Muffix Sassif – TRD

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Содержание

1 Геометрия

1.1 3D

```
struct Pt {
  dbl x, y, z;

  Pt() : x(0), y(0), z(0) {}
  Pt(dbl x_, dbl y_, dbl z_) : x(x_), y(y_), z(z_) {}

  Pt operator-(const Pt& o) const {
    return {x - o.x, y - o.y, z - o.z};
  }
  Pt operator+(const Pt& o) const {
    return {x + o.x, y + o.y, z + o.z};
  }
  Pt operator/(const dbl& a) const {
    return {x / a, y / a, z / a};
  }
  Pt operator*(const dbl& a) const {
    return {x * a, y * a, z * a};
  }
  Pt cross(const Pt& o) const {
    dbl nx = y * o.z - z * o.y;
    dbl ny = z * o.x - x * o.z;
    dbl nz = x * o.y - y * o.x;
    return {nx, ny, nz};
  }
  dbl dot(const Pt &o) const {
    return x * o.x + y * o.y + z * o.z;
  }
  bool operator==(const Pt& o) const {
    return abs(x - o.x) < EPS && abs(y - o.y) < EPS &&
    abs(z - o.z) < EPS;
  }
  dbl dist() {
    return sqrtl(x * x + y * y + z * z);
  }
};

struct Plane {
  dbl a, b, c, d;

  Plane(dbl a_, dbl b_, dbl c_, dbl d_) : a(a_), b(b_),
    c(c_), d(d_) {
    dbl z = sqrtl(a * a + b * b + c * c);
    if (z < EPS) return;
    a /= z, b /= z, c /= z, d /= z;
  }
  dbl get_val(const Pt &p) const {
    // НЕ СТАВИТЬ МОДУЛЬ
    return a * p.x + b * p.y + c * p.z + d;
  }
  dbl dist(const Pt &p) const {
    return abs(get_val(p));
  }
  bool on_plane(const Pt &p) const {
    return dist(p) / sqrtl(a * a + b * b + c * c) < EPS;
  }
  Pt proj(const Pt &p) const {
    dbl t = get_val(p) / (a * a + b * b + c * c);
    return p - Pt(a, b, c) * t;
  }
};

bool on_line(Pt p1, Pt p2, Pt p3) {
  return (p2 - p1).cross(p3 - p1) == Pt(0, 0, 0);
}

Plane get_plane(Pt p1, Pt p2, Pt p3) {
  Pt norm = (p2 - p1).cross(p3 - p1);
  Plane pl(norm.x, norm.y, norm.z, 0);
  pl.d = -pl.get_val(p1);
  return pl;
}

pair<pair<dbl, dbl>, pair<dbl, dbl>> get_xy(dbl a, dbl b
    , dbl c) {
  if (abs(a) > EPS) {
    dbl y1 = 0, y2 = 10;
    return {{(-c - b * y1) / a, y1}, {(-c - b * y2) / a,
     y2}};
  }
  dbl x1 = 0, x2 = 10;
  return {{x1, (-c - a * x1) / b}, {x2, (-c - a * x2) /
    b}};
}

pair<Pt, Pt> intersect(Plane pl1, Plane pl2) {
  if (abs(pl2.a) < EPS && abs(pl2.b) < EPS && abs(pl2.c)
     < EPS) {
    assert(false);
  }
  if (abs(pl2.a) > EPS) {
    dbl nd = pl1.d - pl1.a * pl2.d / pl2.a;
    dbl nc = pl1.c - pl1.a * pl2.c / pl2.a;
    dbl nb = pl1.b - pl1.a * pl2.b / pl2.a;
    if (abs(nc) < EPS && abs(nb) < EPS) {
      // плоскости параллельны (могут совпадать)
      return {Pt(0, 0, 0), Pt(0, 0, 0)};
    }
    auto [yz1, yz2] = get_xy(nb, nc, nd);
    dbl x1 = (-pl2.d - pl2.c * yz1.second - pl2.b * yz1.
    first) / pl2.a;
    dbl x2 = (-pl2.d - pl2.c * yz2.second - pl2.b * yz2.
    first) / pl2.a;
    return {Pt(x1, yz1.first, yz1.second), Pt(x2, yz2.
    first, yz2.second)};
  }
  Plane copy_pl1(pl1.c, pl1.a, pl1.b, pl1.d);
  Plane copy_pl2(pl2.c, pl2.a, pl2.b, pl2.d);
  auto [p1, p2] = intersect(copy_pl1, copy_pl2);
  return {Pt(p1.y, p1.z, p1.x), Pt(p2.y, p2.z, p2.x)};
}

// угол между двумя векторами
dbl get_ang(Pt p1, Pt p2) {
  return acosl(p1.dot(p2) / p1.dist() / p2.dist());
}

// любой перпендикулярный вектор
Pt vector_perp(Pt v) {
  if (abs(v.x) > EPS || abs(v.y) > EPS)
    return {v.y, -v.x, 0};
  return {v.z, 0, -v.x};
}

// плоскость через точку p перпендикулярная вектору v
Plane plane_perp(Pt p, Pt v) {
  Pt v1 = vector_perp(v);
  Pt v2 = v.cross(v1);
  return get_plane(p, v1 + p, v2 + p);
}
```

1.2 База 1 - вектор

```
char sign(dbl x) {
  return x < -EPS ? -1 : x > EPS;
}

struct vctr {
  dbl x, y;
  vctr() {}
  vctr(dbl x, dbl y) : x(x), y(y) {}

  dbl operator%(const vctr &o) const { return x * o.x +
    y * o.y; }
  dbl operator*(const vctr &o) const { return x * o.y -
    y * o.x; }
  vctr operator+(const vctr &o) const { return {x + o.x,
     y + o.y}; }
  vctr operator-(const vctr &o) const { return {x - o.x,
     y - o.y}; }
  vctr operator-() const { return {-x, -y}; }
  vctr operator*(const dbl d) const { return {x * d, y *
     d}; }
  vctr operator/(const dbl d) const { return {x / d, y /
     d}; }
  void operator+=(const vctr &o) { x += o.x, y += o.y; }
  void operator-=(const vctr &o) { x -= o.x, y -= o.y; }
  dbl dist2() const { return x * x + y * y; }
  dbl dist() const { return sqrtl(dist2()); }
  vctr norm() const { return *this / dist(); }
};

dbl angle_between(const vctr &a, const vctr &b) {
  return atan2(b * a, b % a);
}

// y > 0 ? 0 : 1
bool is2plane(const vctr &a) {
  return sign(a.y) < 0 || (sign(a.y) == 0 && sign(a.x) <
     0);
}

bool cmp_angle(const vctr &a, const vctr &b) {
  bool pla = is2plane(a);
  bool plb = is2plane(b);
  if (pla != plb)
```

```
    return pla < plb;
  return sign(a * b) > 0;
}

vctr rotate_ccw(const vctr &a, dbl phi) {
  dbl cs = cos(phi);
  dbl sn = sin(phi);
  return {a.x * cs - a.y * sn, a.y * cs + a.x * sn};
}

vctr rotate_ccw_90(const vctr &a) {
  return {-a.y, a.x};
}
```

1.3 База 2 - прямая

```
struct line {
  dbl a, b, c;

  line() {}
  line(dbl a, dbl b, dbl c) : a(a), b(b), c(c) {}
  line(const vctr A, const vctr B) {
    a = A.y - B.y;
    b = B.x - A.x;
    c = A * B;
    // left halfplane of A->B is positive
    // assert(a != 0 || b != 0);
  }
  void operator*=(dbl x) { a *= x, b *= x, c *= x; }
  void operator/=(dbl x) { a /= x, b /= x, c /= x; }
  dbl get(const vctr P) const { return a * P.x + b * P.y
     + c; }
  vctr anyPoint() const {
    dbl x = -a * c / (a * a + b * b);
    dbl y = -b * c / (a * a + b * b);
    return {x, y};
  }
  void normalize() {
    dbl d = sqrtl(a * a + b * b);
    a /= d, b /= d, c /= d;
  }
};

bool isparallel(line l1, line l2) {
  return sign(l2.a * l1.b - l2.b * l1.a) == 0;
}

vctr intersection(const line &l1, const line &l2) {
  dbl z = l2.a * l1.b - l2.b * l1.a;
  dbl x = (l1.c * l2.b - l2.c * l1.b) / z;
  dbl y = -(l1.c * l2.a - l2.c * l1.a) / z;
  return {x, y};
}

// Серединный перпендикуляр (не биссектриса!)
line bisection(const vctr A, const vctr B) {
  vctr M = (A + B) / 2;
  return line(M, M + rotate_ccw_90(B - A));
}
```

1.4 База 3 - окружность

```
struct circle {
  vctr C;
  dbl r;

  circle() {}
  circle(dbl x, dbl y, dbl r) : C(x, y), r(r) {}
  circle(vctr C, dbl r) : C(C), r(r) {}
  circle(const vctr A, const vctr B) {
    C = (A + B) / 2;
    r = (A - B).dist() / 2;
  }
  circle(const vctr A, const vctr B, const vctr D) {
    line l1 = bisection(A, B);
    line l2 = bisection(B, D);
    C = intersection(l1, l2);
    r = (C - A).dist();
  }

  bool isin(const vctr P) const {
    return sign((C - P).dist2() - r * r) <= 0;
  }
};

vector<vctr> intersection_line_circ(line l, circle c) {
  l.normalize();
  dbl d = abs(l.get(c.C));
  vctr per = vctr(l.a, l.b).norm() * d;
  vctr a = c.C + per;
  if (sign(d - c.r) > 0)
    return {};
  if (sign(l.get(a)) != 0)
    a = c.C - per;
  if (sign(c.r - d) == 0)
    return {a};
  dbl k = sqrtl(c.r * c.r - d * d);
  vctr v = vctr(-l.b, l.a).norm() * k;
  return {a + v, a - v};
}

vector<vctr> intersection_circ_circ(circle A, circle B)
    {
  vctr a = A.C, b = B.C;
  line l(2 * (b.x - a.x),
         2 * (b.y - a.y),
         B.r * B.r - A.r * A.r
             + (a.x * a.x + a.y * a.y)
             - (b.x * b.x + b.y * b.y));
  if (sign(l.a) == 0 && sign(l.b) == 0)
    return {};
  return intersection_line_circ(l, A);
}

vector<vctr> tangent_vctr_circ(vctr v, circle c) {
  dbl d = (c.C - v).dist();
  dbl k = sqrtl(d * d - c.r * c.r);
  circle c2(v.x, v.y, k);
  return intersection_circ_circ(c, c2);
}
```

1.5 Выпуклая оболочка

```
vctr minvctr(INF, INF);

bool cmp_convex_hull(const vctr &a, const vctr &b) {
  vctr A = a - minvctr;
  vctr B = b - minvctr;
  auto sign_prod = sign(A * B);
  if (sign_prod != 0)
    return sign_prod > 0;
  return A.dist2() < B.dist2();
}

// minvctr updates here
vector<vctr> get_convex_hull(vector<vctr> arr) {
  minvctr = {INF, INF};
  for (auto v : arr) {
    auto tmp = v - minvctr;
    if (sign(tmp.y) < 0 || (sign(tmp.y) == 0 && sign(tmp
    .x) < 0))
      minvctr = v;
  }
  vector<vctr> hull;
  sort(arr.begin(), arr.end(), cmp_convex_hull);
  for (vctr &el : arr) {
    while (hull.size() > 1 && sign((hull.back() - hull[
    hull.size() - 2]) * (el - hull.back())) <= 0)
      hull.pop_back();
    hull.push_back(el);
  }
  return hull;
}
```

1.6 Задача 16

```
bool isInSameHalf(vctr p, vctr r1, vctr r2) {
  return sign((r2 - r1) % (p - r1)) >= 0;
}

dbl distPointPoint(vctr a, vctr b) {
  return (a - b).dist();
}

dbl distPointLine(vctr a, vctr l1, vctr l2) {
  line l(l1, l2);
  l.normalize();
  return abs(l.get(a));
}

dbl distPointRay(vctr a, vctr r1, vctr r2) {
  if (!isInSameHalf(a, r1, r2))
    return distPointPoint(a, r1);
  return distPointLine(a, r1, r2);
```

```
}

dbl distPointSeg(vctr a, vctr s1, vctr s2) {
  return max(distPointRay(a, s1, s2),
             distPointRay(a, s2, s1));
}

bool isIntersectionLineLine(line l1, line l2) {
  dbl znam = l1.b * l2.a - l1.a * l2.b;
  return sign(znam) != 0;
}

vctr intersectionLineLine(line l1, line l2) {
  dbl znam = l1.b * l2.a - l1.a * l2.b;
  dbl y = -(l1.c * l2.a - l2.c * l1.a) / znam;
  dbl x = -(l1.c * l2.b - l2.c * l1.b) / -znam;
  return vctr(x, y);
}

vctr getPointOnLine(line l) {
  if (sign(l.b) != 0)
    return vctr(0, -l.c / l.b);
  return vctr(-l.c / l.a, 0);
}

dbl distLineLine(vctr l1a, vctr l1b, vctr l2a, vctr l2b)
     {
  line l1(l1a, l1b);
  line l2(l2a, l2b);
  if (isIntersectionLineLine(l1, l2))
    return 0;
  vctr p = getPointOnLine(l1);
  l2.normalize();
  return abs(l2.get(p));
}

dbl distRayLine(vctr r1, vctr r2, vctr l1, vctr l2) {
  line r(r1, r2);
  line l(l1, l2);
  if (!isIntersectionLineLine(l, r))
    return distLineLine(r1, r2, l1, l2);
  vctr p = intersectionLineLine(l, r);
  if (isInSameHalf(p, r1, r2))
    return 0;
  return distPointLine(r1, l1, l2);
}

dbl distSegLine(vctr s1, vctr s2, vctr l1, vctr l2) {
  return max(distRayLine(s1, s2, l1, l2),
             distRayLine(s2, s1, l1, l2));
}

dbl distRayRay(vctr r1a, vctr r1b, vctr r2a, vctr r2b) {
  line r1(r1a, r1b);
  line r2(r2a, r2b);
  if (!isIntersectionLineLine(r1, r2)) {
    if (isInSameHalf(r1a, r2a, r2b) || isInSameHalf(r2a,
     r1a, r1b))
      return distLineLine(r1a, r1b, r2a, r2b);
    else
      return distPointPoint(r1a, r2a);
  }
  vctr p = intersectionLineLine(r1, r2);
  if (isInSameHalf(p, r1a, r1b) && isInSameHalf(p, r2a,
    r2b))
    return 0;
  return min(distPointRay(r1a, r2a, r2b),
             distPointRay(r2a, r1a, r1b));
}

dbl distSegRay(vctr s1, vctr s2, vctr r1, vctr r2) {
  return max(distRayRay(s1, s2, r1, r2),
             distRayRay(s2, s1, r1, r2));
}

dbl distSegSeg(vctr s1a, vctr s1b, vctr s2a, vctr s2b) {
  return max(distSegRay(s1a, s1b, s2a, s2b),
             distSegRay(s1a, s1b, s2b, s2a));
}
```

