techniques which make full use of race conditions and rider information.

First of all, we believe that it is difficult for an athlete to control his own output power, while controlling the speed can be achieved based on the configuration of a tachometer. Thus, the main body of the modeling at the beginning of this paper depends on the main line of velocity v. The actual power curve is subsequently exhibited using the formula power $P =$ impulse $F \times$ velocity V to obtain the actual power curve.

In the first part of the paper, we model the optimal stamina allocation based on the finish time in two parts. In one part, based on the race course as a whole, a comprehensive model including energy, speed, and wind resistance is established and solved by using the method of generalized polar values. In the other, starting from the slope and type of the course, a model based on physical analysis is established and solved using Newtonian mechanics. As a result, the coping strategies under different rolling road conditions are obtained. Meanwhile, the conclusion is drawn that 12 m/s is the ideal motion speed when the athlete is driving with 350 watts of power .

In the second part of the paper, we take the characteristics of each type of rider into account and define the power curves (endurance curves) for time trial specialists and sprinters by gender. Then we use this to combine model 1 and model 2 (for the men's category) to give the actual power output curves for the individual time trial at the Tokyo 2021 Olympic Games, the individual time trial at the 2021 World Championships in Belgium, and the individual time trial on a self-built track, respectively.

In the third part of the paper, we perform a sensitivity analysis in terms of weather conditions-wind and wind direction, and accuracy of strategy execution. For wind and wind direction, we conclude that the influence of weather is greater at lower altitudes and much less at higher altitudes and in mountainous areas of the race. For the accuracy of strategy execution, we take the possible deviations of the racers to the strategy execution into consideration and improved the original conclusion: the key nodes were chosen close to the landmark stages to ensure the executability. The improved conclusions can be found in Section 7.

Finally, we extended the model to explore how it can be applied to the team model, giving specific ideas and writing a 2-page guide to team leaders for teams to better apply our strategy.

Keywords: Optimization model, Generalized functional analysis, Variational method, Newtonian mechanics, Power curve, Strength allocation strategy

Contents

1 Introduction

1.1 Problem Background

The sports competition such as Olympic Game could be an exploration of human limits. Every single breakthrough in the competition could be an exploration in human cognition.

Meanwhile, the athletes could also convey spiritual inspiration to the human being through the sports competition: with the development of automated machine, the human being seems to be neither strong enough nor efficient enough, but the pursuit of perseverance could inspire the audience to pursue the excellence in more other fields.

The development of information technology has promoted the informatization of sports, making the current sports competition a multi-disciplinary and comprehensive interdisciplinary subject. The informatization of sports refers to the application of science and technology to affect the training effects and the sports performance. Through detailed information and scientific evaluation method, the athletes could enhance training efficiency, improve the scores and prevent them from the occurrence of excessive fatigue or accelerate.

In the bicycle road races, the application of information technology would probably increase the speed. Through analyzing the context of the competition and the information of the rider, the mathematical model could be established to determine the correlation of the riders position on the course and the force the rider applies. The optimal competition strategy could be provided accordingly and help the athletes to achieve better scores.

Figure 1: Bicycle Road Races

1.2 Overview of Our Work

For this problem, our idea is to divide the physical energy allocation model (optimal strategy) into two parts, one part is based on the course consideration, dividing the course into three processes and solving them using generalized function analysis. The other part is based on the road and slope, and is solved mainly using dynamical force analysis. Then according to the model: we give different scenarios and strategies for specific scenarios based on the characteristics of different types of athletes (endurance curves). In the second half of the paper, we do a sensitivity analysis of the wind for the model and conclude that the effect of wind variation is greater when driving on flat roads and less in mountainous sections. And considering that: riders cannot precisely follow the mathematical model to implement the strategy, we fuzzed the results from the model and used landmark road sections as the judgment point for measurement. Finally, we discussed and analyzed the extension of the model to the fleet, and wrote a two-page rider's guide for reference.

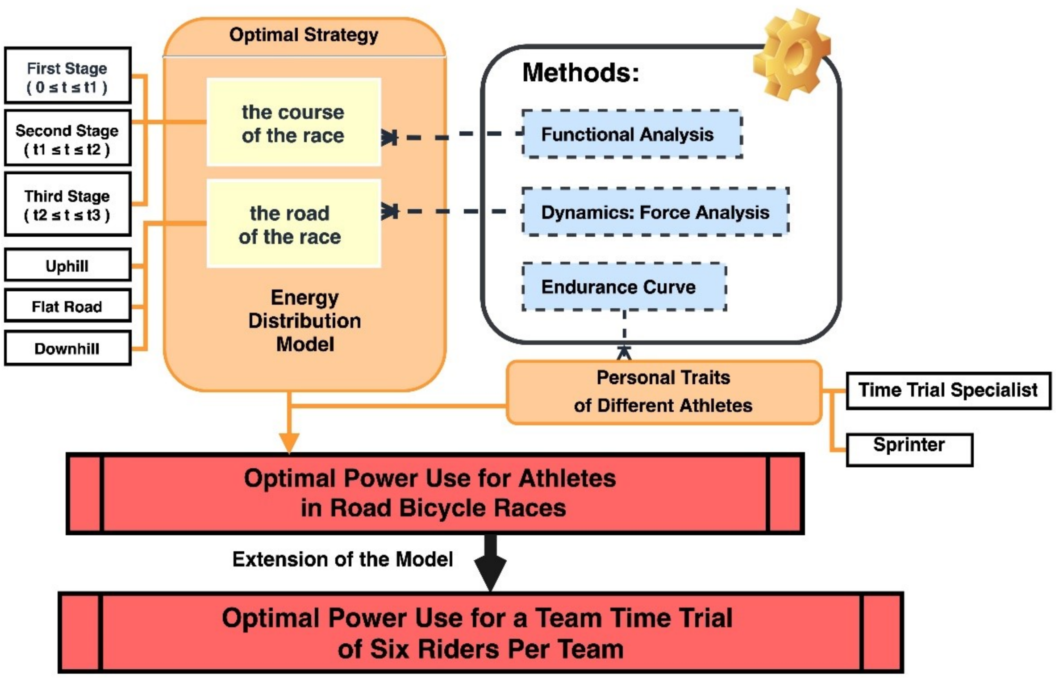


Figure 2: Mind Mapping

2 General Assumptions and Justifications

2.1 Assumptions

- In order to simplify the model and normalize the influence of mass, for model 1: we model the unit mass of the athletes.
- In model 1, to simplify the model, we assume that the air resistance is proportional to the speed.
- In the sensitivity analysis considering wind resistance, we assume that the wind force is always constant.

2.2 Variable Description

The primary notations used in this paper are listed in Table 1.

Table 1: Notations

Symbol	Definition
F	Impulse
$F_{\max}$	Rider's maximum impulse
P	Power
E	Rider energy
E_0	Rider's initial stored energy
σ	Oxygen absorption metabolism recovery energy
γ	Wind resistance coefficient
v	Speed
s, x	Displacement

3 The Models

3.1 Model 1: Bicycle Racer Stamina Distribution Model

3.1.1 Preparing Knowledge and Model Analysis

For bicycle riders: In the course of a cycling race, bicycle riders often have to overcome internal and external resistance to ensure that they can accelerate or reach a certain speed, and this requires some kind of impulse and momentum. There are two sources of this power and impulse:

1. Through the respiratory and circulatory system: through the metabolic action of oxygen to produce energy which is equivalent to the amount of oxygen absorbed.
2. The racer himself: the energy that racers previously stored to finish the race.

To begin with, we could simplify the model: it should be reasonable to assume that the data of each racer remains constant during the race. Secondly, we need to solve the problem of how to reasonably distribute the energy to different stages of the race.

As for bicycle road races: we can roughly divide the race course into three main stages: uphill, flat road, and downhill. Uphill is the time when gravity and resistance are high, and acceleration and climbing in this zone require more energy and cost more resistance. On flat roads, the resistance is mainly from the road itself and the rolling friction of the wheels, and the resistance can be considered moderate. On the downhill, the resistance is less and would naturally accelerate, and the runner requires less energy and power in this stage than in the other two.

