$Muffix \ Sassif-TRD$

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2.5 Минкост (Джонсон)



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1 Геометрия

1.1 2D

```
class Pt:
   def dot(self, other):
       return self.x * other.x + self.y * other.y
   def cross(self, other):
       return self.x * other.y - self.y * other.x
   @staticmethod
   def get_straight(self, other):
       a = self.y - other.y
       b = other.x - self.x
       c = self.cross(other)
       return a, b, c
class Straight:
   def __eq__(self, other):
       if self.b != 0 or other.b != 0:
           return self.a * other.b == other.a * self.b
     and self.c * other.b == other.c * self.b
       val1 = math.sqrt(self.a ** 2 + self.b ** 2)
       val2 = math.sqrt(other.a ** 2 + other.b ** 2)
       a1, c1 = self.a / val1, self.c / val1
       a2, c2 = other.a / val2, other.c / val2
       if (a1 < 0) != (a2 < 0):
            a1, a2, c1, c2 = a1, -a2, c1, -c2
       return a1 == a2 and c1 == c2
   def perpendicular(self, point: Pt):
       return Straight(-self.b, self.a, self.b * point
    .x - self.a * point.y)
   def get_value(self, point):
       return self.a * point.x + self.b * point.y +
    self.c
   def intersection(self, other):
       d = Pt(self.a, self.b).cross(Pt(other.a, other.
       dx = Pt(self.c, self.b).cross(Pt(other.c, other
    .b))
       dy = Pt(self.a, self.c).cross(Pt(other.a, other
    .c))
       return Pt(-dx / d, -dv / d)
```

```
def dist_from_point(self, point):
        val = math.sqrt(self.a ** 2 + self.b ** 2)
       return abs(Straight(self.a / val, self.b / val,
     self.c / val).get_value(point))
    def parallel(self, dist):
        val = math.sqrt(self.a ** 2 + self.b ** 2)
        return Straight(self.a, self.b, self.c - dist *
     val)
    def is_parallel(self, other):
        return self.a * other.b == self.b * other.a
    def is_perpendicular(self, other):
        per = Straight(-self.b, self.a, 0)
       return per.a * other.b == per.b * other.a
class Triangle:
    def intersection medians(self):
        return (self.A + self.B + self.C) / 3
    def intersection_altitudes(self):
        st1 = Straight(self.A, self.B).perpendicular(
    self.C)
        st2 = Straight(self.A, self.C).perpendicular(
    self.B)
        return st1.intersection(st2)
    def intersection_middle_pers(self):
        st1 = Straight(self.A, self.B).perpendicular((
    self.A + self.B) / 2)
        st2 = Straight(self.A, self.C).perpendicular((
    self.A + self.C) / 2)
        return st1.intersection(st2)
class Circle:
    def intersect_straight(self, st):
        pt = st.get_point()
       A = st.a ** 2 + st.b ** 2
       B = 2 * (-st.b * (pt.x - self.center.x) + st.a
    * (pt.y - self.center.y))
        C = (pt.x - self.center.x) ** 2 + (pt.y - self.
    center.y) ** 2 - self.radius ** 2
       D = B ** 2 - 4 * A * C
       if D < 0:
            return []
       D = math.sqrt(D)
```

```
vector = Pt(-st.b, st.a)
   if D == 0:
       t = -B / (2 * A)
       return [pt + t * vector]
   t1 = (-B - D) / (2 * A)
   t2 = (-B + D) / (2 * A)
   return [pt + t1 * vector, pt + t2 * vector]
def intersect_circle(self, other):
   x1, x2 = self.center.x, other.center.x
   y1, y2 = self.center.y, other.center.y
   a = -2 * (x1 - x2)
   b = -2 * (y1 - y2)
   c = (x1 ** 2 - x2 ** 2) + (y1 ** 2 - y2 ** 2) -
 (self.radius ** 2 - other.radius ** 2)
   return self.intersect_straight(Straight(a, b, c
))
def is_own(self, pt):
   return (pt.x - self.center.x) ** 2 + (pt.y -
self.center.y) ** 2 == self.radius ** 2
def tangent_pts(self, pt):
    if self.is_own(pt):
       return [pt]
    cir = Circle(pt, math.sqrt(abs(pt - self.center
) ** 2 - self.radius ** 2))
   return self.intersect_circle(cir)
def dist_by_circle(self, pt1, pt2):
   pt1 -= self.center
   pt2 -= self.center
    ang = (pt1.polar_angle() - pt2.polar_angle()) %
 (2 * math.pi)
    ang = min(ang, 2 * math.pi - ang)
   return self.radius * ang
```

1.2 3D

// TODO

1.3 Выпуклая оболочка

```
Pt start(0, 0);
bool comp(Pt a, Pt b) {
    ll ang = (a - start).cross(b - start);
    if (ang < 0) {</pre>
```

```
return false;
   } else if (ang > 0) {
        return true;
   return abs(start - a) < abs(start - b);</pre>
vector<Pt> convex_hull(vector<Pt> points) {
    int n = points.size();
   start = points[0];
   for (auto x : points) {
        start = min(start, x);
   sort(points.begin(), points.end(), comp);
   vector<Pt> s = {points[0], points[1]};
   for (int i = 2; i < n; i++) {</pre>
        int k = s.size();
        while (k > 1 \&\& (s[k - 1] - s[k - 2]).cross(
    points[i] - s[k - 1]) \le 0) {
            s.pop_back();
            k = s.size();
        }
        s.push_back(points[i]);
   }
   return s;
```

1.4 Касательные из точки

// TODO

1.5 Касательные параллельные прямой

// TODO

2 Графы

2.1 Венгерский алгоритм

```
vector<int> venger(vector<vector<int>> arr) {
   int n = (int) arr.size() - 1;
   vector<int> u(n + 1), v(n + 1), p(n + 1), way(n +
   1);
   for (int i = 1; i <= n; i++) {
      p[0] = i;
      int ind = 0;
      vector<int> minv(n + 1, INF), used(n + 1);
```

```
do {
        used[ind] = 1:
        int ind2 = p[ind], dlt = INF, ind3 = 0;
        for (int j = 1; j \le n; j++)
            if (!used[j]) {
                 int cur = arr[ind2][j] - u[ind2] -
v[j];
                 if (cur < minv[j]) {</pre>
                     minv[j] = cur;
                     wav[i] = ind;
                 if (minv[j] < dlt) {</pre>
                     dlt = minv[j], ind3 = j;
                }
            }
        for (int j = 0; j <= n; j++)</pre>
            if (used[i]) {
                u[p[i]] += dlt;
                 v[j] -= dlt;
            } else {
                 minv[j] -= dlt;
            }
        ind = ind3:
    } while (p[ind] != 0);
    do {
        int ind3 = way[ind];
        p[ind] = p[ind3];
        ind = ind3;
    } while (ind):
vector<int> ans(n + 1);
for (int j = 1; j <= n; j++) {
    ans[p[j]] = j;
}
return ans;
```

