

Conditional Language Modeling with Attention

Chris Dyer



Review: Conditional LMs

A **conditional language model** assigns probabilities to sequences of words, $\mathbf{w} = (w_1, w_2, \dots, w_\ell)$, given some conditioning context, \mathbf{x} .

As with unconditional models, it is again helpful to use the chain rule to decompose this probability:

$$p(\mathbf{w} \mid \mathbf{x}) = \prod_{t=1}^{\ell} p(w_t \mid \mathbf{x}, w_1, w_2, \dots, w_{t-1})$$

*What is the probability of the next word, given the history of previously generated words **and** conditioning context \mathbf{x} ?*

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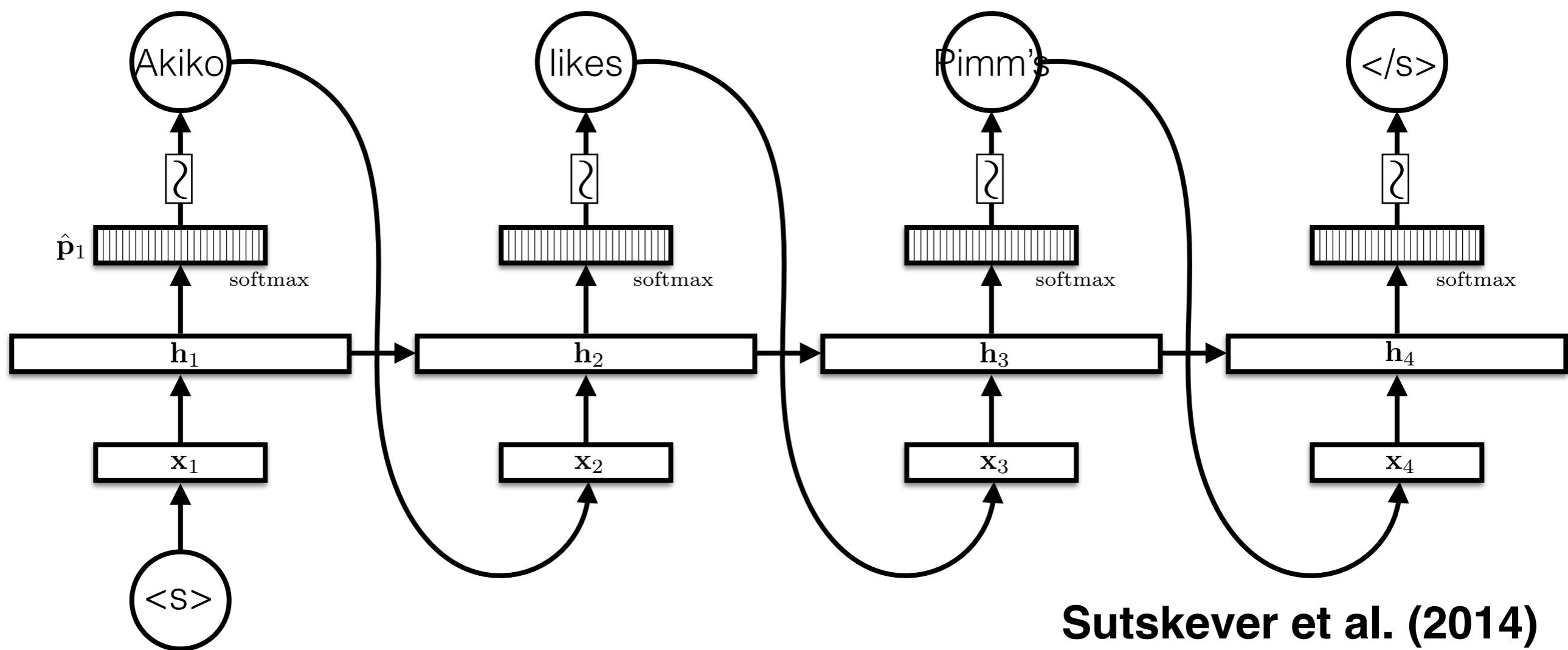
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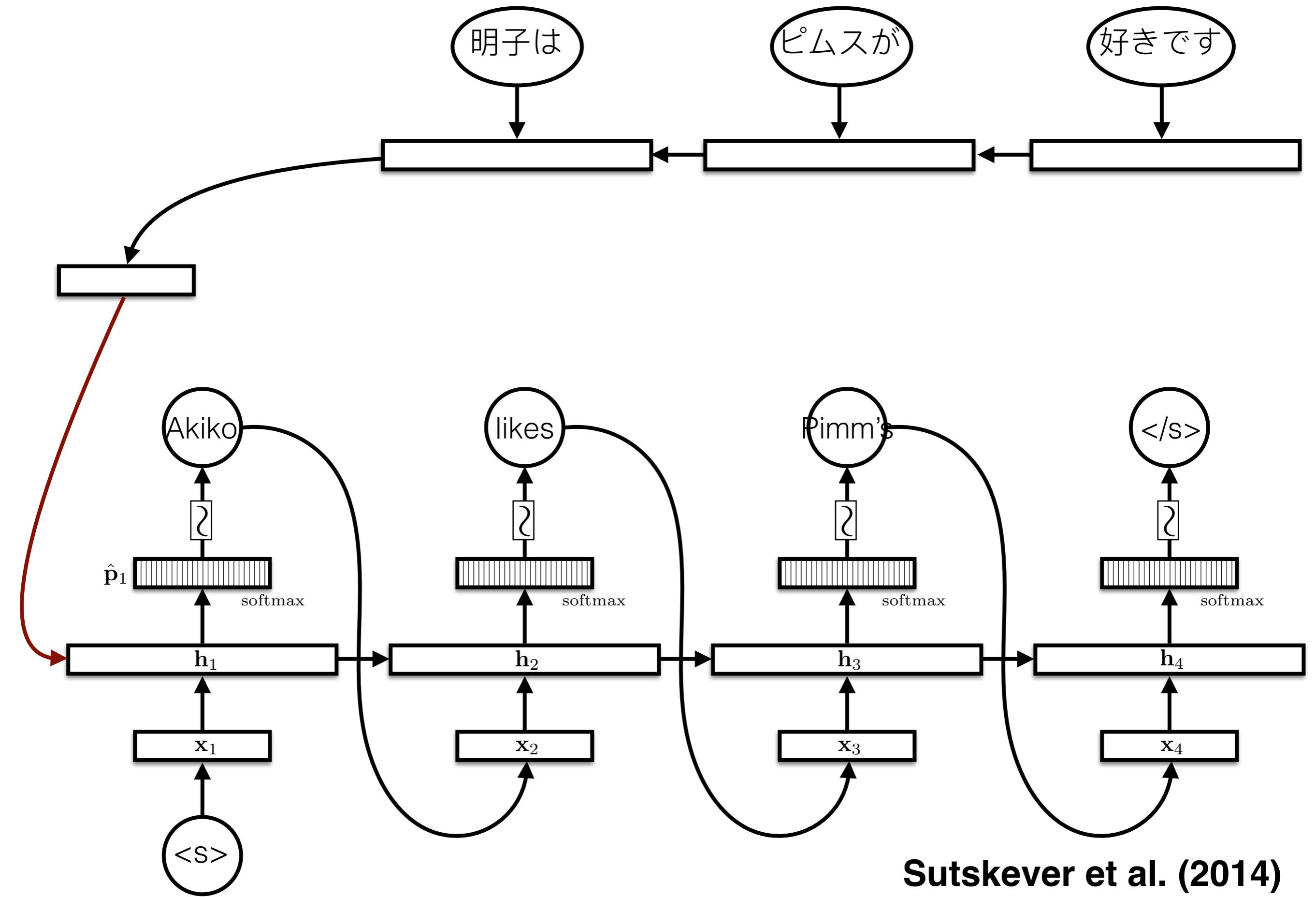
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Sutskever et al. (2014)



Conditioning with vectors

We are compressing a lot of information in a finite-sized vector.

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“You can't cram the meaning of a whole %&!\$# sentence into a single \$&!#* vector!”

Prof. Ray Mooney

Conditioning with vectors

We are compressing a lot of information in a finite-sized vector.

Gradients have a long way to travel. Even LSTMs forget!

Conditioning with vectors

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What is to be done?

Outline of Lecture

- Machine translation with attention
- Image caption generation with attention

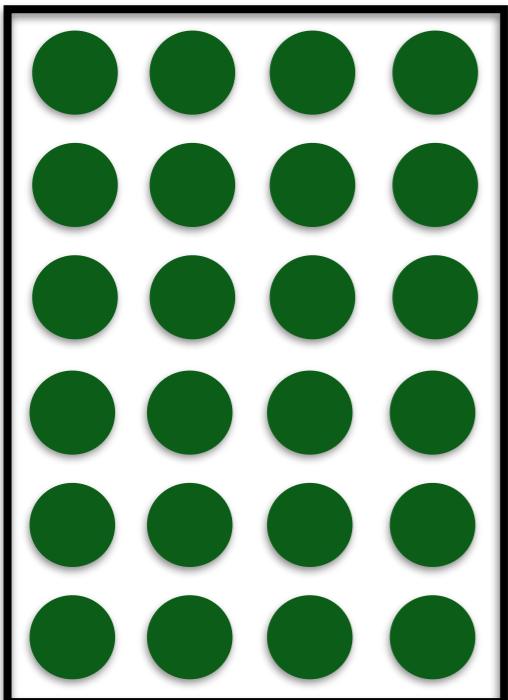
Solving the Vector Problem in Translation

- Represent a source sentence as a matrix
- Generate a target sentence from a matrix
- This will
 - Solve the capacity problem
 - Solve the gradient flow problem

Sentences as Matrices

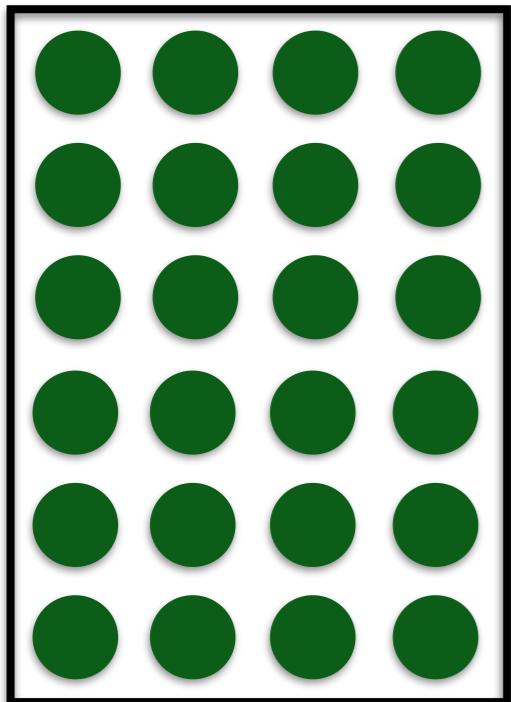
- Problem with the fixed-size vector model
 - Sentences are of different sizes but vectors are of the same size
- Solution: use matrices instead
 - Fixed number of rows, but number of columns depends on the number of words
 - Usually $|\mathbf{f}| = \# \text{cols}$

Sentences as Matrices



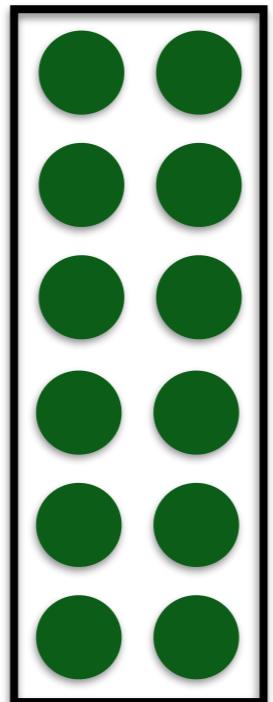
Ich möchte ein Bier

Sentences as Matrices

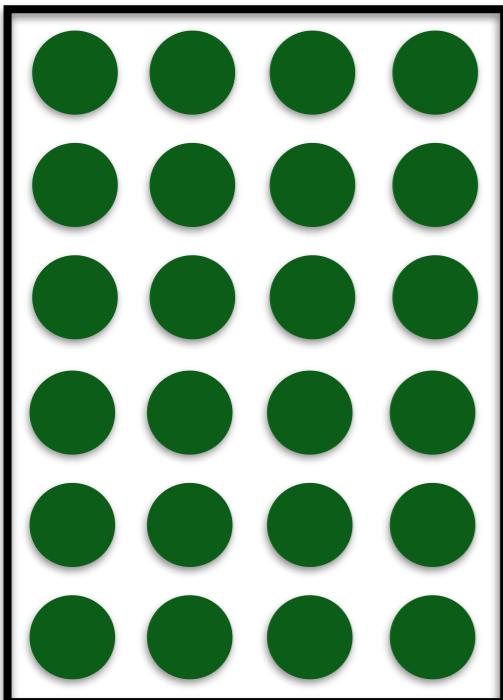


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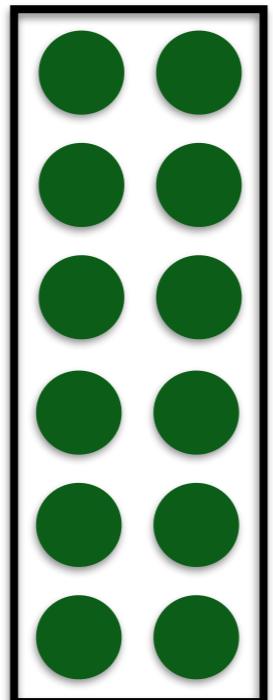
Mach's gut



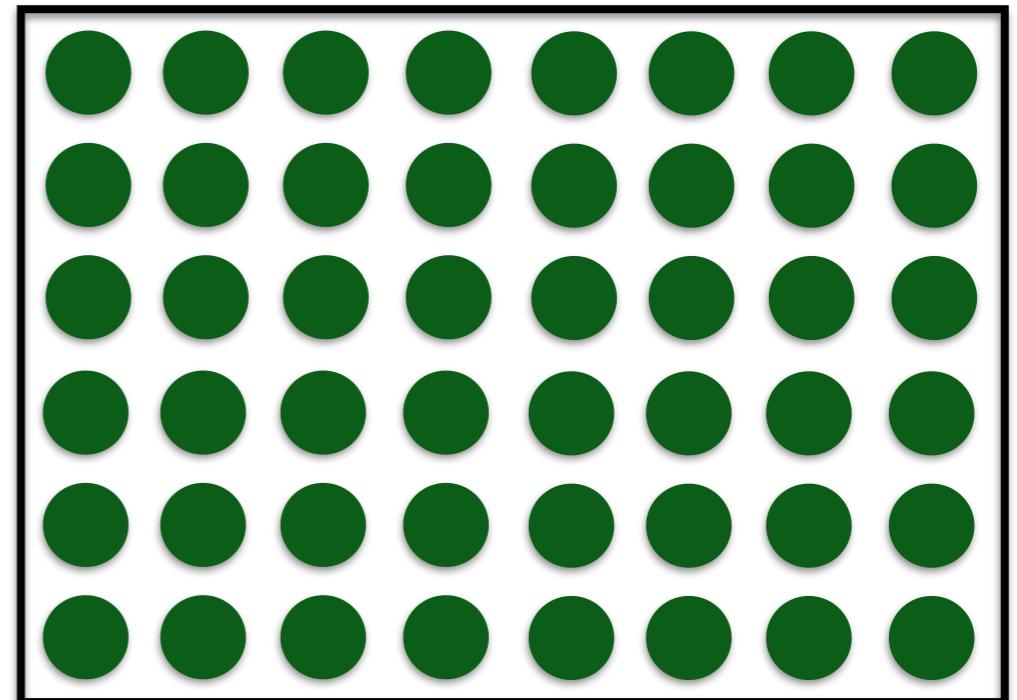
Sentences as Matrices



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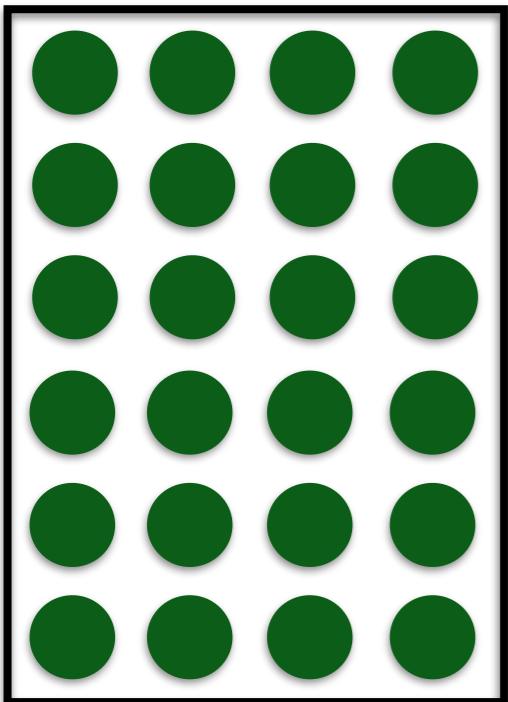


Mach's gut



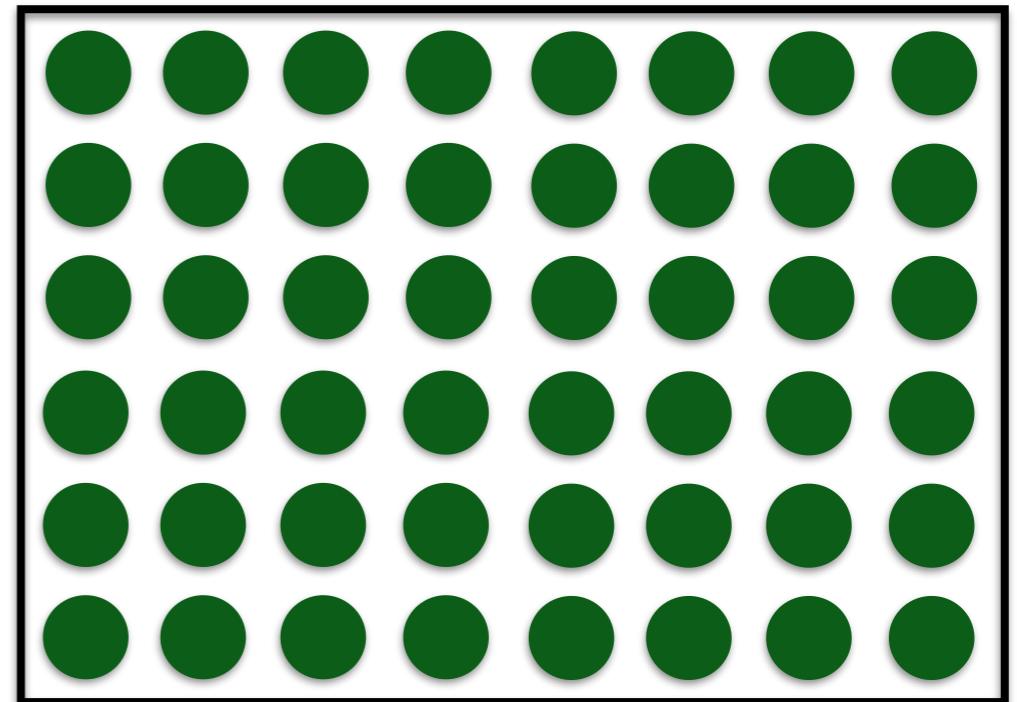
Die Wahrheiten der Menschen sind die unwiderlegbaren Irrtümer

Sentences as Matrices



Ich möchte ein Bier

Mach's gut



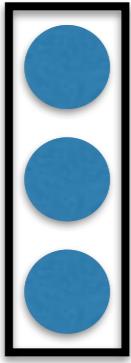
Die Wahrheiten der Menschen sind die unwiderlegbaren Irrtümer

Question: How do we build these matrices?

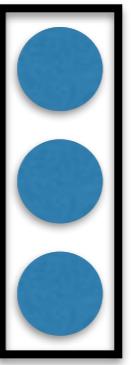
With Concatenation

- Each word type is represented by an n-dimensional vector
- Take all of the vectors for the sentence and concatenate them into a matrix
- Simplest possible model
 - So simple, no one has bothered to publish how well/badly it works!

