

Muffix Sassif – TRD

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1 Геометрия

1.1 3D

```
double eps = 1e-7;
```

```
struct Pt {
    double x;
    double y;
    double z;

    Pt(double x_, double y_, double z_) : x(x_), y(y_), z(z_) {}

    Pt operator-(const Pt& other) const {
        return {x - other.x, y - other.y, z - other.z};
    }

    Pt operator+(const Pt& other) const {
        return {x + other.x, y + other.y, z + other.z};
    }

    Pt operator/(const double& a) const {
        return {x / a, y / a, z / a};
    }

    Pt operator*(const double& a) const {
        return {x * a, y * a, z * a};
    }

    Pt cross(const Pt& p2) const {
        double nx = y * p2.z - z * p2.y;
        double ny = z * p2.x - x * p2.z;
        double nz = x * p2.y - y * p2.x;
        return {nx, ny, nz};
    }

    bool operator==(const Pt& pt) const {
        return abs(x - pt.x) < eps && abs(y - pt.y) < eps &&
            abs(z - pt.z) < eps;
    }

    double dist() {
        return sqrtl(x * x + y * y + z * z);
    }
};

struct Plane {
    double a, b, c, d;
```

```
Plane(double a_, double b_, double c_, double d_) : a(
    a_), b(b_), c(c_), d(d_) {
    double kek = sqrtl(a * a + b * b + c * c);
    if (kek < eps) return;
    a /= kek;
    b /= kek;
    c /= kek;
    d /= kek;
}

double get_val(Pt p) {
    return a * p.x + b * p.y + c * p.z + d;
}

bool on_plane(Pt p) {
    return abs(get_val(p)) / sqrtl(a * a + b * b + c * c)
        < eps;
}

Pt proj(Pt p) {
    double t = (a * p.x + b * p.y + c * p.z + d) / (a *
        a + b * b + c * c);
    return p - Pt(a, b, c) * t;
};

bool on_line(Pt p1, Pt p2, Pt p3) {
    return (p2 - p1).cross(p3 - p1) == Pt(0, 0, 0);
}

Plane get_plane(Pt p1, Pt p2, Pt p3) {
    Pt norm = (p2 - p1).cross(p3 - p1);
    Plane pl(norm.x, norm.y, norm.z, 0);
    pl.d = -pl.get_val(p1);
    return pl;
}

pair<pair<double, double>, pair<double, double>> get_xy(
    double a, double b, double c) {
    if (abs(a) > eps) {
        double y1 = 0, y2 = 10;
        return {{(-c - b * y1) / a, y1}, {(-c - b * y2) / a,
            y2}};
    }
    double x1 = 0, x2 = 10;
    return {{x1, (-c - a * x1) / b}, {x2, (-c - a * x2) /
        b}};
}
```

```
pair<Pt, Pt> intersect(Plane pl1, Plane pl2) {
    if (abs(pl2.a) < eps && abs(pl2.b) < eps && abs(pl2.c)
        < eps) {
        assert(false);
    }
    if (abs(pl2.a) > eps) {
        double nd = pl1.d - pl1.a * pl2.d / pl2.a;
        double nc = pl1.c - pl1.a * pl2.c / pl2.a;
        double nb = pl1.b - pl1.a * pl2.b / pl2.a;
        if (abs(nc) < eps && abs(nb) < eps) {
            // плоскости параллельны (могут совпадать)
            return {Pt(0, 0, 0), Pt(0, 0, 0)};
        }
        auto [yz1, yz2] = get_xy(nb, nc, nd);
        double x1 = (-pl2.d - pl2.c * yz1.second - pl2.b *
            yz1.first) / pl2.a;
        double x2 = (-pl2.d - pl2.c * yz2.second - pl2.b *
            yz2.first) / pl2.a;
        return {Pt(x1, yz1.first, yz1.second), Pt(x2, yz2.
            first, yz2.second)};
    }
    Plane copy_pl1(pl1.c, pl1.a, pl1.b, pl1.d);
    Plane copy_pl2(pl2.c, pl2.a, pl2.b, pl2.d);
    auto [p1, p2] = intersect(copy_pl1, copy_pl2);
    return {Pt(p1.y, p1.z, p1.x), Pt(p2.y, p2.z, p2.x)};
}
```

1.2 Вектор, прямая, окружность

```
//// Вектор ////
```

```
struct vctr {
    dbl x, y;
    vctr() {}
    vctr(dbl x, dbl y) : x(x), y(y) {}

    dbl operator%(const vctr &o) const { return x * o.x +
        y * o.y; }
    dbl operator*(const vctr &o) const { return x * o.y -
        y * o.x; }
    vctr operator+(const vctr &o) const { return {x + o.x,
        y + o.y}; }
    vctr operator-(const vctr &o) const { return {x - o.x,
        y - o.y}; }
    vctr operator-() const { return {-x, -y}; }
    vctr operator*(const dbl d) const { return {x * d, y *
        d}; }
```

```

    vctr operator/(const dbl d) const { return {x / d, y /
        d}; }
    void operator+=(const vctr &o) { x += o.x, y += o.y; }
    void operator-=(const vctr &o) { x -= o.x, y -= o.y; }
    dbl dist2() const { return x * x + y * y; }
    dbl dist() const { return sqrtl(dist2()); }
    vctr norm() const { return *this / dist(); }
};

dbl angle_between(const vctr &a, const vctr &b) {
    return atan2(b * a, b % a);
}

// y > 0 ? 0 : 1
bool is2plane(const vctr &a) {
    return sign(a.y) < 0 || (sign(a.y) == 0 && sign(a.x) <
        0);
}

bool cmp_angle(const vctr &a, const vctr &b) {
    bool pla = is2plane(a);
    bool plb = is2plane(b);
    if (pla != plb)
        return pla < plb;
    return sign(a * b) > 0;
}

//// Прямая ////

struct line {
    dbl a, b, c;

    line() {}
    line(dbl a, dbl b, dbl c) : a(a), b(b), c(c) {}
    line(const vctr A, const vctr B) {
        a = A.y - B.y;
        b = B.x - A.x;
        c = A * B;
        assert(a != 0 || b != 0);
    }

    void operator*=(dbl x) { a *= x, b *= x, c *= x; }
    void operator/=(dbl x) { a /= x, b /= x, c /= x; }
    dbl get(const vctr P) const { return a * P.x + b * P.y
        + c; }
    vctr anyPoint() const {
        dbl x = -a * c / (a * a + b * b);
        dbl y = -b * c / (a * a + b * b);
        return vctr(x, y);
    }
};

```

```

    }
    void normalize() {
        dbl d = sqrtl(a * a + b * b);
        a /= d;
        b /= d;
        c /= d;
    }
};

bool isparallel(line l1, line l2) {
    return vctr(l1.a, l1.b) * vctr(l2.a, l2.b) == 0;
}

vctr intersection(const line &l1, const line &l2) {
    dbl x = (l1.c * l2.b - l2.c * l1.b) / (l2.a * l1.b -
        l2.b * l1.a);
    dbl y = -(l1.c * l2.a - l2.c * l1.a) / (l2.a * l1.b -
        l2.b * l1.a);
    return vctr(x, y);
}

// Серединный перпендикуляр (не биссектриса!)
line bisection(const vctr A, const vctr B) {
    vctr M = (A + B) / 2;
    vctr AB = B - A;
    vctr norm = vctr(AB.y, -AB.x);
    return line(M, M + norm);
}

//// Окружность ////

struct circle {
    dbl x, y, r;

    circle() {}
    circle(dbl x, dbl y, dbl r) : x(x), y(y), r(r) {}
    circle(vctr P, dbl r) : x(P.x), y(P.y), r(r) {}
    circle(const vctr A, const vctr B) {
        vctr C = (A + B) / 2;
        x = C.x, y = C.y;
        r = (A - B).dist() / 2;
    }

    circle(const vctr A, const vctr B, const vctr C) {
        line l1 = bisection(A, B);
        line l2 = bisection(B, C);
        vctr P = intersection(l1, l2);
        x = P.x, y = P.y;
        r = (P - A).dist();
    }
};

```

```

bool isin(const vctr P) const {
    return (vctr(x, y) - P).dist2() <= r * r;
}
vctr cent() const { return vctr(x, y); }
};

```

1.3 Выпуклая оболочка

```

vctr minvctr(INF, INF);

bool cmp_convex_hull(const vctr &a, const vctr &b) {
    vctr A = a - minvctr;
    vctr B = b - minvctr;
    auto sign_prod = sign(A * B);
    if (sign_prod != 0)
        return sign_prod > 0;
    return A.dist2() < B.dist2();
}

// minvctr updates here
vector<vctr> get_convex_hull(vector<vctr> arr) {
    minvctr = rotate_min_vctr(arr);
    vector<vctr> hull;
    sort(arr.begin(), arr.end(), cmp_convex_hull);
    for (vctr &el : arr) {
        while (hull.size() > 1 && sign((hull.back() - hull[
            hull.size() - 2]) * (el - hull.back())) <= 0)
            hull.pop_back();
        hull.push_back(el);
    }
    return hull;
}

```

1.4 Задача 16

```

bool isInSameHalf(vctr p, vctr r1, vctr r2) {
    return sign((r2 - r1) % (p - r1)) >= 0;
}

dbl distPointPoint(vctr a, vctr b) {
    return (a - b).dist();
}

dbl distPointLine(vctr a, vctr l1, vctr l2) {
    line l(l1, l2);
    l.normalize();
}

```

```

    return abs(l.get(a));
}

dbl distPointRay(vctr a, vctr r1, vctr r2) {
    if (!isInSameHalf(a, r1, r2))
        return distPointPoint(a, r1);
    return distPointLine(a, r1, r2);
}

dbl distPointSeg(vctr a, vctr s1, vctr s2) {
    return max(distPointRay(a, s1, s2),
               distPointRay(a, s2, s1));
}

bool isIntersectionLineLine(line l1, line l2) {
    dbl znam = l1.b * l2.a - l1.a * l2.b;
    return sign(znam) != 0;
}

vctr intersectionLineLine(line l1, line l2) {
    dbl znam = l1.b * l2.a - l1.a * l2.b;
    dbl y = -(l1.c * l2.a - l2.c * l1.a) / znam;
    dbl x = -(l1.c * l2.b - l2.c * l1.b) / -znam;
    return vctr(x, y);
}

vctr getPointOnLine(line l) {
    if (sign(l.b) != 0)
        return vctr(0, -l.c / l.b);
    return vctr(-l.c / l.a, 0);
}

dbl distLineLine(vctr l1a, vctr l1b, vctr l2a, vctr l2b) {
    {
        line l1(l1a, l1b);
        line l2(l2a, l2b);
        if (isIntersectionLineLine(l1, l2))
            return 0;
        vctr p = getPointOnLine(l1);
        l2.normalize();
        return abs(l2.get(p));
    }
}

dbl distRayLine(vctr r1, vctr r2, vctr l1, vctr l2) {
    line r(r1, r2);
    line l(l1, l2);
    if (!isIntersectionLineLine(l, r))
        return distLineLine(r1, r2, l1, l2);
    vctr p = intersectionLineLine(l, r);