2.2 Дейкстра за квадрат

// TODO

2.3 Диниц

```
vector<edge> g[MAXN];
pair<int, int> pred[MAXN];
int d[MAXN];
int inds[MAXN];
```

```
bool dfs(int v, int final, int W) {
    if (v == final) {
        return true;
    for (int i = inds[v]; i < (int) g[v].size(); i++) {</pre>
        auto e = g[v][i];
        if (e.f + W \le e.c \&\& d[v] + 1 == d[e.v]) {
            pred[e.v] = {v, i};
            bool flag = dfs(e.v, final, W);
            if (flag) {
                return true;
            inds[v]++;
        } else {
            inds[v]++;
   }
    return false;
bool bfs(int start, int final, int W) {
    fill(d, d + MAXN, INF);
    d[start] = 0;
    deque<int> q = {start};
    while (!q.empty()) {
        int v = q.front();
        q.pop_front();
        for (auto e : g[v]) {
            if (e.f + W \le e.c \&\& d[e.v] > d[v] + 1) {
                d[e.v] = d[v] + 1;
                q.push_back(e.v);
            }
        }
   }
    if (d[final] == INF) {
        return false;
    fill(inds, inds + MAXN, 0);
    while (dfs(start, final, W)) {
        int v = final:
        int x = INF;
        while (v != start) {
            int ind = pred[v].second;
            v = pred[v].first;
            x = min(x, g[v][ind].c - g[v][ind].f);
        v = final;
        while (v != start) {
            int ind = pred[v].second;
            v = pred[v].first;
```

```
g[v][ind].f += x;
            g[g[v][ind].v][g[v][ind].ind].f == x;
   }
   return true;
void Dinic(int start, int final) {
   int W = (1LL << 30);</pre>
   do {
        while (bfs(start, final, W));
        W /= 2;
   } while (W >= 1);
signed main() {
   vector<pair<int, int>> edges;
   for (int i = 0; i < m; i++) {</pre>
        int u, v, c;
        cin >> u >> v >> c;
        edges.emplace_back(u, v);
        g[u].push_back({v, 0, c, (int) g[v].size()});
        g[v].push_back({u, 0, c, (int) g[u].size() -
    1});
   }
   Dinic(1, n);
   int res = 0:
   for (auto e : g[1]) {
        res += e.f:
   }
   vector<int> ans;
   for (int i = 0; i < m; i++) {</pre>
        int u = edges[i].first, v = edges[i].second;
        if ((d[u] != INF && d[v] == INF) || (d[u] ==
    INF && d[v] != INF)) {
            ans.push_back(i + 1);
       }
   }
```

2.4 KCC

```
void dfs1(int v) {
    vis[v] = 1;
    for (auto u : g[v]) {
        if (!vis[u]) {
            dfs1(u);
        }
    }
```

```
topsort.push_back(v);
void dfs2(int v) {
    vis[v] = 1;
    for (auto u : rg[v]) {
        if (!vis[u]) {
            dfs2(u);
        }
   }
    comp.push_back(v);
signed main() {
    for (int i = 1; i <= n; i++)</pre>
        if (!vis[i])
            dfs1(i);
    reverse(topsort.begin(), topsort.end());
    for (int j = 1; j <= n; j++) {</pre>
        int vert = topsort[i - 1];
        if (!vis[vert])
            dfs2(vert);
   }
```

2.5 Минкост (Джонсон)

```
using cost_t = 11;
using flow_t = int;
const int MAXN = 10000;
const int MAXM = 25000 * 2;
const cost_t INFw = 1e12;
const flow_t INFf = 10;
struct Edge {
 int v, u;
 flow_t f, c;
 cost_t w;
};
Edge edg[MAXM];
int esz = 0;
vector<int> graph[MAXN];
11 dist[MAXN];
11 pot[MAXN];
int S, T;
int NUMV;
```

```
int pre[MAXN];
bitset<MAXN> inQ;
flow_t get_flow() {
 int v = T;
  if (pre[v] == -1)
   return 0;
 flow t f = INFf:
  do {
    int ei = pre[v];
    Edge &e = edg[ei];
    f = min(f, e.c - e.f);
   if (f == 0)
      return 0:
    v = e.v;
 } while (v != S);
 v = T:
  do {
   int ei = pre[v];
    edg[ei].f += f;
    edg[ei ^ 1].f -= f;
    v = edg[ei].v;
 } while (v != S);
  return f;
void spfa() {
 fill(dist, dist + NUMV, INFw);
  dist[S] = 0;
  deque<int> Q = {S};
  inQ[S] = true;
  while (!Q.empty()) {
   int v = Q.front();
    Q.pop_front();
    inQ[v] = false;
    cost_t d = dist[v];
    for (int ei : graph[v]) {
      Edge &e = edg[ei];
      if (e.f == e.c)
        continue:
      cost_t w = e.w + pot[v] - pot[e.u];
      if (dist[e.u] <= d + w)</pre>
        continue;
      pre[e.u] = ei;
      dist[e.u] = d + w;
      if (!inQ[e.u]) {
        inQ[e.u] = true;
        Q.push_back(e.u);
   }
```

```
}
  for (int i = 0; i < NUMV; ++i)</pre>
    pot[i] += dist[i];
cost t mincost() {
  spfa(); // pot[i] = 0 // or ford_bellman
  flow_t f = 0;
  while (true) {
    flow_t ff = get_flow();
    if (ff == 0)
      break:
    f += ff:
    spfa(); // or dijkstra
  }
  cost_t res = 0;
  for (int i = 0; i < esz; ++i)</pre>
    res += edg[i].f * edg[i].w;
  res /= 2:
  return res;
}
void add_edge(int v, int u, int c, int w) {
  edg[esz] = \{v, u, 0, c, w\};
  edg[esz + 1] = \{u, v, 0, 0, -w\};
  graph[v].push_back(esz);
  graph[u].push_back(esz + 1);
  esz += 2;
signed main() {
  ios_base::sync_with_stdio(false);
  cin.tie(nullptr);
  int n, m;
  cin >> n >> m;
  S = 0;
  T = n - 1;
  NUMV = n:
  for (int i = 0; i < m; ++i) {</pre>
   int v, u, c, w;
    cin >> v >> u >> c >> w;
    v--, u--;
    add_edge(v, u, c, w);
  cost_t ans = mincost();
  cout << ans;</pre>
```

```
void dfs(int v, int par) {
    vis[v] = 1;
    up[v] = tin[v] = timer++;
    for (auto u : g[v]) {
        if (!vis[u]) {
            dfs(u, v);
            up[v] = min(up[v], up[u]);
        } else if (u != par) {
            up[v] = min(up[v], tin[u]);
        }
        if (up[u] > tin[v]) {
            bridges.emplace_back(v, u);
        }
    }
}
```