3.1.2 Modeling of Overall Race Physical Force Distribution

Firstly, we could analyze the problem without considering the road influence factors of uphill, flat road and downhill temporarily. Instead, we could focus on the overall physical strength of the racers as a whole. Based on the analysis, it is easy to conclude that the cyclist's strength distribution model requires the identification of two key points. The relationship between impulse or power and the racer's speed; The ability of the above two capabilities to supply the impulse and power to do work.

First, let us assume that:

- The maximum impulse force of a bicycle racer is $F_{\max}$ (the maximum power varies from athlete to athlete);
- The bicycle racer is subject to resistance f (related to speed v);
- The energy supplied per unit time by the metabolic action of oxygen σ;
- The initial value of energy stored in each racer's body for the race E_0 (also related to the athletes themselves, different athletes should have different reserves of their own physical strength).

According to Furusawa, Hill, and Parkinson in a 1927 work on kinematics[1]:

Resistance as an instantaneous quantity of change should be independent of the history of the motion and can therefore be expressed as a function of the instantaneous speed u. Based on physics: when the velocity tends to be 0, the drag should also close to 0. For this reason they chose to approximate the drag by a polynomial on the instantaneous velocity v. That is, $f = a_1 v + a_2 v^2 + \cdots$, where $a_1, a_2, \cdots$ are constants.

In professional road cycling races, professional cyclists need to maintain a high rate of motion for a long time, and thus they often have to face great wind resistance during the race. Most of the energy that the drag exerts on their physical strength comes from air resistance. Thus we can reasonably assume that these drag forces should be linear for velocity v. The quadratic term in the drag model about velocity v mainly comes from road friction, and this part is smaller for high speed air resistance. We assume that the air resistance is proportional to the velocity: the scale factor is $\frac{1}{\gamma}$.

Let the time for a cyclist to run the whole race course D with speed $v(t)$ be T, then we have

$$D[v(t)] = \int_0^T v(t)dt. \tag{1}$$

According to Newton's Second Law, the impulse of the athlete is denoted as $F(t)$. It is not difficult to obtain the acceleration,

$$\begin{cases} \dfrac{dv}{dt} = F(t) - \dfrac{v}{\gamma}, \\ v(0) = 0, \\ 0 \le F(t) \le F_{\max}. \end{cases} \tag{2}$$

The energy stored in the athlete's body is $E(t)$, and its rate of change is the difference between the energy provided per unit of time and the value of energy fv consumed by the racer, which could satisfies the conditions of the initial value and the actual situation. Namely,

$$\begin{cases} \frac{dE}{dt} = \sigma - Fv, \\ E(0) = E_0, \\ E(t) \geq 0. \end{cases} \tag{3}$$

This constitutes a generalized extreme value problem under conditions (2) and (3) with $D[v(t)]$ as the objective function, where F, γ, σ, E_0 are considered as a known parameter.

Combining (2) and (3), and using the linear differential equation solution formula gives

$$v(t) = \sqrt{2}e^{-t/\gamma}\Big[\int_0^t (\sigma - \frac{dE}{dt})e^{2s/r}ds\Big]^{1/2}. \tag{4}$$

Plugging (4) into equation (1), the generalized expression is obtained as

$$D[E(t)] = \sqrt{2}\int_0^T e^{-t/\gamma}\Big[\int_0^t (\sigma - \frac{dE}{dt})e^{2s/r}ds\Big]^{1/2}dt. \tag{5}$$

3.1.3 Model Solving and Classification Discussion

Based on the above discussion, we could conclude that it is only necessary to find a suitable $V(t)$ in the above given initial conditions to ensure D can be maximized at a fixed time of T. Then the obtained function $v(t)$ is the optimal velocity profile of the racer, and bringing him into (3) also gives the consumption and distribution profile of energy $E(t)$. However, for this model the generalized function model is more complex and it is difficult to find the analytical solution directly. It is also not an Euler-Lagrange model, so it is also more difficult to make an effective simulation by numerical solution. Consequently, we also need to make a certain degree of simplification of the model, and here we classify the race into two cases for discussion.

Case 1: The course is short and can be ridden with the maximum possible impulse throughout the race:

Putting $F(t) = F_{\max}$ into (2) and combining the initial conditions, we can obtain the solution for speed v as

$$v(t) = F_{\max}\gamma(1 - e^{-t/\gamma}). \tag{6}$$

We then bring it into (3) to obtain

$$\frac{dE}{dt} = \sigma - F_{\max}^2\gamma(1 - e^{-t/\gamma}), \tag{7}$$

$$E(t) = E_0 - (F_{\max}^2\gamma - \sigma) + F_{\max}^2\gamma(1 - e^{-t/\gamma}). \tag{8}$$

In the beginning of the acceleration, due to the speed v is very small, the total power consumption is still relatively small, absorbing a portion of oxygen to replenish $E(t)$, with the speed V rapidly become larger, the racer's energy is gradually consumed, when $t = t_c, E(t) = 0$.

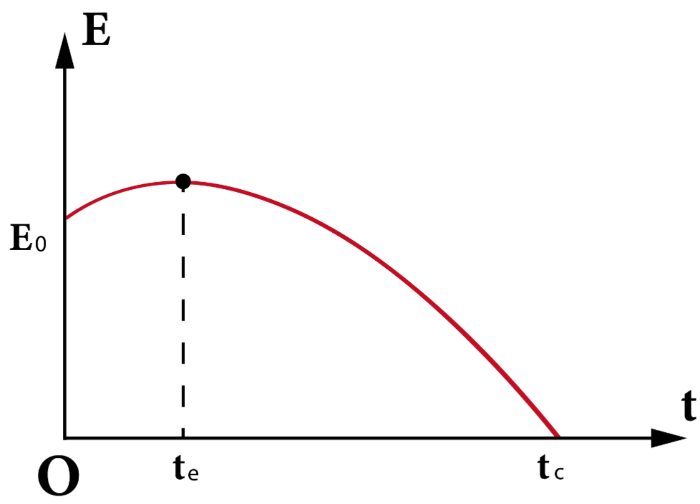


Figure 3: Rider Energy Loss Diagram

Further, consider that in the process of extreme sprinting, the racer cannot continuously exert the maximum impulse after reaching a certain high speed due to the physiological condition, and there must be a decay. We set the decay coefficient as $-\frac{1}{k}(k > 0)$,i.e., $\frac{dF}{dt} = -\frac{1}{k}F_{\max}$. Then, through the linear differential equation formula, it could be turned out that $F(t) = F_{\max}e^{-t/k}$. Bringing in (2), we get the system of equations,

$$\begin{cases} \frac{dv}{dt} + \frac{v}{\gamma} = F_{\max}e^{-t/k}, \quad 0 \leq t \leq T, \\ v(0) = 0. \end{cases} \tag{9}$$

Solving the system of equations yields,

$$v(t) = \frac{\gamma k F_{\max}}{k-\gamma}(e^{-t/k} - e^{-t/\gamma}), \quad k > \gamma, \tag{10}$$

$$s(t) = \frac{\gamma k F_{\max}}{k-\gamma}(k - \gamma - ke^{-t/k} + \gamma e^{-t/\gamma}). \tag{11}$$

At t=t*, the racer's speed reaches its maximum.

Case 2: When the rider's physical strength is not able to sprint the whole course at a high speed.

For this case, we divide the rider's race process into three stages:

- The first stage: $0 \leq\leq t_1$, first accelerate with the maximum impulse to accelerate to a higher speed position.
- The second stage: $t_1 \leq t \leq t_2$, maintain a higher speed cruise to a uniform speed.
- The third stage: $t_2 \leq t \leq T$, total residual energy could be used up in the final stage, the limit of the sprint.