```

```

    line r(r1, r2);
    line l(l1, l2);
    if (!isIntersectionLineLine(l, r))
        return distLineLine(r1, r2, l1, l2);
    vctr p = intersectionLineLine(l, r);

```

```

    if (isInSameHalf(p, r1, r2))
        return 0;
    return distPointLine(r1, l1, l2);
}

dbl distSegLine(vctr s1, vctr s2, vctr l1, vctr l2) {
    return max(distRayLine(s1, s2, l1, l2),
               distRayLine(s2, s1, l1, l2));
}

dbl distRayRay(vctr r1a, vctr r1b, vctr r2a, vctr r2b) {
    line r1(r1a, r1b);
    line r2(r2a, r2b);
    if (!isIntersectionLineLine(r1, r2)) {
        if (isInSameHalf(r1a, r2a, r2b) || isInSameHalf(r2a,
        r1a, r1b))
            return distLineLine(r1a, r1b, r2a, r2b);
        else
            return distPointPoint(r1a, r2a);
    }
    vctr p = intersectionLineLine(r1, r2);
    if (isInSameHalf(p, r1a, r1b) && isInSameHalf(p, r2a,
    r2b))
        return 0;
    return min(distPointRay(r1a, r2a, r2b),
               distPointRay(r2a, r1a, r1b));
}

dbl distSegRay(vctr s1, vctr s2, vctr r1, vctr r2) {
    return max(distRayRay(s1, s2, r1, r2),
               distRayRay(s2, s1, r1, r2));
}

dbl distSegSeg(vctr s1a, vctr s1b, vctr s2a, vctr s2b) {
    return max(distSegRay(s1a, s1b, s2a, s2b),
               distSegRay(s1a, s1b, s2b, s2a));
}

```

1.5 Касательные из точки

```

pair<int, int> tangents_from_point(vector<vctr> &p, vctr
    &a) {
    int n = p.size();
    int logn = 31 - __builtin_clz(n);
    auto findWithSign = [&](int val) {
        int i = 0;
        for (int k = logn; k >= 0; --k) {
            int i1 = (i - (1 << k) + n) % n;

```

```

            int i2 = (i + (1 << k)) % n;
            if (sign((p[i1] - a) * (p[i] - a)) == val)
                i = i1;
            if (sign((p[i2] - a) * (p[i] - a)) == val)
                i = i2;
        }
        return i;
    };
    return {findWithSign(1), findWithSign(-1)};
}

```

1.6 Касательные параллельные прямой

```

// find point with max signed distance to line
int tangent_parallel_line(const vector<vctr> &p, line l)
{
    int n = p.size();
    int i = 0;
    int logn = 31 - __builtin_clz(n);
    for (int k = logn; k >= 0; --k) {
        int i1 = (i - (1 << k) + n) % n;
        int i2 = (i + (1 << k)) % n;
        if (l.get(p[i1]) > l.get(p[i]))
            i = i1;
        if (l.get(p[i2]) > l.get(p[i]))
            i = i2;
    }
    return i;
}

```

1.7 Лежит ли точка в многоугольнике

```

// P starts with minvctr
bool is_point_in_poly(vctr A, vector<vctr> &P) {
    int n = P.size();
    int ind = lower_bound(P.begin(), P.end(), A,
        cmp_convex_hull) - P.begin();
    if (ind == n || ind == 0)
        return false;
    if (ind == 0)
        ind++;
    vctr B = A - P[ind - 1];
    vctr C = P[ind] - P[ind - 1];
    return sign(C * B) >= 0;
}

```

1.8 Минимальная покрывающая окружность

```
mt19937 rnd(179);
```

```
circle MinDisk2(vector<vctr> &p, vctr A, vctr B, int sz)
{
    circle w(A, B);
    for (int i = 0; i < sz; ++i) {
        if (w.isin(p[i]))
            continue;
        w = circle(A, B, p[i]);
    }
    return w;
}
```

```
circle MinDisk1(vector<vctr> &p, vctr A, int sz) {
    shuffle(p.begin(), p.begin() + sz, rnd);
    circle w(A, p[0]);
    for (int i = 1; i < sz; ++i) {
        if (w.isin(p[i]))
            continue;
        w = MinDisk2(p, A, p[i], i);
    }
    return w;
}
```

```
circle MinDisk(vector<vctr> &p) {
    int sz = p.size();
    if (sz == 1)
        return circle(p[0], 0);
    shuffle(p.begin(), p.end(), rnd);
    circle w(p[0], p[1]);
    for (int i = 2; i < sz; ++i) {
        if (w.isin(p[i]))
            continue;
        w = MinDisk1(p, p[i], i);
    }
    return w;
}
```

1.9 Многоугольники

```
// Сдвиг многоугольника, чтобы начинался с минимального
// вектора
```

```
vctr rotate_min_vctr(vector<vctr> &pts) {
    int ind = 0;
    for (int i = 1; i < pts.size(); ++i) {
```

```
        if (is2plane(pts[i] - pts[ind]))
            ind = i;
    }
    rotate(pts.begin(), pts.begin() + ind, pts.end());
    return pts[0];
}

// Список вершин -> список рёбер
vector<vctr> poly_to_edges(const vector<vctr> &A) {
    vector<vctr> edg(A.size());
    for (int i = 0; i < A.size(); ++i)
        edg[i] = A[(i + 1) % A.size()] - A[i];
    return edg;
}
```

1.10 Пересечение полуплоскостей

```
// half plane: ax+by+c > 0
// bounding box MUST have
vector<int> intersection_half_planes_inds(const vector<
    line> &ls) {
    int n = (int)ls.size();
    vector<int> lsi(n);
    iota(lsi.begin(), lsi.end(), 0);
    sort(lsi.begin(), lsi.end(), [&](int i, int j) {
        vctr aa(ls[i].a, ls[i].b);
        vctr bb(ls[j].a, ls[j].b);
        bool pla = is2plane(aa);
        bool plb = is2plane(bb);
        if (pla != plb)
            return pla < plb;
        return aa * bb > 0;
    });
```

```
    vector<line> st;
    vector<int> inds;
    for (int ii = 0; ii < 2 * n; ++ii) {
        int i = lsi[ii % n];
        if (st.empty()) {
            st.push_back(ls[i]);
            inds.push_back(i);
            continue;
        }
        vctr p = intersection(ls[i], st.back());
        bool pp = isparallel(ls[i], st.back());
        bool bad = false;
        while (st.size() >= 2) {
            if (!pp && sign(st[st.size() - 2].get(p)) >= 0)
```

```
                break;
            else if (pp && sign(st.back().get(ls[i].anyPoint())
                ) <= 0) {
                bad = true;
                break;
            }
            st.pop_back();
            inds.pop_back();
            p = intersection(ls[i], st.back());
            pp = isparallel(ls[i], st.back());
        }
        if (!bad) {
            st.push_back(ls[i]);
            inds.push_back(i);
        }
    }
    vector<int> cnt(n, 0);
    for (int i : inds)
        cnt[i]++;
    vector<int> good;
    for (int i : inds) {
        if (cnt[i]-- == 2)
            good.push_back(i);
    }
    return good;
}
```

```
vector<vctr> intersection_half_planes(vector<line> &ls)
{
    vector<int> inter = intersection_half_planes_inds(ls);
    int n = inter.size();
    vector<vctr> pts;
    for (int i = 0; i < n; ++i) {
        int j = (i + 1) % n;
        vctr P = intersection(ls[inter[i]], ls[inter[j]]);
        if (pts.empty() || sign(pts.back().x - P.x) != 0
            || sign(pts.back().y - P.y) != 0)
            pts.push_back(P);
    }
    return pts;
}
```

1.11 Пересечение с окружностью

```
bool is_intersection_line_circ(line l, circle c) {
    l.normalize();
    dbl d = abs(l.get(c.cent()));
    return d < c.r - EPS;
```

```

}

vector<vctr> intersection_line_circ(line l, circle c) {
    l.normalize();
    dbl d = abs(l.get(c.cent()));
    vctr per = vctr(l.a, l.b).norm() * d;
    vctr a = c.cent() + per;
    if (sign(d - c.r) > 0)
        return {};
    if (sign(l.get(a)) != 0)
        a = c.cent() - per;
    if (sign(c.r - d) == 0)
        return {a};
    dbl k = sqrtl(c.r * c.r - d * d);
    vctr par = vctr(-l.b, l.a).norm() * k;
    return {a + par, a - par};
}

```

```

vector<vctr> intersection_circ_circ(circle a, circle b)
{
    line l(2 * (b.x - a.x),
           2 * (b.y - a.y),
           b.r * b.r - a.r * a.r
           + (a.x * a.x + a.y * a.y)
           - (b.x * b.x + b.y * b.y));
    if (sign(l.a) == 0 && sign(l.b) == 0)
        return {};
    return intersection_line_circ(l, a);
}

```

```

vector<vctr> tangent_vctr_circ(vctr v, circle c) {
    dbl d = (c.cent() - v).dist();
    dbl k = sqrtl(d * d - c.r * c.r);
    circle c2(v.x, v.y, k);
    return intersection_circ_circ(c, c2);
}

```

1.12 Проверка на пересечение отрезков

```

bool is_intersection_seg(vctr A, vctr B, vctr C, vctr D)
{
    for (int i = 0; i < 2; ++i) {
        auto l1 = A.x, r1 = B.x, l2 = C.x, r2 = D.x;
        if (l1 > r1) swap(l1, r1);
        if (l2 > r2) swap(l2, r2);
        if (max(l1, l2) > min(r1, r2))
            return false;
        swap(A.x, A.y);
    }
}

```

```

swap(B.x, B.y);
swap(C.x, C.y);
swap(D.x, D.y);
}
for (int _ = 0; _ < 2; ++_) {
    auto v1 = (B - A) * (C - A);
    auto v2 = (B - A) * (D - A);
    if (sign(v1) * sign(v2) == 1)
        return false;
    swap(A, C);
    swap(B, D);
}
return true;
}