2.7 Паросочетания

```
bool dfs(int v, int c) {
   if (used[v] == c) return false;
   used[v] = c;
   for (auto u : g[v]) {
        if (res[u] == -1) {
            res[u] = v;
            return true:
        }
   for (auto u : g[v]) {
        if (dfs(res[u], c)) {
            res[u] = v;
            return true;
        }
   }
    return false;
signed main() {
   for (int i = 0; i < s; ++i) {</pre>
        ans += dfs(i, i + 1);
   }
```

2.8 Точки сочленения

```
void dfs(int v, int par) {
    vis[v] = 1;
    up[v] = tin[v] = timer++;
    int child = 0;
```

```
for (auto u : g[v]) {
    if (!vis[u]) {
        dfs(u, v);
        up[v] = min(up[v], up[u]);
        if (up[u] >= tin[v] && par != -1) {
            points.insert(v);
        }
        child++;
    } else if (u != par) {
        up[v] = min(up[v], tin[u]);
    }
}
if (par == -1 && child >= 2) {
        points.insert(v);
}
```

2.9 Эдмондс-Карп

```
struct edge {
    int v, f, c, ind;
};
vector<edge> g[MAXN];
bool bfs(int start, int final, int W) {
    vector<int> d(MAXN, INF);
    vector<pair<int, int>> pred(MAXN);
    d[start] = 0;
    deque<int> q = {start};
    while (!q.empty()) {
        int v = q.front();
        q.pop_front();
        for (int i = 0; i < (int) g[v].size(); i++) {</pre>
            auto e = g[v][i];
            if (e.f + W \le e.c \&\& d[e.v] > d[v] + 1) {
                d[e.v] = d[v] + 1:
                pred[e.v] = {v, i};
                q.push_back(e.v);
        }
    if (d[final] == INF) {
        return false;
    int v = final;
    int x = INF;
    while (v != start) {
        int ind = pred[v].second;
```

```
v = pred[v].first;
       x = min(x, g[v][ind].c - g[v][ind].f);
   }
   v = final;
   while (v != start) {
       int ind = pred[v].second;
       v = pred[v].first;
       g[v][ind].f += x;
       g[g[v][ind].v][g[v][ind].ind].f == x;
   }
   return true;
signed main() {
   for (int i = 0; i < m; i++) {</pre>
       int u, v, c;
       cin >> u >> v >> c;
       g[u].push_back({v, 0, c, (int) g[v].size()});
       g[v].push_back({u, 0, 0, (int) g[u].size() -
    1});
   }
   int W = (1 << 30);
   do {
       while (bfs(1, n, W));
       W /= 2;
   } while (W >= 1);
   int res = 0:
   for (auto e : g[1]) {
       res += e.f:
   }
```

2.10 Эйлеров цикл

// TODO

3 ДП

3.1 CHT

```
pair<ld, ld> inter(Line a, Line b) {
    ld x = (b.b - a.b) / (a.k - b.k);
    ld y = a.k * x + a.b;
    return {x, y};
}

void add_line(ld k, ld b, vector<Line> &s, vector<pair<
    ld, ld>> &pts) {
```

```
while (s.size() >= 2) {
        pair<ld, ld> x1 = inter(s.back(), s[s.size() -
    2]);
        pair<ld, ld > x2 = inter(s[s.size() - 2], \{k, b
    }):
        if (x1 > x2) {
            break;
        pts.pop_back();
        s.pop_back();
    }
    if (!s.empty()) {
        pts.push_back(inter(s.back(), {k, b}));
    s.push_back({k, b});
ld bin_search(vector<Line> &s, ld x) {
    int 1 = 0, r = s.size();
    while (1 + 1 < r) {
        int m = (r + 1) / 2:
        auto kek = inter(s[m - 1], s[m]);
        if (kek.first >= x) {
            1 = m;
        } else {
            r = m;
    return s[1].k * x + s[1].b:
```

3.2 Li Chao

```
// max

struct Line {
    int k, b;

    int f(int x) {
        return k * x + b;
    }
};

struct ST {
    vector<Line> st;

    ST(int n) {
        Line ln = {OLL, -INF};
        st.resize(4 * n, ln);
}
```

```
}
    void upd(int i, int l, int r, Line ln) {
        int child = 1;
        Line ln1 = ln;
        int m = (1 + r) / 2;
        if (ln.f(m) > st[i].f(m)) {
            if (ln.k < st[i].k) {</pre>
                child = 2;
            ln1 = st[i]:
            st[i] = ln;
       } else {
            if (st[i].k < ln.k) {</pre>
                child = 2;
        }
        if (1 + 1 < r) {
            if (child == 1) {
                upd(i * 2 + 1, 1, m, ln1);
            } else {
                upd(i * 2 + 2, m, r, ln1);
        }
   }
    int res(int i, int l, int r, int x) {
        if (1 + 1 == r) {
            return st[i].f(x);
        int m = (1 + r) / 2;
        int val = st[i].f(x);
        if (x < m) {
            val = max(val, res(i * 2 + 1, 1, m, x));
        } else {
            val = max(val, res(i * 2 + 2, m, r, x));
        return val;
   }
};
```

3.3 SOS-dp

// TODO

3.4 HBΠ

// TODO

$3.5 \quad \text{HOB}\Pi$

// TODO

4 Деревья

4.1 Centroid

```
void sizes(int v, int p) {
    sz[v] = 1;
    for (auto u : g[v]) {
        if (u != p && !used[u]) {
            sizes(u, v);
            sz[v] += sz[u];
       }
   }
int centroid(int v, int p, int n) {
   for (int u : g[v]) {
        if (sz[u] > n / 2 && u != p && !used[u]) {
            return centroid(u, v, n);
       }
   }
    return v;
void dfs(int v, int p) {
    for (auto u : g[v]) {
        if (u != p && !used[u]) {
            dfs(u, v);
       }
   }
}
void solve(int v) {
    sizes(v, -1);
    for (auto u : g[v]) {
        if (!used[u]) {
            . . . . . . . . . . .
            dfs(u, v);
            }
   }
    used[v] = 1;
   for (int u : g[v]) {
        if (!used[u]) {
```

```
solve(centroid(u, v, sz[u]));
}

}
int main() {
    sizes(0, -1);
    solve(centroid(0, -1, n));
}
```

