Muffix Sassif – TRD

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Содержание

1 Геометрия

1.1 3D

```
double eps = 1e-7;

struct Pt {
  double x;
  double y;
  double z;

  Pt(double x_, double y_, double z_) : x(x_), y(y_), z(
    z_) {}

  Pt operator-(const Pt& other) const {
    return {x - other.x, y - other.y, z - other.z};
  }

  Pt operator+(const Pt& other) const {
    return {x + other.x, y + other.y, z + other.z};
  }

  Pt operator/(const double& a) const {
    return {x / a, y / a, z / a};
  }

  Pt operator*(const double& a) const {
    return {x * a, y * a, z * a};
  }

  Pt cross(const Pt& p2) const {
    double nx = y * p2.z - z * p2.y;
    double ny = z * p2.x - x * p2.z;
    double nz = x * p2.y - y * p2.x;
    return {nx, ny, nz};
  }

  bool operator==(const Pt& pt) const {
    return abs(x - pt.x) < eps && abs(y - pt.y) < eps &&
      abs(z - pt.z) < eps;
  }

  double dist() {
    return sqrtl(x * x + y * y + z * z);
  }
};

struct Plane {
  double a, b, c, d;

  Plane(double a_, double b_, double c_, double d_) : a(
    a_), b(b_), c(c_), d(d_) {
    double kek = sqrtl(a * a + b * b + c * c);
    if (kek < eps) return;
    a /= kek;
    b /= kek;
    c /= kek;
    d /= kek;
  }

  double get_val(Pt p) {
    // НЕ СТАВИТЬ МОДУЛЬ
    return a * p.x + b * p.y + c * p.z + d;
  }

  double dist(Pt p) {
    return abs(get_val(p));
  }

  bool on_plane(Pt p) {
    return abs(get_val(p)) / sqrtl(a * a + b * b + c * c
    ) < eps;
  }

  Pt proj(Pt p) {
    double t = (a * p.x + b * p.y + c * p.z + d) / (a *
    a + b * b + c * c);
    return p - Pt(a, b, c) * t;
  }
};

bool on_line(Pt p1, Pt p2, Pt p3) {
  return (p2 - p1).cross(p3 - p1) == Pt(0, 0, 0);
}

Plane get_plane(Pt p1, Pt p2, Pt p3) {
  Pt norm = (p2 - p1).cross(p3 - p1);
  Plane pl(norm.x, norm.y, norm.z, 0);
  pl.d = -pl.get_val(p1);
  return pl;
}

pair<pair<double, double>, pair<double, double>> get_xy(
    double a, double b, double c) {
  if (abs(a) > eps) {
    double y1 = 0, y2 = 10;
    return {{(-c - b * y1) / a, y1}, {(-c - b * y2) / a,
     y2}};
  }
  double x1 = 0, x2 = 10;
  return {{x1, (-c - a * x1) / b}, {x2, (-c - a * x2) /
    b}};
}

pair<Pt, Pt> intersect(Plane pl1, Plane pl2) {
  if (abs(pl2.a) < eps && abs(pl2.b) < eps && abs(pl2.c)
     < eps) {
    assert(false);
  }
  if (abs(pl2.a) > eps) {
    double nd = pl1.d - pl1.a * pl2.d / pl2.a;
    double nc = pl1.c - pl1.a * pl2.c / pl2.a;
    double nb = pl1.b - pl1.a * pl2.b / pl2.a;
    if (abs(nc) < eps && abs(nb) < eps) {
      // плоскости параллельны (могут совпадать)
      return {Pt(0, 0, 0), Pt(0, 0, 0)};
    }
    auto [yz1, yz2] = get_xy(nb, nc, nd);
    double x1 = (-pl2.d - pl2.c * yz1.second - pl2.b *
    yz1.first) / pl2.a;
    double x2 = (-pl2.d - pl2.c * yz2.second - pl2.b *
    yz2.first) / pl2.a;
    return {Pt(x1, yz1.first, yz1.second), Pt(x2, yz2.
    first, yz2.second)};
  }
  Plane copy_pl1(pl1.c, pl1.a, pl1.b, pl1.d);
  Plane copy_pl2(pl2.c, pl2.a, pl2.b, pl2.d);
  auto [p1, p2] = intersect(copy_pl1, copy_pl2);
  return {Pt(p1.y, p1.z, p1.x), Pt(p2.y, p2.z, p2.x)};
}
```

1.2 Вектор, прямая, окружность

```
//// Вектор ////

struct vctr {
  dbl x, y;
  vctr() {}
  vctr(dbl x, dbl y) : x(x), y(y) {}

  dbl operator%(const vctr &o) const { return x * o.x +
    y * o.y; }
  dbl operator*(const vctr &o) const { return x * o.y -
    y * o.x; }
  vctr operator+(const vctr &o) const { return {x + o.x,
     y + o.y}; }
  vctr operator-(const vctr &o) const { return {x - o.x,
     y - o.y}; }
  vctr operator-() const { return {-x, -y}; }
  vctr operator*(const dbl d) const { return {x * d, y *
     d}; }
  vctr operator/(const dbl d) const { return {x / d, y /
     d}; }
  void operator+=(const vctr &o) { x += o.x, y += o.y; }
  void operator-=(const vctr &o) { x -= o.x, y -= o.y; }
  dbl dist2() const { return x * x + y * y; }
  dbl dist() const { return sqrtl(dist2()); }
  vctr norm() const { return *this / dist(); }
};

dbl angle_between(const vctr &a, const vctr &b) {
  return atan2(b * a, b % a);
}

// y > 0 ? 0 : 1
bool is2plane(const vctr &a) {
  return sign(a.y) < 0 || (sign(a.y) == 0 && sign(a.x) <
     0);
}

bool cmp_angle(const vctr &a, const vctr &b) {
  bool pla = is2plane(a);
  bool plb = is2plane(b);
  if (pla != plb)
    return pla < plb;
  return sign(a * b) > 0;
}

//// Прямая ////
```

```
struct line {
  dbl a, b, c;

  line() {}
  line(dbl a, dbl b, dbl c) : a(a), b(b), c(c) {}
  line(const vctr A, const vctr B) {
    a = A.y - B.y;
    b = B.x - A.x;
    c = A * B;
    assert(a != 0 || b != 0);
  }

  void operator*=(dbl x) { a *= x, b *= x, c *= x; }
  void operator/=(dbl x) { a /= x, b /= x, c /= x; }
  dbl get(const vctr P) const { return a * P.x + b * P.y
     + c; }
  vctr anyPoint() const {
    dbl x = -a * c / (a * a + b * b);
    dbl y = -b * c / (a * a + b * b);
    return vctr(x, y);
  }
  void normalize() {
    dbl d = sqrtl(a * a + b * b);
    a /= d;
    b /= d;
    c /= d;
  }
};

bool isparallel(line l1, line l2) {
  return vctr(l1.a, l1.b) * vctr(l2.a, l2.b) == 0;
}

vctr intersection(const line &l1, const line &l2) {
  dbl x = (l1.c * l2.b - l2.c * l1.b) / (l2.a * l1.b -
    l2.b * l1.a);
  dbl y = -(l1.c * l2.a - l2.c * l1.a) / (l2.a * l1.b -
    l2.b * l1.a);
  return vctr(x, y);
}

// Серединный перпендикуляр (не биссектриса!)
line bisection(const vctr A, const vctr B) {
  vctr M = (A + B) / 2;
  vctr AB = B - A;
  vctr norm = vctr(AB.y, -AB.x);
  return line(M, M + norm);
}

//// Окружность ////

struct circle {
  dbl x, y, r;

  circle() {}
  circle(dbl x, dbl y, dbl r) : x(x), y(y), r(r) {}
  circle(vctr P, dbl r) : x(P.x), y(P.y), r(r) {}
  circle(const vctr A, const vctr B) {
    vctr C = (A + B) / 2;
    x = C.x, y = C.y;
    r = (A - B).dist() / 2;
  }
  circle(const vctr A, const vctr B, const vctr C) {
    line l1 = bisection(A, B);
    line l2 = bisection(B, C);
    vctr P = intersection(l1, l2);
    x = P.x, y = P.y;
    r = (P - A).dist();
  }

  bool isin(const vctr P) const {
    return (vctr(x, y) - P).dist2() <= r * r;
  }
  vctr cent() const { return vctr(x, y); }
};
```

1.3 Выпуклая оболочка

```
vctr minvctr(INF, INF);

bool cmp_convex_hull(const vctr &a, const vctr &b) {
  vctr A = a - minvctr;
  vctr B = b - minvctr;
  auto sign_prod = sign(A * B);
  if (sign_prod != 0)
    return sign_prod > 0;
  return A.dist2() < B.dist2();
}

// minvctr updates here
vector<vctr> get_convex_hull(vector<vctr> arr) {
  minvctr = {INF, INF};
  for (auto v : pts) {
    if (is2plane(v - minvctr))
      minvctr = v;
  }
  vector<vctr> hull;
  sort(arr.begin(), arr.end(), cmp_convex_hull);
  for (vctr &el : arr) {
    while (hull.size() > 1 && sign((hull.back() - hull[
    hull.size() - 2]) * (el - hull.back())) <= 0)
      hull.pop_back();
    hull.push_back(el);
  }
  return hull;
}
```