```

1.13 Сумма Минковского

```

vector<vctr> minkowski_sum(const vector<vctr> &A, const
vector<vctr> &B) {
    auto edgA = poly_to_edges(A);
    auto edgB = poly_to_edges(B);
    vector<vctr> edgC(A.size() + B.size());
    merge(edgA.begin(), edgA.end(), edgB.begin(), edgB.end
        (), edgC.begin(), cmp_angle);
    vector<vctr> C(edgC.size());
    C[0] = A[0] + B[0];
    for (int i = 0; i + 1 < C.size(); ++i)
        C[i + 1] = C[i] + edgC[i];
    return C;
}

```

1.14 Формула Эйлера

- V – число вершин выпуклого многогранника (планарного графа)
- E – число рёбер
- F – число граней (если планарный граф, то включая внешнюю)

Тогда $V - E + F = 2$

2 Графы

2.1 2-SAT

```

for (int i = 1; i <= n; ++i) {
    not_v[i] = i + n;
    not_v[i + n] = i;
}
for (int i = 0; i < m; ++i) {
    cin >> u >> v;
    g[not_v[v]].push_back(u);
    g[not_v[u]].push_back(v);
    rg[u].push_back(not_v[v]);
    rg[v].push_back(not_v[u]);
}
// делаем КСС, получаем comp
for (int v = 1; v <= n; ++v) {
    if (comp[v] == comp[not_v[v]]) {
        cout << "UNSATISFIABLE\n";
        return 0;
    }
}
for (int v = 1; v <= n; ++v)
    cout << (comp[v] > comp[not_v[v]] ? v : not_v[v]);

```

2.2 Венгерский алгоритм

```

pair<int, vector<int>> venger(vector<vector<int>> a) {
    // ищет минимальное по стоимости
    // работает только при n <= m
    // a – массив весов (n + 1) × (m + 1)
    // a[0][..] = a[..][0] = 0
    // возвращает ans[i] = j если взяли ребро a[i][j]
    int n = (int) a.size() - 1;
    int m = (int) a[0].size() - 1;
    vector<int> u(n + 1), v(m + 1), p(m + 1), way(m + 1);
    for (int i = 1; i <= n; ++i) {
        p[0] = i;
        int j0 = 0;
        vector<int> minv(m + 1, INF);
        vector<char> used(m + 1, false);
        do {
            used[j0] = true;
            int i0 = p[j0], delta = INF, j1;
            for (int j = 1; j <= m; ++j)
                if (!used[j]) {
                    int cur = a[i0][j] - u[i0] - v[j];
                    if (cur < minv[j])

```

```

        minv[j] = cur, way[j] = j0;
        if (minv[j] < delta)
            delta = minv[j], j1 = j;
    }
    for (int j = 0; j <= m; ++j)
        if (used[j])
            u[p[j]] += delta, v[j] -= delta;
        else
            minv[j] -= delta;
    j0 = j1;
} while (p[j0] != 0);
do {
    int j1 = way[j0];
    p[j0] = p[j1];
    j0 = j1;
} while (j0);
}
int cost = -v[0];
vector<int> ans(n + 1);
for (int j = 1; j <= m; ++j)
    ans[p[j]] = j;
return {cost, ans};
}

```

2.3 Вершинная двусвязность

```

struct edge {
    int u, ind;

    bool operator<(const edge &other) const {
        return u < other.u;
    }
};

```

```
vector<int> stack_;
```

```

void paint(int v, int pr = -1) {
    used[v] = pr;
    up[v] = tin[v] = ++timer;
    for (auto e: g[v]) {
        if (e.u == pr) {
            continue;
        }
        if (!used[e.u]) {
            stack_.push_back(e.ind);
            paint(e.u, v);
            if (up[e.u] >= tin[v]) {
                ++mx_col;
            }
        }
    }
}

```

```

while (true) {
    int cur_edge = stack_.back();
    col[cur_edge] = mx_col;
    stack_.pop_back();
    if (cur_edge == e.ind) {
        break;
    }
}
up[v] = min(up[v], up[e.u]);
} else if (tin[e.u] < tin[v]) {
    stack_.push_back(e.ind);
    up[v] = min(up[v], tin[e.u]);
} else if (up[v] > tin[e.u]) {
    up[v] = up[e.u];
}
}
}

signed main() {
    int n, m;
    cin >> n >> m;
    for (int i = 0; i < m; ++i) {
        int u, v;
        cin >> u >> v;
        g[u].push_back({v, i});
        g[v].push_back({u, i});
    }
    for (int v = 1; v <= n; ++v) {
        sort(all(g[v]));
    }
    for (int v = 1; v <= n; ++v) {
        if (!used[v]) {
            paint(v);
        }
    }
    for (int v = 1; v <= n; ++v) {
        int len = g[v].size();
        for (int i = 1; i < len; ++i) {
            if (col[g[v][i].ind] == 0) {
                col[g[v][i].ind] = col[g[v][i - 1].ind];
            }
        }
    }
}

```

2.4 Дейкстра за квадрат

```

// 0-based
pair<vector<int>, vector<int>> dijkstra(int start, int n) {
    vector<int> dist(n, INF);
    vector<int> pred(n, -1);
    vector<int> used(n);
    dist[start] = 0;
    for (int _ = 0; _ < n; ++_) {
        int v = -1;
        for (int i = 0; i < n; ++i) {
            if (!used[i])
                if (v == -1 || dist[v] > dist[i])
                    v = i;
        }
        if (v == -1) break;
        for (auto [u, w] : g[v]) {
            if (dist[u] > w + dist[v]) {
                dist[u] = w + dist[v];
                pred[u] = v;
            }
        }
        used[v] = 1;
    }
    return {dist, pred};
}

```

2.5 Диниц

```

struct edge {
    int v, f, c, ind;
};

vector<edge> g[MAXN];
pair<int, int> pred[MAXN];
int d[MAXN];
int inds[MAXN];

bool dfs(int v, int final, int W) {
    if (v == final) {
        return true;
    }
    for (int i = inds[v]; i < (int) g[v].size(); i++) {
        auto e = g[v][i];
        if (e.f + W <= e.c && d[v] + 1 == d[e.v]) {
            pred[e.v] = {v, i};
            bool flag = dfs(e.v, final, W);
            if (flag) {
                return true;
            }
        }
    }
}

```



```

    }
    inds[v]++;
} else {
    inds[v]++;
}
}
return false;
}

bool bfs(int start, int final, int W) {
    fill(d, d + MAXN, INF);
    d[start] = 0;
    deque<int> q = {start};
    while (!q.empty()) {
        int v = q.front();
        q.pop_front();
        for (auto e : g[v]) {
            if (e.f + W <= e.c && d[e.v] > d[v] + 1) {
                d[e.v] = d[v] + 1;
                q.push_back(e.v);
            }
        }
    }
    if (d[final] == INF) {
        return false;
    }
    fill(inds, inds + MAXN, 0);
    while (dfs(start, final, W)) {
        int v = final;
        int x = INF;
        while (v != start) {
            int ind = pred[v].second;
            v = pred[v].first;
            x = min(x, g[v][ind].c - g[v][ind].f);
        }
        v = final;
        while (v != start) {
            int ind = pred[v].second;
            v = pred[v].first;
            g[v][ind].f += x;
            g[g[v][ind].v][g[v][ind].ind].f -= x;
        }
    }
    return true;
}

void Dinic(int start, int final) {
    int W = (1LL << 30);
    do {

```

```

        while (bfs(start, final, W));
        W /= 2;
    } while (W >= 1);
}

signed main() {
    int n, m;
    vector<pair<int, int>> edges;
    for (int i = 0; i < m; i++) {
        int u, v, c;
        cin >> u >> v >> c;
        edges.emplace_back(u, v);
        g[u].push_back({v, 0, c, (int) g[v].size()});
        // если ребро - ориентированно,
        // то обратная capacity = 0
        g[v].push_back({u, 0, c, (int) g[u].size() - 1});
    }
    int start = 1, target = n;
    Dinic(start, target);
    int res = 0;
    for (auto e : g[start]) {
        res += e.f;
    }
    vector<int> cut;
    for (int i = 0; i < m; i++) {
        int u = edges[i].first, v = edges[i].second;
        if ((d[u] != INF && d[v] == INF) ||
            (d[u] == INF && d[v] != INF)) {
            cut.push_back(i + 1);
        }
    }
}

```

2.6 KCC

```

void dfs1(int v, vector<int> &topsort) {
    used[v] = 1;
    for (auto u : g[v]) {
        if (!used[u]) {
            dfs1(u, topsort);
        }
    }
    topsort.push_back(v);
}

void dfs2(int v, int col) {
    comp[v] = col;
    for (auto u : rg[v]) {

```

```

        if (!comp[u]) {
            dfs2(u, col);
        }
    }
}

signed main() {
    vector<int> topsort;
    for (int v = 1; v <= n; ++v)
        if (!used[v])
            dfs1(v, topsort);
    reverse(all(topsort));
    for (int j = 1; j <= n; ++j)
        if (!comp[topsort[j - 1]])
            dfs2(topsort[j - 1], j);
}

```

2.7 Минкост (Джонсон)

```

using cost_t = ll;
using flow_t = int;

const int MAXN = 10000;
const int MAXM = 25000 * 2;
const cost_t INFw = 1e12;
const flow_t INFf = 10;

struct Edge {
    int v, u;
    flow_t f, c;
    cost_t w;
};

Edge edg[MAXN];
int esz = 0;
vector<int> graph[MAXN];
ll dist[MAXN];
ll pot[MAXN];
int S, T;
int NUMV;
int pre[MAXN];
bitset<MAXN> inQ;

flow_t get_flow() {
    int v = T;
    if (pre[v] == -1)
        return 0;

```



```

flow_t f = INFf;
do {
    int ei = pre[v];
    Edge &e = edg[ei];
    f = min(f, e.c - e.f);
    if (f == 0)
        return 0;
    v = e.v;
} while (v != S);
v = T;
do {
    int ei = pre[v];
    edg[ei].f += f;
    edg[ei ^ 1].f -= f;
    v = edg[ei].v;
} while (v != S);
return f;
}

void spfa() {
    fill(dist, dist + NUMV, INFw);
    dist[S] = 0;
    deque<int> Q = {S};
    inQ[S] = true;
    while (!Q.empty()) {
        int v = Q.front();
        Q.pop_front();
        inQ[v] = false;
        cost_t d = dist[v];
        for (int ei : graph[v]) {
            Edge &e = edg[ei];
            if (e.f == e.c)
                continue;
            cost_t w = e.w + pot[v] - pot[e.u];
            if (dist[e.u] <= d + w)
                continue;
            pre[e.u] = ei;
            dist[e.u] = d + w;
            if (!inQ[e.u]) {
                inQ[e.u] = true;
                Q.push_back(e.u);
            }
        }
    }
    for (int i = 0; i < NUMV; ++i)
        pot[i] += dist[i];
}

cost_t mincost() {