4.2 HLD

```
const int MAXN = 50500;
const int INF = (int) 1e15;
const int L = 20;
vector<int> g[MAXN];
int sz[MAXN];
int depth[MAXN];
vector<vector<int>> up(MAXN, vector<int>(L + 1));
void dfs(int v, int p) {
   up[v][0] = p;
   for (int i = 1; i <= L; i++) {
        up[v][i] = up[up[v][i - 1]][i - 1];
   for (int u : g[v]) {
       if (u != p) {
            dfs(u, v);
       }
   }
int lca(int u, int v) {
   if (u == v) {
       return u;
   int du = depth[u], dv = depth[v];
   if (du < dv) {
        swap(du, dv);
        swap(u, v);
   for (int i = L; i >= 0; i--) {
       if (du - (int) pow(2, i) >= dv) {
           u = up[u][i];
            du = (int) pow(2, i);
       }
   }
   if (u == v) {
        return u;
```

```
}
    for (int i = L; i >= 0; i--) {
        if (up[u][i] != up[v][i]) {
            u = up[u][i];
            v = up[v][i];
        }
   }
   return up[u][0];
void dfs1(int v, int p) {
    sz[v] = 1;
    for (int u : g[v]) {
        if (u != p) {
            dfs1(u, v);
            sz[v] += sz[u];
        }
   }
}
int cnt = 0;
int nn[MAXN];
int pred[MAXN];
int rup[MAXN];
void dfs2(int v, int p, int root, int dep = 0) {
    depth[v] = dep;
    nn[v] = cnt++;
    pred[v] = p;
    rup[v] = root;
    int mx = 0;
    int vert = -1;
    for (int u : g[v]) {
        if (u != p) {
            if (mx < sz[u]) {</pre>
                mx = sz[u];
                vert = u;
        }
   }
   if (vert != -1) {
        dfs2(vert, v, root, dep + 1);
   }
   for (int u : g[v]) {
        if (u != p && u != vert) {
            dfs2(u, v, u, dep + 1);
        }
   }
```

```
ST st({});
int n;
int mx_path_up(int u, int v) {
    if (depth[u] < depth[v]) {</pre>
        swap(u, v);
    int res = -INF:
    while (true) {
        int root = rup[u];
        if (depth[root] <= depth[v]) {</pre>
            res = max(res, st.rmq(0, 0, n, nn[v], nn[u])
     + 1)):
            break;
        }
        res = max(res, st.rmq(0, 0, n, nn[root], nn[u])
        u = pred[rup[u]];
    }
    return res;
}
int mx_path(int u, int v) {
    int vert = lca(u, v);
    return max(mx_path_up(u, vert), mx_path_up(v, vert)
    );
}
void change(int u, int qd) {
    st.update(0, 0, n, nn[u], qd);
signed main() {
    cin >> n:
    vector<int> hs(n);
    for (auto &x : hs) {
        cin >> x;
    }
    for (int i = 0; i < n - 1; i++) {
        cin >> u1 >> v1:
        g[u1].push_back(v1);
        g[v1].push_back(u1);
    }
    dfs1(1, -1);
    dfs(1, 1);
    dfs2(1, -1, 1);
    vector<int> nhs(n);
    for (int i = 1; i <= n; i++) {</pre>
        nhs[nn[i]] = hs[i - 1];
    }
```

```
st = *new ST(nhs);
char op;
int q;
cin >> q;
while (q--) {
    cin >> op >> v1 >> u1;
    if (op == '?') {
        cout << mx_path(u1, v1) << endl;
    } else {
        change(v1, u1);
    }
}</pre>
```

4.3 Link-cut

```
struct Node {
  Node *ch[2];
  Node *p;
  bool rev;
  int sz;
  Node () {
    ch[0] = nullptr;
    ch[1] = nullptr;
    p = nullptr;
    rev = false;
    sz = 1;
};
int size(Node *v) {
 return (v ? v->sz : 0);
int chnum(Node *v) {
 return v->p->ch[1] == v;
bool isroot(Node *v) {
 return v->p == nullptr || v->p->ch[chnum(v)] != v;
void push(Node *v) {
 if (v->rev) {
    if (v->ch[0])
      v->ch[0]->rev ^= 1;
    if (v->ch[1])
      v->ch[1]->rev ^= 1;
```

```
swap(v->ch[0], v->ch[1]);
    v->rev = false;
 }
void pull(Node *v) {
 v->sz = size(v->ch[1]) + size(v->ch[0]) + 1;
void attach(Node *v, Node *p, int num) {
    p->ch[num] = v;
  if (v)
    v - p = p;
void rotate(Node *v) {
 Node *p = v->p;
 push(p);
  push(v);
  int num = chnum(v);
 Node *u = v - ch[1 - num];
  if (!isroot(v->p))
    attach(v, p->p, chnum(p));
  else
    v->p = p->p;
  attach(u, p, num);
  attach(p, v, 1 - num);
  pull(p);
 pull(v);
void splay(Node *v) {
 push(v):
  while (!isroot(v)) {
    if (!isroot(v->p)) {
      if (chnum(v) == chnum(v->p))
       rotate(v->p);
      else
       rotate(v);
   }
    rotate(v);
 }
}
void expose(Node *v) {
  splay(v);
 v->ch[1] = nullptr;
  pull(v);
  while (v->p != nullptr) {
```

```
Node *p = v - p;
    splay(p);
   attach(v, p, 1);
   pull(p);
   splay(v);
void makeroot(Node *v) {
 expose(v);
 v->rev ^= 1;
 push(v);
void link(Node *v, Node *u) {
 makeroot(v);
 makeroot(u);
 u->p = v;
void cut(Node *v, Node *u) {
 makeroot(u);
 makeroot(v);
 v->ch[1] = nullptr;
 u->p = nullptr;
int get(Node *v, Node *u) {
 makeroot(u):
 makeroot(v);
 Node *w = u;
 while (!isroot(w))
   w = w - > p;
 return (w == v ? size(v) - 1 : -1);
const int MAXN = 100010;
Node *nodes[MAXN];
int main() {
 ios_base::sync_with_stdio(false);
 cin.tie(nullptr);
 cout.tie(nullptr);
 int n, q;
 cin >> n >> q;
 for (int i = 0; i < n; ++i)</pre>
   nodes[i] = new Node();
 while (q--) {
   string s;
   Int a, b;
```

```
cin >> s >> a >> b;
a--, b--;
if (s[0] == 'g')
   cout << get(nodes[a], nodes[b]) << '\n';
else if (s[0] == 'l')
   link(nodes[a], nodes[b]);
else
   cut(nodes[a], nodes[b]);
}</pre>
```