1.7 Калиперы

```
// Диаметр выпуклого многоугольника
int calipers(vector<vctr> &pts) {
  int n = pts.size();
  int a = 0, b = 0;
  for (int i = 1; i < n; ++i) {
    auto &v = pts[i];
    if (tie(v.y, v.x) < tie(pts[a].y, pts[a].x))
      a = i;
    if (tie(v.y, v.x) > tie(pts[b].y, pts[b].x))
      b = i;
  }
  int aa = (a + 1) % n, bb = (b + 1) % n;
  int dist2 = 0;
  for (int i = 0; i < n; ++i) {
    while (sign((pts[aa] - pts[a]) * (pts[bb] - pts[b]))
     > 0)
      b = bb, bb = (b + 1) % n;
    dist2 = max(dist2, (pts[a] - pts[b]).dist2());
    a = aa, aa = (a + 1) % n;
  }
  return dist2;
}
```

1.8 Касательные из точки

```
pair<int, int> tangents_from_point(vector<vctr> &p, vctr
     &a) {
  int n = p.size();
  int logn = 31 - __builtin_clz(n);
  auto findWithSign = [&](int sgn) {
    int i = 0;
    for (int k = logn; k >= 0; --k) {
      int i1 = (i - (1 << k) + n) % n;
      int i2 = (i + (1 << k)) % n;
      if (sign((p[i1] - a) * (p[i] - a)) == sgn)
        i = i1;
      if (sign((p[i2] - a) * (p[i] - a)) == sgn)
        i = i2;
    }
    return i;
  };
  return {findWithSign(1), findWithSign(-1)};
}
```

1.9 Касательные параллельные прямой

```
// find point with max (sgn=1) or min (sgn=-1) signed
    distance to line
int tangent_parallel_line(const vector<vctr> &p, line l,
     int sgn) {
  l *= sgn;
  int n = p.size();
  int i = 0;
  int logn = 31 - __builtin_clz(n);
  for (int k = logn; k >= 0; --k) {
    int i1 = (i - (1 << k) + n) % n;
    int i2 = (i + (1 << k)) % n;
    if (l.get(p[i1]) > l.get(p[i]))
      i = i1;
    if (l.get(p[i2]) > l.get(p[i]))
      i = i2;
  }
  return i;
}
```

1.10 Лежит ли точка в многоугольнике

```
// Выпуклый многоугольник, P[0] = minvctr
bool is_point_in_poly(vctr A, vector<vctr> &P) {
  auto tmp = A - P[0];
  if (sign(tmp.y) < 0 || (sign(tmp.y) == 0 && sign(tmp.x
    ) < 0))
    return false;
  if (sign(tmp.y) == 0 && sign(tmp.x) == 0)
    return true;
  int ind = lower_bound(P.begin(), P.end(), A,
    cmp_convex_hull) - P.begin();
  assert(ind != 0);
  if (ind == P.size())
    return false;
  vctr B = A - P[ind - 1];
  vctr C = P[ind] - P[ind - 1];
  return sign(C * B) >= 0;
}

bool is_point_in_poly_strict(vctr A, vector<vctr> &P) {
  if (sign(A.y - P[0].y) <= 0 || sign((A - P[0]) * (P.
    back() - P[0])) <= 0)
      return false;
  int ind = lower_bound(P.begin(), P.end(), A,
    cmp_convex_hull) - P.begin();
  assert(ind != 0 && ind != P.size());
```

```
  vctr B = A - P[ind - 1];
  vctr C = P[ind] - P[ind - 1];
  return sign(C * B) > 0;
}
```

1.11 Минимальная покрывающая окружность

```
mt19937 rnd(179);

circle MinDisk2(vector<vctr> &p, vctr A, vctr B, int sz)
     {
  circle w(A, B);
  for (int i = 0; i < sz; ++i) {
    if (w.isin(p[i]))
      continue;
    w = circle(A, B, p[i]);
  }
  return w;
}

circle MinDisk1(vector<vctr> &p, vctr A, int sz) {
  shuffle(p.begin(), p.begin() + sz, rnd);
  circle w(A, p[0]);
  for (int i = 1; i < sz; ++i) {
    if (w.isin(p[i]))
      continue;
    w = MinDisk2(p, A, p[i], i);
  }
  return w;
}

circle MinDisk(vector<vctr> &p) {
  int sz = p.size();
  if (sz == 1)
    return circle(p[0], 0);
  shuffle(p.begin(), p.end(), rnd);
  circle w(p[0], p[1]);
  for (int i = 2; i < sz; ++i) {
    if (w.isin(p[i]))
      continue;
    w = MinDisk1(p, p[i], i);
  }
  return w;
}
```

1.12 Пересечение полуплоскостей

```
// half plane: ax+by+c > 0
// bounding box MUST have
vector<int> intersection_half_planes_inds(const vector<
    line> &ls) {
  int n = (int)ls.size();
  vector<int> lsi(n);
  iota(lsi.begin(), lsi.end(), 0);
  sort(lsi.begin(), lsi.end(), [&](int i, int j) {
    vctr aa(ls[i].a, ls[i].b);
    vctr bb(ls[j].a, ls[j].b);
    bool pla = is2plane(aa);
    bool plb = is2plane(bb);
    if (pla != plb)
      return pla < plb;
    return aa * bb > 0;
  });

  vector<line> st;
  vector<int> inds;
  for (int ii = 0; ii < 2 * n; ++ii) {
    int i = lsi[ii % n];
    if (st.empty()) {
      st.push_back(ls[i]);
      inds.push_back(i);
      continue;
    }
    vctr p = intersection(ls[i], st.back());
    bool pp = isparallel(ls[i], st.back());
    bool bad = false;
    while (st.size() >= 2) {
      if (!pp && sign(st[st.size() - 2].get(p)) >= 0)
        break;
      else if (pp && sign(st.back().get(ls[i].anyPoint()
      )) <= 0) {
        bad = true;
        break;
      }
      st.pop_back();
      inds.pop_back();
      p = intersection(ls[i], st.back());
      pp = isparallel(ls[i], st.back());
    }
    if (!bad) {
      st.push_back(ls[i]);
      inds.push_back(i);
    }
  }
  vector<int> cnt(n, 0);
  for (int i : inds)
    cnt[i]++;
  vector<int> good;
  for (int i : inds) {
    if (cnt[i]-- == 2)
      good.push_back(i);
  }
  return good;
}

vector<vctr> intersection_half_planes(vector<line> &ls)
    {
  vector<int> inter = intersection_half_planes_inds(ls);
  int n = inter.size();
  vector<vctr> pts;
  for (int i = 0; i < n; ++i) {
    int j = (i + 1) % n;
    vctr P = intersection(ls[inter[i]], ls[inter[j]]);
    if (pts.empty() || sign(pts.back().x - P.x) != 0
        || sign(pts.back().y - P.y) != 0)
      pts.push_back(P);
  }
  // pts против часовой стрелки, но pts[0] != minvctr
  return pts;
}
```

1.13 Проверка на пересечение отрезков

```
bool is_intersection_seg(vctr A, vctr B, vctr C, vctr D)
    {
  for (int i = 0; i < 2; ++i) {
    auto l1 = A.x, r1 = B.x, l2 = C.x, r2 = D.x;
    if (l1 > r1) swap(l1, r1);
    if (l2 > r2) swap(l2, r2);
    if (max(l1, l2) > min(r1, r2))
      return false;
    swap(A.x, A.y);
    swap(B.x, B.y);
    swap(C.x, C.y);
    swap(D.x, D.y);
  }
  for (int _ = 0; _ < 2; ++_) {
    auto v1 = (B - A) * (C - A);
    auto v2 = (B - A) * (D - A);
    if (sign(v1) * sign(v2) == 1)
      return false;
    swap(A, C);
    swap(B, D);
  }
  return true;
}
```

1.14 Сумма Минковского

```
// Список вершин -> список рёбер
vector<vctr> poly_to_edges(const vector<vctr> &A) {
  vector<vctr> edg(A.size());
  for (int i = 0; i < A.size(); ++i)
    edg[i] = A[(i + 1) % A.size()] - A[i];
  return edg;
}

// A и B начинаются с минимальных вершин
vector<vctr> minkowski_sum(const vector<vctr> &A, const
    vector<vctr> &B) {
  auto edgA = poly_to_edges(A);
  auto edgB = poly_to_edges(B);
  vector<vctr> edgC(A.size() + B.size());
  merge(edgA.begin(), edgA.end(), edgB.begin(), edgB.end
    (), edgC.begin(), cmp_angle);
  // cmp_angle из шаблона вектора
  vector<vctr> C(edgC.size());
  C[0] = A[0] + B[0];
  for (int i = 0; i + 1 < C.size(); ++i)
    C[i + 1] = C[i] + edgC[i];
  return C;
}
```

1.15 Формула Эйлера

- V – число вершин выпуклого многогранника (планарного графа)
- E – число рёбер
- F – число граней (если планарный граф, то включая внешнюю)

Тогда $V - E + F = 2$

2 Графы

2.1 2-SAT

```
for (int i = 1; i <= n; ++i) {
  not_v[i] = i + n;
  not_v[i + n] = i;
}
for (int i = 0; i < m; ++i) {
  cin >> u >> v;
  g[not_v[v]].push_back(u);
  g[not_v[u]].push_back(v);
  rg[u].push_back(not_v[v]);
  rg[v].push_back(not_v[u]);
}
// делаем ККС, получаем comp
for (int v = 1; v <= n; ++v) {
  if (comp[v] == comp[not_v[v]]) {
    cout << "UNSATISFIABLE\n";
    return 0;
  }
}
for (int v = 1; v <= n; ++v)
  cout << (comp[v] > comp[not_v[v]] ? v : not_v[v]);
```

2.2 WeightedMatching

```
// НЕ ЗАБЫТЬ вызвать init(n)
// вершины нумеруются от 1 до n
namespace weighted_matching{
  const int INF = (int)1e9 + 7;
  const int MAXN = 1050; //double of possible N
  struct E{
    int x, y, w;
  };
  int n, m;
  E G[MAXN][MAXN];
  int lab[MAXN], match[MAXN], slack[MAXN], st[MAXN], pa[
    MAXN];
  int flo_from[MAXN][MAXN], S[MAXN], vis[MAXN];
  vector<int> flo[MAXN];
  queue<int> Q;
  void init(int _n) {
    n = _n;
    for(int x = 1; x <= n; ++x)
      for(int y = 1; y <= n; ++y)
        G[x][y] = E{x, y, 0};
  }
  void add_edge(int x, int y, int w) {
    G[x][y].w = G[y][x].w = w;
  }
  int e_delta(E e) {
    return lab[e.x] + lab[e.y] - G[e.x][e.y].w * 2;
  }
  void update_slack(int u, int x) {
    if(!slack[x] || e_delta(G[u][x]) < e_delta(G[slack[x
    ]][x]))
      slack[x] = u;
  }
  void set_slack(int x) {
    slack[x] = 0;
    for(int u = 1; u <= n; ++u)
      if(G[u][x].w > 0 && st[u] != x && S[st[u]] == 0)
        update_slack(u, x);
  }
  void q_push(int x) {
    if(x <= n) Q.push(x);
    else for(int i = 0; i < (int)flo[x].size(); ++i)
        q_push(flo[x][i]);
  }
  void set_st(int x, int b) {
    st[x] = b;
    if(x > n) for(int i = 0; i < (int)flo[x].size(); ++i
    )
        set_st(flo[x][i], b);
  }
  int get_pr(int b, int xr) {
    int pr = find(flo[b].begin(), flo[b].end(), xr) -
    flo[b].begin();
    if(pr & 1) {
      reverse(flo[b].begin() + 1, flo[b].end());
      return (int)flo[b].size() - pr;
    }
    else return pr;
  }
  void set_match(int x, int y) {
    match[x] = G[x][y].y;
    if(x <= n) return;
    E e = G[x][y];
    int xr = flo_from[x][e.x], pr = get_pr(x, xr);
    for(int i = 0; i < pr; ++i) set_match(flo[x][i], flo
    [x][i^1]);
    set_match(xr, y);
    rotate(flo[x].begin(), flo[x].begin() + pr, flo[x].
    end());
  }
  void augment(int x, int y) {
    while(1) {
      int ny = st[match[x]];
      set_match(x, y);
      if(!ny) return;
      set_match(ny, st[pa[ny]]);
      x = st[pa[ny]], y = ny;
    }
  }
  int get_lca(int x, int y) {
    static int t = 0;
    for(++t; x || y; swap(x, y)) {
      if(x == 0) continue;
      if(vis[x] == t) return x;
      vis[x] = t;
      x = st[match[x]];
      if(x) x = st[pa[x]];
    }
    return 0;
  }
  void add_blossom(int x, int l, int y) {
    int b = n + 1;
    while(b <= m && st[b]) ++b;
    if(b > m) ++m;
    lab[b] = 0, S[b] = 0;
    match[b] = match[l];
    flo[b].clear();
    flo[b].push_back(l);
    for(int u = x, v; u != l; u = st[pa[v]])
      flo[b].push_back(u), flo[b].push_back(v = st[match
    [u]]), q_push(v);
    reverse(flo[b].begin() + 1, flo[b].end());
    for(int u = y, v; u != l; u = st[pa[v]])
      flo[b].push_back(u), flo[b].push_back(v = st[match
    [u]]), q_push(v);
    set_st(b, b);
    for(int i = 1; i <= m; ++i) G[b][i].w = G[i][b].w =
    0;
    for(int i = 1; i <= n; ++i) flo_from[b][i] = 0;
    for(int i = 0; i < (int)flo[b].size(); ++i) {
      int us = flo[b][i];
      for(int u = 1; u <= m; ++u)
        if(G[b][u].w == 0 || e_delta(G[us][u]) < e_delta
    (G[b][u]))
          G[b][u] = G[us][u], G[u][b] = G[u][us];
      for(int u = 1; u <= n; ++u)
        if(flo_from[us][u])
          flo_from[b][u] = us;
    }
    set_slack(b);
  }
  void expand_blossom(int b) {
    for(int i = 0; i < (int)flo[b].size(); ++i)
      set_st(flo[b][i], flo[b][i]);
    int xr = flo_from[b][G[b][pa[b]].x], pr = get_pr(b,
    xr);
    for(int i = 0; i < pr; i += 2) {
      int xs = flo[b][i], xns = flo[b][i + 1];
      pa[xs] = G[xns][xs].x;
      S[xs] = 1, S[xns] = 0;
      slack[xs] = 0, set_slack(xns);
      q_push(xns);
    }
    S[xr] = 1, pa[xr] = pa[b];
    for(int i = pr + 1; i < (int)flo[b].size(); ++i) {
      int xs = flo[b][i];
```