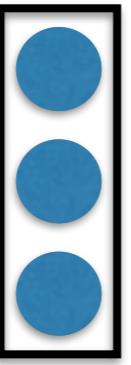
x_1



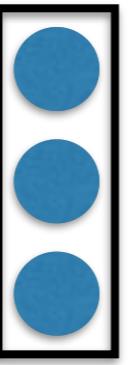
x_2



x_3



x_4



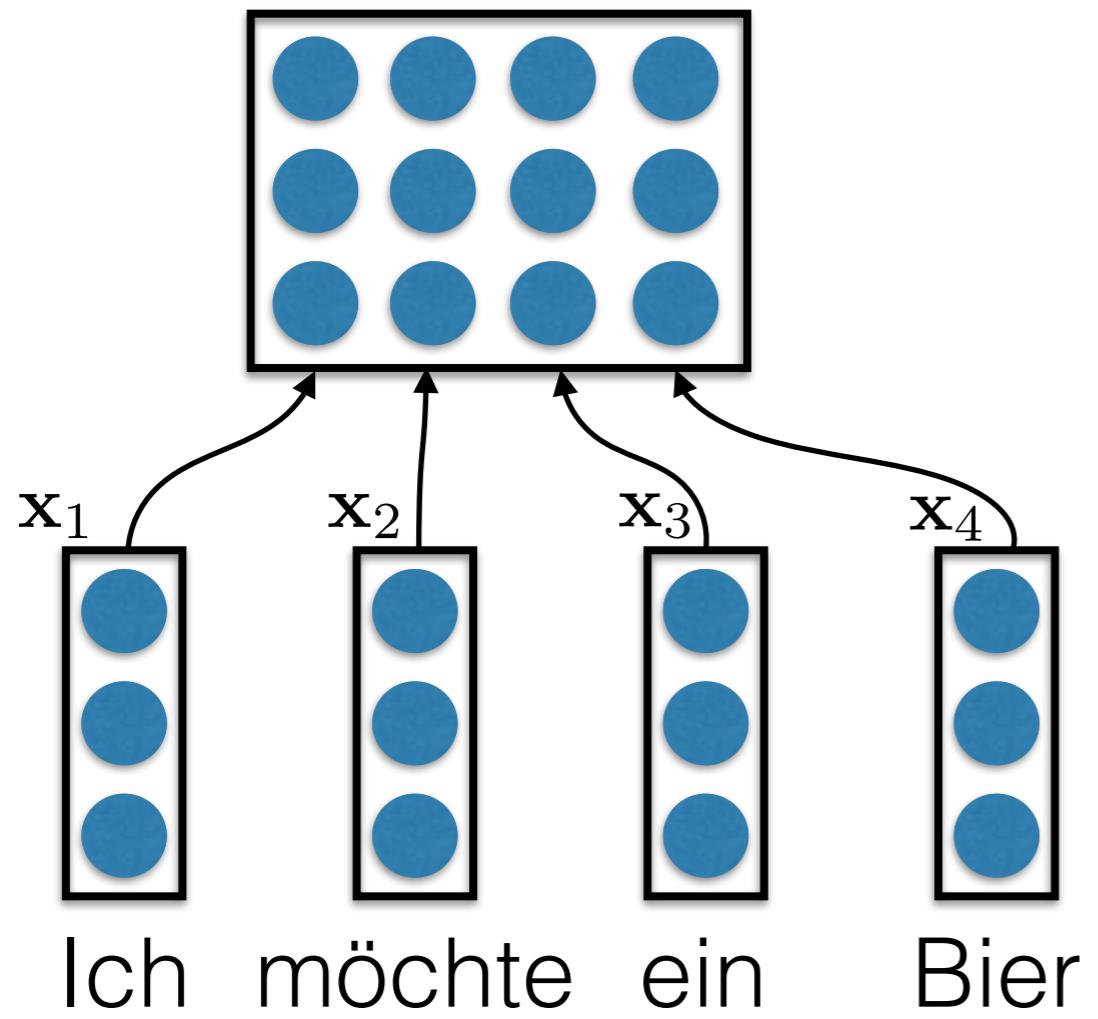
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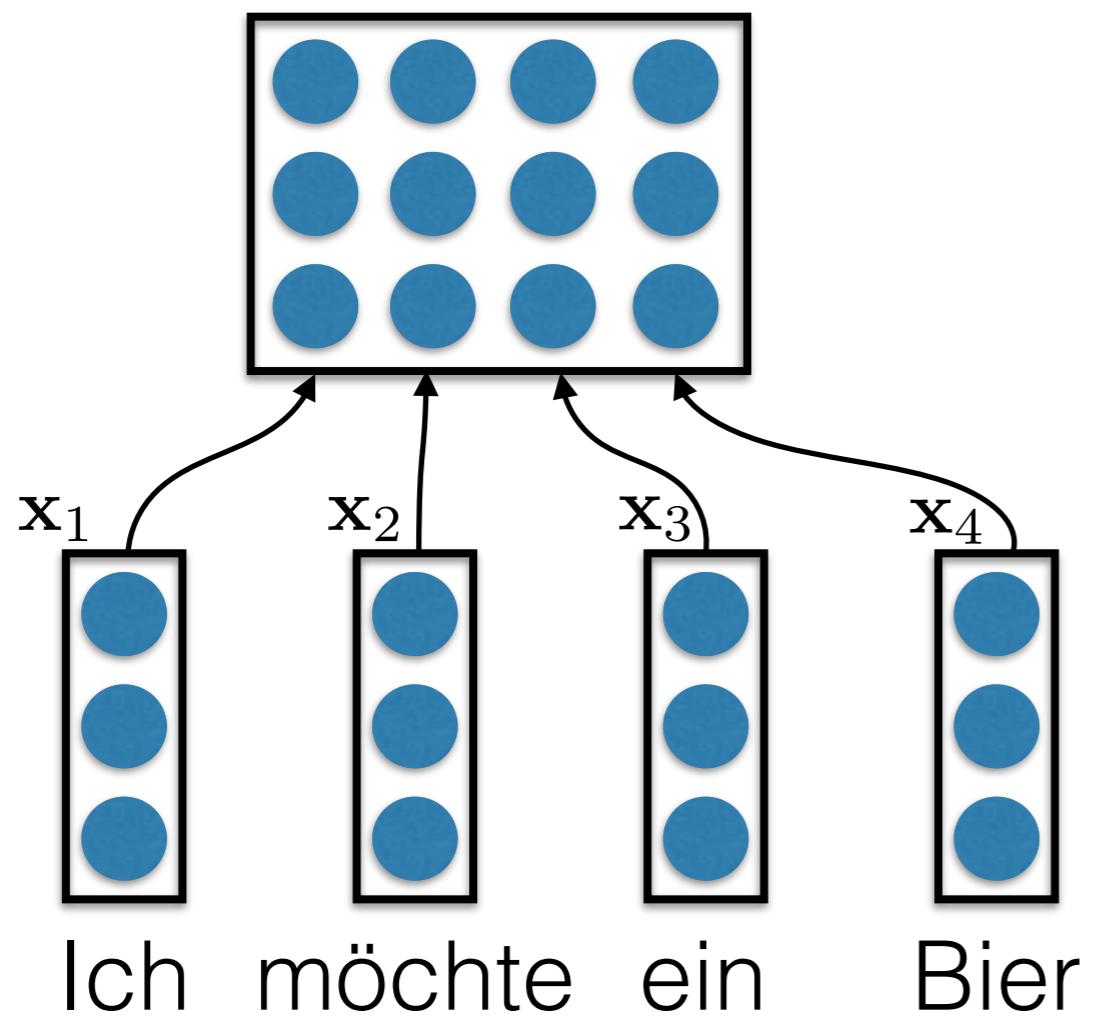
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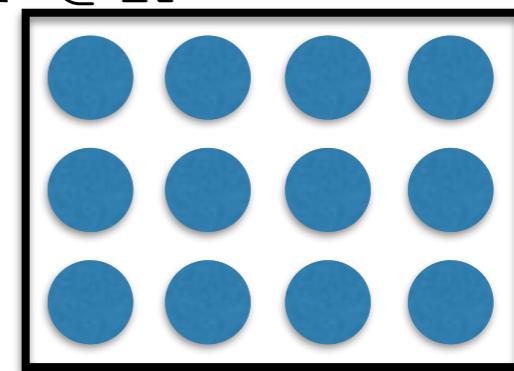
$$\mathbf{f}_i = \mathbf{x}_i$$



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$$\mathbf{F} \in \mathbb{R}^{n \times |f|}$$

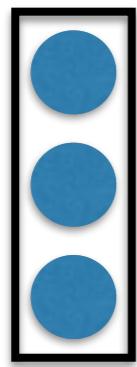


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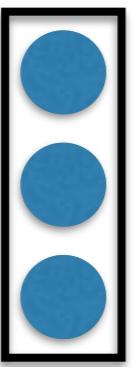
With Convolutional Nets

- Apply convolutional networks to transform the naive concatenated matrix to obtain a context-dependent matrix
- Explored in a recent ICLR submission by Gehring et al., 2016 (from FAIR)
 - Closely related to the neural translation model proposed by Kalchbrenner and Blunsom, 2013
- Note: convnets usually have a “pooling” operation at the top level that results in a fixed-sized representation. For sentences, leave this out.

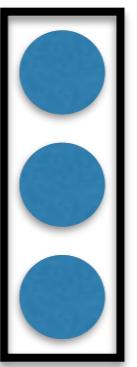
x_1



x_2



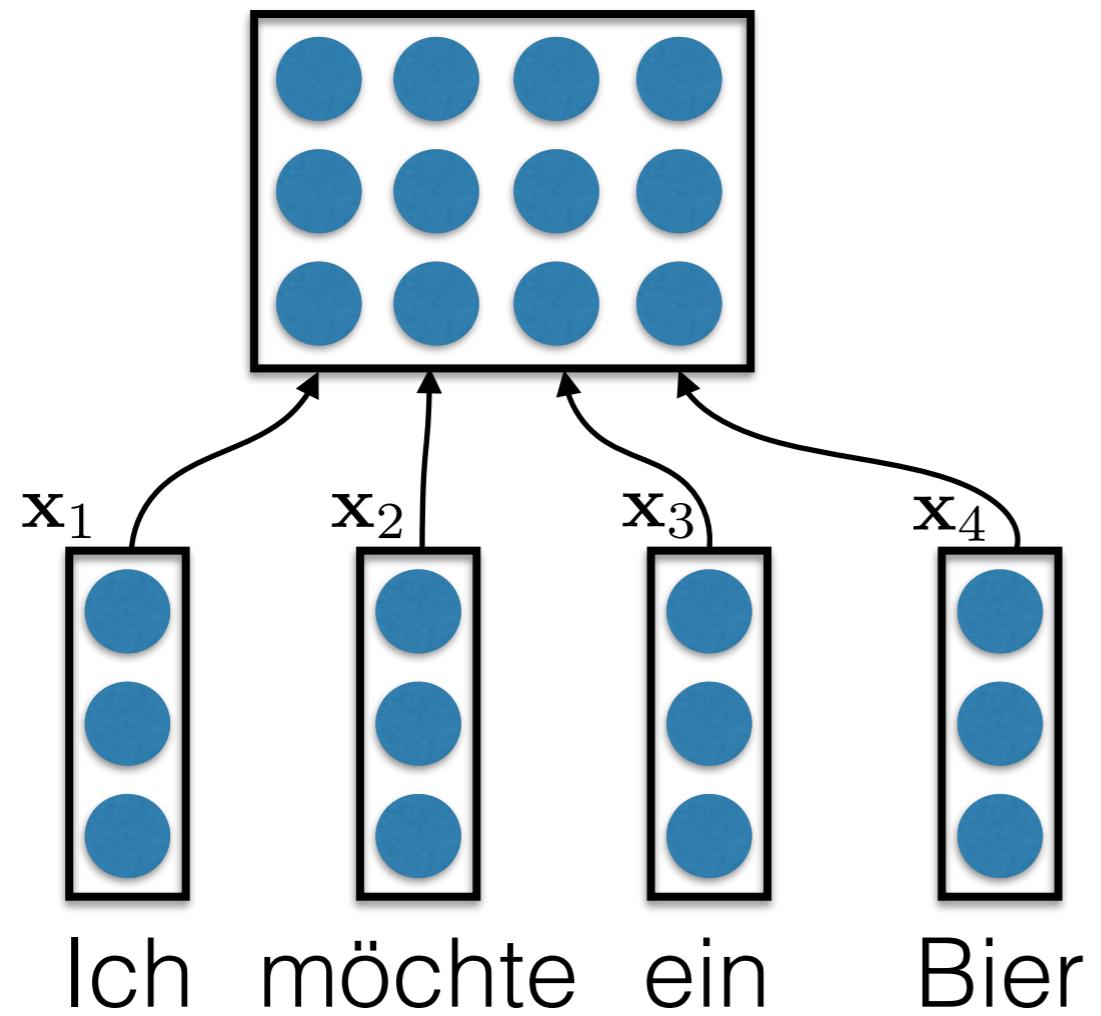
x_3

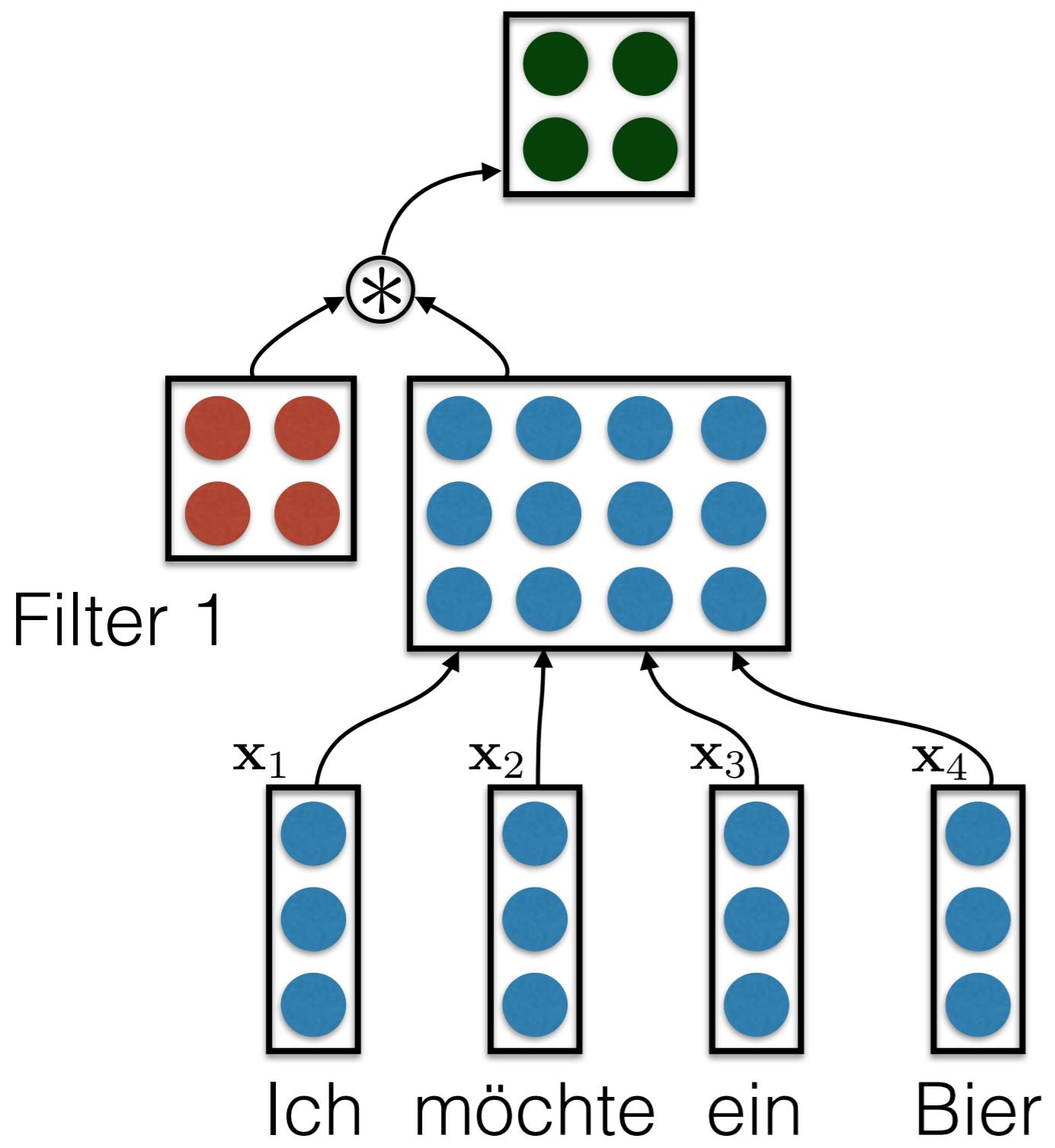


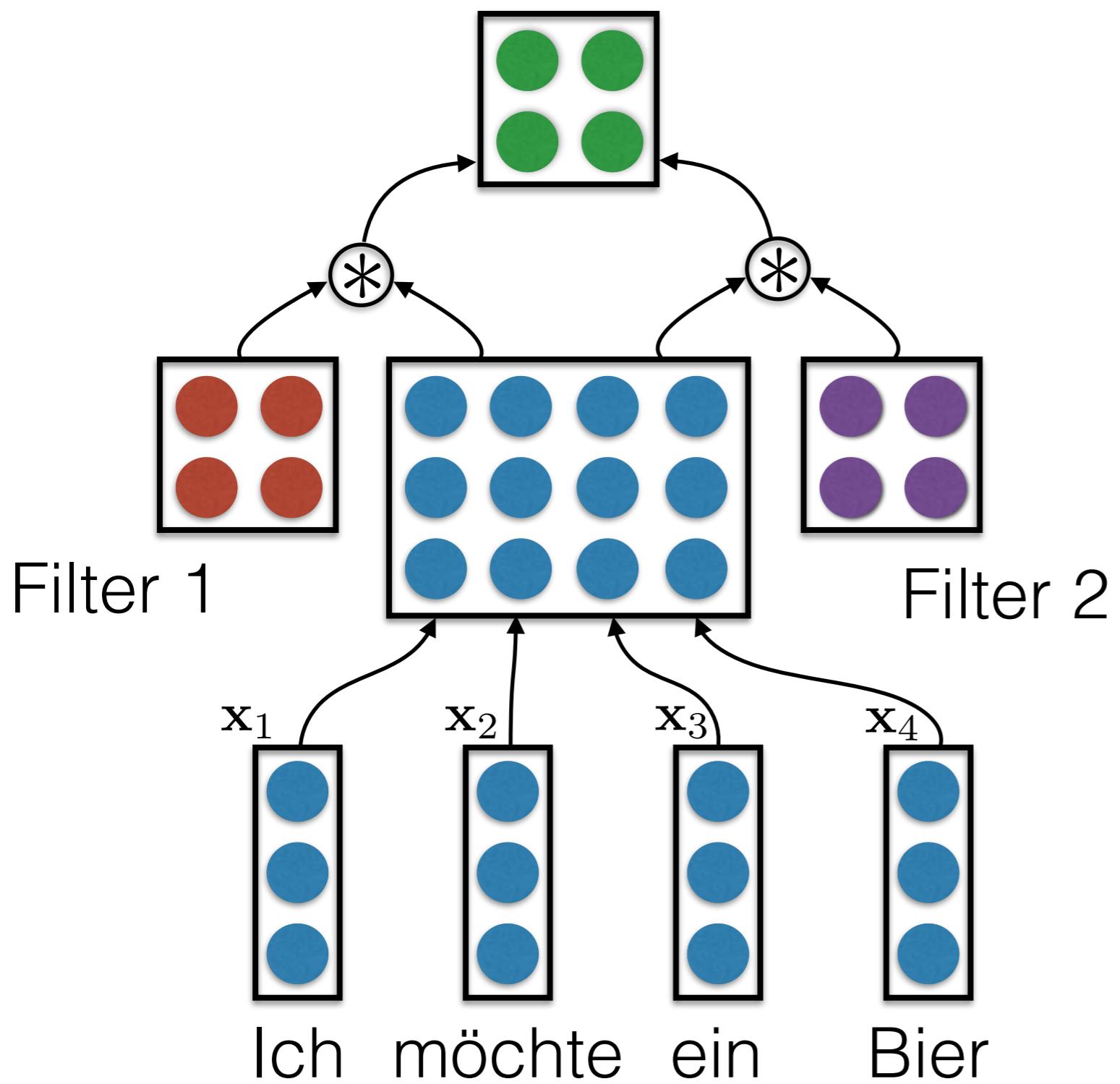
x_4

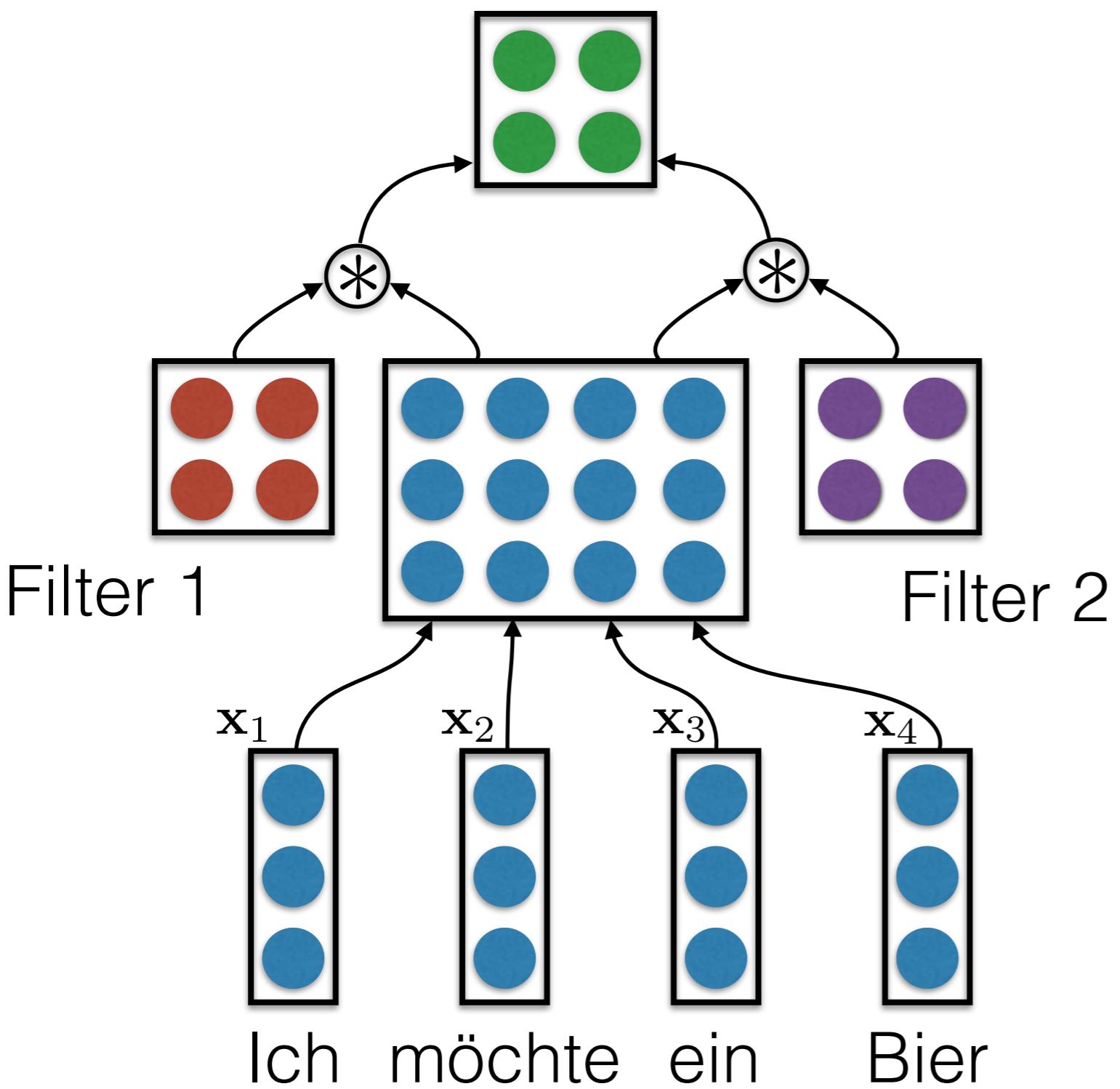


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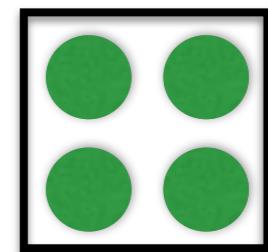








$$\mathbf{F} \in \mathbb{R}^{f(n) \times g(|\mathcal{F}|)}$$

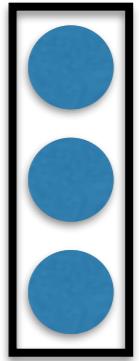


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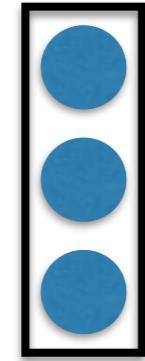
With Bidirectional RNNs

- By far the most widely used matrix representation, due to Bahdanau et al (2015)
- One column per word
- Each column (word) has two halves concatenated together:
 - a “forward representation”, i.e., a word and its left context
 - a “reverse representation”, i.e., a word and its right context
- Implementation: bidirectional RNNs (GRUs or LSTMs) to read **f** from left to right and right to left, concatenate representations