```

```

    spfa(); // pot[i] = 0 // or ford_bellman
    flow_t f = 0;
    while (true) {
        flow_t ff = get_flow();
        if (ff == 0)
            break;
        f += ff;
        spfa(); // or dijkstra
    }
    cost_t res = 0;
    for (int i = 0; i < esz; ++i)
        res += edg[i].f * edg[i].w;
    res /= 2;
    return res;
}

void add_edge(int v, int u, int c, int w) {
    edg[esz] = {v, u, 0, c, w};
    edg[esz + 1] = {u, v, 0, 0, -w};
    graph[v].push_back(esz);
    graph[u].push_back(esz + 1);
    esz += 2;
}

signed main() {
    ios_base::sync_with_stdio(false);
    cin.tie(nullptr);
    int n, m;
    cin >> n >> m;
    S = 0;
    T = n - 1;
    NUMV = n;
    for (int i = 0; i < m; ++i) {
        int v, u, c, w;
        cin >> v >> u >> c >> w;
        v--, u--;
        add_edge(v, u, c, w);
    }
    cost_t ans = mincost();
    cout << ans;
}

```

2.8 Мосты

```

void dfs(int v, int par) {
    vis[v] = 1;
    up[v] = tin[v] = timer++;
    for (auto u : g[v]) {

```

```

        if (!vis[u]) {
            dfs(u, v);
            up[v] = min(up[v], up[u]);
        } else if (u != par) {
            up[v] = min(up[v], tin[u]);
        }
        if (up[u] > tin[v]) {
            bridges.emplace_back(v, u);
        }
    }
}

```

2.9 Паросочетания

```

int dfs(int v, int c) {
    if (used[v] == c) return 0;
    used[v] = c;
    for (auto u : g[v]) {
        if (res[u] == -1) {
            res[u] = v;
            return 1;
        }
    }
    for (auto u : g[v]) {
        if (dfs(res[u], c)) {
            res[u] = v;
            return 1;
        }
    }
    return 0;
}

signed main() {
    // n - в левой доле, m - в правой
    fill(res, res + m, -1);
    for (int i = 0; i < n; ++i) {
        ans += dfs(i, i + 1);
    }
}

```

2.10 Точки сочленения

```

void dfs(int v, int par) {
    vis[v] = 1;
    up[v] = tin[v] = timer++;
    int child = 0;
    for (auto u : g[v]) {

```

```

if (!vis[u]) {
    dfs(u, v);
    up[v] = min(up[v], up[u]);
    if (up[u] >= tin[v] && par != -1) {
        points.insert(v);
    }
    child++;
} else if (u != par) {
    up[v] = min(up[v], tin[u]);
}
}
if (par == -1 && child >= 2) {
    points.insert(v);
}
}
}

```

2.11 Эдмондс-Карп

```

struct edge {
    int v, f, c, ind;
};

vector<edge> g[MAXN];

bool bfs(int start, int final, int W) {
    vector<int> d(MAXN, INF);
    vector<pair<int, int>> pred(MAXN);
    d[start] = 0;
    deque<int> q = {start};
    while (!q.empty()) {
        int v = q.front();
        q.pop_front();
        for (int i = 0; i < (int) g[v].size(); i++) {
            auto e = g[v][i];
            if (e.f + W <= e.c && d[e.v] > d[v] + 1) {
                d[e.v] = d[v] + 1;
                pred[e.v] = {v, i};
                q.push_back(e.v);
            }
        }
    }
    if (d[final] == INF) {
        return false;
    }
    int v = final;
    int x = INF;
    while (v != start) {
        int ind = pred[v].second;

```

```

        v = pred[v].first;
        x = min(x, g[v][ind].c - g[v][ind].f);
    }
    v = final;
    while (v != start) {
        int ind = pred[v].second;
        v = pred[v].first;
        g[v][ind].f += x;
        g[g[v][ind].v][g[v][ind].ind].f -= x;
    }
    return true;
}

signed main() {
    int n, m;
    for (int i = 0; i < m; i++) {
        int u, v, c;
        cin >> u >> v >> c;
        g[u].push_back({v, 0, c, (int) g[v].size()});
        g[v].push_back({u, 0, 0, (int) g[u].size() - 1});
    }
    int start = 1, final = n;
    int W = (1 << 30);
    do {
        while (bfs(start, final, W));
        W /= 2;
    } while (W >= 1);
    int res = 0;
    for (auto e : g[start]) {
        res += e.f;
    }
}

```

2.12 Эйлеров цикл

```

// unconnected graph, deleting edges, set<int> g[N];
for (int v = 0; v < n; v++) {
    if (!g[v].empty()) {
        vector<int> ccl;
        vector<int> s = {v};
        while (!s.empty()) {
            int u = s.back();
            if (g[u].empty()) {
                ccl.pb(u);
                s.pop_back();
            } else {
                int u2 = *g[u].begin();
                g[u].erase(u2);

```

```

                g[u2].erase(u);
                s.pb(u2);
            }
        }
        // ccl[0] = ccl.back()
        // i.e for graph with edges
        // (1,2), (1,3), (2,3) → ccl = [1,2,3,1]
    }
}

```

3 ДП

3.1 СХТ

```

struct Line {
    ld k, b;
};

pair<ld, ld> inter(Line a, Line b) {
    ld x = (b.b - a.b) / (a.k - b.k);
    ld y = a.k * x + a.b;
    return {x, y};
}

void add_line(ld k, ld b, vector<Line> &s, vector<pair<
    ld, ld>> &pts) {
    while (s.size() >= 2) {
        pair<ld, ld> x1 = inter(s.back(), s[s.size() - 2]);
        pair<ld, ld> x2 = inter(s[s.size() - 2], {k, b});
        if (x1 > x2) {
            break;
        }
        pts.pop_back();
        s.pop_back();
    }
    if (!s.empty()) {
        pts.push_back(inter(s.back(), {k, b}));
    }
    s.push_back({k, b});
}

ld bin_search(vector<Line> &s, ld x) {
    int l = 0, r = s.size();
    while (l + 1 < r) {
        int m = (r + l) / 2;
        auto kek = inter(s[m - 1], s[m]);
        if (kek.first >= x) {
            l = m;

```

```

    } else {
        r = m;
    }
}
return s[l].k * x + s[l].b;
}

```

3.2 Li Chao

```

// MAXIMUM
struct Line {
    int k, b;

    int f(int x) {
        return k * x + b;
    }
};

struct ST {
    vector<Line> st;

    ST(int n) {
        Line ln = {0LL, -INF};
        st.resize(4 * n, ln);
    }

    void upd(int i, int l, int r, Line ln) {
        int child = 1;
        Line ln1 = ln;
        int m = (l + r) / 2;
        if (ln.f(m) > st[i].f(m)) {
            if (ln.k < st[i].k) {
                child = 2;
            }
            ln1 = st[i];
            st[i] = ln;
        } else {
            if (st[i].k < ln.k) {
                child = 2;
            }
        }
        if (l + 1 < r) {
            if (child == 1) {
                upd(i * 2 + 1, l, m, ln1);
            } else {
                upd(i * 2 + 2, m, r, ln1);
            }
        }
    }
}

```

```

}

int res(int i, int l, int r, int x) {
    if (l + 1 == r) {
        return st[i].f(x);
    }
    int m = (l + r) / 2;
    int val = st[i].f(x);
    if (x < m) {
        val = max(val, res(i * 2 + 1, l, m, x));
    } else {
        val = max(val, res(i * 2 + 2, m, r, x));
    }
    return val;
}
};

```

3.3 SOS-dp

```

// dp initial fill, a[] is given array, mb extra zeros
for (int i = 0; i < (1 << N); i++) {
    dp[i] = a[i];
}

// Classic SOS-dp, goal: dp[mask] = \sum a[submasks of mask]
for (int i = 0; i < N; i++) {
    for (int mask = 0; mask < (1 << N); mask++) {
        if ((mask >> i) & 1) {
            dp[mask] += dp[mask ^ (1 << i)];
        }
    }
}

// Overmasks SOS-dp, goal: dp[mask] = \sum a[overmasks of mask]
for (int i = 0; i < N; i++) {
    for (int mask = (1 << N) - 1; mask >= 0; mask--) {
        if (((mask >> i) & 1) == 0) {
            dp[mask] += dp[mask ^ (1 << i)];
        }
    }
}

// to inverse SOS-dp (restore original array by SOS-dp array):
// use same code, but -= instead of += in dp transitions

```

3.4 HBI

```

// 0-indexation ({a_0, ..., a_{n-1}})
vector<int> lis(vector<int> a) {
    int n = (int) a.size();
    vector<int> dp(n + 1, INF), ind(n + 1), par(n + 1); //
    INF > all a[i] required
    ind[0] = -INF;
    dp[0] = -INF;
    for (int i = 0; i < n; i++) {
        int l = upper_bound(dp.begin(), dp.end(), a[i]) - dp
            .begin();
        if (dp[l - 1] < a[i] && a[i] < dp[l]) {
            dp[l] = a[i];
            ind[l] = i;
            par[i] = ind[l - 1];
        }
    }
    vector<int> ans; // exact values
    for (int l = n; l >= 0; l--) {
        if (dp[l] < INF) {
            int pi = ind[l];
            ans.resize(1);
            for (int i = 0; i < l; i++) {
                ans[i] = a[pi]; // =pi if need indices
                pi = par[pi];
            }
            reverse(ans.begin(), ans.end());
            return ans;
        }
    }
    return {};
}

```

3.5 HOBII

```

// 1-indexation ({0, a_1, ..., a_n}, {0, b_1, ..., b_m})
vector<int> lcis(vector<int> a, vector<int> b) {
    int n = (int) a.size() - 1, m = (int) b.size() - 1;
    vector<int> dp(m + 1), dp2(m + 1), par(m + 1);
    for (int i = 1; i <= n; i++) {
        int best = 0, best_idx = 0;
        for (int j = 1; j <= m; j++) {
            dp2[j] = dp[j];
            if (a[i] == b[j]) {
                dp2[j] = max(dp2[j], best + 1);
                par[j] = best_idx;
            }
        }
    }
}

```

```

        if (a[i] > b[j] && best < dp[j]) {
            best = dp[j];
            best_idx = j;
        }
    }
    swap(dp, dp2);
}
int pj = 0;
for (int j = 1; j <= m; j++) {
    if (dp[pj] < dp[j]) {
        pj = j;
    }
}
vector<int> ans; // exact values
while (pj > 0) {
    ans.push_back(b[pj]);
    pj = par[pj];
}
reverse(ans.begin(), ans.end());
return ans;
}