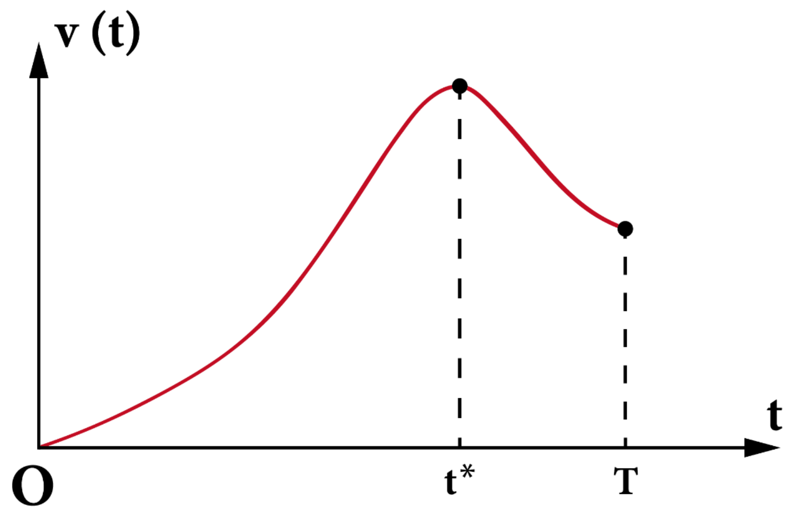


Figure 4: Rider Rate Curve Variation Diagram

For the first stage: we assume that the rider accelerates with the maximum impulse $F_{\max}$ at the beginning in order to reach the uniform speed process of cruising as soon as possible. Then the velocity $v_1(t)$ during this time ($0 \le t \le t_1$, t_1 is determined after) can be obtained from the formula (6),

$$v_1(t) = F_{\max}\gamma(1 - e^{-t/\gamma}). \tag{12}$$

For the third stage ($t_2 \le t \le T$, t_2 determined later): bringing $E(t) = 0$ into (3), equation (2) becomes

$$\frac{1}{2}\frac{dv_3^2}{dt} + \frac{v_3}{\gamma} = \sigma. \tag{13}$$

For (13), using the condition that the velocity is continuous at t_2: $v_3(t_2) = v_2$, we obtain its solution as

$$v_3(t) = \left[(v_3^2 - \sigma t)e^{-2(t-t_2)/\gamma} + \sigma\gamma\right]^{1/2}. \tag{14}$$

For the second stage: $t_1 \le t \le t_2$, we will determine this by maximizing the target generalized polar value $D[v(t)]$. Here,

$$D[v(t)] = \int_0^{t_1} v_1(t)dt + v_2(t_2 - t_1) + \int_{t_2}^{T} v_3(t)dt. \tag{15}$$

To obtain the constraint that $E(t_2) = 0$, using (2) and (3), we get

$$E(t) = E_0 + \sigma t - \frac{1}{2}v^2(t) - \frac{1}{\gamma}\int_0^t v^2(s)ds. \tag{16}$$

Let $t = t_2$ and bring (6) into the above equation, then we have

$$E(t_2) = E_0 + \sigma t - \frac{v_2^2}{2} - \int_0^{t_1} F_{\max}^2\gamma(1 - e^{-t/\gamma})^2 dt - \frac{v_2^2(t_2 - t_1)}{\gamma}. \tag{17}$$

The problem thus boils down to finding v_2, t_2 such that $D[v(t)]$ expressed in (15) is maximized under the condition $E(t_2) = 0$. In the following we solve this problem using the variational solution more commonly used in finding the extrema of generalized functional analysis.

First, we first construct the Hamiltonian function using a constant to be determined λ:

$$I\big(v(t), t_2\big) = D[v(t)] + \frac{\lambda}{2} E(t_2). \tag{18}$$

By omitting the terms on the right-hand side that are unrelated to v_2, t_2, we obtain

$$I\big(v(t), t_2\big) = \int_{t_2}^{T} v_3(t)dt + \frac{\lambda\sigma}{2} t_2 - \frac{\lambda}{4} v_2^2 + (v_2 - \frac{\lambda v_2^2}{2\gamma})(t_2 - t_1). \tag{19}$$

Then, by finding the derivative of the above equation, we obtain the necessary conditions for v_2, t_2 to be the optimal solution as

$$\begin{cases} v_2 = \dfrac{\gamma}{\lambda}, \\ 2\int_{t_2}^{T} \left[(v_2^2 - \sigma\gamma)e^{-2(t-t_2)/\gamma} + \sigma\gamma\right]^{1/2} e^{-2(t-t_2)/\gamma} dt = \lambda. \end{cases} \tag{20}$$

Determination of t_2, t_1, λ.

Using the continuity of $v(t) = t_1$, we obtain from equations (6) and (18) that

$$\lambda F_{\max}(1 - e^{-t_1/\gamma}) = 1. \tag{21}$$

Bringing (20) into (17) and making it 0,

$$E_0 + \sigma t_2 - \frac{\gamma^2}{2\lambda^2} - \int_0^{t_1} F_{\max}^2 \gamma (1 - e^{-t/\gamma})^2 dt - \frac{\gamma}{\lambda^2}(t_2 - t_1) = 0. \tag{22}$$

Also from (20) we obtain

$$2\left[(\gamma^2 - \lambda^2\sigma\gamma)e^{-2(\gamma - t_2)/\gamma} + \lambda^2\sigma\gamma\right]^{1/2} - 2\gamma = \lambda^2\sigma - \gamma. \tag{23}$$

The above parameters can be obtained by combining equations (21) - (23).

3.2 Model 2: The Model of Physical Strength Distribution Based on Road Conditions and Wind Resistance

In the previous section of the model, we have considered how to allocate physical strength over the course as a whole. We did not give riders guidance and countermeasures for specific intersection stages, such as uphill, downhill, and flat roads. The model in this section is designed to address this issue. The factors of road impact on riders could be divided into slope and wind resistance (in the first model wind resistance is simplified to linear, here we will reconsider) that is different for uphill, downhill, and flat roads.

During the bike ride, the overall composition of the rider and the bike is mainly affected by gravity, ground support, air and ground resistance. However, the forward motion of a bicycle is not same as ordinary rigid body motion when cycling due to the unique structure between the human and the bicycle. The hollowing out of the human-vehicle structure changes the maximum head-on area and wind resistance coefficient. At the same time, the wheel rotation, lower limb pedaling wheel rotation and lower limb pedaling will produce the Magnus Effect. In addition, under the condition of outdoor environment during the road cycling condition, wind variations and road surface undulations can complicate the force structure and processes, making the analysis of the processes much more complex than in track cycling.

3.2.1 Mechanical Analysis

In a two-dimensional coordinate system constructed in the direction of motion perpendicular to the ground, the overall force on the cycling-vehicle is mainly composed of gravity Fg, ground support Fs, human-vehicle air resistance Fd, wheel rotation air resistance Fw and tire-ground friction Ff. The pedaling force during the riding process is the guarantee of the forward movement of the human and the vehicle.

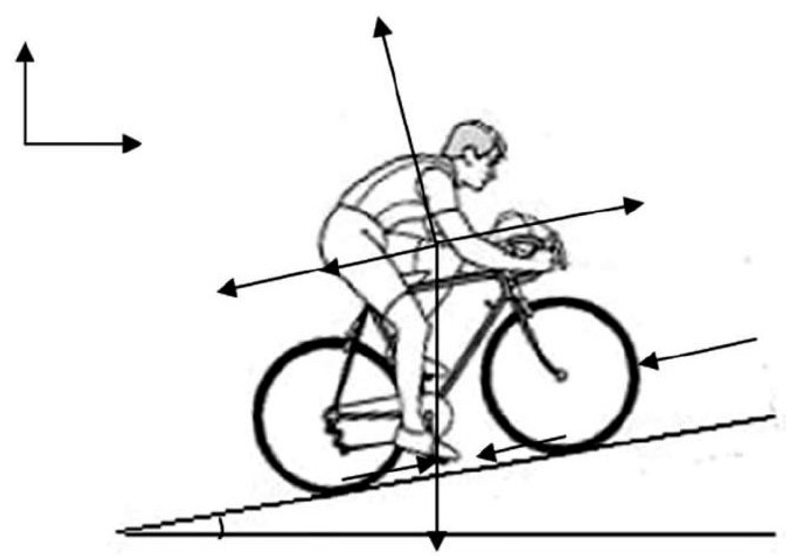


Figure 5: The Sketch Map of the Force Analysis of Cycling

Gravity component: When riding on a non-horizontal road, the gravitational force F_g would be decomposed into the direction of motion of the component force F_g' ; when riding uphill, F_g' and the opposite direction of motion, is the resistance to riding; when riding downhill, F_g' and the same direction of motion, is the riding assistance. The expression is

$$F_g' = F_g \sin\theta = Mg \sin\theta, \tag{24}$$

where M is the total mass of the person and the car and θ is the angle between the ground and the horizontal plane, which takes positive values when riding uphill, negative values when riding downhill, and 0 when riding on a horizontal road.