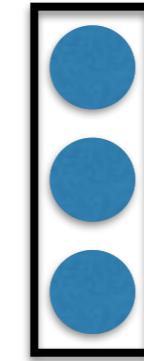
x_1



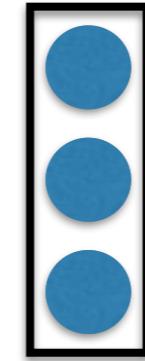
x_2



x_3

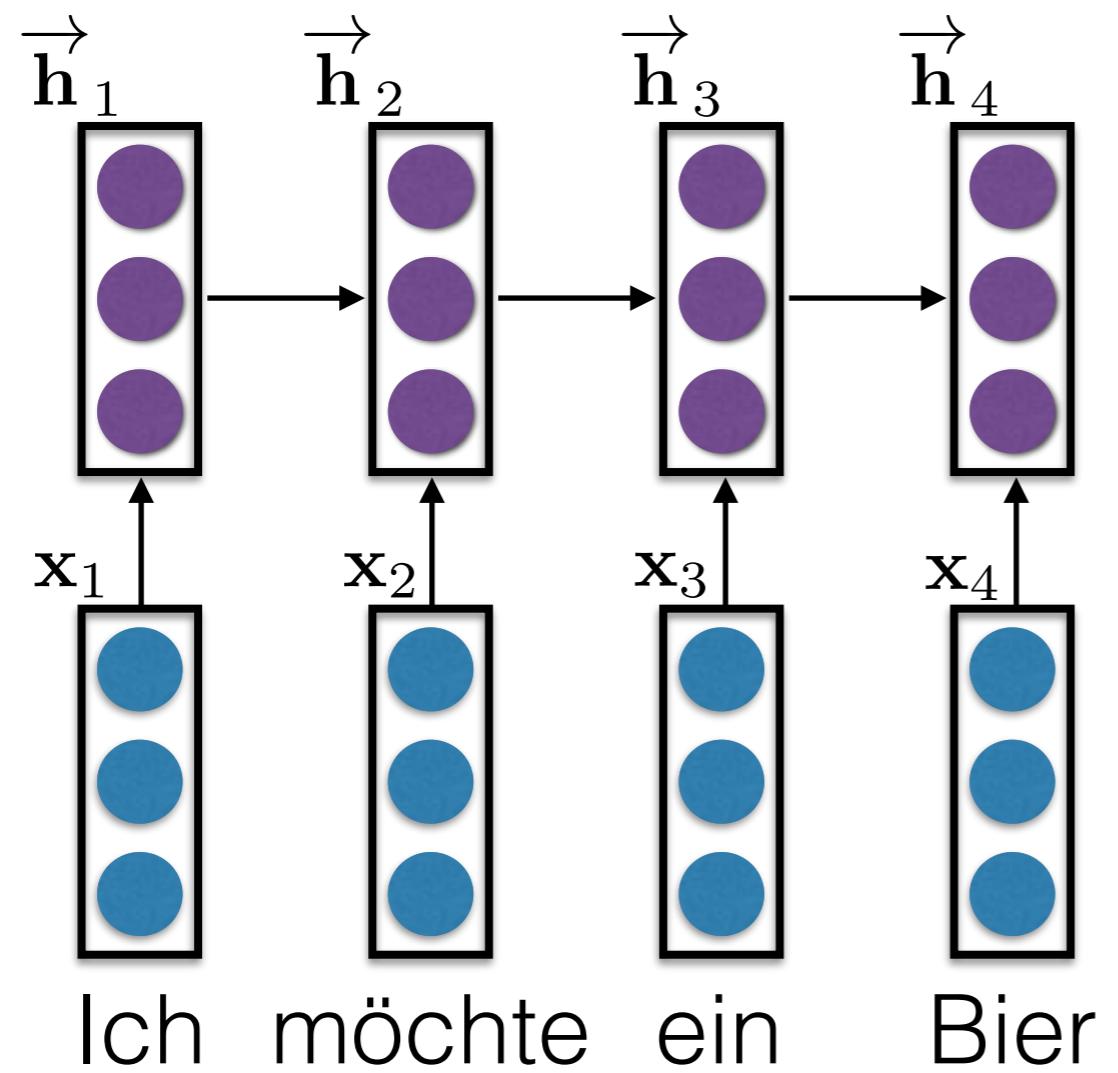


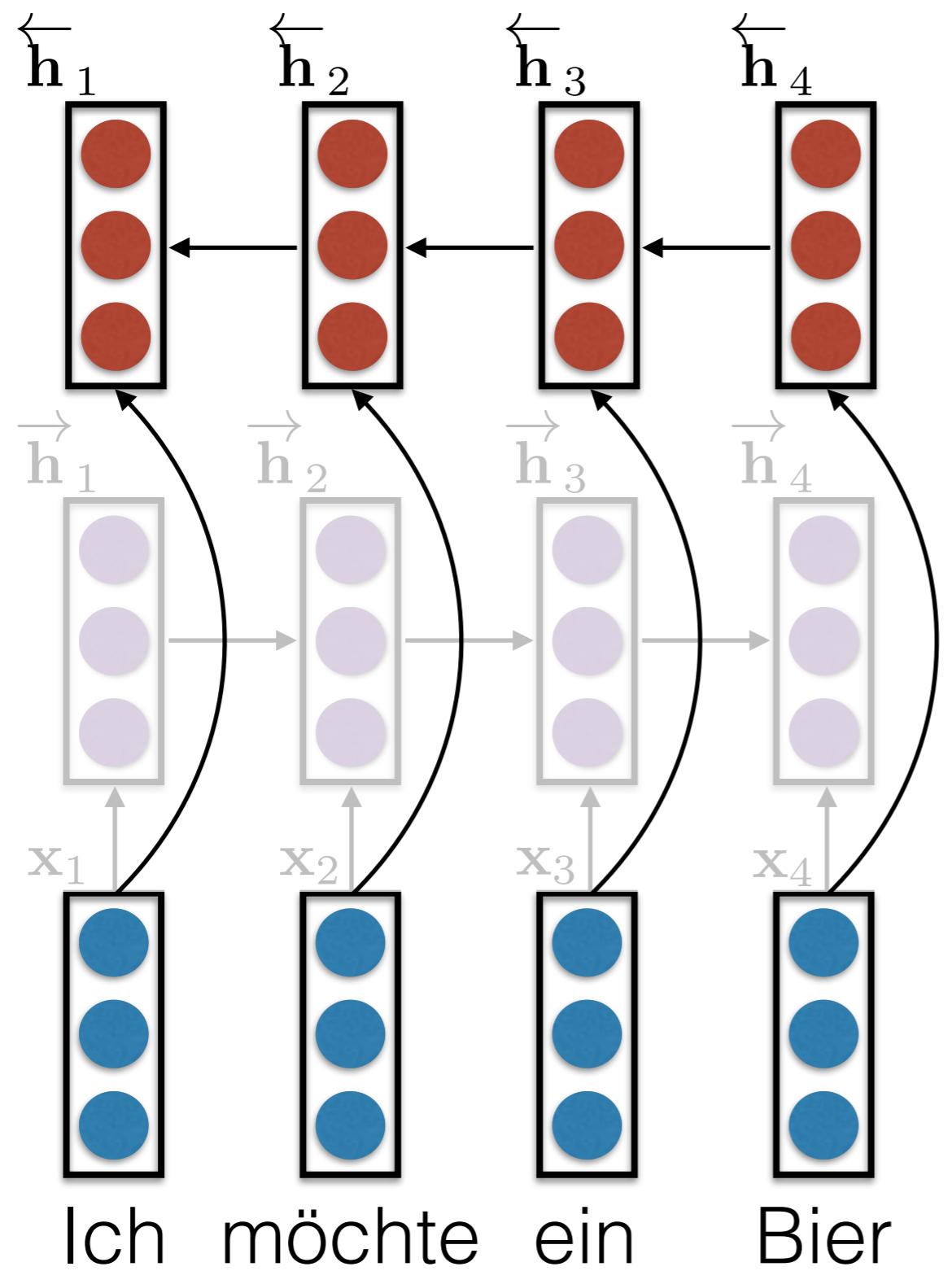
x_4



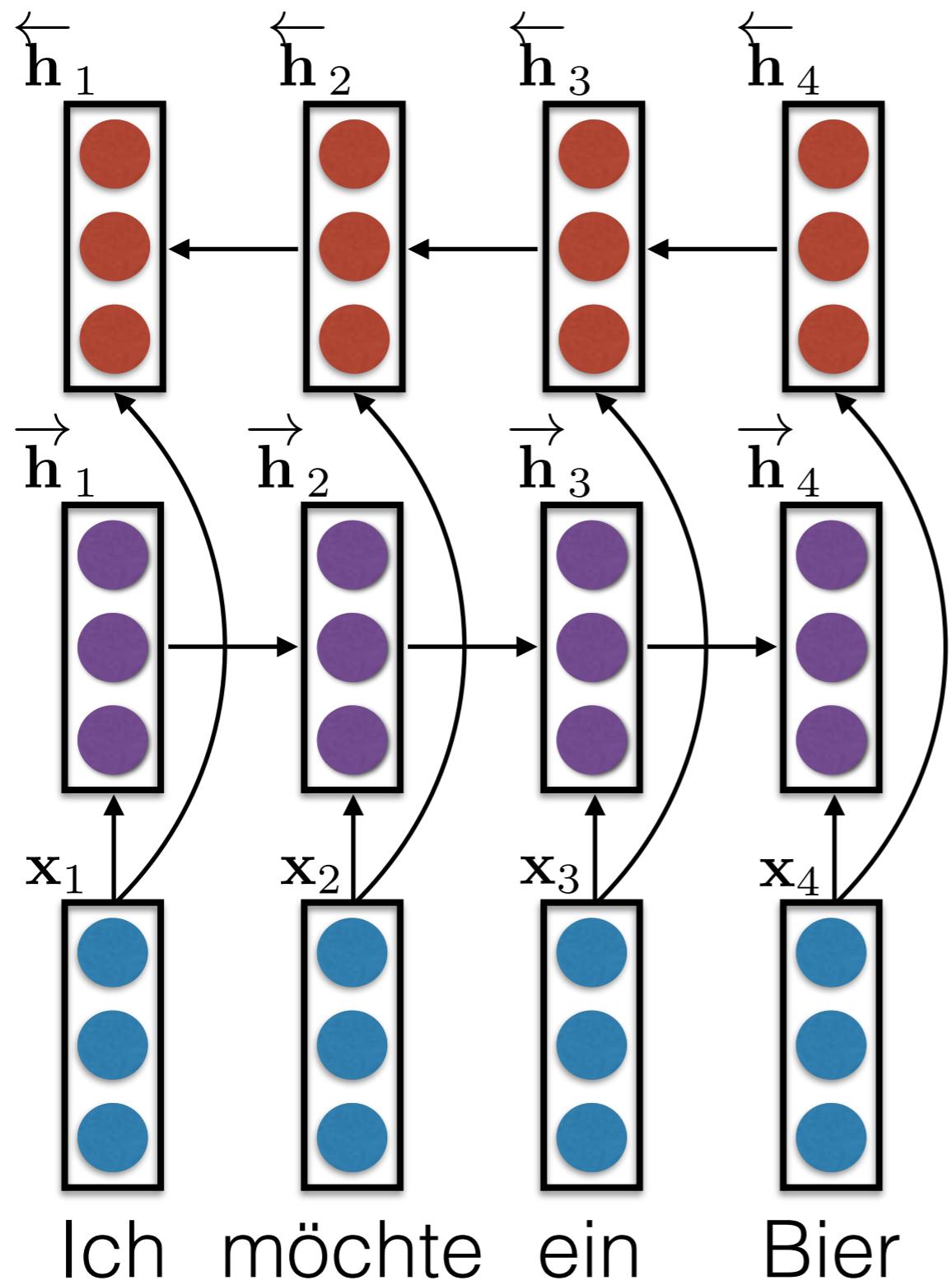
Ich möchte ein

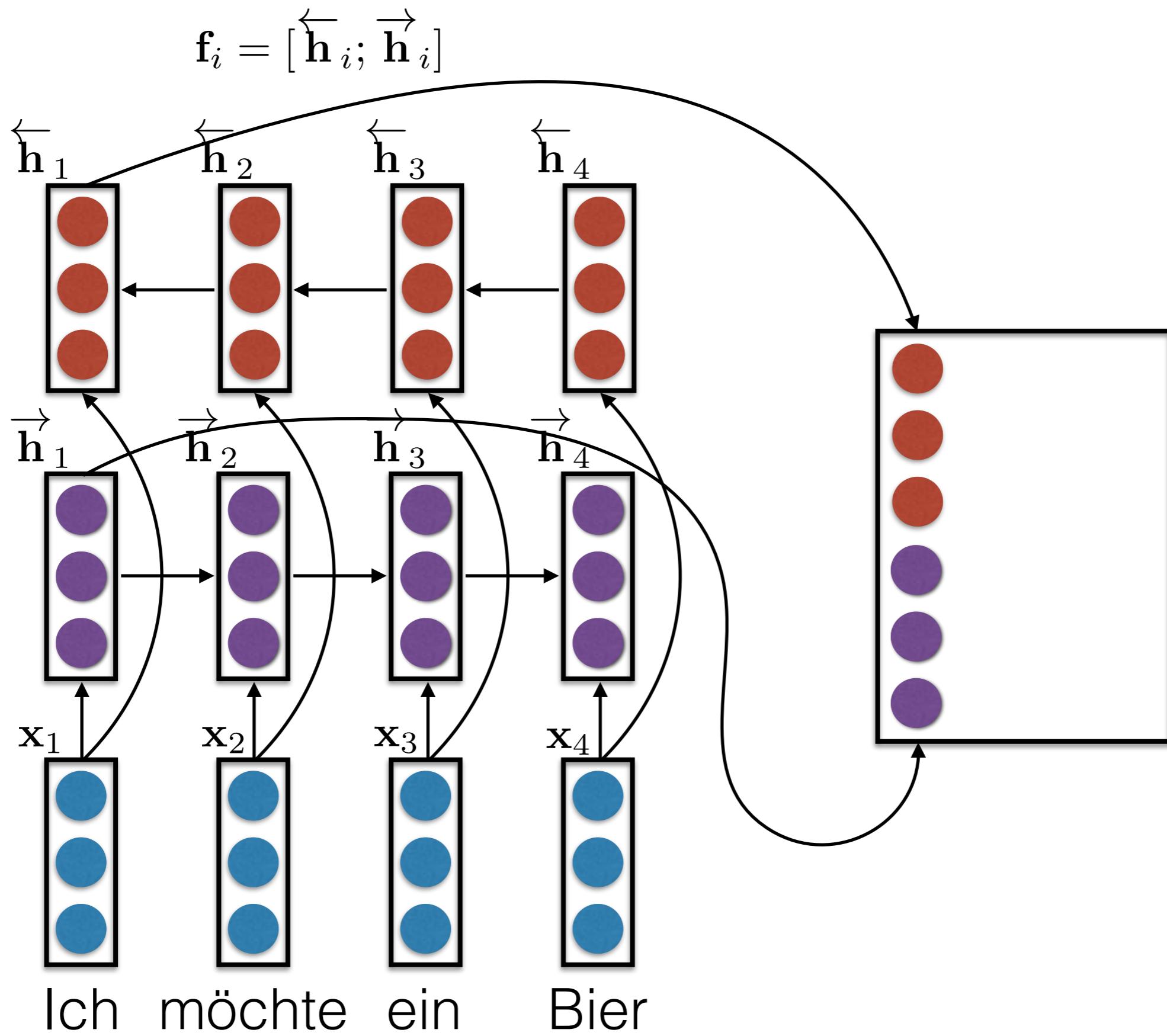
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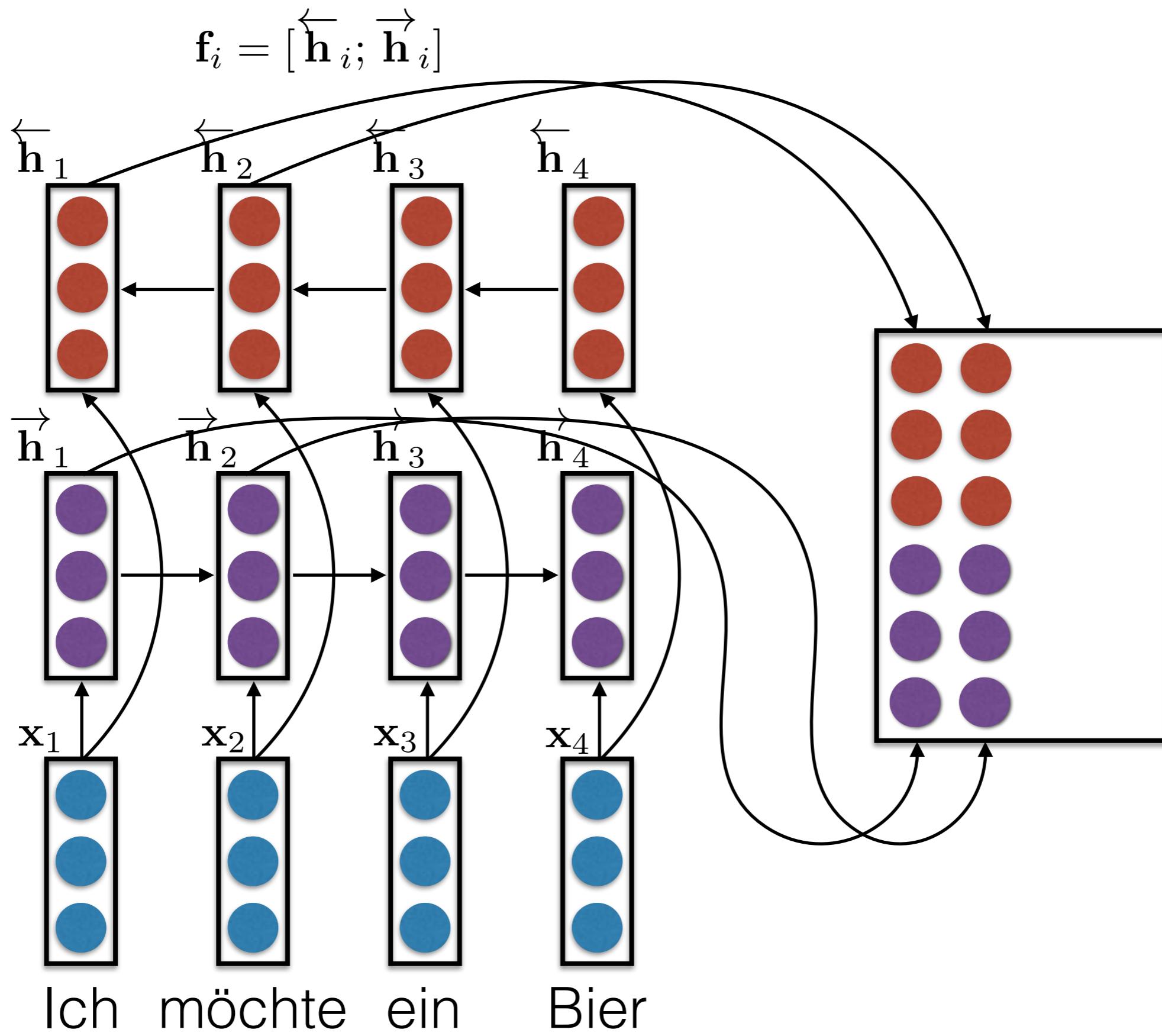




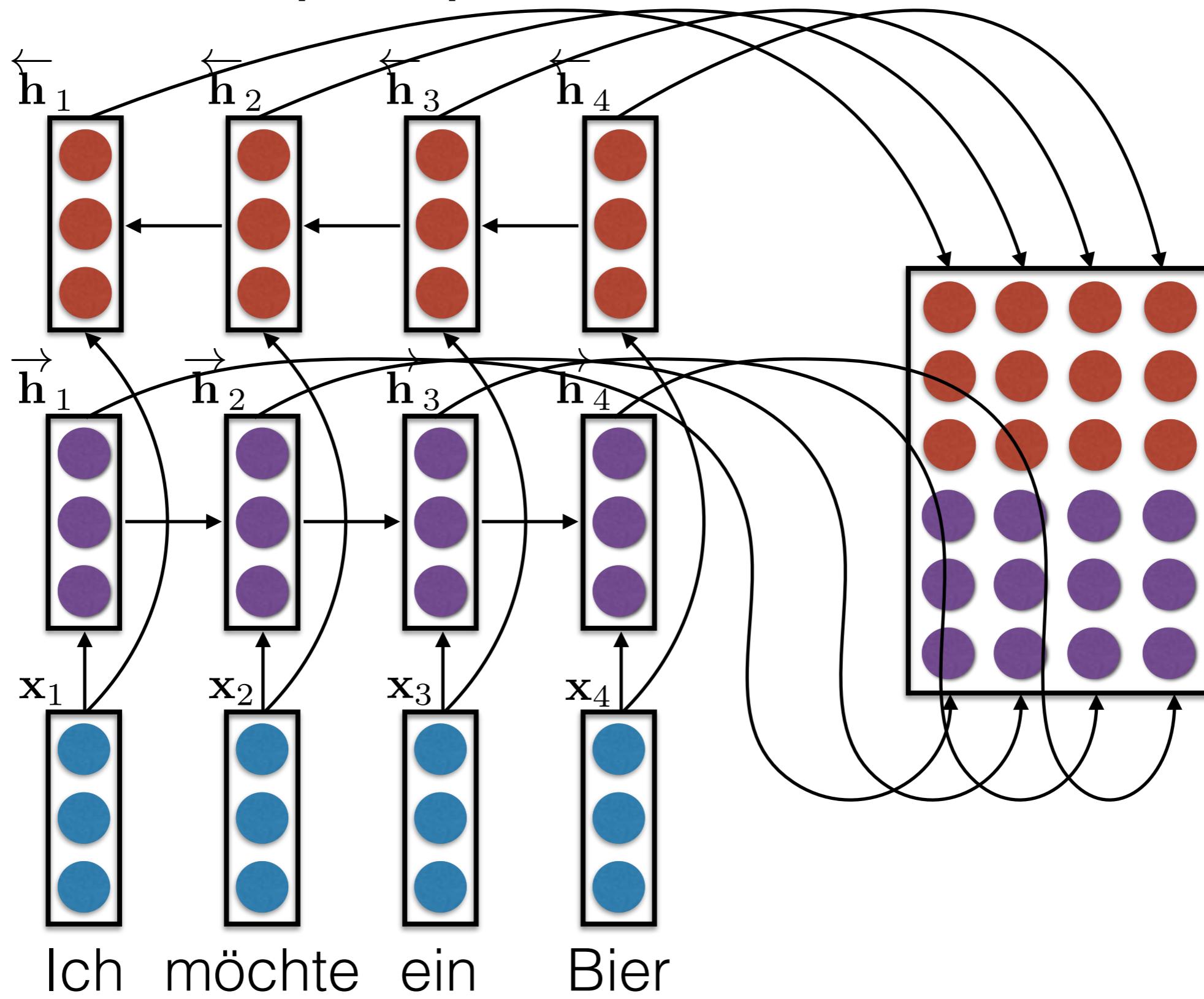
$$\mathbf{f}_i = [\overleftarrow{\mathbf{h}}_i; \overrightarrow{\mathbf{h}}_i]$$