```

4 Деревья

4.1 Centroid

```

void sizes(int v, int p) {
    sz[v] = 1;
    for (auto u : g[v]) {
        if (u != p && !used[u]) {
            sizes(u, v);
            sz[v] += sz[u];
        }
    }
}

int centroid(int v, int p, int n) {
    for (int u : g[v]) {
        if (sz[u] > n / 2 && u != p && !used[u]) {
            return centroid(u, v, n);
        }
    }
    return v;
}

void dfs(int v, int p) {
    // code
    for (auto u : g[v]) {

```

```

        if (u != p && !used[u]) {
            dfs(u, v);
        }
    }
}

void solve(int v) {
    sizes(v, -1);
    // code (?)
    for (auto u : g[v]) {
        if (!used[u]) {
            // code (?)
            dfs(u, v);
            // сохраняем результаты dfs
        }
    }
    // code (сливаем результаты dfs)
    used[v] = 1;
    for (int u : g[v]) {
        if (!used[u]) {
            solve(centroid(u, v, sz[u]));
        }
    }
}

signed main() {
    sizes(0, -1);
    solve(centroid(0, -1, n));
}

```

4.2 HLD

```

const int MAXN = 50500;
const int INF = (int) 1e15;
const int L = 20;
vector<int> g[MAXN];
int sz[MAXN];
int depth[MAXN];

vector<vector<int>> up(MAXN, vector<int>(L + 1));

void dfs(int v, int p) {
    up[v][0] = p;
    for (int i = 1; i <= L; i++) {
        up[v][i] = up[up[v][i - 1]][i - 1];
    }
    for (int u : g[v]) {
        if (u != p) {

```

```

            dfs(u, v);
        }
    }
}

int lca(int u, int v) {
    if (u == v) {
        return u;
    }
    int du = depth[u], dv = depth[v];
    if (du < dv) {
        swap(du, dv);
        swap(u, v);
    }
    for (int i = L; i >= 0; i--) {
        if (du - (int) pow(2, i) >= dv) {
            u = up[u][i];
            du -= (int) pow(2, i);
        }
    }
    if (u == v) {
        return u;
    }
    for (int i = L; i >= 0; i--) {
        if (up[u][i] != up[v][i]) {
            u = up[u][i];
            v = up[v][i];
        }
    }
    return up[u][0];
}

void dfs1(int v, int p) {
    sz[v] = 1;
    for (int u : g[v]) {
        if (u != p) {
            dfs1(u, v);
            sz[v] += sz[u];
        }
    }
}

int cnt = 0;
int nn[MAXN];
int pred[MAXN];
int rup[MAXN];

void dfs2(int v, int p, int root, int dep = 0) {
    depth[v] = dep;

```

```

    nn[v] = cnt++;
    pred[v] = p;
    rup[v] = root;
    int mx = 0;
    int vert = -1;
    for (int u : g[v]) {
        if (u != p) {
            if (mx < sz[u]) {
                mx = sz[u];
                vert = u;
            }
        }
    }
    if (vert != -1) {
        dfs2(vert, v, root, dep + 1);
    }
    for (int u : g[v]) {
        if (u != p && u != vert) {
            dfs2(u, v, u, dep + 1);
        }
    }
}

ST st({});
int n;

int mx_path_up(int u, int v) {
    if (depth[u] < depth[v]) {
        swap(u, v);
    }
    int res = -INF;
    while (true) {
        int root = rup[u];
        if (depth[root] <= depth[v]) {
            res = max(res, st.rmq(0, 0, n, nn[v], nn[u] + 1));
            break;
        }
        res = max(res, st.rmq(0, 0, n, nn[root], nn[u] + 1));
        u = pred[rup[u]];
    }
    return res;
}

int mx_path(int u, int v) {
    int vert = lca(u, v);
    return max(mx_path_up(u, vert), mx_path_up(v, vert));
}

```

```

void change(int u, int qd) {
    st.update(0, 0, n, nn[u], qd);
}

signed main() {
    cin >> n;
    vector<int> hs(n);
    for (auto &x : hs) {
        cin >> x;
    }
    for (int i = 0; i < n - 1; i++) {
        cin >> u1 >> v1;
        g[u1].push_back(v1);
        g[v1].push_back(u1);
    }
    dfs1(1, -1);
    dfs(1, 1);
    dfs2(1, -1, 1);
    vector<int> nhs(n);
    for (int i = 1; i <= n; i++) {
        nhs[nn[i]] = hs[i - 1];
    }
    st = *new ST(nhs);
    char op;
    int q;
    cin >> q;
    while (q--) {
        cin >> op >> v1 >> u1;
        if (op == '?') {
            cout << mx_path(u1, v1) << endl;
        } else {
            change(v1, u1);
        }
    }
}

```

4.3 Link-cut

```

struct Node {
    Node *ch[2];
    Node *p;
    bool rev;
    int sz;

    Node() {
        ch[0] = nullptr;
        ch[1] = nullptr;
        p = nullptr;
    }
}

```

```

    rev = false;
    sz = 1;
}
};

int size(Node *v) {
    return (v ? v->sz : 0);
}

int chnum(Node *v) {
    return v->p->ch[1] == v;
}

bool isroot(Node *v) {
    return v->p == nullptr || v->p->ch[chnum(v)] != v;
}

void push(Node *v) {
    if (v->rev) {
        if (v->ch[0])
            v->ch[0]->rev ^= 1;
        if (v->ch[1])
            v->ch[1]->rev ^= 1;
        swap(v->ch[0], v->ch[1]);
        v->rev = false;
    }
}

void pull(Node *v) {
    v->sz = size(v->ch[1]) + size(v->ch[0]) + 1;
}

void attach(Node *v, Node *p, int num) {
    if (p)
        p->ch[num] = v;
    if (v)
        v->p = p;
}

void rotate(Node *v) {
    Node *p = v->p;
    push(p);
    push(v);
    int num = chnum(v);
    Node *u = v->ch[1 - num];
    if (!isroot(v->p))
        attach(v, p->p, chnum(p));
    else
        v->p = p->p;
}

```

```

    attach(u, p, num);
    attach(p, v, 1 - num);
    pull(p);
    pull(v);
}

void splay(Node *v) {
    push(v);
    while (!isroot(v)) {
        if (!isroot(v->p)) {
            if (chnum(v) == chnum(v->p))
                rotate(v->p);
            else
                rotate(v);
        }
        rotate(v);
    }
}

```

```

void expose(Node *v) {
    splay(v);
    v->ch[1] = nullptr;
    pull(v);
    while (v->p != nullptr) {
        Node *p = v->p;
        splay(p);
        attach(v, p, 1);
        pull(p);
        splay(v);
    }
}

```

```

void makeroot(Node *v) {
    expose(v);
    v->rev ^= 1;
    push(v);
}

```

```

void link(Node *v, Node *u) {
    makeroot(v);
    makeroot(u);
    u->p = v;
}

```

```

void cut(Node *v, Node *u) {
    makeroot(u);
    makeroot(v);
    v->ch[1] = nullptr;
    u->p = nullptr;
}

```

```

}

int get(Node *v, Node *u) {
    makeroot(u);
    makeroot(v);
    Node *w = u;
    while (!isroot(w))
        w = w->p;
    return (w == v ? size(v) - 1 : -1);
}

```

```

const int MAXN = 100010;
Node *nodes[MAXN];

```

```

int main() {
    int n, q;
    cin >> n >> q;
    for (int i = 0; i < n; ++i)
        nodes[i] = new Node();
    while (q--) {
        string s;
        int a, b;
        cin >> s >> a >> b;
        a--, b--;
        if (s[0] == 'g')
            cout << get(nodes[a], nodes[b]) << '\n';
        else if (s[0] == 'l')
            link(nodes[a], nodes[b]);
        else
            cut(nodes[a], nodes[b]);
    }
}

```

5 Другое

5.1 Slope trick

// Дан массив a_n . Сделать минимальное кол-во ± 1 , чтобы a_n стал неубывающим.

```

void solve() {
    int n;
    cin >> n;
    vector<int> a(n);
    for (int i = 0; i < n; i++) {
        cin >> a[i];
    }
    int ans = 0;
}

```

```

multiset<int> now;
for (int i = 0; i < n; i++) {
    now.insert(a[i]);
    ans += (*now.rbegin() - a[i]);
    now.erase(now.find(*now.rbegin()));
    now.insert(a[i]);
}
cout << ans << '\n';
}

```

5.2 attribute_packed

```

struct Kek {
    int a;
    char b;
    // char[3]
    int c;
} __attribute__((packed));
// sizeof = 9 (instead of 12)

```

5.3 ordered_set

```

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>

using namespace __gnu_pbds;

typedef tree<int, null_type, less<>, rb_tree_tag,
            tree_order_statistics_node_update> ordered_set;

//st.find_by_order(index);
//st.order_of_key(key);

```

5.4 pragma

```

#pragma GCC optimize("Ofast,fast-math,unroll-loops,no-
stack-protector,inline")
#pragma GCC target("sse,sse2,sse3,ssse3,sse4,sse4.1,sse4
.2,avx,avx2,abm,mmx,popcnt")

```

5.5 Аллокатор Копелиовича

```

// Код вставить до инклюдов

#include <cassert>

```

```

const int MAX_MEM = 1e8; // ~100mb
int mpos = 0;
char mem[MAX_MEM];

inline void *operator new(std::size_t n) {
    assert((mpos += n) <= MAX_MEM);
    return (void *) (mem + mpos - n);
}

inline void operator delete(void *) noexcept {} // must
have!
inline void operator delete(void *, std::size_t)
noexcept {} // fix!!