Ground friction: When riding on a horizontal road, the total mass of people and the car body in the Y direction form a positive pressure equal to gravity F_g; while riding on a non-horizontal road, the force of gravity in the direction perpendicular to the road would form a positive pressure on the ground F_g'. The pressure squeeze leads to the tire elastic deformation, coupled with the roughness of the tire and the road. Consequently, there exists a relative motion within the body of the car and the road, forming the friction between the wheel and the road. The expression of the friction force between the car body and the ground is

$$F_f = C_r mg\cos\theta, \tag{25}$$

where C_r is the rolling friction coefficient between the wheel tire and the ground. The rolling friction coefficient depends on the material of the road and the tires.

Human-vehicle head-on air resistance: When people-bike riding forward, the front of the air is compressed to produce pressure. The two sides of the surface and the friction of the air generate friction, these effects together form the opposite direction of movement of air resistance. The part of air resistance is located at the center of windward area, and its expression is

$$F_w = \frac{1}{2}C_d A\rho V^2, \tag{26}$$

where C_d is the air resistance coefficient, A is the maximum cross-sectional area of the windward side, ρ is the air density at the altitude and V is the relative velocity of the vehicle body relative to the air flow.

Air resistance of wheel motion: Rotating objects in viscous fluids produce asymmetric fluid dynamic effects. Greenwell (1995) showed that the magnitude of rotational drag depends mainly on the size of the wheel and the shape of the hub, and does not change significantly with the speed of the wheel. At the same time, the air resistance of the rear wheel is reduced by 25% due to the action of the human body and the bicycle's vertical beam. In general, the radii of the front and rear wheels and the spokes of the wheels are the same for road bicycles, and the total air resistance of the front and rear wheels when they rotate is calculated by the formula:

$$F_d = F_{id} + F_{rd} = \frac{1}{2}C_w\rho V^2\pi r^2 + \frac{3}{4}\times\frac{1}{2}C_w\rho V^2\pi r^2 = \frac{7}{8}C_w\rho V^2\pi r^2, \tag{27}$$

where C_w is the air resistance coefficient of the bicycle wheel, r is the wheel radius, ρ is the air density at the altitude and V is the relative velocity of the wheel with respect to the air flow.

3.2.2 Mathematical Modeling

By Newton's second law, the basic kinetic equation of the road bicycle riding process can be established, and the specific expression form is

$$F - F_g' - F_f - F_w - F_d = Ma. \tag{28}$$

Here the impulse F is expressed by the ratio of instantaneous power and speed,

$$\frac{P}{d'(t)} - \frac{1}{2}AC_d\rho d'(t)^2 - MC_f g\cos\theta - Mg\sin\theta - \frac{7}{8}C_w\rho d'(t)^2\pi r^2 = (M + \frac{I_f + I_r}{r^2})d''(t). \tag{29}$$

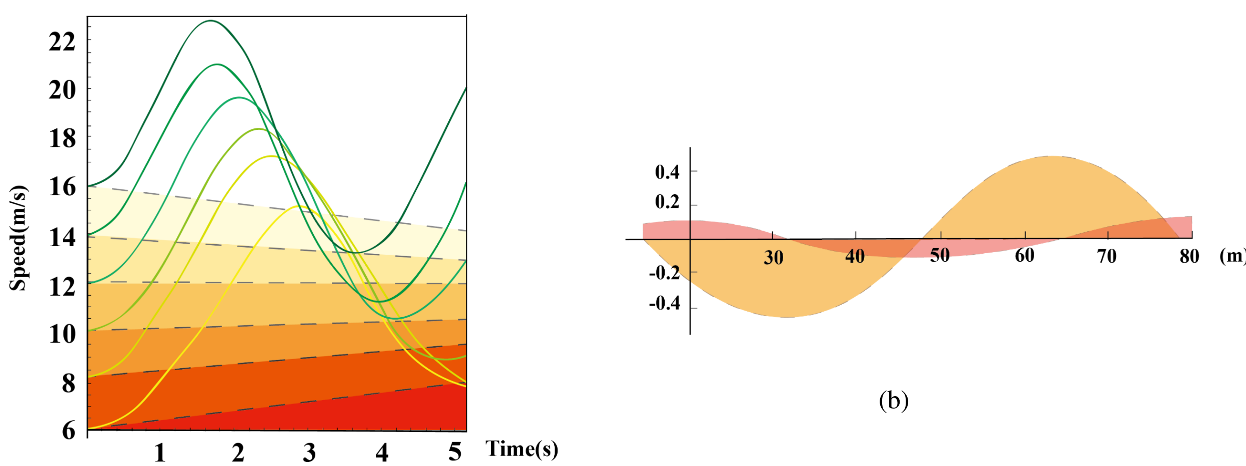
3.2.3 Simulation

The parameters in this model simulation are selected from various literature, as follows:

Table 2: The List of the Initial Parameter of Model

Indicator	Parameter	Value
Environmental indicators	temperature	$20°C$
	atmosphere pressure	1 atm
	air density	$1.266\ kg/m^2$
Human body indicators	body weight	$80\ kg$
	body height	$180\ cm$
	coefficient of air resistance	0.5
	cross-sectional area of windward side	$0.5\ m^2$
Bicycle indicators	wheel radius	$0.35\ m$
	moment of inertia of the wheel	$0.08\ kgm^2$
	coefficient of air resistance of wheel	0.0397
	rolling friction coefficient	0.004

The head-on wind speed is set to $3\ m/s$. The study shows that under normal conditions, the average power output of the athlete is about $350\ W$ when riding on a relatively gentle road surface, and the dashed line in the figure below shows the speed change after the rider enters the horizontal road surface at $6\ m/s$, $8\ m/s$, $10\ m/s$, $12\ m/s$, $14\ m/s$ and $16\ m/s$ with the rated power of $350\ W$. It can be seen that the speed change after the rider enters the horizontal road surface at $6\ m/s$, $8\ m/s$, $10\ m/s$, $12\ m/s$, $14\ m/s$ and $16\ m/s$.



(a)

(b)

Figure 6: The Speed Variation of Riding on Horizontal and Rolling Roads at Rated Power

It can be seen that when riding on the horizontal road, the speed gradually increases when the initial speed is $6\ m/s$, $8\ m/s$ and $10\ m/s$, which is the accelerated riding stage; when the initial speed is $14\ m/s$ and $16\ m/s$, the speed gradually decreases, which is the decelerated riding stage;

when the initial speed is 12 m/s, the speed basically remains constant, and the forward power transmitted by the pedal force is basically equal to the air resistance. Therefore, a speed of 12 m/s is the most effective way to ride through a relatively gentle road surface, and this conclusion is consistent with the average speed of 11.0 - 13.3 m/s of the best athletes in major international road cycling events.

The solid line in 6a is the speed variation curve when riding through the undulating road with the above conditions. 6b shows the continuous undulating road surface and angle variation diagram for riding through, the route equation modeled as $y = 5\sin(x/10)$, $x \in [5\pi, 25\pi]$, the maximum inclination of this route is 5%. The image shows that when the initial speed is 8 m/s, the initial speed is basically the same after each undulating road cycle.The time taken for each cycle was about 5.2 s.

Therefore, in the process of continuous undulating road riding, the corresponding riding speed should be chosen according to the specific length of the road section. When the distance is short, the riding speed can be increased to pass the undulating road section quickly; however, when the distance is longer, the speed can be reduced accordingly to pass the undulating road relatively smoothly.