1.4 Задача 16

```
bool isInSameHalf(vctr p, vctr r1, vctr r2) {
  return sign((r2 - r1) % (p - r1)) >= 0;
}

dbl distPointPoint(vctr a, vctr b) {
  return (a - b).dist();
}

dbl distPointLine(vctr a, vctr l1, vctr l2) {
  line l(l1, l2);
  l.normalize();
  return abs(l.get(a));
}

dbl distPointRay(vctr a, vctr r1, vctr r2) {
  if (!isInSameHalf(a, r1, r2))
    return distPointPoint(a, r1);
  return distPointLine(a, r1, r2);
}

dbl distPointSeg(vctr a, vctr s1, vctr s2) {
  return max(distPointRay(a, s1, s2),
             distPointRay(a, s2, s1));
}

bool isIntersectionLineLine(line l1, line l2) {
  dbl znam = l1.b * l2.a - l1.a * l2.b;
  return sign(znam) != 0;
}

vctr intersectionLineLine(line l1, line l2) {
  dbl znam = l1.b * l2.a - l1.a * l2.b;
  dbl y = -(l1.c * l2.a - l2.c * l1.a) / znam;
  dbl x = -(l1.c * l2.b - l2.c * l1.b) / -znam;
  return vctr(x, y);
}

vctr getPointOnLine(line l) {
  if (sign(l.b) != 0)
    return vctr(0, -l.c / l.b);
  return vctr(-l.c / l.a, 0);
}

dbl distLineLine(vctr l1a, vctr l1b, vctr l2a, vctr l2b)
    {
  line l1(l1a, l1b);
  line l2(l2a, l2b);
  if (isIntersectionLineLine(l1, l2))
    return 0;
  vctr p = getPointOnLine(l1);
  l2.normalize();
  return abs(l2.get(p));
}

dbl distRayLine(vctr r1, vctr r2, vctr l1, vctr l2) {
  line r(r1, r2);
  line l(l1, l2);
  if (!isIntersectionLineLine(l, r))
    return distLineLine(r1, r2, l1, l2);
  vctr p = intersectionLineLine(l, r);
  if (isInSameHalf(p, r1, r2))
    return 0;
  return distPointLine(r1, l1, l2);
}

dbl distSegLine(vctr s1, vctr s2, vctr l1, vctr l2) {
  return max(distRayLine(s1, s2, l1, l2),
```

```
            distRayLine(s2, s1, l1, l2));
}

dbl distRayRay(vctr r1a, vctr r1b, vctr r2a, vctr r2b) {
  line r1(r1a, r1b);
  line r2(r2a, r2b);
  if (!isIntersectionLineLine(r1, r2)) {
    if (isInSameHalf(r1a, r2a, r2b) || isInSameHalf(r2a,
     r1a, r1b))
      return distLineLine(r1a, r1b, r2a, r2b);
    else
      return distPointPoint(r1a, r2a);
  }
  vctr p = intersectionLineLine(r1, r2);
  if (isInSameHalf(p, r1a, r1b) && isInSameHalf(p, r2a,
    r2b))
    return 0;
  return min(distPointRay(r1a, r2a, r2b),
             distPointRay(r2a, r1a, r1b));
}

dbl distSegRay(vctr s1, vctr s2, vctr r1, vctr r2) {
  return max(distRayRay(s1, s2, r1, r2),
             distRayRay(s2, s1, r1, r2));
}

dbl distSegSeg(vctr s1a, vctr s1b, vctr s2a, vctr s2b) {
  return max(distSegRay(s1a, s1b, s2a, s2b),
             distSegRay(s1a, s1b, s2b, s2a));
}
```

1.5 Касательные из точки

```
pair<int, int> tangents_from_point(vector<vctr> &p, vctr
     &a) {
  int n = p.size();
  int logn = 31 - __builtin_clz(n);
  auto findWithSign = [&](int val) {
    int i = 0;
    for (int k = logn; k >= 0; --k) {
      int i1 = (i - (1 << k) + n) % n;
      int i2 = (i + (1 << k)) % n;
      if (sign((p[i1] - a) * (p[i] - a)) == val)
        i = i1;
      if (sign((p[i2] - a) * (p[i] - a)) == val)
        i = i2;
    }
    return i;
  };
  return {findWithSign(1), findWithSign(-1)};
}
```

1.6 Касательные параллельные прямой

```
// find point with max signed distance to line
int tangent_parallel_line(const vector<vctr> &p, line l)
     {
  int n = p.size();
  int i = 0;
  int logn = 31 - __builtin_clz(n);
  for (int k = logn; k >= 0; --k) {
    int i1 = (i - (1 << k) + n) % n;
    int i2 = (i + (1 << k)) % n;
    if (l.get(p[i1]) > l.get(p[i]))
      i = i1;
    if (l.get(p[i2]) > l.get(p[i]))
      i = i2;
  }
  return i;
}
```

1.7 Лежит ли точка в многоугольнике

```
// P starts with minvctr
bool is_point_in_poly(vctr A, vector<vctr> &P) {
  int n = P.size();
  int ind = lower_bound(P.begin(), P.end(), A,
    cmp_convex_hull) - P.begin();
  if (ind == n || ind == 0)
    return false;
  if (ind == 0)
    ind++;
  vctr B = A - P[ind - 1];
  vctr C = P[ind] - P[ind - 1];
  return sign(C * B) >= 0;
}
```

1.8 Минимальная покрывающая окружность

```
mt19937 rnd(179);

circle MinDisk2(vector<vctr> &p, vctr A, vctr B, int sz)
     {
  circle w(A, B);
  for (int i = 0; i < sz; ++i) {
    if (w.isin(p[i]))
      continue;
    w = circle(A, B, p[i]);
  }
  return w;
}

circle MinDisk1(vector<vctr> &p, vctr A, int sz) {
  shuffle(p.begin(), p.begin() + sz, rnd);
  circle w(A, p[0]);
  for (int i = 1; i < sz; ++i) {
    if (w.isin(p[i]))
      continue;
    w = MinDisk2(p, A, p[i], i);
  }
  return w;
}

circle MinDisk(vector<vctr> &p) {
  int sz = p.size();
  if (sz == 1)
    return circle(p[0], 0);
  shuffle(p.begin(), p.end(), rnd);
  circle w(p[0], p[1]);
  for (int i = 2; i < sz; ++i) {
    if (w.isin(p[i]))
      continue;
    w = MinDisk1(p, p[i], i);
  }
  return w;
}
```

1.9 Пересечение полуплоскостей

```
// half plane: ax+by+c > 0
// bounding box MUST have
vector<int> intersection_half_planes_inds(const vector<
    line> &ls) {
  int n = (int)ls.size();
  vector<int> lsi(n);
  iota(lsi.begin(), lsi.end(), 0);
  sort(lsi.begin(), lsi.end(), [&](int i, int j) {
    vctr aa(ls[i].a, ls[i].b);
    vctr bb(ls[j].a, ls[j].b);
    bool pla = is2plane(aa);
    bool plb = is2plane(bb);
    if (pla != plb)
      return pla < plb;
    return aa * bb > 0;
  });

  vector<line> st;
  vector<int> inds;
  for (int ii = 0; ii < 2 * n; ++ii) {
    int i = lsi[ii % n];
    if (st.empty()) {
      st.push_back(ls[i]);
      inds.push_back(i);
      continue;
    }
    vctr p = intersection(ls[i], st.back());
    bool pp = isparallel(ls[i], st.back());
    bool bad = false;
    while (st.size() >= 2) {
      if (!pp && sign(st[st.size() - 2].get(p)) >= 0)
        break;
      else if (pp && sign(st.back().get(ls[i].anyPoint()
      )) <= 0) {
        bad = true;
        break;
      }
      st.pop_back();
      inds.pop_back();
      p = intersection(ls[i], st.back());
      pp = isparallel(ls[i], st.back());
    }
    if (!bad) {
      st.push_back(ls[i]);
```

```
      inds.push_back(i);
    }
  }
  vector<int> cnt(n, 0);
  for (int i : inds)
    cnt[i]++;
  vector<int> good;
  for (int i : inds) {
    if (cnt[i]-- == 2)
      good.push_back(i);
  }
  return good;
}

vector<vctr> intersection_half_planes(vector<line> &ls)
    {
  vector<int> inter = intersection_half_planes_inds(ls);
  int n = inter.size();
  vector<vctr> pts;
  for (int i = 0; i < n; ++i) {
    int j = (i + 1) % n;
    vctr P = intersection(ls[inter[i]], ls[inter[j]]);
    if (pts.empty() || sign(pts.back().x - P.x) != 0
        || sign(pts.back().y - P.y) != 0)
      pts.push_back(P);
  }
  return pts;
}
```

1.10 Пересечение с окружностью

```
bool is_intersection_line_circ(line l, circle c) {
  l.normalize();
  dbl d = abs(l.get(c.cent()));
  return d < c.r - EPS;
}

vector<vctr> intersection_line_circ(line l, circle c) {
  l.normalize();
  dbl d = abs(l.get(c.cent()));
  vctr per = vctr(l.a, l.b).norm() * d;
  vctr a = c.cent() + per;
  if (sign(d - c.r) > 0)
      return {};
  if (sign(l.get(a)) != 0)
      a = c.cent() - per;
  if (sign(c.r - d) == 0)
    return {a};
  dbl k = sqrtl(c.r * c.r - d * d);
  vctr par = vctr(-l.b, l.a).norm() * k;
  return {a + par, a - par};
}

vector<vctr> intersection_circ_circ(circle a, circle b)
    {
  line l(2 * (b.x - a.x),
         2 * (b.y - a.y),
         b.r * b.r - a.r * a.r
             + (a.x * a.x + a.y * a.y)
             - (b.x * b.x + b.y * b.y));
  if (sign(l.a) == 0 && sign(l.b) == 0)
    return {};
  return intersection_line_circ(l, a);
}

vector<vctr> tangent_vctr_circ(vctr v, circle c) {
  dbl d = (c.cent() - v).dist();
  dbl k = sqrtl(d * d - c.r * c.r);
  circle c2(v.x, v.y, k);
  return intersection_circ_circ(c, c2);
}
```

1.11 Проверка на пересечение отрезков

```
bool is_intersection_seg(vctr A, vctr B, vctr C, vctr D)
     {
  for (int i = 0; i < 2; ++i) {
    auto l1 = A.x, r1 = B.x, l2 = C.x, r2 = D.x;
    if (l1 > r1) swap(l1, r1);
    if (l2 > r2) swap(l2, r2);
    if (max(l1, l2) > min(r1, r2))
      return false;
    swap(A.x, A.y);
    swap(B.x, B.y);
    swap(C.x, C.y);
    swap(D.x, D.y);
  }
  for (int _ = 0; _ < 2; ++_) {
    auto v1 = (B - A) * (C - A);
    auto v2 = (B - A) * (D - A);
    if (sign(v1) * sign(v2) == 1)
      return false;
    swap(A, C);
    swap(B, D);
  }
  return true;
}
```

1.12 Сумма Минковского

```
// Список вершин -> список рёбер
vector<vctr> poly_to_edges(const vector<vctr> &A) {
  vector<vctr> edg(A.size());
  for (int i = 0; i < A.size(); ++i)
    edg[i] = A[(i + 1) % A.size()] - A[i];
  return edg;
}

// A и B начинаются с минимальных вершин
vector<vctr> minkowski_sum(const vector<vctr> &A, const
    vector<vctr> &B) {
  auto edgA = poly_to_edges(A);
  auto edgB = poly_to_edges(B);
  vector<vctr> edgC(A.size() + B.size());
  merge(edgA.begin(), edgA.end(), edgB.begin(), edgB.end
    (), edgC.begin(), cmp_angle);
  vector<vctr> C(edgC.size());
  C[0] = A[0] + B[0];
  for (int i = 0; i + 1 < C.size(); ++i)
    C[i + 1] = C[i] + edgC[i];
  return C;
}
```

1.13 Формула Эйлера

- V – число вершин выпуклого многогранника (планарного графа)
- E – число рёбер
- F – число граней (если планарный граф, то включая внешнюю)

Тогда $V - E + F = 2$

2 Графы

2.1 2-SAT

```
for (int i = 1; i <= n; ++i) {
  not_v[i] = i + n;
  not_v[i + n] = i;
}
for (int i = 0; i < m; ++i) {
  cin >> u >> v;
  g[not_v[v]].push_back(u);
  g[not_v[u]].push_back(v);
  rg[u].push_back(not_v[v]);
  rg[v].push_back(not_v[u]);
}
// делаем КСС, получаем comp
for (int v = 1; v <= n; ++v) {
  if (comp[v] == comp[not_v[v]]) {
    cout << "UNSATISFIABLE\n";
    return 0;
  }
}
for (int v = 1; v <= n; ++v)
  cout << (comp[v] > comp[not_v[v]] ? v : not_v[v]);
```

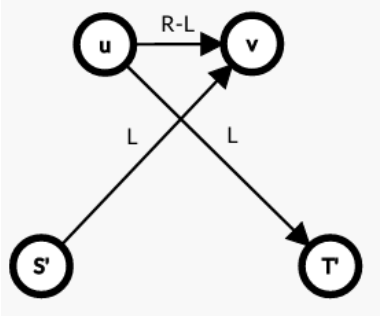
2.2 l-r-capacity-maxflow

Maximum flow problem with minimum capacities

We describe how to find the maximum flow from S' to T' when the edges also constrain the minimum bound of the flow amount

(edges have "minimum capacities"). It can be boiled down to an ordinary max-flow problem.