```

6 Математика

6.1 FFT mod

```

const int MOD = 998244353; //  $7 \cdot 17 \cdot 2^{23} + 1$ 
const int GEN = 3;
//const int MOD = 7340033; //  $7 \cdot 2^{20} + 1$ 
//const int GEN = 5;
//const int MOD = 469762049; //  $7 \cdot 2^{26} + 1$ 
//const int GEN = 30;

```

```

const int LOG = 20;
const int MAXN = 1 << LOG;
int tail[MAXN + 1];
int OMEGA[MAXN + 1];

```

```

int binpow(int x, int p) {
    int res = 1;
    while (p > 0) {
        if (p & 1)
            res = res * 111 * x % MOD;
        x = x * 111 * x % MOD;
        p >>= 1;
    }
    return res;
}

```

```

int omega(int n, int k) {
    return OMEGA[MAXN / n * k];
}

```

```

int gettail(int x, int lg) {
    return tail[x] >> (LOG - lg);
}

```

```

}

void calcomega() {
    long long one = binpow(GEN, (MOD - 1) / MAXN);
    OMEGA[0] = 1;
    for (int i = 1; i < MAXN; ++i) {
        OMEGA[i] = OMEGA[i - 1] * one % MOD;
    }
}

```

```

void calctail() {
    int n = MAXN;
    for (int x = 0; x < n; ++x) {
        int res = 0;
        for (int i = 0; i < LOG; ++i) {
            res += ((x >> i) & 1) << (LOG - i - 1);
        }
        tail[x] = res;
    }
}

```

```

// Without precalc, tail[], OMEGA[]
//
//long long omega(int n, int k) {
//    return binpow(GEN, (MOD - 1) / n * k);
//}
//
//int gettail(int x, int lg) {
//    int res = 0;
//    for (int i = 0; i < lg; ++i)
//        res += ((x >> i) & 1) << (lg - i - 1);
//    return res;
//}

```

```

void fft(vector<int> &A, int lg) {
    int n = 1 << lg;
    for (int i = 0; i < n; ++i) {
        int j = gettail(i, lg);
        if (i < j)
            swap(A[i], A[j]);
    }
    for (int len = 2; len <= n; len *= 2) {
        for (int i = 0; i < n; i += len) {
            for (int j = 0; j < len / 2; ++j) {
                auto v = A[i + j];
                auto u = A[i + j + len / 2] * 111 * omega(len, j) % MOD;
                A[i + j] = (v + u) % MOD;
                A[i + j + len / 2] = (v - u + MOD) % MOD;
            }
        }
    }
}

```

```

    }
}

int inverse(int x) {
    return binpow(x, MOD - 2);
}

void invfft(vector<int> &A, int lg) {
    int n = 1 << lg;
    fft(A, lg);
    for (auto &el : A)
        el = el * 111 * inverse(n % MOD) % MOD;
    reverse(A.begin() + 1, A.end());
}

vector<int> mul(vector<int> A, vector<int> B) {
    int lg = 32 - __builtin_clz(A.size() + B.size() - 1);
    int n = 1 << lg;
    A.resize(n, 0);
    B.resize(n, 0);
    fft(A, lg);
    fft(B, lg);
    for (int i = 0; i < n; ++i)
        A[i] = A[i] * 111 * B[i] % MOD;
    invfft(A, lg);
    return A;
}

signed main() {
    calctail(); // НЕ ЗАБЫТЬ
    calcomega(); // НЕ ЗАБЫТЬ
    int n, m;
    cin >> n >> m;
    vector<int> A(n), B(m);
    for (int &el : A)
        cin >> el;
    for (int &el : B)
        cin >> el;
    auto C = mul(A, B);
    for (auto el : C)
        cout << el << '␣';
}

```

6.2 FFT

```

const double PI = acos(-1);

```



```

const int LOG = 20;
const int MAXN = 1 << LOG;

struct comp {
    double x, y;
    comp() : x(0), y(0) {}
    comp(double x, double y) : x(x), y(y) {}
    comp(int x) : x(x), y(0) {}
    comp operator+(const comp &o) const {
        return {x + o.x, y + o.y};
    }
    comp operator-(const comp &o) const {
        return {x - o.x, y - o.y};
    }
    comp operator*(const comp &o) const {
        return {x * o.x - y * o.y, x * o.y + y * o.x};
    }
    comp operator/(const int k) const {
        return {x / k, y / k};
    }
    comp conj() const {
        return {x, -y};
    }
};

comp OMEGA[MAXN + 10];
int tail[MAXN + 10];

comp omega(int n, int k) {
    return OMEGA[MAXN / n * k];
}

void calcomega() {
    for (int i = 0; i < MAXN; ++i) {
        double x = 2 * PI * i / MAXN;
        OMEGA[i] = {cos(x), sin(x)};
    }
}

void calctail() {
    tail[0] = 0;
    for (int i = 1; i < MAXN; ++i) {
        tail[i] = (tail[i >> 1] >> 1) | ((i & 1) << (LOG - 1));
    }
}

void fft(vector<comp> &A) {
    int n = A.size();
    for (int i = 0; i < n; ++i) {
        if (i < tail[i])
            swap(A[i], A[tail[i]]);
    }
    for (int len = 2; len <= n; len *= 2) {
        for (int i = 0; i < n; i += len) {
            for (int j = 0; j < len / 2; ++j) {
                auto v = A[i + j];
                auto u = A[i + j + len / 2] * omega(len, j);
                A[i + j] = v + u;
                A[i + j + len / 2] = v - u;
            }
        }
    }
}

void fft2(vector<comp> &A, vector<comp> &B) {
    int n = A.size();
    vector<comp> C(n);
    for (int i = 0; i < n; ++i) {
        C[i].x = A[i].x;
        C[i].y = B[i].x;
    }
    fft(C);
    C.push_back(C[0]);
    for (int i = 0; i < n; ++i) {
        A[i] = (C[i] + C[n - i].conj()) / 2;
        B[i] = (C[i] - C[n - i].conj()) / 2 * comp(0, -1);
    }
}

void invfft(vector<comp> &A) {
    fft(A);
    for (auto &el : A)
        el = el / MAXN;
    reverse(A.begin() + 1, A.end());
}

vector<int> mul(vector<int> &a, vector<int> &b) {
    vector<comp> A(MAXN, 0), B(MAXN, 0);
    for (int i = 0; i < (int)a.size(); ++i)
        A[i] = a[i];
    for (int i = 0; i < (int)b.size(); ++i)
        B[i] = b[i];
    fft2(A, B);
    for (int i = 0; i < MAXN; ++i)
        A[i] = A[i] * B[i];
    invfft(A);
    vector<int> c(MAXN);
    for (int i = 0; i < MAXN; ++i) {

```

```

        int x = round(A[i].x);
        c[i] = x;
    }
    while (!c.empty() && c.back() == 0)
        c.pop_back();
    return c;
}

signed main() {
    calcomega(); // НЕ ЗАБЫТЬ
    calctail(); // НЕ ЗАБЫТЬ
    // your code here
}

```

6.3 Гайц

```

vector<vector<int>> gauss(vector<vector<int>> &a) {
    int n = a.size();
    int m = a[0].size();
    // int det = 1;
    for (int col = 0, row = 0; col < m && row < n; ++col) {
        for (int i = row; i < n; ++i) {
            if (a[i][col]) {
                swap(a[i], a[row]);
                if (i != row) {
                    det *= -1;
                }
                break;
            }
        }
        if (!a[row][col])
            continue;
        for (int i = 0; i < n; ++i) {
            if (i != row && a[i][col]) {
                int val = a[i][col] * inv(a[row][col]) % mod;
                for (int j = col; j < m; ++j) {
                    a[i][j] -= val * a[row][j];
                    a[i][j] %= mod;
                }
            }
        }
        ++row;
    }
    // for (int i = 0; i < n; ++i) det = (det * a[i][i]) %
    // mod;
    // det = (det % mod + mod) % mod;
    // result in (-mod, mod)
}

```

```

    return a;
}

pair<int, vector<int>> sle(vector<vector<int>> a, vector<int> b) {
    int n = a.size();
    int m = a[0].size();
    assert(n == b.size());
    for (int i = 0; i < n; ++i) {
        a[i].push_back(b[i]);
    }
    a = gauss(a);
    vector<int> x(m, 0);
    for (int i = n - 1; i >= 0; --i) {
        int leftmost = m;
        for (int j = 0; j < m; ++j) {
            if (a[i][j] != 0) {
                leftmost = j;
                break;
            }
        }
        if (leftmost == m && a[i].back() != 0) return {-1, {}};
        if (leftmost == m) continue;
        int val = a[i].back();
        for (int j = m - 1; j > leftmost; --j) {
            val -= a[i][j] * x[j];
            val %= mod;
        }
        x[leftmost] = (val * inv(a[i][leftmost]) % mod + mod) % mod;
    }
    return {1, x};
}

vector<bitset<N>> gauss_bit(vector<bitset<N>> a, int m) {
    {
        int n = a.size();
        for (int col = 0, row = 0; col < m && row < n; ++col) {
            for (int i = row; i < n; ++i) {
                if (a[i][col]) {
                    swap(a[i], a[row]);
                    break;
                }
            }
            if (!a[row][col])
                continue;
            for (int i = 0; i < n; ++i)

```

```

        if (i != row && a[i][col])
            a[i] ^= a[row];
        ++row;
    }
    return a;
}

```

6.4 Диофантовы уравнения

```

pair<int, int> ext_gcd(int a, int b) {
    int x1 = 1, y1 = 0, x2 = 0, y2 = 1;
    while (b) {
        int k = a / b;
        x1 = x1 - x2 * k;
        y1 = y1 - y2 * k;
        swap(x1, x2);
        swap(y1, y2);
        a %= b;
        swap(a, b);
    }
    return {x1, y1};
}

bool cool_ext_gcd(int a, int b, int c, int &x, int &y) {
    if (b == 0) {
        y = 0;
        if (a == 0) {
            x = 0;
            return c == 0;
        } else {
            x = c / a;
            return c % a == 0;
        }
    }
    auto [x0, y0] = ext_gcd(a, b);
    int g = x0 * a + y0 * b;
    if (c % g != 0)
        return false;
    x0 *= c / g;
    y0 *= c / g;
    int t = b / g;
    int k = (-x0) / t;
    if (x0 + t * k < 0)
        k += t / abs(t);
    x = x0 + t * k;
    y = y0 - (a / g) * k;
    return true;
}

```

6.5 KTO

```

// x = a_i % p_i
vector<vector<int>> r(k, vector<int>(k));
for (int i = 0; i < k; ++i)
    for (int j = 0; j < k; ++j)
        if (i != j)
            r[i][j] = binpow(p[i] % p[j], p[j] - 2, p[j]);
vector<int> x(k);
for (int i = 0; i < k; ++i) {
    x[i] = a[i];
    for (int j = 0; j < i; ++j) {
        x[i] = r[j][i] * (x[i] - x[j]);
        x[i] = x[i] % p[i];
        if (x[i] < 0) x[i] += p[i];
    }
}
int ans = 0;
for (int i = 0; i < k; ++i) {
    int val = x[i];
    for (int j = 0; j < i; ++j) val *= p[j];
    ans += val;
}

```

6.6 Код Грея

```

for (int i = 0; i < (1 << n); i++) {
    gray[i] = i ^ (i >> 1);
}

```

6.7 Линейное решето

```

const int N = 10000000;
int lp[N + 1];
vector<int> pr;
for (int i = 2; i <= N; ++i) {
    if (lp[i] == 0) {
        lp[i] = i;
        pr.push_back(i);
    }
    for (int j = 0; j < (int) pr.size() && pr[j] <= lp[i] && i * pr[j] <= N; ++j)
        lp[i * pr[j]] = pr[j];
}