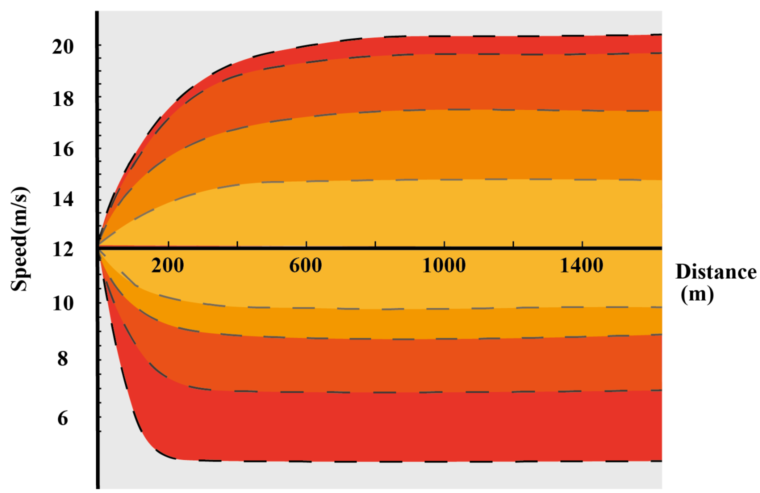


Figure 7: Speed Change Graph for Uphill and Downhill Riding at Rated Power

The velocity profiles of 350 W riding power at an initial speed of 12 m/s through uphill sections with inclination of 2%, 4%, 6%, and 8%, and downhill sections with -2%, -4%, -6%, and -8%, respectively. When riding uphill, the component force of gravity in the direction of motion acts as a resistance, while when riding downhill, it becomes a forward power, and the air resistance is positively related to the speed of riding. Therefore, the speed decreases at the beginning of uphill, but the wind resistance decreases as the speed decreases, the speed increases at the beginning of downhill, and the acceleration of speed will lead to the increase of wind resistance.

From the relationship between the change of inclination and the increase and decrease of speed, the increase of speed decreases with the increase of inclination angle in downhill, and the decrease of speed increases with the increase of inclination angle in uphill. Therefore, the effect on speed is not obvious for both smaller uphill and larger downhill, while the cycling force can be increased to obtain stage advantage for larger uphill and smaller downhill.

4 Practical Applications

4.1 Power Curve

We define two different types of road riders: Time trial Specialist and Sprinter. As the core of team success, Sprinter's performance is often eye-catching. These guys have big, powerful legs and can kick it faster than any in the peloton. They are extremely explosive, producing well above average power for a short period of time, but decay quickly, which is why they tend to start firing at the end of a race. The power curve of this type of player is defined as follows,

$$p_{male}(x) = \frac{3200}{\pi}\tan^{-1}(-0.001x) + 1800,$$
$$p_{female}(x) = \frac{3200}{\pi}\tan^{-1}(-0.001x) + 1700.$$

Time Trial Specialist is the type that specializes in the individual time trial events. These riders are superb going solo, in a skinsuit, with an aero helmet. Built for speed over a medium range with sufficient power to succeed on a flat and through climbs. They have a good grasp of their own strength and can follow highly detailed plans. While they are not as explosive as sprinters, their power output is smoother and more durable.

$$p_{male}(x) = -\frac{1300}{1+e^{-0.003x}} + 1800, \quad p_{female}(x) = -\frac{1250}{1+e^{-0.002x}} + 1700.$$

Considering the gender impact, female professional cyclists are generally inferior to their male counterparts. However, based on the sampling of the world's top sprinters, the explosive power and endurance of female sprinters are lower than that of male competitors of the same level. However, female Time Trial specialists are more durable in a short period of Time, although the peak value of female sprinters is lower than that of male competitors.

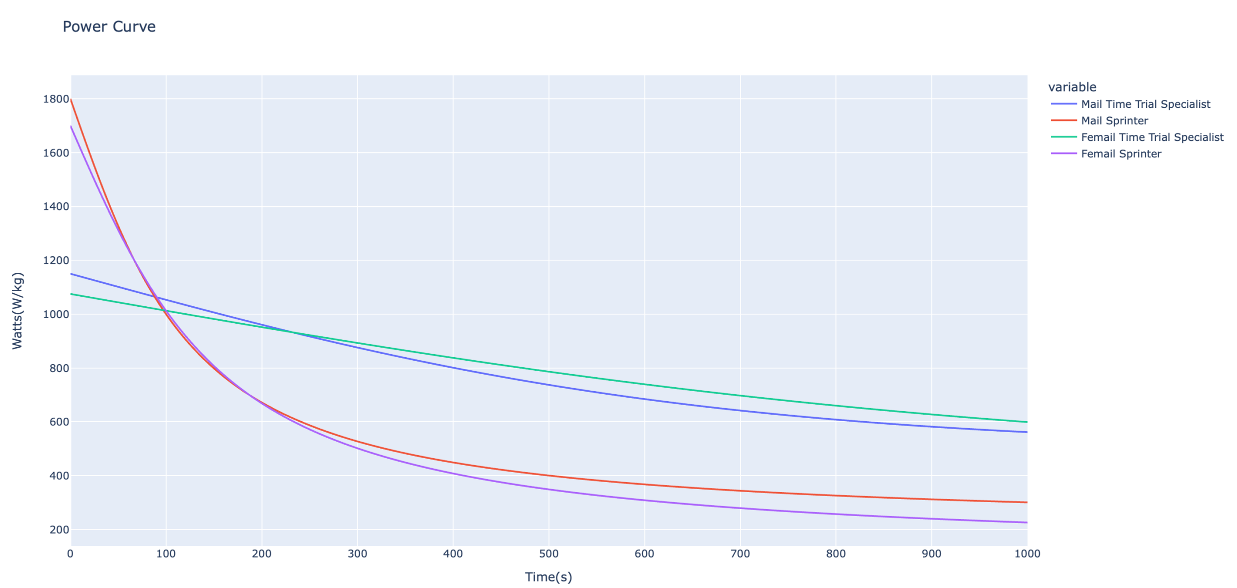
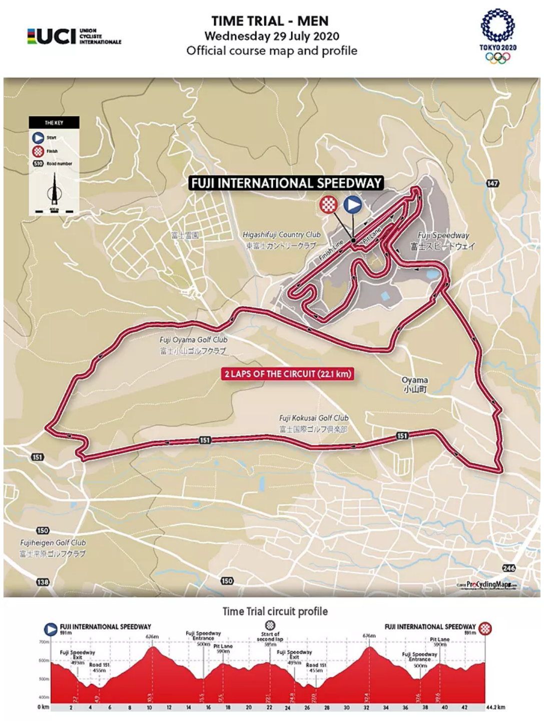


Figure 8: Power Curve

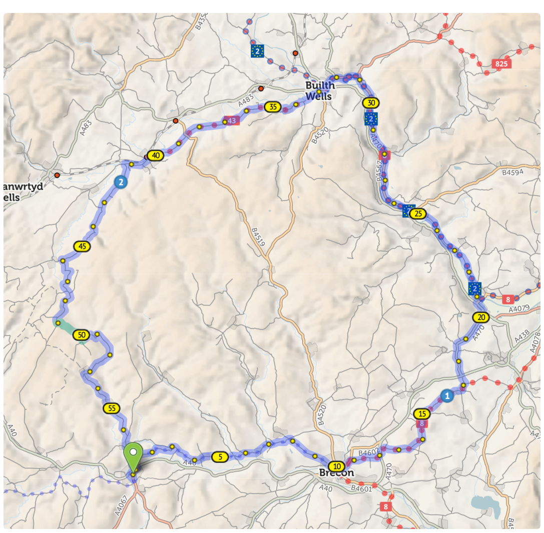
4.2 Race Track Introduction

2021 Japan Tokyo Olympic Games Time Trial: The women and men will race on the same 22.1 km course that starts and finishes at Fuji International Speedway (road race finish), passing through Shibantan, Pit lane and other key courses. The women will complete one lap and the men will complete two laps (44.2 km in total). It will be a wet and hot course with rolling ups and downs and 846 meters of altitude climbing throughout.