Consider an edge from u to v whose capacity is R and minimum capacity is L. To deal with the minimum capacity, create a new vertex S' to T', remove the original edge, and add edges with the following capacities:



Flow network example with minimum capacities

Add such edges for all edges with the minimum capacities. On the resulting graph, accumulate maximum flow in the following order:

- from S' to T'
- from S' to T
- from S to T'
- from S to T

An S–T flow that satisfies the minimum capacities exists if and only if, for all outgoing edges from S' and incoming edges to T', the flow and capacity are equal. (This can be understood by corresponding the flows from S' and T' to the original edges.)

Alternatively, if you just want to know the existence of a flow satisfying the minimum capacities, one can add an edge from T' to S' with infinite capacity and consider the flow from S' to T' once, instead of accumulating flows four times.

2.3 Венгерский алгоритм

```
pair<int, vector<int>> venger(vector<vector<int>> a) {
// ищет минимальное по стоимости
// работает только при n <= m
// a - массив весов (n + 1) × (m + 1)
// a[0][..] = a[..][0] = 0
// возвращает ans[i] = j если взяли ребро a[i][j]
  int n = (int) a.size() - 1;
  int m = (int) a[0].size() - 1;
  vector<int> u(n + 1), v(m + 1), p(m + 1), way(m + 1);
  for (int i = 1; i <= n; ++i) {
    p[0] = i;
    int j0 = 0;
    vector<int> minv(m + 1, INF);
    vector<char> used(m + 1, false);
    do {
      used[j0] = true;
      int i0 = p[j0], delta = INF, j1;
      for (int j = 1; j <= m; ++j)
        if (!used[j]) {
          int cur = a[i0][j] - u[i0] - v[j];
          if (cur < minv[j])
            minv[j] = cur, way[j] = j0;
          if (minv[j] < delta)
            delta = minv[j], j1 = j;
        }
      for (int j = 0; j <= m; ++j)
        if (used[j])
          u[p[j]] += delta, v[j] -= delta;
        else
          minv[j] -= delta;
      j0 = j1;
    } while (p[j0] != 0);
    do {
      int j1 = way[j0];
      p[j0] = p[j1];
      j0 = j1;
    } while (j0);
  }
  int cost = -v[0];
  vector<int> ans(n + 1);
  for (int j = 1; j <= m; ++j)
    ans[p[j]] = j;
  return {cost, ans};
}
```

2.4 Вершинная двусвязность

```
struct edge {
  int u, ind;

  bool operator<(const edge &other) const {
    return u < other.u;
  }
};

vector<int> stack_;

void paint(int v, int pr = -1) {
  used[v] = pr;
  up[v] = tin[v] = ++timer;
  for (auto e: g[v]) {
    if (e.u == pr) {
      continue;
    }
    if (!used[e.u]) {
      stack_.push_back(e.ind);
      paint(e.u, v);
      if (up[e.u] >= tin[v]) {
        ++mx_col;
        while (true) {
          int cur_edge = stack_.back();
          col[cur_edge] = mx_col;
          stack_.pop_back();
          if (cur_edge == e.ind) {
            break;
          }
        }
      }
      up[v] = min(up[v], up[e.u]);
    } else if (tin[e.u] < tin[v]) {
      stack_.push_back(e.ind);
      up[v] = min(up[v], tin[e.u]);
    } else if (up[v] > tin[e.u]) {
      up[v] = up[e.u];
    }
  }
}

signed main() {
  int n, m;
  cin >> n >> m;
  for (int i = 0; i < m; ++i) {
    int u, v;
    cin >> u >> v;
    g[u].push_back({v, i});
    g[v].push_back({u, i});
  }
  for (int v = 1; v <= n; ++v) {
    sort(all(g[v]));
  }
  for (int v = 1; v <= n; ++v) {
    if (!used[v]) {
      paint(v);
    }
  }
  for (int v = 1; v <= n; ++v) {
    int len = g[v].size();
    for (int i = 1; i < len; ++i) {
      if (col[g[v][i].ind] == 0) {
        col[g[v][i].ind] = col[g[v][i - 1].ind];
      }
    }
  }
}
```

2.5 Диниц

```
struct edge {
  int v, f, c, ind;
};

vector<edge> g[MAXN];
pair<int, int> pred[MAXN];
int d[MAXN];
int inds[MAXN];

bool dfs(int v, int final, int W) {
```

```
  if (v == final) {
    return true;
  }
  for (int i = inds[v]; i < (int) g[v].size(); i++) {
    auto e = g[v][i];
    if (e.f + W <= e.c && d[v] + 1 == d[e.v]) {
      pred[e.v] = {v, i};
      bool flag = dfs(e.v, final, W);
      if (flag) {
        return true;
      }
      inds[v]++;
    } else {
      inds[v]++;
    }
  }
  return false;
}

bool bfs(int start, int final, int W) {
  fill(d, d + MAXN, INF);
  d[start] = 0;
  deque<int> q = {start};
  while (!q.empty()) {
    int v = q.front();
    q.pop_front();
    for (auto e : g[v]) {
      if (e.f + W <= e.c && d[e.v] > d[v] + 1) {
        d[e.v] = d[v] + 1;
        q.push_back(e.v);
      }
    }
  }
  if (d[final] == INF) {
    return false;
  }
  fill(inds, inds + MAXN, 0);
  while (dfs(start, final, W)) {
    int v = final;
    int x = INF;
    while (v != start) {
      int ind = pred[v].second;
      v = pred[v].first;
      x = min(x, g[v][ind].c - g[v][ind].f);
    }
    v = final;
    while (v != start) {
      int ind = pred[v].second;
      v = pred[v].first;
      g[v][ind].f += x;
      g[g[v][ind].v][g[v][ind].ind].f -= x;
    }
  }
  return true;
}

void Dinic(int start, int final) {
  int W = (1LL << 30);
  do {
    while (bfs(start, final, W));
    W /= 2;
  } while (W >= 1);
}

signed main() {
  int n, m;
  vector<pair<int, int>> edges;
  for (int i = 0; i < m; i++) {
    int u, v, c;
    cin >> u >> v >> c;
    edges.emplace_back(u, v);
    g[u].push_back({v, 0, c, (int) g[v].size()});
    // если ребро - ориентированно,
    // то обратная capacity = 0
    g[v].push_back({u, 0, c, (int) g[u].size() - 1});
  }
  int start = 1, target = n;
  Dinic(start, target);
  int res = 0;
  for (auto e : g[start]) {
    res += e.f;
  }
  vector<int> cut;
  for (int i = 0; i < m; i++) {
    int u = edges[i].first, v = edges[i].second;
    if ((d[u] != INF && d[v] == INF) ||
        (d[u] == INF && d[v] != INF)) {
      cut.push_back(i + 1);
    }
  }
}
```

2.6 KCC

```
void dfs1(int v, vector<int> &topsort) {
  used[v] = 1;
  for (auto u : g[v]) {
    if (!used[u]) {
      dfs1(u, topsort);
    }
  }
  topsort.push_back(v);
}

void dfs2(int v, int col) {
  comp[v] = col;
  for (auto u : rg[v]) {
    if (!comp[u]) {
      dfs2(u, col);
    }
  }
}

signed main() {
  vector<int> topsort;
  for (int v = 1; v <= n; ++v)
    if (!used[v])
      dfs1(v, topsort);
  reverse(all(topsort));
  for (int j = 1; j <= n; ++j)
    if (!comp[topsort[j - 1]])
      dfs2(topsort[j - 1], j);
}
```

2.7 Минкост (Джонсон)

```
using cost_t = ll;
using flow_t = int;

const int MAXN = 10000;
const int MAXM = 25000 * 2;
const cost_t INFw = 1e12;
const flow_t INFf = 10;

struct Edge {
  int v, u;
  flow_t f, c;
  cost_t w;
};

Edge edg[MAXM];
int esz = 0;
vector<int> graph[MAXN];
ll dist[MAXN];
ll pot[MAXN];
int S, T;
int NUMV;
int pre[MAXN];
bitset<MAXN> inQ;

flow_t get_flow() {
  int v = T;
  if (pre[v] == -1)
    return 0;
  flow_t f = INFf;
  do {
    int ei = pre[v];
    Edge &e = edg[ei];
    f = min(f, e.c - e.f);
    if (f == 0)
      return 0;
    v = e.v;
  } while (v != S);
  v = T;
  do {
    int ei = pre[v];
    edg[ei].f += f;
    edg[ei ^ 1].f -= f;
    v = edg[ei].v;
  } while (v != S);
  return f;
}

void spfa() {
  fill(dist, dist + NUMV, INFw);
  dist[S] = 0;
  deque<int> Q = {S};
```

```
  inQ[S] = true;
  while (!Q.empty()) {
    int v = Q.front();
    Q.pop_front();
    inQ[v] = false;
    cost_t d = dist[v];
    for (int ei : graph[v]) {
      Edge &e = edg[ei];
      if (e.f == e.c)
        continue;
      cost_t w = e.w + pot[v] - pot[e.u];
      if (dist[e.u] <= d + w)
        continue;
      pre[e.u] = ei;
      dist[e.u] = d + w;
      if (!inQ[e.u]) {
        inQ[e.u] = true;
        Q.push_back(e.u);
      }
    }
  }
  for (int i = 0; i < NUMV; ++i)
    pot[i] += dist[i];
}

cost_t mincost() {
  spfa(); // pot[i] = 0 // or ford_bellman
  flow_t f = 0;
  while (true) {
    flow_t ff = get_flow();
    if (ff == 0)
      break;
    f += ff;
    spfa(); // or dijkstra
  }
  cost_t res = 0;
  for (int i = 0; i < esz; ++i)
    res += edg[i].f * edg[i].w;
  res /= 2;
  return res;
}

void add_edge(int v, int u, int c, int w) {
  edg[esz] = {v, u, 0, c, w};
  edg[esz + 1] = {u, v, 0, 0, -w};
  graph[v].push_back(esz);
  graph[u].push_back(esz + 1);
  esz += 2;
}

signed main() {
  ios_base::sync_with_stdio(false);
  cin.tie(nullptr);
  int n, m;
  cin >> n >> m;
  S = 0;
  T = n - 1;
  NUMV = n;
  for (int i = 0; i < m; ++i) {
    int v, u, c, w;
    cin >> v >> u >> c >> w;
    v--, u--;
    add_edge(v, u, c, w);
  }
  cost_t ans = mincost();
  cout << ans;
}
```