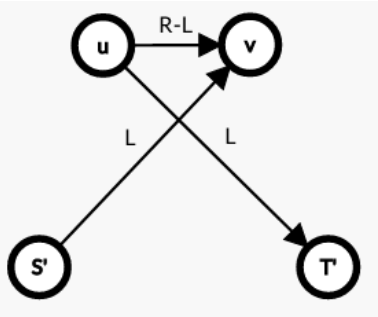
```
      S[xs] = -1, set_slack(xs);
    }
    st[b] = 0;
  }
  bool on_found_edge(E e) {
    int x = st[e.x], y = st[e.y];
    if(S[y] == -1) {
      pa[y] = e.x, S[y] = 1;
      int ny = st[match[y]];
      slack[y] = slack[ny] = 0;
      S[ny] = 0, q_push(ny);
    }
    else if(S[y] == 0) {
      int l = get_lca(x, y);
      if(!l) return augment(x, y), augment(y, x), true;
      else add_blossom(x, l, y);
    }
    return false;
  }
  bool matching() {
    fill(S + 1, S + m + 1, -1);
    fill(slack + 1, slack + m + 1, 0);
    Q = queue<int>();
    for(int x = 1; x <= m; ++x)
      if(st[x] == x && !match[x]) pa[x] = 0, S[x] = 0,
    q_push(x);
    if(Q.empty()) return false;
    while(1) {
      while(Q.size()) {
        int x = Q.front(); Q.pop();
        if(S[st[x]] == 1) continue;
        for(int y = 1; y <= n; ++y) {
          if(G[x][y].w > 0 && st[x] != st[y]) {
            if(e_delta(G[x][y]) == 0) {
              if(on_found_edge(G[x][y])) return true;
            }
            else update_slack(x, st[y]);
          }
        }
      }
      int d = INF;
      for(int b = n + 1; b <= m; ++b)
        if(st[b] == b && S[b] == 1) d = min(d, lab[b] /
    2);
      for(int x = 1; x <= m; ++x)
        if(st[x] == x && slack[x]) {
          if(S[x] == -1) d = min(d, e_delta(G[slack[x]][
    x]));
          else if(S[x] == 0) d = min(d, e_delta(G[slack[
    x]][x]) / 2);
        }
      for(int x = 1; x <= n; ++x) {
        if(S[st[x]] == 0) {
          if(lab[x] <= d) return 0;
          lab[x] -= d;
        }
        else if(S[st[x]] == 1) lab[x] += d;
      }
      for(int b = n + 1; b <= m; ++b)
        if(st[b] == b) {
          if(S[st[b]] == 0) lab[b] += d * 2;
          else if(S[st[b]] == 1) lab[b] -= d * 2;
        }
      Q = queue<int>();
      for(int x = 1; x <= m; ++x)
        if(st[x] == x && slack[x] && st[slack[x]] != x
      && e_delta(G[slack[x]][x]) == 0)
          if(on_found_edge(G[slack[x]][x])) return true;
      for(int b = n + 1; b <= m; ++b)
        if(st[b] == b && S[b] == 1 && lab[b] == 0)
          expand_blossom(b);
    }
    return false;
  }
  pair<int, int> solve(vector<pair<int, int>> &ans) {
    fill(match + 1, match + n + 1, 0);
    m = n;
    int cnt = 0; int sum = 0;
    for(int u = 0; u <= n; ++u) st[u] = u, flo[u].clear
    ();
    int mx = 0;
    for(int x = 1; x <= n; ++x)
      for(int y = 1; y <= n; ++y){
        flo_from[x][y] = (x == y ? x : 0);
        mx = max(mx, G[x][y].w);
      }
    for(int x = 1; x <= n; ++x) lab[x] = mx;
    while(matching()) ++cnt;
    for(int x = 1; x <= n; ++x)
      if(match[x] && match[x] < x) {
        sum += G[x][match[x]].w;
        ans.push_back({x, G[x][match[x]].y});
      }
    return {sum, cnt};
  }
}
```

2.3 l-r-capacity-maxflow

Maximum flow problem with minimum capacities

We describe how to find the maximum flow from S to T when the edges also constrain the minimum bound of the flow amount (edges have "minimum capacities"). It can be boiled down to an ordinary max-flow problem.

Consider an edge from u to v whose capacity is R and minimum capacity is L. To deal with the minimum capacity, create a new vertex S' to T', remove the original edge, and add edges with the following capacities:



Flow network example with minimum capacities

Add such edges for all edges with the minimum capacities. On the resulting graph, accumulate maximum flow in the following order:

- from S' to T'
- from S' to T
- from S to T'
- from S to T

An S–T flow that satisfies the minimum capacities exists if and only if, for all outgoing edges from S' and incoming edges to T', the flow and capacity are equal. (This can be understood by corresponding the flows from S' and T' to the original edges.)

Alternatively, if you just want to know the existence of a flow satisfying the minimum capacities, one can add an edge from T to S with infinite capacity and consider the flow from S' to T' once, instead of accumulating flows four times.

2.4 Венгерский алгоритм

```
pair<int, vector<int>> venger(vector<vector<int>> a) {
// ищет минимальное по стоимости
// работает только при n <= m
// a - массив весов (n+1) × (m+1)
// a[0][..] = a[..][0] = 0
// возвращает ans[i] = j если взяли ребро a[i][j]
  int n = (int) a.size() - 1;
  int m = (int) a[0].size() - 1;
  vector<int> u(n + 1), v(m + 1), p(m + 1), way(m + 1);
  for (int i = 1; i <= n; ++i) {
    p[0] = i;
    int j0 = 0;
    vector<int> minv(m + 1, INF);
    vector<char> used(m + 1, false);
    do {
      used[j0] = true;
      int i0 = p[j0], delta = INF, j1;
```

```
      for (int j = 1; j <= m; ++j)
        if (!used[j]) {
          int cur = a[i0][j] - u[i0] - v[j];
          if (cur < minv[j])
            minv[j] = cur, way[j] = j0;
          if (minv[j] < delta)
            delta = minv[j], j1 = j;
        }
      for (int j = 0; j <= m; ++j)
        if (used[j])
          u[p[j]] += delta, v[j] -= delta;
        else
          minv[j] -= delta;
      j0 = j1;
    } while (p[j0] != 0);
    do {
      int j1 = way[j0];
      p[j0] = p[j1];
      j0 = j1;
    } while (j0);
  }
  int cost = -v[0];
  vector<int> ans(n + 1);
  for (int j = 1; j <= m; ++j)
    ans[p[j]] = j;
  return {cost, ans};
}
```

2.5 Вершинная двусвязность

```
vector<pair<int, int>> graph[MAX_V];
bitset<MAX_V> vis;
int st[MAX_E], col[MAX_E], tin[MAX_V], up[MAX_V];
int sti = 0, cc = 0, tt = 0;

void dfs(int v, int pei) {
  vis[v] = true;
  int upv = tin[v] = tt++;
  for (auto [u, ei] : graph[v]) {
    if (ei == pei) continue;
    if (!vis[u]) {
      int pt = sti;
      st[sti++] = ei;
      dfs(u, ei);
      upv = min(upv, up[u]);
      if (up[u] >= tin[v]) {
        while (sti > pt)
          col[st[--sti]] = cc;
        cc++;
      }
    } else if (tin[u] <= tin[v]) {
      st[sti++] = ei;
      upv = min(upv, tin[u]);
    }
  }
  up[v] = upv;
}

// graph[v].emplace_back(u, i);
// graph[u].emplace_back(v, i);
fill(col, col + m, -1);
for (int v = 0; v < n; ++v) {
  if (!vis[v])
    dfs(v, -1);
}
// col[i] - компонента i-го ребра
// cc - итоговое кол-во компонент
```

2.6 Диниц

```
const int LOG = 29; // масштабирование, =0 если не нужно
struct Edge { int u, f, c, r; };

struct Dinic {
  vector<Edge> graph[MAXN];
  bitset<MAXN> vis;
  int inds[MAXN], dist[MAXN], Q[MAXN];
  int ql, qr, S, T, BIT;
  Dinic() {}

  bool bfs() {
    vis.reset();
    ql = 0, qr = 0;
    dist[S] = 0, vis[S] = true;
    Q[qr++] = S;
    while (ql < qr) {
      int v = Q[ql++];
      for (auto &e : graph[v]) {
        int u = e.u;
        if (vis[u] || e.c - e.f < BIT)
          continue;
        vis[u] = true;
        dist[u] = dist[v] + 1;
        Q[qr++] = u;
        if (u == T) return true;
      }
    }
    return false;
  }

  int dfs(int v, int maxF) {
    if (v == T) return maxF;
    int ans = 0;
    for (int &i = inds[v]; i < graph[v].size(); ++i) {
      auto &e = graph[v][i];
      auto cc = min(maxF - ans, e.c - e.f);
      if (dist[e.u] <= dist[v] || !vis[e.u] || inds[e.u]
     == graph[e.u].size() || cc < BIT)
        continue;
      auto f = dfs(e.u, cc);
      if (f != 0) {
        e.f += f, ans += f;
        graph[e.u][e.r].f -= f;
      }
      // иногда быстрее один иф, иногда другой
      if (maxF - ans < 1) break;
      // if (maxF - ans < BIT) break;
    }
    return ans;
  }

  void run(int s, int t) {
    S = s, T = t;
    assert(S != T);
    for (BIT = (1ll << LOG); BIT > 0; BIT >>= 1) {
      while (bfs()) {
        memset(inds, 0, sizeof inds);
        for (auto &e : graph[S]) {
          if (inds[e.u] == graph[e.u].size() || e.c - e.
    f < BIT)
            continue;
          int f = dfs(e.u, e.c - e.f);
          e.f += f, graph[e.u][e.r].f -= f;
        }
      }
    }
  }

  void addedge(int v, int u, int c) {
    graph[v].push_back({u, 0, c, (int)graph[u].size()});
    // если ориентированно, то обратная capacity = 0
    graph[u].push_back({v, 0, c, (int)graph[v].size() -
    1});
  }
};

void use_example() {
  Dinic dinic;
  for (int i = 0; i < m; ++i) {
    int v, u, c;
    cin >> v >> u >> c;
    v--, u--;
    dinic.addedge(v, u, c);
  }
  dinic.run(s, t);

  long long maxflow = 0;
  for (auto &e : dinic.graph[s])
    maxflow += e.f;

  vector<int> cut;
  for (int i = 0; i < m; i++) {
    auto &e = edges[i];
    if (dinic.vis[e.v] != dinic.vis[e.u])
      cut.push_back(i);
  }
}
```

2.7 КСС

```
void dfs1(int v, vector<char> &used, vector<int> &
    topsort) {
  used[v] = 1;
  for (auto u : g[v]) {
    if (!used[u])
      dfs1(u, used, topsort);
  }
```

```
  topsort.push_back(v);
}

void dfs2(int v, int col, vector<int> &comp) {
  comp[v] = col;
  for (auto u : rg[v]) {
    if (comp[u] == -1)
      dfs2(u, col, comp);
  }
}

signed main() {
  vector<int> topsort;
  topsort.reserve(n);
  vector<char> used(n, 0);
  for (int v = 0; v < n; ++v) {
    if (!used[v])
      dfs1(v, used, topsort);
  }
  reverse(all(topsort));
  int cc = 0;
  vector<int> comp(n, -1);
  for (int i = 0; i < n; ++i) {
    if (comp[topsort[i]] == -1)
      dfs2(topsort[i], cc++, comp);
  }
}
```

2.8 Минкост (Джонсон)

```
using cost_t = ll;
using flow_t = int;

const int MAXN = 10000;
const int MAXM = 25000 * 2;
const cost_t INFw = 1e12;
const flow_t INFf = 10;

struct Edge {
  int v, u;
  flow_t f, c;
  cost_t w;
};

Edge edg[MAXM];
int esz = 0;
vector<int> graph[MAXN];
ll dist[MAXN];
ll pot[MAXN];
int S, T;
int NUMV;
int pre[MAXN];
bitset<MAXN> inQ;

flow_t get_flow() {
  int v = T;
  if (pre[v] == -1)
    return 0;
  flow_t f = INFf;
  do {
    int ei = pre[v];
    Edge &e = edg[ei];
    f = min(f, e.c - e.f);
    if (f == 0)
      return 0;
    v = e.v;
  } while (v != S);
  v = T;
  do {
    int ei = pre[v];
    edg[ei].f += f;
    edg[ei ^ 1].f -= f;
    v = edg[ei].v;
  } while (v != S);
  return f;
}

void spfa() {
  fill(dist, dist + NUMV, INFw);
  dist[S] = 0;
  deque<int> Q = {S};
  inQ[S] = true;
  while (!Q.empty()) {
    int v = Q.front();
    Q.pop_front();
    inQ[v] = false;
    cost_t d = dist[v];
    for (int ei : graph[v]) {
      Edge &e = edg[ei];
      if (e.f == e.c)
        continue;
      cost_t w = e.w + pot[v] - pot[e.u];
      if (dist[e.u] <= d + w)
        continue;
      pre[e.u] = ei;
      dist[e.u] = d + w;
      if (!inQ[e.u]) {
        inQ[e.u] = true;
        Q.push_back(e.u);
      }
    }
  }
  for (int i = 0; i < NUMV; ++i)
    pot[i] += dist[i];
}

cost_t mincost() {
  spfa(); // pot[i] = 0 // or ford_bellman
  flow_t f = 0;
  while (true) {
    flow_t ff = get_flow();
    if (ff == 0)
      break;
    f += ff;
    spfa(); // or dijkstra
  }
  cost_t res = 0;
  for (int i = 0; i < esz; ++i)
    res += edg[i].f * edg[i].w;
  res /= 2;
  return res;
}

void add_edge(int v, int u, int c, int w) {
  edg[esz] = {v, u, 0, c, w};
  edg[esz + 1] = {u, v, 0, 0, -w};
  graph[v].push_back(esz);
  graph[u].push_back(esz + 1);
  esz += 2;
}

signed main() {
  ios_base::sync_with_stdio(false);
  cin.tie(nullptr);
  int n, m;
  cin >> n >> m;
  S = 0;
  T = n - 1;
  NUMV = n;
  for (int i = 0; i < m; ++i) {
    int v, u, c, w;
    cin >> v >> u >> c >> w;
    v--, u--;
    add_edge(v, u, c, w);
  }
  cost_t ans = mincost();
  cout << ans;
}
```

2.9 Мосты

```
// graph[v][i] = {u, edge_i}
void dfs(int v, int pi = -1) {
  vis[v] = 1;
  up[v] = tin[v] = timer++;
  for (auto [u, ei] : g[v]) {
    if (!vis[u]) {
      dfs(u, ei);
      up[v] = min(up[v], up[u]);
    } else if (ei != pi)
      up[v] = min(up[v], tin[u]);
    if (up[u] > tin[v]) {
      bridges.emplace_back(v, u);
      is_bridge[ei] = 1;
    }
  }
}
```

2.10 Паросочетания

```
int dfs(int v, int c) {
  if (used[v] == c) return 0;
  used[v] = c;
  for (auto u : g[v]) {
    if (res[u] == -1) {
```

```
      res[u] = v;
      return 1;
    }
  }
  for (auto u : g[v]) {
    if (dfs(res[u], c)) {
      res[u] = v;
      return 1;
    }
  }
  return 0;
}

signed main() {
  // n - в левой доле, m - в правой
  fill(res, res + m, -1);
  for (int i = 0; i < n; ++i) {
    ans += dfs(i, i + 1);
  }
}
```