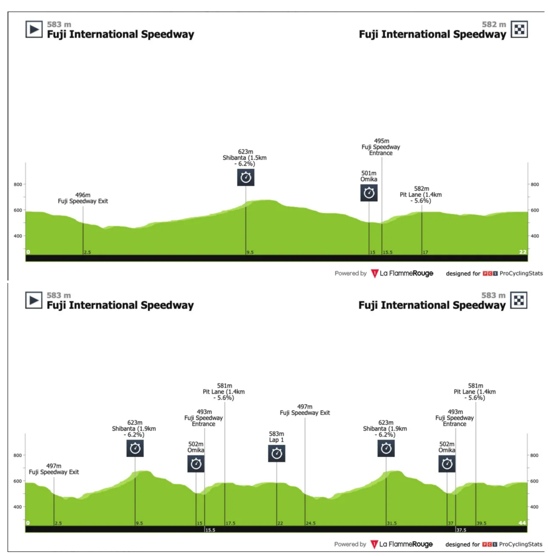


(a) Mt. Fuji Track for the Tokyo Olympics in Japan

(b) UCI World Championship Individual Time Trial in Belgium



(c) Self-built Course Map



(d) Tokyo Olympics ITT (Individual Time Trial) Terrain Map

Figure 9: Three ITT Course Maps

2021 UCI World Championship Time Trial in Belgium: Unlike the complex, winding course with lots of ups and downs and climbs of the big group races, the course of the individual time trial at the World Championships is largely technically uncomplicated and largely flat. The elite men's individual time trial course is 43.3 kilometers with only 78 meters of elevation climbing, while the women's course will have fewer switchbacks with 30.3 kilometers and 54 meters of elevation climbing.

Self-built course: Mountain individual time trial: Our track started in Brecon, a very charming, very Welsh town with its winding streets and slow river crossings. With mountains and moorland, standing stones and castles, lively waterfalls and vibrant communities, the Brecon has masses to offer residents and visitors. It has a long and colourful history and a rich and varied mythology and culture , which is definitely a good route place for cyclist enjoying beautiful scenery.

The course is 61.4 km in length, with 87% of it carefully constructed but about 4% unpaved. Due to the stability of the road conditions, this kind of track is easy to achieve their full potential, and also easy to ensure their own safety.

The track is a complex course with eight steep uphill sections ranging from 101 m to 433 m above sea level, with a maximum gradient of 21%. This continuous up and down hill will undoubtedly take a lot of energy from the athletes, and it will be a great challenge to the overall ability of the athletes.

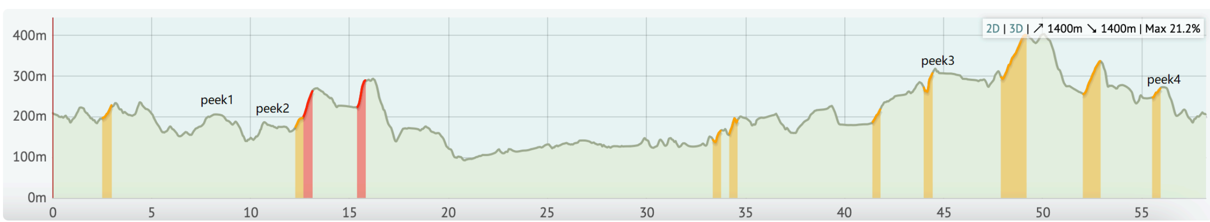


Figure 10: Self-built Course 2D Diagram

Rnk	Rider	Team	UCI	Pnt	Time	Avg
1	ROGLIČ Primož	Slovenia	350	300	0:55:04	48.160
2	DUMOULIN Tom	Netherlands	250	220	1:01	47.287
3	DENNIS Rohan	Australia	200	170	1:04	47.245
4	KÜNG Stefan	Switzerland	150	130	,,	47.245
5	GANNA Filippo	Italy	125	100	1:05	47.231
6	VAN AERT Wout	Belgium	100	90	1:40	46.745
7	ASGREEN Kasper	Denmark	85	80	1:48	46.635
8	URÁN Rigoberto	Colombia	70	70	2:14	46.283
9	EVENEPOEL Remco	Belgium	60	60	2:17	46.242
10	BEVIN Patrick	New Zealand	50	50	2:20	46.202
11	BETTIOL Alberto	Italy	40	40	2:34	46.015
12	THOMAS Geraint	Great Britain	30	30	2:42	45.909
13	HOULE Hugo	Canada	25	20	2:52	45.777
14	DE BOD Stefan	South Africa	20	15	2:53	45.764
15	SCHACHMANN Maximilian	Germany	15	10	3:29	45.295

Figure 11: Actual Results of ITT in Tokyo Olympics

4.3 Optimal Strategy

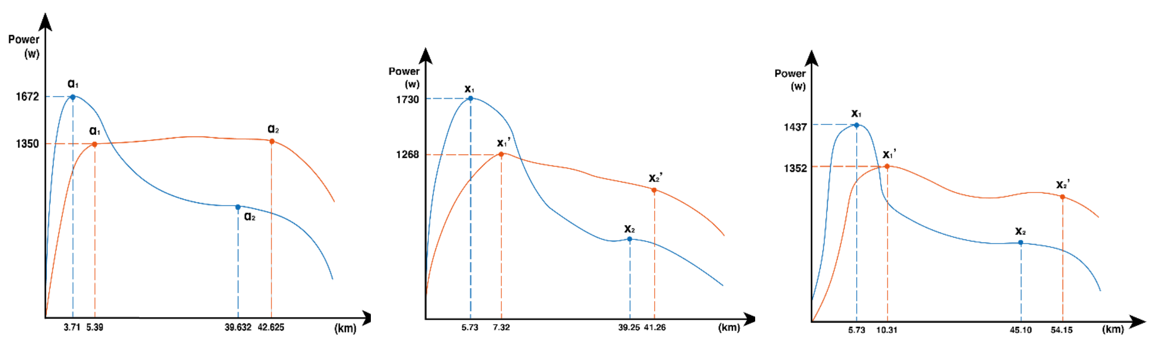
For the sake of discussion, we give strategies mainly for the men's race. First we discuss the Tokyo Olympics.

For the Tokyo Olympics: we give the time trial specialist the maximum tractive force $F_{\text{max}} = 15$ N/kg (for a mass of 1 kg) that can be exerted on undulating roads, the attenuation coefficient of wind resistance is given as $r = 1.42$, and the initial energy $E_0 = 2$ million kJ. For a sprinter: set the maximum traction force $F_{\text{max}} = 20$ N/kg that he can exert on undulating roads, given the same wind resistance, and a lower initial energy of 1.25 million kJ.

Similarly, we are given individual parameter data for the Belgian World Championships (flat roads): time trial specialists: $F_{\text{max}} = 20$ N/kg (flat roads), sprinters: $F_{\text{max}} = 25$ N/kg.

And the personal parameter data of the self-built track (high altitude): time trial expert $F_{\text{max}} = 10$ N/kg, sprinter: $F_{\text{max}} = 13$ N/kg.

The parameters were brought into model 1 case 2 (For case 1, we estimate the limit sprint distance to be 5-10km, and the individual time trial distance is much larger than this, which is obviously not the case) to obtain s_1 and s_2, and normalized to the actual track in proportion to them. The actual output power curve is made using the power $P = Fv$ (F is determined by model 1 and considers the same decay as the power curve) and the results are as follows.



(a) Tokyo (b) Belgium (c) Self-built

Figure 12: Results Graph

Red line represents: power output curve for time trial specialists, while blue represents: power output curve for sprinters.

x_1: point to end the acceleration phase

x_2: point at the end of the cruising full sprint

5 Sensitivity Analysis

5.1 Weather

In both our model 1 and model 2, the effect from air resistance/wind resistance is introduced. At high speeds, the wind tends to act as a convective drag that impedes the rider's forward motion, while downwind actually consumes little of the impact curve for the rider. In the following we will briefly calculate the effect of increasing the wind intensity rate consumption based on model 2.