5 Другое

5.1 attribute packed

// TODO

5.2 ordered set

5.3 pragma

5.4 Аллокатор Копелиовича

// TODO

6 Математика

6.1 2-SAT

// TODO

6.2 FFT mod

```
#include <bits/stdc++.h>
using namespace std;
typedef long long 11;
#define int long long
const int MOD = 998244353; // 7*17 * 2^23 + 1
const int GEN = 3;
//const int MOD = 7340033; // 7 * 2^20 + 1
//const int GEN = 5;
//const int MOD = 469762049; // 7 * 2^26 + 1
//const int GEN = 30:
const int LOG = 20;
const int MAXN = 1 << LOG;</pre>
int tail[MAXN + 1];
int OMEGA[MAXN + 1];
int binpow(int x, int p) {
    int res = 1;
    while (p > 0) {
        if (p & 1)
            res = res * 111 * x % MOD;
        x = x * 111 * x % MOD;
        p >>= 1:
   }
    return res;
int omega(int n, int k) {
    return OMEGA[MAXN / n * k];
int gettail(int x, int lg) {
    return tail[x] >> (LOG - lg);
void calcomega() {
    11 one = binpow(GEN, (MOD - 1) / MAXN);
    OMEGA[O] = 1:
    for (int i = 1; i < MAXN; ++i) {</pre>
        OMEGA[i] = OMEGA[i - 1] * one % MOD;
   }
```

```
void calctail() {
    int n = MAXN;
    for (int x = 0; x < n; ++x) {
        int res = 0:
        for (int i = 0; i < LOG; ++i) {</pre>
            res += ((x >> i) & 1) << (LOG - i - 1);
        tail[x] = res;
    }
}
// Without precalc, tail[], OMEGA[]
//ll omega(int n, int k) {
      return binpow(GEN, (MOD - 1) / n * k);
//}
//
//int gettail(int x, int lg) {
      int res = 0;
      for (int i = 0; i < lg; ++i)
          res += ((x >> i) & 1) << (lg - i - 1);
      return res;
//}
void fft(vector<int> &A, int lg) {
    int n = 1 \ll lg;
    for (int i = 0; i < n; ++i) {</pre>
        int j = gettail(i, lg);
        if (i < j)
            swap(A[i], A[j]);
    }
    for (int len = 2; len <= n; len *= 2) {</pre>
        for (int i = 0; i < n; i += len) {</pre>
            for (int j = 0; j < len / 2; ++j) {
                 auto v = A[i + j];
                auto u = A[i + j + len / 2] * 111 *
    omega(len, j) % MOD;
                A[i + j] = (v + u) \% MOD;
                A[i + j + len / 2] = (v - u + MOD) %
    MOD;
            }
        }
    }
int inverse(int x) {
    return binpow(x, MOD - 2);
}
```

```
void invfft(vector<int> &A, int lg) {
    int n = 1 \ll lg;
    fft(A, lg);
    for (auto &el : A)
        el = el * 1ll * inverse(n % MOD) % MOD;
    reverse(A.begin() + 1, A.end());
vector<int> mul(vector<int> A, vector<int> B) {
    int lg = 32 - __builtin_clz(A.size() + B.size() -
    int n = 1 << lg;</pre>
    A.resize(n, 0);
    B.resize(n. 0):
    fft(A, lg);
    fft(B, lg);
    for (int i = 0; i < n; ++i)</pre>
        A[i] = A[i] * 111 * B[i] % MOD;
    invfft(A, lg);
    return A;
signed main() {
    ios_base::sync_with_stdio(false);
    cin.tie(nullptr);
    calctail();
    calcomega();
    int n, m;
    cin >> n >> m;
    vector<int> A(n), B(m);
    for (int &el : A)
        cin >> el:
    for (int &el : B)
        cin >> el:
    auto C = mul(A, B);
    for (auto el : C)
        cout << el << ',,';
```