2.8 Мосты

```
void dfs(int v, int par) {
  vis[v] = 1;
  up[v] = tin[v] = timer++;
  for (auto u : g[v]) {
    if (!vis[u]) {
      dfs(u, v);
      up[v] = min(up[v], up[u]);
    } else if (u != par) {
      up[v] = min(up[v], tin[u]);
    }
    if (up[u] > tin[v]) {
      bridges.emplace_back(v, u);
    }
  }
}
```

2.9 Паросочетания

```
int dfs(int v, int c) {
  if (used[v] == c) return 0;
  used[v] = c;
  for (auto u : g[v]) {
    if (res[u] == -1) {
      res[u] = v;
      return 1;
    }
  }
  for (auto u : g[v]) {
    if (dfs(res[u], c)) {
      res[u] = v;
      return 1;
    }
  }
  return 0;
}

signed main() {
  // n - в левой доле, m - в правой
  fill(res, res + m, -1);
  for (int i = 0; i < n; ++i) {
    ans += dfs(i, i + 1);
  }
}
```

2.10 Точки сочленения

```
void dfs(int v, int par) {
  vis[v] = 1;
  up[v] = tin[v] = timer++;
  int child = 0;
  for (auto u : g[v]) {
    if (!vis[u]) {
      dfs(u, v);
      up[v] = min(up[v], up[u]);
      if (up[u] >= tin[v] && par != -1) {
        points.insert(v);
      }
      child++;
    } else if (u != par) {
      up[v] = min(up[v], tin[u]);
    }
  }
  if (par == -1 && child >= 2) {
    points.insert(v);
  }
}
```

2.11 Эдмондс-Карп

```
struct edge {
  int v, f, c, ind;
};

vector<edge> g[MAXN];

bool bfs(int start, int final, int W) {
  vector<int> d(MAXN, INF);
  vector<pair<int, int>> pred(MAXN);
  d[start] = 0;
  deque<int> q = {start};
  while (!q.empty()) {
    int v = q.front();
    q.pop_front();
    for (int i = 0; i < (int) g[v].size(); i++) {
      auto e = g[v][i];
      if (e.f + W <= e.c && d[e.v] > d[v] + 1) {
        d[e.v] = d[v] + 1;
        pred[e.v] = {v, i};
        q.push_back(e.v);
      }
    }
  }
  if (d[final] == INF) {
    return false;
  }
  int v = final;
  int x = INF;
  while (v != start) {
    int ind = pred[v].second;
    v = pred[v].first;
    x = min(x, g[v][ind].c - g[v][ind].f);
  }
  v = final;
  while (v != start) {
```

```
    int ind = pred[v].second;
    v = pred[v].first;
    g[v][ind].f += x;
    g[g[v][ind].v][g[v][ind].ind].f -= x;
  }
  return true;
}

signed main() {
  int n, m;
  for (int i = 0; i < m; i++) {
    int u, v, c;
    cin >> u >> v >> c;
    g[u].push_back({v, 0, c, (int) g[v].size()});
    g[v].push_back({u, 0, 0, (int) g[u].size() - 1});
  }
  int start = 1, final = n;
  int W = (1 << 30);
  do {
    while (bfs(start, final, W));
    W /= 2;
  } while (W >= 1);
  int res = 0;
  for (auto e : g[start]) {
    res += e.f;
  }
}
```

2.12 Эйлеров цикл

```
// unconnected graph, deleting edges, set<int> g[N];
for (int v = 0; v < n; v++) {
  if (!g[v].empty()) {
    vector<int> ccl;
    vector<int> s = {v};
    while (!s.empty()) {
      int u = s.back();
      if (g[u].empty()) {
        ccl.pb(u);
        s.pop_back();
      } else {
        int u2 = *g[u].begin();
        g[u].erase(u2);
        g[u2].erase(u);
        s.pb(u2);
      }
    }
    // ccl[0] = ccl.back()
    // i.e for graph with edges
    (1,2),(1,3),(2,3) → ccl = [1,2,3,1]
  }
}
```

3 ДП

3.1 CHT

```
struct line {
    int k, b;
    int eval(int x) {
      return k * x + b;
    }
};
struct part {
    line a;
    double x;
};
double intersection(line a, line b) {
  return (a.b - b.b) / (double) (b.k - a.k);
}
struct ConvexHull {
    // for min: k decreasing (non-increasing)
    // for max: k increasing (non-decreasing)
    vector<part> st;

    void add(line a) {
      if (!st.empty() && st.back().a.k == a.k) {
        if (st.back().a.b < a.b) st.pop_back(); // for
    max
//        if (st.back().a.b > a.b) st.pop_back(); // for
     min
        else return;
      }
      while (st.size() > 1 &&
             intersection(st[st.size() - 2].a, a) <= st[
    st.size() - 2].x)
        st.pop_back();
      if (!st.empty()) st.back().x = intersection(st.
    back().a, a);
      st.push_back({a, INF});
      // INF = max intersection point
    }

    int get_val(int x) {
      int l = -1, r = (int) st.size() - 1;
      while (r - l > 1) {
        int m = (l + r) / 2;
        if (st[m].x < x) l = m;
        else r = m;
      }
      return st[r].a.eval(x);
    }
};
```

3.2 Li Chao

```
// MAXIMUM
struct Line {
  int k, b;

  int f(int x) {
    return k * x + b;
  }
};
```

```
struct ST {
  vector<Line> st;

  ST(int n) {
    Line ln = {0LL, -INF};
    st.resize(4 * n, ln);
  }

  void upd(int i, int l, int r, Line ln) {
    int child = 1;
    Line ln1 = ln;
    int m = (l + r) / 2;
    if (ln.f(m) > st[i].f(m)) {
      if (ln.k < st[i].k) {
        child = 2;
      }
      ln1 = st[i];
      st[i] = ln;
    } else {
      if (st[i].k < ln.k) {
        child = 2;
      }
    }
    if (l + 1 < r) {
      if (child == 1) {
        upd(i * 2 + 1, l, m, ln1);
      } else {
        upd(i * 2 + 2, m, r, ln1);
      }
    }
  }

  int res(int i, int l, int r, int x) {
    if (l + 1 == r) {
      return st[i].f(x);
    }
    int m = (l + r) / 2;
    int val = st[i].f(x);
    if (x < m) {
      val = max(val, res(i * 2 + 1, l, m, x));
    } else {
      val = max(val, res(i * 2 + 2, m, r, x));
    }
    return val;
  }
};
```

3.3 SOS-dp

```
// dp initial fill, a[] is given array, mb extra zeros
for (int i = 0; i < (1 << N); i++) {
  dp[i] = a[i];
}

// Classic SOS-dp, goal: dp[mask] = \sum a[submasks of
    mask]
for (int i = 0; i < N; i++) {
  for (int mask = 0; mask < (1 << N); mask++) {
    if ((mask >> i) & 1) {
```

```
      dp[mask] += dp[mask ^ (1 << i)];
    }
  }
}

// Overmasks SOS-dp, goal: dp[mask] = \sum a[overmasks
    of mask]
for (int i = 0; i < N; i++) {
  for (int mask = (1 << N) - 1; mask >= 0; mask--) {
    if (((mask >> i) & 1) == 0) {
      dp[mask] += dp[mask ^ (1 << i)];
    }
  }
}

// to inverse SOS-dp (restore original array by SOS-dp
    array):
// use same code, but -= instead of += in dp transitions
```

3.4 НВП

```
// 0-indexation ({a_0, ..., a_{n-1}})
vector<int> lis(vector<int> a) {
  int n = (int) a.size();
  vector<int> dp(n + 1, INF), ind(n + 1), par(n + 1); //
     INF > all a[i] required
  ind[0] = -INF;
  dp[0] = -INF;
  for (int i = 0; i < n; i++) {
    int l = upper_bound(dp.begin(), dp.end(), a[i]) - dp
    .begin();
    if (dp[l - 1] < a[i] && a[i] < dp[l]) {
      dp[l] = a[i];
      ind[l] = i;
      par[i] = ind[l - 1];
    }
  }
  vector<int> ans; // exact values
  for (int l = n; l >= 0; l--) {
    if (dp[l] < INF) {
      int pi = ind[l];
      ans.resize(l);
      for (int i = 0; i < l; i++) {
        ans[i] = a[pi]; // =pi if need indices
        pi = par[pi];
      }
      reverse(ans.begin(), ans.end());
      return ans;
    }
  }
  return {};
}
```

3.5 НОВП

```
// 1-indexation ({0, a_1, ..., a_n}, {0, b_1, ..., b_m})
vector<int> lcis(vector<int> a, vector<int> b) {
  int n = (int) a.size() - 1, m = (int) b.size() - 1;
  vector<int> dp(m + 1), dp2(m + 1), par(m + 1);
  for (int i = 1; i <= n; i++) {
    int best = 0, best_idx = 0;
    for (int j = 1; j <= m; j++) {
      dp2[j] = dp[j];
      if (a[i] == b[j]) {
        dp2[j] = max(dp2[j], best + 1);
        par[j] = best_idx;
      }
      if (a[i] > b[j] && best < dp[j]) {
        best = dp[j];
        best_idx = j;
      }
    }
    swap(dp, dp2);
  }
  int pj = 0;
  for (int j = 1; j <= m; j++) {
    if (dp[pj] < dp[j]) {
      pj = j;
    }
  }
  vector<int> ans; // exact values
  while (pj > 0) {
    ans.push_back(b[pj]);
    pj = par[pj];
  }
  reverse(ans.begin(), ans.end());
  return ans;
}
```

4 Деревья

4.1 Centroid

```
int levels[MAXN];
int szs[MAXN];
int cent_par[MAXN];

int calcsizes(int v, int p) {
  int sz = 1;
  for (int u : graph[v]) {
    if (u != p && levels[u] == 0)
      sz += calcsizes(u, v);
  }
  return szs[v] = sz;
}

void centroid(int v, int lvl=1, int p=-1) {
  int sz = calcsizes(v, -1);
  int nxt = v, prv;
  while (nxt != -1) {
    prv = v, v = nxt, nxt = -1;
    for (int u : graph[v]) {
      if (u != prv && levels[u] == 0 && szs[u] * 2 >= sz
      )
        nxt = u;
    }
  }
  levels[v] = lvl;
  cent_par[v] = p;
  for (int u : graph[v]) {
    if (levels[u] == 0)
      centroid(u, lvl + 1, v);
  }
  // calc smth for centroid v
}
```