In model 2: the total drag is mainly from wind resistance, wheel resistance, tire resistance, and gravity resistance (climbing).

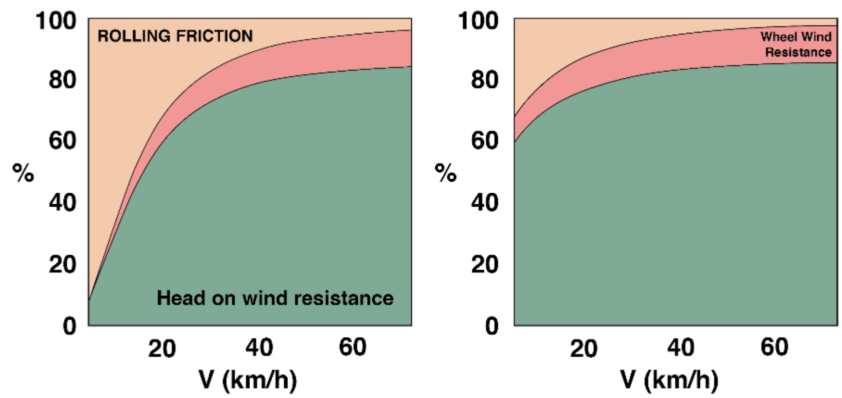


Figure 13: The Effect of Wind on Flat Road Riding (Left: windless; Right: 15 km/h headwind)

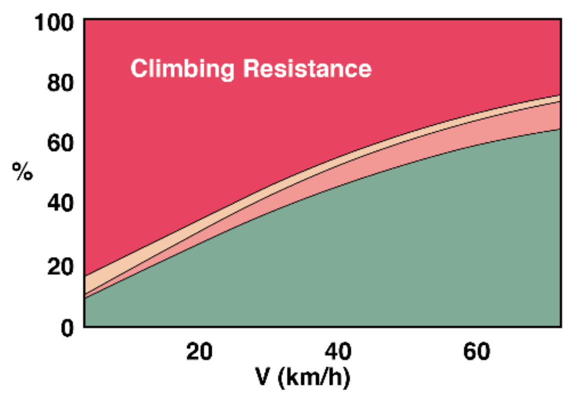


Figure 14: The Effect of Wind on Climbing (0.05 gradient, 15 km/h headwind)

The left figure shows the windless condition, and the right figure shows the 15 km/h headwind. The horizontal axis is the riding speed and the vertical axis is the percentage of total drag from each source of drag. When a 0.05 gradient is added to this (below), the percentage of wind resistance decreases significantly, and the athlete's power consumption mainly comes from climbing. Therefore, the main influence of wind resistance on the model lies mainly in the low altitude race area, while the influence factor on the high altitude area and mountain race is not significant.

5.2 Accuracy

The strategy of following the mathematical model's precisely is obviously unreliable for the rider. We have taken this into account: in fact, when communicating results and strategies to teams, we often use key sections and iconic track building points to facilitate this. The original calculated strategy key points are shifted towards the standard track points, so that the overall plan is accurate, although there is a shortfall in finish time.

All strategies will be presented together in the Conclusion section 7.

6 Model Expansion - Team Model

In the team race, each team only counts for the fourth place finish. This means that for a team of six, two riders are there to block the wind and disrupt the other team's pace. For the sake of space, we will show the main ideas here. We divide the six-man team into two parts: part one: four people, for the players who will be counted in the final score. The second part: two people, one person is the main wind blocker, mainly responsible for blocking the wind for four people, while the other person is the deputy wind blocker, and the first and the main wind blocker together with the wind blocker, the middle and later waiting for the opportunity to attack and disrupt the rhythm and position of other teams. We will build an optimization model on the need for an existing model, with the objective function of the time spent in fourth place. To simplify the model: our last four to cross the line are treated as a whole. For the optimization model, we have to determine two parameters, first the distance point of the main blocker from the team and second: the distance point of the secondary blocker (attacker) from the team.

$$
\text{Objective Function:} \quad \min T(x_3, x_4)
$$

$$
s.t. \begin{cases} \text{Strategy for model 1,about} x_3,\, x_4 \text{ (after changing the wind resistance coefficient),} \\ \text{Strategy for model 2,about} x_3,\, x_4 \text{ (after changing the wind resistance coefficient),} \\ \text{Power curves for different types of personnel.} \end{cases} \tag{30}
$$

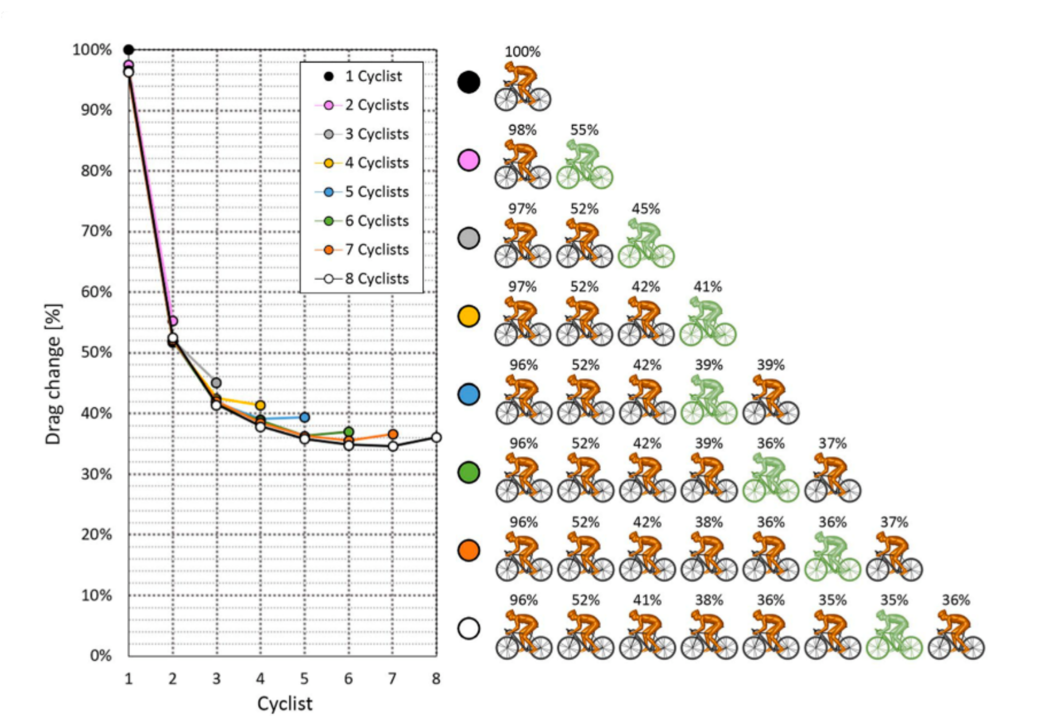


Figure 15: Relationship between Wind Resistance and Team Size

7 Conclusion

Taking the men's group as example.

For the Tokyo Olympics.

Time trial specialists: Accelerate from the start to Fuji Speed Way Exit in the first stage to reach a maximum power of 1350 watts, cruise to Pit Lane in the second lap at average power in the second stage and finally sprint at full power.

Sprinters: Accelerate from the start to the Fuji Speed Way Exit in the first stage to reach a maximum power of 1672 watts, then cruise to the Fuji Speed Way Entrance on the second lap with as much average power as possible in the second stage, and then sprint at full power.

Both: In the Shibanta and Omika areas of the intermediate stage, the long and undulating stages can be passed with appropriate deceleration. In the flat areas such as LAP1 and Pit Lane, you can speed up appropriately.

For the Belgian bid race.

Time Trial Experts: Stage 1: Accelerate from the start to 7.32 km to a maximum power of 1268 watts. Stage 2: Cruise at cruising speed to 42.68 km. Final full sprint.

For sprinters: Stage 1: Accelerate from the start to the 5.73 km mark to reach a maximum power of 1732 watts, Stage 2: Drive at cruising speed to: 39.26 km and finally sprint at full power.

For self-built course: refer to Fig.10.