6.3 FFT

```
const double PI = acos(-1);
const int LOG = 19;
const int MAXN = 1 << LOG;

struct comp {
   double x, y;
   comp() : x(0), y(0) {}
   comp(double x, double y) : x(x), y(y) {}</pre>
```

```
comp(int x) : x(x), y(0) {}
    comp operator+(const comp &o) const {
        return comp(x + o.x, y + o.y);
    comp operator-(const comp &o) const {
        return comp(x - o.x, y - o.y);
    comp operator*(const comp &o) const {
        return comp(x * o.x - y * o.y, x * o.y + y * o.
    x);
    }
    comp operator/(const int k) const {
        return comp(x / k, y / k);
    comp conj() const {
        return comp(x, -y);
   }
};
comp OMEGA[MAXN + 10];
int tail[MAXN + 10];
comp omega(int n, int k) {
    return OMEGA[MAXN / n * k];
void calcomega() {
    for (int i = 0; i < MAXN; ++i) {</pre>
        double x = 2 * PI * i / MAXN;
        OMEGA[i] = {cos(x), sin(x)};
   }
}
void calctail() {
    tail[0] = 0;
   for (int i = 1; i < MAXN; ++i) {</pre>
        tail[i] = (tail[i >> 1] >> 1) | ((i & 1) << (
    LOG - 1));
   }
}
void fft(vector<comp> &A) {
    int n = A.size();
   for (int i = 0; i < n; ++i) {</pre>
        if (i < tail[i])</pre>
            swap(A[i], A[tail[i]]);
   }
    for (int len = 2; len <= n; len *= 2) {</pre>
        for (int i = 0; i < n; i += len) {</pre>
            for (int j = 0; j < len / 2; ++j) {
```

```
auto v = A[i + j];
                 auto u = A[i + j + len / 2] * omega(len)
    , j);
                A[i + j] = v + u;
                A[i + j + len / 2] = v - u;
            }
        }
    }
}
void fft2(vector<comp> &A, vector<comp> &B) {
    int n = A.size();
    vector<comp> C(n);
    for (int i = 0: i < n: ++i) {</pre>
        C[i].x = A[i].x;
        C[i].y = B[i].x;
    }
    fft(C);
    C.push_back(C[0]);
    for (int i = 0; i < n; ++i) {</pre>
        A[i] = (C[i] + C[n - i].conj()) / 2;
        B[i] = (C[i] - C[n - i].conj()) / 2 * comp(0,
    -1);
    }
void invfft(vector<comp> &A) {
    fft(A):
    for (auto &el : A)
        el = el / MAXN:
    reverse(A.begin() + 1, A.end());
vector<int> mul(vector<int> &a, vector<int> &b) {
    vector<comp> A(MAXN, 0), B(MAXN, 0);
    for (int i = 0; i < (int)a.size(); ++i)</pre>
        A[i] = a[i]:
    for (int i = 0; i < (int)b.size(); ++i)</pre>
        B[i] = b[i];
    fft2(A, B);
    for (int i = 0; i < MAXN; ++i)</pre>
        A[i] = A[i] * B[i];
    invfft(A);
    vector<int> c(MAXN);
    for (int i = 0; i < MAXN; ++i) {</pre>
        int x = round(A[i].x);
        c[i] = x;
    }
    while (!c.empty() && c.back() == 0)
        c.pop_back();
    return c;
```

```
signed main() {
    ios_base::sync_with_stdio(false);
    cin.tie(nullptr);
    calcomega();
    calctail();
    string sa, sb;
    cin >> sa >> sb;
   reverse(sa.begin(), sa.end());
    reverse(sb.begin(), sb.end());
    vector<int> a(sa.size()), b(sb.size());
   bool minus = false;
   if (sa.back() == '-') {
        minus ^= true:
        sa.pop_back();
   if (sb.back() == '-') {
        minus ^= true;
        sb.pop_back();
   for (int i = 0; i < (int)sa.size(); ++i)</pre>
        a[i] = sa[i] - '0';
   for (int i = 0; i < (int)sb.size(); ++i)</pre>
        b[i] = sb[i] - '0';
   auto c = mul(a, b);
   int shift = 0:
   for (int i = 0; i < (int)c.size(); ++i) {</pre>
        int x = c[i] + shift;
        c[i] = x % 10;
        shift = x / 10;
   }
   while (shift > 0) {
        c.push_back(shift % 10);
        shift \neq 10;
   while (!c.empty() && c.back() == 0)
        c.pop_back();
   if (c.empty()) {
        cout << 0:
        return 0;
   }
   if (minus)
        cout << '-':
   for (int i = c.size() - 1; i >= 0; --i)
        cout << c[i];
```

```
6.4 Γaycc
```

```
// TODO битовый
// TODO дабловый
// TODO по модулю
```

5.5 Диофантовы уравнения

```
def bezout(a, b):
    x, xx, y, yy = 1, 0, 0, 1
    while b:
        q = a // b
        a, b = b, a % b
        x, xx = xx, x - xx*q
        y, yy = yy, y - yy*q
    return (a, x, y)

a, b, c = map(int, input().split())
d, k, l = bezout(a, b)
q = c // d
x, y = q*k, q*l
if c % d == 0:
    x -= x // (b//d) * (b//d)
    y = (c-a*x) // b
```

6.6 KTO

// TODO

6.7 Код Грея

// TODO

6.8 Линейное решето

// TODO

6.9 Ро-Поллард

// TODO

6.10 Символ Якоби, Лежандра

// TODO

7 Строки

7.1 Z-функция

```
int main() {
    vector<int> z(n, 0);
    z[0] = n;
    int 1 = 0, r = 0;
    for (int i = 1; i < n; i++) {
        if (i < r) {
            z[i] = min(z[i - 1], r - i);
        }
        while (i + z[i] < n && s[z[i]] == s[i + z[i]])
    {
        z[i]++;
      }
      if (i + z[i] > r) {
        1 = i;
        r = i + z[i];
      }
}
```

7.2 Ахо-Корасик

```
int cntv = 1;
void add(string &s) {
    static int cnt_s = 1;
   int v = 0:
   for (char el : s) {
        if (go[v][el - 'a'] == 0) {
            go[v][el - 'a'] = cntv;
            par[cntv] = v;
            par_c[cntv] = el;
            cntv++;
        v = go[v][el - 'a'];
   }
    term[v].push_back(cnt_s++);
void bfs() {
    deque < int > q = {0};
   while (!q.empty()) {
        int v = q.front();
        q.pop_front();
        if (v > 0) {
            if (par[v] == 0) {
```

7.3 Префикс-функция

```
int main() {
    vector<int> pref(n, 0);
    int ans = 0;
    for (int i = 1; i < n; i++) {
        while (ans > 0 && s[ans] != s[i]) {
            ans = pref[ans - 1];
        }
        if (s[i] == s[ans]) {
            ans++;
        }
        pref[i] = ans;
    }
}
```

7.4 Суффиксный автомат

```
struct Node {
    int go[26];
    int suf;
    int len;
};
Node verts[MAXN];
int cnt_v = 1;
int max_v = 0;

void add(char c) {
    c -= 'a';
    int nv = cnt_v++;
    verts[max_v].go[c] = nv;
```

```
verts[nv].len = verts[max_v].len + 1;
int v = max v:
while (v != 0) {
    if (verts[verts[v].suf].go[c] == 0) {
        v = verts[v].suf;
        verts[v].go[c] = nv;
        verts[nv].len = max(verts[nv].len, verts[v
1.len + 1):
        continue;
    int vv = verts[v].suf, uu = verts[vv].go[c];
    if (verts[vv].len + 1 == verts[uu].len) {
        verts[nv].suf = uu:
        break:
    }
    int v2 = cnt_v++;
    for (int c2 = 0; c2 < 26; c2++) {</pre>
        verts[v2].go[c2] = verts[uu].go[c2];
    int to = verts[vv].go[c];
        if (verts[vv].go[c] == to) {
            verts[vv].go[c] = v2;
            verts[v2].len = max(verts[v2].len,
verts[vv].len + 1);
        } else {
            break:
        vv = verts[vv].suf:
    } while (vv != 0);
    if (verts[vv].go[c] == to) {
        verts[vv].go[c] = v2;
        verts[v2].len = max(verts[v2].len, verts[vv
l.len + 1):
    }
    verts[v2].suf = verts[uu].suf;
    verts[uu].suf = verts[nv].suf = v2;
    break;
}
max_v = nv;
```