4.2 HLD

```
int par[MAXN], sizes[MAXN];
int pathup[MAXN];
int tin[MAXN], tout[MAXN];
int timer;

int dfs1_hld(int v, int p) {
  par[v] = p;
  int sz = 1;
  for (int i = 0; i < graph[v].size(); ++i) {
    int u = graph[v][i];
    if (u == p) {
      swap(graph[v][i--], graph[v].back());
      graph[v].pop_back();
      continue;
    }
    sz += dfs1_hld(u, v);
  }
  return sizes[v] = sz;
}

void dfs2_hld(int v, int up) {
  tin[v] = timer++;
  pathup[v] = up;
  if (graph[v].empty()) {
    tout[v] = timer;
    return;
  }
  for (int i = 1; i < graph[v].size(); ++i) {
    if (sizes[graph[v][i]] > sizes[graph[v][0]])
      swap(graph[v][i], graph[v][0]);
  }
  dfs2_hld(graph[v][0], up);
  for (int i = 1; i < graph[v].size(); ++i)
    dfs2_hld(graph[v][i], graph[v][i]);
  tout[v] = timer;
}

bool is_ancestor(int v, int p) {
  return tin[p] <= tin[v] && tout[v] <= tout[p];
}

// get_hld полностью аналогичный
void update_hld(int v, int u, int ARG) {
  for (int _ = 0; _ < 2; ++_) {
    while (!is_ancestor(u, pathup[v])) {
      int vup = pathup[v];
```

```
      ST.update(0, 0, timer, tin[vup], tin[v] + 1, ARG);
      v = par[vup];
    }
    swap(v, u);
  }
  if (tin[v] > tin[u])
    swap(v, u);
  // v = lca
  ST.update(0, 0, timer, tin[v], tin[u] + 1, ARG);
}

signed main() {
  dfs1_hld(0, -1);
  dfs2_hld(0, 0);
  ST.build();
  // your code here
}
```

4.3 Link-cut

```
struct Node {
  Node *ch[2];
  Node *p;
  bool rev;
  int sz;

  Node() {
    ch[0] = nullptr;
    ch[1] = nullptr;
    p = nullptr;
    rev = false;
    sz = 1;
  }
};

int size(Node *v) {
  return (v ? v->sz : 0);
}

int chnum(Node *v) {
  return v->p->ch[1] == v;
}

bool isroot(Node *v) {
  return v->p == nullptr || v->p->ch[chnum(v)] != v;
}

void push(Node *v) {
  if (v->rev) {
    if (v->ch[0])
      v->ch[0]->rev ^= 1;
    if (v->ch[1])
      v->ch[1]->rev ^= 1;
    swap(v->ch[0], v->ch[1]);
    v->rev = false;
  }
}

void pull(Node *v) {
  v->sz = size(v->ch[1]) + size(v->ch[0]) + 1;
}

void attach(Node *v, Node *p, int num) {
  if (p)
    p->ch[num] = v;
  if (v)
    v->p = p;
}

void rotate(Node *v) {
  Node *p = v->p;
  push(p);
  push(v);
  int num = chnum(v);
  Node *u = v->ch[1 - num];
  if (!isroot(v->p))
    attach(v, p->p, chnum(p));
  else
    v->p = p->p;
  attach(u, p, num);
  attach(p, v, 1 - num);
  pull(p);
  pull(v);
}

void splay(Node *v) {
  push(v);
  while (!isroot(v)) {
    if (!isroot(v->p)) {
      if (chnum(v) == chnum(v->p))
        rotate(v->p);
      else
        rotate(v);
    }
    rotate(v);
  }
}

void expose(Node *v) {
  splay(v);
  v->ch[1] = nullptr;
  pull(v);
  while (v->p != nullptr) {
    Node *p = v->p;
    splay(p);
    attach(v, p, 1);
    pull(p);
    splay(v);
  }
}

void makeroot(Node *v) {
  expose(v);
  v->rev ^= 1;
  push(v);
}

void link(Node *v, Node *u) {
  makeroot(v);
  u->p = v;
}

void cut(Node *v, Node *u) {
  makeroot(u);
  makeroot(v);
  v->ch[1] = nullptr;
  u->p = nullptr;
}

int get(Node *v, Node *u) {
  makeroot(u);
  makeroot(v);
  Node *w = u;
  while (!isroot(w))
    w = w->p;
  return (w == v ? size(v) - 1 : -1);
}

const int MAXN = 100010;
Node *nodes[MAXN];

int main() {
  int n, q;
  cin >> n >> q;
  for (int i = 0; i < n; ++i)
    nodes[i] = new Node();
  while (q--) {
    string s;
    int a, b;
    cin >> s >> a >> b;
    a--, b--;
    if (s[0] == 'g')
      cout << get(nodes[a], nodes[b]) << '\n';
    else if (s[0] == 'l')
      link(nodes[a], nodes[b]);
    else
      cut(nodes[a], nodes[b]);
  }
}
```

5 Другое

5.1 Slope trick

```
// Дан массив a_n. Сделать минимальное кол-во ±1, чтобы
    a_n стал неубывающим.

void solve() {
    int n;
    cin >> n;
    vector<int> a(n);
    for (int i = 0; i < n; i++) {
        cin >> a[i];
    }
    int ans = 0;
    multiset<int> now;
```

```
    for (int i = 0; i < n; i++) {
        now.insert(a[i]);
        ans += (*now.rbegin() - a[i]);
        now.erase(now.find(*now.rbegin()));
        now.insert(a[i]);
    }
    cout << ans << '\n';
}
```

5.2 attribute_packed

```
struct Kek {
  int a;
  char b;
  // char[3]
  int c;
} __attribute__((packed));
// sizeof = 9 (instead of 12)
```

5.3 custom_bitset

```
struct custom_bitset {
    vector<uint64_t> bits;
    int64_t b, n;

    custom_bitset(int64_t _b = 0) {
        init(_b);
    }

    void init(int64_t _b) {
        b = _b;
        n = (b + 63) / 64;
        bits.assign(n, 0);
    }

    void clear() {
        b = n = 0;
        bits.clear();
    }

    void reset() {
        bits.assign(n, 0);
    }

    void _clean() {
        // Reset all bits after `b`.
        if (b != 64 * n)
            bits.back() &= (1LLU << (b - 64 * (n - 1)))
    - 1;
    }

    bool get(int64_t index) const {
        return bits[index / 64] >> (index % 64) & 1;
    }

    void set(int64_t index, bool value) {
        assert(0 <= index && index < b);
        bits[index / 64] &= ~(1LLU << (index % 64));
        bits[index / 64] |= uint64_t(value) << (index %
    64);
    }

    // Simulates `bs |= bs << shift;`
    void or_shift(int64_t shift) {
        int64_t div = shift / 64, mod = shift % 64;

        if (mod == 0) {
            for (int64_t i = n - 1; i >= div; i--)
                bits[i] |= bits[i - div];

            return;
        }

        for (int64_t i = n - 1; i >= div + 1; i--)
            bits[i] |= bits[i - (div + 1)] >> (64 - mod)
      | bits[i - div] << mod;

        if (div < n)
            bits[div] |= bits[0] << mod;

        _clean();
    }

    // Simulates `bs |= bs >> shift;`
    void or_shift_down(int64_t shift) {
        int64_t div = shift / 64, mod = shift % 64;

        if (mod == 0) {
            for (int64_t i = div; i < n; i++)
                bits[i - div] |= bits[i];

            return;
        }

        for (int64_t i = 0; i < n - (div + 1); i++)
            bits[i] |= bits[i + (div + 1)] << (64 - mod)
      | bits[i + div] >> mod;

        if (div < n)
            bits[n - div - 1] |= bits[n - 1] >> mod;

        _clean();
    }

    int64_t find_first() const {
        for (int i = 0; i < n; i++)
            if (bits[i] != 0)
                return 64 * i + __builtin_ctzll(bits[i])
    ;

        return -1;
    }

    custom_bitset &operator&=(const custom_bitset &other
    ) {
        assert(b == other.b);
        for (int i = 0; i < n; i++)
            bits[i] &= other.bits[i];

        return *this;
    }

    custom_bitset &operator|=(const custom_bitset &other
    ) {
        assert(b == other.b);

        for (int i = 0; i < n; i++)
            bits[i] |= other.bits[i];

        return *this;
    }

    custom_bitset &operator^=(const custom_bitset &other
    ) {
        assert(b == other.b);

        for (int i = 0; i < n; i++)
            bits[i] ^= other.bits[i];

        return *this;
    }
};
```

5.4 ordered_set

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>

using namespace __gnu_pbds;

typedef tree<int, null_type, less<>, rb_tree_tag,
    tree_order_statistics_node_update> ordered_set;

//st.find_by_order(index);
//st.order_of_key(key);
```

5.5 pragma

```
#pragma GCC optimize("Ofast,fast-math,unroll-loops,no-
    stack-protector,inline")
#pragma GCC target("sse,sse2,sse3,ssse3,sse4,sse4.1,sse4
    .2,avx,avx2,abm,mmx,popcnt")
```

5.6 Аллокатор Копелиовича

```
// Код вставить до инклюдов

#include <cassert>

const int MAX_MEM = 1e8; // ~100mb
int mpos = 0;
char mem[MAX_MEM];
```

```
inline void *operator new(std::size_t n) {
  assert((mpos += n) <= MAX_MEM);
  return (void *)(mem + mpos - n);
}

inline void operator delete(void *) noexcept {} // must
    have!
inline void operator delete(void *, std::size_t)
    noexcept {} // fix!!
```

6 Математика

6.1 A div B < C div D

```
char sign(ll x) {
  return x < 0 ? -1 : x > 0;
}

// -1 = less, 0 = equal, 1 = greater
char compare(ll a, ll b, ll c, ll d) {
  if (a / b != c / d)
    return sign(a / b - c / d);
  a = a % b;
  c = c % d;
  if (a == 0)
    return -sign(c) * sign(d);
  if (c == 0)
    return sign(a) * sign(b);
  return is_less(d, c, b, a) * sign(a) * sign(b) * sign(
    c) * sign(d);
}
```

6.2 FFT mod

```
const int MOD = 998244353; // 7·17·2^23 + 1
const int GEN = 3;
//const int MOD = 7340033; // 7·2^20 + 1
//const int GEN = 5;
//const int MOD = 469762049; // 7·2^26 + 1
//const int GEN = 30;

const int LOG = 20;
const int MAXN = 1 << LOG;
int tail[MAXN + 1];
int OMEGA[MAXN + 1];

int binpow(int x, int p) {
  int res = 1;
  while (p > 0) {
    if (p & 1)
      res = res * 1ll * x % MOD;
    x = x * 1ll * x % MOD;
    p >>= 1;
  }
  return res;
}

int omega(int n, int k) {
  return OMEGA[MAXN / n * k];
}

int gettail(int x, int lg) {
  return tail[x] >> (LOG - lg);
}

void calcomega() {
  long long one = binpow(GEN, (MOD - 1) / MAXN);
  OMEGA[0] = 1;
  for (int i = 1; i < MAXN; ++i) {
    OMEGA[i] = OMEGA[i - 1] * one % MOD;
  }
}

void calctail() {
  int n = MAXN;
  for (int x = 0; x < n; ++x) {
    int res = 0;
    for (int i = 0; i < LOG; ++i) {
      res += ((x >> i) & 1) << (LOG - i - 1);
    }
    tail[x] = res;
  }
}