2.11 Точки сочленения

```
void dfs(int v, int par) {
  vis[v] = 1;
  up[v] = tin[v] = timer++;
  int child = 0;
  for (auto u : g[v]) {
    if (!vis[u]) {
      dfs(u, v);
      up[v] = min(up[v], up[u]);
      if (up[u] >= tin[v] && par != -1) {
        points.insert(v);
      }
      child++;
    } else if (u != par) {
      up[v] = min(up[v], tin[u]);
    }
  }
  if (par == -1 && child >= 2) {
    points.insert(v);
  }
}
```

2.12 Эдмондс-Карп

```
struct edge {
  int v, f, c, ind;
};

vector<edge> g[MAXN];

bool bfs(int start, int final, int W) {
  vector<int> d(MAXN, INF);
  vector<pair<int, int>> pred(MAXN);
  d[start] = 0;
  deque<int> q = {start};
  while (!q.empty()) {
    int v = q.front();
    q.pop_front();
    for (int i = 0; i < (int) g[v].size(); i++) {
      auto e = g[v][i];
      if (e.f + W <= e.c && d[e.v] > d[v] + 1) {
        d[e.v] = d[v] + 1;
        pred[e.v] = {v, i};
        q.push_back(e.v);
      }
    }
  }
  if (d[final] == INF) {
    return false;
  }
  int v = final;
  int x = INF;
  while (v != start) {
    int ind = pred[v].second;
    v = pred[v].first;
    x = min(x, g[v][ind].c - g[v][ind].f);
  }
  v = final;
  while (v != start) {
    int ind = pred[v].second;
    v = pred[v].first;
    g[v][ind].f += x;
    g[g[v][ind].v][g[v][ind].ind].f -= x;
  }
  return true;
}

signed main() {
  int n, m;
  for (int i = 0; i < m; i++) {
    int u, v, c;
    cin >> u >> v >> c;
    g[u].push_back({v, 0, c, (int) g[v].size()});
    g[v].push_back({u, 0, 0, (int) g[u].size() - 1});
  }
  int start = 1, final = n;
  int W = (1 << 30);
  do {
    while (bfs(start, final, W));
    W /= 2;
  } while (W >= 1);
  int res = 0;
  for (auto e : g[start]) {
    res += e.f;
  }
}
```

2.13 Эйлеров цикл

```
// Эйлеров путь/цикл в компоненте связности s. Возвращае
    т индексы рёбер. Если пути/цикла нет, то алгос найд
    ёт фигню.
// Если неориентированный граф, то edges[ei] и edges[ei
    ^ 1] - обратные друг к другу рёбра.
// edges[graph[v][i]] = {v, u}
vector<int> eulerpath1(int s, vector<vector<int>> &graph
    , vector<pair<int, int>> &edges, vector<char> &used
    , vector<int> &start) {
  vector<pair<int, int>> st = {{-1, s}};
  vector<int> res;
  while (!st.empty()) {
    auto [ei, v] = st.back();
    while (start[v] < graph[v].size() && used[graph[v][
    start[v]]])
      start[v]++;
    if (start[v] == graph[v].size()) {
      if (ei != -1) res.push_back(ei);
      st.pop_back();
    } else {
      int ej = graph[v][start[v]++];
      used[ej] = true;
      used[ej ^ 1] = true; // Удалить если ориент. граф
      st.emplace_back(ej, edges[ej].second);
    }
  }
  reverse(all(res));
  return res;
}

vector<char> used(edges.size(), false);
vector<int> start(graph.size(), 0);
for (int v = 0; v < graph.size(); ++v) {
  // Если ориентированный граф, второе условие заменить
    на cnt_in[v] >= cnt_out[v]
  if (start[v]==graph[v].size() || graph[v].size()%2==0)
    continue;
  auto path = eulerpath1(v, graph, edges, used, start);
}
for (int v = 0; v < graph.size(); ++v) {
  if (start[v] == graph[v].size())
    continue;
  auto cycle = eulerpath1(v, graph, edges, used, start);
}
```

3 ДП

3.1 CHT

```
struct line {
    int k, b;
    int eval(int x) {
      return k * x + b;
    }
};
struct part {
    line a;
    double x;
};
double intersection(line a, line b) {
  return (a.b - b.b) / (double) (b.k - a.k);
}
struct ConvexHull {
```

```
    // for min: k decreasing (non-increasing)
    // for max: k increasing (non-decreasing)
    vector<part> st;

    void add(line a) {
      if (!st.empty() && st.back().a.k == a.k) {
        if (st.back().a.b < a.b) st.pop_back(); // for
    max
//        if (st.back().a.b > a.b) st.pop_back(); // for
     min
        else return;
      }
      while (st.size() > 1 &&
             intersection(st[st.size() - 2].a, a) <= st[
    st.size() - 2].x)
        st.pop_back();
      if (!st.empty()) st.back().x = intersection(st.
    back().a, a);
      st.push_back({a, INFINITY}); // C++ define
    }

    int get_val(int x) {
      if (st.empty()) {
        return -INF; // min possible value, for max
//        return INF; // max possible value, for min
      }
      int l = -1, r = (int) st.size() - 1;
      while (r - l > 1) {
        int m = (l + r) / 2;
        if (st[m].x < x) l = m;
        else r = m;
      }
      return st[r].a.eval(x);
    }
};
```

3.2 Li Chao

```
// MAXIMUM
struct Line {
  int k, b;

  int f(int x) {
    return k * x + b;
  }
};

struct ST {
  vector<Line> st;

  ST(int n) {
    Line ln = {0LL, -INF};
    st.resize(4 * n, ln);
  }

  void upd(int i, int l, int r, Line ln) {
    int child = 1;
    Line ln1 = ln;
    int m = (l + r) / 2;
    if (ln.f(m) > st[i].f(m)) {
      if (ln.k < st[i].k) {
        child = 2;
      }
      ln1 = st[i];
      st[i] = ln;
    } else {
      if (st[i].k < ln.k) {
        child = 2;
      }
    }
    if (l + 1 < r) {
      if (child == 1) {
        upd(i * 2 + 1, l, m, ln1);
      } else {
        upd(i * 2 + 2, m, r, ln1);
      }
    }
  }

  int res(int i, int l, int r, int x) {
    if (l + 1 == r) {
      return st[i].f(x);
    }
    int m = (l + r) / 2;
    int val = st[i].f(x);
    if (x < m) {
      val = max(val, res(i * 2 + 1, l, m, x));
    } else {
      val = max(val, res(i * 2 + 2, m, r, x));
    }
    return val;
  }
};
```

3.3 SOS-dp

```
// dp initial fill, a[] is given array, mb extra zeros
for (int i = 0; i < (1 << N); i++) {
  dp[i] = a[i];
}

// Classic SOS-dp, goal: dp[mask] = \sum a[submasks of
    mask]
for (int i = 0; i < N; i++) {
  for (int mask = 0; mask < (1 << N); mask++) {
    if ((mask >> i) & 1) {
      dp[mask] += dp[mask ^ (1 << i)];
    }
  }
}

// Overmasks SOS-dp, goal: dp[mask] = \sum a[overmasks
    of mask]
for (int i = 0; i < N; i++) {
  for (int mask = (1 << N) - 1; mask >= 0; mask--) {
    if (((mask >> i) & 1) == 0) {
      dp[mask] += dp[mask ^ (1 << i)];
    }
  }
}

// to inverse SOS-dp (restore original array by SOS-dp
    array):
// use same code, but -= instead of += in dp transitions
```

3.4 НВП

```
// 0-indexation ({a_0, ..., a_{n-1}})
vector<int> lis(vector<int> a) {
  int n = (int) a.size();
  vector<int> dp(n + 1, INF), ind(n + 1), par(n + 1); //
     INF > all a[i] required
  ind[0] = -INF;
  dp[0] = -INF;
  for (int i = 0; i < n; i++) {
    int l = upper_bound(dp.begin(), dp.end(), a[i]) - dp
    .begin();
    if (dp[l - 1] < a[i] && a[i] < dp[l]) {
      dp[l] = a[i];
      ind[l] = i;
      par[i] = ind[l - 1];
    }
  }
  vector<int> ans; // exact values
  for (int l = n; l >= 0; l--) {
    if (dp[l] < INF) {
      int pi = ind[l];
      ans.resize(l);
      for (int i = 0; i < l; i++) {
        ans[i] = a[pi]; // =pi if need indices
        pi = par[pi];
      }
      reverse(ans.begin(), ans.end());
      return ans;
    }
  }
  return {};
}
```

3.5 НОВП

```
// 1-indexation ({0, a_1, ..., a_n}, {0, b_1, ..., b_m})
vector<int> lcis(vector<int> a, vector<int> b) {
  int n = (int) a.size() - 1, m = (int) b.size() - 1;
  vector<int> dp(m + 1), dp2(m + 1), par(m + 1);
  for (int i = 1; i <= n; i++) {
    int best = 0, best_idx = 0;
    for (int j = 1; j <= m; j++) {
      dp2[j] = dp[j];
      if (a[i] == b[j]) {
        dp2[j] = max(dp2[j], best + 1);
        par[j] = best_idx;
      }
      if (a[i] > b[j] && best < dp[j]) {
        best = dp[j];
```

```
        best_idx = j;
      }
    }
    swap(dp, dp2);
  }
  int pj = 0;
  for (int j = 1; j <= m; j++) {
    if (dp[pj] < dp[j]) {
      pj = j;
    }
  }
  vector<int> ans; // exact values
  while (pj > 0) {
    ans.push_back(b[pj]);
    pj = par[pj];
  }
  reverse(ans.begin(), ans.end());
  return ans;
}
```

4 Деревья

4.1 Centroid

```
int levels[MAXN];
int szs[MAXN];
int cent_par[MAXN];

int calcsizes(int v, int p) {
  int sz = 1;
  for (int u : graph[v]) {
    if (u != p && levels[u] == 0)
      sz += calcsizes(u, v);
  }
  return szs[v] = sz;
}

void centroid(int v, int lvl=1, int p=-1) {
  int sz = calcsizes(v, -1);
  int nxt = v, prv;
  while (nxt != -1) {
    prv = v, v = nxt, nxt = -1;
    for (int u : graph[v]) {
      if (u != prv && levels[u] == 0 && szs[u] * 2 >= sz
    )
        nxt = u;
    }
  }
  levels[v] = lvl;
  cent_par[v] = p;
  for (int u : graph[v]) {
    if (levels[u] == 0)
      centroid(u, lvl + 1, v);
  }
  // calc smth for centroid v
}
```

4.2 HLD

```
int par[MAXN], sizes[MAXN];
int pathup[MAXN];
int tin[MAXN], tout[MAXN];
int timer;

int dfs1_hld(int v, int p) {
  par[v] = p;
  int sz = 1;
  for (int i = 0; i < graph[v].size(); ++i) {
    int u = graph[v][i];
    if (u == p) {
      swap(graph[v][i--], graph[v].back());
      graph[v].pop_back();
      continue;
    }
    sz += dfs1_hld(u, v);
  }
  return sizes[v] = sz;
}

void dfs2_hld(int v, int up) {
  tin[v] = timer++;
  pathup[v] = up;
  if (graph[v].empty()) {
    tout[v] = timer;
    return;
  }
  for (int i = 1; i < graph[v].size(); ++i) {
    if (sizes[graph[v][i]] > sizes[graph[v][0]])
      swap(graph[v][i], graph[v][0]);
  }
  dfs2_hld(graph[v][0], up);
  for (int i = 1; i < graph[v].size(); ++i)
    dfs2_hld(graph[v][i], graph[v][i]);
  tout[v] = timer;
}

bool is_ancestor(int v, int p) {
  return tin[p] <= tin[v] && tout[v] <= tout[p];
}

// get_hld полностью аналогичный
void update_hld(int v, int u, int ARG) {
  for (int _ = 0; _ < 2; ++_) {
    while (!is_ancestor(u, pathup[v])) {
      int vup = pathup[v];
      ST.update(0, 0, timer, tin[vup], tin[v] + 1, ARG);
      v = par[vup];
    }
    swap(v, u);
  }
  if (tin[v] > tin[u])
    swap(v, u);
  // v = lca
  ST.update(0, 0, timer, tin[v], tin[u] + 1, ARG);
}

signed main() {
  dfs1_hld(0, -1);
  dfs2_hld(0, 0);
  ST.build();
  // your code here
}
```

4.3 Link-cut

```
struct Node {
  Node *ch[2];
  Node *p;
  bool rev;
  int sz;

  Node() {
    ch[0] = nullptr;
    ch[1] = nullptr;
    p = nullptr;
    rev = false;
    sz = 1;
  }
};

int size(Node *v) {
  return (v ? v->sz : 0);
}

int chnum(Node *v) {
  return v->p->ch[1] == v;
}

bool isroot(Node *v) {
  return v->p == nullptr || v->p->ch[chnum(v)] != v;
}

void push(Node *v) {
  if (v->rev) {
    if (v->ch[0])
      v->ch[0]->rev ^= 1;
    if (v->ch[1])
      v->ch[1]->rev ^= 1;
    swap(v->ch[0], v->ch[1]);
    v->rev = false;
  }
}

void pull(Node *v) {
  v->sz = size(v->ch[1]) + size(v->ch[0]) + 1;
}

void attach(Node *v, Node *p, int num) {
  if (p)
    p->ch[num] = v;
  if (v)
    v->p = p;
}

void rotate(Node *v) {
  Node *p = v->p;
```

```
  push(p);
  push(v);
  int num = chnum(v);
  Node *u = v->ch[1 - num];
  if (!isroot(v->p))
    attach(v, p->p, chnum(p));
  else
    v->p = p->p;
  attach(u, p, num);
  attach(p, v, 1 - num);
  pull(p);
  pull(v);
}

void splay(Node *v) {
  push(v);
  while (!isroot(v)) {
    if (!isroot(v->p)) {
      if (chnum(v) == chnum(v->p))
        rotate(v->p);
      else
        rotate(v);
    }
    rotate(v);
  }
}

void expose(Node *v) {
  splay(v);
  v->ch[1] = nullptr;
  pull(v);
  while (v->p != nullptr) {
    Node *p = v->p;
    splay(p);
    attach(v, p, 1);
    pull(p);
    splay(v);
  }
}

void makeroot(Node *v) {
  expose(v);
  v->rev ^= 1;
  push(v);
}

void link(Node *v, Node *u) {
  makeroot(v);
  makeroot(u);
  u->p = v;
}

void cut(Node *v, Node *u) {
  makeroot(u);
  makeroot(v);
  v->ch[1] = nullptr;
  u->p = nullptr;
}

int get(Node *v, Node *u) {
  makeroot(u);
  makeroot(v);
  Node *w = u;
  while (!isroot(w))
    w = w->p;
  return (w == v ? size(v) - 1 : -1);
}

const int MAXN = 100010;
Node *nodes[MAXN];

int main() {
  int n, q;
  cin >> n >> q;
  for (int i = 0; i < n; ++i)
    nodes[i] = new Node();
  while (q--) {
    string s;
    int a, b;
    cin >> s >> a >> b;
    a--, b--;
    if (s[0] == 'g')
      cout << get(nodes[a], nodes[b]) << '\n';
    else if (s[0] == 'l')
      link(nodes[a], nodes[b]);
    else
      cut(nodes[a], nodes[b]);
  }
}
```