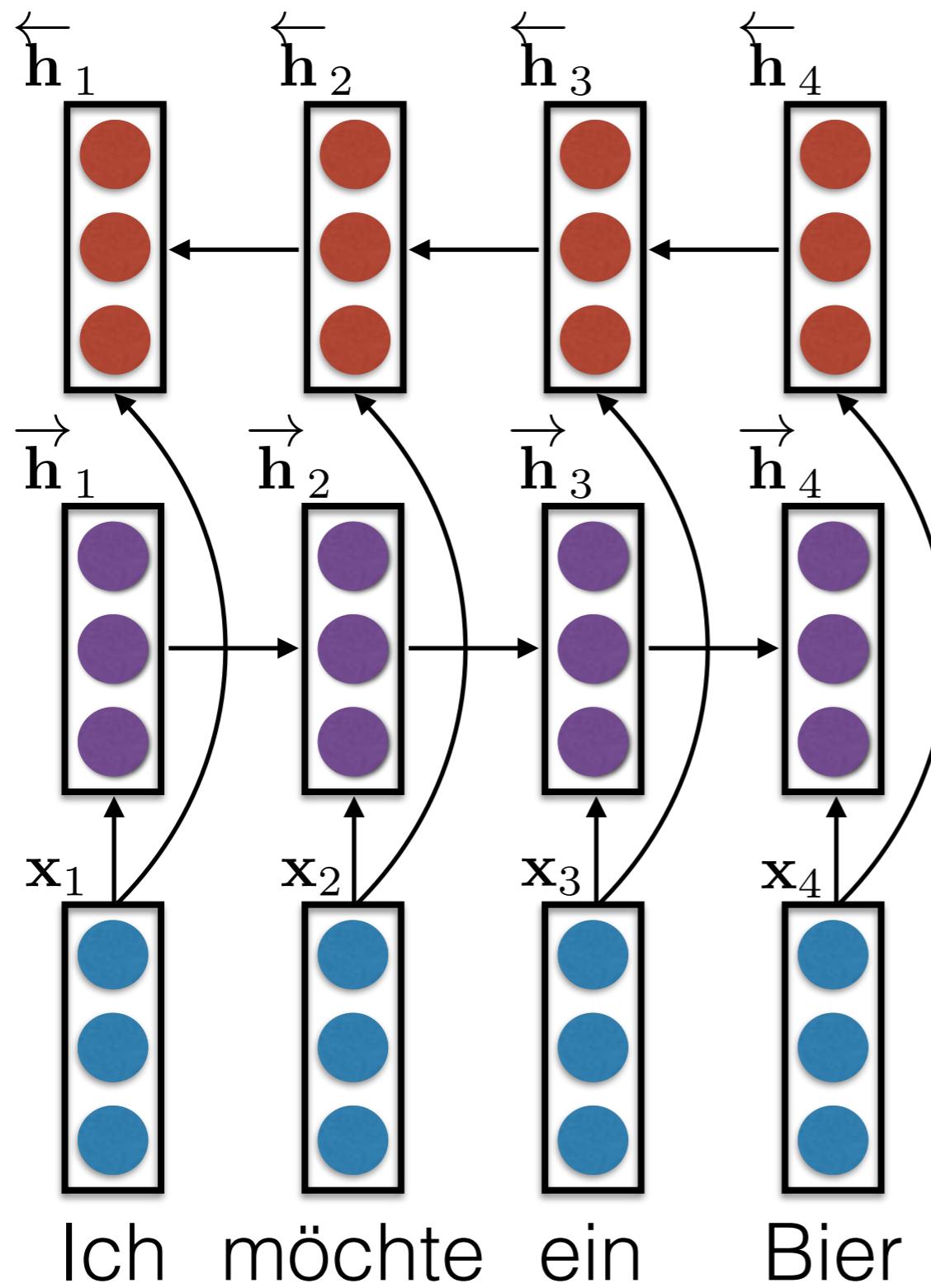




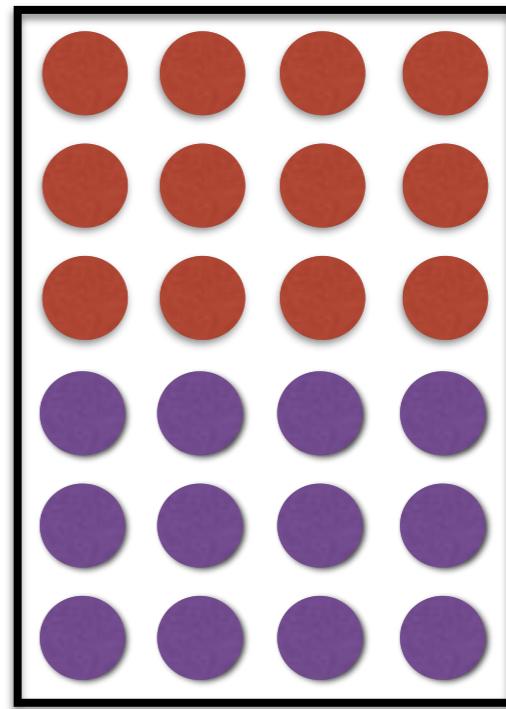
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$$\mathbf{F} \in \mathbb{R}^{2n \times |f|}$$



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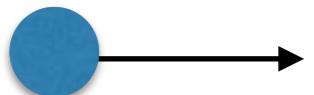
Where are we in 2017?

- There are lots of ways to construct **F**
 - Very little systematic work comparing them
 - There are many more undiscovered things out there
 - convolutions are particularly interesting and under-explored
 - syntactic information can help (Sennrich & Haddow, 2016; Nadejde et al., 2017), but many more integration strategies are possible
 - try something with phrase types instead of word types?

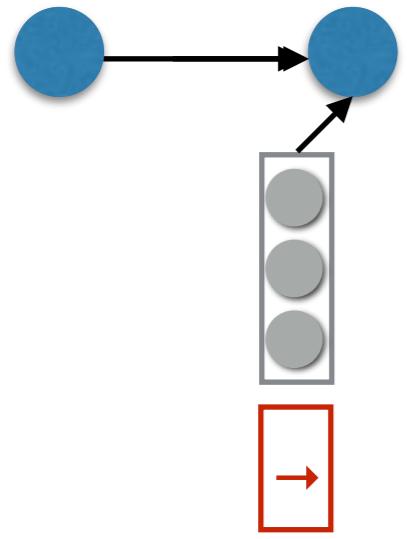
Multi-word expressions are a pain in the neck .

Generation from Matrices

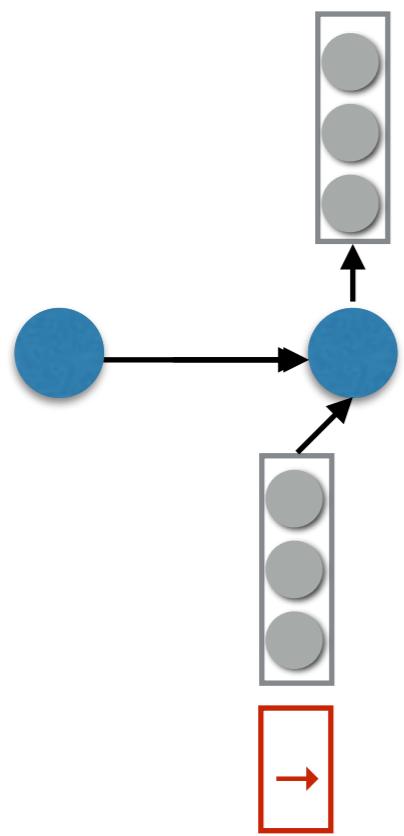
- We have a matrix \mathbf{F} representing the input, now we need to generate from it
- Bahdanau et al. (2015) were the first to propose using **attention** for translating from matrix-encoded sentences
- High-level idea
 - Generate the output sentence word by word using an RNN
 - At each output position t , the RNN receives **two** inputs (in addition to any recurrent inputs)
 - a fixed-size vector embedding of the previously generated output symbol e_{t-1}
 - a fixed-size vector encoding a “view” of the input matrix
 - How do we get a fixed-size vector from a matrix that changes over time?
 - Bahdanau et al: do a weighted sum of the columns of \mathbf{F} (i.e., words) based on how important they are *at the current time step*. (i.e., just a matrix-vector product $\mathbf{F}\mathbf{a}_t$)
 - The weighting of the input columns at each time-step (\mathbf{a}_t) is called **attention**



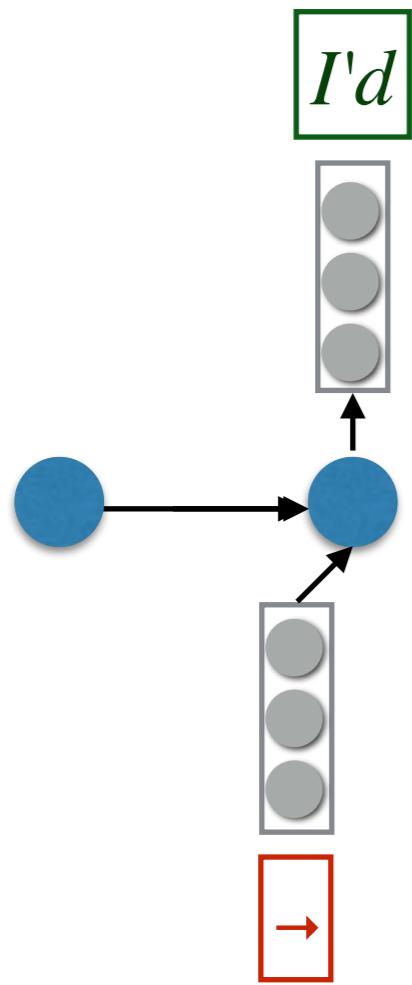
Recall RNNs...



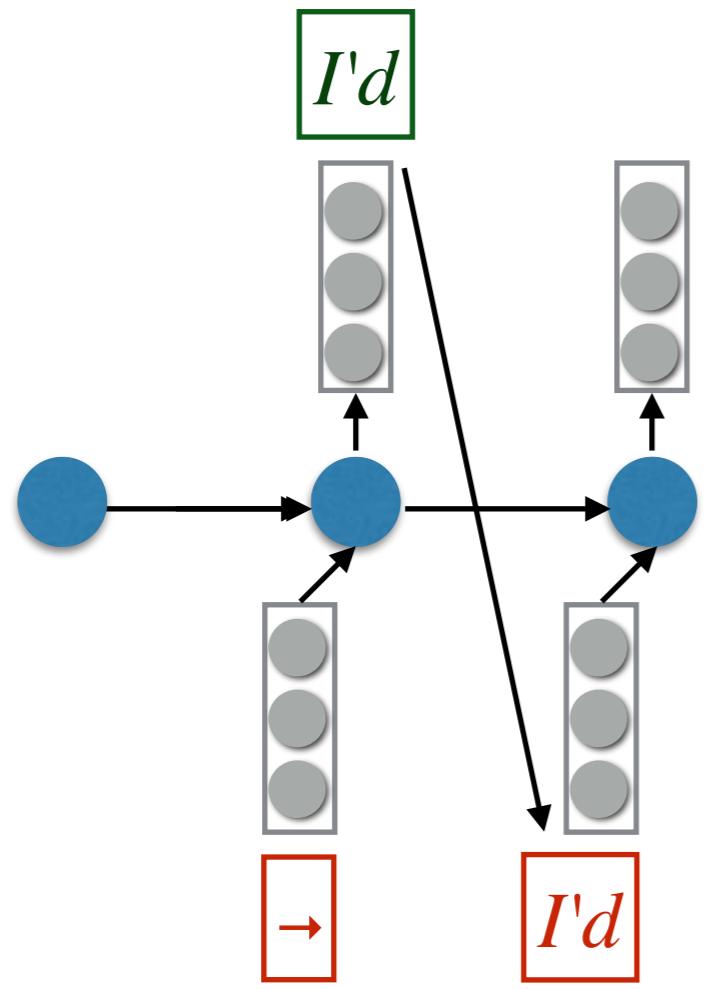
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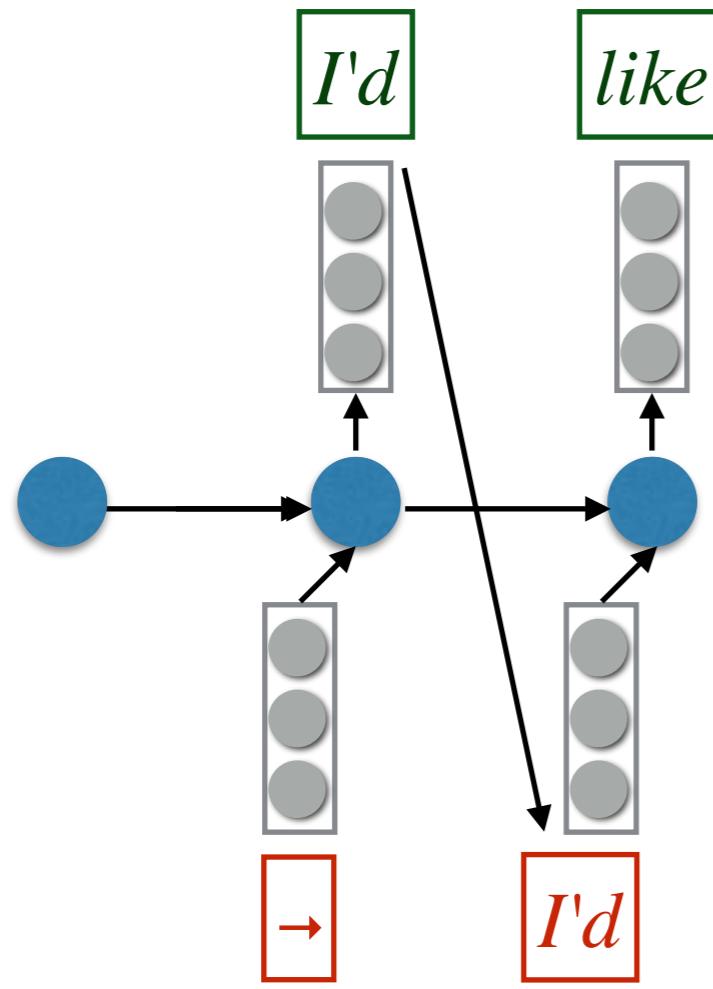
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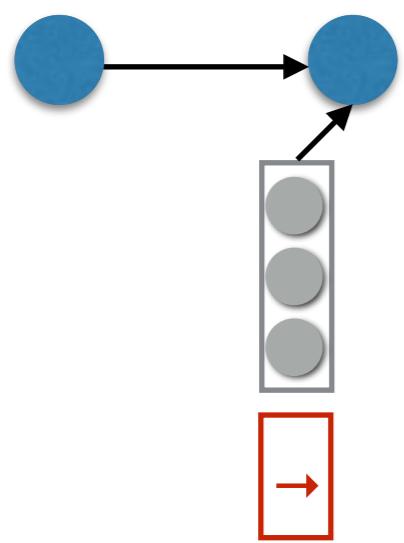
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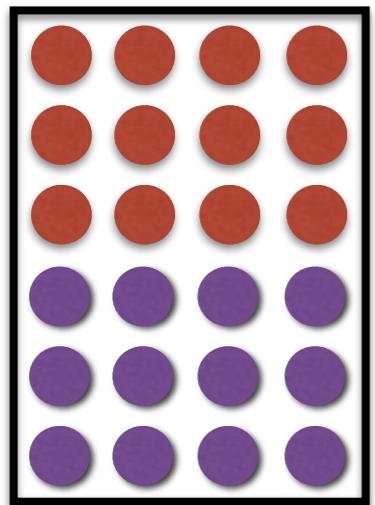
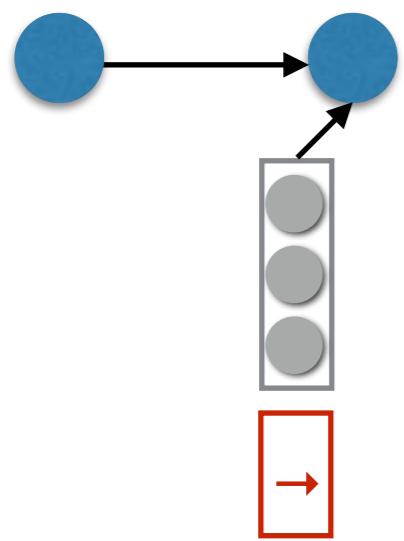


Recall RNNs...

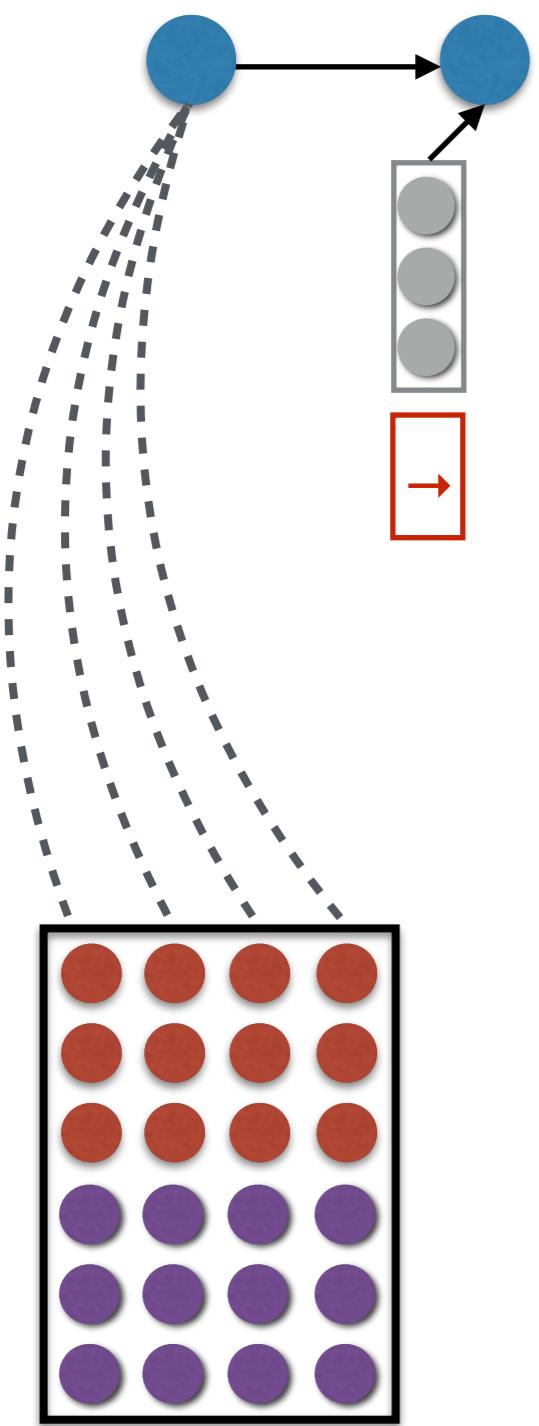


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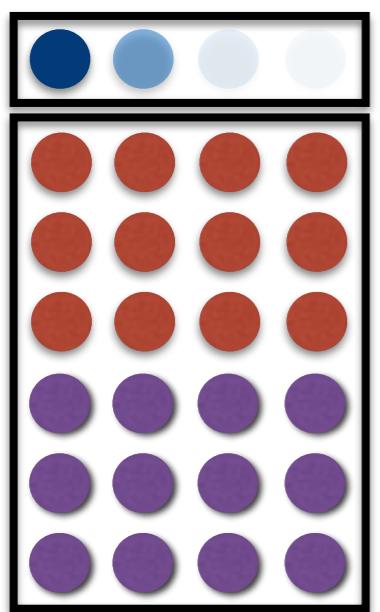
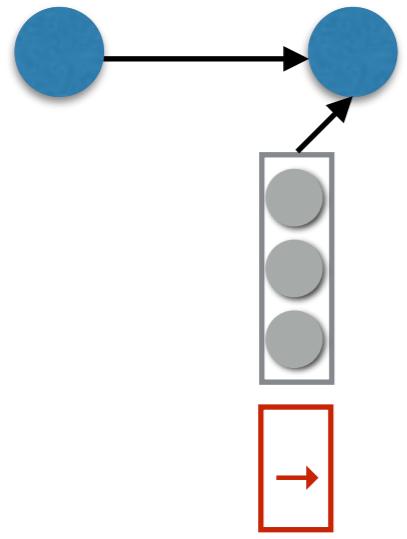