// Without precalc, tail[], OMEGA[]
//
//long long omega(int n, int k) {
//    return binpow(GEN, (MOD - 1) / n * k);
//}
//
//int gettail(int x, int lg) {
//    int res = 0;
//    for (int i = 0; i < lg; ++i)
//        res += ((x >> i) & 1) << (lg - i - 1);
//    return res;
//}

void fft(vector<int> &A, int lg) {
  int n = 1 << lg;
  for (int i = 0; i < n; ++i) {
    int j = gettail(i, lg);
    if (i < j)
      swap(A[i], A[j]);
  }
  for (int len = 2; len <= n; len *= 2) {
    for (int i = 0; i < n; i += len) {
      for (int j = 0; j < len / 2; ++j) {
        auto v = A[i + j];
        auto u = A[i + j + len / 2] * 1ll * omega(len, j
    ) % MOD;
        A[i + j] = (v + u) % MOD;
        A[i + j + len / 2] = (v - u + MOD) % MOD;
      }
    }
  }
}

int inverse(int x) {
  return binpow(x, MOD - 2);
}

void invfft(vector<int> &A, int lg) {
  int n = 1 << lg;
  fft(A, lg);
  for (auto &el : A)
    el = el * 1ll * inverse(n % MOD) % MOD;
  reverse(A.begin() + 1, A.end());
}

vector<int> mul(vector<int> A, vector<int> B) {
  if (A.empty() || B.empty())
    return {};
  int lg = 32 - __builtin_clz(A.size() + B.size() - 1);
  int n = 1 << lg;
  A.resize(n, 0);
  B.resize(n, 0);
  fft(A, lg);
  fft(B, lg);
  for (int i = 0; i < n; ++i)
    A[i] = A[i] * 1ll * B[i] % MOD;
  invfft(A, lg);
  return A;
}

signed main() {
  calctail(); // НЕ ЗАБЫТЬ
  calcomega(); // НЕ ЗАБЫТЬ
  // your code here
}
```

6.3 FFT

```
const double PI = acos(-1);
const int LOG = 20;
const int MAXN = 1 << LOG;

//using comp = complex<double>;
struct comp {
  double x, y;
  comp() : x(0), y(0) {}
  comp(double x, double y) : x(x), y(y) {}
  comp(int x) : x(x), y(0) {}
  comp operator+(const comp &o) const { return {x + o.x,
      y + o.y}; }
  comp operator-(const comp &o) const { return {x - o.x,
      y - o.y}; }
  comp operator*(const comp &o) const { return {x * o.x
    - y * o.y, x * o.y + y * o.x}; }
  comp operator/(const int k) const { return {x / k, y /
      k}; }
  comp conj() const { return {x, -y}; }
};

comp OMEGA[MAXN + 10];
```

```
int tail[MAXN + 10];

comp omega(int n, int k) {
  return OMEGA[MAXN / n * k];
}

int gettail(int x, int lg) {
  return tail[x] >> (LOG - lg);
}

void calcomega() {
  for (int i = 0; i < MAXN; ++i) {
    double x = 2 * PI * i / MAXN;
    OMEGA[i] = {cos(x), sin(x)};
  }
}

void calctail() {
  tail[0] = 0;
  for (int i = 1; i < MAXN; ++i)
    tail[i] = (tail[i >> 1] >> 1) | ((i & 1) << (LOG -
    1));
}

void fft(vector<comp> &A, int lg) {
  int n = A.size();
  for (int i = 0; i < n; ++i) {
    int j = gettail(i, lg);
    if (i < j)
      swap(A[i], A[j]);
  }
  for (int len = 2; len <= n; len *= 2) {
    for (int i = 0; i < n; i += len) {
      for (int j = 0; j < len / 2; ++j) {
        auto v = A[i + j];
        auto u = A[i + j + len / 2] * omega(len, j);
        A[i + j] = v + u;
        A[i + j + len / 2] = v - u;
      }
    }
  }
}

void fft2(vector<comp> &A, vector<comp> &B, int lg) {
  int n = A.size();
  vector<comp> C(n);
  for (int i = 0; i < n; ++i) {
    C[i].x = A[i].x;
    C[i].y = B[i].x;
  }
  fft(C, lg);
  C.push_back(C[0]);
  for (int i = 0; i < n; ++i) {
    A[i] = (C[i] + C[n - i].conj()) / 2;
    B[i] = (C[i] - C[n - i].conj()) / 2 * comp(0, -1);
  }
}

void invfft(vector<comp> &A, int lg) {
  int n = 1 << lg;
  fft(A, lg);
  for (auto &el : A)
    el = el / n;
  reverse(A.begin() + 1, A.end());
}

vector<int> mul(vector<int> &a, vector<int> &b) {
  if (a.empty() || b.empty())
    return {};
  int lg = 32 - __builtin_clz(a.size() + b.size() - 1);
  int n = 1 << lg;
  vector<comp> A(n, 0), B(n, 0);
  for (int i = 0; i < a.size(); ++i)
    A[i] = a[i];
  for (int i = 0; i < b.size(); ++i)
    B[i] = b[i];
//  fft2(A, B, lg);
  fft(A, lg);
  fft(B, lg);
  for (int i = 0; i < n; ++i)
    A[i] = A[i] * B[i];
  invfft(A, lg);
  vector<int> c(n);
  for (int i = 0; i < n; ++i)
    c[i] = round(A[i].x);
  while (!c.empty() && c.back() == 0)
    c.pop_back();
  return c;
}

signed main() {
  calcomega(); // НЕ ЗАБЫТЬ
  calctail(); // НЕ ЗАБЫТЬ
  // your code here
}
```

6.4 Floor Sum

```
int floor_sum(int n, int div, int mul, int add) {
    // sum_{i=0}^{n-1} floor((add + i*mul)/div)
    int ans = 0;
    ans += (n * (n - 1) / 2) * (mul / div);
    mul %= div;
    ans += n * (add / div);
    add %= div;
    int l = mul * n + add;
    if (l >= div)
        ans += floor_sum(l / div, mul, div, l % div);
    return ans;
}
```

6.5 Гаусс

```
vector<vector<int>> gauss(vector<vector<int>> &a) {
  int n = a.size();
  int m = a[0].size();
//  int det = 1;
  for (int col = 0, row = 0; col < m && row < n; ++col)
    {
    for (int i = row; i < n; ++i) {
      if (a[i][col]) {
        swap(a[i], a[row]);
        if (i != row) {
//          det *= -1;
        }
        break;
      }
    }
    if (!a[row][col])
      continue;
    for (int i = 0; i < n; ++i) {
      if (i != row && a[i][col]) {
        int val = a[i][col] * inv(a[row][col]) % mod;
        for (int j = col; j < m; ++j) {
          a[i][j] -= val * a[row][j];
          a[i][j] %= mod;
        }
      }
    }
    ++row;
  }
//  for (int i = 0; i < n; ++i) det = (det * a[i][i]) %
    mod;
//  det = (det % mod + mod) % mod;
// result in (-mod, mod)
  return a;
}

pair<int, vector<int>> sle(vector<vector<int>> a, vector
    <int> b) {
  int n = a.size();
  int m = a[0].size();
  assert(n == b.size());
  for (int i = 0; i < n; ++i) {
    a[i].push_back(b[i]);
  }
  a = gauss(a);
  vector<int> x(m, 0);
  for (int i = n - 1; i >= 0; --i) {
    int leftmost = m;
    for (int j = 0; j < m; ++j) {
      if (a[i][j] != 0) {
        leftmost = j;
        break;
      }
    }
    if (leftmost == m && a[i].back() != 0) return {-1,
    {}};
    if (leftmost == m) continue;
    int val = a[i].back();
    for (int j = m - 1; j > leftmost; --j) {
      val -= a[i][j] * x[j];
      val %= mod;
    }
    x[leftmost] = (val * inv(a[i][leftmost]) % mod + mod
    ) % mod;
  }
```

```
  return {1, x};
}

vector<bitset<N>> gauss_bit(vector<bitset<N>> a, int m)
    {
  int n = a.size();
  for (int col = 0, row = 0; col < m && row < n; ++col)
    {
    for (int i = row; i < n; ++i) {
      if (a[i][col]) {
        swap(a[i], a[row]);
        break;
      }
    }
    if (!a[row][col])
      continue;
    for (int i = 0; i < n; ++i)
      if (i != row && a[i][col])
        a[i] ^= a[row];
    ++row;
  }
  return a;
}
```

6.6 Диофантовы уравнения

```
pair<int, int> ext_gcd(int a, int b) {
  int x1 = 1, y1 = 0, x2 = 0, y2 = 1;
  while (b) {
    int k = a / b;
    x1 = x1 - x2 * k;
    y1 = y1 - y2 * k;
    swap(x1, x2);
    swap(y1, y2);
    a %= b;
    swap(a, b);
  }
  return {x1, y1};
}

bool cool_ext_gcd(int a, int b, int c, int &x, int &y) {
  if (b == 0) {
    y = 0;
    if (a == 0) {
      x = 0;
      return c == 0;
    } else {
      x = c / a;
      return c % a == 0;
    }
  }
  auto [x0, y0] = ext_gcd(a, b);
  int g = x0 * a + y0 * b;
  if (c % g != 0)
    return false;
  x0 *= c / g;
  y0 *= c / g;
  int t = b / g;
  int k = (-x0) / t;
  if (x0 + t * k < 0)
    k += t / abs(t);
  x = x0 + t * k;
  y = y0 - (a / g) * k;
  return true;
}
```

6.7 КТО

```
// x = a_i % p_i
vector<vector<int>> r(k, vector<int>(k));
for (int i = 0; i < k; ++i)
  for (int j = 0; j < k; ++j)
    if (i != j)
      r[i][j] = binpow(p[i] % p[j], p[j] - 2, p[j]);
vector<int> x(k);
for (int i = 0; i < k; ++i) {
  x[i] = a[i];
  for (int j = 0; j < i; ++j) {
    x[i] = r[j][i] * (x[i] - x[j]);
    x[i] = x[i] % p[i];
    if (x[i] < 0) x[i] += p[i];
  }
}
int ans = 0;
for (int i = 0; i < k; ++i) {
  int val = x[i];
  for (int j = 0; j < i; ++j) val *= p[j];
  ans += val;
}
```

6.8 Код Грея