```

6.8 Миллер Рабин

```
// assuming '#define int long long' is ON (replace 'int'
// with 'long long' if not)
// works for all n < 2^64
const int MAGIC[7] = {2, 325, 9375, 28178, 450775,
9780504, 1795265022};

int bpow(__int128 a, int x, int mod) {
    a %= mod;
    __int128 ans = 1;
    while (x) {
        if (x % 2) {
            ans *= a;
            ans %= mod;
        }
        a *= a;
        a %= mod;
        x /= 2;
    }
    return (int) ans;
}

bool is_prime(int n) {
    if (n == 1) return false;
    if (n <= 3) return true;
    if (n % 2 == 0 || n % 3 == 0) return false;
    int s = __builtin_ctzll(n - 1), d = n >> s; //
    n - 1 = 2^s · d
    for (auto a : MAGIC) {
        if (a % n == 0) {
            continue;
        }
        int x = bpow(a, d, n);
        for (int _ = 0; _ < s; _++) {
            int y = bpow(x, 2, n);
            if (y == 1 && x != 1 && x != n - 1) {
                return false;
            }
            x = y;
        }
        if (x != 1) {
            return false;
        }
    }
    return true;
}
```

6.9 Ро-Поллард

```
typedef long long ll;

ll mult(ll a, ll b, ll mod) {
    return (__int128)a * b % mod;
}

ll f(ll x, ll c, ll mod) {
    return (mult(x, x, mod) + c) % mod;
}

ll rho(ll n, ll x0=2, ll c=1) {
    ll x = x0;
    ll y = x0;
    ll g = 1;
    while (g == 1) {
        x = f(x, c, n);
        y = f(y, c, n);
        y = f(y, c, n);
        g = gcd(abs(x - y), n);
    }
    return g;
}

mt19937_64 rnd(time(nullptr));

void factor(int n, vector<int> &pr) {
    if (n == 4) {
        factor(2, pr);
        factor(2, pr);
        return;
    }
    if (n == 1) {
        return;
    }
    if (is_prime(n)) {
        pr.push_back(n);
        return;
    }
    int d = rho(n, rnd() % (n - 2) + 2, rnd() % 3 + 1);
    factor(n / d, pr);
    factor(d, pr);
}
```

7 Строки

7.1 Z-функция

```
vector<int> z_func(string s) {
    int n = s.size();
    vector<int> z(n, 0);
    z[0] = n;
    int l = 0, r = 0;
    for (int i = 1; i < n; i++) {
        if (i < r) {
            z[i] = min(z[i - l], r - i);
        }
        while (i + z[i] < n && s[z[i]] == s[i + z[i]]) {
            z[i]++;
        }
        if (i + z[i] > r) {
            l = i;
            r = i + z[i];
        }
    }
    return z;
}
```

7.2 Ахо-Корасик

```
int go[MAXN][ALPH];
vector<int> term[MAXN];
int par[MAXN], suf[MAXN];
char par_c[MAXN];
vector<int> g[MAXN];

int cntv = 1;

void add(string &s) {
    static int cnt_s = 1;
    int v = 0;
    for (char el : s) {
        if (go[v][el - 'a'] == 0) {
            go[v][el - 'a'] = cntv;
            par[cntv] = v;
            par_c[cntv] = el;
            cntv++;
        }
        v = go[v][el - 'a'];
    }
    term[v].push_back(cnt_s++);
}

void bfs() {
    deque<int> q = {0};
    while (!q.empty()) {
```

```

int v = q.front();
q.pop_front();
if (v > 0) {
    if (par[v] == 0) {
        suf[v] = 0;
    } else {
        suf[v] = go[suf[par[v]]][par_c[v] - 'a'];
    }
    g[suf[v]].push_back(v);
}
for (int c = 0; c < 26; c++) {
    if (go[v][c] == 0) {
        go[v][c] = go[suf[v]][c];
    } else {
        q.push_back(go[v][c]);
    }
}
}
}

```

7.3 Муффиксный Сассив

```

vector<int> build_suff_arr(string s) {
    s.push_back('#');
    int n = s.size();
    vector<int> suf(n), c(n);
    vector<int> cnt(MAX);
    for (int i = 0; i < n; i++) {
        cnt[s[i] - '#']++;
    }
    vector<int> pos(MAX);
    for (int i = 1; i < MAX; i++) {
        pos[i] = pos[i - 1] + cnt[i - 1];
    }
    for (int i = 0; i < n; i++) {
        suf[pos[s[i] - '#']++] = i;
    }
    int cls = -1;
    for (int i = 0; i < n; i++) {
        if (i == 0 || s[suf[i]] != s[suf[i - 1]]) {
            cls++;
        }
        c[suf[i]] = cls;
    }
    for (int L = 1; L < n; L *= 2) {
        fill(cnt.begin(), cnt.end(), 0);
        for (int i = 0; i < n; i++) {
            cnt[c[i]]++;

```

```

}
pos[0] = 0;
for (int i = 1; i < n; i++) {
    pos[i] = pos[i - 1] + cnt[i - 1];
}
for (int i = 0; i < n; i++) {
    suf[i] = (suf[i] - L + n) % n;
}
vector<int> new_suf(n), new_c(n);
for (int i = 0; i < n; i++) {
    int where = pos[c[suf[i]]];
    new_suf[where] = suf[i];
    pos[c[suf[i]]]++;
}
cls = -1;
for (int i = 0; i < n; i++) {
    if (i == 0) {
        cls++;
        new_c[new_suf[i]] = cls;
        continue;
    }
    pair<int, int> prev = {c[new_suf[i - 1]], c[(
new_suf[i - 1] + L) % n]};
    pair<int, int> now = {c[new_suf[i]], c[(new_suf[i]
+ L) % n]};
    if (prev != now) {
        cls++;
    }
    new_c[new_suf[i]] = cls;
}
swap(c, new_c);
swap(suf, new_suf);
}
vector<int> res;
for (int i = 1; i < n; i++) {
    res.push_back(suf[i]);
}
return res;
}

vector<int> kasai(string s, vector<int> sa) {
    int n = s.size(), k = 0;
    vector<int> lcp(n, 0);
    vector<int> rank(n, 0);
    for (int i = 0; i < n; i++) rank[sa[i]] = i;
    for (int i = 0; i < n; i++, k ? k-- : 0) {
        if (rank[i] == n - 1) {
            k = 0;
            continue;

```

```

}
int j = sa[rank[i] + 1];
while (i + k < n && j + k < n && s[i + k] == s[j + k]) k++;
lcp[rank[i]] = k;
}
return lcp;
}

```

7.4 Префикс-функция

```

vector<int> prefix_func(string s) {
    int n = s.size();
    vector<int> pref(n, 0);
    int ans = 0;
    for (int i = 1; i < n; i++) {
        while (ans > 0 && s[ans] != s[i]) {
            ans = pref[ans - 1];
        }
        if (s[i] == s[ans]) {
            ans++;
        }
        pref[i] = ans;
    }
    return pref;
}

```

7.5 Суффиксный автомат

```

// Суфавтомат с подсчётом кол-ва различных подстрок

const int SIGMA = 26;
int ans = 0;

struct Node {
    int go[SIGMA];
    int s, p;
    int len;

    Node() {
        fill(go, go + SIGMA, -1);
        s = -1, p = -1;
        len = 0;
    }
};

int add(int A, int ch, vector<Node> &sa) {

```

```

int B = sa.size();
sa.emplace_back();
sa[B].p = A;
sa[B].s = 0;
sa[B].len = sa[A].len + 1;
for (; A != -1; A = sa[A].s) {
    if (sa[A].go[ch] == -1) {
        sa[A].go[ch] = B;
        continue;
    }
    int C = sa[A].go[ch];
    if (sa[C].p == A) {
        sa[B].s = C;
        break;
    }
    int D = sa.size();
    sa.emplace_back();
    sa[D].s = sa[C].s;
    sa[D].p = A;
    sa[D].len = sa[A].len + 1;
    sa[C].s = D;
    sa[B].s = D;
    copy(sa[C].go, sa[C].go + SIGMA, sa[D].go);
    for (; A != -1 && sa[A].go[ch] == C; A = sa[A].s)
        sa[A].go[ch] = D;
    break;
}
ans += sa[B].len - sa[sa[B].s].len;
return B;
}

```

```

signed main() {
    string s;
    cin >> s;
    vector<Node> sa(1);
    int A = 0;
    for (char c : s)
        A = add(A, c - 'a', sa);
    cout << ans;
}

```

8 Структуры данных

8.1 Disjoint Sparse Table

```

int tree[LOG][MAXN];
int floorlog2[MAXN]; // i ? (31 - __builtin_clz(i)) : 0

```

```

void build(vector<int> &a) {
    int n = a.size();
    copy(a.begin(), a.end(), tree[0]);
    for (int lg = 1; lg < LOG; ++lg) {
        int len = 1 << lg;
        auto &lvl = tree[lg];
        for (int m = len; m < n; m += len * 2) {
            lvl[m - 1] = a[m - 1];
            lvl[m] = a[m];
            for (int i = m - 2; i >= m - len; --i)
                lvl[i] = min(lvl[i + 1], a[i]);
            for (int i = m + 1; i < m + len && i < n; ++i)
                lvl[i] = min(lvl[i - 1], a[i]);
        }
    }
    for (int i = 2; i <= n; ++i)
        floorlog2[i] = floorlog2[i / 2] + 1;
}

// a[l..r)
int get(int l, int r) {
    r--;
    int i = floorlog2[l ^ r];
    return min(tree[i][l], tree[i][r]);
}

```

8.2 Segment Tree Beats

```

// min=, sum
struct ST {
    vector<int> st, mx, mx_cnt, sec_mx;

    ST(int n) {
        st.resize(n * 4, 0);
        mx.resize(n * 4, 0);
        mx_cnt.resize(n * 4, 0);
        sec_mx.resize(n * 4, 0);
        build(0, 0, n);
    }

    void upd_from_children(int v) {
        st[v] = st[v * 2 + 1] + st[v * 2 + 2];
        mx[v] = max(mx[v * 2 + 1], mx[v * 2 + 2]);
        mx_cnt[v] = 0;
        sec_mx[v] = max(sec_mx[v * 2 + 1], sec_mx[v * 2 + 2]);
        if (mx[v * 2 + 1] == mx[v]) {
            mx_cnt[v] += mx_cnt[v * 2 + 1];