Ich möchte ein Bier



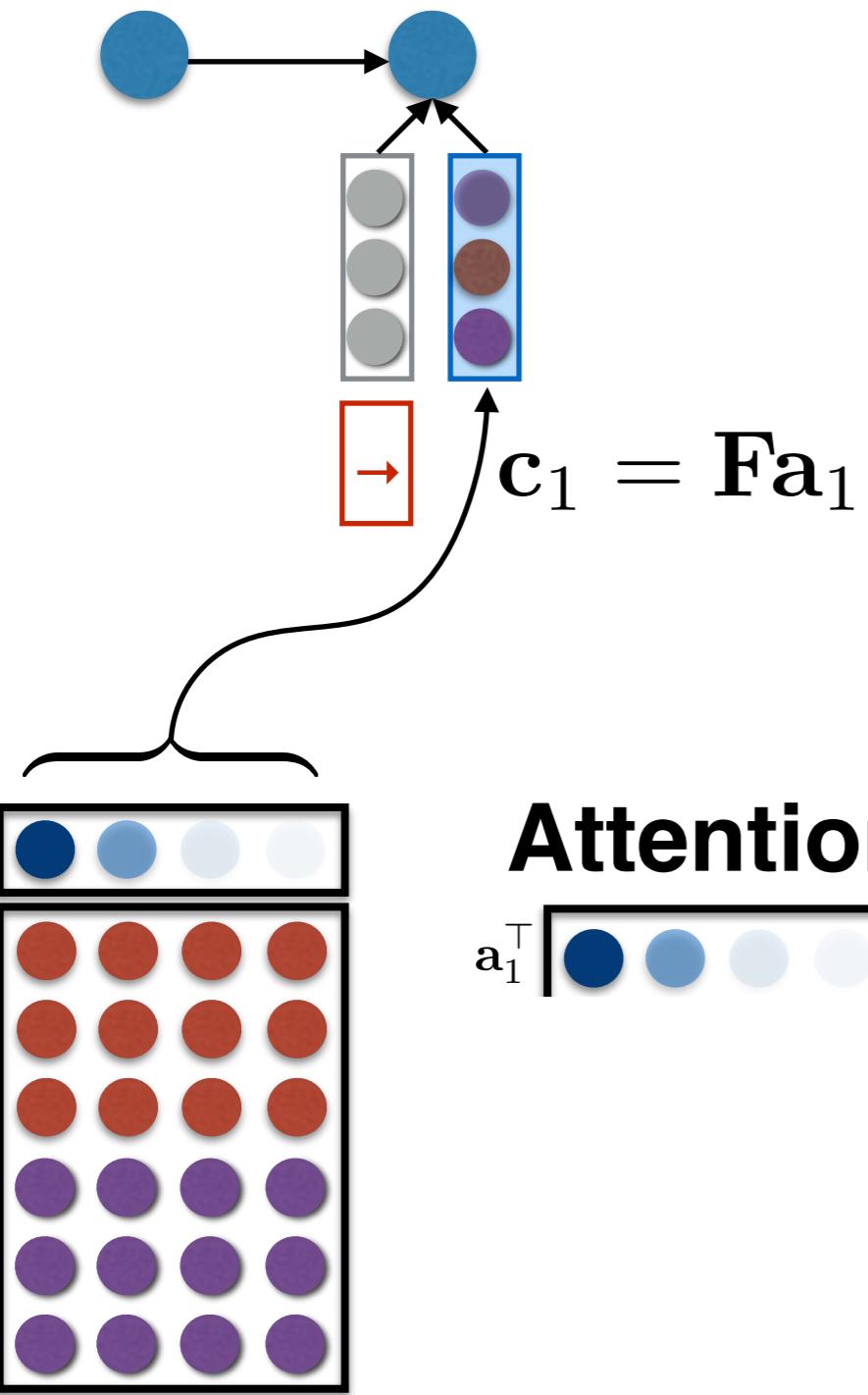
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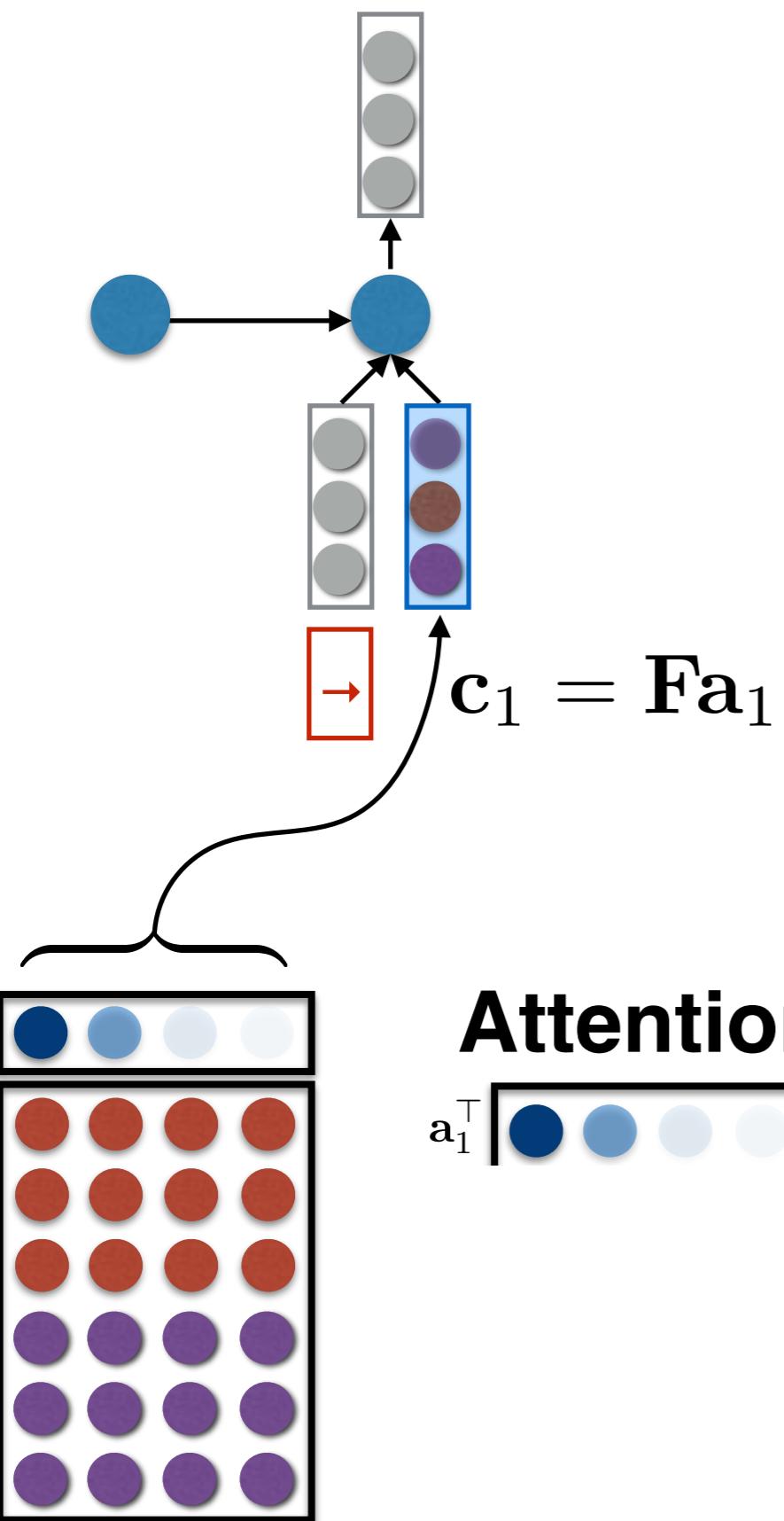
Attention history:

$$a_1^\top \boxed{\text{dark blue} \quad \text{light blue} \quad \text{light gray} \quad \text{white}}$$

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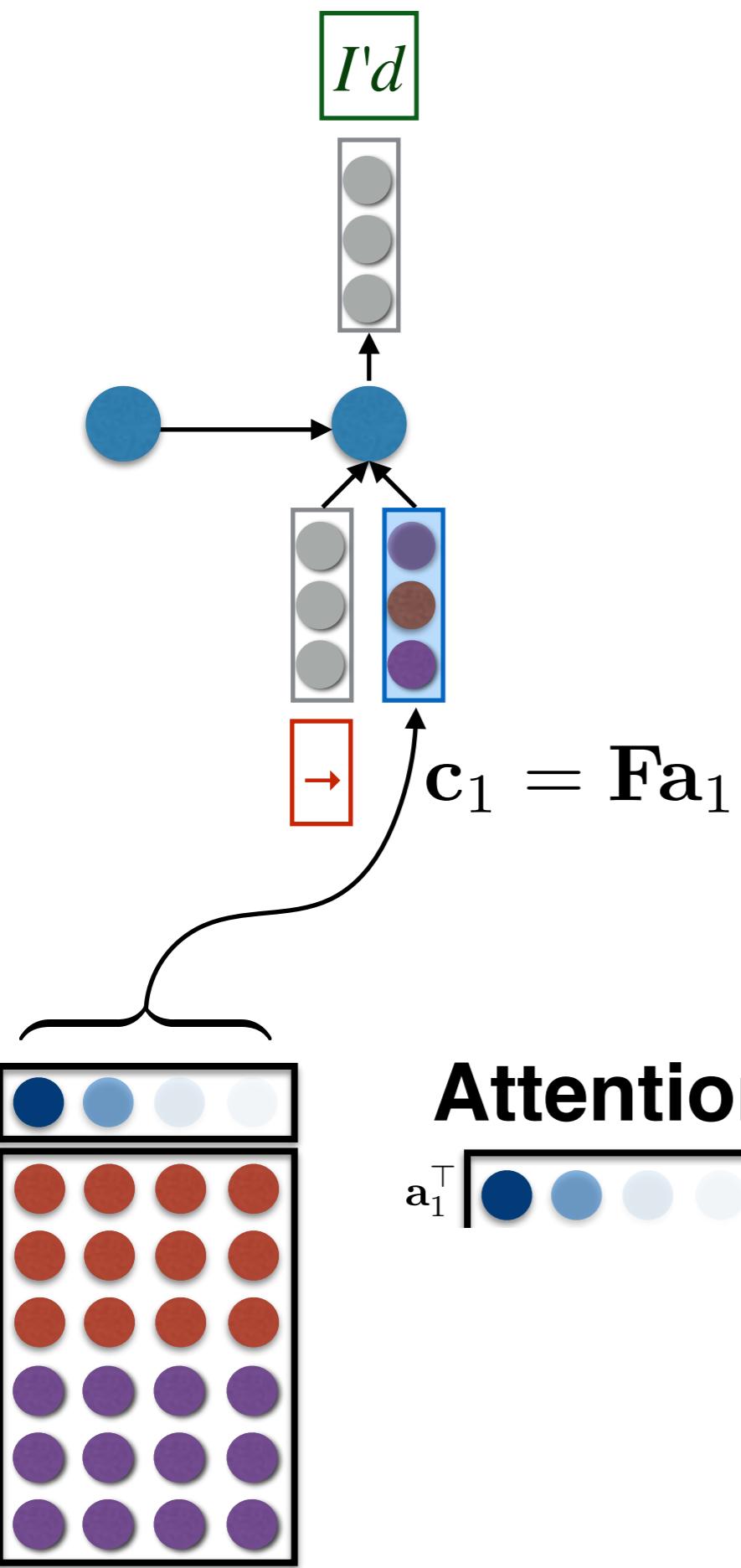
Ich möchte ein Bier



Attention history:

$$a_1^\top \boxed{\text{blue} \quad \text{light blue} \quad \text{light grey} \quad \text{white}}$$

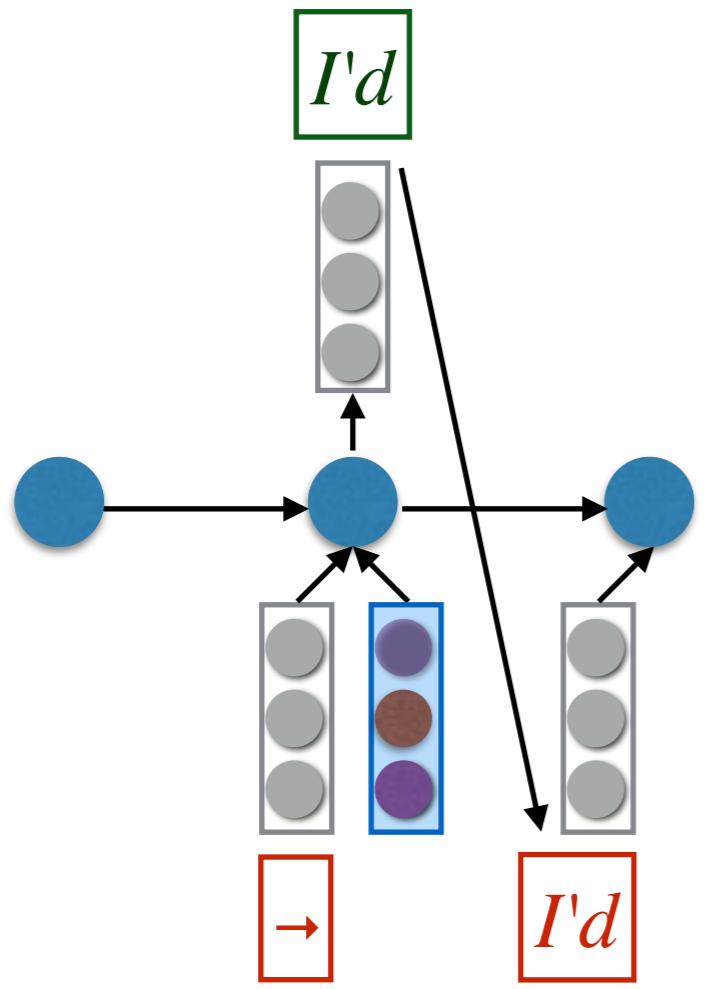
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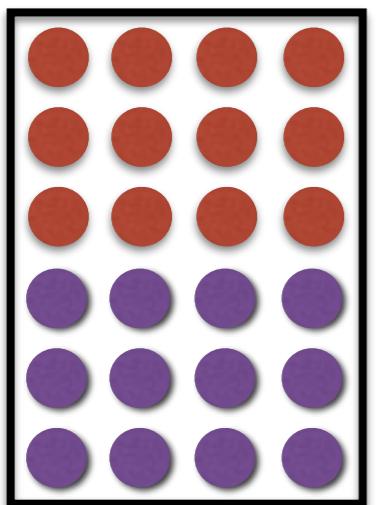
Attention history:

$$a_1^\top \boxed{\text{blue} \quad \text{blue} \quad \text{light blue} \quad \text{white}}$$

Ich möchte ein Bier

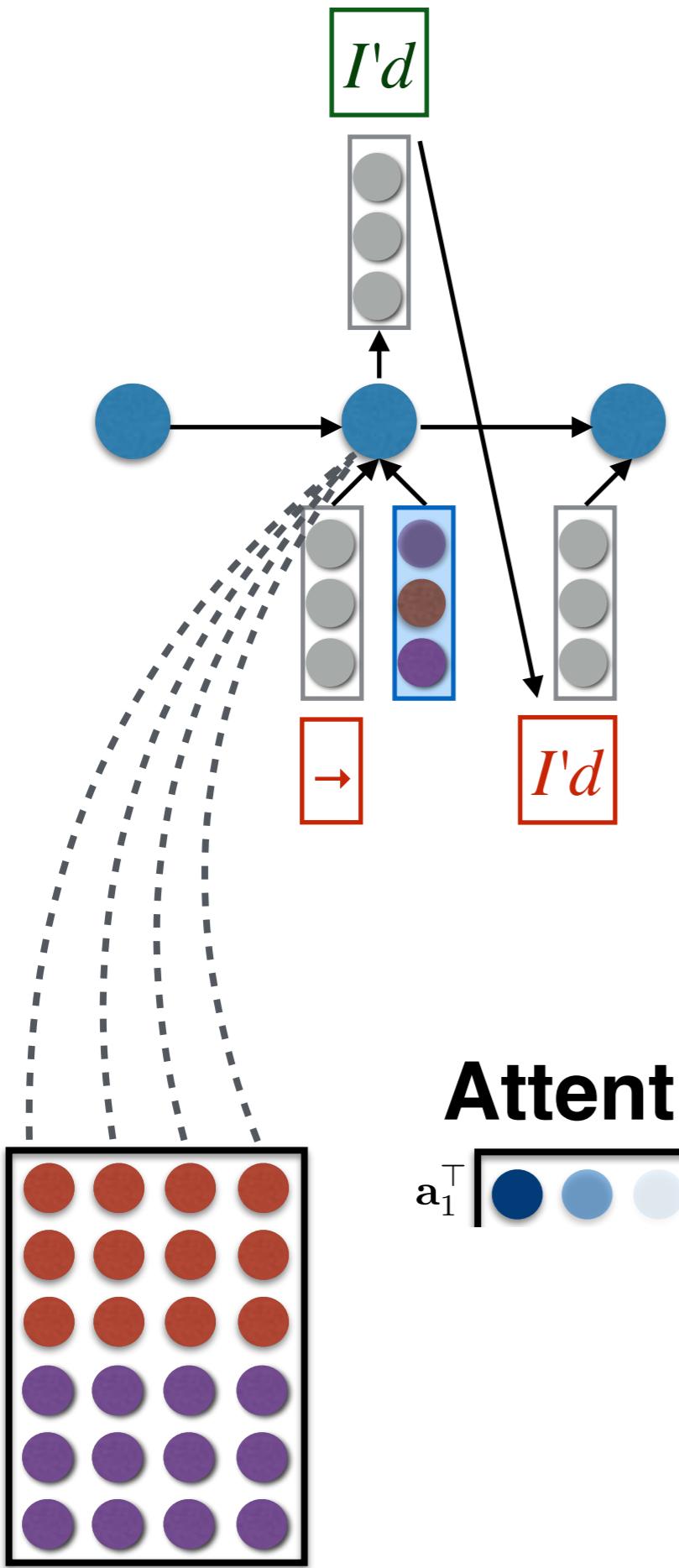


Attention history:



$$a_1^\top [\text{blue} \text{ } \text{blue} \text{ } \text{light blue} \text{ } \text{light blue}]$$

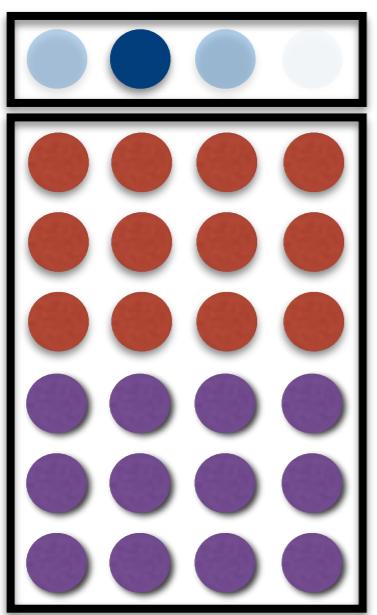
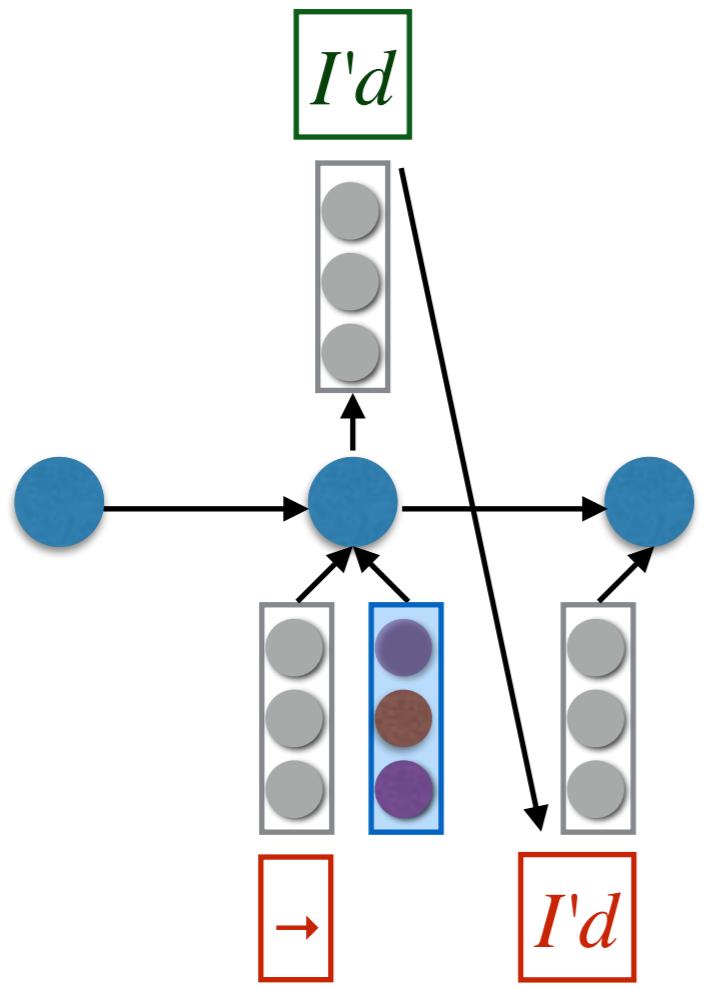
Ich möchte ein Bier



Attention history:

$$a_1^\top [\text{blue dot} \quad \text{blue dot} \quad \text{light blue dot} \quad \text{light blue dot}]$$

Ich möchte ein Bier

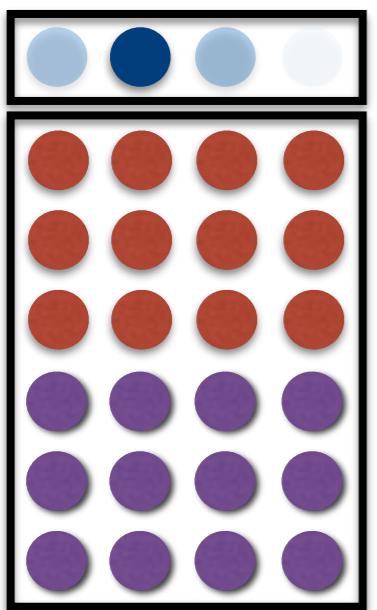
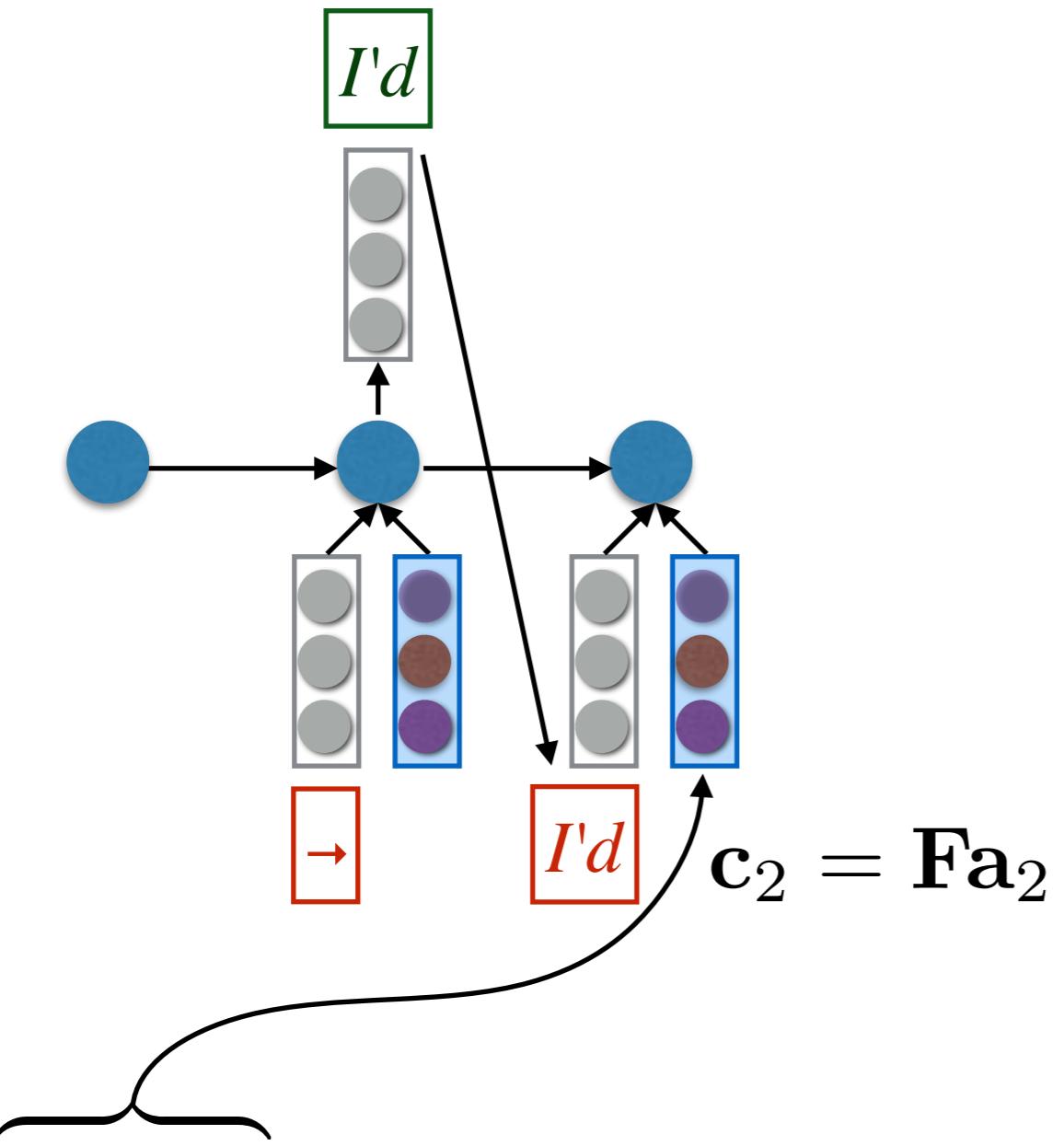


Attention history:

$$a_1^\top \begin{bmatrix} \text{dark blue} & \text{light blue} & \text{white} & \text{white} \end{bmatrix}$$

$$a_2^\top \begin{bmatrix} \text{light blue} & \text{dark blue} & \text{light blue} & \text{white} \end{bmatrix}$$