```
for (int i = 0; i < (1 << n); i++) {
  gray[i] = i ^ (i >> 1);
}
```

6.9 Линейное решето

```
const int N = 10000000;
int lp[N + 1];
vector<int> pr;
for (int i = 2; i <= N; ++i) {
  if (lp[i] == 0) {
    lp[i] = i;
    pr.push_back(i);
  }
  for (int j = 0; j < (int) pr.size() && pr[j] <= lp[i]
    && i * pr[j] <= N; ++j)
    lp[i * pr[j]] = pr[j];
}
```

6.10 Миллер Рабин

```
// assuming '#define int long long' is ON (replace 'int'
     with 'long long' if not)
// works for all n < 2^64
const int MAGIC[7] = {2, 325, 9375, 28178, 450775,
    9780504, 1795265022};

int bpow(__int128 a, int x, int mod) {
  a %= mod;
  __int128 ans = 1;
  while (x) {
    if (x % 2) {
      ans *= a;
      ans %= mod;
    }
    a *= a;
    a %= mod;
    x /= 2;
  }
  return (int) ans;
}

bool is_prime(int n) {
  if (n == 1) return false;
  if (n <= 3) return true;
  if (n % 2 == 0 || n % 3 == 0) return false;
  int s = __builtin_ctzll(n - 1), d = n >> s; //
    n - 1 = 2^s * d
  for (auto a : MAGIC) {
    if (a % n == 0) {
      continue;
    }
    int x = bpow(a, d, n);
    for (int _ = 0; _ < s; _++) {
      int y = bpow(x, 2, n);
      if (y == 1 && x != 1 && x != n - 1) {
        return false;
      }
      x = y;
    }
    if (x != 1) {
      return false;
    }
  }
  return true;
}
```

6.11 Ро-Поллард

```
typedef long long ll;

ll mult(ll a, ll b, ll mod) {
  return (__int128)a * b % mod;
}

ll f(ll x, ll c, ll mod) {
  return (mult(x, x, mod) + c) % mod;
}
```

```
ll rho(ll n, ll x0=2, ll c=1) {
  ll x = x0;
  ll y = x0;
  ll g = 1;
  while (g == 1) {
    x = f(x, c, n);
    y = f(y, c, n);
    y = f(y, c, n);
    g = gcd(abs(x - y), n);
  }
  return g;
}

mt19937_64 rnd(time(nullptr));

void factor(int n, vector<int> &pr) {
  if (n == 4) {
    factor(2, pr);
    factor(2, pr);
    return;
  }
  if (n == 1) {
    return;
  }
  if (is_prime(n)) {
    pr.push_back(n);
    return;
  }
  int d = rho(n, rnd() % (n - 2) + 2, rnd() % 3 + 1);
  factor(n / d, pr);
  factor(d, pr);
}
```

7 Строки

7.1 Z-функция

```
vector<int> z_func(string s) {
  int n = s.size();
  vector<int> z(n, 0);
  z[0] = n;
  int l = 0, r = 0;
  for (int i = 1; i < n; i++) {
    if (i < r) {
      z[i] = min(z[i - l], r - i);
    }
    while (i + z[i] < n && s[z[i]] == s[i + z[i]]) {
      z[i]++;
    }
    if (i + z[i] > r) {
      l = i;
      r = i + z[i];
    }
  }
  return z;
}
```

7.2 eertree

```
int len[MAXN], suf[MAXN];
int go[MAXN][ALPH];
char s[MAXN];

int n, last, sz;

void init() {
  n = 0, last = 0;
  s[n++] = -1;
  suf[0] = 1; // root of suflink tree = 1
  len[1] = -1;
  sz = 2;
}

int get_link(int v) {
  while (s[n - len[v] - 2] != s[n - 1])
    v = suf[v];
  return v;
}

void add_char(char c) {
  c -= 'a';
  s[n++] = c;
  last = get_link(last);
  if (!go[last][c]) {
    len[sz] = len[last] + 2;
    suf[sz] = go[get_link(suf[last])][c];
    go[last][c] = sz++;
  }
  last = go[last][c]; // cur v = last
}
```

7.3 Ахо-Корасик

```
int go[MAXN][ALPH];
vector<int> term[MAXN];
int par[MAXN], suf[MAXN];
char par_c[MAXN];
vector<int> g[MAXN];

int cntv = 1;

void add(string &s) {
  static int cnt_s = 1;
  int v = 0;
  for (char el: s) {
    if (go[v][el - 'a'] == 0) {
      go[v][el - 'a'] = cntv;
      par[cntv] = v;
      par_c[cntv] = el;
      cntv++;
    }
    v = go[v][el - 'a'];
  }
  term[v].push_back(cnt_s++);
}

void bfs() {
  deque<int> q = {0};
  while (!q.empty()) {
    int v = q.front();
    q.pop_front();
    if (v > 0) {
      if (par[v] == 0) {
        suf[v] = 0;
      } else {
        suf[v] = go[suf[par[v]]][par_c[v] - 'a'];
      }
      g[suf[v]].push_back(v);
    }
    for (int c = 0; c < 26; c++) {
      if (go[v][c] == 0) {
        go[v][c] = go[suf[v]][c];
      } else {
        q.push_back(go[v][c]);
      }
    }
  }
}
```

7.4 Муффиксный Сассив

```
vector<int> build_suff_arr(string &s) {
  // Remove, if you want to sort cyclic shifts
  s += (char) (1);
  int n = s.size();
  vector<int> a(n);
  iota(all(a), 0);
  stable_sort(all(a), [&](int i, int j) {
      return s[i] < s[j];
  });
  vector<int> c(n);
  int cc = 0;
  for (int i = 0; i < n; i++) {
    if (i == 0 || s[a[i]] != s[a[i - 1]])
      c[a[i]] = cc++;
    else
      c[a[i]] = c[a[i - 1]];
  }
  for (int L = 1; L < n; L *= 2) {
    vector<int> cnt(n);
    for (auto i: c) cnt[i]++;
    if (*min_element(all(cnt)) > 0) break;
    vector<int> pref(n);
    for (int i = 1; i < n; i++)
      pref[i] = pref[i - 1] + cnt[i - 1];
    vector<int> na(n);
    for (int i = 0; i < n; i++) {
      int pos = (a[i] - L + n) % n;
      na[pref[c[pos]]++] = pos;
    }
    a = na;
    vector<int> nc(n);
    cc = 0;
    for (int i = 0; i < n; i++) {
      if (i == 0 || c[a[i]] != c[a[i - 1]] ||
```

```
          c[(a[i] + L) % n] != c[(a[i - 1] + L) % n])
        nc[a[i]] = cc++;
      else
        nc[a[i]] = nc[a[i - 1]];
    }
    c = nc;
  }
  // Remove, if you want to sort cyclic shifts
  a.erase(a.begin());
  s.pop_back();
  return a;
}

vector<int> kasai(string s, vector<int> sa) {
  // lcp[i] = lcp(sa[i], sa[i + 1])
  int n = s.size(), k = 0;
  vector<int> lcp(n, 0);
  vector<int> rank(n, 0);
  for (int i = 0; i < n; i++) rank[sa[i]] = i;
  for (int i = 0; i < n; i++, k ? k-- : 0) {
    if (rank[i] == n - 1) {
      k = 0;
      continue;
    }
    int j = sa[rank[i] + 1];
    while (i + k < n && j + k < n && s[i + k] == s[j + k
    ]) k++;
    lcp[rank[i]] = k;
  }
  return lcp;
}
```

7.5 Префикс-функция

```
vector<int> prefix_func(string s) {
  int n = s.size();
  vector<int> pref(n, 0);
  int ans = 0;
  for (int i = 1; i < n; i++) {
    while (ans > 0 && s[ans] != s[i]) {
      ans = pref[ans - 1];
    }
    if (s[i] == s[ans]) {
      ans++;
    }
    pref[i] = ans;
  }
  return pref;
}
```

7.6 Суффиксный автомат

```
// Суфавтомат с подсчётом кол-ва различных подстрок

const int SIGMA = 26;
int ans = 0;

struct Node {
  int go[SIGMA];
  int s, p;
  int len;

  Node() {
    fill(go, go + SIGMA, -1);
    s = -1, p = -1;
    len = 0;
  }
};

int add(int A, int ch, vector<Node> &sa) {
  int B = sa.size();
  sa.emplace_back();
  sa[B].p = A;
  sa[B].s = 0;
  sa[B].len = sa[A].len + 1;
  for (; A != -1; A = sa[A].s) {
    if (sa[A].go[ch] == -1) {
      sa[A].go[ch] = B;
      continue;
    }
    int C = sa[A].go[ch];
    if (sa[C].p == A) {
      sa[B].s = C;
      break;
    }
    int D = sa.size();
    sa.emplace_back();
    sa[D].s = sa[C].s;
    sa[D].p = A;
    sa[D].len = sa[A].len + 1;
    sa[C].s = D;
    sa[B].s = D;
    copy(sa[C].go, sa[C].go + SIGMA, sa[D].go);
    for (; A != -1 && sa[A].go[ch] == C; A = sa[A].s)
      sa[A].go[ch] = D;
    break;
  }
  ans += sa[B].len - sa[sa[B].s].len;
  return B;
}

signed main() {
  string s;
  cin >> s;
  vector<Node> sa(1);
  int A = 0;
  for (char c : s)
    A = add(A, c - 'a', sa);
  cout << ans;
}
```

8 Структуры данных

8.1 Disjoint Sparse Table

```
int tree[LOG][MAXN];
int floorlog2[MAXN]; // i ? (31 - __builtin_clz(i)) : 0

void build(vector<int> &a) {
  int n = a.size();
  copy(a.begin(), a.end(), tree[0]);
  for (int lg = 1; lg < LOG; ++lg) {
    int len = 1 << lg;
    auto &lvl = tree[lg];
    for (int m = len; m < n; m += len * 2) {
      lvl[m - 1] = a[m - 1];
      lvl[m] = a[m];
      for (int i = m - 2; i >= m - len; --i)
        lvl[i] = min(lvl[i + 1], a[i]);
      for (int i = m + 1; i < m + len && i < n; ++i)
        lvl[i] = min(lvl[i - 1], a[i]);
    }
  }
  for (int i = 2; i <= n; ++i)
    floorlog2[i] = floorlog2[i / 2] + 1;
}

// a[l..r)
int get(int l, int r) {
  r--;
  int i = floorlog2[l ^ r];
  return min(tree[i][l], tree[i][r]);
}
```

8.2 Segment Tree Beats

```
// min=, sum
struct ST {
  vector<int> st, mx, mx_cnt, sec_mx;