5 Другое

5.1 Fast mod

```
// Быстрое взятие по НЕ константному модулю (в 2-4 раза
    быстрее)
struct FastMod {
  ull b, m;
  FastMod(ull b) : b(b), m(-1ULL / b) {}
  ull mod(ull a) const {
      ull r = a - (ull)((__uint128_t(m) * a) >> 64) * b;
      return r;  // r in [0, 2b) // ≈ x3.5 speed
      return r >= b ? r - b : r; // ≈ x3 speed
  }
}; // Usage:
// FastMod F(m);
// ull x_mod_m = F.mod(x);
```

5.2 Slope trick

```
// Дан массив a_n. Сделать минимальное кол-во ±1, чтобы
    a_n стал неубывающим.

void solve() {
    int n;
    cin >> n;
    vector<int> a(n);
    for (int i = 0; i < n; i++) {
        cin >> a[i];
    }
    int ans = 0;
    multiset<int> now;
    for (int i = 0; i < n; i++) {
        now.insert(a[i]);
        ans += (*now.rbegin() - a[i]);
        now.erase(now.find(*now.rbegin()));
        now.insert(a[i]);
    }
    cout << ans << '\n';
}
```

5.3 attribute_packed

```
struct Kek {
  int a;
  char b;
  // char[3]
  int c;
} __attribute__((packed));
// sizeof = 9 (instead of 12)
```

5.4 custom_bitset

```
// __builtin_ctz = Count Trailing Zeroes
// __builtin_clz = Count Leading Zeroes
// both are UB in gcc when pass 0
struct custom_bitset {
  vector<uint64_t> bits;
  int b, n;

  custom_bitset(int _b = 0) {
    init(_b);
  }
  void init(int _b) {
    b = _b, n = (b + 63) / 64;
    bits.assign(n, 0);
  }
  void clear() {
    b = n = 0;
    bits.clear();
  }
  void reset() {
    bits.assign(n, 0);
  }
  void _clean() {
    // Reset all bits after `b`.
    if (b != 64 * n)
      bits.back() &= (1LLU << (b - 64 * (n - 1))) - 1;
  }
  bool get(int i) const {
    return bits[i / 64] >> (i % 64) & 1;
  }
  void set(int i, bool value) {
    // assert(0 <= i && i < b);
```

```
  bits[i / 64] &= ~(1LLU << (i % 64));
  bits[i / 64] |= uint64_t(value) << (i % 64);
}

// Simulates `bs |= bs << shift;`
// `|=` can be replaced with `^=`, `&=`, `=`
void or_shift_left(int shift) {
  int div = shift / 64, mod = shift % 64;
  if (mod == 0) {
    for (int i = n - 1; i >= div; i--)
      bits[i] |= bits[i - div];
  } else {
    for (int i = n - 1; i >= div + 1; i--)
      bits[i] |= bits[i - (div + 1)] >> (64 - mod) |
   bits[i - div] << mod;
    if (div < n)
      bits[div] |= bits[0] << mod;
  }
  // if `&=`, `=`
  //fill(bits.begin(), bits.begin() + min(div, n), 0);
  _clean();
}

// Simulates `bs |= bs >> shift;`
// `|=` can be replaced with `^=`, `&=`, `=`
void or_shift_right(int shift) {
  int div = shift / 64, mod = shift % 64;
  if (mod == 0) {
    for (int i = div; i < n; i++)
      bits[i - div] |= bits[i];
  } else {
    for (int i = 0; i < n - (div + 1); i++)
      bits[i] |= bits[i + (div + 1)] << (64 - mod) |
   bits[i + div] >> mod;
    if (div < n)
      bits[n - div - 1] |= bits[n - 1] >> mod;
  }
  // if `&=`, `=`
  //fill(bits.end() - min(div, n), bits.end(), 0);
  _clean();
}

// find min j, that j >= i and bs[j] = 1;
int find_next(int i) {
  if (i >= b) return b;
  int div = i / 64, mod = i % 64;
  auto x = bits[div] >> mod;
  if (x != 0)
    return i + __builtin_ctzll(x);
  for (auto k = div + 1; k < n; ++k) {
    if (bits[k] != 0)
      return 64 * k + __builtin_ctzll(bits[k]);
  }
  return b;
}

// `|=` can be replaced with `&=`, `^=`
custom_bitset &operator|=(const custom_bitset &other){
  // assert(b == other.b);
  for (int i = 0; i < n; i++)
    bits[i] |= other.bits[i];
  return *this;
 }
};
```

5.5 ordered_set

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>

using namespace __gnu_pbds;

typedef tree<int, null_type, less<>, rb_tree_tag,
    tree_order_statistics_node_update> ordered_set;

//st.find_by_order(index);
//st.order_of_key(key);
```

5.6 pragma

```
#pragma GCC optimize("Ofast,fast-math,unroll-loops,no-
    stack-protector,inline")
#pragma GCC target("sse,sse2,sse3,ssse3,sse4,sse4.1,sse4
    .2,avx,avx2,abm,mmx,popcnt")
```

5.7 Аллокатор Копелиовича

```
// Код вставить до инклюдов

#include <cassert>

const int MAX_MEM = 1e8; // ~100mb
int mpos = 0;
char mem[MAX_MEM];

inline void *operator new(std::size_t n) {
  mpos += n;
//  assert(mpos <= MAX_MEM);
  return (void *)(mem + mpos - n);
}

inline void operator delete(void *) noexcept {} // must
    have!
inline void operator delete(void *, std::size_t)
    noexcept {} // fix!!
```

5.8 Альфа-бета отсечение

```
int alphabeta(int player, int alpha, int beta, int depth
    ) {
  if (depth == 0) {
    // return current position score
  }
  if (player == 0) { // maximization player
    int val = -INF;
    for (auto move : possible_moves) {
      val = max(val, alphabeta(1, alpha, beta, depth -
     1));
      if (val > beta) break;
      alpha = max(alpha, val);
    }
    return val;
  } else {
    int val = INF;
    for (auto move : possible_moves) {
      val = min(val, alphabeta(0, alpha, beta, depth -
     1));
      if (val < alpha) break;
      beta = min(beta, val);
    }
    return val;
  }
}
```

5.9 Отжиг

```
const double lambda = 0.999;
double temprature = 1;
mt19937 rnd(777);

double gen_rand_01() {
  return rnd() / (double) UINT32_MAX;
}
bool f(int delta) {
  return exp(-delta / temprature) > gen_rand_01();
}
void make_change() {
  temprature *= lambda;
  // calc change score
  if (change_score <= 0 || f(change_score)) {
    score += change_score;
    // make change
  }
}
```

6 Математика

6.1 AdivB cmp CdivD

```
char sign(ll x) {
  return x < 0 ? -1 : x > 0;
}

// -1 = less, 0 = equal, 1 = greater
char compare(ll a, ll b, ll c, ll d) {
  if (a / b != c / d)
    return sign(a / b - c / d);
  a = a % b;
  c = c % d;
  if (a == 0)
    return -sign(c) * sign(d);
```

```
  if (c == 0)
    return sign(a) * sign(b);
  return compare(d, c, b, a) * sign(a) * sign(b) * sign(
    c) * sign(d);
}
```

6.2 FFT mod

```
const int MOD = 998244353; // 7·17·2^23 + 1
const int G = 3;
//const int MOD = 7340033; // 7·2^20 + 1
//const int G = 5;
//const int MOD = 469762049; // 7·2^26 + 1
//const int G = 30;
const int MAXLOG = 23;
int W[(1 << MAXLOG) + 10];
bool nttinit = false;
vector<int> pws;

// int add(), int sub(), int mul(),
// int binpow(), int inv()

void initNTT() {
  if (nttinit) return;
  nttinit = true;
  assert((MOD - 1) % (1 << MAXLOG) == 0);
  pws.push_back(binpow(G, (MOD - 1) / (1 << MAXLOG)));
  for (int i = 0; i < MAXLOG - 1; ++i)
    pws.push_back(mul(pws.back(), pws.back()));
  assert(pws.back() == MOD - 1);
  W[0] = 1;
  for (int i = 1; i < (1 << MAXLOG); ++i)
    W[i] = mul(W[i - 1], pws[0]);
}

void ntt(int n, vector<int> &a, bool rev) {
  initNTT();
  int lg = 31 - __builtin_clz(n);
  vector<int> rv(n);
  for (int i = 1; i < n; ++i) {
    rv[i] = (rv[i >> 1] >> 1) ^ ((i & 1) << (lg - 1));
    if (rv[i] > i) swap(a[i], a[rv[i]]);
  }
  int num = MAXLOG - 1;
  for (int len = 1; len < n; len *= 2, --num) {
    for (int i = 0; i < n; i += 2 * len) {
      for (int j = 0; j < len; ++j) {
        int u = a[i + j];
        int v = mul(W[j << num], a[i + j + len]);
        a[i + j] = add(u, v);
        a[i + j + len] = sub(u, v);
      }
    }
  }
  if (rev) {
    int invn = binpow(n, MOD - 2);
    for (int i = 0; i < n; ++i) a[i] = mul(a[i], invn);
    reverse(a.begin() + 1, a.end());
  }
}

vector<int> conv(vector<int> a, vector<int> b) {
  if (a.empty() || b.empty())
    return {};
  int lg = 32 - __builtin_clz(a.size() + b.size() - 1);
  int n = 1 << lg;
  a.resize(n);
  b.resize(n);
  ntt(n, a, false);
  ntt(n, b, false);
  for (int i = 0; i < n; ++i)
    a[i] = mul(a[i], b[i]);
  ntt(n, a, true);
  while (a.size() > 1 && a.back() == 0)
    a.pop_back();
  return a;
}

vector<int> add(vector<int> a, vector<int> b) {
  a.resize(max(a.size(), b.size()));
  for (int i = 0; i < (int) b.size(); ++i)
    a[i] = add(a[i], b[i]);
  return a;
}

vector<int> sub(vector<int> a, vector<int> b) {
  a.resize(max(a.size(), b.size()));
  for (int i = 0; i < (int) b.size(); ++i)
    a[i] = sub(a[i], b[i]);
  return a;
}

vector<int> inv(const vector<int> &a, int need) {
  vector<int> b = {inv(a[0])};
  while ((int) b.size() < need) {
    vector<int> a1 = a;
    int m = b.size();
    a1.resize(min((int) a1.size(), 2 * m));
    b = conv(b, sub({2}, conv(a1, b)));
    b.resize(2 * m);
  }
  b.resize(need);
  return b;
}

vector<int> div(vector<int> a, vector<int> b) {
  if (count(all(a), 0) == a.size())
    return {0};
  assert(a.back() != 0 && b.back() != 0);
  int n = a.size() - 1;
  int m = b.size() - 1;
  if (n < m)
    return {0};
  reverse(all(a));
  reverse(all(b));
  a.resize(n - m + 1);
  b.resize(n - m + 1);
  vector<int> c = inv(b, b.size());
  vector<int> q = conv(a, c);
  q.resize(n - m + 1);
  reverse(all(q));
  return q;
}

vector<int> mod(vector<int> a, vector<int> b) {
  auto res = sub(a, conv(b, div(a, b)));
  while (res.size() > 1 && res.back() == 0)
    res.pop_back();
  return res;
}

vector<int> multipoint(vector<int> a, vector<int> x) {
  int n = x.size();
  vector<vector<int>> tree(2 * n);
  for (int i = 0; i < n; ++i)
    tree[i + n] = {x[i], MOD - 1};
  for (int i = n - 1; i; --i)
    tree[i] = conv(tree[2 * i], tree[2 * i + 1]);
  tree[1] = mod(a, tree[1]);
  for (int i = 2; i < 2 * n; ++i)
    tree[i] = mod(tree[i >> 1], tree[i]);
  vector<int> res(n);
  for (int i = 0; i < n; ++i)
    res[i] = tree[i + n][0];
  return res;
}

vector<int> deriv(vector<int> a) {
  for (int i = 1; i < (int) a.size(); ++i)
    a[i - 1] = mul(i, a[i]);
  a.back() = 0;
  if (a.size() > 1)
    a.pop_back();
  return a;
}

vector<int> integ(vector<int> a) {
  a.push_back(0);
  for (int i = (int) a.size() - 1; i; --i)
    a[i] = mul(a[i - 1], inv(i));
  a[0] = 0;
  return a;
}

vector<int> log(vector<int> a, int n) {
  assert(a[0] == 1);
  auto res = integ(conv(deriv(a), inv(a, n)));
  res.resize(n);
  return res;
}

vector<int> exp(vector<int> a, int need) {
  assert(a[0] == 0);
  vector<int> b = {1};
  while ((int) b.size() < need) {
    vector<int> a1 = a;
    int m = b.size();
    a1.resize(min((int) a1.size(), 2 * m));
```

```
    a1[0] = add(a1[0], 1);
    b = conv(b, sub(a1, log(b, 2 * m)));
    b.resize(2 * m);
  }
  b.resize(need);
  return b;
}
```

6.3 FFT

```
const double PI = acos(-1);
const int LOG = 20;
const int MAXN = 1 << LOG;

//using comp = complex<double>;
struct comp {
  double x, y;
  comp() : x(0), y(0) {}
  comp(double x, double y) : x(x), y(y) {}
  comp(int x) : x(x), y(0) {}
  comp operator+(const comp &o) const { return {x + o.x,
     y + o.y}; }
  comp operator-(const comp &o) const { return {x - o.x,
     y - o.y}; }
  comp operator*(const comp &o) const { return {x * o.x
    - y * o.y, x * o.y + y * o.x}; }
  comp operator/(const int k) const { return {x / k, y /
     k}; }
  comp conj() const { return {x, -y}; }
};

comp OMEGA[MAXN + 10];
int tail[MAXN + 10];

comp omega(int n, int k) {
  return OMEGA[MAXN / n * k];
}

int gettail(int x, int lg) {
  return tail[x] >> (LOG - lg);
}

void calcomega() {
  for (int i = 0; i < MAXN; ++i) {
    double x = 2 * PI * i / MAXN;
    OMEGA[i] = {cos(x), sin(x)};
  }
}

void calctail() {
  tail[0] = 0;
  for (int i = 1; i < MAXN; ++i)
    tail[i] = (tail[i >> 1] >> 1) | ((i & 1) << (LOG -
    1));
}

void fft(vector<comp> &A, int lg) {
  int n = A.size();
  for (int i = 0; i < n; ++i) {
    int j = gettail(i, lg);
    if (i < j)
      swap(A[i], A[j]);
  }
  for (int len = 2; len <= n; len *= 2) {
    for (int i = 0; i < n; i += len) {
      for (int j = 0; j < len / 2; ++j) {
        auto v = A[i + j];
        auto u = A[i + j + len / 2] * omega(len, j);
        A[i + j] = v + u;
        A[i + j + len / 2] = v - u;
      }
    }
  }
}

void fft2(vector<comp> &A, vector<comp> &B, int lg) {
  int n = A.size();
  vector<comp> C(n);
  for (int i = 0; i < n; ++i) {
    C[i].x = A[i].x;
    C[i].y = B[i].x;
  }
  fft(C, lg);
  C.push_back(C[0]);
  for (int i = 0; i < n; ++i) {
    A[i] = (C[i] + C[n - i].conj()) / 2;
    B[i] = (C[i] - C[n - i].conj()) / 2 * comp(0, -1);
  }
}

void invfft(vector<comp> &A, int lg) {
  int n = 1 << lg;
  fft(A, lg);
  for (auto &el : A)
    el = el / n;
  reverse(A.begin() + 1, A.end());
}

vector<int> mul(vector<int> &a, vector<int> &b) {
  if (a.empty() || b.empty())
    return {};
  int lg = 32 - __builtin_clz(a.size() + b.size() - 1);
  int n = 1 << lg;
  vector<comp> A(n, 0), B(n, 0);
  for (int i = 0; i < a.size(); ++i)
    A[i] = a[i];
  for (int i = 0; i < b.size(); ++i)
    B[i] = b[i];
//  fft2(A, B, lg);
  fft(A, lg);
  fft(B, lg);
  for (int i = 0; i < n; ++i)
    A[i] = A[i] * B[i];
  invfft(A, lg);
  vector<int> c(n);
  for (int i = 0; i < n; ++i)
    c[i] = round(A[i].x);
  while (!c.empty() && c.back() == 0)
    c.pop_back();
  return c;
}

signed main() {
  calcomega(); // НЕ ЗАБЫТЬ
  calctail(); // НЕ ЗАБЫТЬ
  // your code here
}
```

6.4 Floor Sum