For time trial specialists: acceleration from the start to peek 2 to reach maximum power of 1352 watts, second stage at cruising speed to peek 4 and final full sprint.

For sprinters: accelerate from the start to peek 1 to reach a maximum power of 1437 watts, travel at cruising speed to peek 2 in the second stage and finish with a full sprint.

Both: Decelerate appropriately on the long course with more undulations in the middle, and decelerate appropriately on the short course with undulations in the middle.

8 Rider's Race Guidance for a Directeur Sportif

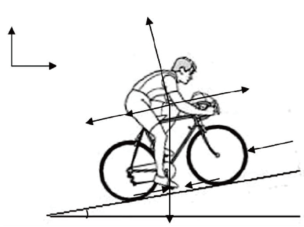
Rider's Race Guidance

for Time Trial Specialist / Sprinter

The road cycling races could be an exciting experience. It could be challenging and thrilling, stimulating your adrenaline epinephrine, and making you feel great. You could also gain precious friendship and receive support from them. Additionally, you would always be infected by the atmosphere of the road cycling races. Many people wanna pursue a better score, and consequently train very hard.

Admittedly, the more experienced you are in the competition, the better understand on your body and therefore the more likely to reach a better score. However, it is also quite essential to learn theoretically to help you make better decision, receive a better score and prevent you from getting injured.

By reading the following instruction, you could have a basic understanding of the road cycling process and a general idea about how to optimal power use in road bicycle races. Since the report is for directeur and the riders, we have tried to explain in a simpler way to make sure everyone without mathematical background could understand. If you are ready, start your reading!



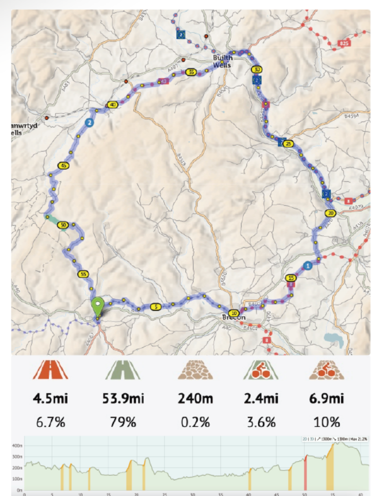
The cycling is a comprehensive process of exerting various forces. The forces mainly contains the gravity, the support forces from the road, and the resistance frim the air and ground. Different from the rigid motion, the unique structure could modify the maximum area facing the wind and factor of wind resistance.

Compared to the cycling race on the indoor track, the outdoor condition in the road cycling races would be much more complex due to the alternation of the wind power and direction, as well as the up and down on the road surface.

Having a basic understanding of the road cycling races, we could move on to the guideline. In this guideline, we would include two kind of athletes. The athletes and the directeur could choose different strategy based on the personal condition and the type of race you would take.

Take the Tokyo Olympic Individual Time Trial of Male as an example:

For **time trial specialist**, it is suggested that the athletes should **accelerate to the maximum speed** at the beginning of the race, till **7.3km** away from the beginning point. Then, you should **keep the uniform motion** until **41.265 km** from the starting point. Then in the final stage, you should **speed up to your speed limit**.



self-designed training course for mountain time trials

For **springter**, the optimal strategy should be speed up the maximum speed after starting the race until **5.73km** away from the starting point. Afterwards, the springter could start keeping the uniform motion until **39.247km**. Then, you should sprint to the finish line.

Additionally, here is some tips for you to reach a better score under real situation that is changing rapidly.

- **Speed up with the wind**
- **Slow down against the wind**
- For mountain time trials:
a course has been designed that can be used for training. Reasonably training could **improve the ability that the athletes judge the length of the course** and could also **simulate complex conditions for athletes to learn how to react**.

Rider's Race Guidance for Time Trial Specialist/Sprinter

A broad summary of the model

The results is based on the model of optimal power distribution in road bicycle races, and the primary model mainly consider two parts: the comprehensive course of the road cycling race and the road included in the road cycling race.

You might be curious about the reason why we choose these two factors: actually, these two factors is derived from the actual condition during the road cycling races.

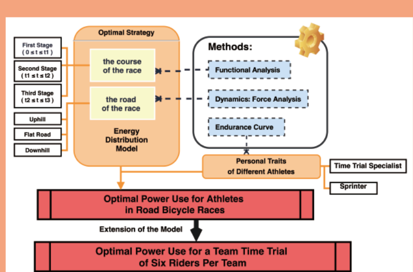
Road cycling is always a long and arduous process. We are all ordinary human being, not machine, so it is impossible for us to keep a rapid speed during the whole process. Sometimes you may feel exhausted or inspired, especially at final stage, and then even break through the personal limitation. However, it would be important for the riders to reasonably allocate when to speed up.

For the road factor, it could be understood through the following case. When riding the bicycle, you might always feel difficult to go through the bumpy, stony road. However, when passing by a smooth road, you might feel it easy. Sometimes you might even need to control your speed to make sure you would not slip down. Consequently, we regard the condition of the road as an important factor. If you live by the seaside, you will always feel that the direction and strength of the sea breeze will greatly affect you go forward. Therefore, the factor of wind direction and the wind power is also taken into consideration in our model which could turn out the aforementioned conclusion.

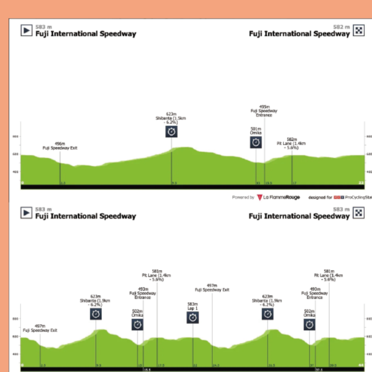
Model is always based on several ideal situation to make sure having focused on an certain problems or factors. In this model, we also need to make some assumption. We model the unit mass of athletes and assume the air resistance is proportional to the speed.

We could also understand that athletes in the intensified races would be extraordinarily nervous. The athletes could not always exactly follow our instruction. However, this factor has also been taken into our consideration. So what you need to do is just follow the instruction and keep training to as faster as possible. Under this condition, we assume that the wind force is constant, focusing on the problem itself without taking too much factors into consideration and complicate the porblem.

Theoretically, this model could analysis is suitable for all kind of different riders. Actually, we have applied our model to several famous races including 2021 Olympic Time Trial course in Tokyo, Japan, 2021 UCI World Championship time trial course in Flanders, Belgium and an own-designed course which has a complicated road situation. If you need to learn more about the model and how the conclusion made, you could refer to our article.



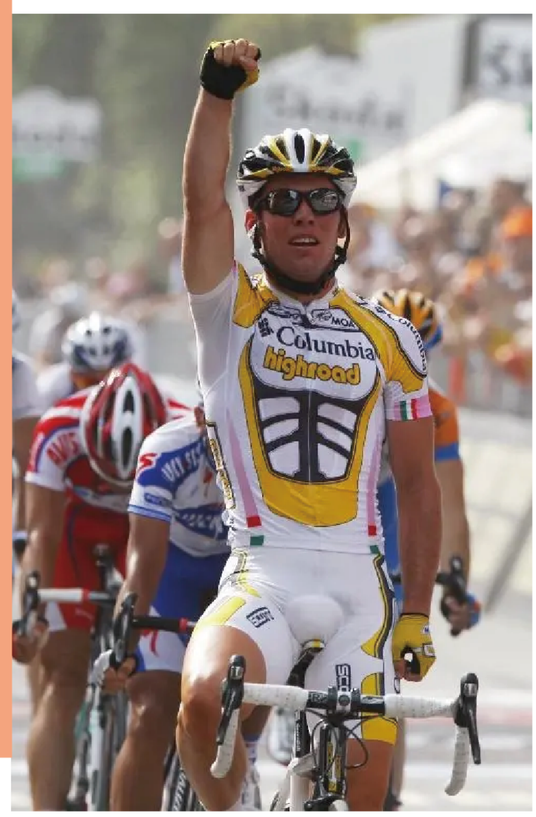
Flow Chart of the Model: an easier way to understand



One of the Applied Case: Fuji International Speedway

TIME TRIAL SPECIALIST:

SPRINGTER:



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