7.5 Суффиксный массив

```
vector<int> build_suff_arr(string s) {
    s.push_back('#');
    int n = s.size();
    vector<int> suf(n), c(n);
    vector<int> cnt(MAX);
```

```
for (int i = 0; i < n; i++) {</pre>
    cnt[s[i] - '#']++;
vector<int> pos(MAX);
for (int i = 1; i < MAX; i++) {</pre>
    pos[i] = pos[i - 1] + cnt[i - 1];
for (int i = 0; i < n; i++) {</pre>
    suf[pos[s[i] - '#']++] = i;
}
int cls = -1;
for (int i = 0; i < n; i++) {</pre>
    if (i == 0 || s[suf[i]] != s[suf[i - 1]]) {
        cls++:
    }
    c[suf[i]] = cls;
for (int L = 1; L < n; L *= 2) {</pre>
    fill(cnt.begin(), cnt.end(), 0);
    for (int i = 0; i < n; i++) {</pre>
         cnt[c[i]]++:
    }
    pos[0] = 0;
    for (int i = 1; i < n; i++) {</pre>
         pos[i] = pos[i - 1] + cnt[i - 1];
    }
    for (int i = 0; i < n; i++) {</pre>
         suf[i] = (suf[i] - L + n) \% n;
    vector<int> new_suf(n), new_c(n);
    for (int i = 0; i < n; i++) {</pre>
         int where = pos[c[suf[i]]];
        new_suf[where] = suf[i];
         pos[c[suf[i]]]++;
    }
    cls = -1;
    for (int i = 0; i < n; i++) {</pre>
        if (i == 0) {
             cls++;
            new_c[new_suf[i]] = cls;
             continue;
        pair<int, int> prev = {c[new_suf[i - 1]], c
[(new_suf[i - 1] + L) % n]};
        pair<int, int> now = {c[new_suf[i]], c[(
new_suf[i] + L) % n]};
        if (prev != now) {
             cls++;
        new_c[new_suf[i]] = cls;
```

```
swap(c, new_c);
        swap(suf, new_suf);
    vector<int> res;
    for (int i = 1; i < n; i++) {</pre>
        res.push_back(suf[i]);
    return res;
vector<int> lcp_neighboring(string &s, vector<int> &suf
    ) {
    int n = s.size();
    vector<int> lcp(n), where(n);
    for (int i = 0; i < n; i++) {</pre>
        where[suf[i]] = i;
    }
    int k = 0;
    for (int j = 0; j < n; j++) {</pre>
        int pos = where[j];
        if (pos == n - 1) {
            k = 0;
            lcp[pos] = 0;
        } else {
            k = max(OLL, k - 1);
            while (s[j + k] == s[suf[pos + 1] + k]) {
                k++;
                if (j + k >= n || suf[pos + 1] + k >= n
    ) {
                     break;
                }
            lcp[pos] = k;
        }
    }
    return lcp;
int sol(int k, string s) {
    int n = s.size();
    vector<int> suf = build_suff_arr(s);
    vector<int> lcp = lcp_neighboring(s, suf);
    vector<int> where(n);
    for (int i = 0; i < n; i++) {</pre>
        where[suf[i]] = i;
    Sparse_Table st(lcp);
    int ans = 0;
    for (int i = 0; i < n - k; i++) {</pre>
```

```
ans += st.rmq(where[i], where[i + k]);
}
return ans;
}
```

В Структуры данных

8.1 Disjoint Sparse Table

```
_____
```

// TODO

8.2 Segment Tree Beats

```
// min=, sum
struct ST {
    vector<int> st, mx, mx_cnt, sec_mx;
    ST(int n) {
        st.resize(n * 4, 0);
        mx.resize(n * 4, 0);
        mx_cnt.resize(n * 4, 0);
        sec_mx.resize(n * 4, 0);
        build(0, 0, n);
   }
    void upd_from_children(int v) {
        st[v] = st[v * 2 + 1] + st[v * 2 + 2];
        mx[v] = max(mx[v * 2 + 1], mx[v * 2 + 2]);
        mx_cnt[v] = 0;
        sec_mx[v] = max(sec_mx[v * 2 + 1], sec_mx[v * 2
     + 2]);
        if (mx[v * 2 + 1] == mx[v]) {
            mx_cnt[v] += mx_cnt[v * 2 + 1];
       } else {
            sec_mx[v] = max(sec_mx[v], mx[v * 2 + 1]);
        if (mx[v * 2 + 2] == mx[v]) {
            mx_cnt[v] += mx_cnt[v * 2 + 2];
            sec_mx[v] = max(sec_mx[v], mx[v * 2 + 2]);
   }
    void build(int i, int l, int r) {
       if (1 + 1 == r) {
            st[i] = mx[i] = 0;
            mx_cnt[i] = 1;
```

```
sec_mx[i] = -INF;
        return:
    }
    int m = (r + 1) / 2;
    build(i * 2 + 1, 1, m);
    build(i * 2 + 2, m, r);
    upd_from_children(i);
}
void push_min_eq(int v, int val) {
    if (mx[v] > val) {
        st[v] -= (mx[v] - val) * mx_cnt[v];
        mx[v] = val;
    }
}
void push(int i) {
    push_min_eq(i * 2 + 1, mx[i]);
    push_min_eq(i * 2 + 2, mx[i]);
}
void update(int i, int l, int r, int ql, int qr,
int val) {
    if (mx[i] <= val) {</pre>
        return:
    }
    if (ql == 1 && qr == r && sec_mx[i] < val) {</pre>
        push_min_eq(i, val);
        return;
    }
    push(i);
    int m = (r + 1) / 2;
    if (qr <= m) {
        update(i * 2 + 1, 1, m, ql, qr, val);
    } else if (ql >= m) {
        update(i * 2 + 2, m, r, ql, qr, val);
        update(i * 2 + 1, 1, m, ql, m, val);
        update(i * 2 + 2, m, r, m, qr, val);
    }
    upd_from_children(i);
}
int sum(int i, int l, int r, int ql, int qr) {
    if (1 == q1 && r == qr) {
        return st[i];
    }
    push(i):
    int m = (r + 1) / 2;
    if (qr <= m) {</pre>
```

```
return sum(i * 2 + 1, 1, m, q1, qr);
}
if (q1 >= m) {
    return sum(i * 2 + 2, m, r, q1, qr);
}
return sum(i * 2 + 1, 1, m, q1, m) + sum(i * 2 + 2, m, r, m, qr);
}
};
```