```
int floor_sum(int n, int d, int m, int a) {
    // sum_{i=0}^{n-1} floor((a + i*m)/d), only non-
    negative integers!
    int ans = 0;
    ans += (n * (n - 1) / 2) * (m / d);
    m %= d;
    ans += n * (a / d);
    a %= d;
    int l = m * n + a;
    if (l >= d)
        ans += floor_sum(l / d, m, d, l % d);
    return ans;
}
```

6.5 OR XOR AND Свёртки

```
vector<int> or_conv(int n, vector<int> a, vector<int> b)
     { // |a| = |b| = 2^n
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < (1 << n); j++) {
            if ((j >> i) & 1) {
                a[j] = (a[j] + a[j ^ (1 << i)]) % MOD;
                b[j] = (b[j] + b[j ^ (1 << i)]) % MOD;
            }
        }
    }
    vector<int> c(1 << n);
    for (int i = 0; i < (1 << n); i++) {
        c[i] = (a[i] * b[i]) % MOD;
    }
    for (int i = n - 1; i >= 0; i--) {
        for (int j = (1 << n) - 1; j >= 0; j--) {
            if ((j >> i) & 1) {
                c[j] = (c[j] - c[j ^ (1 << i)] + MOD) %
    MOD;
            }
        }
    }
    return c;
}

vector<int> and_conv(int n, vector<int> a, vector<int> b
    ) { // |a| = |b| = 2^n
    for (int i = 1; i < (1 << n); i *= 2) {
        for (int j = 0; j < (1 << n); j += i * 2) {
            for (int k = 0; k < i; k++) {
```

```
                a[j + k] = (a[j + k] + a[i + j + k]) %
    MOD;
                b[j + k] = (b[j + k] + b[i + j + k]) %
    MOD;
            }
        }
    }
    vector<int> c(1 << n);
    for (int i = 0; i < (1 << n); i++)
        c[i] = (a[i] * b[i]) % MOD;
    for (int i = 1; i < (1 << n); i *= 2) {
        for (int j = 0; j < (1 << n); j += i * 2) {
            for (int k = 0; k < i; k++) {
                c[j + k] = (c[j + k] - c[i + j + k] +
    MOD) % MOD;
            }
        }
    }
    return c;
}

const int inv2 = (MOD + 1) / 2;
vector<int> xor_conv(int n, vector<int> a, vector<int> b
    ) { // |a| = |b| = 2^n
    for (int i = 1; i < (1 << n); i *= 2) {
        for (int j = 0; j < (1 << n); j++) {
            if ((j & i) == 0) {
                int x = a[j], y = a[j | i];
                a[j] = (x + y) % MOD, a[j | i] = (x - y
    + MOD) % MOD;
                x = b[j], y = b[j | i];
                b[j] = (x + y) % MOD, b[j | i] = (x - y
    + MOD) % MOD;
            }
        }
    }
    vector<int> c(1 << n);
    for (int i = 0; i < (1 << n); i++)
        c[i] = (a[i] * b[i]) % MOD;
    for (int i = 1; i < (1 << n); i *= 2) {
        for (int j = 0; j < (1 << n); j++) {
            if ((j & i) == 0) {
                int x = c[j], y = c[j | i];
                c[j] = (inv2 * (x + y)) % MOD, c[j | i]
    = (inv2 * (x - y + MOD)) % MOD;
            }
        }
    }
    return c;
}
```

6.6 berlekamp-massey-and-friends

```
int getkfps(vector<ll> p, vector<ll> q, ll k) {
    // assert(q[0] != 0);
    while (k) {
        auto f = q;
        for (int i = 1; i < (int) f.size(); i += 2) {
            f[i] = (MOD - f[i] % MOD) % MOD;
        }
        auto p2 = (vector<ll>) convMod<MOD>(p, f);
        auto q2 = (vector<ll>) convMod<MOD>(q, f);
        p.clear(), q.clear();
        for (int i = k % 2; i < (int) p2.size(); i += 2)
     {
            p.pb(p2[i]);
        }
        for (int i = 0; i < (int) q2.size(); i += 2) {
            q.pb(q2[i]);
        }
        k >>= 1;
    }
    return (int) ((p[0] * inverse(q[0])) % MOD);
}

// vals - initials values of reccurence, c - result of
    berlekamp on vals
int getk(vector<ll> &vals, vector<ll> c, ll k) {
    int d = (int) c.size() - 1;
    c[0] = MOD - 1;
    while (c.back() == 0) {
        c.pop_back();
    }
    for (auto &el: c) {
        el = (MOD - el % MOD) % MOD;
    }
    vector<ll> p(d);
    copy(vals.begin(), vals.begin() + d, p.begin());
    p = (vector<ll>) convMod<MOD>(p, c);
    p.resize(d);
    return getkfps(p, c, k);
}

vector<ll> mod_poly_slow(vector<ll> &p, vector<ll> &q) {
    int n = (int) p.size(), m = (int) q.size();
    if (n < m) {
        return p;
    }
    vector<ll> r = p;
    for (int i = n - 1; i >= m - 1; --i) {
        int coef = (r[i] * 1ll * inverse(q[m - 1])) %
    MOD;
        for (int j = 0; j < m; ++j) {
            r[i - j] = (r[i - j] - coef * 1ll * q[m - 1
    - j] % MOD + MOD) % MOD;
        }
    }
    while (!r.empty() && r.back() == 0) {
        r.pop_back();
    }
    return r;
}

int kth_term(vector<ll> &a, vector<ll> s, ll k) {
    // a in 0-index, s in 1-index
    int n = (int) s.size() - 1;
    assert((int) a.size() >= n);
    // a[0]..a[n-1] -- first values
    // a[i] = a[i-1] \cdot s[1] + \ldots + a[i-n] \cdot
    s[n]
    // find a[k]

    // return getk(a, s, k);

    vector<ll> f(n + 1);
    for (int i = 1; i <= n; i++) {
        f[n - i] = (MOD - s[i]) % MOD;
    }
    f[n] = 1;
    vector<ll> res = {1}, w = {0, 1};
    while (k) {
        if (k % 2) {
            res = (vector<ll>) convMod<MOD>(res, w); //
    mul
            res = mod_poly_slow(res, f);
        }
        w = (vector<ll>) convMod<MOD>(w, w); // mul
        w = mod_poly_slow(w, f);
        k /= 2;
    }
    int ans = 0;
    for (int i = 0; i < (int) res.size(); i++) {
        ans = (ans + a[i] * 1ll * res[i] % MOD) % MOD;
    }
    return ans;
}

vector<ll> berlekamp_massey(vector<ll> a) {
    // given a[0]...a[n], returns sequence s[1]..s[k] s.
    t a[i] = a[i-1] \cdot s[1] + \ldots + a[i-k] \cdot
    s[k]
    vector<ll> ls, cur;
    int lf = 0, d = 0;
    for (int i = 0; i < a.size(); ++i) {
        ll t = 0;
        for (int j = 0; j < cur.size(); ++j) {
            t = (t + 1ll * a[i - j - 1] * cur[j]) % MOD;
        }
        if ((t - a[i]) % MOD == 0)continue;
        if (cur.empty()) {
            cur.resize(i + 1);
            lf = i;
            d = (t - a[i]) % MOD;
            continue;
        }
        ll k = -(a[i] - t) * inverse(d) % MOD;
        vector<ll> c(i - lf - 1);
        c.push_back(k);
        for (auto &j: ls)
            c.push_back(-j * k % MOD);
        if (c.size() < cur.size())
            c.resize(cur.size());
        for (int j = 0; j < cur.size(); ++j) {
            c[j] = (c[j] + cur[j]) % MOD;
        }
        if (i - lf + (int) ls.size() >= (int) cur.size()
    ) {
```

```
            tie(ls, lf, d) = make_tuple(cur, i, (t - a[i
    ]) % MOD);
        }
        cur = c;
    }
    cur.insert(cur.begin(), 0); // fictive s[0] = 0
    for (auto &i: cur)
        i = (i % MOD + MOD) % MOD;
    return cur;
}
```

6.7 convMod

```
typedef complex<double> comp;

template<int M>
vector<ll> convMod(const vector<ll> &a, const vector<ll>
     &b) {
    if (a.empty() || b.empty()) return {};
    vector<ll> res((int)a.size() + (int)b.size() - 1);
    int lg = 32 - __builtin_clz((int)res.size()), n = 1
    << lg, cut = int(sqrt(M));
    vector<comp> L(n), R(n), outs(n), outl(n);
    for (int i = 0; i < a.size(); i++) L[i] = comp((int)
     a[i] / cut, (int) a[i] % cut);
    for (int i = 0; i < b.size(); i++) R[i] = comp((int)
     b[i] / cut, (int) b[i] % cut);
    fft(L, lg), fft(R, lg);
    for (int i = 0; i < n; i++) {
        int j = -i & (n - 1);
        outl[j] = (L[i] + conj(L[j])) * R[i] / (2.0 * n)
    ;
        outs[j] = (L[i] - conj(L[j])) * R[i] / (2.0 * n)
     / 1i;
    }
    fft(outl, lg), fft(outs, lg);
    for (int i = 0; i < res.size(); i++) {
        ll av = ll(real(outl[i]) + .5), cv = ll(imag(
    outs[i]) + .5);
        ll bv = ll(imag(outl[i]) + .5) + ll(real(outs[i
    ]) + .5);
        res[i] = ((av % M * cut + bv) % M * cut + cv) %
    M;
    }
    return res;
}
```

6.8 min25 sieve

```
ll min25_sieve(ll n) {
    // given n, calculate prefix sums of some
    multiplicative function f
    // at all points of type floor(n/k) in O(n^{3/4}/log
    (n)), n up to 1e11 is ok
    // in particular you can find f(1) + ... + f(n)
    // also, calculation can be done for primes only, i.
    e prefix sum of f(i)*I{i is prime}
    // to do that, do not run last stage of algorithm

    vector<ll> v;
    v.reserve((int) sqrt(n) * 2 + 7);
    ll sq = 0;
    {
        ll k = 1;
        while (k * k <= n) {
            v.push_back(k);
            ++k;
        }
        --k;
        sq = k;
        if (k * k == n)--k;
        while (k >= 1) {
            v.push_back(n / k);
            --k;
        }
    }
    auto geti = [&](ll x) {
        // returns i, such that v[i] = x
        if (x <= sq) return x - 1;
        return (int) v.size() - (n / x);
    };
    // OP1: f(ab) = f(a)f(b) for coprime a, b; f(p) = p^
    T; f(p^k) can be calculated in O(1); we denote f(p^
    k) = g(p, k) (p is prime) for all k
    // OP2: f also can be any fully multiplicative
    function, f(ab) = f(a)f(b) for all a,b; you need to
     calc pref sum of f fast, so only prime case is
    useful

    auto g = [&](ll p, int k) {
        if (k == 1) {
            return p - 1; // polynomial, for primes-only
     can be any fully multiplicative function
        }
        return p + k; // any function, g(p^k)
    };

    auto f = [&](ll x) {
        return g(x, 1);
    };

    auto pref = [&](ll x) {
        // return sum_{i=1..x} g(i, 1), i.e 1^T + 2^T +
    ... + x^T
        return x * (x + 1) / 2;
    };

    vector<ll> s0(v.size()), s1(v.size()); // for all
    degrees separately
    for (int i = 0; i < (int) v.size(); i++) {
        s0[i] = v[i] % M;
        s1[i] = (((v[i] % M) * ((v[i] + 1) % M) % M) *
    (((M + 1) / 2) % M)) % M; // pref for g(p,1),
    degrees separately
        // s[i] = pref(v[i]) - 1  for primes
    }
    vector<ll> used_primes;
    used_primes.reserve((int) sqrt(n) + 7);
    for (ll p = 2; p * p <= n; ++p) {
        if (s0[p - 1] == s0[p - 2]) continue;
        // p is prime
        used_primes.push_back(p);
        for (int i = (int) v.size() - 1; i >= 0; --i) {
            if (v[i] < p * p) break; // very important,
    dont remove!
            s0[i] += M - ((s0[geti(v[i] / p)] + M - s0[p
     - 2]) % M * (1)) % M; // p^0
            s0[i] %= M;
            s1[i] += M - ((s1[geti(v[i] / p)] + M - s1[p
     - 2]) % M * (p)) % M; // p^1
            s1[i] %= M;
            // s[i] += M - ((s[geti(v[i] / p)] + M - s[p
    -2]) % M * f(p)) % M;
        }
    }

    // PRIMES ONLY calculation is done
    // desired answer for v[i] is in s[i]
    // in particular \sum_{i=1}^n f(i)*I{i is prime} is
    in s.back()
    // now last stage for default calculation

    vector<ll> s(v.size());
    for (int i = 0; i < v.size(); i++) {
        s[i] = (M - s0[i] % M + s1[i]) % M; // combine
    polynomial by degrees with needed coeffs
    }

    vector<ll> r = s;

    for (int ui = (int) used_primes.size() - 1; ui >= 0;
     --ui) { // ui >= 1, sum for odd numbers only
        ll p = used_primes[ui];
        for (int i = (int) v.size() - 1; i >= 0; --i) {
            if (v[i] < p * p) break; // very important,
    dont remove!
            for (ll c = 1, pc = p; pc * p <= v[i]; c++,
    pc *= p) { // pc = p^c
                r[i] += g(p, c + 1) % M + ((g(p, c) % M)
     * ((M + r[geti(v[i] / pc)] - s[geti(p)]) % M)) % M
    ;
                r[i] %= M;
            }
        }
    }