```

```

        } else {
            sec_mx[v] = max(sec_mx[v], mx[v * 2 + 1]);
        }
        if (mx[v * 2 + 2] == mx[v]) {
            mx_cnt[v] += mx_cnt[v * 2 + 2];
        } else {
            sec_mx[v] = max(sec_mx[v], mx[v * 2 + 2]);
        }
    }

    void build(int i, int l, int r) {
        if (l + 1 == r) {
            st[i] = mx[i] = 0;
            mx_cnt[i] = 1;
            sec_mx[i] = -INF;
            return;
        }
        int m = (r + 1) / 2;
        build(i * 2 + 1, l, m);
        build(i * 2 + 2, m, r);
        upd_from_children(i);
    }

    void push_min_eq(int v, int val) {
        if (mx[v] > val) {
            st[v] -= (mx[v] - val) * mx_cnt[v];
            mx[v] = val;
        }
    }

    void push(int i) {
        push_min_eq(i * 2 + 1, mx[i]);
        push_min_eq(i * 2 + 2, mx[i]);
    }

    void update(int i, int l, int r, int ql, int qr, int val) {
        if (mx[i] <= val) {
            return;
        }
        if (ql == l && qr == r && sec_mx[i] < val) {
            push_min_eq(i, val);
            return;
        }
        push(i);
        int m = (r + 1) / 2;
        if (qr <= m) {
            update(i * 2 + 1, l, m, ql, qr, val);
        } else if (ql >= m) {

```

```

    update(i * 2 + 2, m, r, ql, qr, val);
} else {
    update(i * 2 + 1, l, m, ql, m, val);
    update(i * 2 + 2, m, r, m, qr, val);
}
upd_from_children(i);
}

int sum(int i, int l, int r, int ql, int qr) {
    if (l == ql && r == qr) {
        return st[i];
    }
    push(i);
    int m = (r + 1) / 2;
    if (qr <= m) {
        return sum(i * 2 + 1, l, m, ql, qr);
    }
    if (ql >= m) {
        return sum(i * 2 + 2, m, r, ql, qr);
    }
    return sum(i * 2 + 1, l, m, ql, m) + sum(i * 2 + 2,
        m, r, m, qr);
}
};

```

8.3 ДД по неявному

```

pair<Node *, Node *> split(Node *t, int k) {
    if (!now)
        return {nullptr, nullptr};
    int szl = size(t->l);
    if (k <= szl) {
        auto [l, r] = split(t->l, k);
        t->l = r;
        pull(t);
        return {l, t};
    } else {
        auto [l, r] = split(t->r, k - szl - 1);
        t->r = l;
        pull(t);
        return {t, r};
    }
}

```

```

Node *merge(Node *l, Node *r) {
    if (!l)
        return r;
    if (!r)

```

```

        return l;
    if (l->y < r->y) {
        l->r = merge(l->r, r);
        pull(l);
        return l;
    } else {
        r->l = merge(l, r->l);
        pull(r);
        return r;
    }
}

void insert(Node *&root, int pos, int val) {
    Node *new_v = new Node(val);
    auto [l, r] = split(root, pos);
    root = merge(merge(l, new_v), r);
}

void erase(Node *&root, int pos) {
    auto [lm, r] = split(root, pos + 1);
    auto [l, m] = split(lm, pos);
    root = merge(l, r);
}

```

```

int sum(Node *v) {
    return v ? v->sm : 0;
}

```

```

// query [l, r]
int query(Node *&root, int ql, int qr) {
    auto [lm, r] = split(root, qr);
    auto [l, m] = split(lm, ql);
    int res = sum(m);
    root = merge(merge(l, m), r);
    return res;
}

```

8.4 ДД

```

pair<Node *, Node *> split(Node *t, int x) {
    if (!t)
        return {nullptr, nullptr};
    if (x <= t->x) {
        auto [l, r] = split(t->l, x);
        t->l = r;
        pull(t);
        return {l, t};
    }

```

```

} else {
    auto [l, r] = split(t->r, x);
    t->r = l;
    pull(t);
    return {t, r};
}
}

```

```

Node *merge(Node *l, Node *r) {
    if (!l)
        return r;
    if (!r)
        return l;
    if (l->y < r->y) {
        l->r = merge(l->r, r);
        pull(l);
        return l;
    } else {
        r->l = merge(l, r->l);
        pull(r);
        return r;
    }
}

```

```

void insert(Node *&root, int val) {
    Node *new_v = new Node(val);
    auto [l, r] = split(root, val);
    root = merge(merge(l, new_v), r);
}

```

```

void erase(Node *&root, int val) {
    auto [lm, r] = split(root, val + 1);
    auto [l, m] = split(lm, val);
    root = merge(l, r);
}

```

```

int sum(Node *v) {
    return v ? v->sm : 0;
}

```

```

// query [l, r]
int query(Node *&root, int ql, int qr) {
    auto [lm, r] = split(root, qr);
    auto [l, m] = split(lm, ql);
    int res = sum(m);
    root = merge(merge(l, m), r);
    return res;
}

```

8.5 Персистентное ДД по неявному

```
mt19937 rnd(228);
```

```
struct Node;
int size(Node *);
int sum(Node *);
```

```
struct Node {
    Node *l, *r;
    int val, sz, sm;

    Node(int val) : val(val), sz(1), sm(val) {
        l = r = nullptr;
    }
    Node(int val, Node *l, Node *r) : val(val), l(l), r(r) {
        {
            sz = 1 + size(l) + size(r);
            sm = val + sum(l) + sum(r);
        }
    };
};
```

```
int size(Node *v) {
    return v ? v->sz : 0;
}
```

```
int sum(Node *v) {
    return v ? v->sm : 0;
}
```

```
pair<Node *, Node *> split(Node *t, int x) {
    if (!t)
        return {nullptr, nullptr};
    int lsz = size(t->l);
    if (lsz >= x) {
        auto [l, r] = split(t->l, x);
        auto v = new Node(t->val, r, t->r);
        return {l, v};
    } else {
        auto [l, r] = split(t->r, x - lsz - 1);
        auto v = new Node(t->val, t->l, l);
        return {v, r};
    }
}
```

```
bool chooseleft(int lsz, int rsz) {
    return rnd() % (lsz + rsz) < lsz;
}
```

```
Node *merge(Node *l, Node *r) {
    if (!l)
        return r;
    if (!r)
        return l;
    if (chooseleft(l->sz, r->sz)) {
        auto rr = merge(l->r, r);
        auto v = new Node(l->val, l->l, rr);
        return v;
    } else {
        auto ll = merge(l, r->l);
        auto v = new Node(r->val, ll, r->r);
        return v;
    }
}
```

```
Node *insert(Node *root, int pos, int val) {
    Node *new_v = new Node(val);
    auto [l, r] = split(root, pos);
    return merge(merge(l, new_v), r);
}
```

```
Node *erase(Node *root, int pos) {
    auto [lm, r] = split(root, pos + 1);
    auto [l, m] = split(lm, pos);
    return merge(l, r);
}
```

```
// query [l, r)
pair<int, Node *> query(Node *root, int ql, int qr) {
    auto [lm, r] = split(root, qr);
    auto [l, m] = split(lm, ql);
    int res = sum(m);
    auto new_root = merge(merge(l, m), r);
    return {res, new_root};
}
```

8.6 Персистентное ДО

```
// left: v ? v->l : nullptr (same for right)
// sum: v ? v->sm : 0
```

```
// v can be nullptr. returns new root of subtree
Node *update(Node *v, int l, int r, int qi, int qx) {
    if (qi < l || r <= qi)
        return v;
    if (l + 1 == r)
        return new Node(qx);
}
```

```
int m = (l + r) / 2;
Node *u = new Node();
u->l = update(left(v), l, m, qi, qx);
u->r = update(right(v), m, r, qi, qx);
u->sm = sum(u->l) + sum(u->r);
return u;
}
```

```
int get(Node *v, int l, int r, int ql, int qr) {
    if (!v || qr <= l || r <= ql)
        return 0;
    if (ql <= l && r <= qr)
        return v->sm;
    int m = (l + r) / 2;
    auto a = get(v->l, l, m, ql, qr);
    auto b = get(v->r, m, r, ql, qr);
    return a + b;
}
```

8.7 Спарсы

```
int tree[LOG][MAXN];
int floorlog2[MAXN]; // i ? (31 - __builtin_clz(i)) : 0
```

```
void build(vector<int> &a) {
    int n = a.size();
    copy(a.begin(), a.end(), tree[0]);
    for (int i = 1; i < LOG; ++i) {
        int len = 1 << (i - 1);
        for (int j = 0; j + len < n; ++j)
            tree[i][j] = min(tree[i - 1][j], tree[i - 1][j + len]);
    }
    for (int i = 2; i <= n; ++i)
        floorlog2[i] = floorlog2[i / 2] + 1;
}
```

```
// min a[l..r)
int get(int l, int r) {
    int i = floorlog2[r - l];
    return min(tree[i][l], tree[i][r - (1 << i)]);
}
```

8.8 Фенвик (+ на отрезке)

```
// a[l..r) += x
void update(int l, int r, int x) {
}
```



```

    T1.add(l, x);
    T1.add(r, -x);
    T2.add(l, -x * l);
    T2.add(r, x * r);
}

// sum a[0..pos)
int rsq(int pos) {
    return T1.rsq(pos) * pos + T2.rsq(pos);
}

// sum a[l..r)
int sum(int l, int r) {
    return rsq(r) - rsq(l);
}

```

8.9 Фенвик

// Нумерация с 0

```

struct Fenwick {
    int n;
    vector<int> f;

    Fenwick(int n) : n(n) {
        f.resize(n + 1);
    }

    // a[i] += x
    void add(int i, int x) {
        for (++i; i <= n; i += i & -i)
            f[i] += x;
    }

    // sum a[0..i)
    int get(int i) {
        int ans = 0;
        for (; i > 0; i -= i & -i)
            ans += f[i];
        return ans;
    }

    // a[..] > 0; find max k: sum a[0..k) <= x
    int max_not_more(int x) {
        int cur = 0;
        for (int i = 20; i >= 0; --i) {
            int len = 1 << i;
            if (cur + len <= n && f[cur + len] <= x) {

```

```

                cur += len;
                x -= f[cur];
            }
        }
        return cur;
    }
};

// sum a[x1..x2)[y1..y2)[z1..x2)
int sum_3d(int x1, int x2, int y1, int y2, int z1, int
            z2) {
    int ans = get(x2, y2, z2);
    ans -= get(x1, y2, z2) + get(x2, y1, z2) + get(x2, y2,
        z1);
    ans += get(x1, y1, z2) + get(x1, y2, z1) + get(x2, y1,
        z1);
    ans -= get(x1, y1, z1);
    return ans;
}

```