  ST(int n) {
    st.resize(n * 4, 0);
    mx.resize(n * 4, 0);
    mx_cnt.resize(n * 4, 0);
    sec_mx.resize(n * 4, 0);
    build(0, 0, n);
  }

  void upd_from_children(int v) {
    st[v] = st[v * 2 + 1] + st[v * 2 + 2];
    mx[v] = max(mx[v * 2 + 1], mx[v * 2 + 2]);
    mx_cnt[v] = 0;
    sec_mx[v] = max(sec_mx[v * 2 + 1], sec_mx[v * 2 +
    2]);
    if (mx[v * 2 + 1] == mx[v]) {
      mx_cnt[v] += mx_cnt[v * 2 + 1];
    } else {
      sec_mx[v] = max(sec_mx[v], mx[v * 2 + 1]);
    }
    if (mx[v * 2 + 2] == mx[v]) {
      mx_cnt[v] += mx_cnt[v * 2 + 2];
    } else {
      sec_mx[v] = max(sec_mx[v], mx[v * 2 + 2]);
    }
```

```
}

void build(int i, int l, int r) {
  if (l + 1 == r) {
    st[i] = mx[i] = 0;
    mx_cnt[i] = 1;
    sec_mx[i] = -INF;
    return;
  }
  int m = (r + l) / 2;
  build(i * 2 + 1, l, m);
  build(i * 2 + 2, m, r);
  upd_from_children(i);
}

void push_min_eq(int v, int val) {
  if (mx[v] > val) {
    st[v] -= (mx[v] - val) * mx_cnt[v];
    mx[v] = val;
  }
}

void push(int i) {
  push_min_eq(i * 2 + 1, mx[i]);
  push_min_eq(i * 2 + 2, mx[i]);
}

void update(int i, int l, int r, int ql, int qr, int
   val) {
  if (mx[i] <= val) {
    return;
  }
  if (ql == l && qr == r && sec_mx[i] < val) {
    push_min_eq(i, val);
    return;
  }
  push(i);
  int m = (r + l) / 2;
  if (qr <= m) {
    update(i * 2 + 1, l, m, ql, qr, val);
  } else if (ql >= m) {
    update(i * 2 + 2, m, r, ql, qr, val);
  } else {
    update(i * 2 + 1, l, m, ql, m, val);
    update(i * 2 + 2, m, r, m, qr, val);
  }
  upd_from_children(i);
}

int sum(int i, int l, int r, int ql, int qr) {
  if (l == ql && r == qr) {
    return st[i];
  }
  push(i);
  int m = (r + l) / 2;
  if (qr <= m) {
    return sum(i * 2 + 1, l, m, ql, qr);
  }
  if (ql >= m) {
    return sum(i * 2 + 2, m, r, ql, qr);
  }
  return sum(i * 2 + 1, l, m, ql, m) + sum(i * 2 + 2,
    m, r, m, qr);
  }
};
```

8.3 ДД по неявному

```
pair<Node *, Node *> split(Node *t, int k) {
  if (!now)
    return {nullptr, nullptr};
  int szl = size(t->l);
  if (k <= szl) {
    auto [l, r] = split(t->l, k);
    t->l = r;
    pull(t);
    return {l, t};
  } else {
    auto [l, r] = split(t->r, k - szl - 1);
    t->r = l;
    pull(t);
    return {t, r};
  }
}

Node *merge(Node *l, Node *r) {
  if (!l)
    return r;
  if (!r)
    return l;
  if (l->y < r->y) {
    l->r = merge(l->r, r);
    pull(l);
    return l;
  } else {
    r->l = merge(l, r->l);
    pull(r);
    return r;
  }
}

void insert(Node *&root, int pos, int val) {
  Node *new_v = new Node(val);
  auto [l, r] = split(root, pos);
  root = merge(merge(l, new_v), r);
}

void erase(Node *&root, int pos) {
  auto [lm, r] = split(root, pos + 1);
  auto [l, m] = split(lm, pos);
  root = merge(l, r);
}


int sum(Node *v) {
  return v ? v->sm : 0;
}

// query [l, r)
int query(Node *&root, int ql, int qr) {
  auto [lm, r] = split(root, qr);
  auto [l, m] = split(lm, ql);
  int res = sum(m);
  root = merge(merge(l, m), r);
  return res;
}
```

8.4 ДД

```
pair<Node *, Node *> split(Node *t, int x) {
  if (!t)
    return {nullptr, nullptr};
  if (x <= t->x) {
    auto [l, r] = split(t->l, x);
    t->l = r;
    pull(t);
    return {l, t};
  } else {
    auto [l, r] = split(t->r, x);
    t->r = l;
    pull(t);
    return {t, r};
  }
}

Node *merge(Node *l, Node *r) {
  if (!l)
    return r;
  if (!r)
    return l;
  if (l->y < r->y) {
    l->r = merge(l->r, r);
    pull(l);
    return l;
  } else {
    r->l = merge(l, r->l);
    pull(r);
    return r;
  }
}

void insert(Node *&root, int val) {
  Node *new_v = new Node(val);
  auto [l, r] = split(root, val);
  root = merge(merge(l, new_v), r);
}

void erase(Node *&root, int val) {
  auto [lm, r] = split(root, val + 1);
  auto [l, m] = split(lm, val);
  root = merge(l, r);
}

int sum(Node *v) {
  return v ? v->sm : 0;
}

// query [l, r)
```

```
int query(Node *&root, int ql, int qr) {
  auto [lm, r] = split(root, qr);
  auto [l, m] = split(lm, ql);
  int res = sum(m);
  root = merge(merge(l, m), r);
  return res;
}
```

8.5 Персистентное ДД по неявному

```
mt19937 rnd(228);

struct Node;
int size(Node *);
int sum(Node *);

struct Node {
  Node *l, *r;
  int val, sz, sm;

  Node(int val) : val(val), sz(1), sm(val) {
    l = r = nullptr;
  }
  Node(int val, Node *l, Node *r) : val(val), l(l), r(r)
     {
    sz = 1 + size(l) + size(r);
    sm = val + sum(l) + sum(r);
  }
};

int size(Node *v) {
  return v ? v->sz : 0;
}

int sum(Node *v) {
  return v ? v->sm : 0;
}

pair<Node *, Node *> split(Node *t, int x) {
  if (!t)
    return {nullptr, nullptr};
  int lsz = size(t->l);
  if (lsz >= x) {
    auto [l, r] = split(t->l, x);
    auto v = new Node(t->val, r, t->r);
    return {l, v};
  } else {
    auto [l, r] = split(t->r, x - lsz - 1);
    auto v = new Node(t->val, t->l, l);
    return {v, r};
  }
}

bool chooseleft(int lsz, int rsz) {
  return rnd() % (lsz + rsz) < lsz;
}

Node *merge(Node *l, Node *r) {
  if (!l)
    return r;
  if (!r)
    return l;
  if (chooseleft(l->sz, r->sz)) {
    auto rr = merge(l->r, r);
    auto v = new Node(l->val, l->l, rr);
    return v;
  } else {
    auto ll = merge(l, r->l);
    auto v = new Node(r->val, ll, r->r);
    return v;
  }
}

Node *insert(Node *root, int pos, int val) {
  Node *new_v = new Node(val);
  auto [l, r] = split(root, pos);
  return merge(merge(l, new_v), r);
}

Node *erase(Node *root, int pos) {
  auto [lm, r] = split(root, pos + 1);
  auto [l, m] = split(lm, pos);
  return merge(l, r);
}

// query [l, r)
pair<int, Node *> query(Node *root, int ql, int qr) {
  auto [lm, r] = split(root, qr);
  auto [l, m] = split(lm, ql);
  int res = sum(m);
  auto new_root = merge(merge(l, m), r);
  return {res, new_root};
}
```

8.6 Персистентное ДО

```
// left: v ? v->l : nullptr (same for right)
// sum: v ? v->sm : 0

// v can be nullptr. returns new root of subtree
Node *update(Node *v, int l, int r, int qi, int qx) {
  if (qi < l || r <= qi)
    return v;
  if (l + 1 == r)
    return new Node(qx);
  int m = (l + r) / 2;
  Node *u = new Node();
  u->l = update(left(v), l, m, qi, qx);
  u->r = update(right(v), m, r, qi, qx);
  u->sm = sum(u->l) + sum(u->r);
  return u;
}

int get(Node *v, int l, int r, int ql, int qr) {
  if (!v || qr <= l || r <= ql)
    return 0;
  if (ql <= l && r <= qr)
    return v->sm;
  int m = (l + r) / 2;
  auto a = get(v->l, l, m, ql, qr);
  auto b = get(v->r, m, r, ql, qr);
  return a + b;
}
```

8.7 Спарсы

```
int tree[LOG][MAXN];
int floorlog2[MAXN]; // i ? (31 - __builtin_clz(i)) : 0

void build(vector<int> &a) {
  int n = a.size();
  copy(a.begin(), a.end(), tree[0]);
  for (int i = 1; i < LOG; ++i) {
    int len = 1 << (i - 1);
    for (int j = 0; j + len < n; ++j)
      tree[i][j] = min(tree[i - 1][j], tree[i - 1][j +
     len]);
  }
  for (int i = 2; i <= n; ++i)
    floorlog2[i] = floorlog2[i / 2] + 1;
}

// min a[l..r)
int get(int l, int r) {
  int i = floorlog2[r - l];
  return min(tree[i][l], tree[i][r - (1 << i)]);
}
```

8.8 Фенвик (+ на отрезке)

```
// a[l..r) += x
void update(int l, int r, int x) {
  T1.add(l, x);
  T1.add(r, -x);
  T2.add(l, -x * l);
  T2.add(r, x * r);
}

// sum a[0..pos)
int rsq(int pos) {
  return T1.rsq(pos) * pos + T2.rsq(pos);
}

// sum a[l..r)
int sum(int l, int r) {
  return rsq(r) - rsq(l);
}
```

8.9 Фенвик

```
// Нумерация с 0

struct Fenwick {
  int n;
```

```
  vector<int> f;

  Fenwick(int n) : n(n) {
    f.resize(n + 1);
  }

  // a[i] += x
  void add(int i, int x) {
    for (++i; i <= n; i += i & -i)
      f[i] += x;
  }

  // sum a[0..i)
  int get(int i) {
    int ans = 0;
    for (; i > 0; i -= i & -i)
      ans += f[i];
    return ans;
  }

  // a[..] > 0; find max k: sum a[0..k) <= x
  int max_not_more(int x) {
    int cur = 0;
    for (int i = 20; i >= 0; --i) {
      int len = 1 << i;
      if (cur + len <= n && f[cur + len] <= x) {
        cur += len;
        x -= f[cur];
      }
    }
    return cur;
  }
};

// sum a[x1..x2)[y1..y2)[z1..x2)
int sum_3d(int x1, int x2, int y1, int y2, int z1, int
    z2) {
  int ans = get(x2, y2, z2);
  ans -= get(x1, y2, z2) + get(x2, y1, z2) + get(x2, y2,
     z1);
  ans += get(x1, y1, z2) + get(x1, y2, z1) + get(x2, y1,
     z1);
  ans -= get(x1, y1, z1);
  return ans;
}
```