    // done, answer for v[i] is r[i]+1 (f(1)=1)
    // in particular \sum_{i=2}^n f(i) is in r.back()
    // therefore \sum_{i=1}^n f(i) is r.back() + 1
    return r.back() + 1 - g(1, 1); // since f(1)=1 for
    real, not g(1,1): 1 is not prime
}
```

6.9 sqrt mod

```
// p is prime
// -1 if no solution
// x = sqrt(a,p) ⟹ x^2 = a and (-x)^2 = a
// O(log n) if p ≡ 3 mod 4 else O(log^2 n)
// should be changed if const p
ll sqrt(ll a, ll p) {
  a %= p;
  if (a < 0) a += p;
  if (a == 0) return 0;
  if (binpow(a, (p - 1) / 2, p) != 1)
    return -1; // no solution
  if (p % 4 == 3) return binpow(a, (p + 1) / 4, p);
  ll s = p - 1, n = 2;
  int r = 0, m;
  while (s % 2 == 0) ++r, s /= 2;
  while (binpow(n, (p - 1) / 2, p) != p - 1) ++n;
  ll x = binpow(a, (s + 1) / 2, p);
  ll b = binpow(a, s, p), g = binpow(n, s, p);
  for (;; r = m) {
    ll t = b;
    for (m = 0; m < r && t != 1; ++m) t = t * t % p;
    if (m == 0) return x;
    ll gs = binpow(g, 1LL << (r - m - 1), p);
    g = gs * gs % p;
    x = x * gs % p;
    b = b * g % p;
  }
}
```

6.10 Гаусс

```
vector<vector<int>> gauss(vector<vector<int>> &a) {
  int n = a.size();
  int m = a[0].size();
//  int det = 1;
  for (int col = 0, row = 0; col < m && row < n; ++col)
    {
    for (int i = row; i < n; ++i) {
      if (a[i][col]) {
        swap(a[i], a[row]);
        if (i != row) {
//          det *= -1;
        }
        break;
      }
    }
    if (!a[row][col])
      continue;
    for (int i = 0; i < n; ++i) {
      if (i != row && a[i][col]) {
        int val = a[i][col] * inv(a[row][col]) % mod;
        for (int j = col; j < m; ++j) {
          a[i][j] -= val * a[row][j];
          a[i][j] %= mod;
        }
      }
    }
    ++row;
  }
//  for (int i = 0; i < n; ++i) det = (det * a[i][i]) %
    mod;
//  det = (det % mod + mod) % mod;
// result in (-mod, mod)
  return a;
}

pair<int, vector<int>> sle(vector<vector<int>> a, vector
    <int> b) {
  int n = a.size();
  int m = a[0].size();
  assert(n == b.size());
  for (int i = 0; i < n; ++i) {
    a[i].push_back(b[i]);
  }
  a = gauss(a);
  vector<int> x(m, 0);
  for (int i = n - 1; i >= 0; --i) {
    int leftmost = m;
    for (int j = 0; j < m; ++j) {
      if (a[i][j] != 0) {
        leftmost = j;
        break;
      }
    }
    if (leftmost == m && a[i].back() != 0) return {-1,
    {}};
    if (leftmost == m) continue;
    int val = a[i].back();
    for (int j = m - 1; j > leftmost; --j) {
      val -= a[i][j] * x[j];
      val %= mod;
    }
    x[leftmost] = (val * inv(a[i][leftmost]) % mod + mod
    ) % mod;
  }
  return {1, x};
}

vector<bitset<N>> gauss_bit(vector<bitset<N>> a, int m)
    {
  int n = a.size();
  for (int col = 0, row = 0; col < m && row < n; ++col)
    {
    for (int i = row; i < n; ++i) {
      if (a[i][col]) {
        swap(a[i], a[row]);
        break;
      }
    }
    if (!a[row][col])
      continue;
    for (int i = 0; i < n; ++i)
      if (i != row && a[i][col])
        a[i] ^= a[row];
    ++row;
  }
  return a;
}
```

6.11 Диофантовы уравнения

```
// ax + by = ±gcd if a < 0 or b < 0
pair<int, int> ext_gcd(int a, int b) {
  int x1 = 1, y1 = 0, x2 = 0, y2 = 1;
  while (b) {
    int k = a / b;
    x1 -= x2 * k;
    y1 -= y2 * k;
    a %= b;
    swap(x1, x2), swap(y1, y2), swap(a, b);
  }
  return {x1, y1};
}

// solve ax + by = c with minimum x ≥ 0
bool cool_ext_gcd(int a, int b, int c, int &x, int &y) {
    if (b == 0) {
        y = 0;
        if (a == 0)
            return x = 0, c == 0;
        return x = c / a, c % a == 0;
    }
    auto [x0, y0] = ext_gcd(a, b);
    int g = (ll)x0 * a + (ll)y0 * b;
    if (c % g != 0) return false;
    x = (ll)x0 * (c / g) % (b / g);
    if (x < 0) x += abs(b / g);
    y = (c - (ll)a * x) / b;
    return true;
}
```

6.12 КТО

```
// ans % p_i = a_i
vector<vector<int>> r(k, vector<int>(k));
for (int i = 0; i < k; ++i)
  for (int j = 0; j < k; ++j)
    if (i != j)
      r[i][j] = binpow(p[i] % p[j], p[j] - 2, p[j]);
vector<int> x(k);
for (int i = 0; i < k; ++i) {
  x[i] = a[i];
  for (int j = 0; j < i; ++j) {
    x[i] = r[j][i] * (x[i] - x[j]);
    x[i] = x[i] % p[i];
    if (x[i] < 0) x[i] += p[i];
  }
}
int ans = 0;
for (int i = 0; i < k; ++i) {
  int val = x[i];
  for (int j = 0; j < i; ++j) val *= p[j];
  ans += val;
}
```

6.13 Код Грея

```
for (int i = 0; i < (1 << n); i++) {
  gray[i] = i ^ (i >> 1);
}
```

6.14 Линейное решето

```
const int N = 10000000;
int lp[N + 1];
vector<int> pr;
for (int i = 2; i <= N; ++i) {
  if (lp[i] == 0) {
    lp[i] = i;
    pr.push_back(i);
  }
  for (int j = 0; j < (int) pr.size() && pr[j] <= lp[i]
    && i * pr[j] <= N; ++j)
    lp[i * pr[j]] = pr[j];
}
```

6.15 Миллер Рабин

```
// works for all n < 2^64
const ll MAGIC[7] = {2, 325, 9375, 28178, 450775,
    9780504, 1795265022};

bool is_prime(ll n) {
  if (n == 1) return false;
  if (n <= 3) return true;
  if (n % 2 == 0 || n % 3 == 0) return false;
  ll s = __builtin_ctzll(n - 1), d = n >> s; //
    n - 1 = 2^s * d
  for (auto a : MAGIC) {
    if (a % n == 0) {
      continue;
    }
    ll x = binpow(a, d, n); // a -> __int128 in binpow
    for (int _ = 0; _ < s; _++) {
      ll y = binpow(x, 2, n);  // x -> __int128 in
    binpow
      if (y == 1 && x != 1 && x != n - 1) {
        return false;
      }
      x = y;
    }
    if (x != 1) {
      return false;
    }
  }
  return true;
}
```

6.16 Мёбиус

```
vector<int> mu(n + 1);
mu[1] = 1;
for (int x = 1; x <= n; x++) {
  for (int y = x + x; y <= n; y += x) mu[y] -= mu[x];
}
```

6.17 Подсчёт прогулок

```
int count_walks_1(int b1, int b2, int p, int q) {
    // counting walks from (0, 0) to (p, q)
    // each turn x += 1 or y += 1
    // without touching y = x + b1 and y = x + b2
    // b1 < 0 < b2 must hold
    // O((p + q) / (b2 - b1))
    if (min(p, q) < 0) return 0;
    ll ans = C(p, p + q);
    ar(2) F = {p, q}, S = {p, q};
    int cf = mod - 1;
    while (true) {
        F[1] -= b1;
        swap(F[0], F[1]);
        F[1] += b1;
        S[1] -= b2;
        swap(S[0], S[1]);
        S[1] += b2;
        swap(F, S);
        int wf = C(F[0], F[0] + F[1]);
        int ws = C(S[0], S[0] + S[1]);
        ans += (cf * (ll) ((wf + ws) % mod)) % mod;
        if (wf == 0 && ws == 0) break;
        cf = mod - cf;
    }
    ans %= mod;
    return (int) ans;
}

int count_walks_2(int b1, int b2, int p, int q) {
    // counting walks from (0, 0) to (p, q)
    // each turn x += 1 and (y -= 1 or y += 1)
    // without touching y = b1 and y = b2
    // b1 < 0 < b2 must hold
    // O(p / (b2 - b1))
    if (abs(p) % 2 != abs(q) % 2) return 0;
    int p0 = (p - q) / 2, q0 = (p + q) / 2;
    return count_walks_1(b1, b2, p0, q0);
}
```

6.18 Ро-Поллард

```
typedef long long ll;

ll mult(ll a, ll b, ll mod) {
  return (__int128)a * b % mod;
}

ll f(ll x, ll c, ll mod) {
  return (mult(x, x, mod) + c) % mod;
}

ll rho(ll n, ll x0=2, ll c=1) {
  ll x = x0;
  ll y = x0;
  ll g = 1;
  while (g == 1) {
    x = f(x, c, n);
    y = f(y, c, n);
    y = f(y, c, n);
    g = gcd(abs(x - y), n);
  }
  return g;
}

mt19937_64 rnd(time(nullptr));

void factor(int n, vector<int> &pr) {
  if (n == 4) {
    factor(2, pr);
    factor(2, pr);
    return;
  }
  if (n == 1) {
    return;
  }
  if (is_prime(n)) {
    pr.push_back(n);
    return;
  }
  int d = rho(n, rnd() % (n - 2) + 2, rnd() % 3 + 1);
  factor(n / d, pr);
  factor(d, pr);
}
```

7 Строки

7.1 Z-функция

```
vector<int> z_func(string s) {
  int n = s.size();
  vector<int> z(n, 0);
  z[0] = n;
  int l = 0, r = 0;
  for (int i = 1; i < n; i++) {
    if (i < r) {
      z[i] = min(z[i - l], r - i);
    }
    while (i + z[i] < n && s[z[i]] == s[i + z[i]]) {
      z[i]++;
    }
    if (i + z[i] > r) {
      l = i;
      r = i + z[i];
    }
  }
  return z;
}
```

7.2 eertree

```
int len[MAXN], suf[MAXN];
int go[MAXN][ALPH];
char s[MAXN];

int n, last, sz;

void init() {
  n = 0, last = 0;
  s[n++] = -1;
  suf[0] = 1; // root of suflink tree = 1
  len[1] = -1;
  sz = 2;
}

int get_link(int v) {
  while (s[n - len[v] - 2] != s[n - 1])
    v = suf[v];
  return v;
}

void add_char(char c) {
  c -= 'a';
  s[n++] = c;
  last = get_link(last);
  if (!go[last][c]) {
    len[sz] = len[last] + 2;
    suf[sz] = go[get_link(suf[last])][c];
    go[last][c] = sz++;
  }
  last = go[last][c]; // cur v = last
}
```

7.3 Ахо-Корасик

```
int go[MAXN][ALPH];
vector<int> term[MAXN];
int par[MAXN], suf[MAXN];
char par_c[MAXN];
vector<int> g[MAXN];

int cntv = 1;

void add(string &s) {
  static int cnt_s = 1;
  int v = 0;
  for (char el: s) {
    if (go[v][el - 'a'] == 0) {
      go[v][el - 'a'] = cntv;
      par[cntv] = v;
      par_c[cntv] = el;
      cntv++;
    }
    v = go[v][el - 'a'];
  }
  term[v].push_back(cnt_s++);
}

void bfs() {
  deque<int> q = {0};
  while (!q.empty()) {
    int v = q.front();
    q.pop_front();
    if (v > 0) {
      if (par[v] == 0) {
        suf[v] = 0;
      } else {
        suf[v] = go[suf[par[v]]][par_c[v] - 'a'];
      }
      g[suf[v]].push_back(v);
    }
    for (int c = 0; c < ALPH; c++) {
      if (go[v][c] == 0) {
        go[v][c] = go[suf[v]][c];
      } else {
        q.push_back(go[v][c]);
      }
    }
  }
}
```

7.4 Муффиксный Сассив

```
vector<int> build_suff_arr(string &s) {
  // Remove, if you want to sort cyclic shifts
  s += (char) (1);
  int n = s.size();
  vector<int> a(n);
  iota(all(a), 0);
  stable_sort(all(a), [&](int i, int j) {
      return s[i] < s[j];
  });
  vector<int> c(n);
  int cc = 0;
  for (int i = 0; i < n; i++) {
    if (i == 0 || s[a[i]] != s[a[i - 1]])
      c[a[i]] = cc++;
    else
      c[a[i]] = c[a[i - 1]];
  }
  for (int L = 1; L < n; L *= 2) {
    vector<int> cnt(n);
    for (auto i: c) cnt[i]++;
    if (*min_element(all(cnt)) > 0) break;
    vector<int> pref(n);
    for (int i = 1; i < n; i++)
      pref[i] = pref[i - 1] + cnt[i - 1];
    vector<int> na(n);
    for (int i = 0; i < n; i++) {
      int pos = (a[i] - L + n) % n;
      na[pref[c[pos]]++] = pos;
    }
    a = na;
    vector<int> nc(n);
    cc = 0;
    for (int i = 0; i < n; i++) {
      if (i == 0 || c[a[i]] != c[a[i - 1]] ||
          c[(a[i] + L) % n] != c[(a[i - 1] + L) % n])
        nc[a[i]] = cc++;
      else
        nc[a[i]] = nc[a[i - 1]];
    }
    c = nc;
  }
  // Remove, if you want to sort cyclic shifts
  a.erase(a.begin());
  s.pop_back();
  return a;
}

vector<int> kasai(string s, vector<int> sa) {
  // lcp[i] = lcp(sa[i], sa[i + 1])
  int n = s.size(), k = 0;
  vector<int> lcp(n, 0);
  vector<int> rank(n, 0);
  for (int i = 0; i < n; i++) rank[sa[i]] = i;
  for (int i = 0; i < n; i++, k ? k-- : 0) {
    if (rank[i] == n - 1) {
      k = 0;
      continue;
    }
    int j = sa[rank[i] + 1];
    while (i + k < n && j + k < n && s[i + k] == s[j + k
    ]) k++;
    lcp[rank[i]] = k;
  }
  return lcp;
}
```

7.5 Префикс-функция

```
vector<int> prefix_func(string s) {
  int n = s.size();
  vector<int> pref(n, 0);
  int ans = 0;
  for (int i = 1; i < n; i++) {
    while (ans > 0 && s[ans] != s[i]) {
      ans = pref[ans - 1];
    }
    if (s[i] == s[ans]) {
      ans++;
    }
    pref[i] = ans;
  }
  return pref;
}
```

7.6 Суффиксный автомат