Ich möchte ein Bier

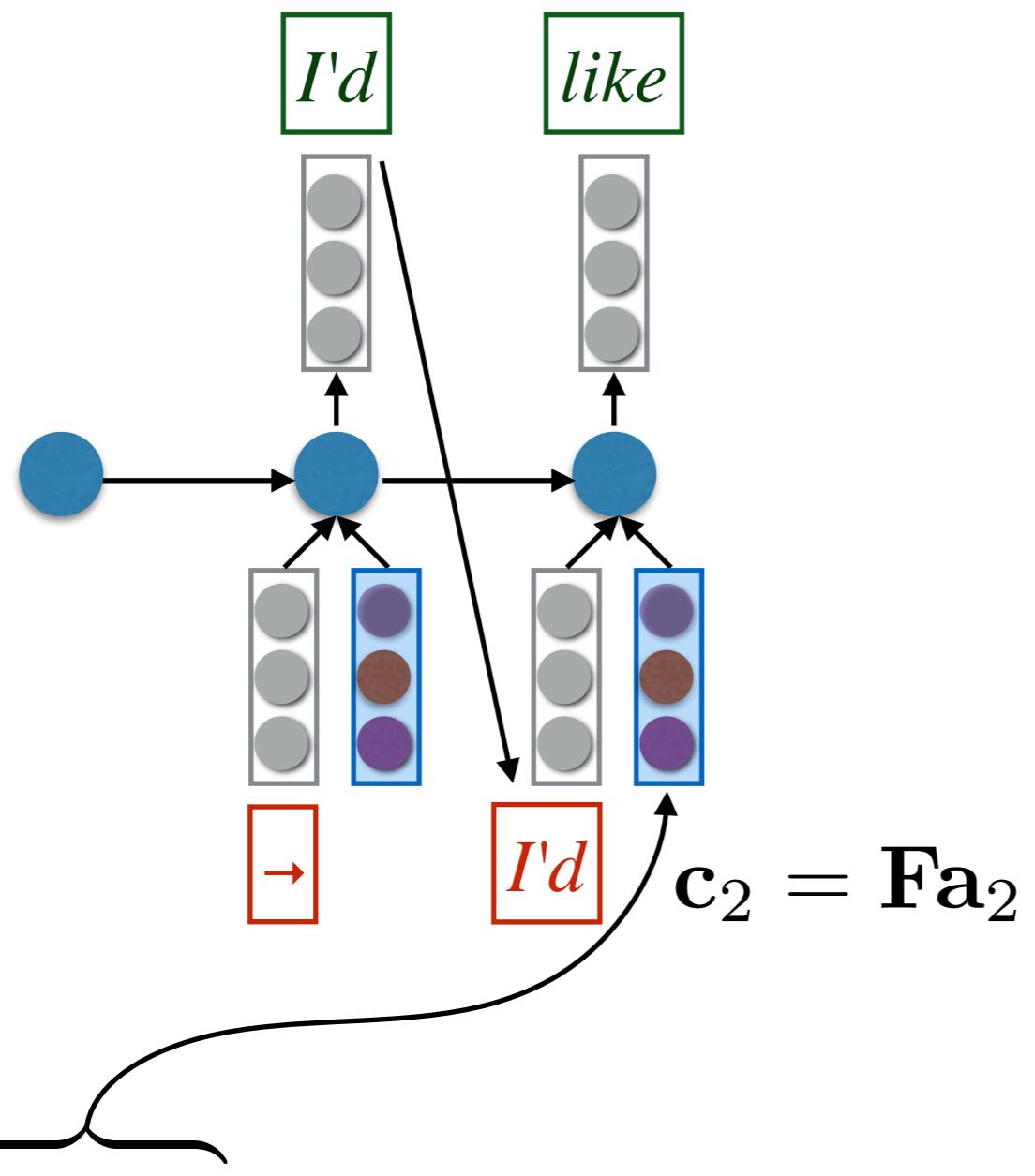


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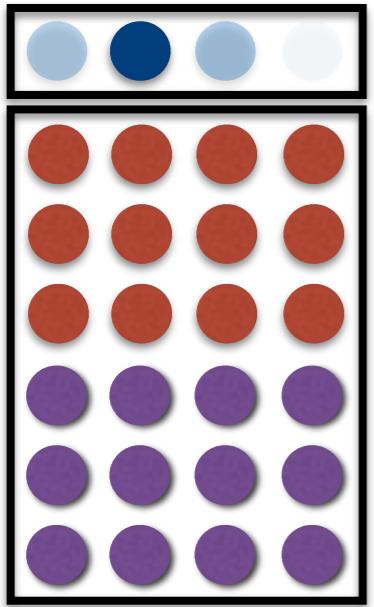
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$$a_2^\top \begin{bmatrix} \text{light blue} \\ \text{dark blue} \\ \text{light blue} \\ \text{white} \end{bmatrix}$$

Ich möchte ein Bier



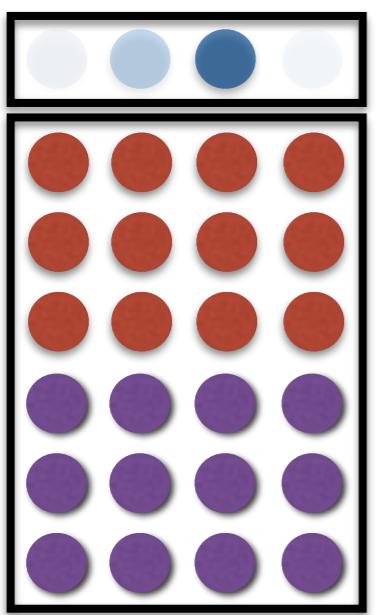
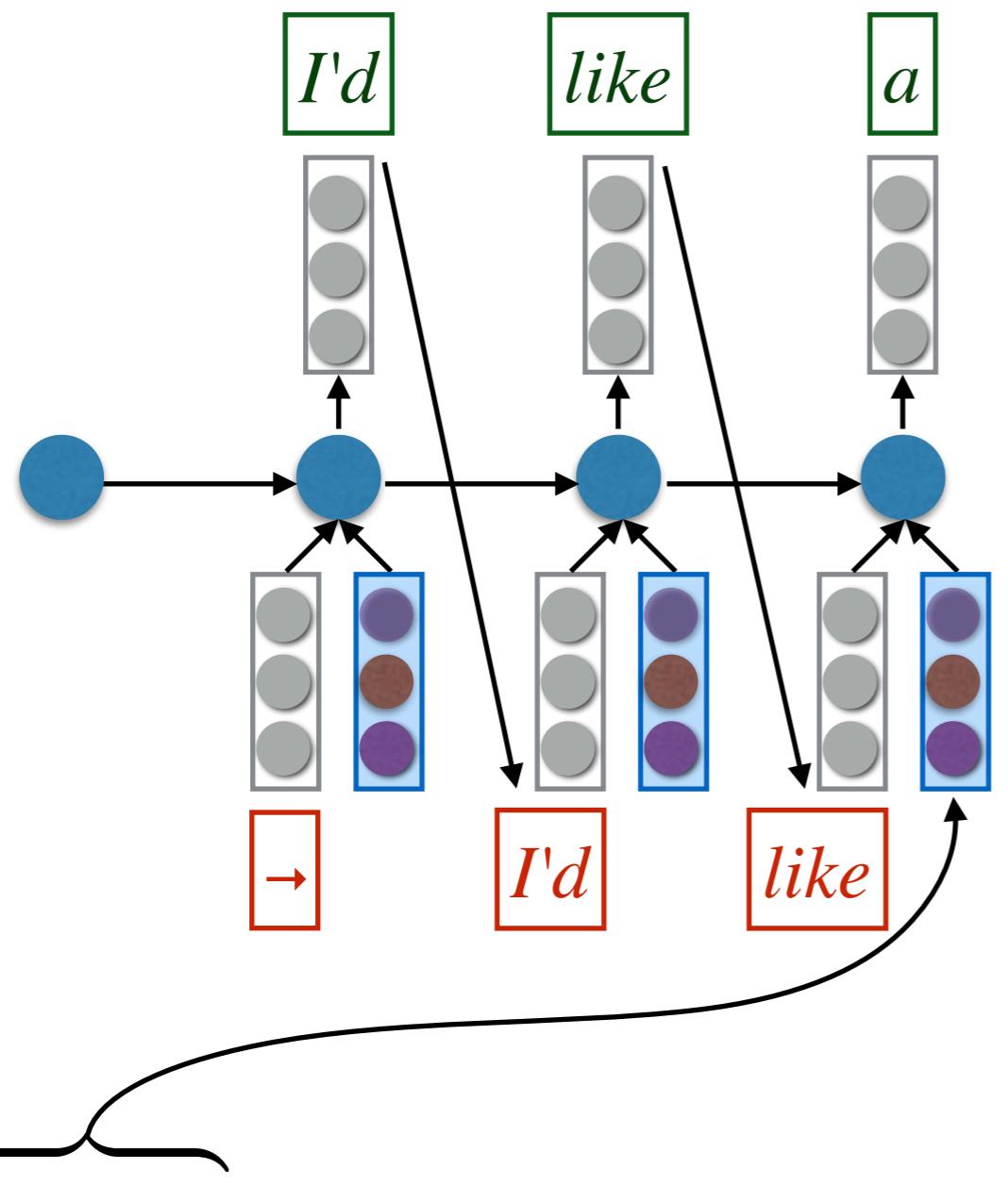
Attention history:



$$a_1^\top \quad \begin{matrix} \text{red} & \text{red} & \text{red} & \text{red} \\ \text{red} & \text{red} & \text{red} & \text{red} \\ \text{red} & \text{red} & \text{red} & \text{red} \\ \text{purple} & \text{purple} & \text{purple} & \text{purple} \\ \text{purple} & \text{purple} & \text{purple} & \text{purple} \end{matrix}$$

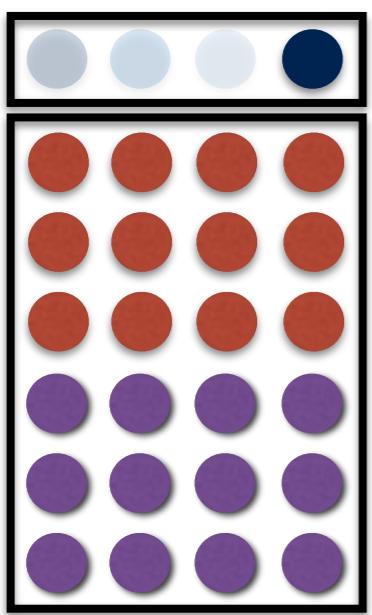
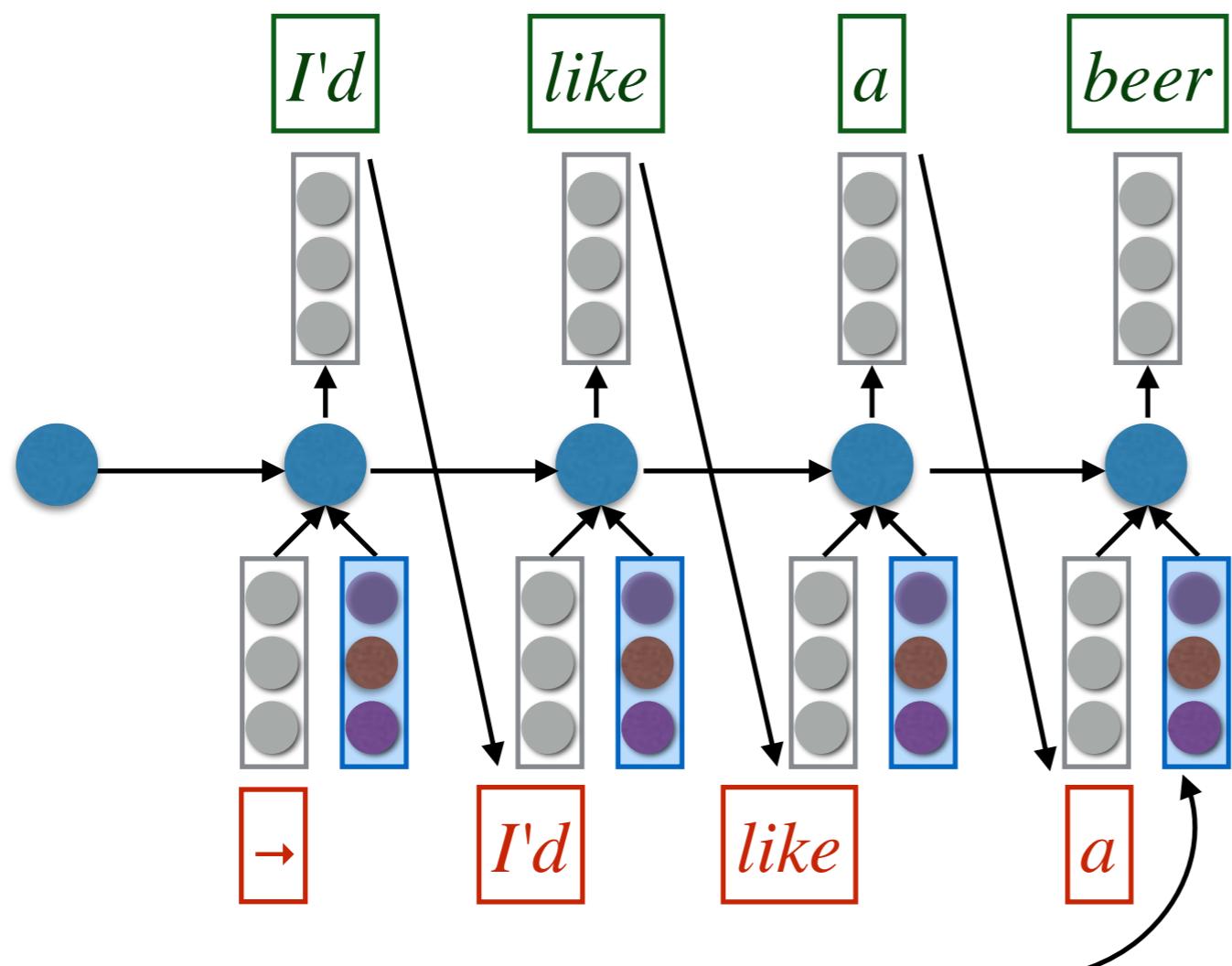
$$a_2^\top \quad \begin{matrix} \text{red} & \text{red} & \text{red} & \text{red} \\ \text{red} & \text{red} & \text{red} & \text{red} \\ \text{red} & \text{red} & \text{red} & \text{red} \\ \text{purple} & \text{purple} & \text{purple} & \text{purple} \\ \text{purple} & \text{purple} & \text{purple} & \text{purple} \end{matrix}$$

Ich möchte ein Bier



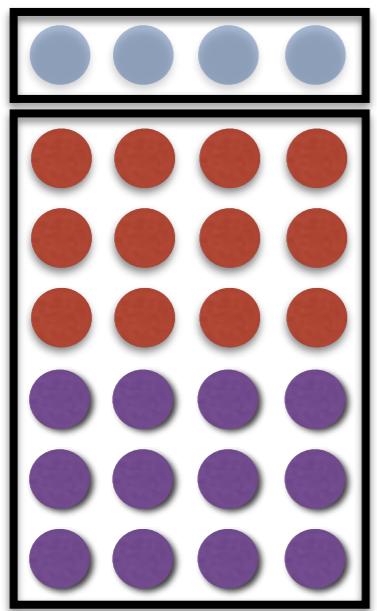
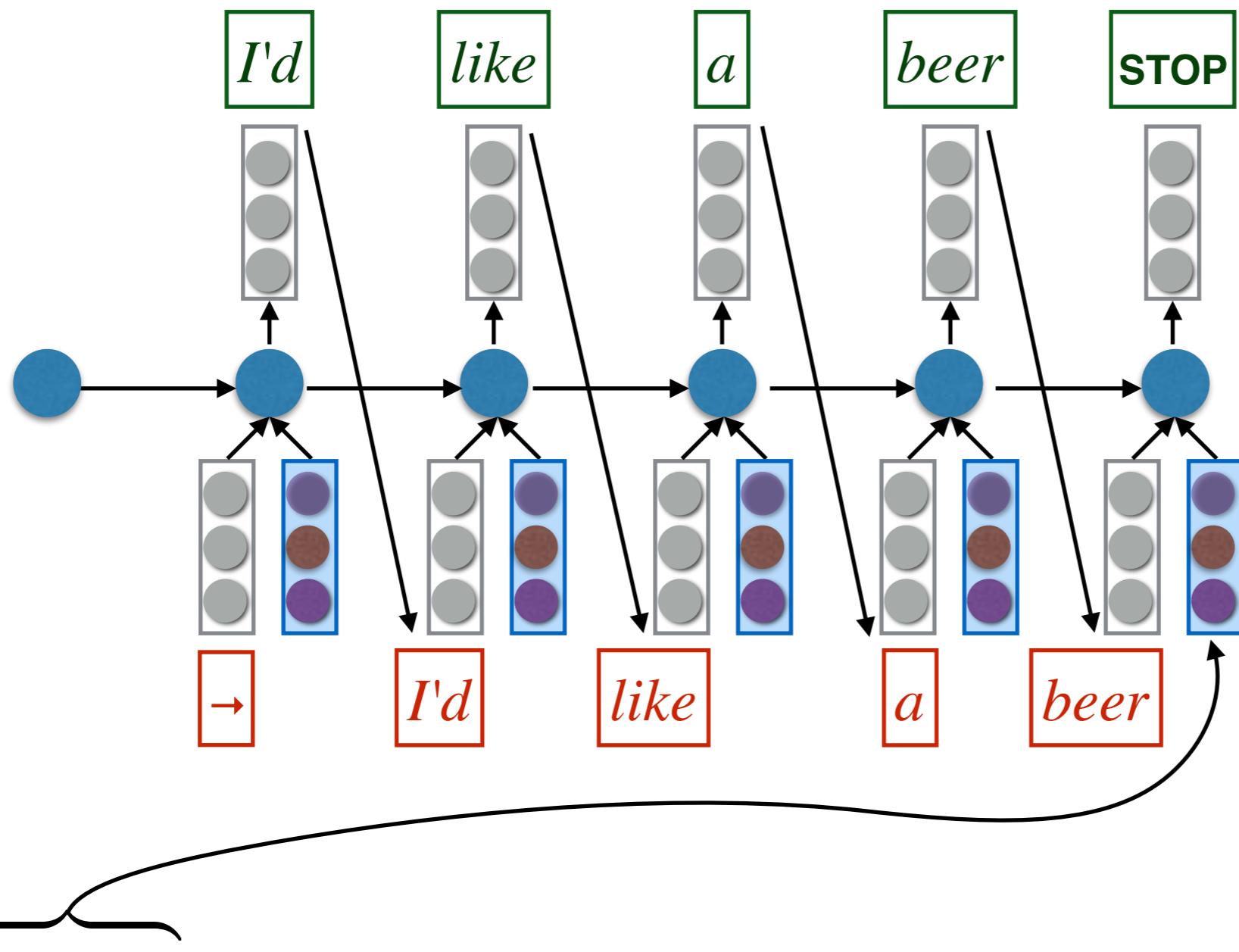
Attention history:

Ich möchte ein Bier

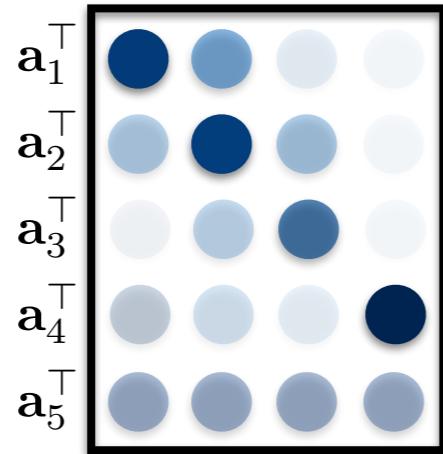


Attention history:

Ich möchte ein Bier



Attention history:



Ich möchte ein Bier

Attention

- How do we know what to attend to at each time-step?
- That is, how do we compute \mathbf{a}_t ?