8.3 ДД по неявному

```
pair<Node *, Node *> split(Node *now, int k) {
    if (!now) {
        return {nullptr, nullptr};
    if (size(now->1) + 1 <= k) {</pre>
        auto ans = split(now->r, k - 1 - size(now->l));
        now->r = ans.first;
        update_size(now);
        return {now, ans.second};
    }
    auto ans = split(now->1, k);
    now->1 = ans.second:
    update_size(now);
    return {ans.first, now};
Node *merge(Node *1, Node *r) {
    if (!1) {
        return r;
    }
    if (!r) {
        return 1;
    }
    if (1->y <= r->y) {
        auto ans = merge(1->r, r);
        1->r = ans;
        update_size(1);
        return 1;
    auto ans = merge(1, r->1);
    r->1 = ans:
    update_size(r);
    return r;
Node *insert(Node *root, int pos) {
    auto r = split(root, pos);
```

```
Node *nn = new Node(pos);
root = merge(r.first, nn);
root = merge(root, r.second);
return root;
}

Node *to_begin(Node *root, int 1, int r) {
   auto a = split(root, 1);
   auto b = split(a.second, r - 1);
   return merge(b.first, merge(a.first, b.second));
}
```

8.4 ДД

```
pair<Node *, Node *> split(Node *now, ll x) {
    if (!now) {
        return {nullptr, nullptr};
   }
    if (now->x <= x) {
        auto ans = split(now->r, x);
        now->r = ans.first;
        update_sum(now);
        return {now, ans.second};
    auto ans = split(now->1, x);
    now->1 = ans.second;
    update_sum(now);
    return {ans.first, now};
Node *merge(Node *1, Node *r) {
    if (!1) {
        return r;
   }
    if (!r) {
        return 1;
   if (1->y <= r->y) {
        auto ans = merge(1->r, r);
       1->r = ans;
        update_sum(1);
       return 1;
    auto ans = merge(1, r->1);
    r->1 = ans:
    update_sum(r);
    return r;
```

```
Node *insert(Node *root, ll val) {
    Node *new_v = new Node(val);
    auto ans = split(root, val);
    return merge(merge(ans.first, new_v), ans.second);
}
Node *del(Node *root, 11 val) {
    auto ans = split(root, val);
    auto ans1 = split(ans.first, val - 1);
    return merge(ans1.first, ans.second);
ll get_sum(Node *root, ll l, ll r) {
    if (!root) {
        return 0:
    auto ans = split(root, 1 - 1);
    auto ans2 = split(ans.second, r);
    if (!ans2.first) {
        merge(merge(ans.first, ans2.first), ans2.second
    ):
        return 0;
    }
    11 res = ans2.first->sum_;
    merge(merge(ans.first, ans2.first), ans2.second);
    return res;
```

8.5 Персистентное ДД

// TODO

8.6 Персистентное ДО

```
struct ST {
    vector<Node *> roots;
    int n;

ST(vector<int> &a) {
        start = a;
        n = a.size();
        roots.push_back(nullptr);
        build(roots[0], 0, n);
}

void update(Node *&now, Node *old, int l, int r,
    int pos, int qd) {
        if (l + 1 == r) {
```

```
now = new_node(qd);
    return;
}
now = new_node();
int m = (1 + r) / 2;
if (pos < m) {
    now->r = old->r;
    update(now->l, old->l, l, m, pos, qd);
} else {
    now->l = old->l;
    update(now->r, old->r, m, r, pos, qd);
}
now->sm = now->l->sm + now->r->sm;
}
};
```

8.7 Спарсы

```
struct Sparse_Table {
    vector<vector<int>> st;
    vector<int> max2;
    Sparse_Table(vector<int> &a) {
         int n = a.size();
         st.emplace_back();
         for (int i = 0; i < n; i++) {</pre>
             st[0].push_back(a[i]);
         for (int i = 1; (1 << i) <= n; i++) {
             st.emplace_back();
             for (int p = 0; p + (1 << i) <= n; p++) {</pre>
                 st[i].push_back(min(st[i - 1][p], st[i
    -1][p + (1 << (i - 1))]);
        \max 2. resize(n + 1);
        \max 2\lceil 1 \rceil = 0:
        for (int i = 2; i <= n; i++) {</pre>
             \max 2[i] = \max 2[i / 2] + 1;
        }
    }
    int rmq(int 1, int r) {
        r++;
        int i = max2[r - 1];
        return min(st[i][1], st[i][r - (1 << i)]);</pre>
    }
};
```

8.8 Фенвик (pref += x)

```
// a[left..right] += delta; get_sum a[1..pos]
//
//void update(left, right, delta)
// T1.add(left, delta)
// T2.add(right + 1, -delta);
// T2.add(left, delta * (left - 1))
// T2.add(right + 1, -delta * right);
//
//int getSum(pos)
// return T1.sum(pos) * pos - T2.sum(pos)
```

8.9 Фенвик

```
// Нумерация с 1
struct Fenwick_tree {
    vector<vector<int>>> ft;
    Fenwick tree(int n) {
        ft.resize(n + 1, vector<vector<int>>(n + 1,
    vector < int > (n + 1));
    void upd(int x, int y, int z, int d) {
        for (int x1 = x; x1 < ft.size(); x1 += x1 & -x1</pre>
    ) {
            for (int y1 = y; y1 < ft[x1].size(); y1 +=</pre>
    y1 & -y1) {
                for (int z1 = z; z1 < ft[x1][y1].size()</pre>
    ; z1 += z1 \& -z1) {
                    ft[x1][y1][z1] += d;
   }
    int rsq(int x, int y, int z) {
        int ans = 0:
        for (int x1 = x; x1 > 0; x1 -= x1 & -x1) {
            for (int y1 = y; y1 > 0; y1 -= y1 & -y1) {
                for (int z1 = z; z1 > 0; z1 -= z1 & -z1
    ) {
                    ans += ft[x1][y1][z1];
                }
       }
        return ans;
```

```
int sum_3d(int x1, int x2, int y1, int y2, int z1,
int z2) {
```

```
int ans = rsq(x2, y2, z1 - 1) + rsq(x1 - 1, y2,
z2) - rsq(x1 - 1, y2, z1 - 1);
ans += rsq(x2, y1 - 1, z2);
ans -= rsq(x2, y1 - 1, z1 - 1) + rsq(x1 - 1, y1
- 1, z2) - rsq(x1 - 1, y1 - 1, z1 - 1);
```

```
return rsq(x2, y2, z2) - ans;
}
};
```