```
// Суфавтомат с подсчётом кол-ва различных подстрок

const int SIGMA = 26;
int ans = 0;
```

```cpp
struct Node {
  int go[SIGMA];
  int s, p;
  int len;

  Node() {
    fill(go, go + SIGMA, -1);
    s = -1, p = -1;
    len = 0;
  }
};

int add(int A, int ch, vector<Node> &sa) {
  int B = sa.size();
  sa.emplace_back();
  sa[B].p = A;
  sa[B].s = 0;
  sa[B].len = sa[A].len + 1;
  for (; A != -1; A = sa[A].s) {
    if (sa[A].go[ch] == -1) {
      sa[A].go[ch] = B;
      continue;
    }
    int C = sa[A].go[ch];
    if (sa[C].p == A) {
      sa[B].s = C;
      break;
    }
    int D = sa.size();
    sa.emplace_back();
    sa[D].s = sa[C].s;
    sa[D].p = A;
    sa[D].len = sa[A].len + 1;
    sa[C].s = D;
    sa[B].s = D;
    copy(sa[C].go, sa[C].go + SIGMA, sa[D].go);
    for (; A != -1 && sa[A].go[ch] == C; A = sa[A].s)
      sa[A].go[ch] = D;
    break;
  }
  ans += sa[B].len - sa[sa[B].s].len;
  return B;
}

signed main() {
  string s;
  cin >> s;
  vector<Node> sa(1);
  int A = 0;
  for (char c : s)
    A = add(A, c - 'a', sa);
  cout << ans;
}
```

8 Структуры данных

8.1 Disjoint Sparse Table

```cpp
// MAXN дополнить до степени двойки (или n*2)
int tree[LOG][MAXN];
int floorlog2[MAXN]; // i ? (31 - __builtin_clz(i)) : 0

void build(vector<int> &a) {
  int n = a.size();
  copy(a.begin(), a.end(), tree[0]);
  for (int lg = 1; lg < LOG; ++lg) {
    int len = 1 << lg;
    auto &lvl = tree[lg];
    for (int m = len; m < n; m += len * 2) {
      lvl[m - 1] = a[m - 1];
      lvl[m] = a[m];
      for (int i = m - 2; i >= m - len; --i)
        lvl[i] = min(lvl[i + 1], a[i]);
      for (int i = m + 1; i < m + len && i < n; ++i)
        lvl[i] = min(lvl[i - 1], a[i]);
    }
  }
  for (int i = 2; i < min(MAXN, n * 2); ++i)
    floorlog2[i] = floorlog2[i / 2] + 1;
}

// a[l..r)
int get(int l, int r) {
  r--;
  int i = floorlog2[l ^ r];
  return min(tree[i][l], tree[i][r]);
}
```

8.2 Segment Tree Beats

```cpp
// %=, =, sum
// mx[v], all_equal[v]
// break: mx[v] < x
// tag: all_equal[v] == true, запрос становится =mx[v]%x

// min=, max=, =, +=, sum, mn, mx
// также как и для min=, sum
// для max= храним mn[v], sec_mn[v]

// +=, gcd
// храним gcd разностей какого-то остовного дерева
// храним any_value[v] = любое значение на отрезке
// gcd(l...r) = gcd(any_value[v], gcd[v])
// при сливании добавляем к gcd значение |a_v[l] - a_v[r
    ]|

// min=, sum
struct ST {
  vector<int> st, mx, mx_cnt, sec_mx;

  ST(vector<int> &a) {
    int n = a.size();
    st.resize(n * 4), mx.resize(n * 4);
    mx_cnt.resize(n * 4, 0), sec_mx.resize(n * 4, 0);
    build(0, 0, n, a);
  }

  void upd_from_children(int v) {
    st[v] = st[v * 2 + 1] + st[v * 2 + 2];
    mx[v] = max(mx[v * 2 + 1], mx[v * 2 + 2]);
    mx_cnt[v] = 0;
    sec_mx[v] = max(sec_mx[v * 2 + 1], sec_mx[v * 2 +
    2]);
    if (mx[v * 2 + 1] == mx[v]) {
      mx_cnt[v] += mx_cnt[v * 2 + 1];
    } else {
      sec_mx[v] = max(sec_mx[v], mx[v * 2 + 1]);
    }
    if (mx[v * 2 + 2] == mx[v]) {
      mx_cnt[v] += mx_cnt[v * 2 + 2];
    } else {
      sec_mx[v] = max(sec_mx[v], mx[v * 2 + 2]);
    }
  }

  void build(int i, int l, int r, vector<int> &a) {
    if (l + 1 == r) {
      st[i] = mx[i] = a[l];
      mx_cnt[i] = 1;
      sec_mx[i] = -INF;
      return;
    }
    int m = (r + l) / 2;
    build(i * 2 + 1, l, m, a);
    build(i * 2 + 2, m, r, a);
    upd_from_children(i);
  }

  void push_min_eq(int v, int val) {
    if (mx[v] > val) {
      st[v] -= (mx[v] - val) * mx_cnt[v];
      mx[v] = val;
    }
  }

  void push(int i, int l, int r) {
    if (l + 1 < r) {
      push_min_eq(i * 2 + 1, mx[i]);
      push_min_eq(i * 2 + 2, mx[i]);
    }
  }

  void update(int i, int l, int r, int ql, int qr, int
    val) {
    if (qr <= l || r <= ql || mx[i] <= val) {
      return;
    }
    if (ql <= l && r <= qr && sec_mx[i] < val) {
      push_min_eq(i, val);
      return;
    }
    push(i, l, r);
    int m = (l + r) / 2;
    update(i * 2 + 1, l, m, ql, qr, val);
    update(i * 2 + 2, m, r, ql, qr, val);
    upd_from_children(i);
  }
```

```
  int sum(int i, int l, int r, int ql, int qr) {
    if (qr <= l || r <= ql) {
      return 0;
    }
    push(i, l, r);
    if (ql <= l && r <= qr) {
      return st[i];
    }
    int m = (l + r) / 2;
    return sum(i * 2 + 1, l, m, ql, qr) + sum(i * 2 + 2,
     m, r, ql, qr);
  }
};
```

8.3 ДД по неявному

```
// потому что nds[0].sz == 0 и sz не изменяется в push
int size(int t) { return nds[t].sz; }

pair<int, int> split(int t, int k) {
  if (!t) return {0, 0};
  push(t);
  int szl = size(nds[t].l);
  if (k <= szl) {
    auto [l, r] = split(nds[t].l, k);
    nds[t].l = r;
    pull(t);
    return {l, t};
  } else {
    auto [l, r] = split(nds[r].r, k - szl - 1);
    nds[t].r = l;
    pull(t);
    return {t, r};
  }
}

// всё остальное ровно как в обычном ДД
// не забыть обновлять sz в pull
// инициализация sz=0 в Node() и sz=1 в Node(...)
```

8.4 ДД

```
// insert (key, val), erase key, max(val) for key ∈ [l, r),
    val+= for key ∈ [l, r)
struct Node {
  int l, r;
  int x, y;
  int val, mx, mod;

  // value of empty set
  Node() : val(-INF), mx(-INF) {
    l = r = 0, mod = 0;
  }
  Node(int x, int val) : x(x), val(val), mx(val) {
    l = r = 0, mod = 0, y = rnd();
  }
};

Node nds[MAX];
int ndsz = 1; // nds[0] means empty

void push(int t) {
  if (!t || nds[t].mod == 0) return;
  nds[t].val += nds[t].mod;
  nds[t].mx += nds[t].mod;
  if (nds[t].l) nds[nds[t].l].mod += nds[t].mod;
  if (nds[t].r) nds[nds[t].r].mod += nds[t].mod;
  nds[t].mod = 0;
}

int getmx(int t) {
  push(t); // delete if sure (faster)
  return nds[t].mx;
}

void pull(int t) {
  if (!t) return;
  push(t), push(nds[t].l), push(nds[t].r); // must have
  nds[t].mx = max(nds[t].val, max(getmx(nds[t].l), getmx
     (nds[t].r)));
}

pair<int, int> split(int t, int x) {
  if (!t) return {0, 0};
  push(t);
  if (x <= nds[t].x) {
    auto [l, r] = split(nds[t].l, x);
    nds[t].l = r;
    pull(t);
    return {l, t};
  } else {
    auto [l, r] = split(nds[t].r, x);
    nds[t].r = l;
    pull(t);
    return {t, r};
  }
}

int merge(int l, int r) {
  push(l), push(r);
  if (!l) return r;
  if (!r) return l;
  if (nds[l].y < nds[r].y) {
    nds[l].r = merge(nds[l].r, r);
    pull(l);
    return l;
  } else {
    nds[r].l = merge(l, nds[r].l);
    pull(r);
    return r;
  }
}

void insert(int &root, int x, int val) {
  nds[ndsz++] = Node(x, val);
  auto [l, r] = split(root, x);
  root = merge(merge(l, ndsz - 1), r);
}

// erase all equal to x
void erase(int &root, int x) {
  auto [lm, r] = split(root, x + 1);
  auto [l, m] = split(lm, x);
  root = merge(l, r);
}

// query [l, r)
int query(int &root, int ql, int qr) {
  auto [lm, r] = split(root, qr);
  auto [l, m] = split(lm, ql);
  int res = getmx(m);
  root = merge(merge(l, m), r);
  return res;
}

// update [l, r)
void update(int &root, int ql, int qr, int qx) {
  auto [lm, r] = split(root, qr);
  auto [l, m] = split(lm, ql);
  if (m) nds[m].mod += qx;
  root = merge(merge(l, m), r);
}
```

8.5 Персистентное ДД по неявному

```
struct Node;
int size(int);
int sum(int);

struct Node {
  int l, r;
  int val, sz, sm;

  Node() : val(0), sz(0), sm(0) {}
  Node(int val, int l, int r) : val(val), l(l), r(r) {
    sz = 1 + size(l) + size(r);
    sm = val + sum(l) + sum(r);
  }
};

Node nds[MAX];
int ndsz = 1;

int size(int t) { return nds[t].sz; }
int sum(int t) { return nds[t].sm; }

int newNode(int val, int l, int r) {
  nds[ndsz++] = newNode(val, l, r);
  return ndsz - 1;
}

pair<int, int> split(int t, int x) {
  if (!t) return {0, 0};
  int szl = size(nds[t].l);
  if (szl >= x) {
```

```
    auto [l, r] = split(nds[t].l, x);
    int v = newNode(nds[t].val, r, nds[t].r);
    return {l, v};
  } else {
    auto [l, r] = split(nds[t].r, x - szl - 1);
    int v = newNode(nds[t].val, nds[t].l, l);
    return {v, r};
  }
}

bool chooseleft(int szl, int szr) {
  return rnd() % (szl + szr) < szl;
}

int merge(int l, int r) {
  if (!l) return r;
  if (!r) return l;
  if (chooseleft(nds[l].sz, nds[r].sz)) {
    int rr = merge(nds[l].r, r);
    int v = newNode(nds[l].val, nds[l].l, rr);
    return v;
  } else {
    int ll = merge(l, nds[r].l);
    int v = newNode(nds[r].val, ll, nds[r].r);
    return v;
  }
}

int insert(int root, int ponds[t].s, int val) {
  int new_v = newNode(val, 0, 0);
  auto [l, r] = split(root, pos);
  return merge(merge(l, new_v), r);
}

int erase(int root, int pos) {
  auto [lm, r] = split(root, pos + 1);
  auto [l, m] = split(lm, pos);
  return merge(l, r);
}

// query [l, r)
pair<int, int> query(int root, int ql, int qr) {
  auto [lm, r] = split(root, qr);
  auto [l, m] = split(lm, ql);
  int res = sum(m);
  auto new_root = merge(merge(l, m), r);
  return {res, new_root};
}
```

8.6 Персистентное ДО

```
Node nds[MAX];
int ndsz = 1;
// nds[0] is default (empty) value

int sum(int v) { return nds[v].sm; }

// returns new root of subtree
int update(int v, int l, int r, int qi, int qx) {
  if (qi < l || r <= qi) return v;
  if (l + 1 == r) {
    nds[ndsz++] = Node(qx);
    return ndsz - 1;
  }
  int m = (l + r) / 2;
  int u = ndsz++;
  nds[u].l = update(nds[v].l, l, m, qi, qx);
  nds[u].r = update(nds[v].r, m, r, qi, qx);
  nds[u].sm = sum(nds[u].l) + sum(nds[u].r);
  return u;
}

int get(int v, int l, int r, int ql, int qr) {
  if (!v || qr <= l || r <= ql) return 0;
  if (ql <= l && r <= qr) return nds[v].sm;
  int m = (l + r) / 2;
  auto a = get(nds[v].l, l, m, ql, qr);
  auto b = get(nds[v].r, m, r, ql, qr);
  return a + b;
}
```

8.7 Спарсы

```
int tree[LOG][MAXN];
int floorlog2[MAXN]; // i ? (31 - __builtin_clz(i)) : 0

void build(vector<int> &a) {
  int n = a.size();
  copy(a.begin(), a.end(), tree[0]);
  for (int i = 1; i < LOG; ++i) {
    int len = 1 << (i - 1);
    for (int j = 0; j + len < n; ++j)
      tree[i][j] = min(tree[i - 1][j], tree[i - 1][j +
    len]);
  }
  for (int i = 2; i <= n; ++i)
    floorlog2[i] = floorlog2[i / 2] + 1;
}

// min a[l..r)
int get(int l, int r) {
  int i = floorlog2[r - l];
  return min(tree[i][l], tree[i][r - (1 << i)]);
}
```

8.8 Фенвик (+ на отрезке)

```
// a[l..r) += x
void update(int l, int r, int x) {
  T1.add(l, x);
  T1.add(r, -x);
  T2.add(l, -x * l);
  T2.add(r, x * r);
}

// sum a[0..i)
int get(int i) {
  return T1.get(i) * i + T2.get(i);
}
```

8.9 Фенвик

```
// Нумерация с 0

struct Fenwick {
  int n;
  vector<int> f;

  Fenwick(int n) : n(n) {
    f.resize(n + 1);
  }

  // a[i] += x
  void add(int i, int x) {
    for (++i; i <= n; i += i & -i)
      f[i] += x;
  }

  // sum a[0..i)
  int get(int i) {
    int ans = 0;
    for (; i > 0; i -= i & -i)
      ans += f[i];
    return ans;
  }

  // a[..] > 0; find max k: sum a[0..k) <= x
  int max_not_more(int x) {
    int cur = 0;
    for (int i = 20; i >= 0; --i) {
      int len = 1 << i;
      if (cur + len <= n && f[cur + len] <= x) {
        cur += len;
        x -= f[cur];
      }
    }
    return cur;
  }
};

// sum a[x1..x2)[y1..y2)[z1..x2)
int sum_3d(int x1, int x2, int y1, int y2, int z1, int
    z2) {
  int ans = get(x2, y2, z2);
  ans -= get(x1, y2, z2) + get(x2, y1, z2) + get(x2, y2,
     z1);
  ans += get(x1, y1, z2) + get(x1, y2, z1) + get(x2, y1,
     z1);
  ans -= get(x1, y1, z1);
  return ans;
}
```