Computing Attention

- At each time step (one time step = one output word), we want to be able to “attend” to different words in the source sentence
 - We need a weight for every column: this is an $|\mathbf{f}|$ -length vector \mathbf{a}_t
 - Here is a simplified version of Bahdanau et al.’s solution
 - Use an RNN to predict model output, call the hidden states \mathbf{s}_t
(\mathbf{s}_t has a fixed dimensionality, call it m)

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 - At time t compute the **expected input embedding** $\mathbf{r}_t = \mathbf{V}\mathbf{s}_{t-1}$ (\mathbf{V} is a learned parameter)
 - Take the dot product with every column in the source matrix to compute the **attention energy**. $\mathbf{u}_t = \mathbf{F}^\top \mathbf{r}_t$ (called \mathbf{e}_t in the paper)
(Since \mathbf{F} has $|\mathbf{f}|$ columns, \mathbf{u}_t has $|\mathbf{f}|$ rows)

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 - Exponentiate and normalize to 1: $\mathbf{a}_t = \text{softmax}(\mathbf{u}_t)$
(called α_t in the paper)
 - Finally, the **input source vector** for time t is $\mathbf{c}_t = \mathbf{F}\mathbf{a}_t$

Nonlinear Attention-Energy Model

- In the actual model, Bahdanau et al. replace the dot product between the columns of \mathbf{F} and \mathbf{r}_t with an MLP:

$$\mathbf{u}_t = \mathbf{F}^\top \mathbf{r}_t \quad (\text{simple model})$$

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- Here, \mathbf{W} and \mathbf{v} are learned parameters of appropriate dimension and + “broadcasts” over the $|\mathbf{f}|$ columns in \mathbf{WF}
- This can learn more complex interactions
 - It is unclear if the added complexity is necessary for good performance

Putting it all together

$\mathbf{F} = \text{EncodeAsMatrix}(f)$ (Part 1 of lecture)

$e_0 = \langle s \rangle$

$s_0 = w$ (Learned initial state; Bahdanau uses $U^{\leftarrow} h_1$)

$t = 0$

while $e_t \neq \langle /s \rangle$:

$t = t + 1$

$\mathbf{r}_t = \mathbf{V}s_{t-1}$

$\mathbf{u}_t = \mathbf{v}^\top \tanh(\mathbf{W}\mathbf{F} + \mathbf{r}_t)$

$\mathbf{a}_t = \text{softmax}(\mathbf{u}_t)$

$\mathbf{c}_t = \mathbf{F}\mathbf{a}_t$

$\mathbf{s}_t = \text{RNN}(\mathbf{s}_{t-1}, [\mathbf{e}_{t-1}; \mathbf{c}_t])$ (\mathbf{e}_{t-1} is a learned embedding of e_t)

$\mathbf{y}_t = \text{softmax}(\mathbf{P}\mathbf{s}_t + \mathbf{b})$ (\mathbf{P} and \mathbf{b} are learned parameters)

$e_t \mid e_{<t} \sim \text{Categorical}(\mathbf{y}_t)$

}

(Compute attention; part 2 of lecture)

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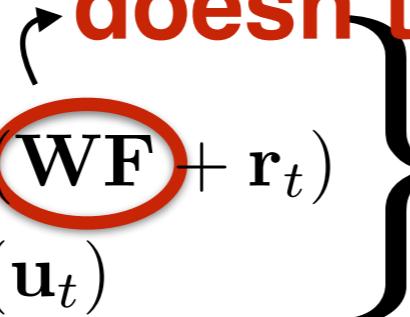
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$e_t | e_{<t} \sim \text{Categorical}(\mathbf{y}_t)$

doesn't depend on output decisions



Putting it all together

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$s_0 = w$ (Learned initial state; Bahdanau uses $U^{\leftarrow} h_1$)

$t = 0$

$X = WF$

while $e_t \neq \langle /s \rangle$:

$t = t + 1$

$r_t = Vs_{t-1}$

$u_t = v^\top \tanh(WF + r_t)$

$a_t = \text{softmax}(u_t)$

$c_t = Fa_t$

$s_t = \text{RNN}(s_{t-1}, [e_{t-1}; c_t])$

} (Compute attention; part 2 of lecture)

$y_t = \text{softmax}(Ps_t + b)$

(P and b are learned parameters)

$e_t | e_{<t} \sim \text{Categorical}(y_t)$

Putting it all together

$\mathbf{F} = \text{EncodeAsMatrix}(f)$ (Part 1 of lecture)

$e_0 = \langle s \rangle$

$s_0 = w$ (Learned initial state; Bahdanau uses $U^{\leftarrow} h_1$)

$t = 0$

$\mathbf{X} = \mathbf{WF}$

while $e_t \neq \langle /s \rangle$:

$t = t + 1$

$\mathbf{r}_t = \mathbf{Vs}_{t-1}$

$\mathbf{u}_t = \mathbf{v}^\top \tanh(\mathbf{X} + \mathbf{r}_t)$

$\mathbf{a}_t = \text{softmax}(\mathbf{u}_t)$

$\mathbf{c}_t = \mathbf{Fa}_t$

$\mathbf{s}_t = \text{RNN}(\mathbf{s}_{t-1}, [\mathbf{e}_{t-1}; \mathbf{c}_t])$

$\mathbf{y}_t = \text{softmax}(\mathbf{Ps}_t + \mathbf{b})$

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}

(Compute attention; part 2 of lecture)

(\mathbf{e}_{t-1} is a learned embedding of e_t)

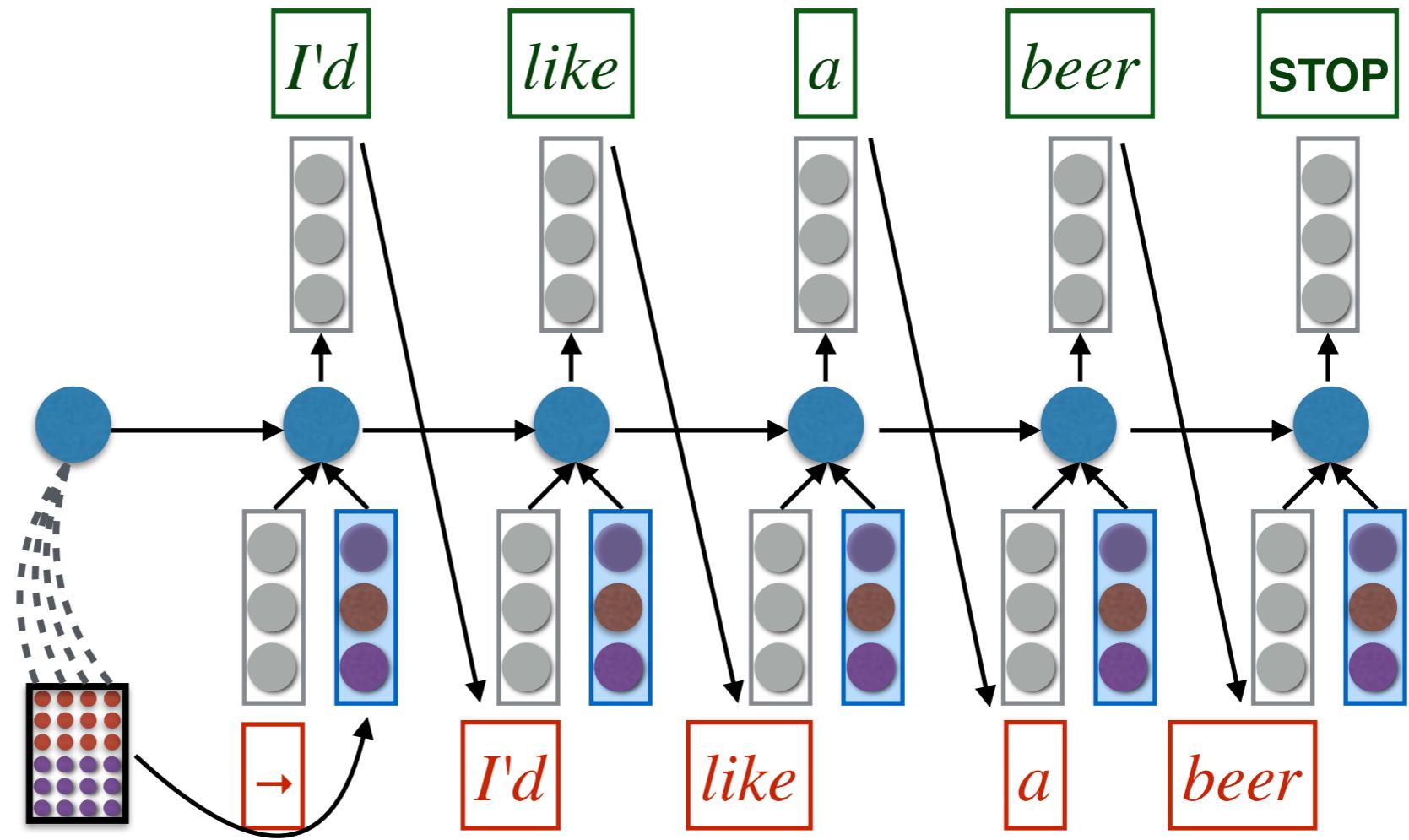
(\mathbf{P} and \mathbf{b} are learned parameters)

Attention in MT

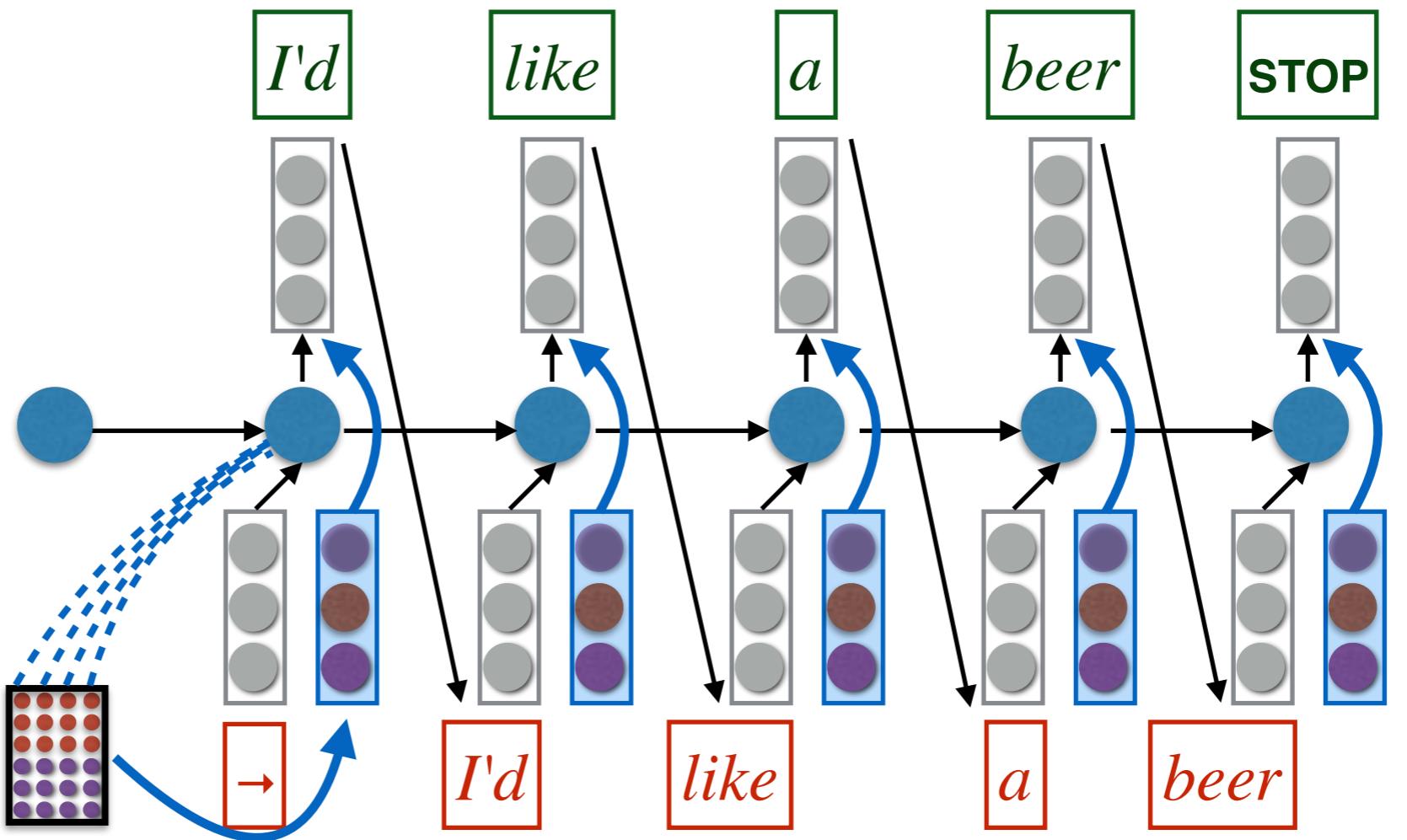
Add attention to seq2seq translation: **+11 BLEU**

Model Variant

“Early binding”



“Late binding”



Model Variant

- What are the relative advantages of early binding versus late binding?

Summary

- Attention is closely related to “pooling” operations in convnets (and other architectures)
- Bahdanau’s attention model seems to only cares about “content”
 - No obvious bias in favor of diagonals, short jumps, fertility, etc.
 - Some work has begun to add other “structural” biases (Luong et al., 2015; Cohn et al., 2016), but there are lots more opportunities
- Attention weights provide interpretation you can look at

L' accord sur la zone économique européenne a été signé en août 1992.

The agreement on the European Economic Area was signed in August 1992.

<end>

(a)

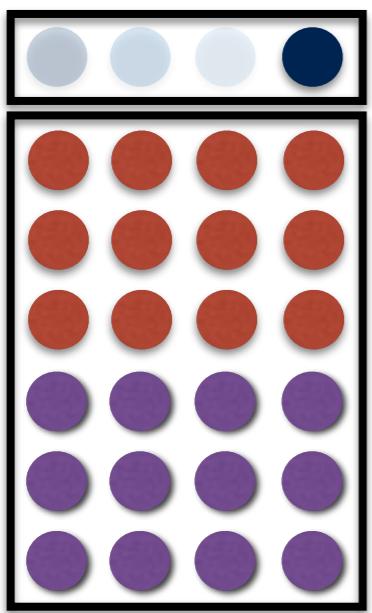
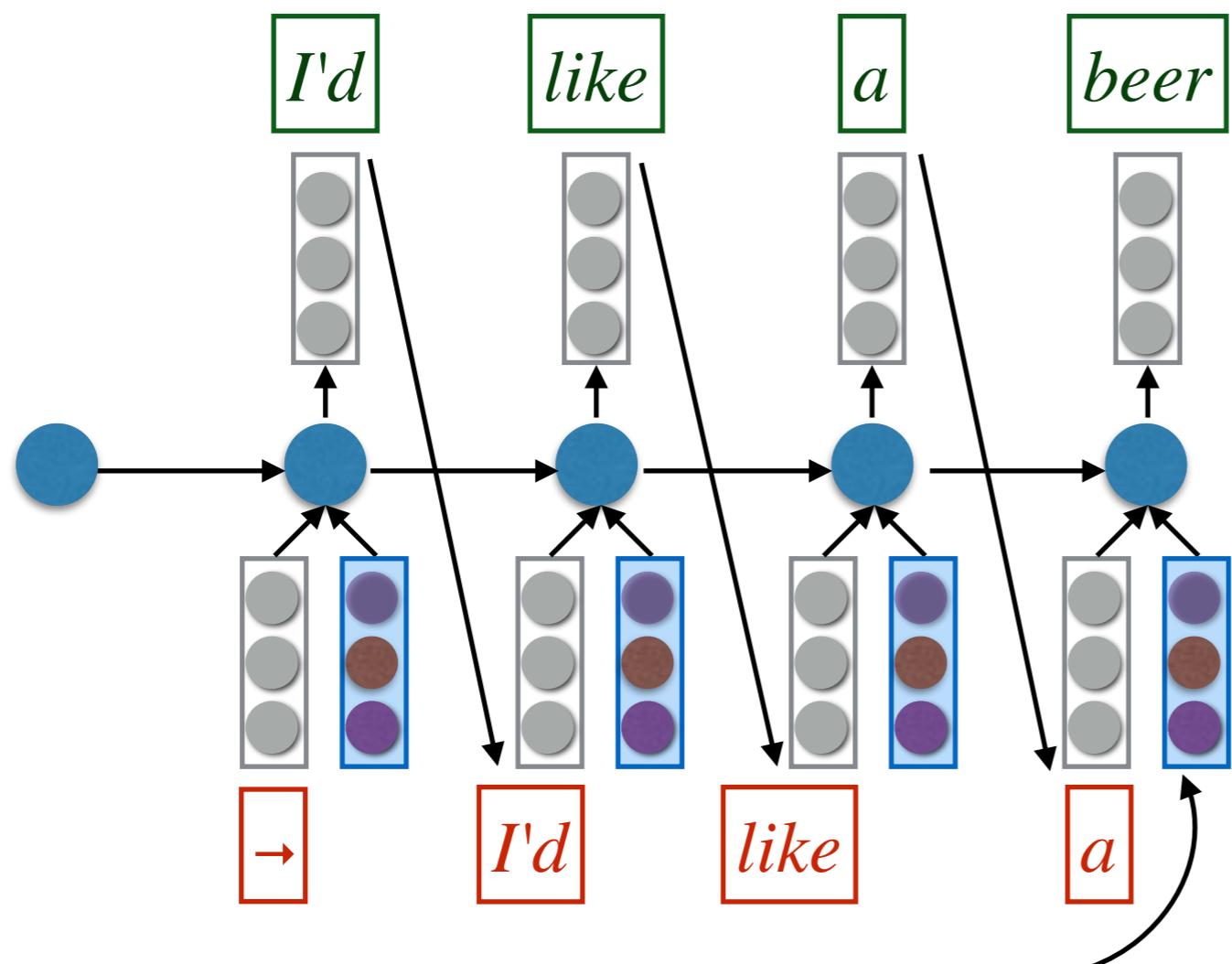
Il convient de noter que l'environnement marin est le moins connu de l'environnement.

It should be noted that the marine environment is the least known of environments.

<end>

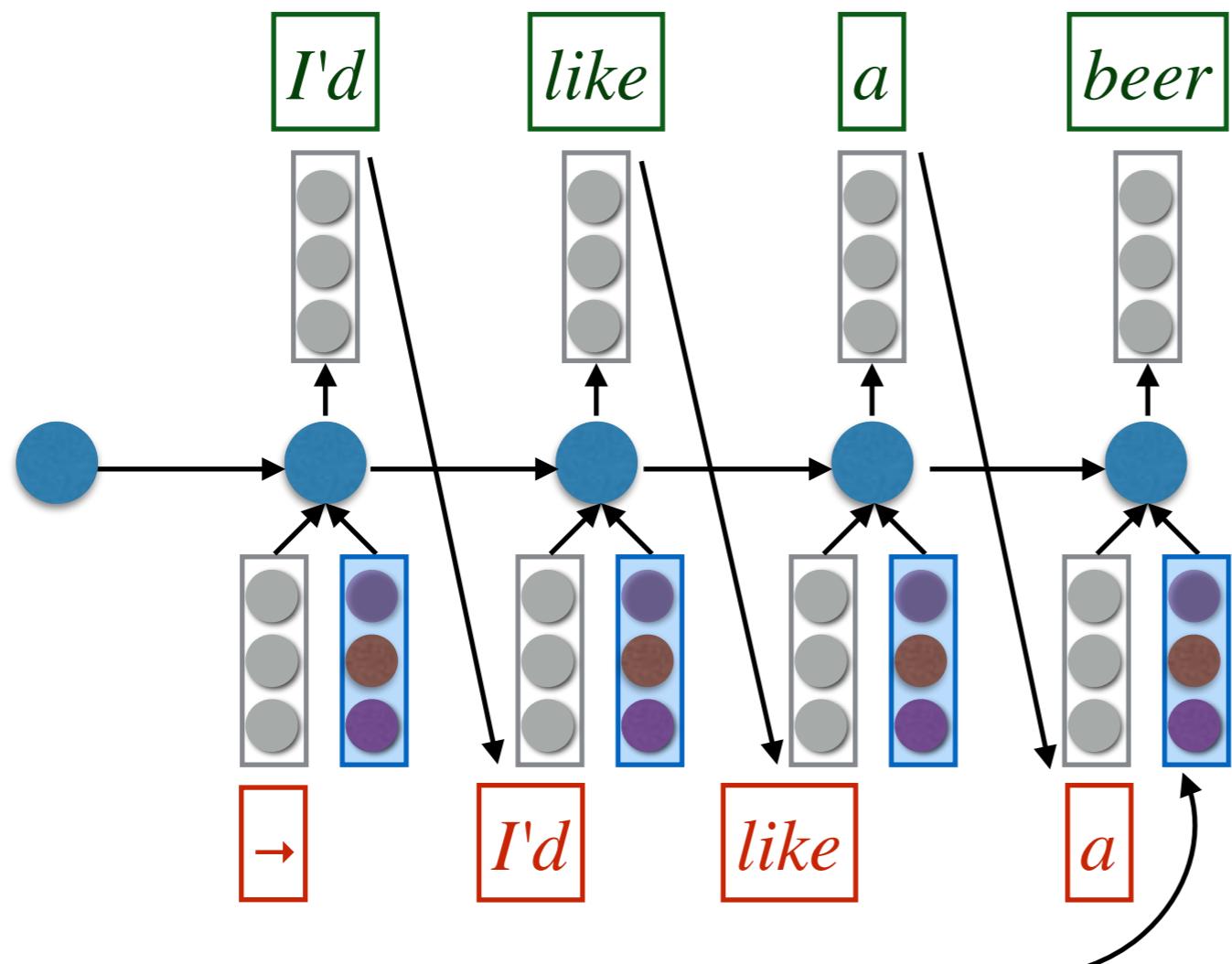
(b)

A word about gradients

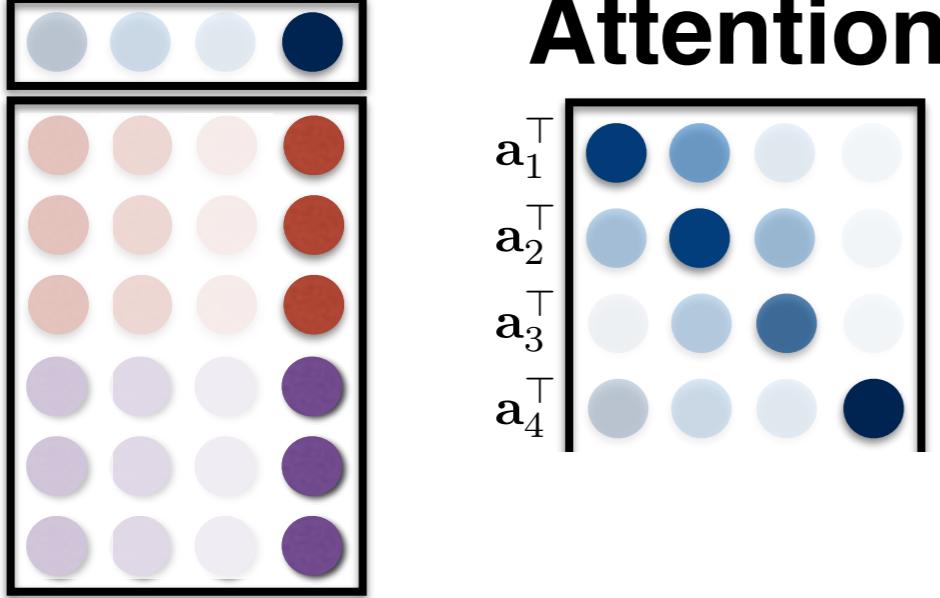


Attention history:

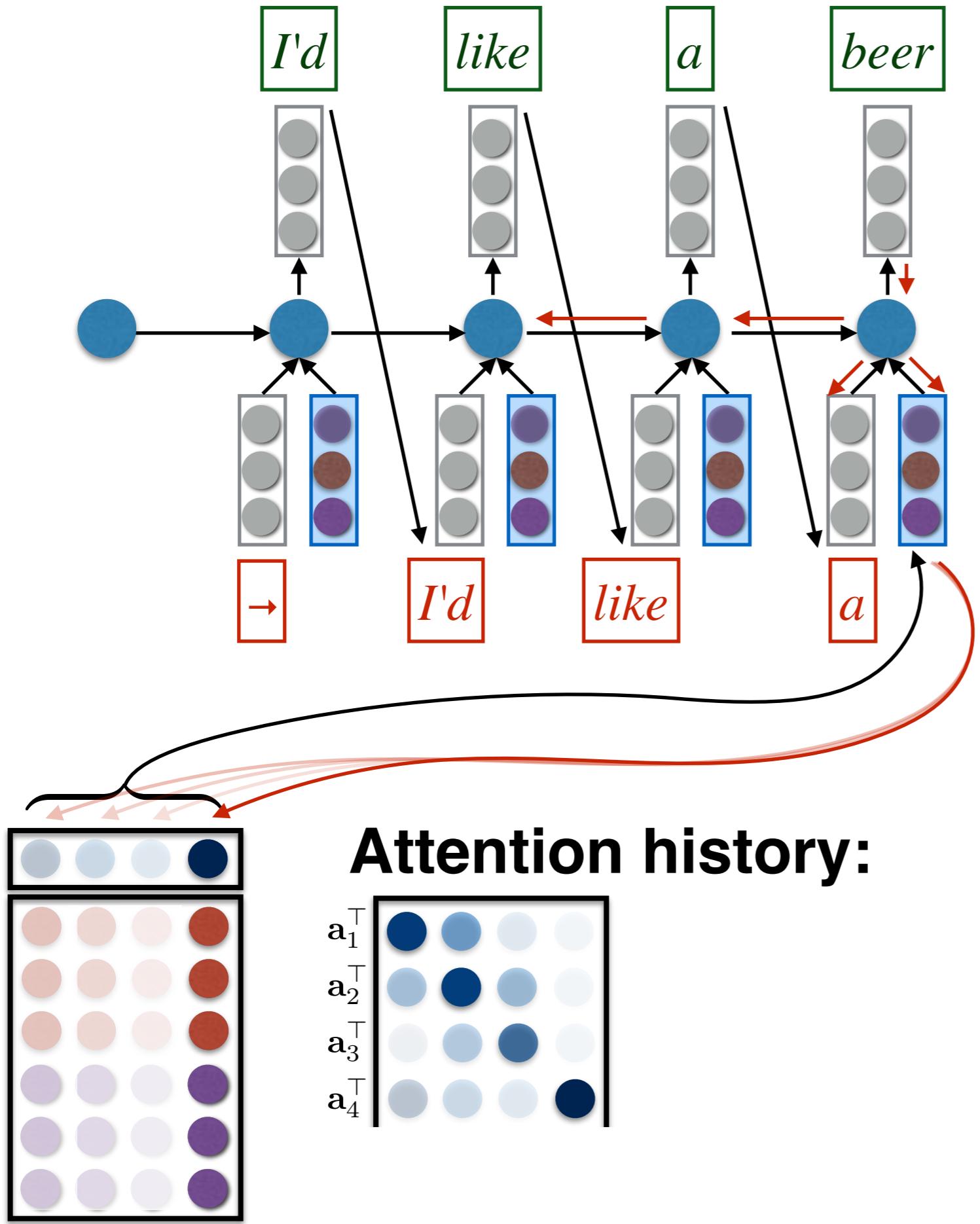
Ich möchte ein Bier



Attention history:



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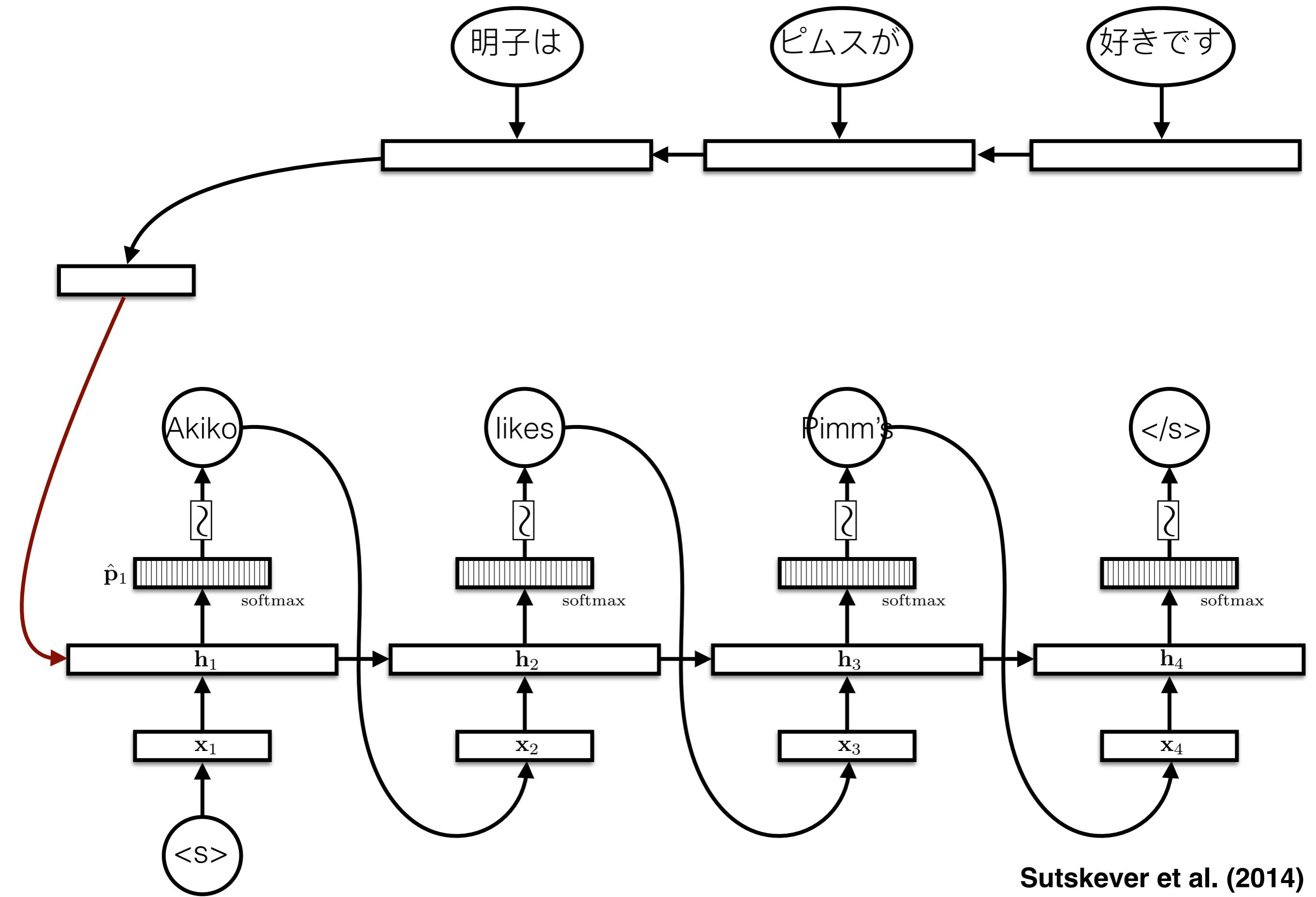


Attention and Translation

- Cho's question: does a translator read and memorize the input sentence/document and then generate the output?
 - Compressing the entire input sentence into a vector basically says “memorize the sentence”
 - Common sense experience says translators refer back and forth to the input. (also backed up by eye-tracking studies)
- Should humans be a model for machines?

Outline of Lecture

- Machine translation with attention
- Image caption generation with attention



Sutskever et al. (2014)

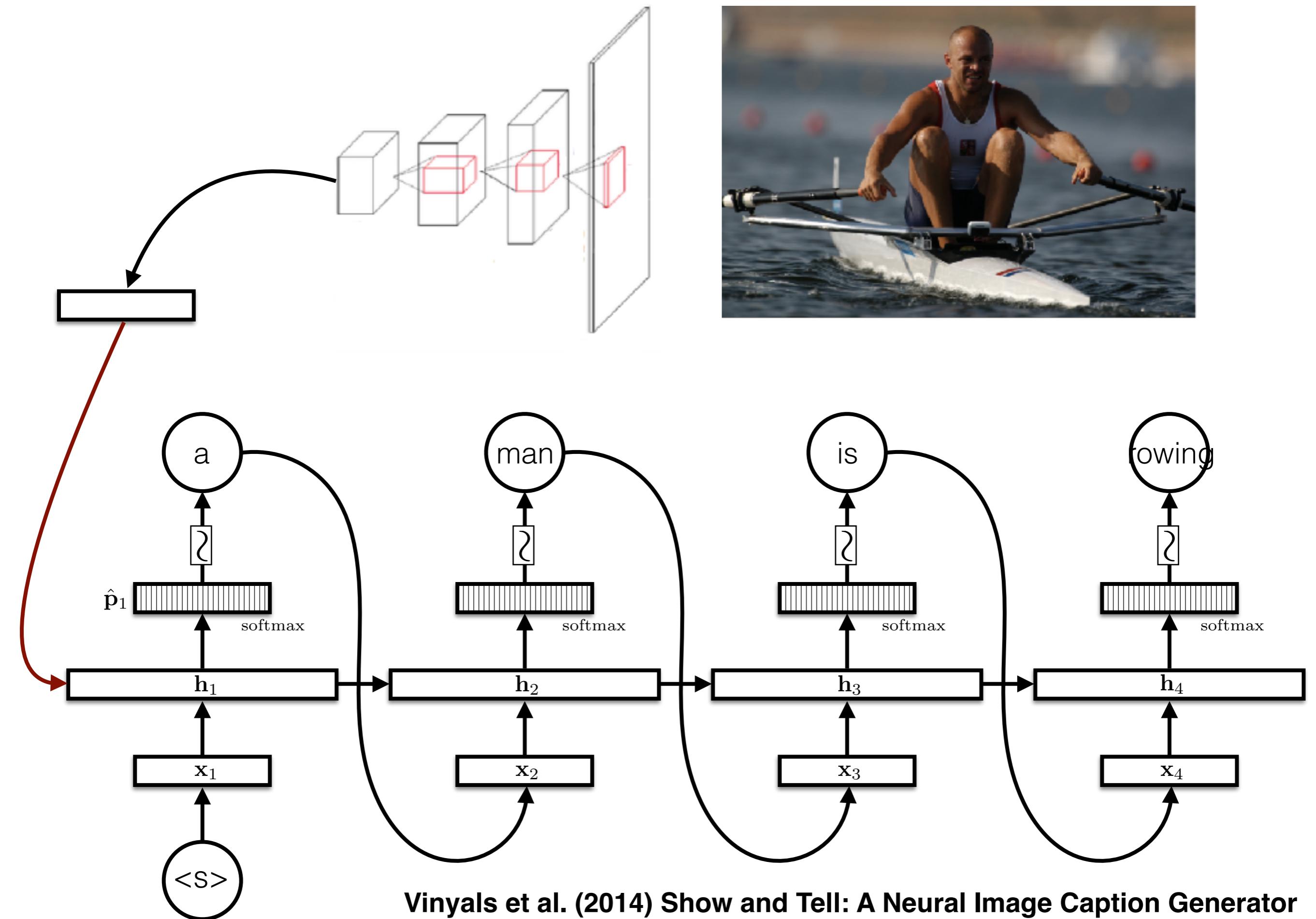


Image Caption Generation

- Can attention help caption modeling?

Show, Attend and Tell: Neural Image Caption Generation with Visual Attention

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Jimmy Lei Ba

Ryan Kiros

Kyunghyun Cho

Aaron Courville

Ruslan Salakhutdinov

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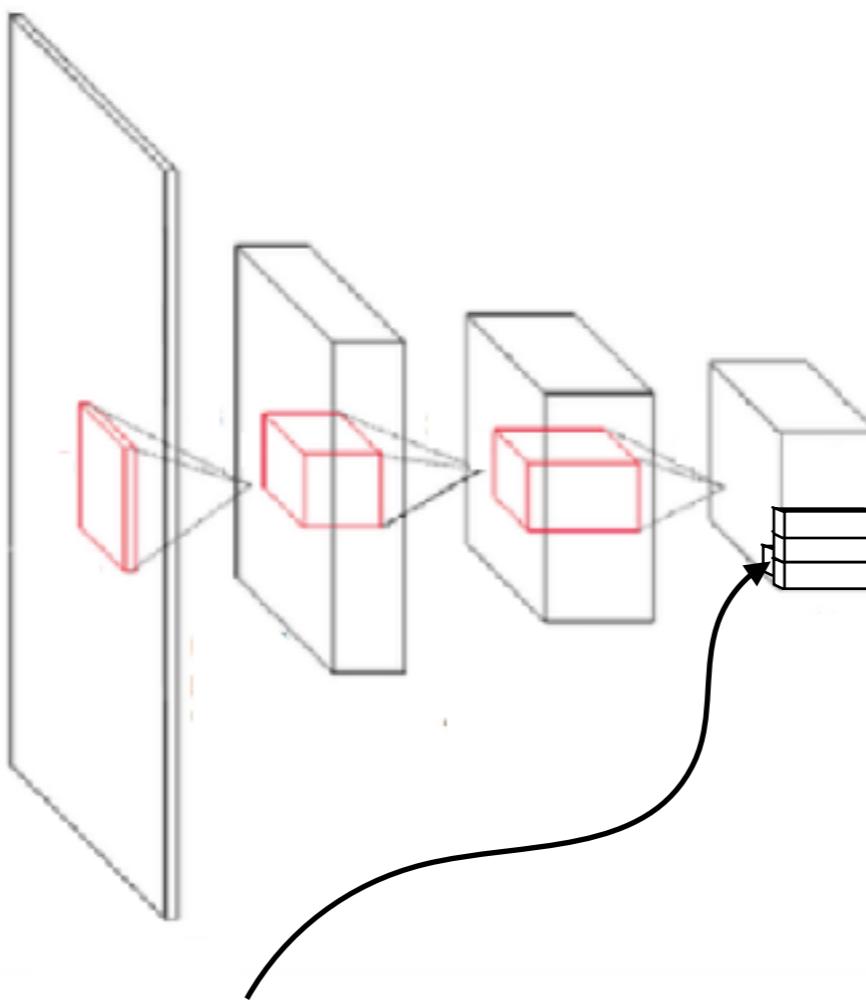
AARON.COURVILLE@UMONTREAL.CA

RSALAKHU@CS.TORONTO.EDU

ZEMEL@CS.TORONTO.EDU

FIND-ME@THE.WEB

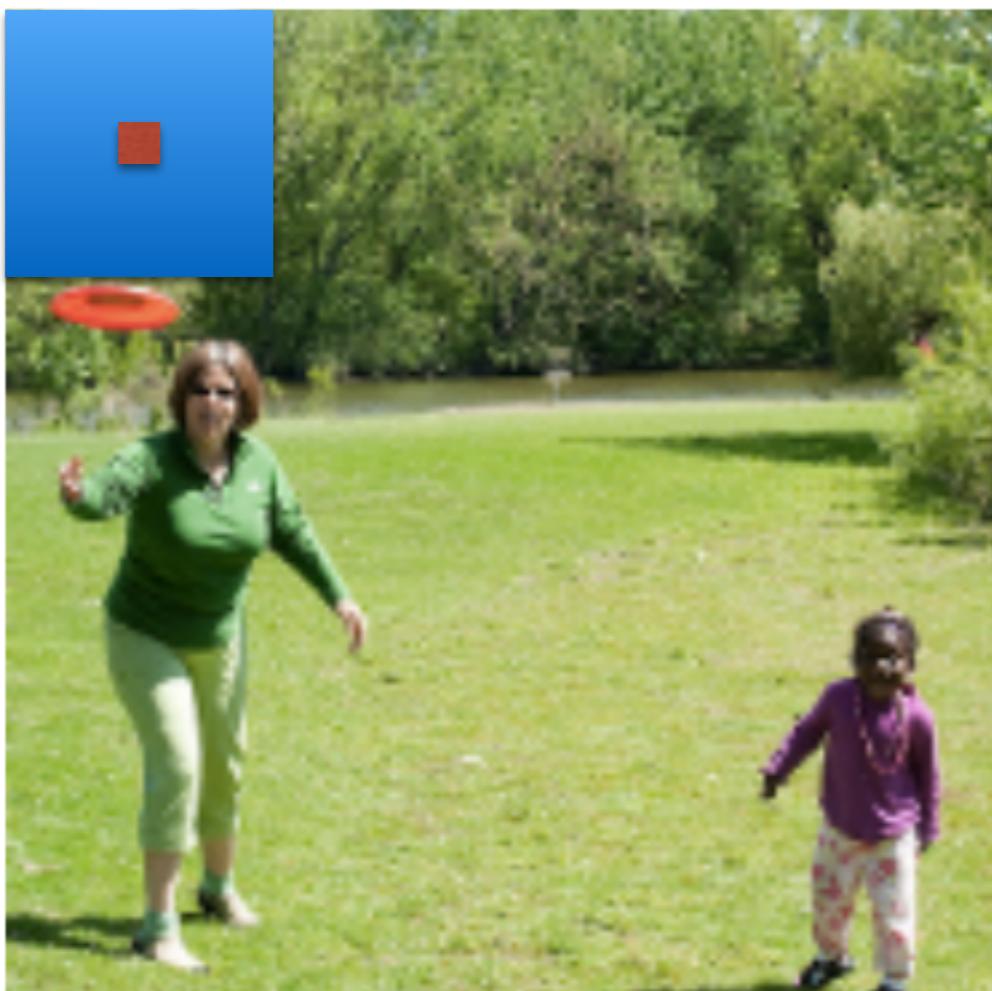
Regions in ConvNets



Each point in a “higher” level of a convnet defines spatially localised feature vectors(/matrices).

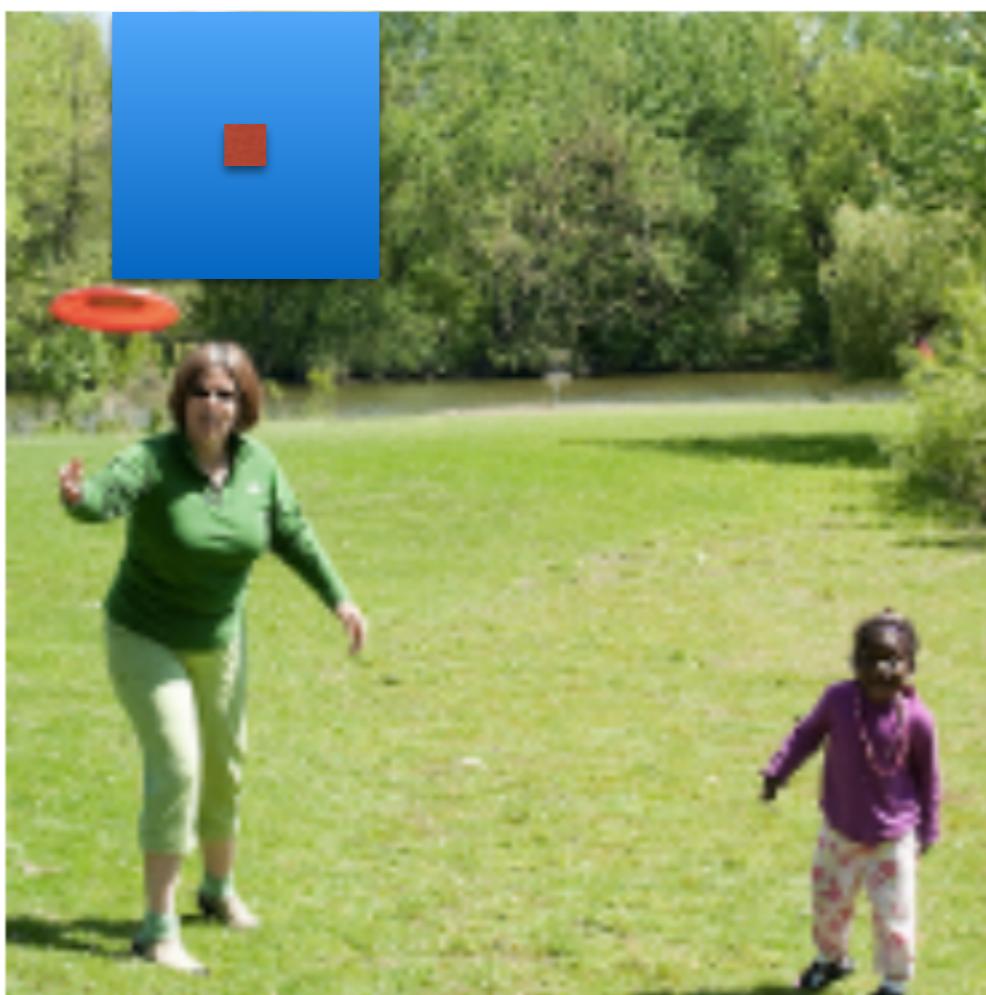
Xu et al. calls these “annotation vectors”, \mathbf{a}_i , $i \in \{1, \dots, L\}$

\mathbf{a}_1



$$\mathbf{F} = \begin{bmatrix} | \\ \mathbf{a}_1 \\ | \end{bmatrix}$$

\mathbf{a}_2



$$\mathbf{F} = \begin{bmatrix} | & | \\ \mathbf{a}_1 & \mathbf{a}_2 \\ | & | \end{bmatrix}$$

a₃



$$\mathbf{F} = \begin{bmatrix} | & | & | \\ \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 & \cdots \\ | & | & | \end{bmatrix}$$

Attention

- Attention “weights” (a_t) are computed using exactly the same technique as discussed above

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- Deterministic soft attention (Bahdanau et al., 2014)

$$\mathbf{c}_t = \mathbf{F}\mathbf{a}_t \quad (\text{weighted average})$$

- Stochastic hard attention (Xu et al., 2015)

$$s_t \sim \text{Categorical}(\mathbf{a}_t)$$

$$\mathbf{c}_t = \mathbf{F}_{:,s_t} \quad (\text{sample a column})$$

- What are the benefits of this model?
- What are the challenges of learning the parameters of this model?

Learning Hard Attention

$$\begin{aligned}\mathcal{L} &= -\log p(\mathbf{w} \mid \mathbf{x}) \\ &= -\log \sum_{\mathbf{s}} p(\mathbf{w}, \mathbf{s} \mid \mathbf{x}) \\ &= -\log \sum_{\mathbf{s}} p(\mathbf{s} \mid \mathbf{x}) p(\mathbf{w} \mid \mathbf{x}, \mathbf{s})\end{aligned}$$

Learning Hard Attention

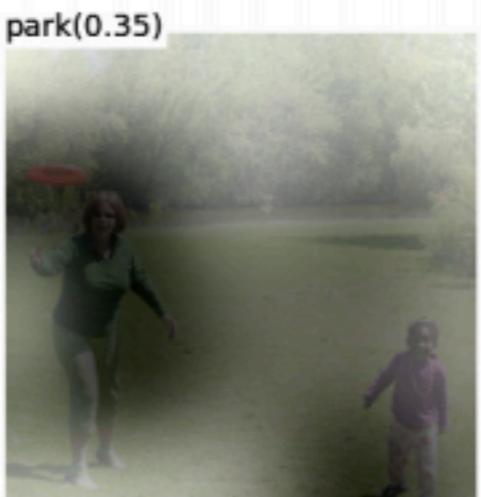
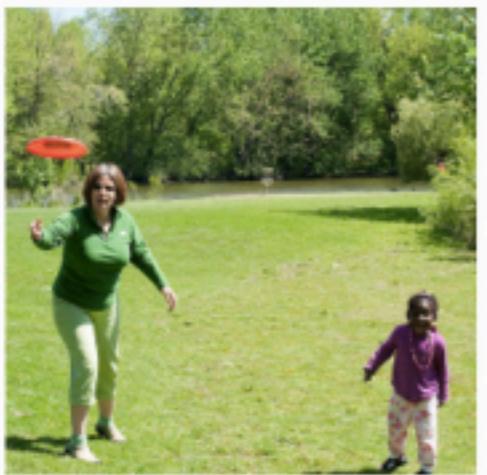
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Learning Hard Attention

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Learning Hard Attention

- Sample N sequences of attention decisions from the model
- The gradient is the probability of the gradient of the probability of this sequence scaled by the log probability of generating the target words using that sequence of attention decisions
- This is equivalent to using the REINFORCE algorithm (Williams, 1992) using the log probability of the observed words as a “reward function”. REINFORCE a policy gradient algorithm used for reinforcement learning.





A woman holding a clock in her hand.



A large white bird standing in a forest.

Attention in Captioning

Add soft attention to image captioning: **+2 BLEU**

Add hard attention to image captioning: **+4 BLEU**

Summary

- Significant performance improvements
 - Better performance over vector-based encodings
 - Better performance with smaller training data sets
- Model interpretability
- Better gradient flow
- Better capacity (especially obvious for translation)

Questions?